İST292 STATISTICS LESSON 7 EXAMPLES

Chi-Square Test Examples

Example 1: Suppose a team of researchers at the University of California assign 900 patients to four test groups for the administering of Alzaret, a drug used in the treatment of Alzheimer's disease (a fictitious (uydurma, hayali) drug), and obtained the following results:

Table 1.

		Same medication: administered by four methods.				
		Method 1	**Method 2**	**Method 3**	**Method 4**	**Total**
Level of patient improvement	Major improvement	50	55	50	25	180
	Slight improvement	120	75	100	65	360
	No improvement	80	70	150	60	360
Total		250	200	300	150	n=900 patients

Are the four populations homogeneous, equally proportioned with respect to patient improvement, or not? (test at 0.05 significance level).

Solution:

Firstly, state the hypotheses. **For the chi-square test of homogeneity,**

H_0: The four methods are homogeneous with respect to patient improvement. (In effect this means there is no difference among the four methods, that is, each will result in the same levels of patient improvement.)

H_1: The four groups are not homogeneous with respect to patient improvement. (This means one or more methods is more effective than the others.)

Since R=3 and C=4, the degrees of freedom for chi-square are (R-1)(C-1)=(2)(3)=6 and α=0.05, we would reject H_0 if $\chi^2_P > \chi^2_{0.05,6} = 12.59$.

The expected frequencies are computed and shown as in Table 2.

Table 2. Expected Frequencies are given in parenthesis.

		Same medication: administered by four methods.				
		Method 1	**Method 2**	**Method 3**	**Method 4**	**Total**
Level of patient improvement	**Major improvement**	50 (50)	55 (40)	50 (60)	25 (30)	180
	Slight improvement	120 (100)	75 (80)	100 (120)	65 (60)	360
	No improvement	80 (100)	70 (80)	150 (120)	60 (60)	360
Total		250	200	300	150	n=900 patients

The test statistic is $\chi_P^2 = \sum_{i=1}^{R}\sum_{j=1}^{C}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$ and computations:

$$\chi_P^2 = \sum_{i=1}^{3}\sum_{j=1}^{4}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(50-50)^2}{50}+\frac{(120-100)^2}{100}+\frac{(80-100)^2}{100}+\frac{(55-40)^2}{40}$$
$$+\frac{(75-80)^2}{80}+\frac{(70-80)^2}{80}+\frac{(50-60)^2}{60}+\frac{(100-120)^2}{120}$$
$$+\frac{(150-120)^2}{120}+\frac{(25-30)^2}{30}+\frac{(65-60)^2}{60}+\frac{(60-60)^2}{60}=28.94$$

Conclusions: Since $\chi_P^2 = 28.94 > \chi^2_{0.05,6} = 12.59$, we reject the null hypothesis. Because the χ^2 value of the sample (28.94) exceeded the cutoff value of 12.59, we reject H_0. ***This data supports the alternative hypothesis,*** H_1, ***that the populations are not all homogeneous (not equally proportioned) with respect to patient improvement.*** Stated another way, the samples provide evidence that in the four populations, levels of patient improvement are different, that the fluctuation in sample results is not merely chance fluctuation, but fluctuation due to actual differences in patient improvement among the four treatment groups.

Example 2: It has long been known that offenders (suçlular) who commit (suç işlemek) misdemeanors (hafif suçları) and felonies (ağır suç, cinayet) often commit crimes under the influence of drugs. A criminologist wants to examine the drug of choice for drug-involved offenders who committed crimes for which they were arrested while under the influence. A total of 140 offenders (some were arrested for felonies while others were arrested for misdemeanors) are sampled and each was asked to indicate the nature of their arrest and which drug was in their system at the time of their offense(suç)/arrest. The following data indicate how many arrestees (tutuklu) were using any given category of drug:

Table 3. The numbers of arrestees using any given category of drug.

