BBM105: Chapter 4 Binary Numbers, Boolean Logic, and Gates

Slides From: Invitation to Computer Science, C++ Version

Objectives

In this course, you will learn about:

- The binary numbering system
- Boolean logic and gates
- Circuits

The Binary Numbering System

- A computer's internal storage techniques are different from the way people represent information in daily lives

- Information inside a digital computer is stored as a collection of **binary data**

Binary Representation of Numeric and Textual Information

- Binary numbering system
  - Base-2
  - Built from ones and zeros
  - Each position is a power of 2

  $1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
- Decimal numbering system
  - Base-10
  - Each position is a power of 10

  $3052 = 3 \times 10^3 + 0 \times 10^2 + 5 \times 10^1 + 2 \times 10^0$

Binary Representation of Numeric and Textual Information (continued)

- Representing integers
  - Decimal integers are converted to binary integers
  - Given k bits, the largest unsigned integer is $2^k - 1$
    - Given 4 bits, the largest is $2^4-1 = 15$

Binary Representation of Numeric and Textual Information (continued)

- Signed integers must also represent the sign (positive or negative) - ***Sign/Magnitude notation***
  - **0**000 = +0
  - 0001 = +1
  - 0010 = +2
  - ...
  - **1**000 = -0
  - 1001 = -1
  - 1111 = -7

Binary Representation of Numeric and Textual Information (continued)

- Signed integers must also represent the sign (positive or negative) – ***Two's Complement!!***
  - Convert Decimal to Two's Complement (to 4 bit)
    - e.g. We have 6 and -6 decimal numbers.
    - For 6 → 0110. It's ok for positive numbers!
    - For -6 → we have positive 0110.
      - In 2nd step, invert all the bits 0110 → 1001
      - In 3rd step, add 1 to 1001 → 1010. Now we have -6!

Binary Representation of Numeric and Textual Information (continued)

- Signed integers must also represent the sign (positive or negative) – ***Two's Complement!!***
  - Convert Two's Complement to Decimal(to 4 bit)
    - e.g. We have 0110 and 1010 decimal numbers.
    - For 0110 → First bit is 0. So this is a positive number!
      - 0110 = 0x2^3 + 1x 2^2 + 1x2^1+0x2^0 = 6
    - For 1010 → First bit is 1. So this is a negative number!
      - Invert all bits again 1010 → 0101
      - Add 1 to the results 0101 + 1 → 0110
      - 0110 = 0x2^3 + 1x 2^2 + 1x2^1+0x2^0 = 6
      - The number was negative, because of first bit! = -6

Binary Representation of Numeric and Textual Information (continued)

- Characters are mapped onto binary numbers
  - ASCII code set
    - 8 bits per character; 256 character codes
  - UNICODE code set
    - 16 bits per character; 65,536 character codes
- Text strings are sequences of characters in some encoding

Binary Representation of Textual Information (cont'd)

ASCII
8 bits long

Decimal	Binary	Val.
48	00110000	0
49	00110001	1
50	00110010	2
51	00110011	3
52	00110100	4
53	00110101	5
54	00110110	6
55	00110111	7
56	00111000	8
57	00111001	9
58	00111010	:
59	00111011	;
60	00111100	<
61	00111101	=
62	00111110	>
63	00111111	?
64	01000000	@
65	01000001	A
66	01000010	B

Dec.	Unicode	Charac.
0x30	0x0030	[0]
0x31	0x0031	[1]
0x32	0x0032	[2]
0x33	0x0033	[3]
0x34	0x0034	[4]
0x35	0x0035	[5]
0x36	0x0036	[6]
0x37	0x0037	[7]
0x38	0x0038	[8]
0x39	0x0039	[9]
0x3A	0x003A	[:]
0x3B	0x003B	[;]
0x3C	0x003C	[<]
0x3D	0x003D	[=]
0x3E	0x003E	[>]
0x3F	0x003F	[?]
0x40	0x0040	[@]
0x41	0x0041	[A]
0x42	0x0042	[B]

Unicode
16 bits long

Partial listings only!

