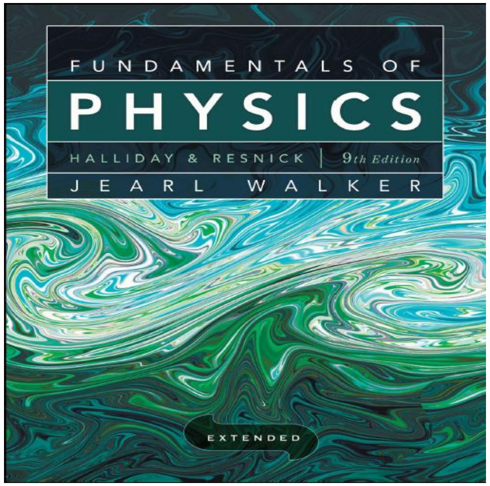
FİZ 137-25
CHAPTER 14

OSCILLATIONS



Dr. Şeyda ÇOLAK

2018 - 2019
FALL SEMESTER

Concept

- Definition of ***Simple Harmonic Oscillation***

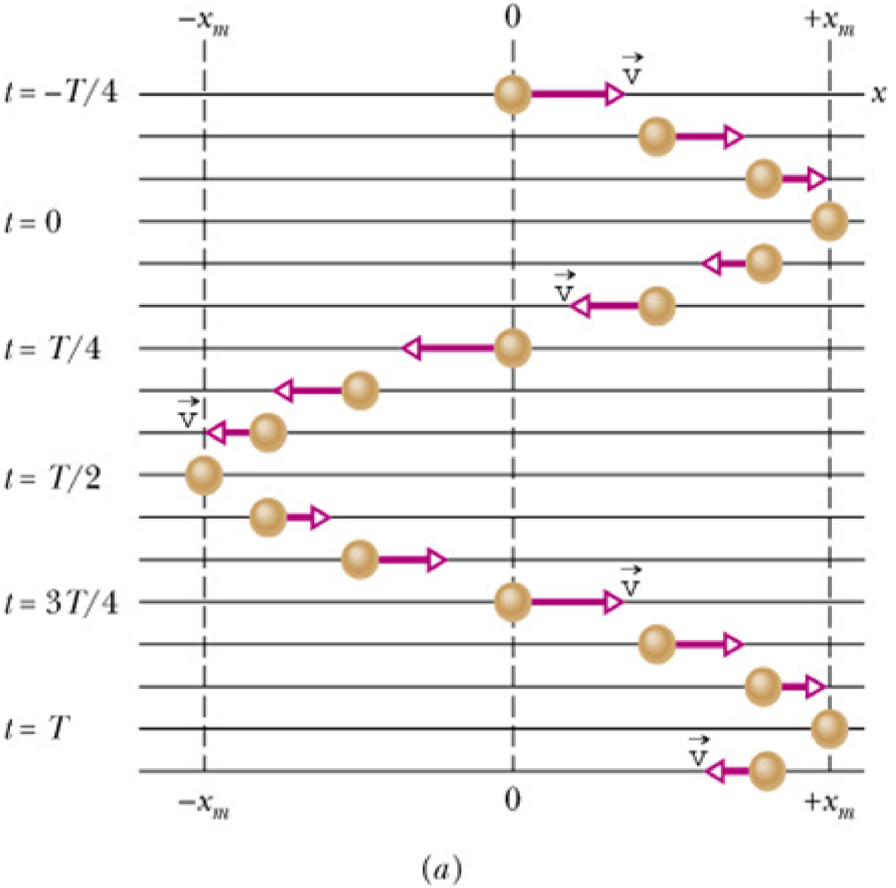
- ***Displacement***, ***velocity*** and ***acceleration*** vectors for a simple harmonic oscillator

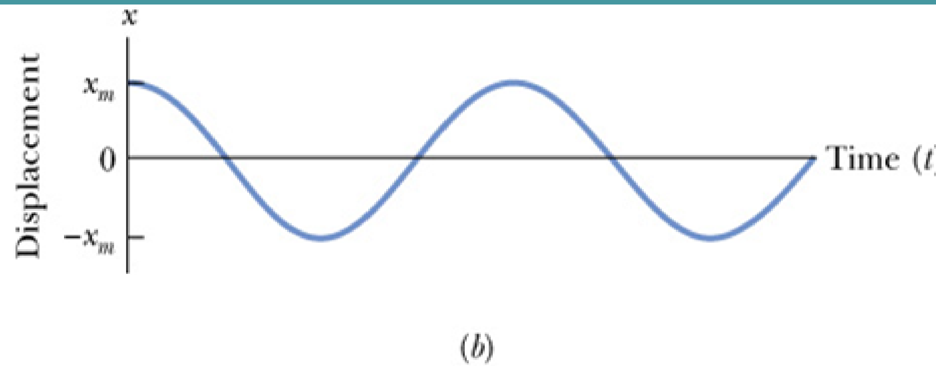
- ***Energy*** of a simple harmonic oscillator

- Examples of simple harmonic oscillators such as:

  - **spring - mass system**
  - **simple pendulum**
  - **physical pendulum**
  - **torsion pendulum**

SIMPLE HARMONIC MOTION (SHM)