The types of Offense	Alcohol	Marijuana	Opiates (uyku ilacı, uyuşturucu ilaç)	Other	**Total**
Misdemeanors	29	25	18	8	80
Felons	11	15	22	12	60
Total	40	40	40	20	**140**

The question for this hypothesis test is whether there are any preferences among the four possible choices for these two groups. Are any of the drugs reported more or less often than would be expected simply by chance? (**test at 0.05 significance level).**

Solution:

Firstly, state the hypotheses and select the α level (α=0.05). ***For the chi-square test for independence,***

H_0: There is no relationship between offense type (misdemeanor versus felony) and the type of drugs that being used at the time of arrest.
H_1: There is a relationship between offense type (misdemeanor versus felony) and the type of drugs that being used at the time of arrest.

Since R=2 and C=4, the degrees of freedom for chi-square are (R-1)(C-1)=(1)(3)=3 and α=0.05, we would reject H_0 if $\chi^2_P > \chi^2_{0.05,3} = 7.81$.

The expected frequencies are computed and shown as in Table 4.

Table 4. Expected Frequencies.

	Alcohol	Marijuana	Opiates	Other	**Total**
Misdemeanors	$(40\times80)/140$	$(40\times80)/140$	$(40\times80)/140$	$(20\times80)/140$	80
Felons	$(40\times60)/140$	$(40\times60)/140$	$(40\times60)/140$	$(20\times60)/140$	60
Total	40	40	40	20	**140**

	Alcohol	Marijuana	Opiates	Other	**Total**
Misdemeanors	22.9	22.9	22.9	11.4	80
Felons	17.1	17.1	17.1	8.6	60
Total	40	40	40	20	**140**

The test statistic is $\chi^2_P = \sum_{i=1}^{R}\sum_{j=1}^{C}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$ and computations:

$$\chi^2_P = \sum_{i=1}^{2}\sum_{j=1}^{4}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} = \frac{(29-22.9)^2}{22.9}+\frac{(25-22.9)^2}{22.9}+\frac{(18-22.9)^2}{22.9}+\frac{(8-11.4)^2}{11.4}$$
$$+\frac{(11-17.1)^2}{17.1}+\frac{(15-17.1)^2}{17.1}+\frac{(22-17.1)^2}{17.1}+\frac{(12-8.6)^2}{8.6} = 9.07$$

Conclusions: Since $\chi^2_P = 9.07 > \chi^2_{0.05,3} = 7.81$, we reject the null hypothesis. Thus, there is a relationship between offense type (misdemeanor versus felony) and the type of drugs that being used at the time of arrest.

The strengths of relationship between offense type (misdemeanor versus felony) and the type of drugs used by arrestees could be measured by using Cramer's V (Row variable: Nominal, Column Variable: Nominal). A measure of association independent of sample size. This statistic is a modification of the Phi statistic so that it is appropriate for larger than 2×2 tables.

$$V = \sqrt{\frac{\chi_P^2}{n\left(min\left(R,C\right)-1\right)}} = \sqrt{\frac{\chi_P^2}{n\,min\left(R-1,C-1\right)}} = \sqrt{\frac{9.07}{140\times \min\left(2-1,4-1\right)}} = \sqrt{\frac{9.07}{140}} = 0.25$$

It could be mentioned that there is not a strong (moderate relationship) relationship (%25) between offense type (misdemeanor versus felony) and the type of drugs used by arrestees at the time of arrest.

Example 3: The following represent mortality data for two groups of patients receiving different treatments, A and B. Is there a relationship between treatment and mortality? Test at 0.05 significance level.

Table 5. Mortality data.

		Outcome		
		Dead	Alive	**Total**
Treatment/Exposure	A	41	216	257
	B	64	180	244
Total		105	396	**501**

Solution:

Firstly, state the hypotheses and select the α level (α=0.05). ***For the chi-square test for independence,***

H_0: There is no relationship between treatment and mortality.
H_1: There is a relationship between treatment and mortality.

