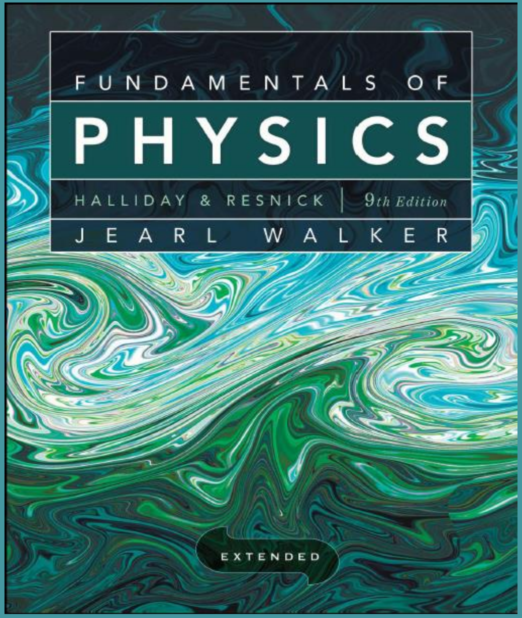
FİZ 137 - 25
CHAPTER 8

POTENTIAL ENERGY AND CONSERVATION OF ENERGY



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2018 - 2019

In this chapter we will introduce the following concepts:

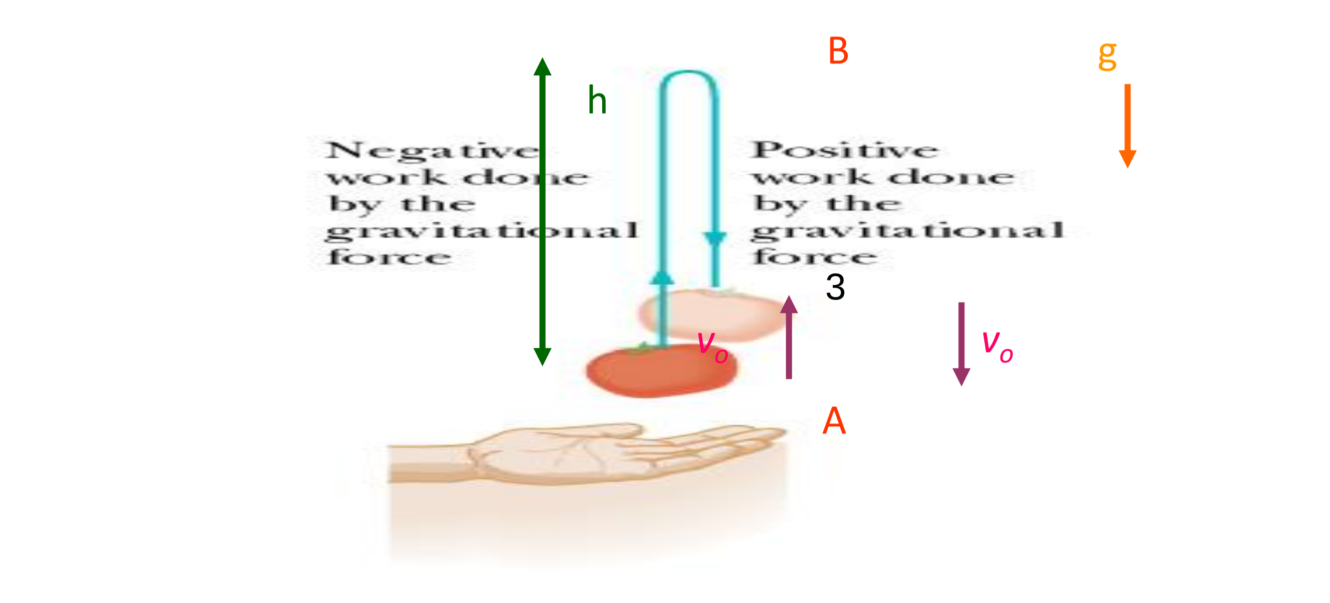
- **Potential Energy**
- **Conservative and non-conservative forces**
- **Mechanical Energy**
- **Conservation of Mechanical Energy**
- **Conservation of Energy Theorem**

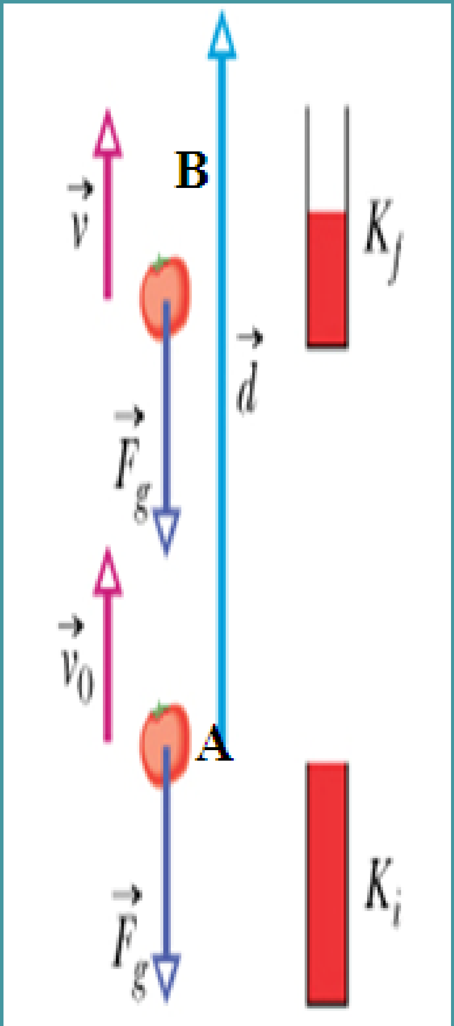
The approach is mathematically simpler

Work and Potential Energy

Consider the tomato of mass *m* shown in the figure. The tomato is taken together with the earth as the system we wish to study. The tomato is thrown upwards with initial speed $\mathbf{v_o}$ at point A. Under the action of the gravitational force it slows down and stops completely at point B. Then the tomato falls back and by the time it reaches point A its speed has reached the original value v_o.

System = Tomato + Earth





Work Done by the Gravitational Force:

Consider a tomato of mass m that is thrown upwards at point A with initial speed v_o. As the tomato rises, it slows down by the gravitational force F_g so that at point B its has a smaller speed v.

The work $W_g(A \to B)$ done by the gravitational force on the tomato as it travels from point A to point B is:

$$\boxed{W_g(A \to B) = mgd\cos 180° = -mgd}$$

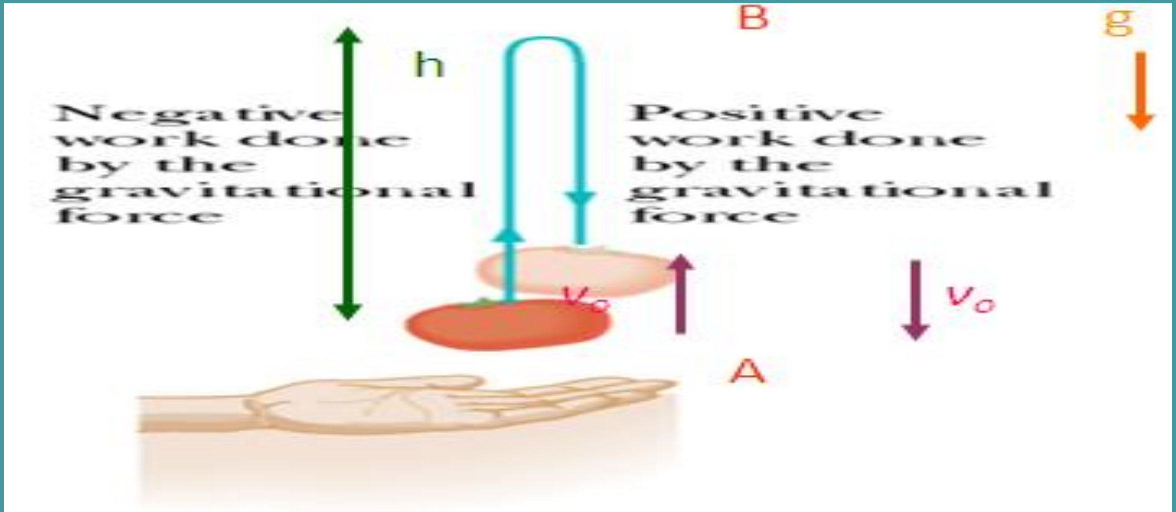
The work $W_g(B \to A)$ done by the gravitational force on the tomato as it travels from point B to point A is:

$$\boxed{W_g(B \to A) = mgd\cos 0° = mgd}$$

$$\boxed{\Delta K = K_f - K_i = W_{net}}$$

During the trip from **A** $\rightarrow$ **B** the gravitational force F_g does negative work $W_1 = -mgh$. ***Energy is transferred by F_g from the kinetic energy of the tomato to the gravitational potential energy U of the tomato-earth system.***