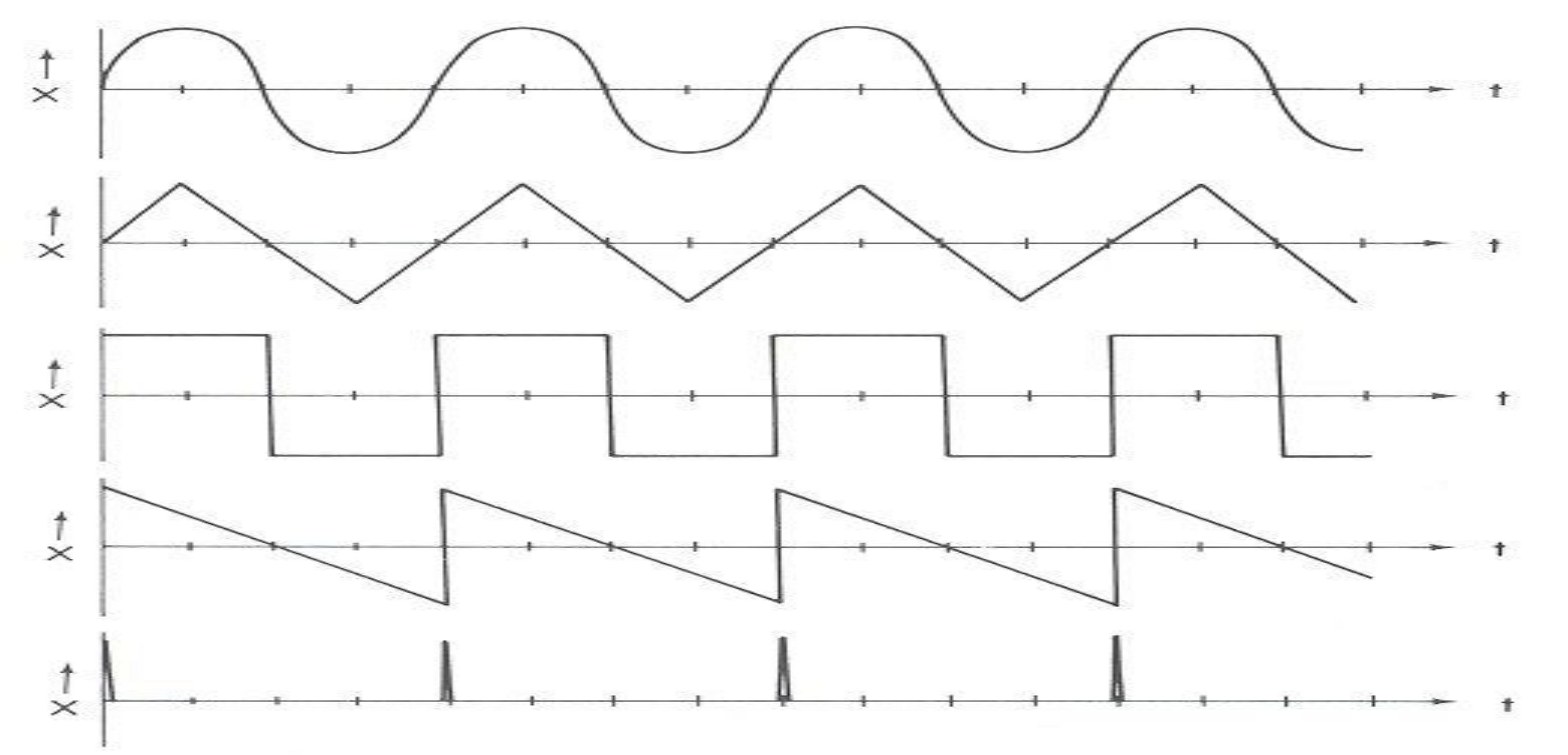


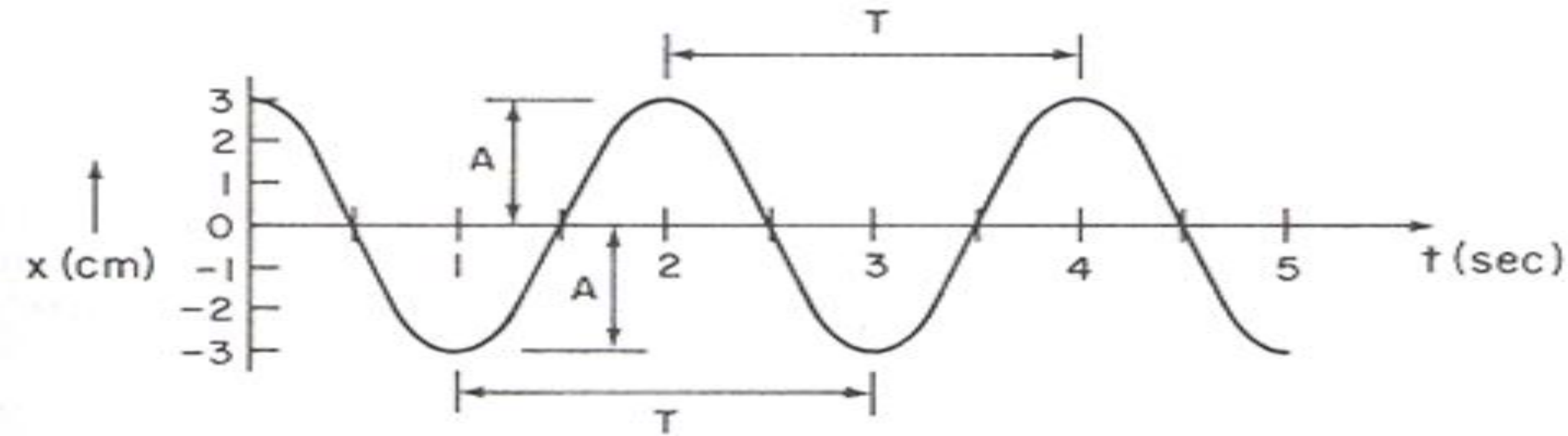


$$x(t) = x_m \cos(\omega t + \phi)$$

Figure shows a sequence of "*snapshots*" of a simple oscillating system, ***a particle moving repeatedly back and forth about the origin of an x-axis***. Any motion that repeats itself at regular intervals is called ***periodic motion*** or ***harmonic motion***.

The motion is periodic i.e. it repeats in time. The time needed to complete one repetition is known as the period (symbol T, units: s). The number of repetitions per unit time is called the frequency (symbol f , unit hertz) $\boxed{f = \frac{1}{T}}$





The displacement of the particle is given by the equation: $\boxed{x(t) = x_m \cos(\omega t + \phi)}$

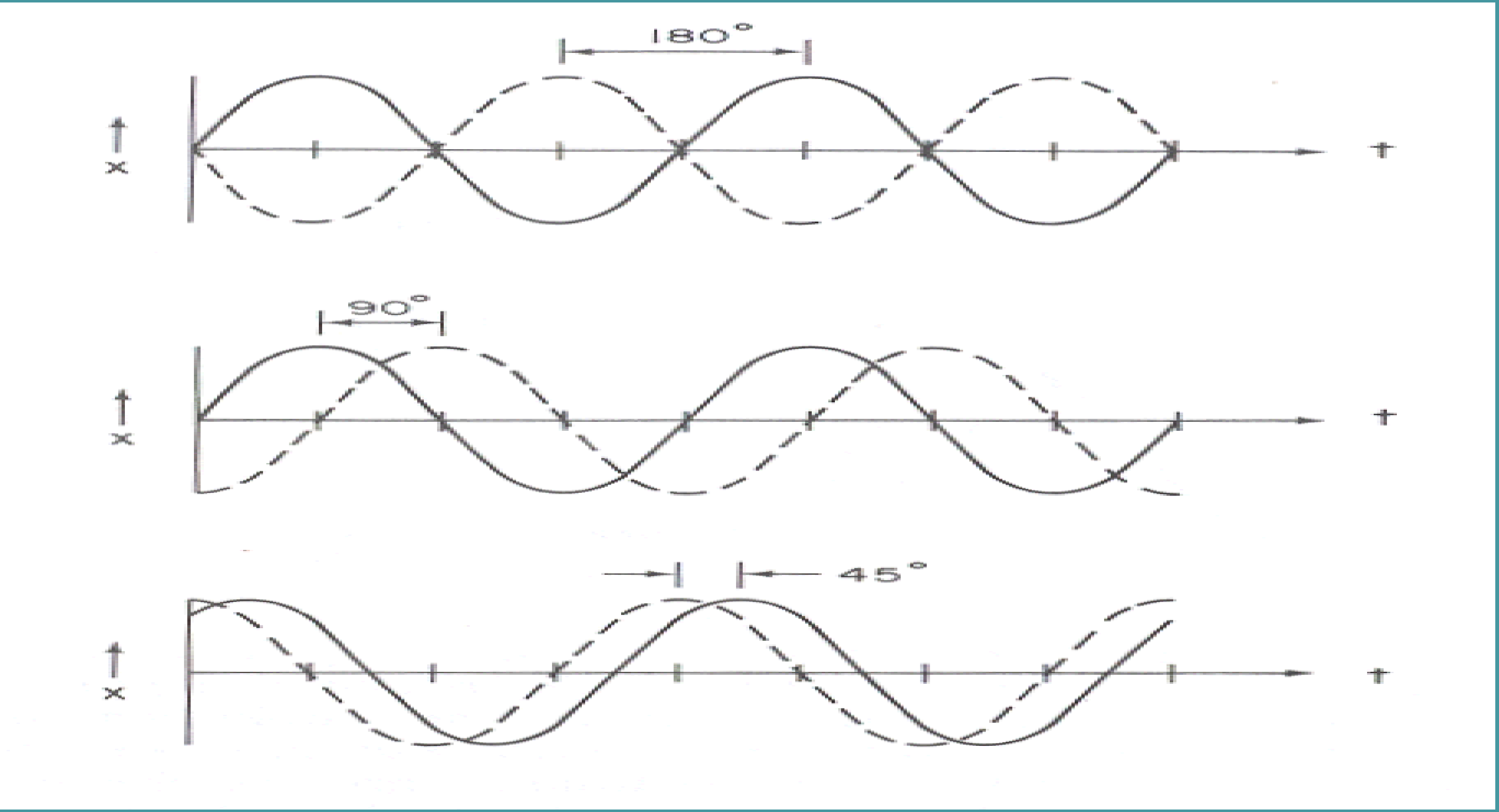
The quantity x_m is called the amplitude of the motion. It gives the maximum possible displacement of the oscillating object

The quantity ω is called the angular frequency of the oscillator. It is given by the equation:

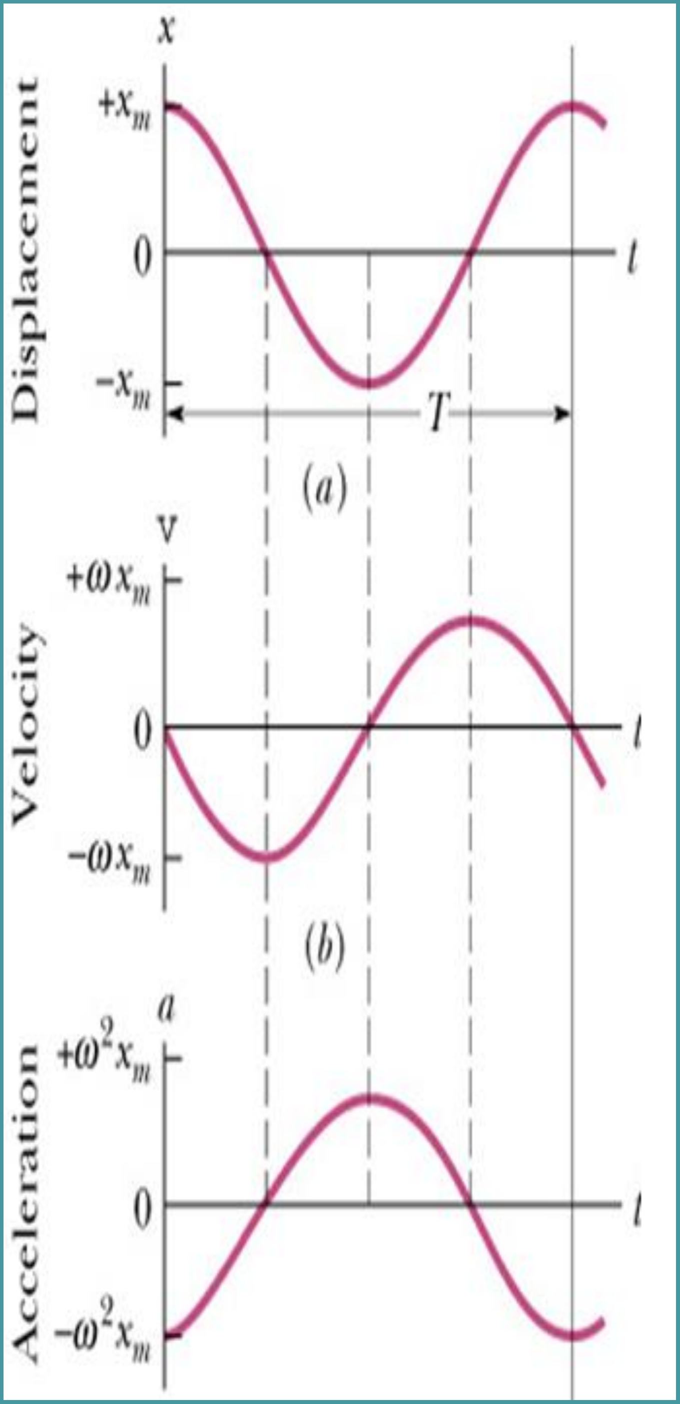
$$\boxed{x(t) = x_m \cos(\omega t + \phi)}$$