Since R=2 and C=2, the degrees of freedom for chi-square are (R-1)(C-1)=(1)(1)=1 and α=0.05, we would reject H_0 if $\chi_P^2 > \chi_{0.05,1}^2 = 3.84$.

The expected frequencies are computed and shown as in Table 6.

Table 6. Expected Frequencies for mortality data are given in parentheses.

		Outcome		
		Dead	Alive	**Total**
Treatment/Exposure	A	41 (53.86)	216 (203.14)	257
	B	64 (51.14)	180 (192.86)	244
Total		105	396	**501**

The test statistic is $\chi_P^2 = \sum_{i=1}^{R}\sum_{j=1}^{C}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}}$ and computations:

$$\chi_P^2 = \sum_{i=1}^{2}\sum_{j=1}^{4}\frac{\left(O_{ij}-E_{ij}\right)^2}{E_{ij}} = \frac{\left(41-53.86\right)^2}{53.86} + \frac{\left(216-203.14\right)^2}{203.14} + \frac{\left(64-51.14\right)^2}{51.14} + \frac{\left(180-192.86\right)^2}{192.86}$$
$$= 3.07 + 0.81 + 3.23 + 0.85$$
$$= 7.96$$

Conclusions: Since $\chi_P^2 = 7.96 > \chi_{0.05,1}^2 = 3.84$, we reject the null hypothesis. Thus, there is a relationship between treatment and mortality.

The strengths of relationship between treatment and mortality could be measured by using **Phi or Pearson's Contingency Coefficient,** both of these measures of association coefficients independent of the sample size.

Phi coefficient is $\phi = \sqrt{\frac{\chi_P^2}{n}} = \sqrt{\frac{7.96}{501}} = 0.1260$

and Pearson's Contingency Coefficient is $C = \sqrt{\frac{\chi_P^2}{\chi_P^2 + n}} = \sqrt{\frac{7.96}{7.96 + 501}} = 0.1250$

It could be mentioned that there is not a strong (weak) relationship (% 12.50) between treatment and mortality.

Example 4: (Software testing example) Are differences in success proportions for techniques 1 and 2 significantly different for these 25 targets? Test at 5% level.

Table 7. Software testing data.

		Technique 2		
		Yes	**No**	**Total**
Technique 1	**Yes**	3	5	8
	No	7	10	17
	Total	10	15	**25**

Solution:
Firstly, state the hypotheses. **For the chi-square test of homogeneity,**

H_0: There are not differences in success proportions for techniques 1 and 2 significantly different for these 25 targets.
H_1: There are differences in success proportions for techniques 1 and 2 significantly different for these 25 targets.

Since observed count for $O_{11} = 3 < 5$ **we can use Fisher Exact test.** Moreover, as it is a 2×2 Table and 2 cells (50.0%) have expected count less than 5, we can use Fisher Exact test.

Table 8. Expected Frequencies for Software testing data.

		Technique 2		
		Yes	**No**	**Total**
Technique 1	**Yes**	3.2	4.8	8
	No	6.8	10.2	17
	Total	10	15	**25**

$$P_r(3,7,5,10) = \frac{10!15!8!17!}{25!3!7!5!10!} = 0.3332 \qquad P_r(2,8,6,9) = \frac{10!15!8!17!}{25!2!8!6!9!} = 0.2082$$

$$P_r(1,9,7,8) = \frac{10!15!8!17!}{25!1!9!7!8!} = 0.0595 \qquad P_r(0,10,8,7) = \frac{10!15!8!17!}{25!0!10!8!7!} = 0.0059$$

Tail probability = 0.3332+0.2082+0.0595+0.0059 = 0.6068

p-value≅0.607>0.05 H_0 is accepted that means it does not matter we choose Technique 1 or Technique 2 as they have same performance.

