

Master's Thesis

Deep Reinforcement Learning for Optimal Self-Triggered Control

Guidance

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February 2021

Abstract

In this thesis, we construct the mathematical models of students who write the master's thesis and develop efficient algorithms for writing the master's thesis based on the models. We show that the proposed algorithms generate the thesis 65536 times more efficiently than writing by oneself.

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1 Introduction

2 Preliminaries

2.1 Background of Deep Reinforcement Learning

Consider a malkov decision process M given with tuple $M = \{S, A, T, d_0, r, \gamma\}$. Here, S, A denotes state, action set, and $T(s'|s, a)$ express transition probability. Also, $d_0, r(s, a), \gamma \in [0, 1]$ are distribution of initial state, reward, discount factor respectively.

Now, the purpose of reinforcement learning is to find a policy such that

$$\pi^* = \operatorname{argmax}_{\pi} J(\pi) \quad (1)$$

where evaluation function $J(\pi)$ and (state) value function $V^\pi(s)$ is given as following:

$$V^\pi(s) = \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) |_{a_t=\pi(s_t)}, s_0 = s \quad (2)$$

$$J(\pi) = \mathbb{E}_{s_0 \sim d_0} [V^\pi(s_0)] \quad (3)$$

Here, we define Q function, which is useful tool for analyzing reinforcement learning. Q function is given as

$$\begin{aligned} Q^\pi(s, a) &= r(s, a) + \gamma \sum_{t=1}^{\infty} \gamma^t r(s_t, a_t) |_{a_t=\pi(s_t)} \\ &= r(s, a) + \gamma V^\pi(s'). \end{aligned} \quad (4)$$

As showed in (4), Q function express the value when agent select action a freely and choose action according to the policy π from next step. Thus, the Q -function is also known as the action value function.

2.2 Policy Iteration

There is an algorithm for achieving (1), called the policy iteration method. It consists in repeating the following two steps.

1. Policy Evaluation: Find (or approximate) action value function $Q^\pi(s, a)$.
2. Policy Improvement: Update policy as $\pi(s) = \operatorname{argmax}_a Q^\pi(s, a)$.

It is known that the optimal policy π^* can be obtained by repeating the above two steps (Measure Improvement Theorem).

2.3 Algorithms adapted to the settings of state and action space

In the case that both the state space and the action space take discrete values, it is easy to obtain $\pi(s) = \underset{a}{\operatorname{argmax}} Q^\pi(s, a)$, which appeared in the previous section, by storing $Q^\pi(s, a)$ in a table.

Now, what about the case where the state space is continuous? Since the state s takes a continuous value, it cannot be stored in a table. Therefore, Minh et al.[2] took the approach of approximating $Q^\pi(s, a)$ by parametrizing it using a neural network. Since the action space is discrete, it is still possible to obtain $\underset{a}{\operatorname{argmax}} Q^\pi(s, a)$.

Finally, in the case where both state and action space is continuous, the problem is that it is very expensive to obtain $\underset{a}{\operatorname{argmax}} Q^\pi(s, a)$. Thus, up to this point, the policy π has been determined by the Q-function, but this approach cannot be taken when both spaces are continuous. Therefore, the policy function is often parameterized as π_θ and the parameter θ is updated by gradient ascent.

2.4 Deterministic Policy Gradient Method

Silver et al.[3] finds the gradient for the evaluation function $J(\pi_\theta)$, even if the policy $\pi(s)$ is defined as deterministic policy. This gradient is known as deterministic policy gradient(DPG), and it is calculate as following:

$$\nabla_\theta J(\pi_\theta) = \mathbb{E}_{s \sim \rho^{\pi_\theta}} [\nabla_\theta \pi_\theta(s) \nabla_a Q^{\pi_\theta}(s, a)|_{a=\pi_\theta(s)}] \quad (5)$$

where,

$$\rho^{\pi_\theta}(s) = \int_S \sum_{t=0}^{\infty} \gamma^t d_0(s_0) \Pr(s_0 \rightarrow s, t, \pi_\theta) ds_0 \quad (6)$$

is discounted distribution.

DDPG(Deep DPG) is a deep reinforcement learning algorithm which utilize this policy gradient. It adopts an Actor-Critic structure, and learns a critic network $Q(s, a|\omega)$ which approximates Q^{π_θ} , and an actor network $\pi(s|\theta) = \pi_\theta$ which represents a measure π , respectively. The update algorithm of actor and critic is described below.

DDPG uses mini-batch learning. First, update idea of critic is shown. The purpose of critic is to approximate Q^π . Because Q function is able to be decomposed like (4), $Q(s, a|\omega)$ should also be updated to satisfy this relation. For that, parameter ω is updated to the direction where it minimize Temporal Difference(TD) error:

$$\text{TD} = Q(s, a|\omega) - \{r(s, a) + \gamma Q(s, \pi(s)|\omega)\} \quad (7)$$

Since it is difficult to optimize for whole (s, a) at once, DDPG uses the mean squared error for the mini-batch E as the loss function and reduces it.