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

Phase Difference



$$x(t) = x_m \cos\left(\omega t + \phi\right)$$



$$\boxed{x(t) = x_m \cos(\omega t + \phi)}$$

The quantity ϕ is called the phase angle of the oscillator. The value of ϕ is determined from the displacement $x(0)$ and the velocity $v(0)$ at $t = 0$. In fig.a $x(t)$ is plotted versus t for $\phi = 0$. $x(t) = x_m \cos \omega t$

Velocity of SHM

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right] = -\omega x_m \sin(\omega t + \phi)$$

The quantity ωx_m is called the velocity amplitude v_m

It expresses the maximum possible value of $v(t)$

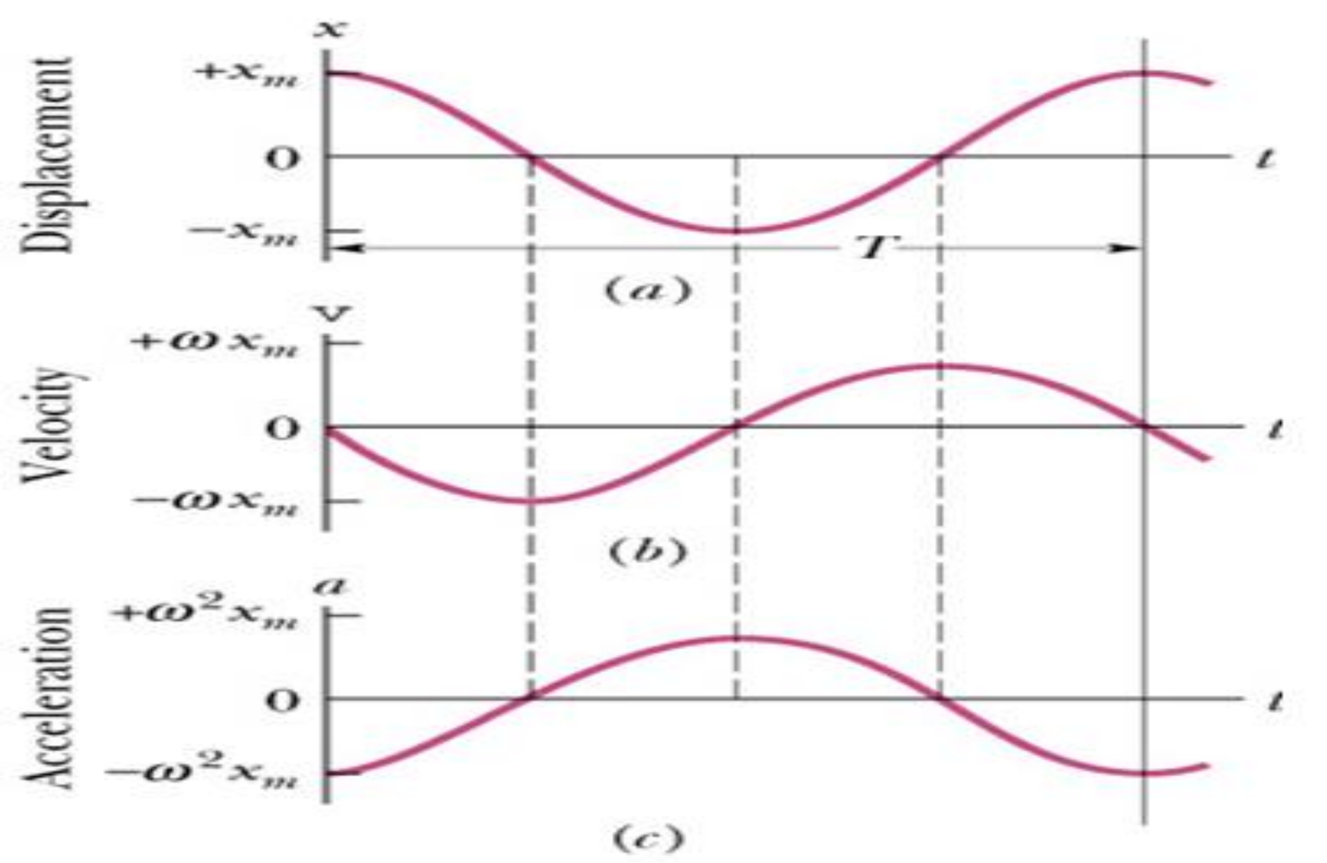
In fig.b the velocity $v(t)$ is plotted versus t for $\phi = 0$.

$$\boxed{v(t) = -\omega x_m \sin \omega t}$$

Acceleration of SHM: $\boxed{a(t)=\frac{dv(t)}{dt}}=\frac{d}{dt}\left[-\omega x_m \sin(\omega t+\phi)\right]=-\omega^2 x_m \cos\omega t=-\omega^2 x$

The quantity $\omega^2 x_m$ is called the acceleration amplitude a_m. It expresses the maximum possible value of a(t). In fig.c the acceleration a(t) is plotted versus t for $\phi=0$.

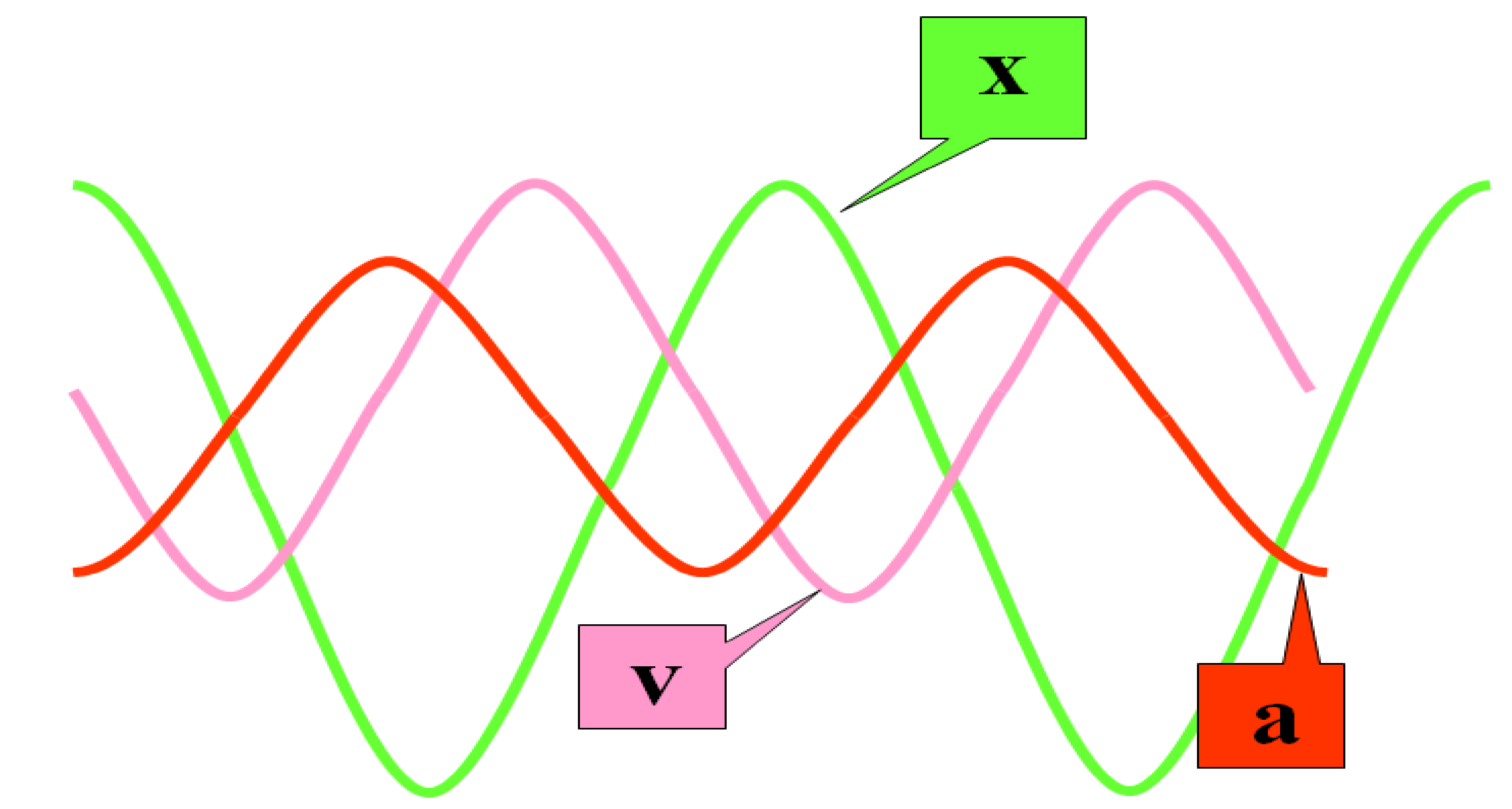
$$\boxed{a(t)=-\omega^2 x_m \cos\omega t}$$