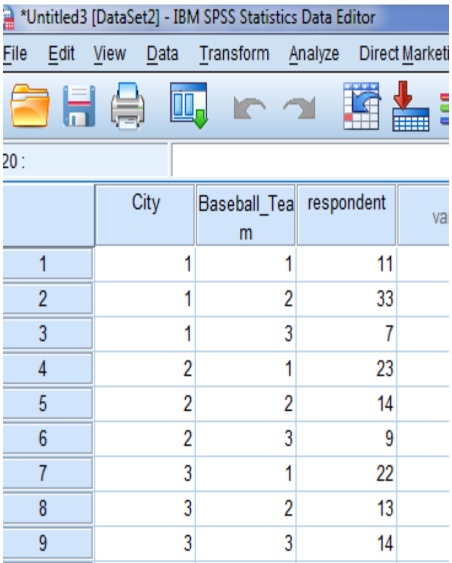
SPSS APPLICATIONS

Example: A typical cross-tabulation table comparing the two hypothetical variables "City of Residence (İkamet Etme, Oturma)" with "Favorite Baseball Team" is shown below. Are city of residence and being a fan of that city's Baseball team independent? The cells of the Table given in below report the frequency counts of respondents in each cell.

		What is Your Favorite Baseball Team?			
		Toronto Blue Jays	Boston Red Socks	New York Yankees	**Totals**
In What City Do You Reside?	Boston, MA	11	33	7	51
	Montreal, Canada	23	14	9	46
	Montpellier, VT	22	13	14	49
	Totals	56	60	30	**n=146**

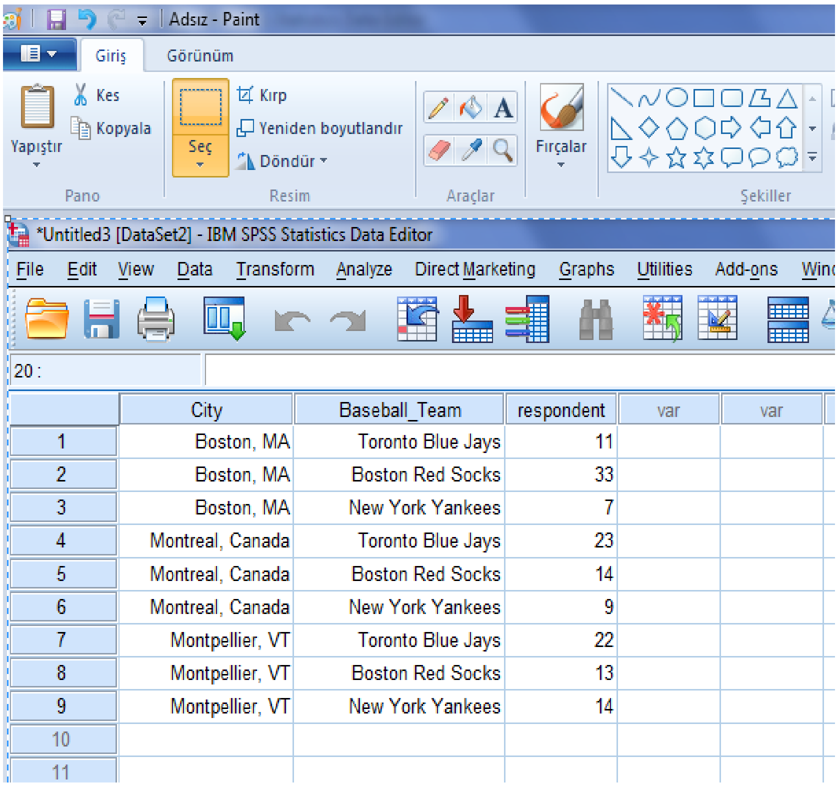
H*$_0$*: *City of residence and* being a fan of that city's Baseball team *are independent.*
H*$_1$*: *City of residence and* being a fan of that city's Baseball team *are not independent.*

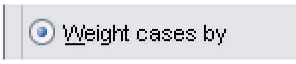
City column shows categories of row, Baseball-Team column shows categories of column and respondent column shows the data in each related cells.



