HACETTEPE UNIVERSITY

	Mathematics I Midterm Exam Instructor: M. Türkyılmazoğlu	
Code : *Math 123* Acad.Year : *2016-2017* Semester *Fall* Date : *14.12.2016* Time : *9:00 - 11:00* Duration : *110 minutes*	Last Name : Name : Student No : Section : Signature : **4** Questions on **4** Pages Total **50** Points	

1	2	3	4	5	

1. Evaluate the limit if exists.

(a) (6 pts) $\lim_{x \to 1} \ln(2 - [|x|])$

(b) (6 pts) Determine α and β such that $\lim_{x \to \infty} \ln\left(\frac{(1+x+x^2)^{\alpha}}{(x^3+4x)^{\beta}}\right) = 0$

2. Evaluate y' and y'' from the following equations.

(a) (6 pts) $y = x^2 \tan(1/x)$

(b) (6 pts) $xy^2 + \sin((x-1)y) = 4$

3. (a) (4 pts) Show that the equation $x + \cos x - 3x^2 = 0$ has at least a real root.

(b) (8 pts) Sketch the graph of $f(x) = \frac{3x^2}{x^2+3}$. Comment on the invertibility of f.

4. (a) (7 pts) Evaluate the integral $\int_1^3 x^{-[\![x]\!]} dx$.

(b) (7 pts) Evaluate the integral $\int_0^{-1+e} \ln^2(1+x) dx$.

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18-22 September 2016, Ohrid, Republic of Macedonia
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2015-2016 MATH 123 30.11.2015 (110 minutes) Fall (Computer Engineering)
(Prof. Dr. Mustafa TÜRKYILMAZOĞLU)

1) Evaluate the limit if exists.

(a) (6 pts) $\lim_{x\to 1} e^{1-[|x|]}$ $\lim_{x\to 1^+} e^{1-1} = 1$ $\lim_{x\to 1^-} e^{1-0} = e$ Limit does not exist.

(b) (6pts) $\lim_{x\to\infty} \frac{\cos x}{1+x} = 0$ $-1 \le \cos x \le 1$ by sandwich theorem.

$$\frac{-1}{1+x} \le \frac{\cos x}{1+x} \le \frac{1}{1+x}$$

($1+x>0$ since $x\to\infty$)

2) Evaluate y' and y'' from the following equations.

(a) (6pts) $y = x^{5/3} - 5x^{2/3}$ $y' = \frac{5}{3}x^{2/3} - 5\cdot\frac{2}{3}\cdot x^{-1/3}$

(b) (6pts) $x+y = e^{xy}$ $1+y' = e^{xy}(y + x\cdot y')$ $\Rightarrow y' = \cdots$

3) (a) (4 pts) A mobile phone rings with a loud function $f(t) = at^4 - 12t$ in time t. What should be the parameter a if you want to switch it off at the peak loud time of $t=1$. Here f takes its max at $t=1$. So $t=1$ is a critical point. That is, $f'(1)=0$

$f'(t) = 4at^3 - 12$ $f'(1) = 4a - 12 = 0 \Rightarrow a = 3$

(b) (8pts) Sketch the graph of $f(x) = x|x|$. Comment on the invertibility of f.

4) (a) (7pts) Evaluate the integral $\int_0^3 [|x|]^2\,dx$ $\int_0^1 0^2\,dx + \int_1^2 1^2\,dx + \int_2^3 2^2\,dx = 5$

(b) (7pts) Evaluate the integral $\int x^3 \ln^2 x\,dx$

$$\left[\begin{matrix}(\ln x)^2 = u & x^3\,dx = dv \\ du = \frac{2\ln x}{x} & \frac{x^4}{4} = v\end{matrix}\right] = \frac{x^4(\ln x)^2}{4} - \frac{1}{2}\int x^3 \ln x\,dx \quad \left[\begin{matrix}\ln x = u & x^3\,dx = dv \\ 1/x\,dx = du & \frac{x^4}{4} = v\end{matrix}\right]$$

$$= \frac{x^4(\ln x)^2}{4} - \frac{1}{2}\left[\frac{\ln x\cdot x^4}{4} - \frac{1}{4}\int x^3\,dx\right] = \frac{x^4(\ln x)^2}{4} - \frac{1}{8}\ln x\cdot x^4 + \frac{1}{8}\cdot\frac{x^4}{4} + C$$

3) (12pts) Sketch the graph of $4x^3 - 3x^4$. comment on the invertibility of f.