$$x(t) = x_m \cos(\omega t + \phi)$$

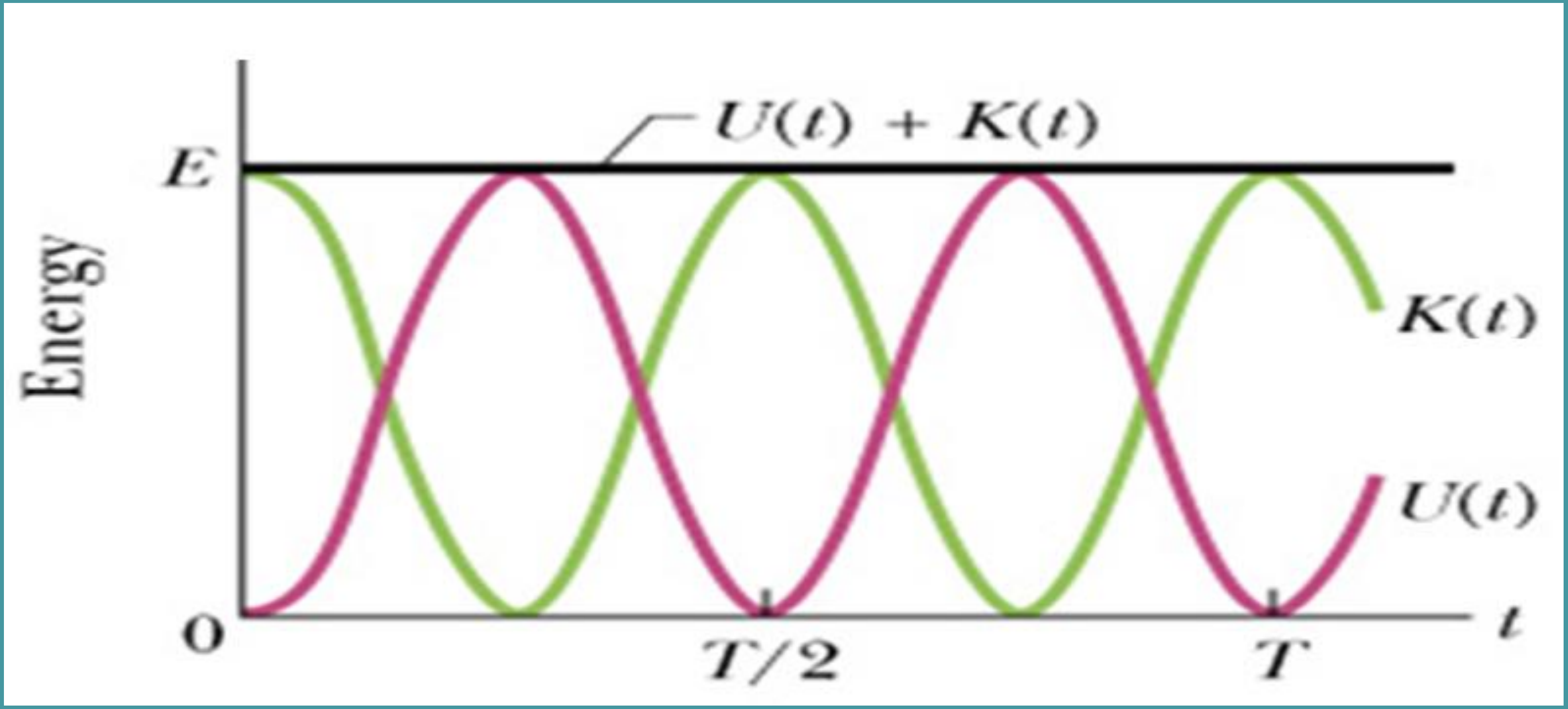
$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 x_m \cos \omega t$$



Energy in Simple Harmonic Motion

The mechanical energy E of a SHM is the sum of its potential and kinetic energies U and K.



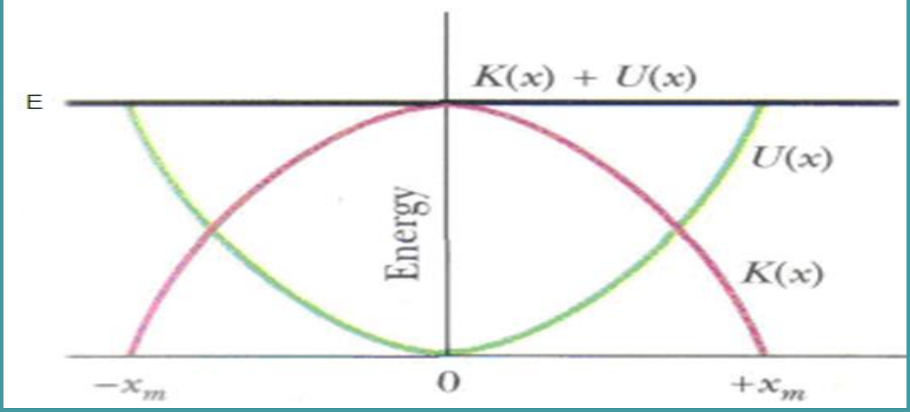
Energy in Simple Harmonic Motion

Potential energy $U=\frac{1}{2}kx^2=\frac{1}{2}kx_m^2\cos^2(\omega t+\phi)$

Kinetic energy $K=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2x_m^2\sin^2(\omega t+\phi)=\frac{1}{2}m\frac{k}{m}x_m^2\sin^2(\omega t+\phi)$

Mechanical energy $E=U+K=\frac{1}{2}kx_m^2\left[\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right]=\frac{1}{2}kx_m^2$