Now, the above method of updating critic is supervised learning itself. Therefore, the data in the mini-batch must be i.i.d.. If the mini-batch E uses the data of the last N steps experienced by the agent, they are no longer independent. Hence, the agent stores

the empirical data in a storage, called replay buffer, and randomly selects N data from it to make a mini-batch to increase the variance of the mini-batch.

Next update law of actor are shown. Because actor is the representation of policy function $\pi(s)$, policy gradient is used for its update. However, since correct Q -function as in equation (5) cannot be used in DDPG, approximated policy gradient

$$\mathbb{E}_{s \sim \rho^\pi} [\nabla_\theta \pi_\theta(s) \nabla_a Q(s, a | \omega) |_{a=\pi_\theta(s)}] \simeq \nabla_\theta J(\pi_\theta) \quad (8)$$

is used. Furthermore, an approximation to the expectation is made like following:

$$\mathbb{E}_{s \sim \rho^\pi} [\nabla_\theta \pi_\theta(s) \nabla_a Q(s, a | \omega) |_{a=\pi_\theta(s)}] \simeq \frac{1}{N} \sum_{s \in E} [\nabla_\theta \pi_\theta(s) \nabla_a Q(s, a | \omega) |_{a=\pi_\theta(s)}]. \quad (9)$$

Therefore, the accuracy of the approximation of the policy gradient may be greatly degraded by the accuracy of critic approximation and the distribution of mini-batch.

3 Problem Formulation

3.1 Self-Triggered Control

We consider control system like Fig. 1. Here, the control target is a continuous-time

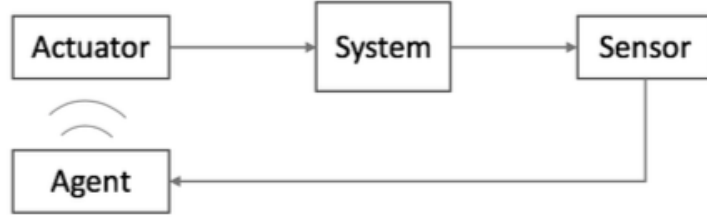


Figure 1: control system

system as in Equation (10)

$$\dot{s} = h(s, u) \quad (10)$$

Now, we consider a feedback control for system (10). In this paper, we call the observation of the state variable s and the sending of the input signal to the actuator as "interaction". In the self-triggered control, the agent does not make interaction continuously, but after a communication interval τ seconds determined by the agent itself. In order to express it mathematically, we assume that the agent's control law $\pi(s)$ is a vector-valued function consisting of two elements, where the first element represents the input u sent to the actuator and the second element represents the interval $\tau(sec)$. The input u sent in the previous interaction is added until the time of the next interaction (ZOH control).

3.2 Optimal Self-Triggered Control

The optimal self-triggered control law π^* is defined as follows

$$\pi^* = \underset{\pi}{\operatorname{argmax}} J(\pi) \quad (11)$$

$$J(\pi) = \mathbb{E}_{s_0 \in d_0} [V^\pi(s_0)] \quad (12)$$

$$V^\pi(s_0) = \sum_{i=0}^{\infty} \gamma^i r_i^\pi \quad (13)$$

$$r_i^\pi = - \int_{T_i}^{T_{i+1}} s(t)^\top Q s(t) dt + \tau_i a_i^\top R a_i + \lambda \tau_i, \quad T_i = \sum_{l=0}^i \tau_l \quad (14)$$

where i denote the number of interactions, and a_i and τ_i be the outputs of the measures π for the i th interaction, respectively.

4 Reinforcement Learning for Optimal Self-Triggered Control

In reinforcement learning, the agent is expected to perform interactions step by step. The self-triggered control problem considered in this paper, where the control target is given as a continuous-time system, satisfies the assumption of reinforcement learning by considering the communication with the actuator as one step.

4.1 Model Settings

In the self-triggered control, the agent needs to decide the input signal u and the interval τ at each step. In this paper, the control law is given as a state feedback. Therefore, the policy function is given as follows:

$$\pi(s) = [u(s) \quad \tau(s)]^\top \quad (15)$$

We use DDPG as reinforcement learning algorithm. As described in section 2, actor and critic is expressed as neural networks respectively. Experiments have shown that learning diverges when using a general network, so here we use the special network. The architecture of 2 networks is in Fig 2.

The activation function "original" shown in Fig 2 is defined as $0.99 \times \text{sigmoid} + 0.01$ to meet upper and lower limits of interval.