$D_f = (-\infty, \infty)$ intercepts: $x=0$, $y=0$ $\quad y = 4x^3 - 3x^4 = 0 \iff x^3(4-3x)=0$
$\iff x=0 \quad x=4/3$

$f'(x) = 12x^2 - 12x^3 = 0$

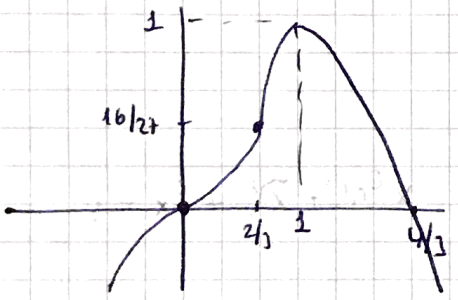
$12x^2(1-x)=0 \qquad x=0,1$

$f''(x) = 24x - 36x^2 = 0$

$12x(2-3x)=0 \qquad x=0, 2/3$

		0		2/3		1	
f'	+		+		+		−
f''	−		+		−		−
f	↗		↗		↗		↘

$(0,0) \qquad (2/3, 16/27) \qquad (1,1)$



4) (a) (7pts) Evaluate the integral $\int x^2 \ln x \, dx$

$\ln x = u \qquad x^2 dx = dv$

$\frac{dx}{x} = du \qquad \frac{x^3}{3} = v$

$$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{dx}{x} = \frac{x^3 \cdot \ln x}{3} - \frac{1}{3}\int x^2 dx = \frac{x^3 \cdot \ln x}{3} - \frac{1}{9}x^3 + c$$

(b) (7 pts) Evaluate $\int_0^{\pi/2} \sin^2 x \, dx$

$$\int_0^{\pi/2} \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4}\Big|_0^{\pi/2} = \pi/4 - 0 = \pi/4$$

Semester

Date

Time

Duration : 120 minutes

1. Evaluate the limit if exists.

(a) (6 pts) $\lim_{x\to 0} \frac{\sin\lfloor x \rfloor}{x}$.

$$\lim_{x\to 0^+} \frac{\sin\lfloor x \rfloor}{x} = \lim_{x\to 0^+} \frac{\sin 0}{x} = 0$$

$$\lim_{x\to 0^-} \frac{\sin\lfloor x \rfloor}{x} = \lim_{x\to 0^-} \frac{\sin(-1)}{x} = \infty$$

(b) (6 pts) $\lim_{x\to\infty} \frac{\sqrt{9x^9 - x}}{x^3 + 1}$.

$$\frac{x^{\frac{9}{2}}\sqrt{9 - \frac{1}{x^8}}}{x^3\left(1 + \frac{1}{x^3}\right)} \to \infty$$

2. Evaluate y' and y'' from the following equations.

(a) (6 pts) $y = x^3 + \sqrt[3]{x} + \ln^3 x$

$$y' = 3x^2 + \frac{1}{3}x^{-\frac{2}{3}} + 3\frac{\ln^2 x}{x}$$

$$y'' = 6x - \frac{2}{9}x^{-\frac{5}{3}} + 3\frac{2\frac{\ln x}{x} \cdot x - \ln^2 x}{x^2}$$

(b) (6 pts) $y = e^{x-y}$

$$y' = e^{x-y}(1-y') = e^{x-y} - e^{x-y}y'$$

$$y' = \frac{e^{x-y}}{1+e^{x-y}}$$

$$y'' = \frac{e^{x-y}(1-y')(1+e^{x-y}) - e^{x-y}\,e^{x-y}(1-y')}{(1+e^{x-y})^2}$$

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	Mathematics I Final Exam Instructor: M. Türkyılmazoğlu	
Code : *Math 123* Acad.Year : *2013-2014* Semester : *Fall* Date : *06.01.2014* Time : *13:00 - 15:00* Duration : *120 minutes*	Last Name : Name : Student No : Section : Signature :	**5** Questions on **5** Pages Total **50** Points

1	2	3	4	5		

1. Evaluate the limit if it exists

(a) (5pts) $\lim_{x\to 0}[|\frac{\sin x}{x}|]$

(b) (5pts) $\lim_{x\to\infty}\frac{\ln x}{x^9}$

2. If $y(x)$ is given in implicit form as $\sin(x - 3y) = 1 - e^{2x+y}$

(a) (5pts) Evaluate y'(0)

(b) (5pts) Evaluate y"(x)

3. (10pts) Sketch
Explain why this fu

3. (**10pts**) Sketch the graph of $y = x^5 - 5x$ using the curve sketching strategy. Explain why this function is not invertible

4. Let R be the plane region bounded by $y = x^2$ and $y = x^3$.

(a) (**4pts**) A three-dimensional object on the base R is formed such the each cross-section perpendicular to the $x-$axis is a semi circular region. Find the volume of this object

(b) (**4pts**) Find the volume of object formed by revolving the region R about the $x-$axis

(c) (**2pts**) Express the arc length of the region R in terms of integrals

each cross-

5. (a) (**5pts**) Investigate the convergence/divergence of $\sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$

(b) (**5pts**) Find the Taylor series expansion of $f(x) = \sin(2x)$ at $x = 0$. From the expansion find the Taylor expansion of $\cos(2x)$.