Binary Representation of Images

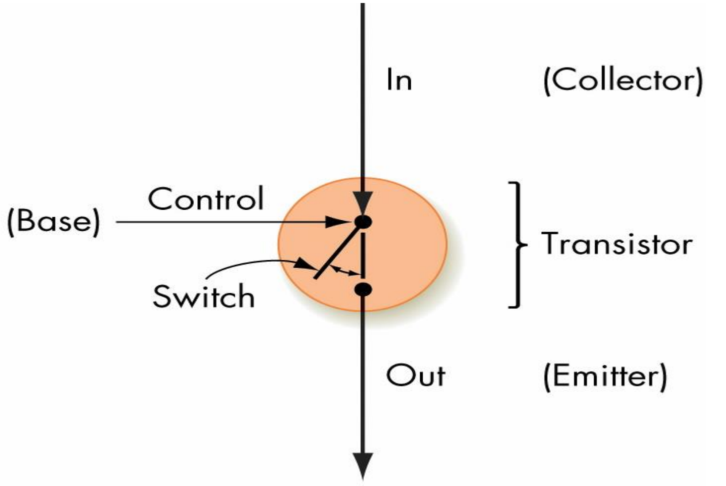
- Representing image data
  - Images are sampled by reading color and intensity values at even intervals across the image
  - Each sampled point is a pixel
  - Image quality depends on number of bits at each pixel
  - More image information: http://cat.xula.edu/tutorials/imaging/grayscale.php
  - Most color format for storing color: RGB encoding scheme

The Reliability of Binary Representation

- Electronic devices are most reliable in a bistable environment
- Bistable environment
  - Distinguishing only two electronic states
    - Current flowing or not
    - Direction of flow
- Computers are bistable: hence binary representations

Binary Storage Devices (continued)

- Transistors
  - Solid-state switches: either permits or blocks current flow
  - A control input causes state change
  - Constructed from semiconductors



Boolean Logic and Gates: Boolean Logic

- Boolean logic describes operations on true/false values
- True/false maps easily onto bistable environment
- Boolean logic operations on electronic signals may be built out of transistors and other electronic devices

Boolean Logic (continued)

- Boolean operations
  - a AND b
    - True only when a is true and b is true
  - a OR b
    - True when either a is true or b is true, or both are true
  - NOT a
    - True when a is false, and vice versa

Boolean Logic (continued)

- Boolean expressions
  - Constructed by combining together Boolean operations
    - Example: (a AND b) OR ((NOT b) AND (NOT a))
- Truth tables capture the output/value of a Boolean expression
  - A column for each input plus the output
  - A row for each combination of input values

Boolean Logic (continued)

- Example:

  (a AND b) OR ((NOT b) and (NOT a))

a	*b*	*Value*
0	0	1
0	1	0
1	0	0
1	1	1

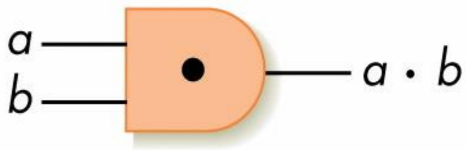
Gates

- Gates
  - Hardware devices built from transistors to mimic Boolean logic
- AND gate
  - Two input lines, one output line
  - Outputs 1 when both inputs are 1

Gates (continued)

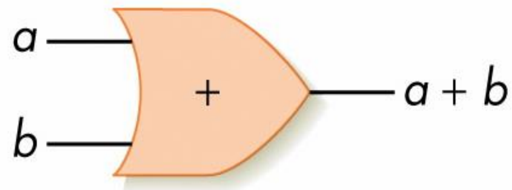
- OR gate
  - Two input lines, one output line
  - Outputs a 1 when either input is 1
- NOT gate
  - One input line, one output line
  - Outputs a 1 when input is 0 and vice versa

AND gate



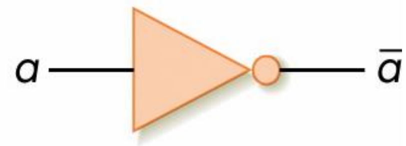
a	b	$a \cdot b$
0	0	0
0	1	0
1	0	0
1	1	1

OR gate



a	b	$a + b$
0	0	0
0	1	1
1	0	1
1	1	1

NOT gate



a	$\bar{a}$
0	1
1	0

Figure 4.15

The Three Basic Gates and Their Symbols

Gates (continued)

