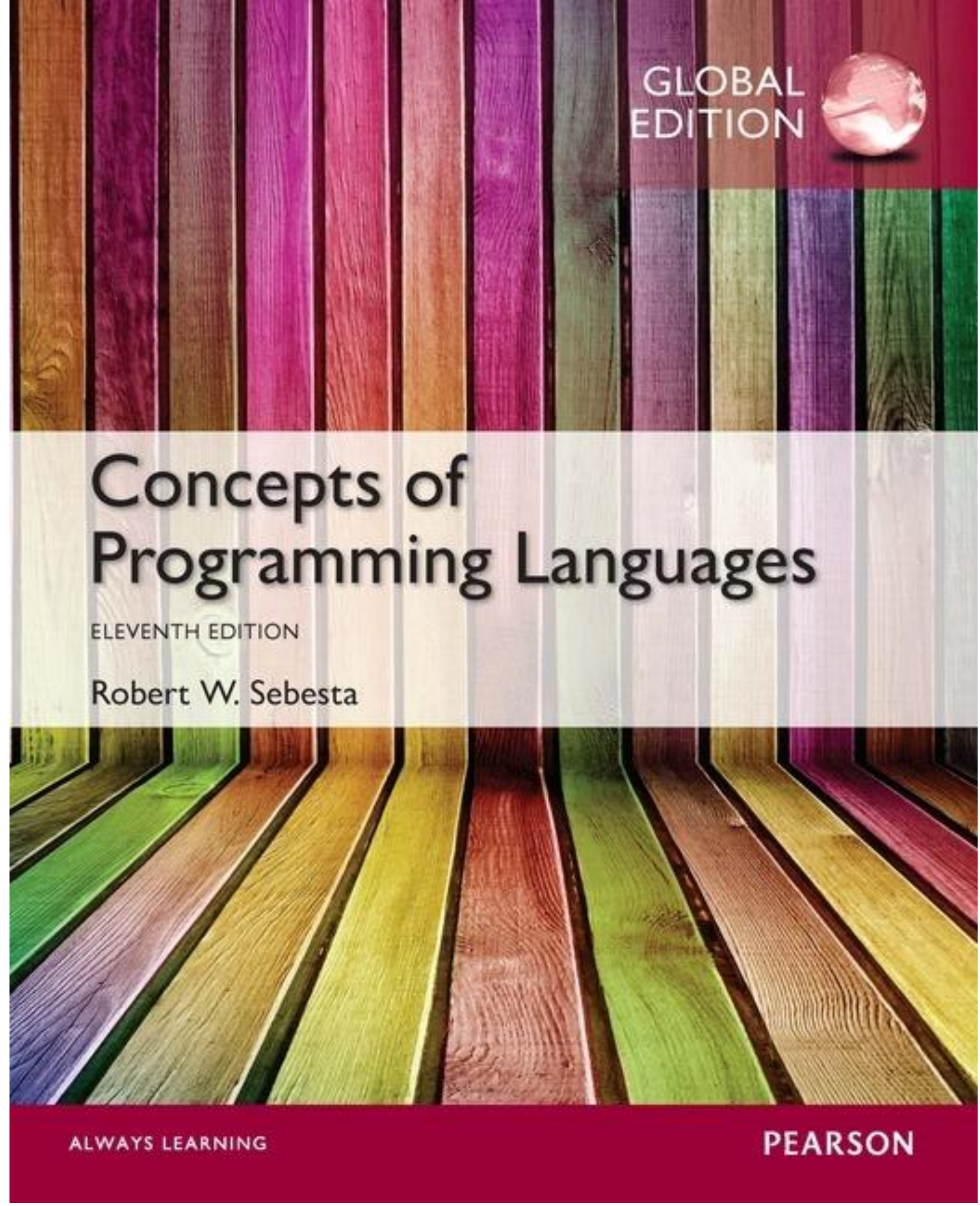
Chapter 16

Logic Programming Languages



Chapter 16 Topics

- Introduction
- A Brief Introduction to Predicate Calculus
- Predicate Calculus and Proving Theorems
- An Overview of Logic Programming
- The Origins of Prolog
- The Basic Elements of Prolog
- Deficiencies of Prolog
- Applications of Logic Programming

Introduction

- Programs in logic languages are expressed in a form of symbolic logic
- Use a logical inferencing process to produce results
- *Declarative* rather that *procedural*:
  - Only specification of *results* are stated (not detailed *procedures* for producing them)

Proposition

- A logical statement that may or may not be true
  - Consists of objects and relationships of objects to each other

Symbolic Logic

- Logic which can be used for the basic needs of formal logic:
  - Express propositions
  - Express relationships between propositions
  - Describe how new propositions can be inferred from other propositions
- Particular form of symbolic logic used for logic programming called *predicate calculus*