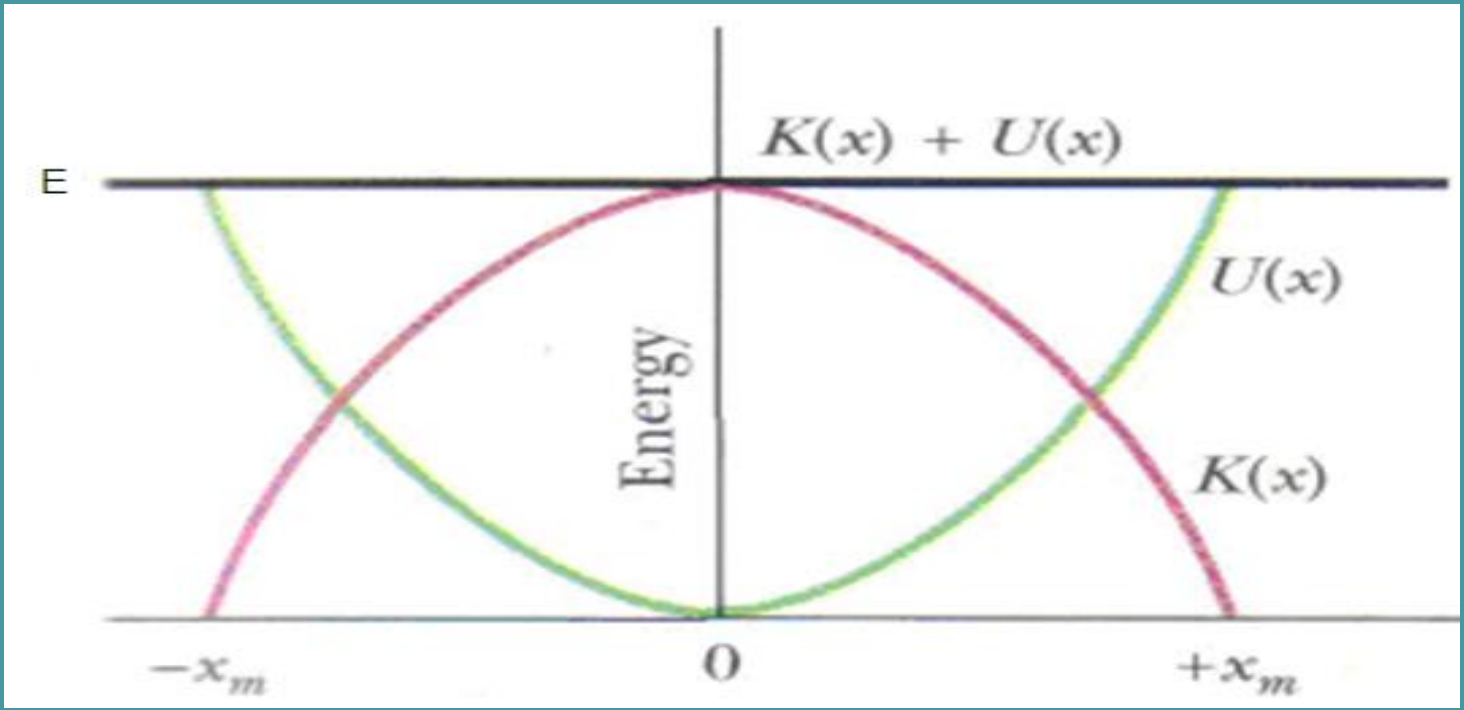
In the figure we plot the potential energy U (green line), the kinetic energy K (red line) and the mechanical energy E (black line) versus time t. While U and K vary with time, the energy E is a constant. The energy of the oscillating object transfers back and forth between potential and kinetic energy, while the sum of the two remains constant



$$\omega=\sqrt{\frac{k}{m}}$$

$$x(t)=x_m\cos(\omega t+\phi)$$

$$v(t)=-\omega x_m\sin(\omega t+\phi)$$



$$U = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

$$K = \frac{1}{2} k\, x_m^2 \sin^2(\omega t + \phi)$$

$$E = U + K = \frac{1}{2} k x_m^2 \left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2} k x_m^2$$

The Force Law for Simple Harmonic Motion

We saw that the acceleration of an object undergoing SHM is: $a = -\omega^2 x$

If we apply Newton's second law we get: $F = ma = -m\omega^2 x = -\left(m\omega^2\right)x$

Simple harmonic motion occurs when the force acting on an object is paroportional to the disaplacement but opposite in sign. The force can be written as: $\boxed{F = -Cx}$

where C is a constant. If we compare the two expressions for F we have:

$m\omega^2 = C \rightarrow$ and $\omega = \frac{2\pi}{T}$

$$\omega^2 = \frac{C}{m}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{C}}}$$

SHM potential energy, kinetic energy, mass dampers

Many tall buildings have *mass dampers*, which are anti-sway devices to prevent them from oscillating in a wind. The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say, eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

Suppose the block has mass $m = 2.72 \times 10^5$ kg and is designed to oscillate at frequency $f = 10.0$ Hz and with amplitude $x_m = 20.0$ cm.

(a) What is the total mechanical energy E of the spring–block system?

KEY IDEA

The mechanical energy E (the sum of the kinetic energy $K = \frac{1}{2}mv^2$ of the block and the potential energy $U = \frac{1}{2}kx^2$ of the spring) is constant throughout the motion of the oscillator. Thus, we can evaluate E at any point during the motion.

Calculations: Because we are given amplitude x_m of the oscillations, let's evaluate E when the block is at position $x = x_m$, where it has velocity $v = 0$. However, to evaluate U at that point, we first need to find the spring constant k. From Eq. 15-12 ($\omega = \sqrt{k/m}$) and Eq. 15-5 ($\omega = 2\pi f$), we find

$$\begin{aligned} k &= m\omega^2 = m(2\pi f)^2 \\ &= (2.72 \times 10^5 \text{ kg})(2\pi)^2(10.0 \text{ Hz})^2 \\ &= 1.073 \times 10^9 \text{ N/m}. \end{aligned}$$

We can now evaluate E as

$$\begin{aligned} E &= K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \\ &= 0 + \tfrac{1}{2}(1.073 \times 10^9 \text{ N/m})(0.20 \text{ m})^2 \\ &= 2.147 \times 10^7 \text{ J} \approx 2.1 \times 10^7 \text{ J}. \qquad \text{(Answer)} \end{aligned}$$

(b) What is the block's speed as it passes through the equilibrium point?

Calculations: We want the speed at $x = 0$, where the potential energy is $U = \frac{1}{2}kx^2 = 0$ and the mechanical energy is entirely kinetic energy. So, we can write

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

$$2.147 \times 10^7 \text{ J} = \tfrac{1}{2}(2.72 \times 10^5 \text{ kg})v^2 + 0,$$

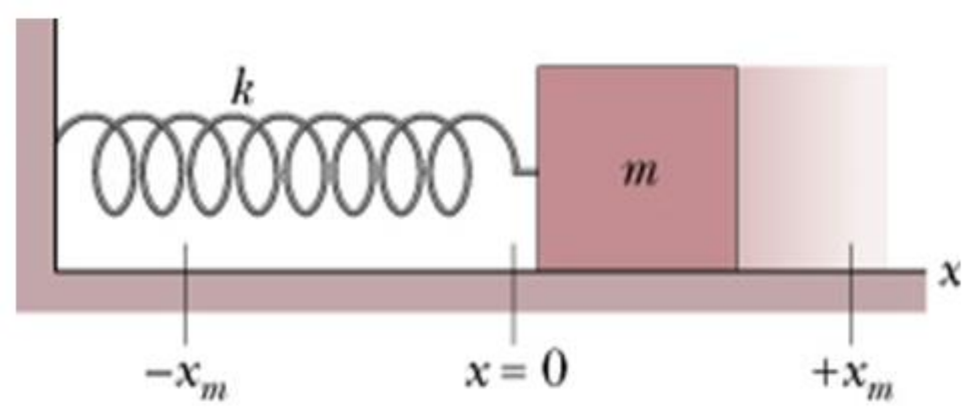
or

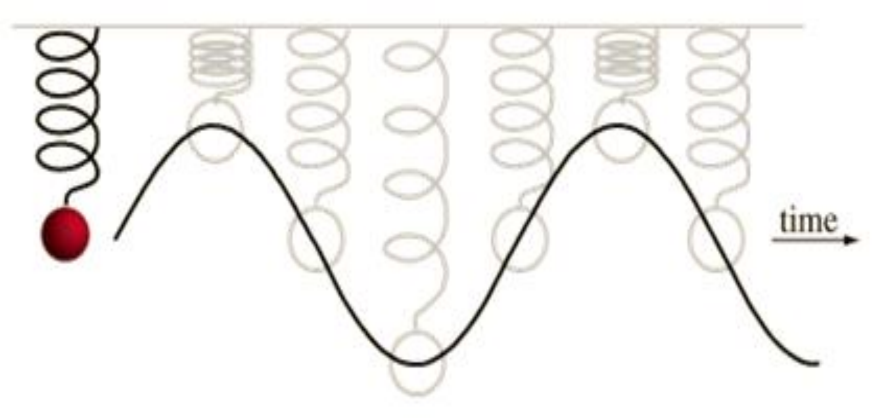
$$v = 12.6 \text{ m/s}. \qquad \text{(Answer)}$$

Because E is entirely kinetic energy, this is the maximum speed v_m.

Mass + Spring System

Consider the motion of a mass m attached to a spring of srping constant k than moves on a frictionless horizontal floor as shown in the figure.





Restoring Force

Simple harmonic motion is the motion executed by a particle of mass m subject to a force that is proportional to the displacement of the particle but opposite in sign.

Mass + Spring System

$$F = -\left(m\omega^2\right)x \qquad F = -Cx \qquad F = -kx.$$

The net force F on m is given by Hooke's law: $F = -kx$. If we compare this equation with the expression $F = -Cx$ we identify the constant C with the sping constant k.

