Basic Linear Algebra - Exam I - April 13, 2020
Department of Computer Engineering

Student Name and Number: _____

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Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

[25pt] 1. a) Let $A = \begin{bmatrix} a & 0 & 0 \\ 2 & b & 0 \\ 3 & 1 & c \end{bmatrix} \cdot \begin{bmatrix} d & -1 & 3 \\ 0 & e & 1 \\ 0 & 0 & f \end{bmatrix}$

Suppose that $AX = 0$ has a nontrivial solution. Show that at least one of a, b, c, d, e, f must be zero.

b) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Show that the matrix $(X - A)^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - A^2$ has no solutions among 2×2 matrices.

[25pt] 2. Let $A = \begin{bmatrix} 1 & 2 & 0 & 4 & 2 \\ 2 & 3 & -1 & 5 & 6 \\ 0 & 2 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 & 0 \\ 1 & 2 & 0 & 4 & 1 \end{bmatrix}$. Compute the following determinants:

(a) $|A|$

(b) $|2A^{-1}|$

(c) $|D|$ if $A \xrightarrow{R_2 \leftrightarrow R_3} B \xrightarrow{100R_3 + R_4} C \xrightarrow{-2R_1} D$

(d) $|B^T D^{-1}|$

[25pt] 3. Determine the values of a for which the following system has,

(a) no solution, (b) infinitely many solutions, (c) a unique solution. Find the general solution in (b) and find the unique solution in (c).

$$x + y + 7z = -7$$
$$2x + 3y + 17z = -16$$
$$x + 2y + (a^2 + 1)z = 3a$$

[25pt] 4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) Find A^{-1} by using elementary row operations.

(b) Find $\operatorname{adj}(A)$ by using A^{-1}.

(c) Solve the system $AX = B$ if $B = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$.

GOOD LUCK