During the trip from **B** $\rightarrow$ **A** the transfer is reversed. The work W_2 done by F_g is positive as $W_2 = +mgh$. ***Thus, the gravitational force transfers energy from the gravitational potential energy U of the tomato-earth system to the kinetic energy of the tomato.***

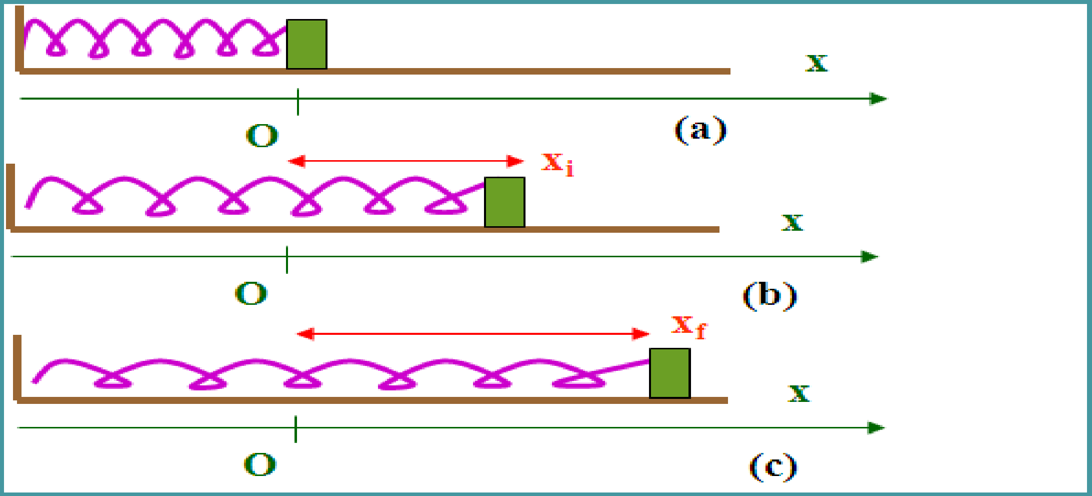


System = Tomato + Earth

The change in the potential energy U is defined as:

$$\boxed{\Delta U = -W}$$

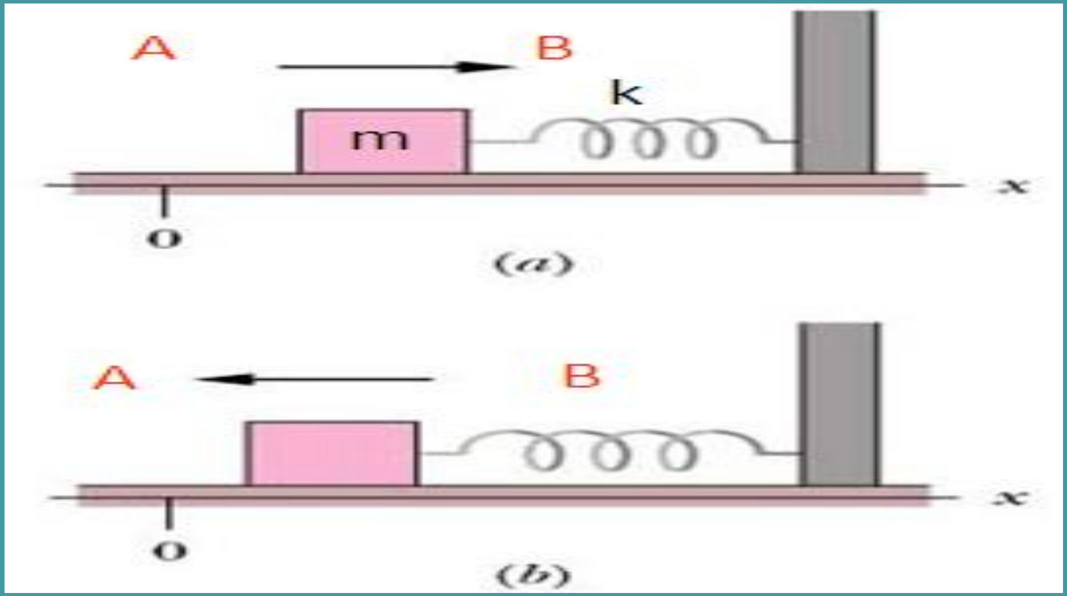
Mass + Spring = System



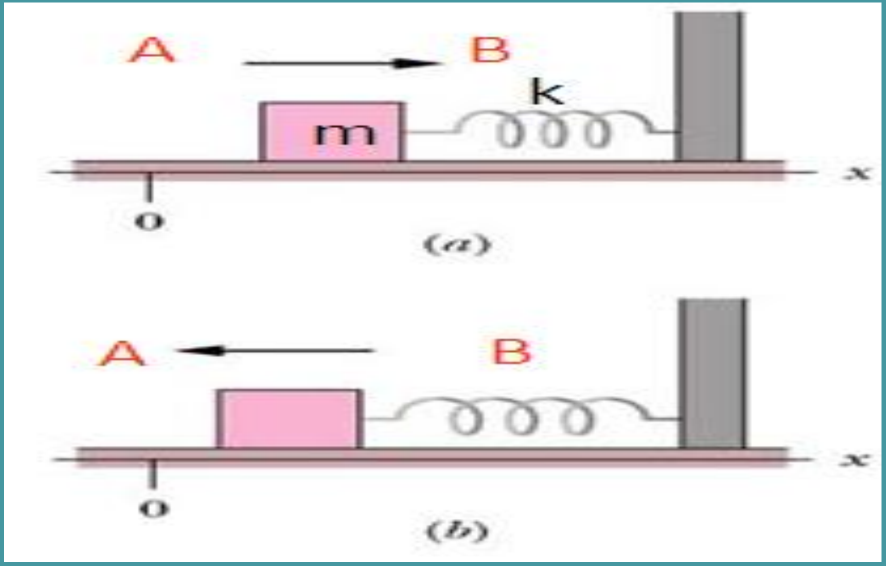
We will use the expression: $W_s = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx = -k\int_{x_i}^{x_f} xdx$

$W_s = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$ Quite often we start we a relaxed spring $(x_i = 0)$ and we either stretch or compress the spring by an amount x $(x_f = \pm x)$. In this case $\boxed{W_s = -\frac{kx^2}{2}}$

System = Mass + Spring



Consider the mass *m* is attached to a spring with the spring constant *k*, as shown in the figure. *The mass is taken together with the spring as the system.* The mass is given an initial speed v_o at point A. Under the action of the spring force it slows down and stops completely at point B which corresponds to a spring compression *x*. Then the mass reverses the direction of its motion and by the time it reaches point A its speed has reached the original value v_o.



$$\Delta U = -W$$

During the trip from A ➔ B the spring force ***F_s* does negative work *$W_1 = -kx^2/2$***. **Energy is transferred by *F_s* from the kinetic energy of the mass to the potential energy *U* of the mass-spring system.**

During the trip from B ➔ A the transfer is reversed. The work ***W_2*** done by ***F_s* is positive** as ***$W_2 = kx^2/2$***. **The spring force transfers energy from the potential energy *U* of the mass+spring system to the kinetic energy of the mass.**