	City	Baseball_Team	respondent
1	1	1	11
2	1	2	33
3	1	3	7
4	2	1	23
5	2	2	14
6	2	3	9
7	3	1	22
8	3	2	13
9	3	3	14

The categories of row and column variable are labeled from variable view.



Before starting analysis from Data → Weight Cases send respondent column to Frequency Variable section, then OK . Here also must be clicked 

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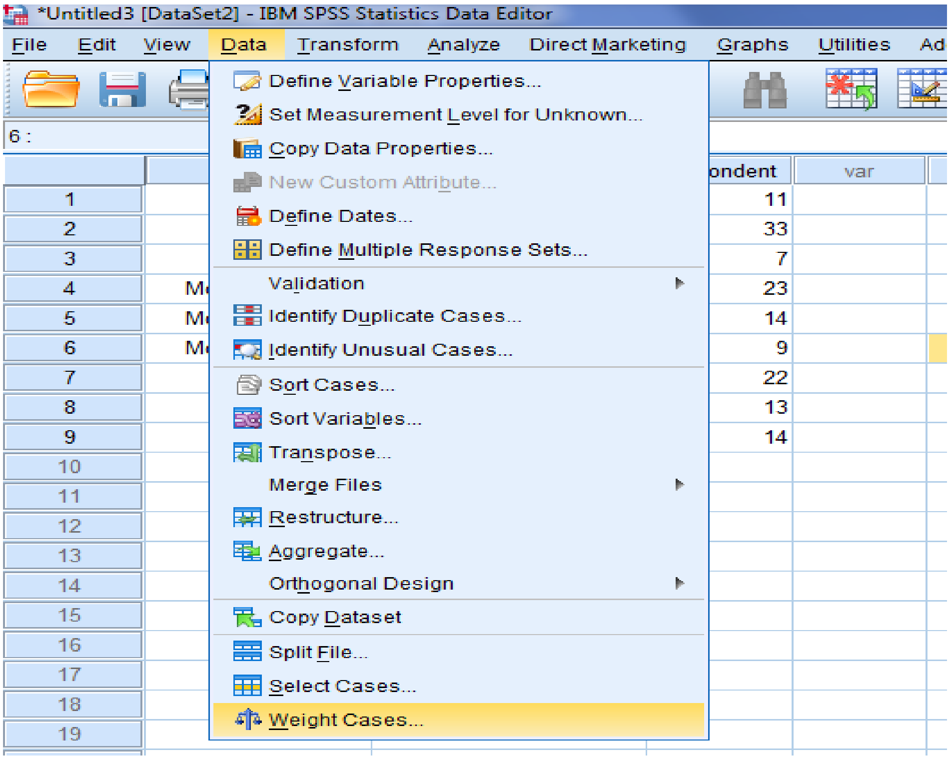
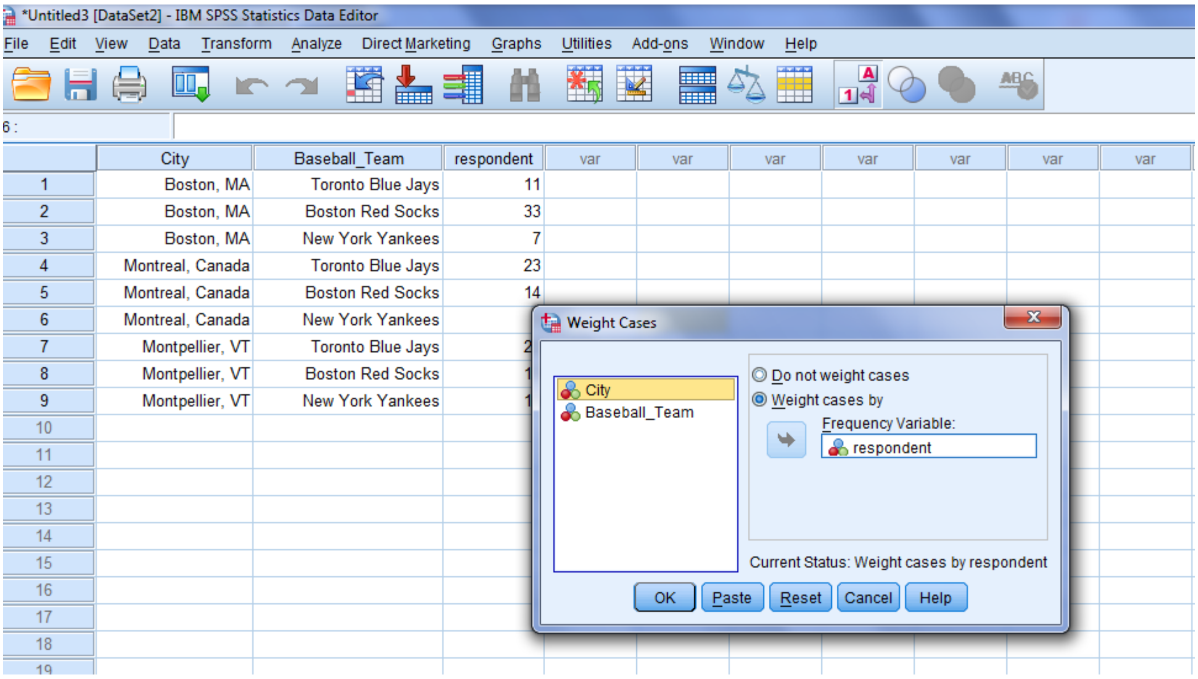
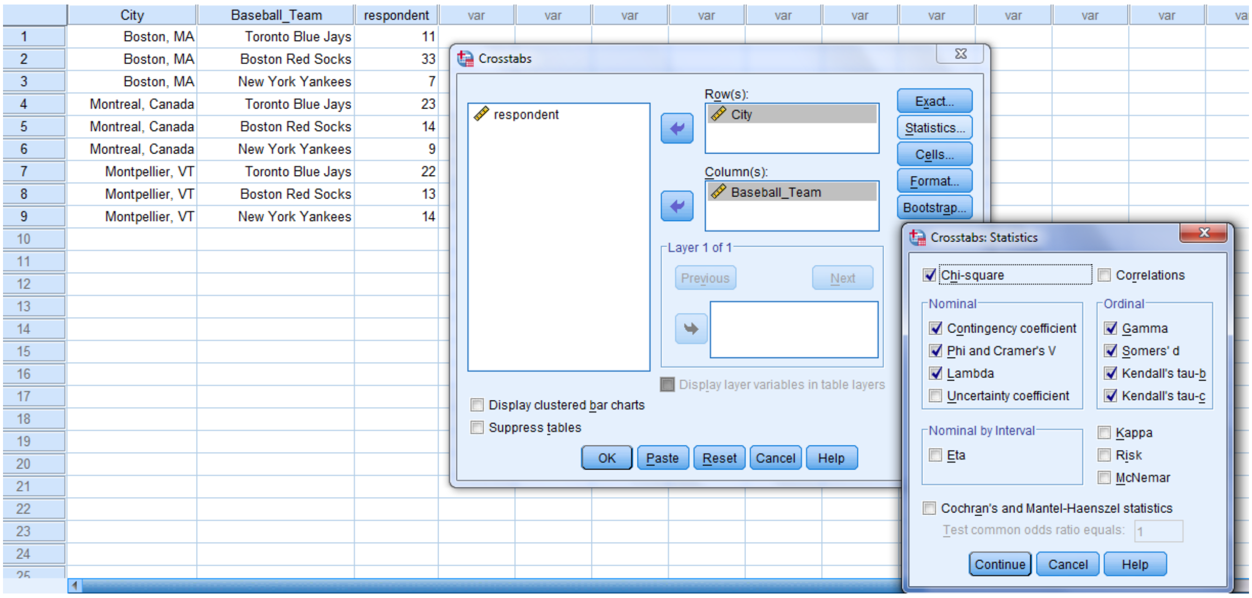
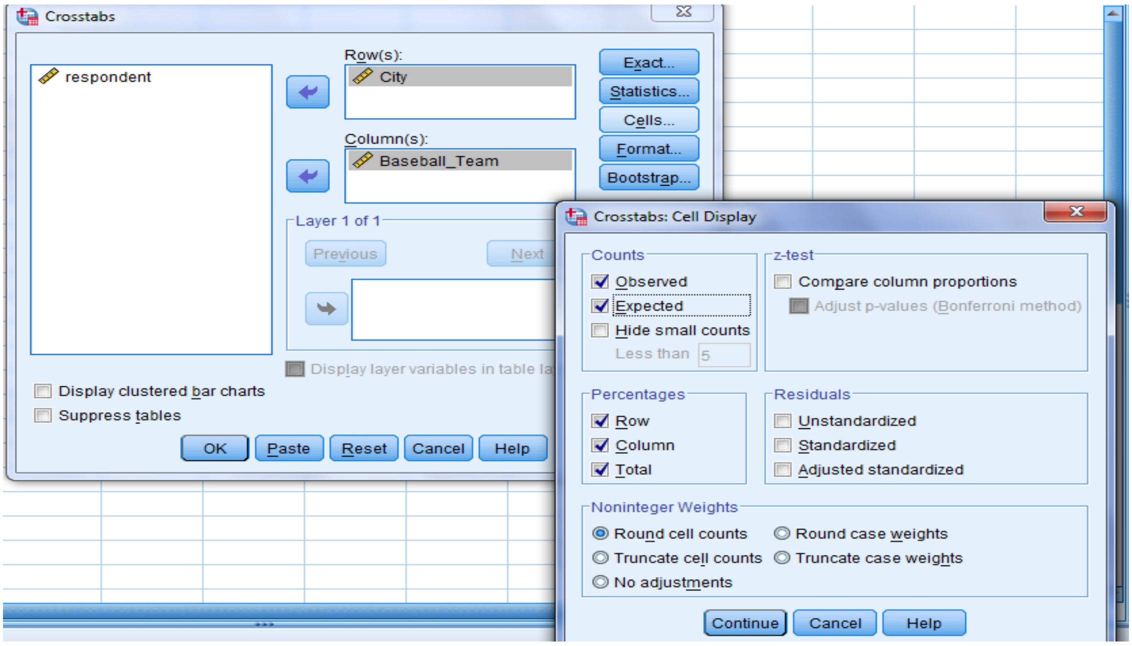