Object Representation

- Objects in propositions are represented by simple terms: either constants or variables
- *Constant*: a symbol that represents an object
- *Variable*: a symbol that can represent different objects at different times
  - Different from variables in imperative languages

Compound Terms

- *Atomic propositions* consist of compound terms
- *Compound term*: one element of a mathematical relation, written like a mathematical function
  - Mathematical function is a mapping
  - Can be written as a table

Parts of a Compound Term

- Compound term composed of two parts
  - Functor: function symbol that names the relationship
  - Ordered list of parameters (tuple)
- Examples:

```
student(jon)
like(seth, OSX)
like(nick, windows)
like(jim, linux)
```

Forms of a Proposition

- Propositions can be stated in two forms:
  - *Fact*: proposition is assumed to be true
  - *Query*: truth of proposition is to be determined
- Compound proposition:
  - Have two or more atomic propositions
  - Propositions are connected by operators

Logical Operators

Name	Symbol	Example	Meaning
negation	$\neg$	$\neg$ a	not a
conjunction	$\cap$	a $\cap$ b	a and b
disjunction	$\cup$	a $\cup$ b	a or b
equivalence	$\equiv$	a $\equiv$ b	a is equivalent to b
implication	$\supset$ $\subset$	a $\supset$ b a $\subset$ b	a implies b b implies a

Quantifiers

Name	Example	Meaning
universal	$\forall X.P$	For all X, P is true
existential	$\exists X.P$	There exists a value of X such that P is true

Clausal Form

- Too many ways to state the same thing
- Use a standard form for propositions
- *Clausal form*:
  - $B_1 \cup B_2 \cup \ldots \cup B_n \subset A_1 \cap A_2 \cap \ldots \cap A_m$
  - means if all the As are true, then at least one B is true
- *Antecedent*: right side
- *Consequent*: left side

- All predicate calculus propositions can be algorithmically converted to clausal form.

Example Clausal Forms

$$\text{likes(bob, trout)} \subset \text{likes(bob, fish)} \cap \text{fish(trout)}$$

if bob likes fish and a trout is a fish, then bob likes trout.

$$\text{father(louis, al)} \cup \text{father(louis, violet)} \subset$$
$$\text{father(al, bob)} \cap \text{mother(violet, bob)} \cap \text{grandfather(louis, bob)}$$

if al is bob's father and violet is bob's mother and
louis is bob's grandfather, then
louis is either al's father or violet's father

Predicate Calculus and Proving Theorems

- A use of propositions is to discover new theorems that can be inferred from known axioms and theorems
- *Resolution*: an inference principle that allows inferred propositions to be computed from given propositions

Concept of Resolution

Suppose there are two propositions of the form:

$$P_1 \subset P_2$$
$$Q_1 \subset Q_2$$

Suppose P1 is identical to Q2

$$T \subset P_2$$
$$Q_1 \subset T$$

We can write

$$Q_1 \subset P_2$$

Concept of Resolution

$$\text{older(joanne, jake)} \subset \text{mother(joanne, jake)}$$
$$\text{wiser(joanne, jake)} \subset \text{older(joanne, jake)}$$

$$\text{wiser(joanne, jake)} \subset \text{mother(joanne, jake)}$$

The mechanics:

- Terms of the left sides are OR'd
- Terms of the right sides are AND'd.
- Any term that appears on both sides is removed.

Concept of Resolution

$$\text{father(bob, jake)} \cup \text{mother(bob, jake)} \subset \text{parent(bob, jake)}$$
$$\text{grandfather(bob, fred)} \subset \text{father(bob, jake)} \cap \text{father(jake, fred)}$$

$$\text{mother(bob, jake)} \cup \text{grandfather(bob, fred)} \subset$$
$$\text{parent(bob, jake)} \cap \text{father(jake, fred)}$$

The mechanics:

- Terms of the left sides are OR'd
- Terms of the right sides are AND'd.
- Any term that appears on both sides is removed.

Resolution

- *Unification*: finding values for variables in propositions that allows matching process to succeed
- *Instantiation*: assigning temporary values to variables to allow unification to succeed
- After instantiating a variable with a value, if matching fails, may need to *backtrack* and instantiate with a different value

Proof by Contradiction

- *Hypotheses*: a set of pertinent propositions
- *Goal*: negation of theorem stated as a proposition
- Theorem is proved by finding an inconsistency

Theorem Proving

- Basis for logic programming
- When propositions used for resolution, only restricted form can be used
- *Horn clause* – can have only two forms
  - *Headed*: single atomic proposition on left side
  - *Headless*: empty left side (used to state facts)
- Most propositions can be stated as Horn clauses

Overview of Logic Programming

- Declarative semantics
  - There is a simple way to determine the meaning of each statement
  - Simpler than the semantics of imperative languages
- Programming is nonprocedural
  - Programs do not state now a result is to be computed, but rather the form of the result
  - *Programming in both imperative and functional languages is procedural*, which means that the programmer instructs the computer on exactly how the computation is to be done.

