BBM 205 - Discrete Structures Final Exam
Date: 7.1.2016, Time: 9:30 - 11:00 (90 minutes)

Please solve 5 out of 6 questions. Circle the questions you answered.

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Ögrenci No /Student ID:
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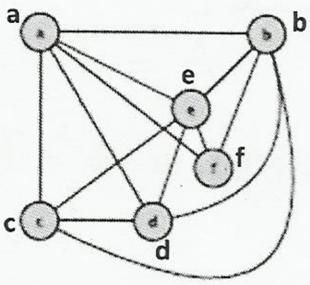
Question	1	2	3	4	5	6	Total
Points	20	20	20	20	20	20	100
Grade							

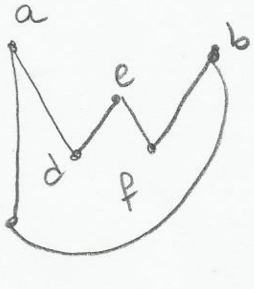
1. (20 points) Consider the following graph G.



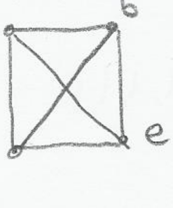
   (a) Find a Hamilton path in G.
   (b) Find a subgraph of G that is isomorphic to K_4.
   (c) Find a subgraph of G that is homeomorphic to K_5. Is G planar?
   (d) Is G Eulerian? Justify your answer~~s~~.
   (e) Find two non-isomorphic spanning trees of G.
   (f) Give a list of 5 non-negative integers that cannot be the degree list of a graph on 5 vertices.

You can use the back of this page to answer Question 1.

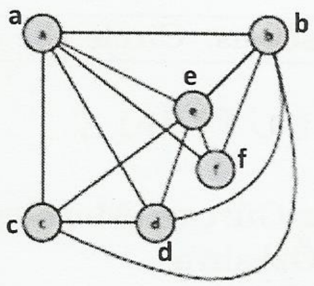
a)



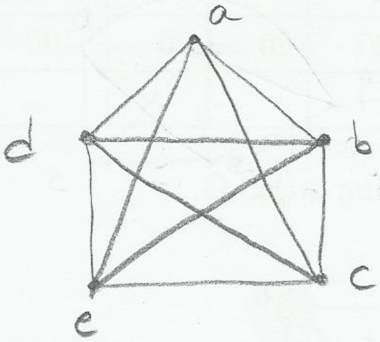
b)



(The figure copied from Page 1)



c)

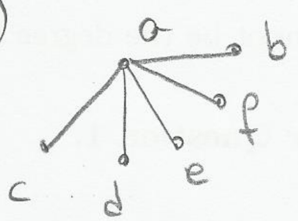


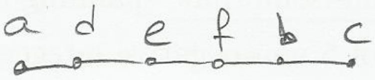
$\cong K_5$

By Kuratowski's Thm., G is not planar.

d) There are vertices a, b, e, f that have an odd degree. Therefore, G is not Eulerian.

e)





f) $\{1, 1, 1, 1, 1\}$

2. (20 points) Find the solution of the recurrence relation

$$a_n = 8a_{n-1} - 16a_{n-2}$$

with initial conditions $a_0 = 1$, $a_1 = 7$.

$a_n - 8a_{n-1} + 16a_{n-2} = 0$

$t^2 - 8t + 16 = 0$

$(t-4)^2 = 0$

$S_n = A \cdot 4^n + B \cdot n 4^n$

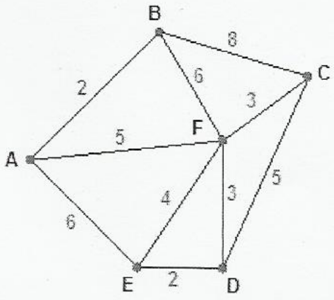
$S_0 = 1 = A$

$S_1 = 7 = 4A + 4B$

$\therefore B = \frac{3}{4}$

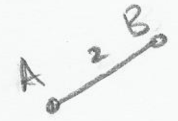
$a_n = S_n = 4^n + \frac{3}{4} n 4^n$

3. (20 points) Find a minimum spanning tree by using Prim's Algorithm for the following graph. SHOW EACH STEP OF THE ALGORITHM IN DETAIL.