- Abstraction in hardware design
  - Map hardware devices to Boolean logic
  - Design more complex devices in terms of logic, not electronics
  - Conversion from logic to hardware design may be automated

Building Computer Circuits: Introduction

- A circuit is a collection of logic gates:
  - Transforms a set of binary inputs into a set of binary outputs
  - Values of the outputs depend only on the current values of the inputs
- Combinational circuits have no cycles in them (no outputs feed back into their own inputs)

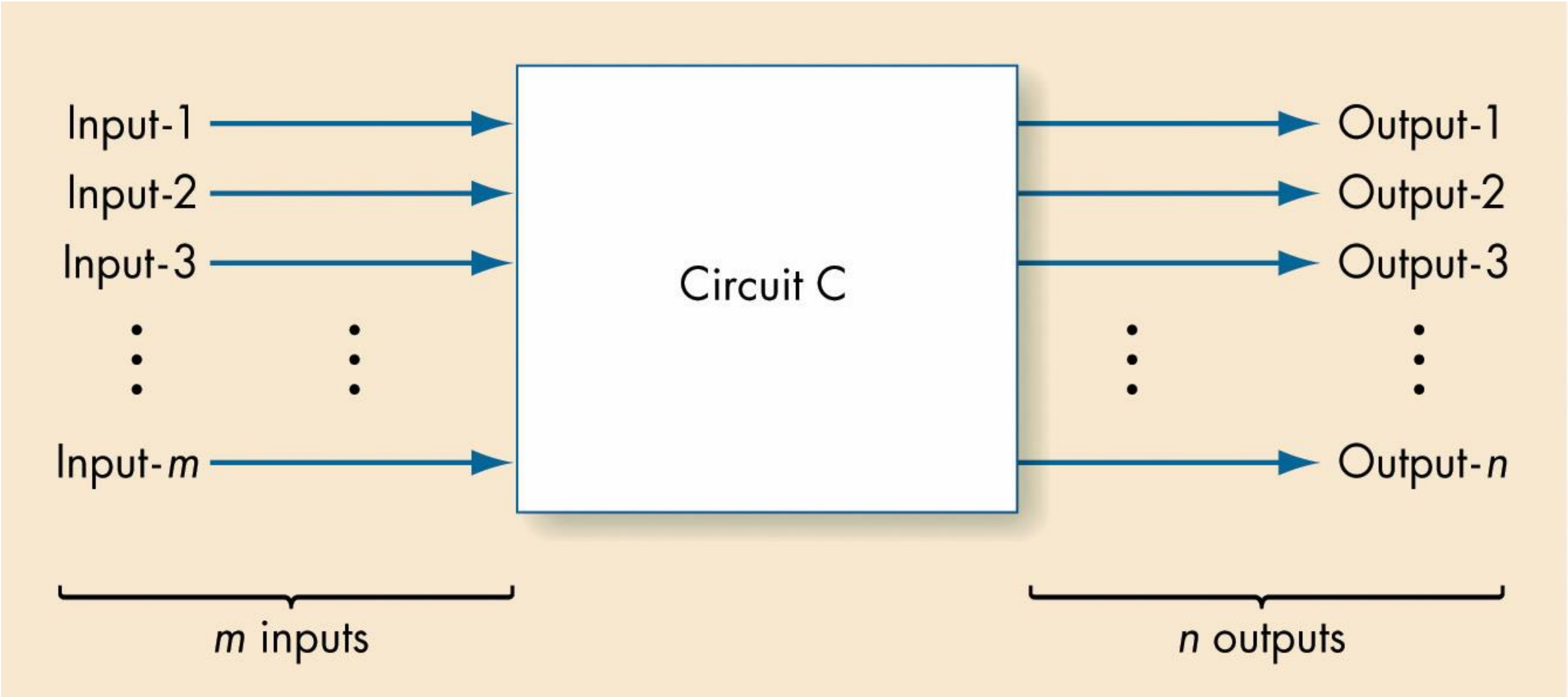


Figure 4.19

Diagram of a Typical Computer Circuit

A Circuit Construction Algorithm

- Sum-of-products algorithm is one way to design circuits:
  - Truth table to Boolean expression to gate layout