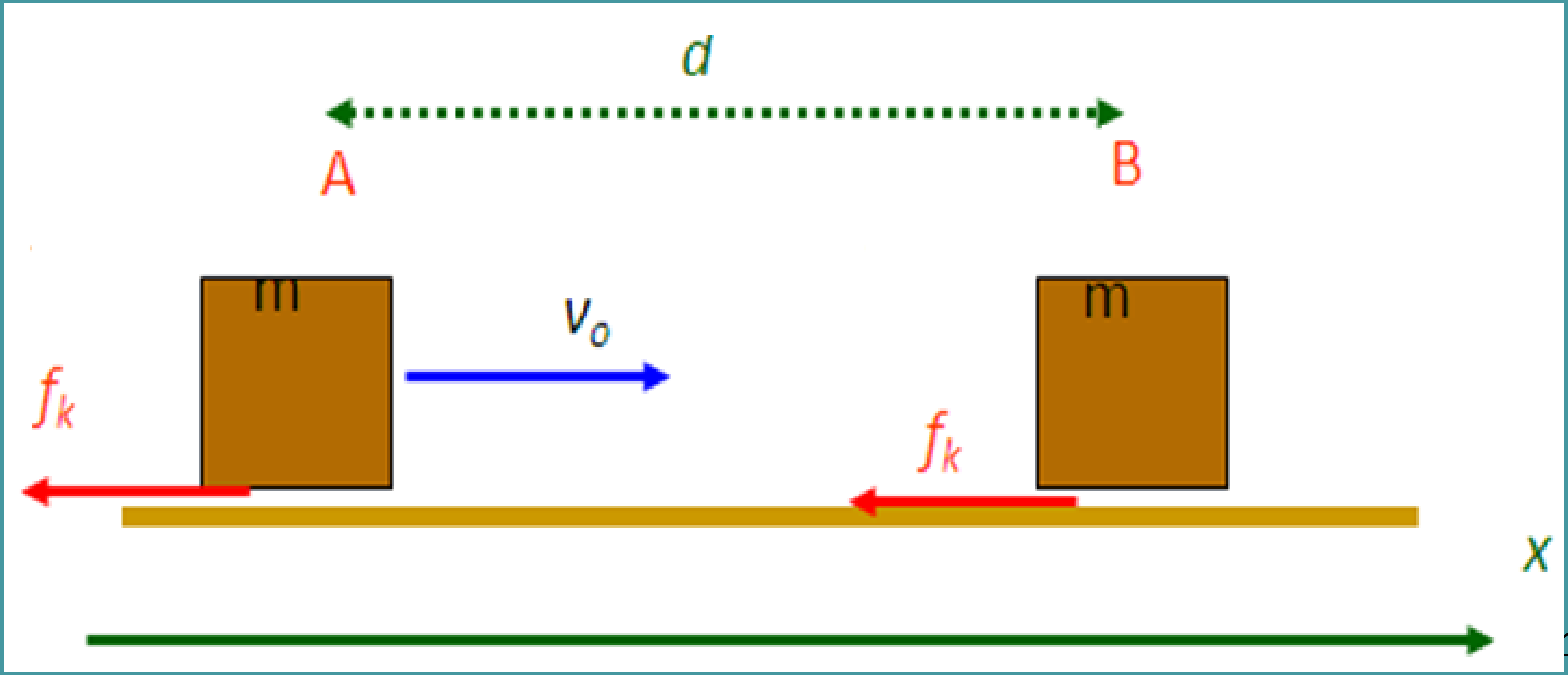
Conservative and Non-Conservative Forces

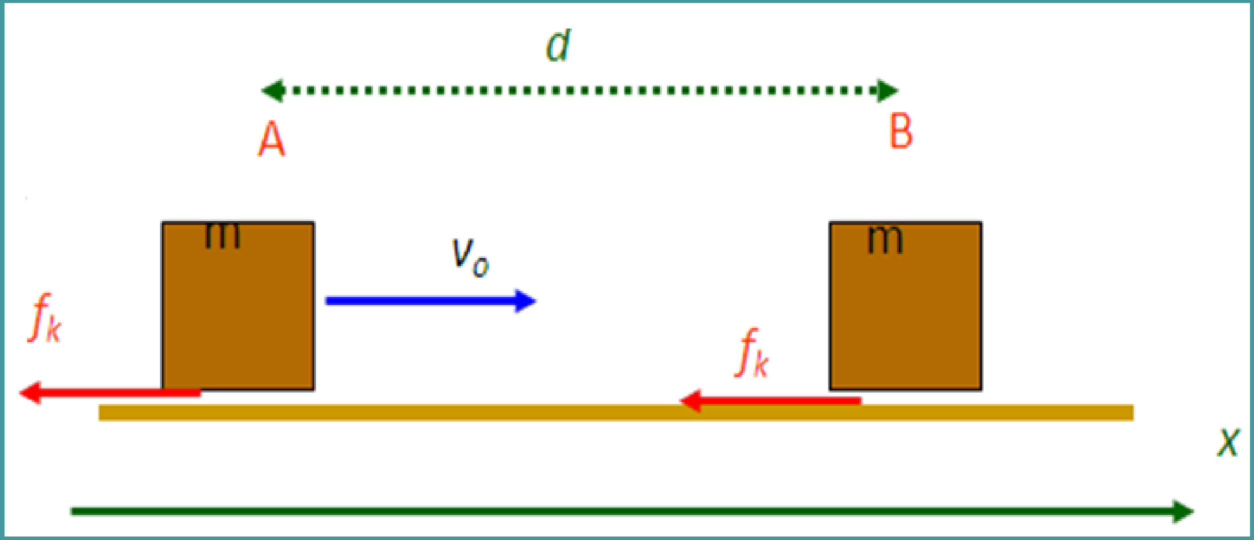
The **gravitational force** and the **spring force** are called "**conservative**" because ***they can transfer energy from the kinetic energy of part of the system to potential energy and vice versa.***

Frictional forces on the other hand is called

"non-conservative".

Consider a system that consists of a block of mass ***m*** and the floor on which it rests. The block starts to move on a horizontal floor with initial speed v_o at point A. The coefficient of kinetic friction between the floor and the block is μ_k. The block will slow down by the kinetic friction f_k and will stop at point B after it has travelled a distance of ***d***.





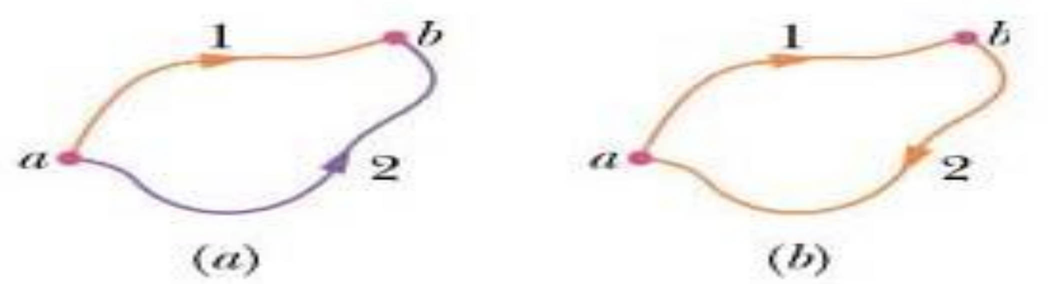
- During the trip from point A ➔ B the frictional force has done work $W_f = -(\mu_k mg)d$.
- The frictional force transfers energy from the kinetic energy of the block to a type of energy called **«thermal energy»**.
- **This energy transfer cannot be reversed.** Thermal energy cannot be transferred back to kinetic energy of the block by the kinetic friction.

This is the mark of non-conservative forces.

The work done by a conservative force **depends only on the position of the initial and final positions (two points), not on the path that is followed.**

The total work done by a conservative force through a round trip is zero.

Path Independence of Conservative Forces

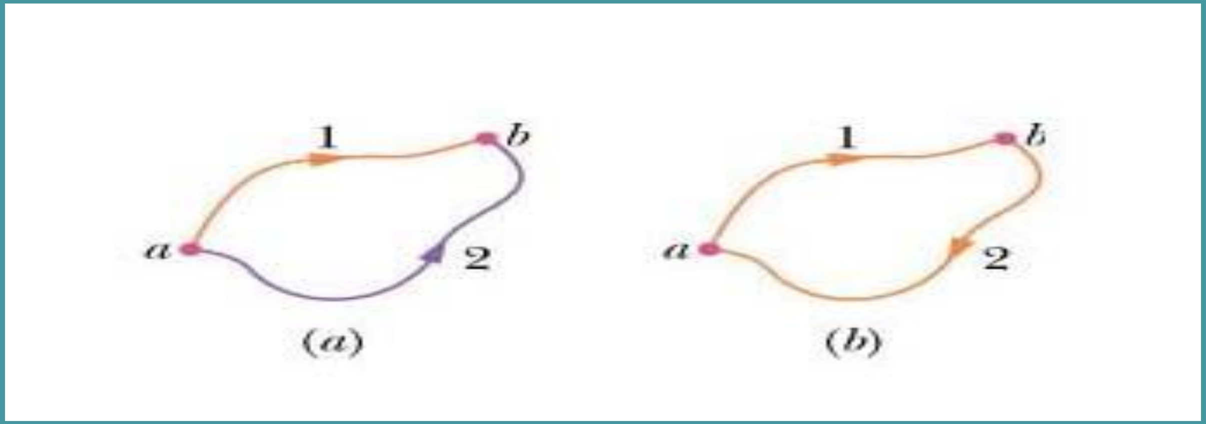


A force is conservative if the net work done on a particle during a round trip is always equal to zero.