Analyze → Descriptive Statistics → Cross Tabs then City variable is under Row(s) and Baseball-Team variable under Column (s). Click Statistics then you can choose nominal and ordinal coefficients and Chi-Square.



Example of Calculations of Expected Frequencies, Table Percentages, Row Percentages and Column Percentages in SPSS (Analyze → Descriptive Statistics → Cross Tabs then City variable is under Row(s) and Baseball-Team variable under Column (s). Click Cells)



OUTPUTS

Outputs of Expected Frequencies, Table Percentages, Row Percentages and Column Percentages

City * Baseball_Team Crosstabulation

			Baseball_Team			Total
			Toronto Blue Jays	Boston Red Socks	New York Yankees	
City	Boston, MA	Count	11	33	7	51
		Expected Count	19,6	21,0	10,5	51,0
		% within City	21,6%	64,7%	13,7%	100,0%
		% within Baseball_Team	19,6%	55,0%	23,3%	34,9%
		% of Total	7,5%	22,6%	4,8%	34,9%
	Montreal, Canada	Count	23	14	9	46
		Expected Count	17,6	18,9	9,5	46,0
		% within City	50,0%	30,4%	19,6%	100,0%
		% within Baseball_Team	41,1%	23,3%	30,0%	31,5%
		% of Total	15,8%	9,6%	6,2%	31,5%
	Montpellier, VT	Count	22	13	14	49
		Expected Count	18,8	20,1	10,1	49,0
		% within City	44,9%	26,5%	28,6%	100,0%
		% within Baseball_Team	39,3%	21,7%	46,7%	33,6%
		% of Total	15,1%	8,9%	9,6%	33,6%
Total		Count	56	60	30	146
		Expected Count	56,0	60,0	30,0	146,0
		% within City	38,4%	41,1%	20,5%	100,0%
		% within Baseball_Team	100,0%	100,0%	100,0%	100,0%
		% of Total	38,4%	41,1%	20,5%	100,0%