We can then calculate the angular frequancy ω and the period T.

$$\omega = \sqrt{\frac{C}{m}} = \sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{C}} = 2\pi\sqrt{\frac{m}{k}}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \qquad \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Mass + Spring System

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \text{N/m}}{0.68\ \text{kg}}} = 9.78\ \text{rad/s}$$

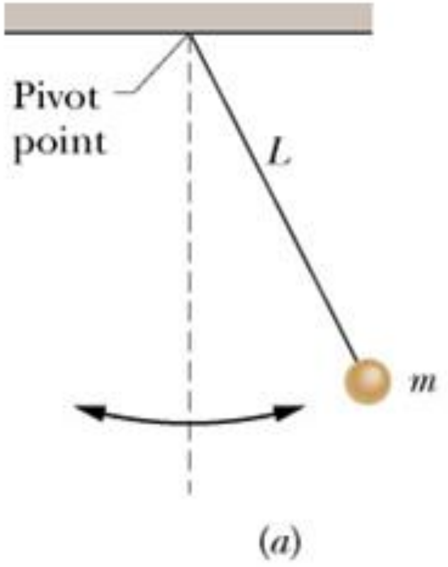
$$\approx 9.8\ \text{rad/s}. \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

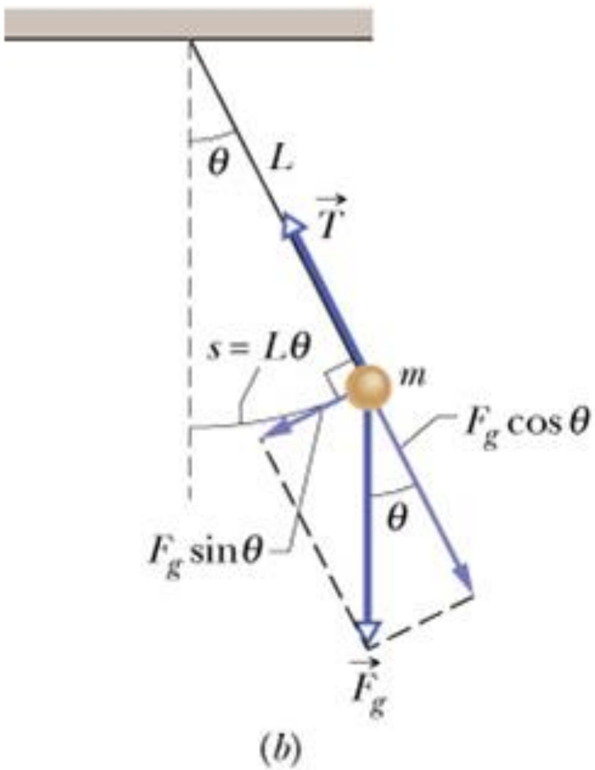
$$f = \frac{\omega}{2\pi} = \frac{9.78\ \text{rad/s}}{2\pi\ \text{rad}} = 1.56\ \text{Hz} \approx 1.6\ \text{Hz}. \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\ \text{Hz}} = 0.64\ \text{s} = 640\ \text{ms}. \qquad \text{(Answer)}$$



(a)



(b)

The Simple Pendulum

A simple pendulum consists of a particle of mass m suspended by a string of length L from a pivot point. If the mass is disturbed from its equilibrium position the net force acting on it is such that the system exectutes simple harmonic motion.

There are two forces acting on m: The gravitational force and the tension from the string. The net torque of these forces is:

$\boxed{\tau = -r_{\perp}F_g} = \boxed{-Lmg\sin\theta}$ Here θ is the angle that the thread makes with the vertical. If $\theta \ll 1$ (say less than 5 °) then we can make the following approximation: $\sin\theta \approx \theta$ where θ is expressed in radians. With this approximation the torque τ is:

$$\boxed{\tau \approx -(Lmg)\theta}$$

In the **small angle approximation** we assumed that **$\theta << 1$** and used the approximation: **$\sin\theta \cong \theta$.**

θ (degrees)	θ (radians)	$\sin\theta$
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259
20	0.349	0.342

Conclusion: If we keep $\theta < 10°$ we make less that 1 % error

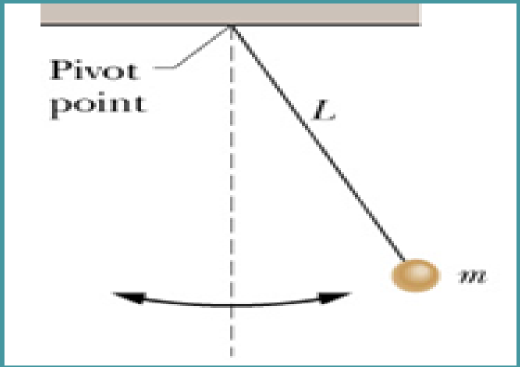
If we compare the expression for τ $\quad \tau \approx -(Lmg)\theta$

$F = -Cx \longrightarrow \tau = -C\theta \qquad \boxed{C = mgL}$

$\omega = \sqrt{\frac{C}{m}} \qquad T = 2\pi\sqrt{\frac{m}{C}} \longrightarrow \omega = \sqrt{\frac{C}{I}} \qquad T = 2\pi\sqrt{\frac{I}{C}}$

The rotational inertia I about the pivot point is equal to $\boxed{I = mL^2}$

Thus $\boxed{T = 2\pi\sqrt{\frac{I}{mgL}}} = 2\pi\sqrt{\frac{mL^2}{mgL}}$



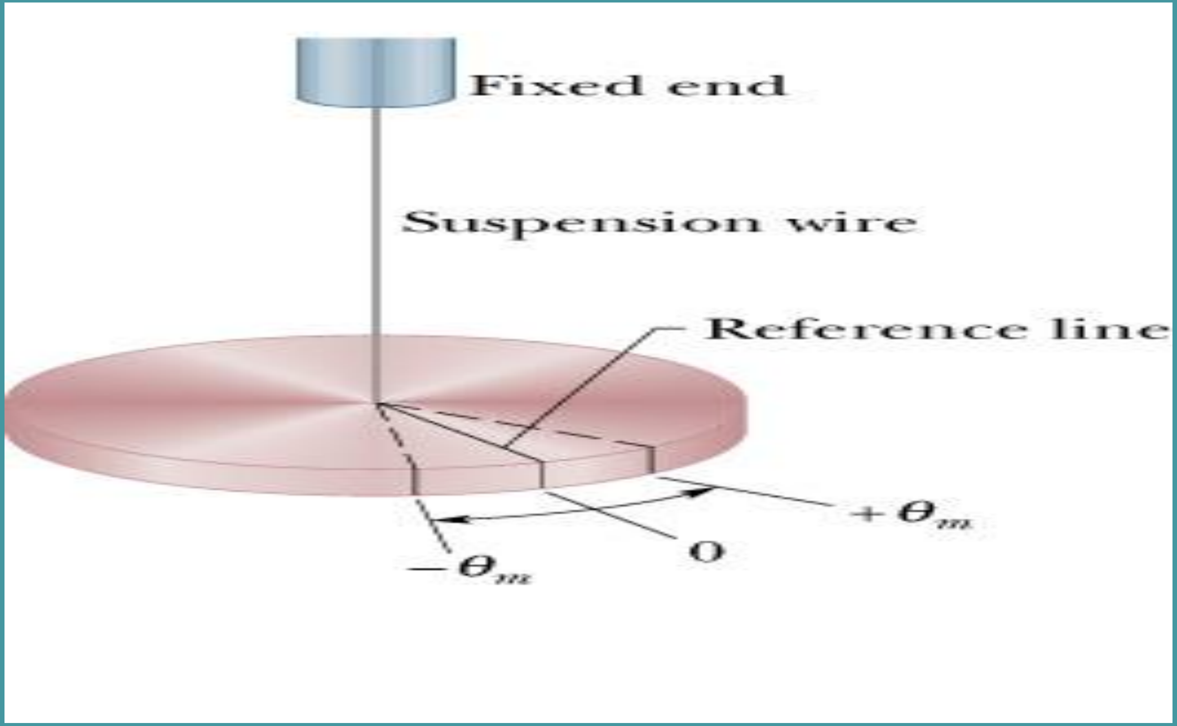
$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

(Simple Pendulum)

Torsion Pendulum

It consists of a disc of rotational inertia I suspended from a wire that twists as m rotates by an angle θ. The wire exerts on the disc a restoring torque $\boxed{\tau = -\kappa\theta}$

This is the angular form of Hooke's law. The constant κ is called the torsion constant of the wire.



$$\boxed{\tau = -\kappa\theta}$$

If we compare the expression $\tau = -\kappa\theta$ for the torque with the force equation $F = -Cx$ we realize that we identify the constant C with the torsion constant κ. We can thus readily determine the angular frequency ω and the period T of the oscillation.

$$\omega = \sqrt{\frac{C}{I}} = \sqrt{\frac{\kappa}{I}} \qquad T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{I}{\kappa}}$$

We note that I is the rotational inertia of the disc about an axis that coincides with the wire. The angle θ is given by the equation:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$C = \kappa$$

$$\boxed{\theta(t) = \theta_m \cos(\omega t + \phi)}$$

Figure 15-8a shows a thin rod whose length L is 12.4 cm and whose mass m is 135 g, suspended at its midpoint from a long wire. Its period T_a of angular SHM is measured to be 2.53 s. An irregularly shaped object, which we call object X, is then hung from the same wire, as in Fig. 15-8b, and its period T_b is found to be 4.76 s. What is the rotational inertia of object X about its suspension axis?