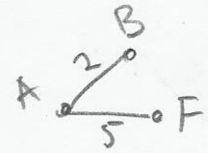


(There can be different order of steps.)

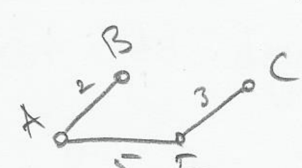
Start by picking the edge AB, since it has the minimum weight 2.



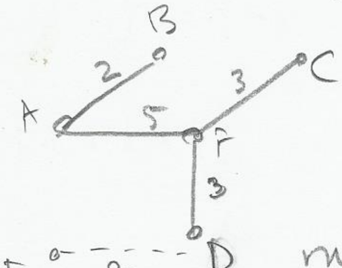
$\min\{6,5,6,8\} = 5$. Add the edge AF.



$\min\{6,4,8,3\} = 3$. Add the edge FC



$\min\{6,4,3,5\} = 3$ Add the edge FD.



$\min\{6,4,2\} = 2$ Add the edge ED.

Done.

4. (20 points) Show that $2^n \geq n^2$ for every integer $n \geq 4$.

Use induction:

Base step: $n=4$

$2^4 \geq 4^2$ since $16=16$.

Ind. Hypo. Assume that $2^{n-1} \geq (n-1)^2$. (*)

Ind. Step: To show $2^n \geq n^2$, multiply each side of (*) with 2:

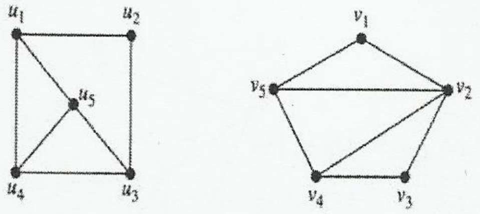
$$\underbrace{2 \cdot 2^{n-1}}_{=2^n} \geq \underbrace{2(n-1)^2}_{\geq 2n^2-4n+2}$$

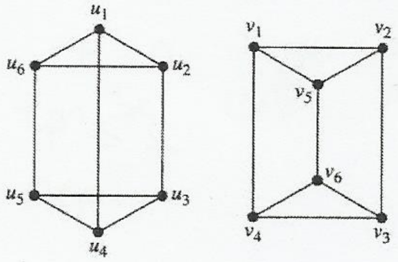
$$2n^2-4n+2 = n^2 + \underbrace{n^2-4n+2}_{\geq 0 \text{ when } n \geq 4}$$

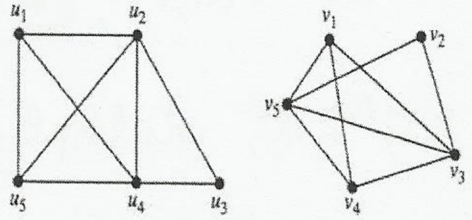
Therefore,

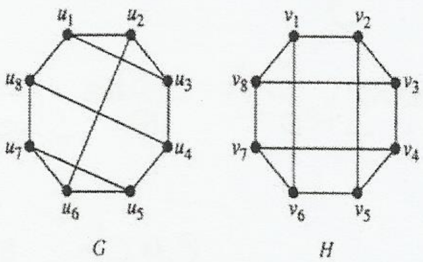
$2^n \geq n^2$. ■

5. (20 points) Determine whether the given pair of graphs is isomorphic or not. Justify your answer.

a)


b)


c)


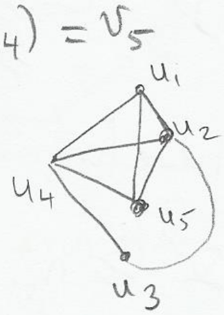
d)