Comments About Percentages in City * Baseball_Team Crosstabulation (Examples)

21.6% of people who live in Boston, MA are also fan of Toronto Blue Jays. (Example for % within City)

21.7% of people who are fan of Boston Red Socks also live in Montpellier, VT. (Example for % within Baseball_Team)

31.5% people live in Montreal, Canada. (Example of % of Total for Row)

20.5% of people are fan of New York Yankees. (Example of % of Total for Column)

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	19,351[a]	4	,001
Likelihood Ratio	19,331	4	,001
Linear-by-Linear Association	,338	1	,561
N of Valid Cases	146		

a. 0 cells (0,0%) have expected count less than 5. The minimum expected count is 9,45.

Are city of residence and being a fan of that city's Baseball team independent? Test the hypotheses in above.

Since 0 cells (0,0%) have expected count less than 5, we can use the Pearson Chi-Square results as it gives a p-value=0.001<0.05 (or $\chi^2_P = 19.351 > \chi^2_{0.05,4} = 9.48773$), H_0 is rejected. City of residence and being a fan of that city's Baseball team are not independent at the 0.05 significance level.

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Nominal by Nominal	Phi	,364			,001
	Cramer's V	,257			,001
	Contingency Coefficient	,342			,001
Ordinal by Ordinal	Kendall's tau-b	-,070	,074	-,950	,342
	Kendall's tau-c	-,068	,072	-,950	,342
	Gamma	-,103	,108	-,950	,342
N of Valid Cases		146			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

Since both city of residence and being a fan of that city's Baseball team variables are nominal variables we will look the coefficients under nominal by nominal. We can look Cramer's V (25.7%) or Contingency Coefficient (34.2%) values. It could be mentioned that there is not a strong (moderate relationship) relationship between city of residence and being a fan of that city baseball team

We can test the significance of this correlation:

H_0: The relationship between city of residence and being a fan of that city's baseball team is not important.

H_1: The relationship between city of residence and being a fan of that city's baseball team is important.

Since p-value (Approx. Sig)=0.001<0.05, we reject the null hypothesis, then H_0 is rejected that we accept this is an statistically important relationship between city of residence and being a fan of that city's baseball team.

SPSS Application of Fisher Exact Test's (2×2 Crosstabs)

SPSS Application for Example 4

OUTPUTS:

Technique1 * Technique2 Crosstabulation

Count

		Technique2		Total
		yes	no	
Technique1	yes	3	5	8
	no	7	10	17
Total		10	15	25

Technique1 * Technique2 Crosstabulation

			Technique2		Total
			yes	no	
Technique1	yes	Count	3	5	8
		Expected Count	3,2	4,8	8,0
	no	Count	7	10	17
		Expected Count	6,8	10,2	17,0
Total		Count	10	15	25
		Expected Count	10,0	15,0	25,0

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	,031[a]	1	,861		
Continuity Correction[b]	,000	1	1,000		
Likelihood Ratio	,031	1	,861		
Fisher's Exact Test				1,000	,607
Linear-by-Linear Association	,029	1	,864		
N of Valid Cases	25				

a. 2 cells (50,0%) have expected count less than 5. The minimum expected count is 3,20.
b. Computed only for a 2x2 table

The p-value for **Fisher's test** is given as 0.607. Since p-value is greater than α ($p=0.607>\alpha=0.05$) null hypothesis H_0 is accepted. There are no differences in success proportions for techniques 1 and 2 for these 25 targets.