KEY IDEA

The rotational inertia of either the rod or object X is related to the measured period by Eq. 15-23.

Calculations: In Table 10-2e, the rotational inertia of a thin rod about a perpendicular axis through its midpoint is given as $\frac{1}{12}mL^2$. Thus, we have, for the rod in Fig. 15-8a,

$$I_a = \tfrac{1}{12}mL^2 = (\tfrac{1}{12})(0.135\text{ kg})(0.124\text{ m})^2$$
$$= 1.73 \times 10^{-4}\text{ kg}\cdot\text{m}^2.$$

Now let us write Eq. 15-23 twice, once for the rod and once for object X:

$$T_a = 2\pi\sqrt{\frac{I_a}{\kappa}} \quad\text{and}\quad T_b = 2\pi\sqrt{\frac{I_b}{\kappa}}.$$

The constant κ, which is a property of the wire, is the same for both figures; only the periods and the rotational inertias differ.

Let us square each of these equations, divide the second by the first, and solve the resulting equation for I_b. The result is

$$I_b = I_a\frac{T_b^2}{T_a^2} = (1.73 \times 10^{-4}\text{ kg}\cdot\text{m}^2)\frac{(4.76\text{ s})^2}{(2.53\text{ s})^2}$$
$$= 6.12 \times 10^{-4}\text{ kg}\cdot\text{m}^2. \qquad \text{(Answer)}$$

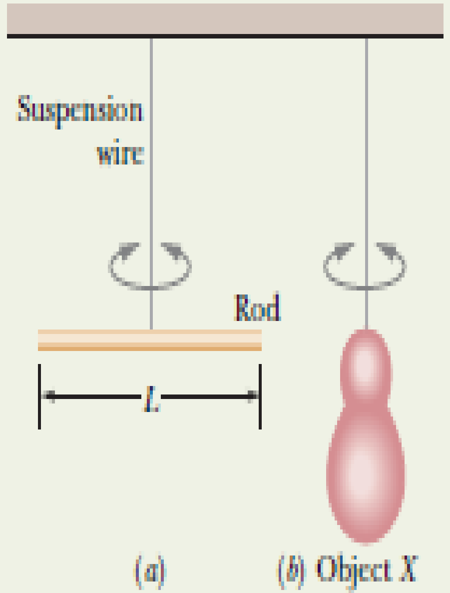
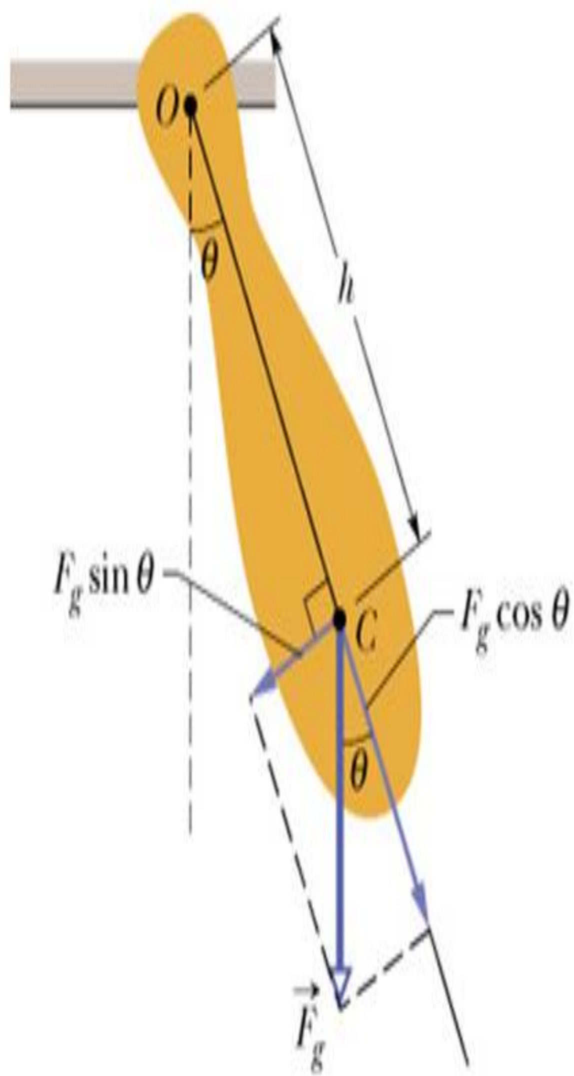


Fig. 15-8 Two torsion pendulums, consisting of (a) a wire and a rod and (b) the same wire and an irregularly shaped object.

Physical Pendulum

A physical pendulum is an extened rigid body that is suspended from a fixed point O and oscillates under the influence of gravity



The net torque $\boxed{\tau = -mgh\sin\theta}$ Here h is the distance between point O and the center of mass C of the suspended body.

If we make the small angle approximation $\theta \ll 1$, we have:

$\boxed{\tau \approx -(mgh)\theta.}$ If we compare the torques with the force equation $F = -Cx$ we realize that we identify the constant C with the term hmg. We can thus readily determine period T of the oscillation.

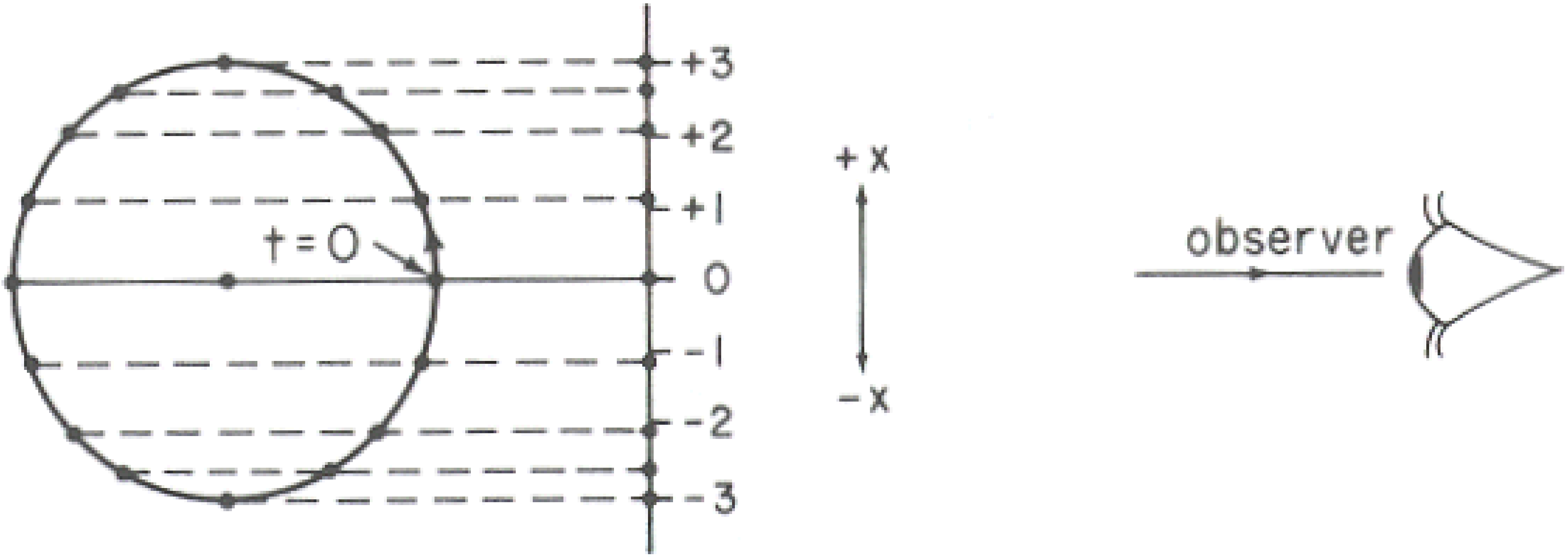
$$\boxed{C = mgh}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{I}{mgh}}}$$