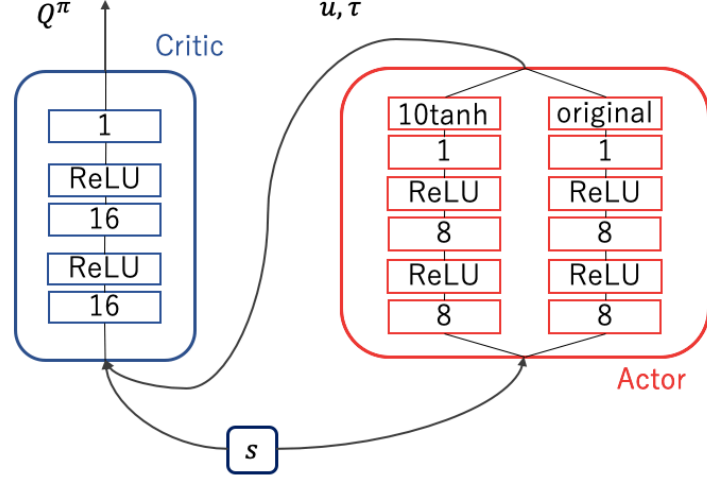


Figure 2: Agent Model

5 Consideration

5.1 Linear Case

First, we adopt reinforcement learning to self-triggered control for linear system. The control object is

$$\dot{s} = As + Bu = \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix} s + \begin{bmatrix} 2 \\ 4 \end{bmatrix} u. \quad (16)$$

5.1.1 Initial Policy

For learning efficiency, we use π_{init} such that

$$\pi_{\text{init}} = [-Ks \quad 0.01] \quad (17)$$

where K is a feedback gain calculated by Linear Quadratic Regulator. This policy stabilize the system (really??).

5.1.2 Learned Policy

5.2 Non-Linear Case

In this subsection, we investigate whether the self-triggered control law can be learned by reinforcement learning even when the control target is extended to non-linear systems, especially control affine systems. We consider an inverted pendulum, whose state-space representation is

$$\frac{d}{dt} \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \frac{3g}{2l} \sin \theta + \frac{3}{ml^2} a \end{pmatrix}. \quad (18)$$

Therefore, for an inverted pendulum, the state variable s is considered to be $\begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$.

5.2.1 Initial Policy

For learning efficiency, we use π_{init} such that

$$\pi_{\text{init}} = \begin{bmatrix} -Ks & 0.2 \end{bmatrix} \quad (19)$$

where K is a feedback gain calculated by Linear Quadratic Regulator. This policy stabilize the system (really??).

5.2.2 Learned Policy

Fig 3 shows a control path with learned policy, stating from $s_0 = [3., 3.]$.

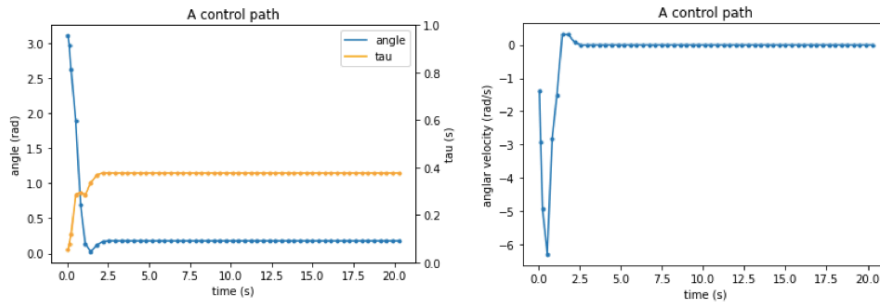


Figure 3: A control path with learned policy π_{RL}

The left panel of Figure 3 plots the change of angle and communication interval τ against the control time. The time of the communication is marked with a dot. From this figure, we can see that the communication interval is determined flexibly at each interaction.

Now, we confirm whether the learned policy π_{RL} enlarge the evaluation function $J(\pi)$ compared from the initial policy π_{init} . Since $J(\pi) = \mathbb{E}_{s_0}[V^\pi(s_0)]$, we generate 500 initial states s_0 according to the initial state distribution d_0 , and average the results of $V^\pi(s_0)$. We conduct this approximation of expectation 1000 times. From the result

$$J(\pi_{\text{init}}) = 4.23 \pm 0.083, \quad J(\pi_{\text{RL}}) = 37.12 \pm 0.071, \quad (20)$$

we can confirm $J(\pi_{\text{init}}) < J(\pi_{\text{RL}})$. It can be concluded that the policy has been improved.

5.3 Optimality1

Is the policy π_{RL} described in the previous section an optimal? If it is an optimal solution, the policy gradient must be $\mathbf{0}$. Therefore from equation (5), a condition

$$\nabla_a Q^{\pi_{\text{RL}}}(s, a)|_{a=\pi_{\text{RL}}(s)} = \mathbf{0} \quad (21)$$

should be met.