Example: Sorting a List

- Describe the characteristics of a sorted list, not the process of rearranging a list

  sort(old_list, new_list) $\subset$ permute (old_list, new_list) $\cap$ sorted (new_list)

  sorted (list) $\subset \forall_j$ such that $1 \leq j < n$, list(j) $\leq$ list (j+1)

The Origins of Prolog

- University of Aix-Marseille (Calmerauer & Roussel)
  - Natural language processing
- University of Edinburgh (Kowalski)
  - Automated theorem proving

Terms

- This book uses the Edinburgh syntax of Prolog
- *Term*: a constant, variable, or structure
- *Constant*: an atom or an integer
- *Atom*: symbolic value of Prolog
- Atom consists of either:
  - a string of letters, digits, and underscores beginning with a lowercase letter
  - a string of printable ASCII characters delimited by apostrophes

Terms: Variables and Structures

- *Variable*: any string of letters, digits, and underscores beginning with an uppercase letter
- *Instantiation*: binding of a variable to a value
  - Lasts only as long as it takes to satisfy one complete goal
- *Structure*: represents atomic proposition

  functor (*parameter list*)

Fact Statements

- Used for the hypotheses
- Headless Horn clauses

```
female(shelley).
male(bill).
father(bill, jake).
father(bill, shelley).
mother(mary, jake).
mother(mary, shelley).
```

Notice that every Prolog statement is terminated by a period.

Rule Statements

- Used for the hypotheses
- Headed Horn clause
- Right side: *antecedent* (*if* part)
  - May be single term or conjunction
- Left side: *consequent* (***then*** part)
  - Must be single term
- *Conjunction*: multiple terms separated by logical AND operations (implied)

```
female(shelley), child(shelley).
```

Rule Statements

- The general form of the Prolog headed Horn clause statement is

  consequence :- antecedent_expression.

"consequence can be concluded if the antecedent expression is true or can be made to be true by some instantiation of its variables."

Example Rules

```
ancestor(mary,shelley):- mother(mary,shelley).
```

- Can use variables (*universal objects*) to generalize meaning:

```
parent(X,Y):- mother(X,Y).
parent(X,Y):- father(X,Y).
grandparent(X,Z):- parent(X,Y), parent(Y,Z).
```

Here X, Y, Z are variables (they start with uppercase letters)

Goal Statements

- For theorem proving, theorem is in form of proposition that we want system to prove or disprove – *goal statement*
- Same format as headless Horn

```
man(fred)
```

- Conjunctive propositions and propositions with variables also legal goals

```
father(X, mike)
```

Inferencing Process of Prolog

- Queries are called goals
- If a goal is a compound proposition, each of the facts is a subgoal
- To prove a goal is true, must find a chain of inference rules and/or facts. For goal Q:

```
P2 :- P1
P3 :- P2
...
Q :- Pn
```

- Process of proving a subgoal called matching, satisfying, or resolution

Approaches

- *Matching* is the process of proving a proposition
- Proving a subgoal is called *satisfying* the subgoal
- *Bottom-up resolution, forward chaining*
  - Begin with facts and rules of database and attempt to find sequence that leads to goal
  - Works well with a large set of possibly correct answers
- *Top-down resolution, backward chaining*
  - Begin with goal and attempt to find sequence that leads to set of facts in database
  - Works well with a small set of possibly correct answers
- Prolog implementations use backward chaining

Subgoal Strategies

- When goal has more than one subgoal, can use either
  - Depth-first search: find a complete proof for the first subgoal before working on others
  - Breadth-first search: work on all subgoals in parallel
- Prolog uses depth-first search
  - Can be done with fewer computer resources

Backtracking

- With a goal with multiple subgoals, if fail to show truth of one of subgoals, reconsider previous subgoal to find an alternative solution: *backtracking*
- Begin search where previous search left off
- Can take lots of time and space because may find all possible proofs to every subgoal

Simple Arithmetic

- Prolog supports integer variables and integer arithmetic
- **`is`** operator: takes an arithmetic expression as right operand and variable as left operand

```
A is B / 17 + C
```

- Not the same as an assignment statement!
  - The following is illegal (Left side variable cannot be previously instantiated)

```
Sum is Sum + Number.
```

Example

```
speed(ford,100).
speed(chevy,105).
speed(dodge,95).
speed(volvo,80).
time(ford,20).
time(chevy,21).
time(dodge,24).
time(volvo,24).
distance(X,Y) :-   speed(X,Speed),
                     time(X,Time),
                     Y is Speed * Time.
```