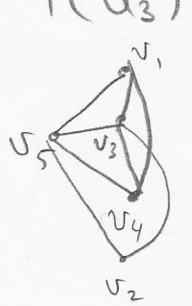
G H

a) not isomorphic : v_2 has degree 4. No such vertex in the first graph.

b) Isomorphic:
$f(u_1) = v_5$, $f(u_6) = v_1$, $f(u_2) = v_2$,
$f(u_4) = v_6$, $f(u_5) = v_4$, $f(u_3) = v_3$.

c) isomorphic:
$f(u_4) = v_5$ and $f(u_3) = v_2$ (Only possibility)





$f(u_1) = v_1$
$f(u_5) = v_4$
$f(u_2) = v_3$

d) not isomorphic:
Many reasons are possible.
One reason is there is a K_3 (triangle) in the first graph, but no triangle in the second graph.

6. (20 points) Encode (Find a code for) the sentence HAVE A HAPPY NEW YEAR using Huffman Code.

HAVE␣A␣HAPPY␣NEW␣YEAR

Frequencies:

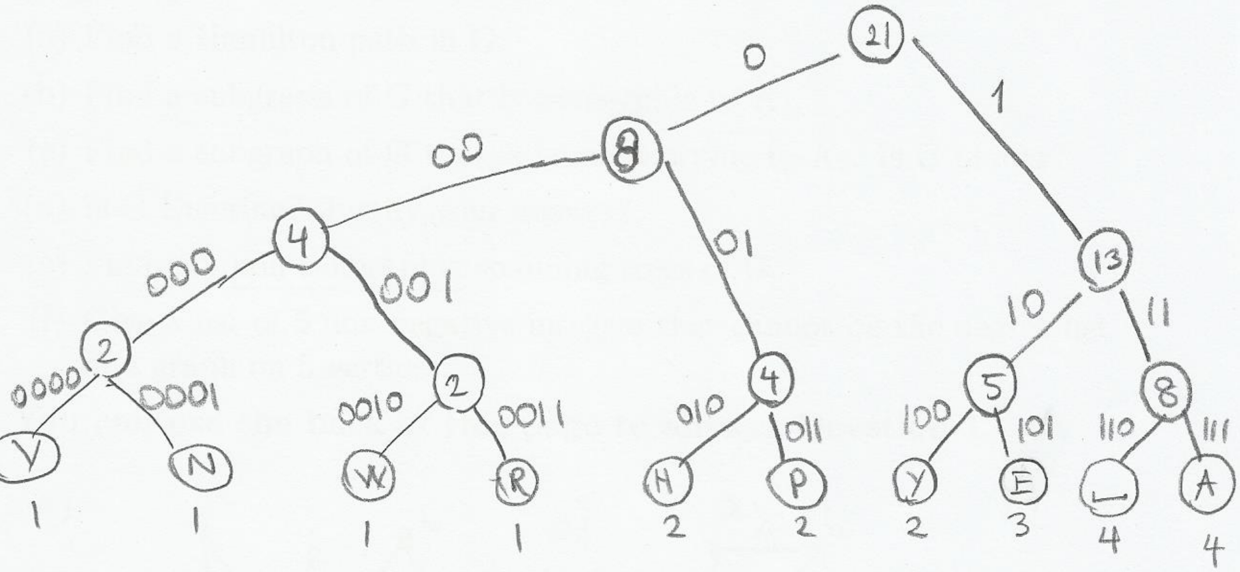
H - 2 P - 2

A - 4 Y - 2

V - 1 N - 1

E - 3 W - 1

␣ - 4 R - 1



V = 0000
N = 0001
W = 0010

R = 0011
H = 010
P = 011

Y = 100
E = 101
␣ = 110

A = 111

Use these codes to encode the given sentence