Sum-of-Products Algorithm for Constructing Circuits

1. Construct the truth table describing the behavior of the desired circuit
2. While there is still an output column in the truth table, do steps 3 through 6
3. Select an output column
4. Subexpression construction using AND and NOT gates
5. Subexpression combination using OR gates
6. Circuit diagram production
7. Done

Figure 4.21

The Sum-of-Products Circuit Construction Algorithm

A Circuit Construction Algorithm (continued)

- Sum-of-products algorithm
  - Truth table captures every input/output possible for circuit
  - Repeat process for each output line
    - Build a Boolean expression using AND and NOT for each 1 of the output line
    - Combine together all the expressions with ORs
    - Build circuit from whole Boolean expression

A Compare-for-equality Circuit

- Compare-for-equality circuit
  - CE compares two unsigned binary integers for equality
  - Built by combining together 1-bit comparison circuits (1-CE)
  - Integers are equal if corresponding bits are equal (AND together 1-CD circuits for each pair of bits)

A Compare-for-equality Circuit (continued)

- 1-CE circuit truth table

a	*b*	*Output*
0	0	**1**
0	1	0
1	0	0
1	1	**1**

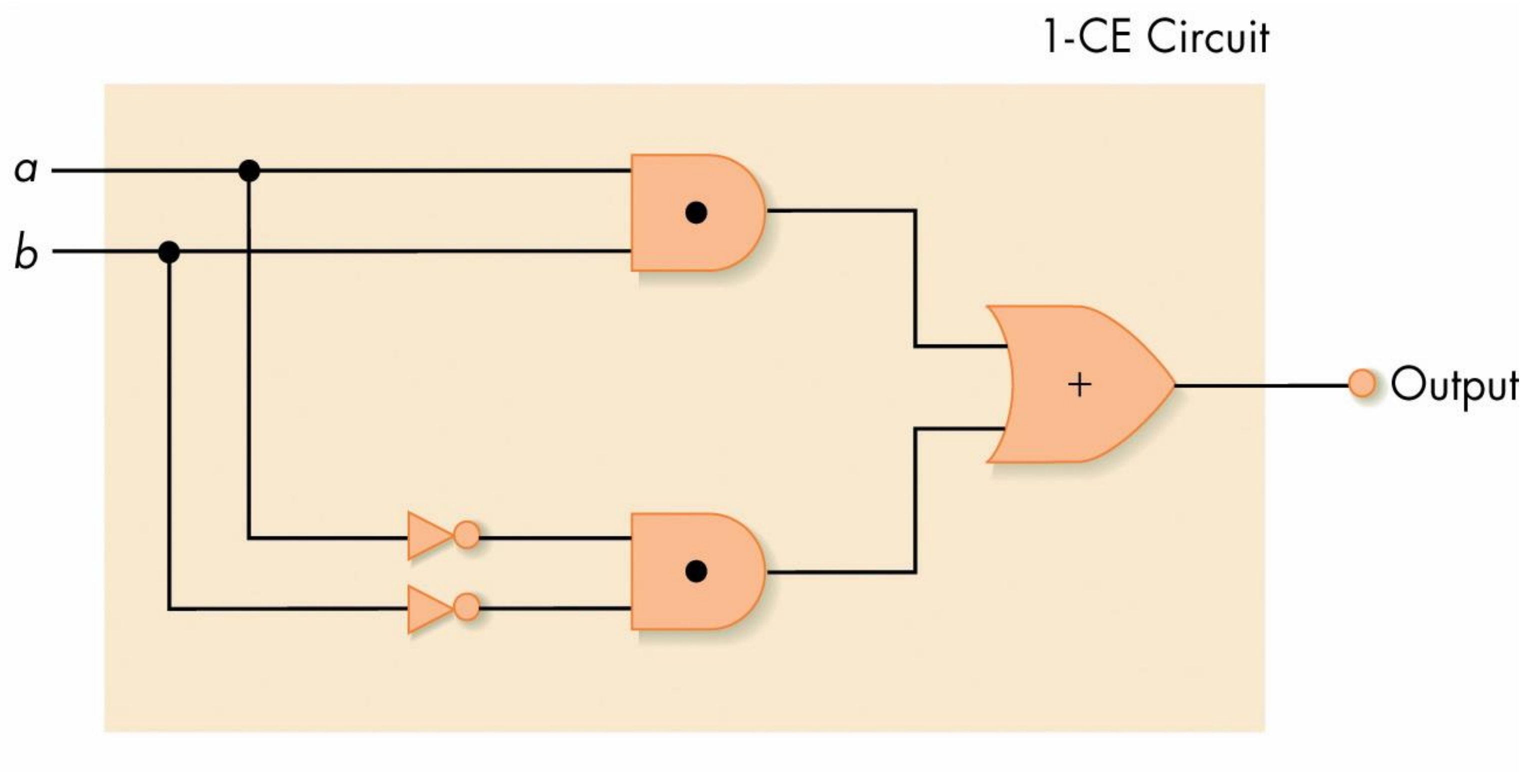


Figure 4.22

One-Bit Compare for Equality Circuit

A Compare-for-equality Circuit (continued)

- 1-CE Boolean expression
  - First case: (NOT a) AND (NOT b)
  - Second case: a AND b
  - Combined:

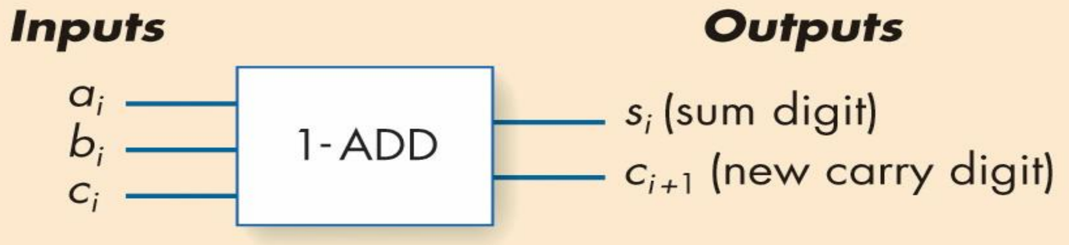