$$\boxed{W_{net} = 0}$$

In the examples of the tomato-earth and mass-spring systems

$$\boxed{W_{ab,1} = W_{ab,2}}$$



$$\boxed{W_{net} = 0}$$

Proof

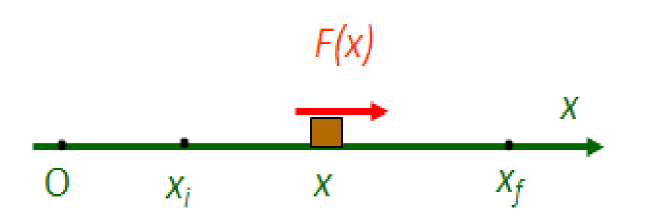
$$W_{net} = W_{ab,1} + W_{ba,2} = 0 \quad \text{(Fig.b)}$$

$$W_{ab,1} = -\,W_{ba,2} \quad \text{(eqs.1)}$$

$$W_{ab,2} = -\,W_{ba,2} \quad \text{(eqs.2) (Fig.a)}$$

$$\boxed{W_{ab,1} = W_{ab,2}}$$

Determining Potential Energy Values:



In this section we will discuss a method that can be used to determine the difference in potential energy ΔU of a conservative force F between points x_f and x_i on the x-axis if we know $F(x)$

A conservative force F moves an object along the x-axis from an initial point x_i to a final point x_f. The work W that the force F does on the object is given by :

$$\boxed{W = \int_{x_i}^{x_f} F(x)dx}$$

The corresponding change in potential energy ΔU was defined as:

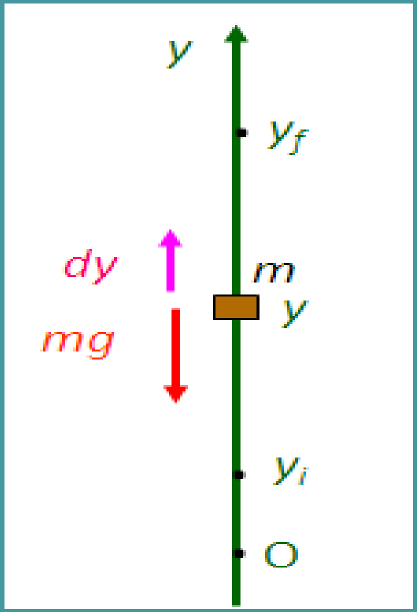
$$\boxed{\Delta U = -W}$$

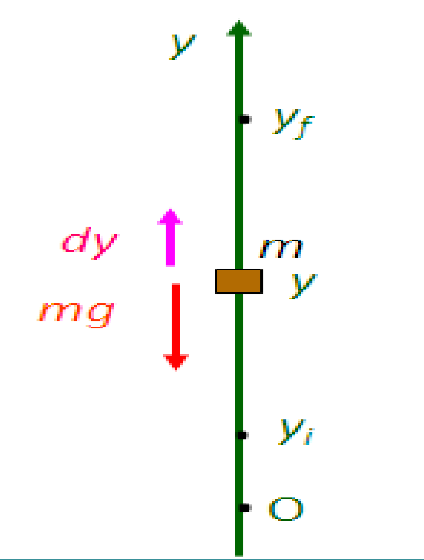
Therefore the expression for ΔU becomes:

$$\boxed{\Delta U = -\int_{x_i}^{x_f} F(x)dx}$$

Gravitational Potential energy:

Consider a particle of mass m moving vertically along the y-axis from point y_i to point y_f. At the same time the gravitational force does work W on the particle which changes the potential energy of the particle-earth system. We use the result of the previous section to calculate ΔU





$$\Delta U = -\int_{y_i}^{y_f} F(x)dy \quad F = -mg \quad \rightarrow \quad \Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg\int_{y_i}^{y_f} dy = mg\left[y\right]_{y_i}^{y_f}$$

$\Delta U = mg\left(y_f - y_i\right) = mg\Delta y$ We assign the final point y_f to be the "generic" point y on the y-axis whose potential is $U(y)$. $\rightarrow$ $U(y) - U_i = mg\left(y - y_i\right)$

Since only changes in the potential are physically menaingful, this allows us to define arbitrarily y_i and U_i The most convenient choice is:

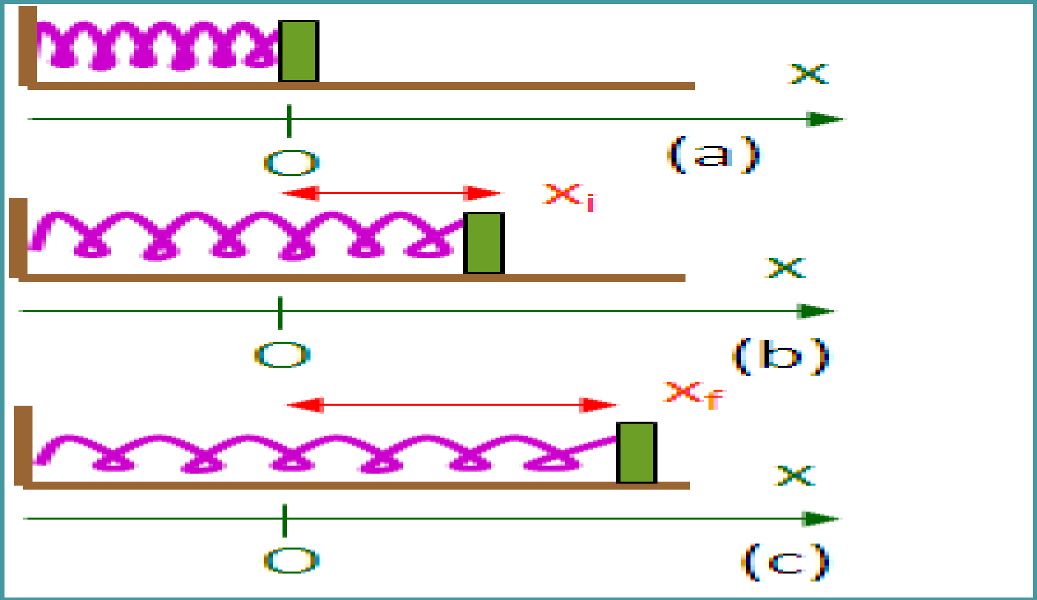
$y_i = 0$, $U_i = 0$ This particular choice gives: $\boxed{U(y) = mgy}$

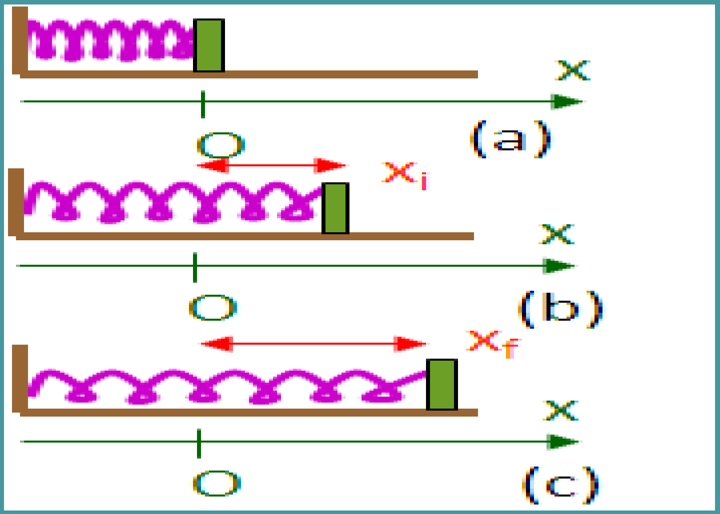
Potential Energy of a spring:

Consider the block-mass system shown in the figure.