Now, during the reinforcement learning process, the policy keeps changing at each step, so $Q^\pi(s, a)$, which is the approximation target of critic, also keeps changing. In addition, the empirical data used for critic's learning includes data of experience with policies different from the one to be evaluated. Thus, there is no guarantee that $Q^{\pi_{\text{RL}}}(s, a)$ is approximated correctly at each step. Therefore, we learned $Q^{\pi_{\text{RL}}}(s, a)$ by supervised learning of critic using only the data generated by interaction with a fixed policy. Let $Q_{\text{ap}}^{\pi_{\text{RL}}}(s, a)$ denotes this approximated function.

If π_{RL} is the optimal policy, $\nabla_a Q_{\text{ap}}^{\pi_{\text{RL}}}(s, a)|_{\pi_{\text{RL}}}$ should be $\mathbf{0}$. However, when this is calculated in practice, we have

$$\nabla_a Q_{\text{ap}}^{\pi_{\text{RL}}}(s, a) = [0.03, 0.07] . \quad (22)$$

Therefore, we find π_{RL} is not the optimal policy.

5.4 Optimality2

Is the policy π_{RL} described in the previous section an optimal? If it is an optimal solution, the policy gradient must be $\mathbf{0}$. Therefore from equation (5), a condition

$$\nabla_a Q^{\pi_{\text{RL}}}(s, a)|_{a=\pi_{\text{RL}}(s)} = \mathbf{0} \quad (23)$$

should be met.

In order to check whether the policy π_{textrmRL} satisfies the condition (??), we want to know $Q^{\pi_{\text{RL}}}(s, a)$. Then, we obtain the approximation of value function $V^{\pi_{\text{RL}}}(s) = \sum_{i=0}^N \gamma^i r(s_i, \pi_{\text{RL}}(s_i))$ with N step simulation:

$$V_{\text{ap}}^{\pi_{\text{RL}}}(s) = \sum_{i=0}^N \gamma^i r(s_i, \pi_{\text{RL}}(s_i)). \quad (24)$$

Next, the approximation of $Q^{\pi_{\text{RL}}}$ is calculated with $V_{\text{ap}}^{\pi_{\text{RL}}}$ as following:

$$Q_{\text{apd}}^{\pi_{\text{RL}}}(s, a) = r(s, a) + \gamma V_{\text{ap}}^{\pi_{\text{RL}}}(s') \quad (25)$$

Since $\gamma < 1$, $Q_{\text{apd}}^{\pi_{\text{RL}}}(s, a)$ is a good approximation if the number of simulation steps N is sufficiently large. However, to compute the policy gradient, we need to compute $\nabla_a Q^{\pi_{\text{RL}}}(s, a)$, while $Q_{\text{apd}}^{\pi_{\text{RL}}}$ can only be computed the value at (s, a) . Then, we approximate the function $Q_{\text{apd}}^{\pi_{\text{RL}}}(s, a)$ by supervised learning with $Q_{\text{apd}}^{\pi_{\text{textrmRL}}}(s, a)$ as the teacher data. This function is defined as $Q_{\text{ap}}^{\pi_{\text{RL}}}$.

Now, when we calculate the policy gradient using $Q_{\text{ap}}^{\pi_{\text{RL}}}$, we obtain

$$\mathbb{E}_{s \sim \rho^{\pi_{\text{RL}}}} [\nabla_{\theta} \pi_{\text{RL}}(s) \nabla_a Q^{\pi_{\text{RL}}}(s, a)|_{a=\pi_{\text{RL}}(s)}] \neq \mathbf{0}, \quad (26)$$

which means that the policy π_{RL} is not an optimal measure. We will discuss the reason for this.

5.5 Approximation Accuracy of Critic

方策 π_{RL} は, critic ネットワーク $Q(s, a|\omega)$ に対して

$$\mathbb{E}_{s \sim \rho^{\pi_{\text{RL}}}} [\nabla_{\theta} \pi_{\text{RL}}(s) \nabla_a Q(s, a|\omega)|_{a=\pi_{\text{RL}}(s)}] = \mathbf{0} \quad (27)$$

となったため, 近似勾配に対しては最適な方策であると言える. 従って, critic の近似精度の低さが最適方策を学習できなかった原因であると考えられる.

6 Conclusion

Acknowledgments

The author would like to express his sincere gratitude to Professor Yoshito Ohta and Assistant Professor Kenji Kashima for their helpful advices.

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A Appendix

This is an appendix. This is a citation [?].

Table 1: This is a table.

	A	B
C	70	80
D	100	0

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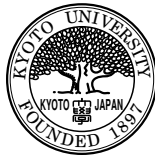
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