EXERCISES

1. A washing machine in a laundromat breakdowns on average of three times per month. Using the Poisson probability distribution formula, find the probability that during the next month this machine will have,

i) exactly two breakdowns
ii) at most one breakdowns.

Solution:

Since we know that $f(x)=\dfrac{e^{-\lambda}\lambda^{x}}{x!}$ and here $\lambda=3$ then;

i) $P(X=2)=e^{-3}\dfrac{3^{2}}{2!}=0.2240$

ii) $P(X=0)+P(X=1)=e^{-3}\dfrac{3^{0}}{0!}+e^{-3}\dfrac{3^{1}}{1!}=0.1992$

Binomial Distribution

2. If we toss a coin 20 times,
i) what is the probability of getting exactly 10 heads?
ii) what is the probability of getting of 2 or fewer heads?

Solution:

i) Since we know that $f(x)=\binom{n}{x}p^{x}(1-p)^{n-x}$ $x=0,1,2\cdots,n$ we can write

$$P(X=10)=\binom{20}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{20-10}=\binom{20}{10}\left(\frac{1}{2}\right)^{20}=0.1762$$

ii)

$$P(X\le 2)=\sum_{x=0}^{2}\binom{20}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{20-x}=\binom{20}{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{20-0}+\binom{20}{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{20-1}+\binom{20}{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{20-2}$$

$$=\left(\frac{1}{2}\right)^{20}+20\times\left(\frac{1}{2}\right)^{20}+190\times\left(\frac{1}{2}\right)^{20}$$

$$=2.012\times10^{-4}$$

Geometric Distribution

3. A test of weld strength involves loading welded joints until a fracture occurs. For a certain type of weld, 80% of the fractures occur in the weld itself, while the other 20% occur in the beam. A number of welds are tested and the tests are independent. Let X be the number of tests up to and including the first test that results in a beam fracture.

i) Find the probability of $P(X\ge 3)$

ii) Find mean, variance and standard deviation of X.

Solution:

Since we know that $f(x)=(1-p)^{x-1}p \qquad x=1,2,\cdots$

i) $P(X\geq 3)=1-P(X<3)=1-\left[P(X=1)+P(X=2)\right]=1-\left[(0.8)^{1-1}(0.2)+(0.8)^{2-1}(0.2)\right]$
$=0.64$

ii) $E(X)=\dfrac{1}{p}=\dfrac{1}{0.2}=5$

$$V(X)=\frac{(1-p)}{p^2}=\frac{(1-0.2)}{(0.2)^2}=20 \;\; and \;\; standard \;\; deviation=\sqrt{V(X)}=\sqrt{20}=4.47$$

Negative Binomial Distribution

4. In the American League Championship Series (ALCS) the Yankees play the Red Sox. The team that records its 4th win wins the series. Suppose P(Yankees win a game)=0.6 and that the games are won or lost independently of each other. Find the probability P(Yankees win in 7 games).

Solution:

Since we know that $f(x)=\binom{x-1}{r-1}(1-p)^{x-r}p^r \qquad x=r,\mathrm{r}+1,\mathrm{r}+2,\cdots$

$$P(X=7)=\binom{7-1}{4-1}(1-0.6)^{7-4}(0.6)^4=\binom{6}{3}(0.4)^3(0.6)^4=0.166$$

5. Bob is high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw is 0.70. During the season,

i) what is the probability that Bob makes his third free throw on his fifth shot?
ii) what is the probability that Bob makes his first free throw on his fifth shot?

Solution:

i) *Negative Binomial Distribution* and $P(\mathit{free\,throw})=p=0.70$

$$P(X=5)=\binom{5-1}{3-1}(1-0.70)^{5-3}(0.70)^3=\binom{4}{2}(0.30)^2(0.70)^3=0.18522$$

ii) *Geometric Distribution* and $P(X=5)=(1-0.70)^{5-1}(0.70)=(0.30)^4(0.70)=0.00567$

Hypergeometric Distrubition

6. A watch of 10 rocker cover gaskets (conta) contains 4 defective gaskets. If we draw samples of size 3 without replacement from the batch of 10,

i) Find the probability that a sample contains 2 defective gaskets.
ii) Find the mean and variance of the probability distribution of X.

Solution:

Since we know that $f(x)=\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ and here K=4 x=2 N=10 n=3

i) $P(X=2)=\frac{\binom{4}{2}\binom{10-4}{3-2}}{\binom{10}{3}}=0.3$

ii) $E(X)=np=n\frac{K}{N}=3\times\frac{4}{10}=1.2$

$$V(X)=np(1-p)\left(\frac{N-n}{N-1}\right)=n\frac{K}{N}\left(\frac{N-K}{N}\right)\left(\frac{N-n}{N-1}\right)=3\times\frac{4}{10}\times\frac{6}{10}\times\frac{7}{9}=0.56$$

7. A deck (deste) of cards contains 20 cards; 6 red cards and 14 black cards. 5 cards are drawn randomly without replacement. What is the probability that exactly 4 red cards are drown?

Solution:

$P(X=4)=\frac{\binom{6}{4}\binom{20-6}{5-4}}{\binom{20}{5}}=0.0135$ **(**Here, X denotes the number of red cards in the sample.)

8. Suppose that in a certain shipment of 250 Pentium processors there are 17 defective processors. A quality control consultant randomly collects 5 processors for inspection to determine whether or not they are defective. Let X denotes the number of defectives in the sample. Find the probability of exactly 3 defectives in the sample.

Solution:

$$P(X=3)=\frac{\binom{17}{3}\binom{250-17}{5-3}}{\binom{250}{5}}$$