The block moves from point x_i to point x_f. At the same time the spring force does work W on the block which changes the potential energy of the block-spring system by an amount

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx = -k\int_{x_i}^{x_f} xdx \qquad \Delta U = -W \rightarrow$$





$$\Delta U = k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = \frac{kx_f^2}{2} - \frac{kx_i^2}{2}$$

We assign the final point x_f to be the "generic" point x on the x-axis whose potential is $U(x)$. $\rightarrow$ $U(x) - U_i = \frac{kx^2}{2} - \frac{kx_i^2}{2}$

Since only changes in the potential are physically menaingful, this allows us to define arbitrarily x_i and U_i The most convenient choice is:

$y_i = 0$, $U_i = 0$ This particular choice gives: $\boxed{U = \frac{kx^2}{2}}$

Conservation of Mechanical Energy:

Mechanical energy of a system is defined as the sum of potential and kinetic energies

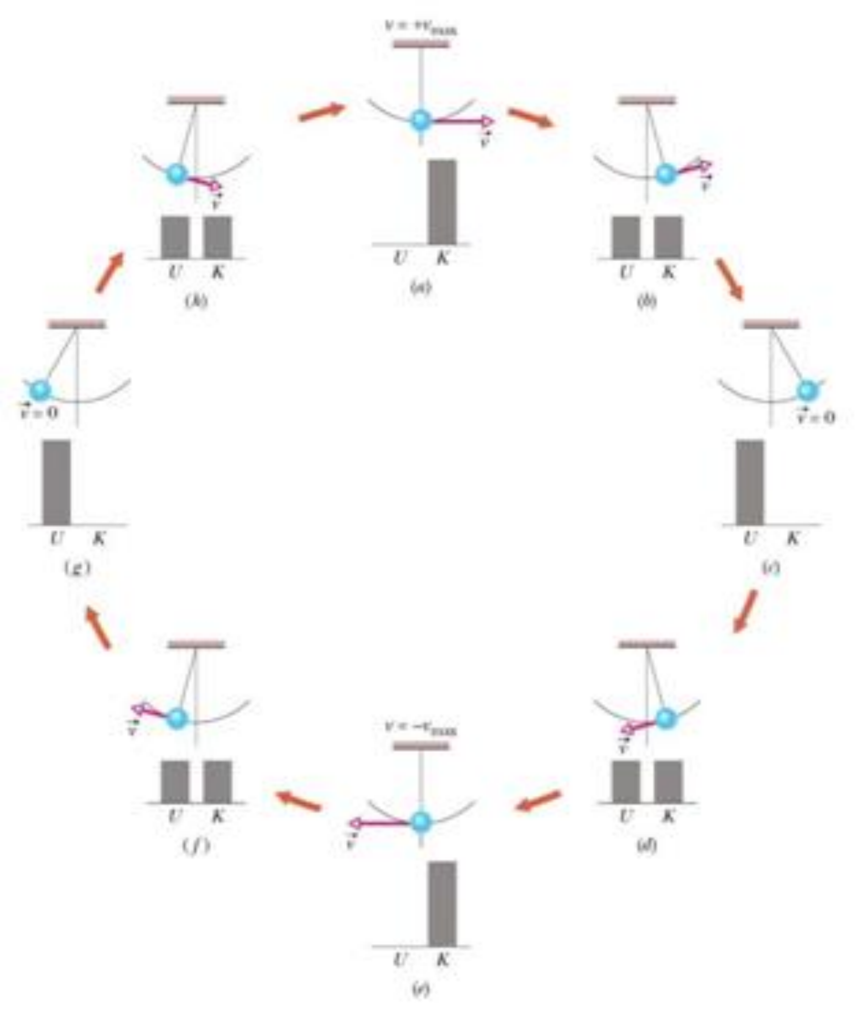
$E_{mech} = K + U$ We assume that the system is isolated i.e. no external forces change the energy of the system. We also assume that all the forces in the system are conservative. When an interal force does work W on an object of the system this changes the kinetic energy by $\Delta K = W$ (eqs.1) This amount of work also changes the potential energy of the system by an amount $\Delta U = -W$ (eqs.2)

If we compare equations 1 and 2 we have: $\Delta K = -\Delta U \rightarrow$

$\boxed{K_2 - K_1 = -(U_2 - U_1)} \rightarrow K_1 + U_1 = K_2 + U_2$ This equation is known as the principle of conservation of mechanical energy. It can be summarized as:

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$$

An example consists of a pendulum of mass *m* moving under the action of the gravitational force



The total mechanical energy of the *pendulum - earth system* remains constant. As the pendulum swings, the total energy ***E*** is transferred back and forth between kinetic energy ***K*** of the bob and potential energy ***U*** of the bob-earth system.

We assume that *U* is zero at the lowest point of the pendulum orbit. *K* is maximum in frame a, and e (*U* is minimum there). *U* is maximum in frames c and g (*K* is minimum there)

***In the presence of friction*, the mechanical energy of the system will decrease.**

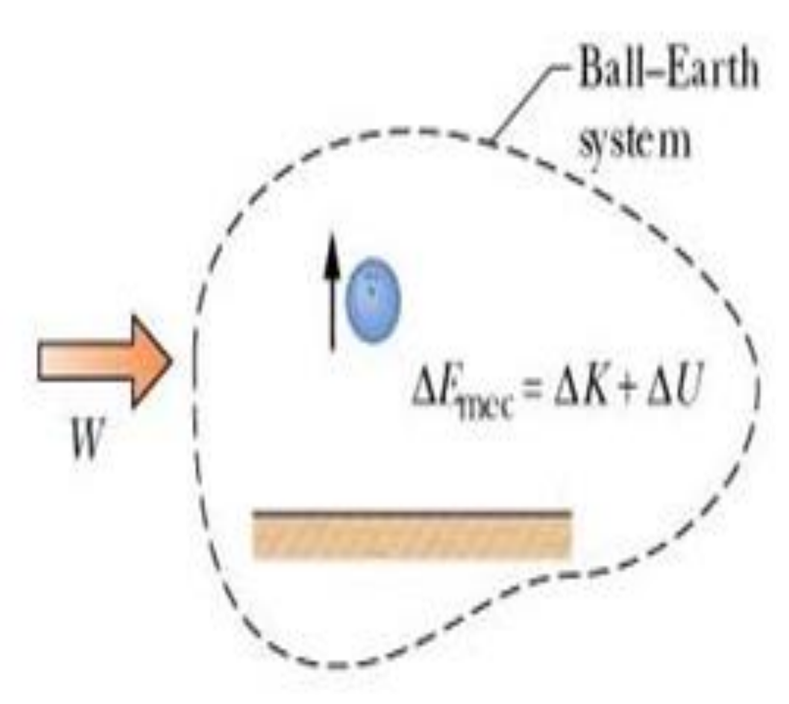
The change in mechanical energy will be **negative** and will equal the **work done by friction.**

$$\Delta E_{mech} = E_{mech,final} - E_{mech,initial} = -fd$$

For an isolated system in which the forces are a mixture of conservative and non conservative forces the principle takes the following form

$$\boxed{\Delta E_{mech} = W_{nc}}$$

Here, W_{nc} is defined as the work of all the non-consrvative forces of the system



Work Done on a System by an External Force

Up to this point we have considered only isolated systems in which no external forces were present. We will now consider a system in which there are forces external to the system.

The system under study is a bowling ball being hurled by a player. The system consists of the ball and the earth taken together. ***The force exerted on the ball by the player is an external force.*** In this case the mechanical energy E_{mec} of the system is **not constant**. Instead it changes by an amount equal to the work W done by the external force according to the equation: $W = \Delta E_{mec} = \Delta K + \Delta U$

Finding the Force $F(x)$ analytically from the potential energy $U(x)$

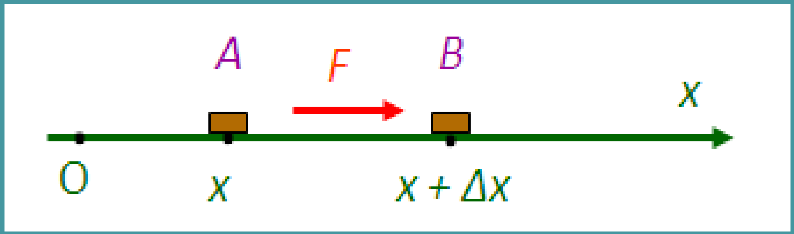
Consider an object that moves along the x-axis under the influence of an unknown force F whose potential energy U(x) we know at all points of the x-axis. The object moves from point A (coordinate x) to a close by point B (coordinate $x + \Delta x$). The force does work W on the object given by the equation:

$W = F\Delta x$ eqs.1

The work of the force changes the potential energy U of the system by the amount:

$\Delta U = -W$ eqs.2 If we combine equations 1 and 2 we get:

$F = -\dfrac{\Delta U}{\Delta x}$ We take the limit as $\Delta x \to 0$ and we end up with the equation:



$$\boxed{F(x) = -\frac{dU(x)}{dx}}$$

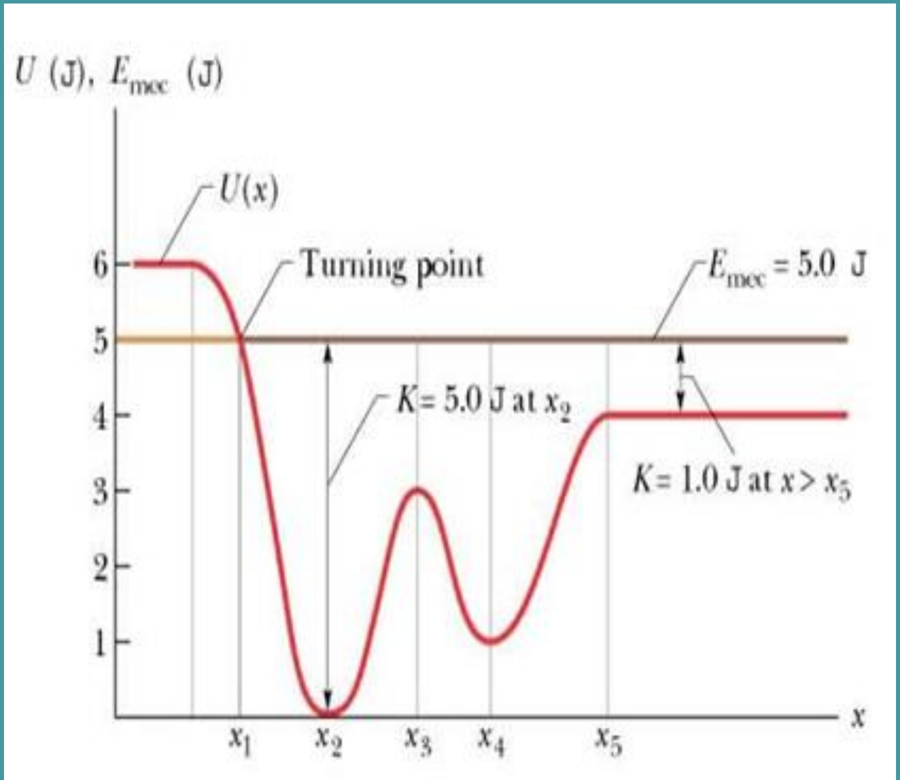
The Potential Energy Curve

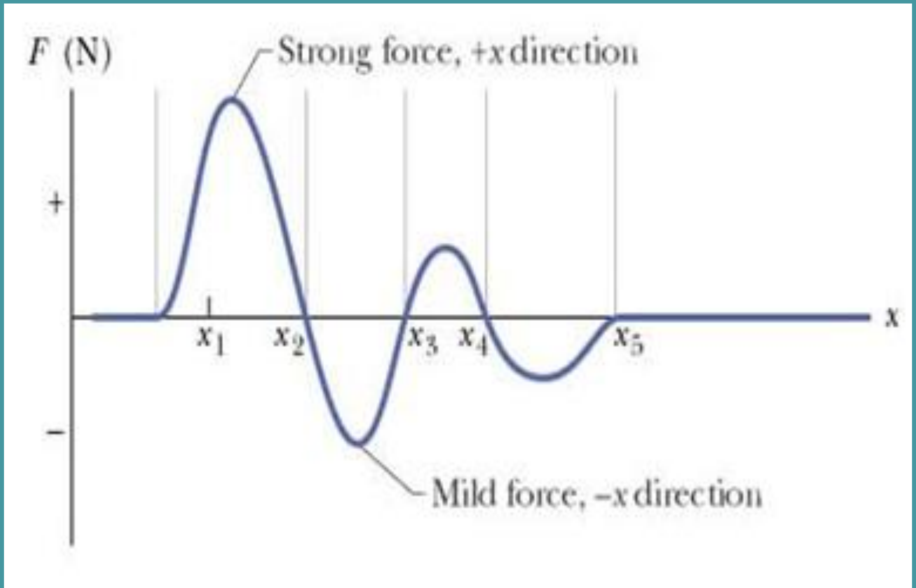
If we plot the potential energy ***U*** versus ***x*** for a force ***F*** that acts along the ***x*-axis** we can glean a wealth of information about the motion of a particle on which **F** is acting.

The first parameter that we can determine is the force *F(x)* using the equation:

$$F(x) = -\frac{dU(x)}{dx}$$

$$F(x) = -\frac{dU(x)}{dx}$$

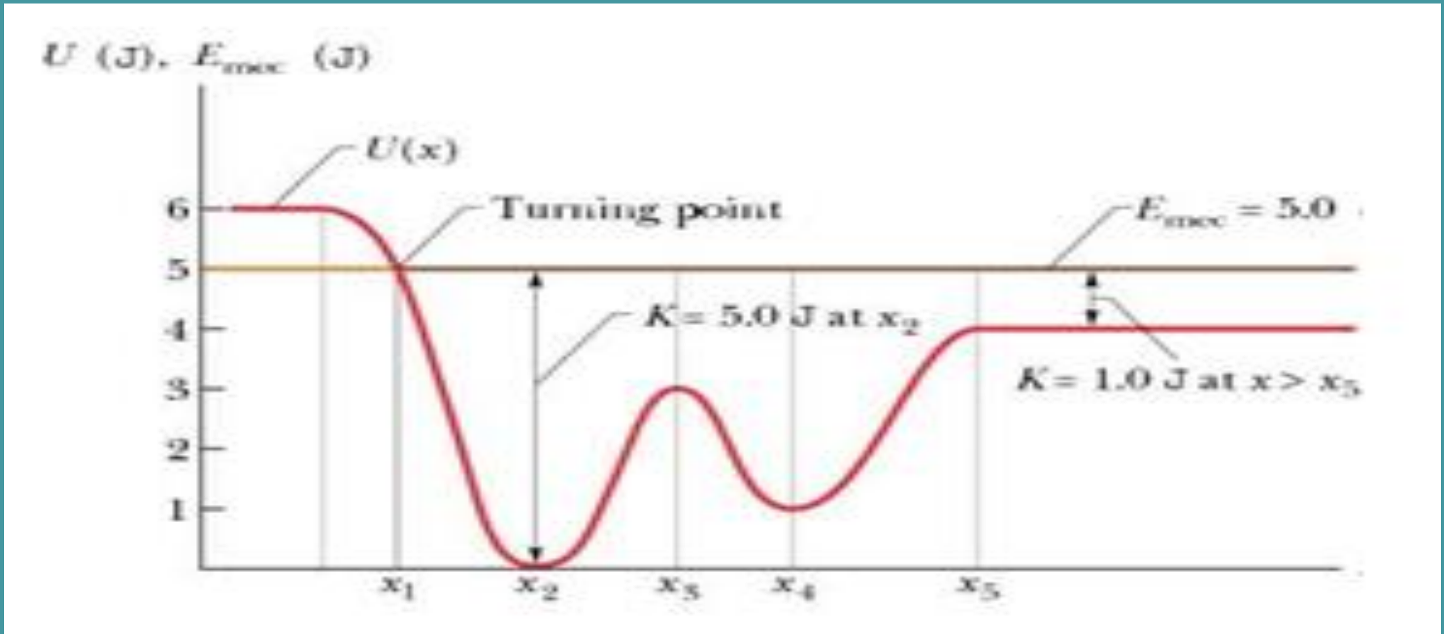




At x_2, x_3 and x_4 points the slope of the *U(x)* - *x* curve is zero, thus **F = 0**.

The slope ***dU/dx*** between x_3 and x_4 is negative; Thus **F > 0** for the this interval.