    ((NOT a) AND (NOT b)) OR (a AND b)

An Addition Circuit

- Addition circuit
  - Adds two unsigned binary integers, setting output bits and an overflow
  - Built from 1-bit adders (1-ADD)
  - Starting with rightmost bits, each pair produces
    - A value for that order
    - A carry bit for next place to the left

An Addition Circuit (continued)

- 1-ADD truth table
  - Input
    - One bit from each input integer
    - One carry bit (always zero for rightmost bit)
  - Output
    - One bit for output place value
    - One “carry” bit



Inputs			Outputs	
a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Figure 4.24

The 1-ADD Circuit and Truth Table

An Addition Circuit (continued)

- Building the full adder
  - Put rightmost bits into 1-ADD, with zero for the input carry
  - Send 1-ADD's output value to output, and put its carry value as input to 1-ADD for next bits to left
  - Repeat process for all bits

Control Circuits

- Do not perform computations
- Choose order of operations or select among data values
- Major types of controls circuits
  - Multiplexors
    - Select one of inputs to send to output
  - Decoders
    - Sends a 1 on one output line, based on what input line indicates

Control Circuits (continued)

- Multiplexor form
  - 2^N regular input lines
  - N selector input lines
  - 1 output line
- Multiplexor purpose
  - Given a code number for some input, selects that input to pass along to its output
  - Used to choose the right input value to send to a computational circuit

