Problem-01:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow ABd / Aa / a$$

$$B \rightarrow Be / b$$

Solution-

The grammar after eliminating left recursion is-

$$A \rightarrow aA'$$

$$A' \rightarrow BdA' / aA' / \in$$

$$B \rightarrow bB$$

$$B'\to eB'\ /\in$$

Problem-02:

Consider the following grammar and eliminate left recursion-

$$E \rightarrow E + E / E \times E / a$$

Solution-

The grammar after eliminating left recursion is-

$$E \rightarrow aA$$

$$A \rightarrow +EA / xEA / \in$$

Problem-03:

Consider the following grammar and eliminate left recursion-

$$E \rightarrow E + T / T$$

$$T \to T \; x \; F \; / \; F$$

$$F \mathop{\rightarrow} id$$

Solution-

The grammar after eliminating left recursion is-

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' / \in$$

$$T \rightarrow FT'$$

$$T' \rightarrow xFT' / \in$$

$$F \rightarrow id$$

Problem-04:

Consider the following grammar and eliminate left recursion-

$$S \rightarrow (L) / a$$

$$L \rightarrow L$$
, S/S

Solution-

The grammar after eliminating left recursion is-

$$S \rightarrow (L) / a$$

$$L \rightarrow SL'$$

$$L' \to ,SL' \,/ \in$$

Problem-05:

Consider the following grammar and eliminate left recursion-

$$S \rightarrow S0S1S / 01$$

Solution-

The grammar after eliminating left recursion is-

$$S \rightarrow 01A$$

$$A \rightarrow 0S1SA / \in$$

Problem-06:

Consider the following grammar and eliminate left recursion-

$$S \rightarrow A$$

$$A \rightarrow Ad / Ae / aB / ac$$

$$B \rightarrow bBc / f$$

Solution-

The grammar after eliminating left recursion is-

$$S \rightarrow A$$

$$A \rightarrow aBA' / acA'$$

$$A' \rightarrow dA' / eA' / \in$$

$$B \rightarrow bBc / f$$

Problem-07:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow AA\alpha / \beta$$

Solution-

The grammar after eliminating left recursion is-

$$A \rightarrow \beta A'$$

$$A' \rightarrow A\alpha A' / \in$$

Problem-08:

Consider the following grammar and eliminate left recursion-

$$A \rightarrow Ba / Aa / c$$

$$B \rightarrow Bb / Ab / d$$

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from A \rightarrow Ba / Aa / c

Eliminating left recursion from here, we get-

$$A \rightarrow BaA' / cA'$$

$$A' \rightarrow aA' / \in$$

Now, given grammar becomes-

$$A \rightarrow BaA' / cA'$$

$$A' \rightarrow aA' / \in$$

$$B \rightarrow Bb / Ab / d$$

Step-02:

Substituting the productions of A in $B \rightarrow Ab$, we get the following grammar-

$$A \rightarrow BaA' / cA'$$

$$A' \rightarrow aA' / \in$$

$$B \rightarrow Bb / BaA'b / cA'b / d$$

Step-03:

Now, eliminating left recursion from the productions of B, we get the following grammar-

$$A \rightarrow BaA' / cA'$$

$$A' \rightarrow aA' / \in$$

$$B \rightarrow cA'bB' / dB'$$

$$B' \rightarrow bB' / aA'bB' / \in$$

This is the final grammar after eliminating left recursion.

Problem-09:

Consider the following grammar and eliminate left recursion-

$$X \rightarrow XSb / Sa / b$$

$$S \rightarrow Sb / Xa / a$$

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $X \rightarrow XSb / Sa / b$

Eliminating left recursion from here, we get-

$$X \rightarrow SaX' / bX'$$

$$X' \rightarrow SbX' / \in$$

Now, given grammar becomes-

$$X \rightarrow SaX' / bX'$$

$$X' \rightarrow SbX' / \in$$

$$S \rightarrow Sb / Xa / a$$

Step-02:

Substituting the productions of X in $S \rightarrow Xa$, we get the following grammar-

$$X \rightarrow SaX' / bX'$$

$$X' \rightarrow SbX' / \in$$

$$S \rightarrow Sb / SaX'a / bX'a / a$$

Step-03:

Now, eliminating left recursion from the productions of S, we get the following grammar-

$$X \rightarrow SaX' / bX'$$

$$X' \rightarrow SbX' / \in$$

$$S \rightarrow bX'aS' / aS'$$

$$S' \rightarrow bS' / aX'aS' / \in$$

This is the final grammar after eliminating left recursion.

Problem-10:

Consider the following grammar and eliminate left recursion-

$$S \rightarrow Aa/b$$

$$A \rightarrow Ac / Sd / \in$$

Solution-

This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $S \rightarrow Aa/b$

This is already free from left recursion.

Step-02:

Substituting the productions of S in $A \rightarrow Sd$, we get the following grammar-

$$S \rightarrow Aa / b$$

$$A \rightarrow Ac / Aad / bd / \in$$

Step-03:

Now, eliminating left recursion from the productions of A, we get the following grammar-

$$S \rightarrow Aa / b$$

$$A \rightarrow bdA' / A'$$

$$A' \rightarrow cA' / adA' / \in$$

This is the final grammar after eliminating left recursion.