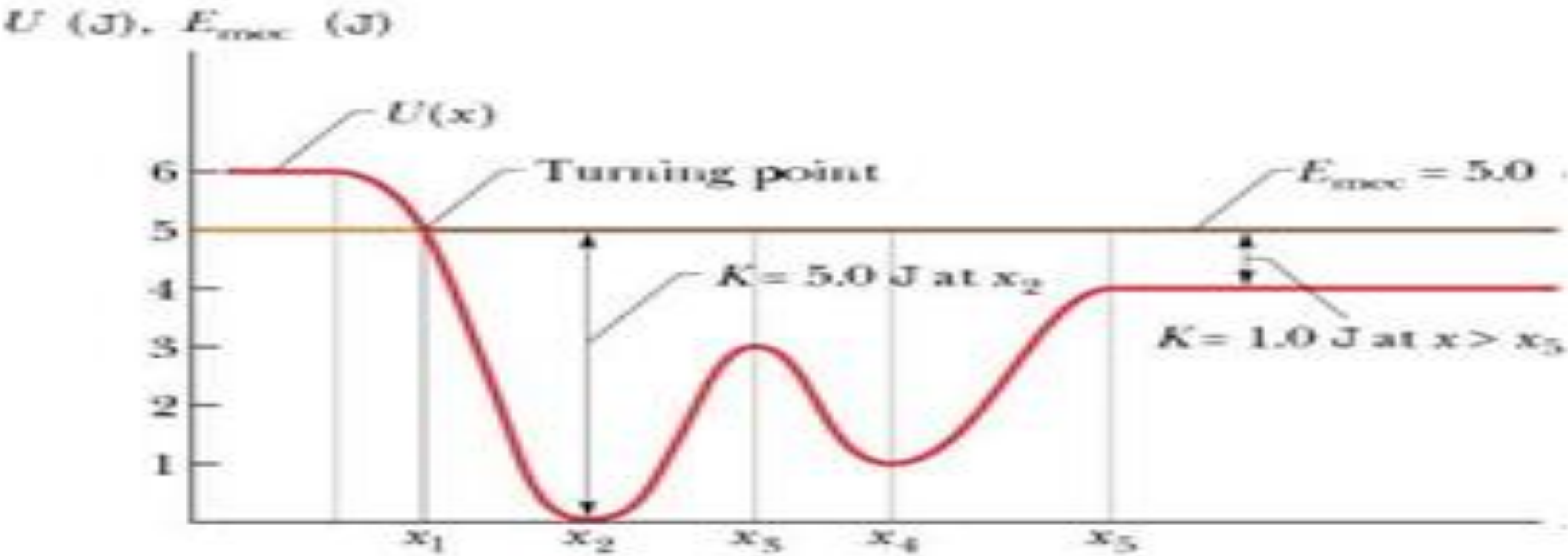
The slope ***dU/dx*** between x_2 and x_3 is positive; Thus **F < 0** for the same interval .



Turning Points:

The total mechanical energy is $E_{mec} = K(x) + U(x)$
This energy is constant (equal to 5 J in the figure) and is thus represented by a horizontal line. We can slolve this equation for $K(x)$ and get:

$K(x) = E_{mec} - U(x)$ At any point x on the x-axis we can read the value of $U(x)$. Then we can solve the equation above and determine K



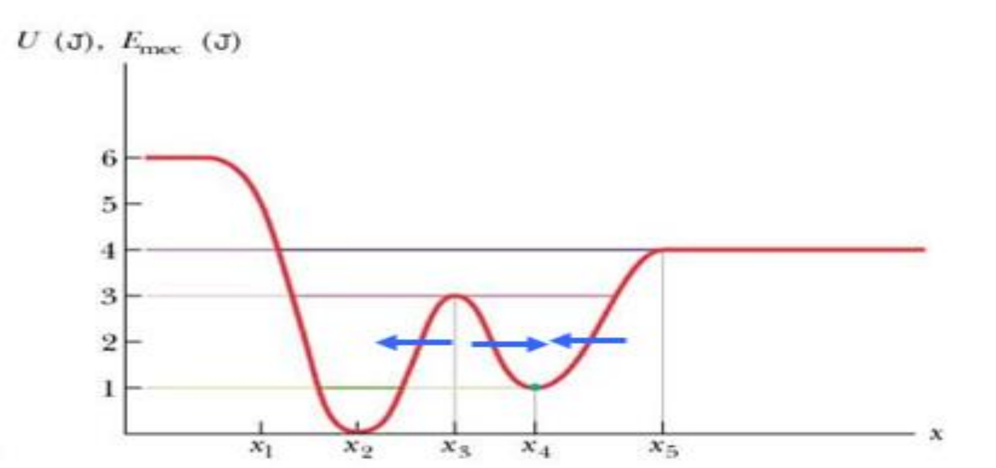
From the definition of $K = \dfrac{mv^2}{2}$ the kinetic energy cannot be negative.

This property of K allows us to determine which regions of the x-axis motion is allowed. $K(x) =$ $K(x) = E_{mec} - U(x)$

If $K > 0 \rightarrow E_{mech} - U(x) > 0 \rightarrow U(x) < E_{mec}$ Motion is allowed

If $K < 0 \rightarrow E_{mech} - U(x) < 0 \rightarrow U(x) > E_{mec}$ Motion is forbidden

The points at which: $E_{mec} = U(x)$ are known as turning points for the motion. For example x_1 is the turning point for the U versus x plot above. At the turning point $K = 0$



The blue arrows in the figure indicate the direction of the force F as determined from the equation:

$$F(x) = -\frac{dU(x)}{dx}$$

Positions of Stable Equilibrium: An example is point x_4 where U has a minimum. If we arrange E_{mec} = 1 J then K = 0 at point x_4. A particle with E_{mec} = 1 J is stationary at x_4. If we displace slightly the particle either to the right or to the left of x_4, the force tends to bring it back to the equilibrium position. ***This equilibrium is stable.***

Positions of Unstable Equilibrium: An example is point x_3 where U has a maximum. If we arrange E_{mec} = 3 J then K = 0 at point x_3. A particle with E_{mec} = 3 J is stationary at x_3. If we displace slightly the particle either to the right or to the left of x_3 the force tends to take it further away from the equilibrium position. ***This equilibrium is unstable.***

4. A force on a particle is conservative if:

A. its work equals the change in the kinetic energy of the particle
B. it obeys Newton's second law
C. it obeys Newton's third law
D. its work depends on the end points of every motion, not on the path between
E. it is not a frictional force

ans: D

16. A 6.0-kg block is released from rest 80 m above the ground. When it has fallen 60 m its kinetic energy is approximately:

A. 4800 J

B. 3500 J

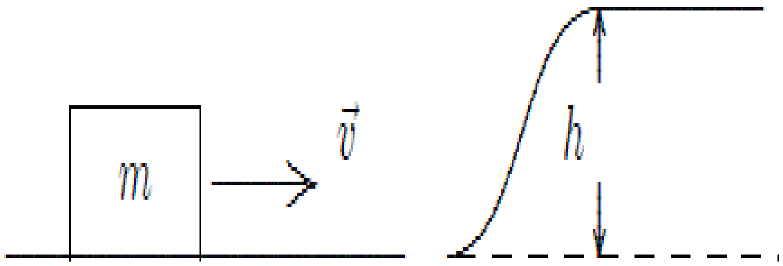
C. 1200 J

D. 120 J

E. 60 J

ans: B

20. For a block of mass m to slide without friction up the rise of height h shown, it must have a minimum initial kinetic energy of:



A. gh

B. mgh

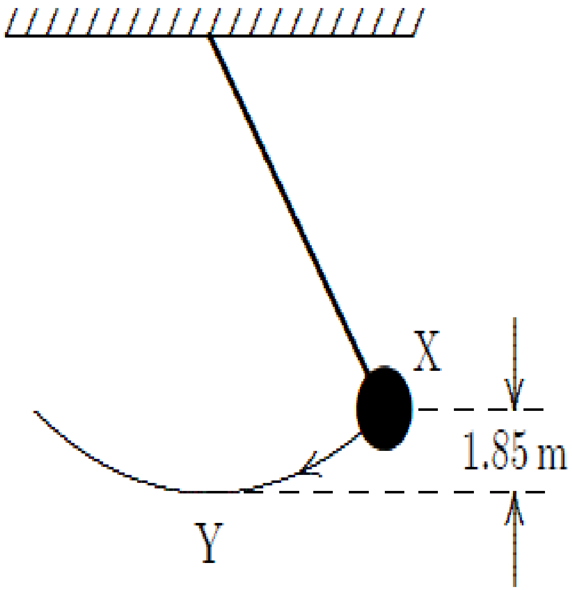
C. $gh/2$

D. $mgh/2$

E. $2mgh$

ans: B

22. A simple pendulum consists of a 2.0-kg mass attached to a string. It is released from rest at X as shown. Its speed at the lowest point Y is about:



A. $0.90\,\mathrm{m/s}$
B. $\sqrt{3.6}\,\mathrm{m/s}$
C. $3.6\,\mathrm{m/s}$
D. $6.0\,\mathrm{m/s}$
E. $36\,\mathrm{m/s}$

ans: D

24. A particle moves along the x axis under the influence of a stationary object. The net force on the particle is given by $F = (8\,\mathrm{N/m^3})x^3$. If the potential energy is taken to be zero for $x = 0$ then the potential energy is given by:

A. $(2\,\mathrm{J/m^4})x^4$

B. $(-2\,\mathrm{J/m^4})x^4$

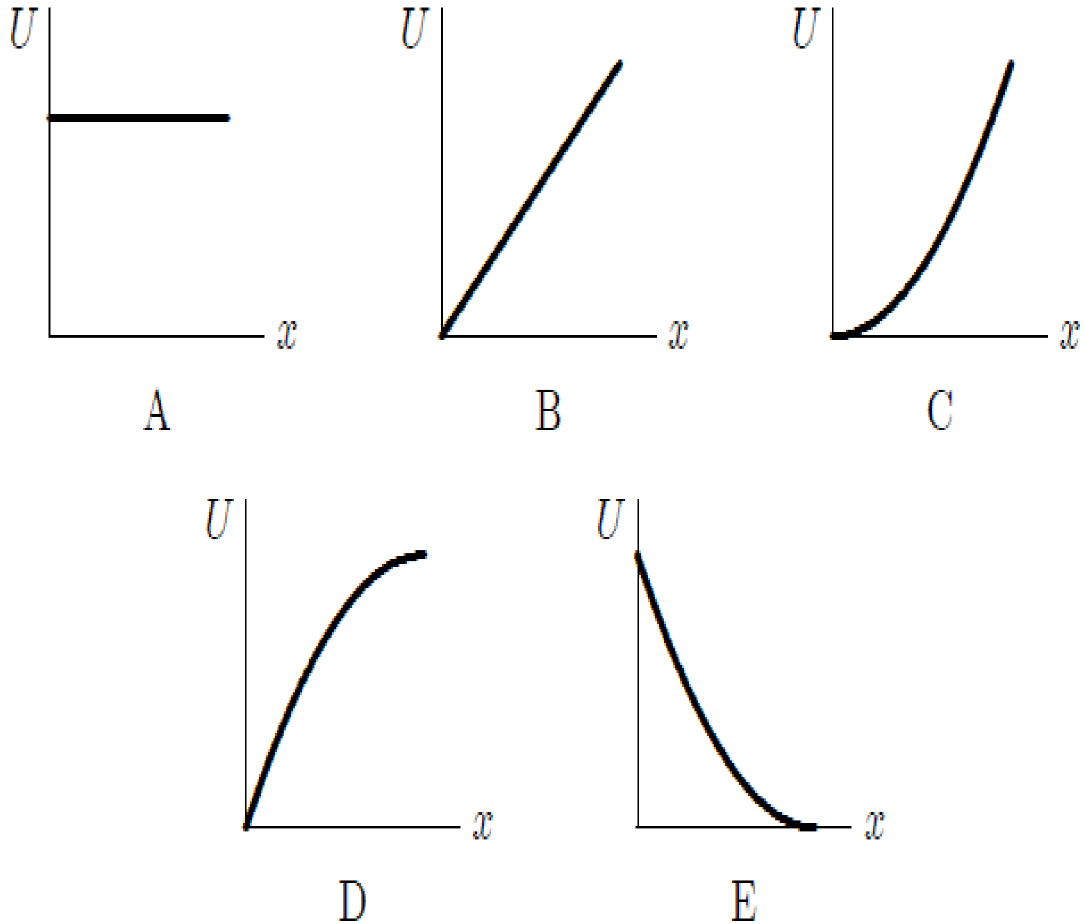
C. $(24\,\mathrm{J/m^2}x^2$

D. $(-24\,\mathrm{J/m^2})x^2$

E. $5\,\mathrm{J} - (2\,\mathrm{J/m^4})x^4$

ans: B

26. Which of the five graphs correctly shows the potential energy of a spring as a function of its elongation x?



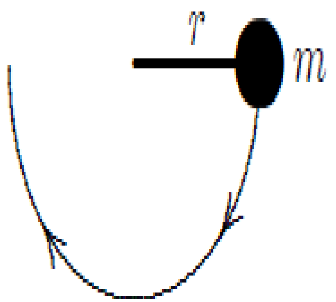
ans: C

30. A 0.50-kg block attached to an ideal spring with a spring constant of 80 N/m oscillates on a horizontal frictionless surface. The total mechanical energy is 0.12 J. The greatest speed of the block is:

A. 0.15 m/s
B. 0.24 m/s
C. 0.49 m/s
D. 0.69 m/s
E. 1.46 m/s

ans: D

36. A small object of mass m, on the end of a light cord, is held horizontally at a distance r from a fixed support as shown. The object is then released. What is the tension force of the cord when the object is at the lowest point of its swing?



A. $mg/2$

B. mg

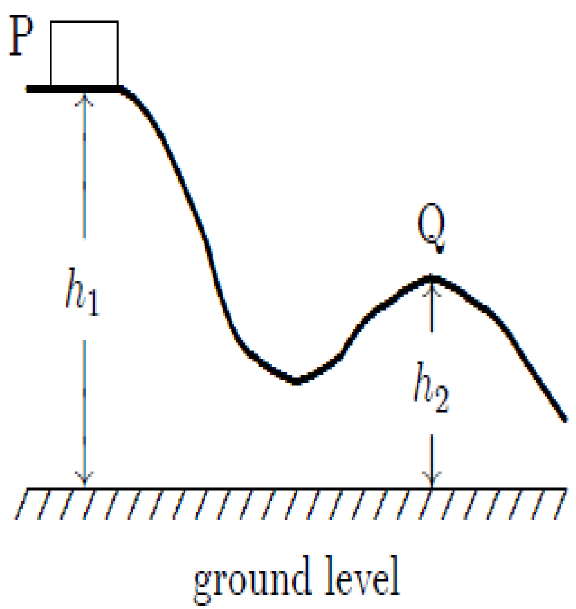
C. $2mg$

D. $3mg$

E. mgr

ans: D

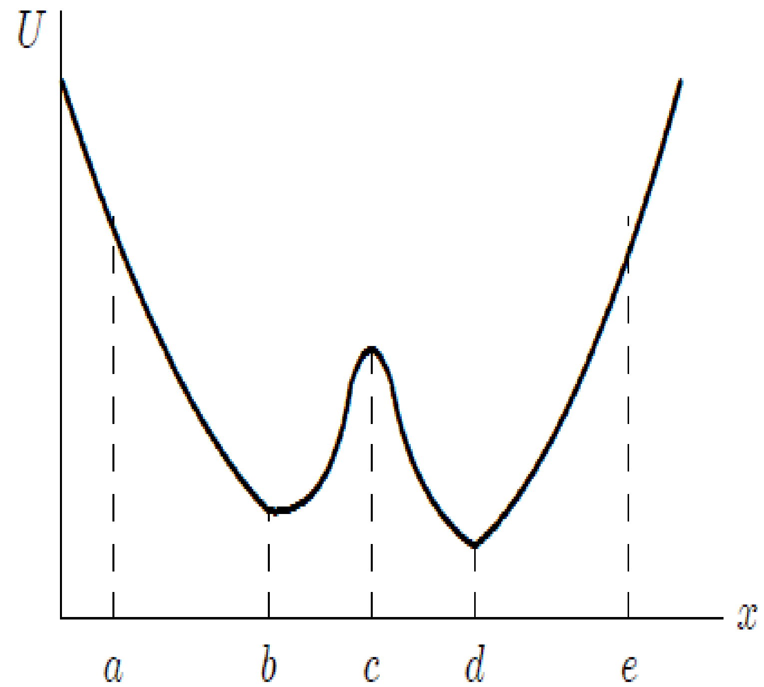
38. A block is released from rest at point P and slides along the frictionless track shown. At point Q, its speed is:



A. $2g\sqrt{h_1 - h_2}$
B. $2g(h_1 - h_2)$
C. $(h_1 - h_2)/2g$
D. $\sqrt{2g(h_1 - h_2)}$
E. $(h_1 - h_2)^2/2g$

ans: D

43. A particle is released from rest at the point $x = a$ and moves along the x axis subject to the potential energy function $U(x)$ shown. The particle:



A. moves to a point to the left of $x = e$, stops, and remains at rest
B. moves to a point to $x = e$, then moves to the left
C. moves to infinity at varying speed
D. moves to $x = b$, where it remains at rest
E. moves to $x = e$ and then to $x = d$, where it remains at rest

ans: B

52. The potential energy of a body of mass m is given by $U = -mgx + \frac{1}{2}kx^2$. The corresponding force is:

A. $-mgx^2/2 + kx^3/6$

B. $mgx^2/2 - kx^3/6$

C. $-mg + kx/2$

D. $-mg + kx$

E. $mg - kx$

ans: E

57. Objects A and B interact with each other via both conservative and nonconservative forces. Let K_A and K_B be the kinetic energies, U be the potential energy, and E_{int} be the thermal energy. If no external agent does work on the objects then:

A. $K_A + U$ is conserved
B. $K_A + U + E_{\text{int}}$ is conserved
C. $K_A + K_B + E_{\text{int}}$ is conserved
D. $K_A + K_B + U$ is conserved
E. $K_A + K_B + U + E_{\text{int}}$ is conserved

ans: E