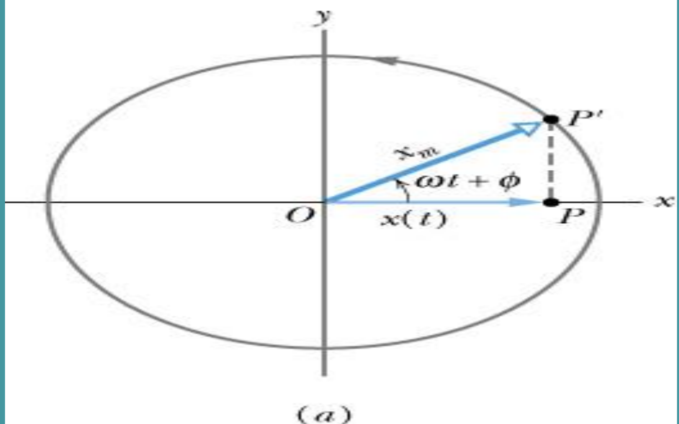
Here I is the rotational inertia about an axis through O. $I = I_{com} + mh^2$

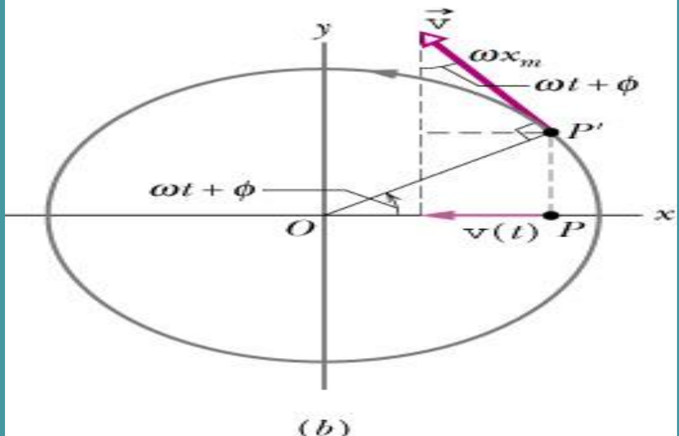
Simple Harmonic Motion and Uniform Circular Motion

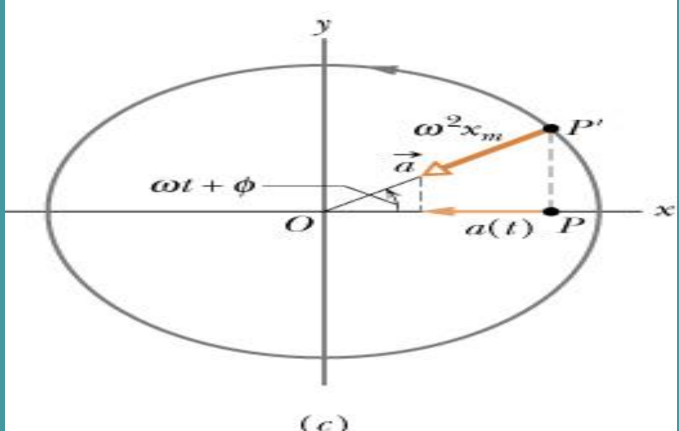
Simple Harmonic Motion is the projection of Uniform Circular Motion



Conclusion: **Whether we look at the displacement ,the velocity, or the acceleration , the projection of uniform circular motion on the x-axis diameter is SHM**







Simple Harmonic Motion and Uniform Circular Motion

Consider an object moving on a circular path of radius x_m with a uniform speed v. If we project the position of the moving particle at point P′ on the x-axis we get point P.

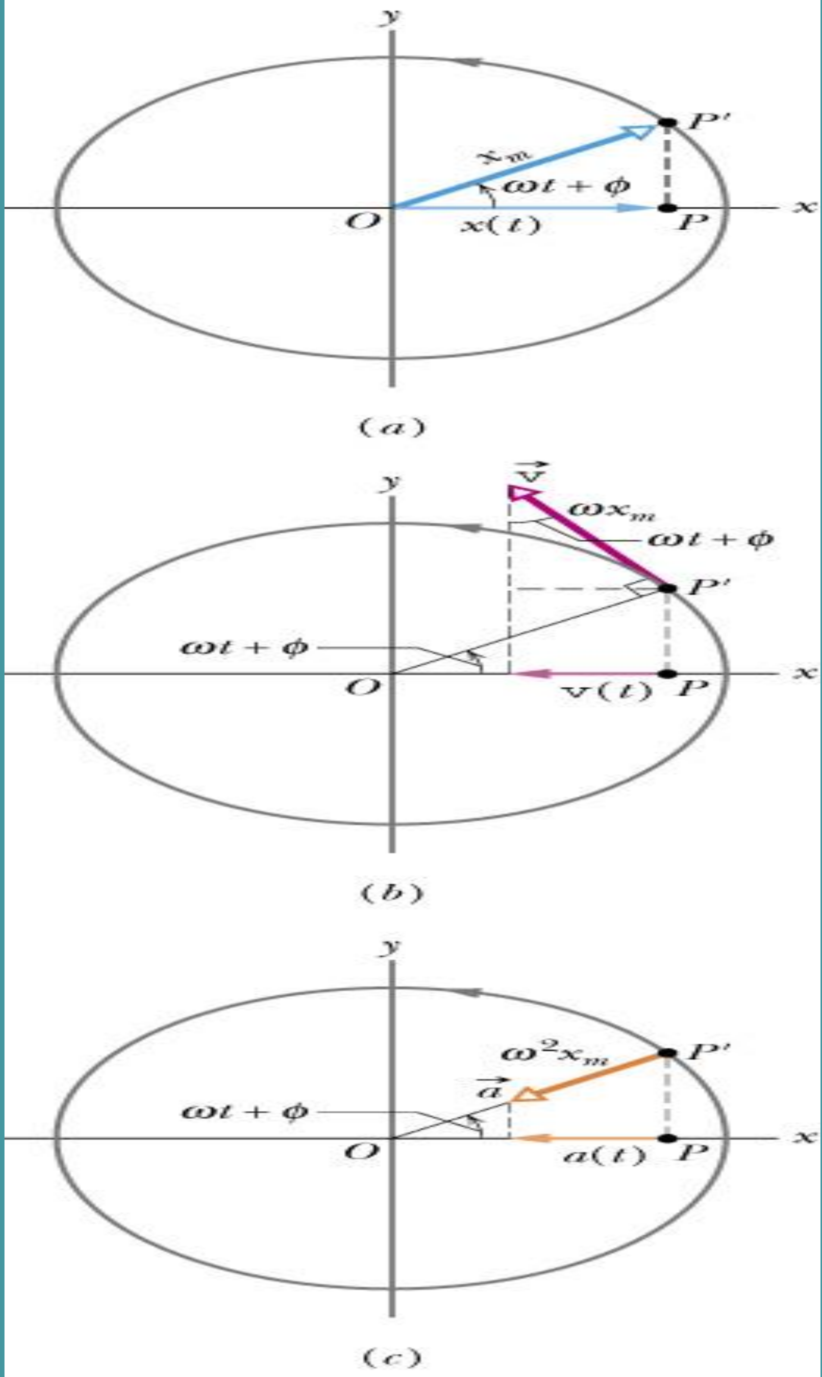
The coordinate of P is: $\boxed{x(t) = x_m \cos(\omega t + \phi)}$.

While point P′ executes unifrom circular motion its projection P moves along the x-axis with simple harmonic motion.

The speed v of point P′ is equal to ωx_m. The direction of the velocity vector is along the tangent to the circular path. If we project the velocity $\vec{v}$ on the x-axis we get: $\boxed{v(t) = -\omega x_m \sin(\omega t + \phi)}$

The acceleration $\vec{a}$ points along the center O. If we project $\vec{a}$ along the x-axis we get: $\boxed{a(t) = -\omega^2 x_m \cos(\omega t + \phi)}$

Simple Harmonic Motion is the <u>projection</u> of Uniform Circular Motion



$$x(t) = x_m \cos(\omega t + \phi)$$

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

3. In simple harmonic motion, the magnitude of the acceleration is:

A. constant
B. proportional to the displacement
C. inversely proportional to the displacement
D. greatest when the velocity is greatest
E. never greater than g

ans: B

40. A block attached to a spring undergoes simple harmonic motion on a horizontal frictionless surface. Its total energy is 50 J. When the displacement is half the amplitude, the kinetic energy is:

A. zero
B. 12.5 J
C. 25 J
D. 37.5 J
E. 50 J

ans: D

49. Three physical pendulums, with masses m_1, $m_2 = 2m_1$, and $m_3 = 3m_1$, have the same shape and size and are suspended at the same point. Rank them according to their periods, from shortest to longest.

A. 1, 2, 3
B. 3, 2, 1
C. 2, 3, 1
D. 2, 1, 3
E. All the same

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgh}}}$$

ans: E