A query: `distance(chevy, Chevy_Distance).`

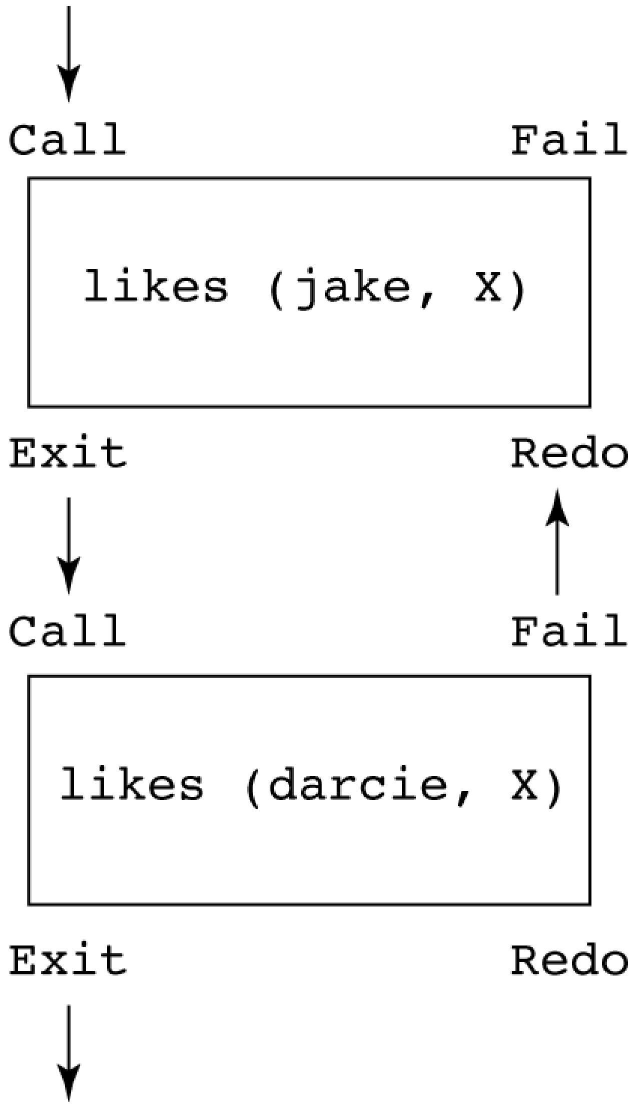
Trace

- Built-in structure that displays instantiations at each step
- *Tracing model* of execution – four events:
  - *Call* (beginning of attempt to satisfy goal)
  - *Exit* (when a goal has been satisfied)
  - *Redo* (when backtrack occurs)
  - *Fail* (when goal fails)

Example

```
likes(jake,chocolate).
likes(jake, apricots).
likes(darcie, licorice).
likes(darcie, apricots).

trace.
likes(jake, X), likes(darcie, X).
(1)  1 Call: likes(jake, _0)?
(1) 1 Exit: likes(jake, chocolate)
(2) 1 Call: likes(darcie, chocolate)?
(2) 1 Fail: likes(darcie, chocolate)
(1)  1 Redo: likes(jake, _0)?
(1) 1 Exit: likes(jake, apricots)
(3) 1 Call: likes(darcie, apricots)?
(3) 1 Exit: likes(darcie, apricots)
X = apricots
```



List Structures

- Other basic data structure (besides atomic propositions we have already seen): list
- *List* is a sequence of any number of elements
- Elements can be atoms, atomic propositions, or other terms (including other lists)

```
[apple, prune, grape, kumquat]
[]              (empty list)
[X | Y]         (head X and tail Y)
```

Append Example

```
append([], List, List).
append([Head | List_1], List_2, [Head | List_3]) :-
               append (List_1, List_2, List_3).
```

More Examples

```
reverse([], []).
reverse([Head | Tail], List) :-
  reverse (Tail, Result),
          append (Result, [Head], List).


member(Element, [Element | _]).
member(Element, [_ | List]) :-
                       member(Element, List).
```

The underscore character means an anonymous variable—it means we do not care what instantiation it might get from unification

Deficiencies of Prolog

- Resolution order control
  - In a pure logic programming environment, the order of attempted matches is nondeterministic and all matches would be attempted concurrently
- The closed-world assumption
  - The only knowledge is what is in the database
- The negation problem
  - Anything not stated in the database is assumed to be false
- Intrinsic limitations
  - It is easy to state a sort process in logic, but difficult to actually do—it doesn’t know how to sort

Applications of Logic Programming

- Relational database management systems
- Expert systems
- Natural language processing

Summary

- Symbolic logic provides basis for logic programming
- Logic programs should be nonprocedural
- Prolog statements are facts, rules, or goals
- Resolution is the primary activity of a Prolog interpreter
- Although there are a number of drawbacks with the current state of logic programming it has been used in a number of areas