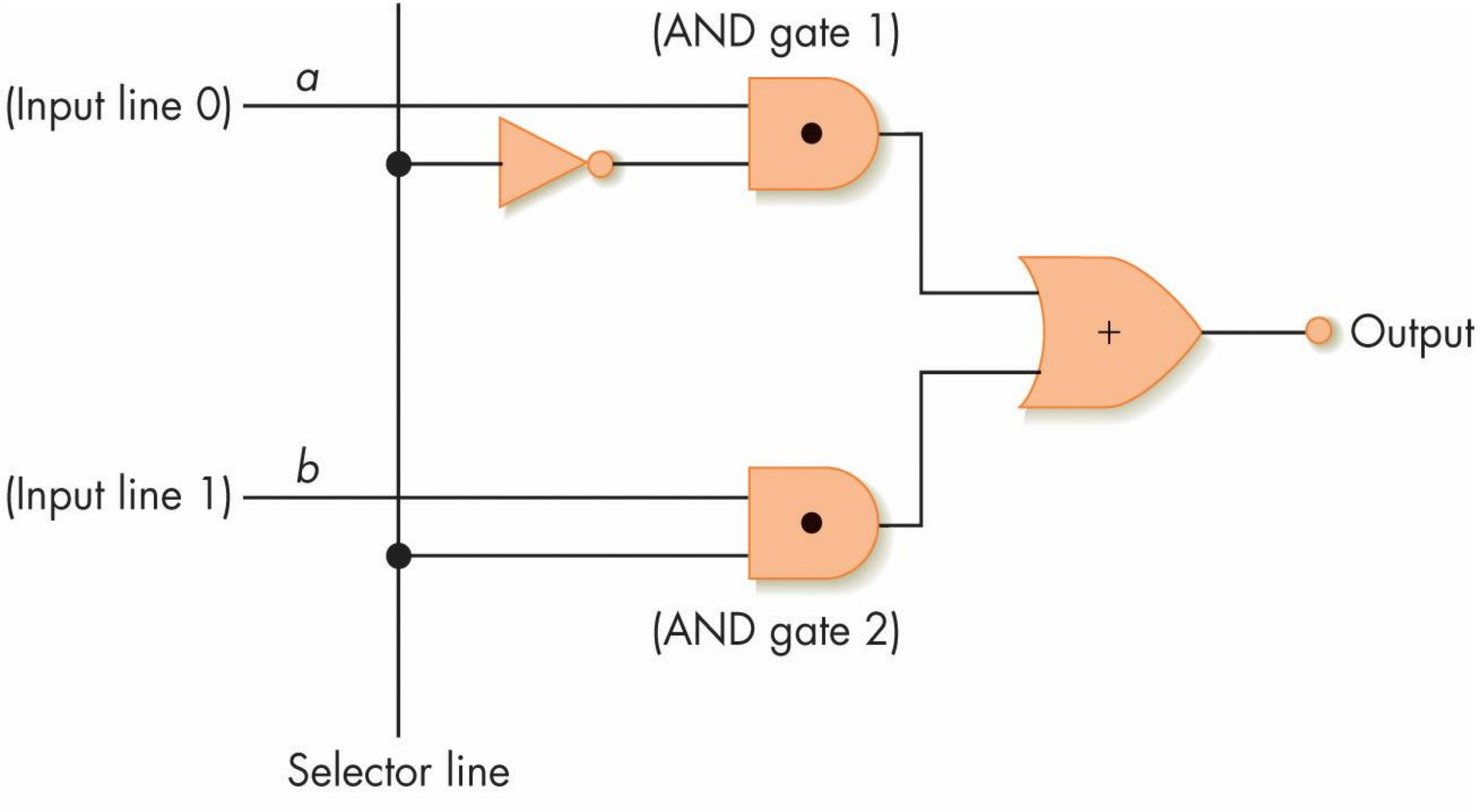


Figure 4.28

A Two-Input Multiplexor Circuit

Control Circuits (continued)

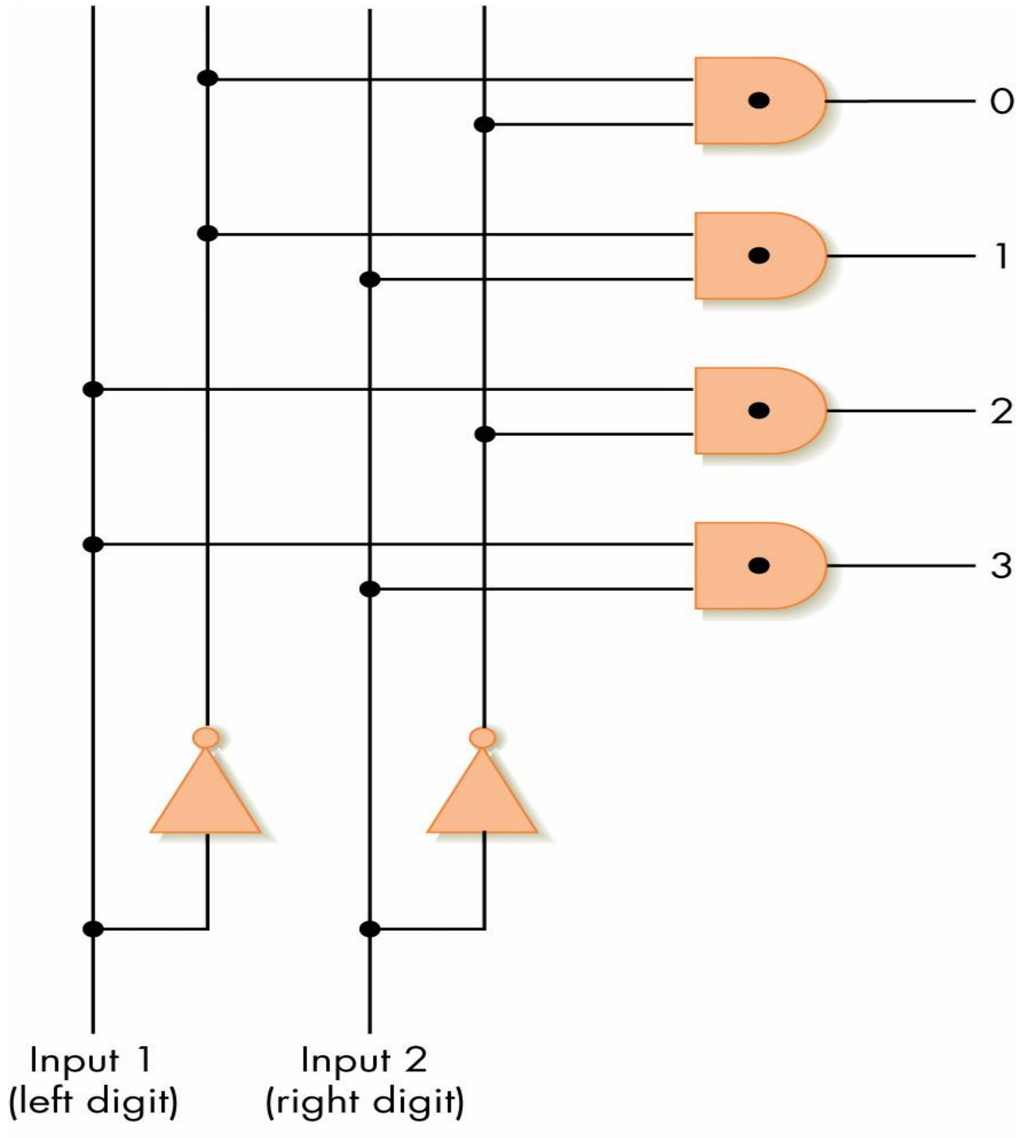
- Decoder
  - Form
    - N input lines
    - 2^N output lines
  - N input lines indicate a binary number, which is used to select one of the output lines
  - Selected output sends a 1, all others send 0

Control Circuits (continued)

- Decoder purpose
  - Given a number code for some operation, trigger just that operation to take place
  - Numbers might be codes for arithmetic: add, subtract, etc.
  - Decoder signals which operation takes place next

Figure 4.29

A 2-to-4 Decoder Circuit



Summary

- Digital computers use binary representations of data: numbers, text, multimedia
- Binary values create a bistable environment, making computers reliable
- Boolean logic maps easily onto electronic hardware
- Circuits are constructed using Boolean expressions as an abstraction
- Computational and control circuits may be built from Boolean gates