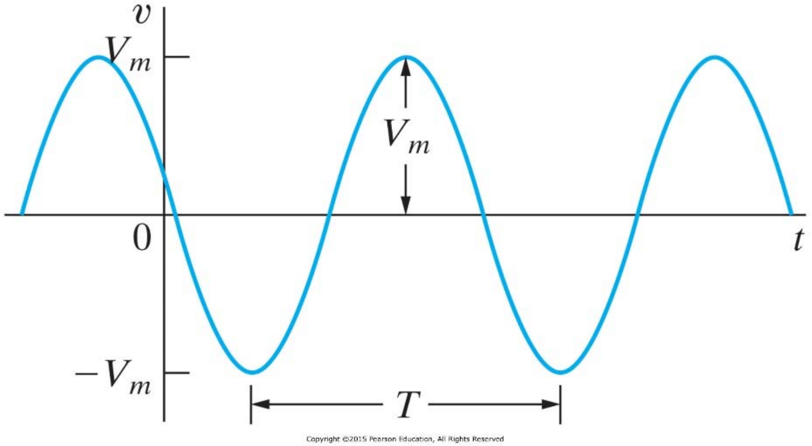
ELE 296 Basic Electric Circuits and Electronics

- Sinusoidal Steady State Analysis
  - •Sinusoidal Source
  - •Phasor Concept
  - •Kirchhoff Laws in the Frequency Domain
  - •Source Transformation itFD
  - •Thévenin–Norton Equivalent Circuits itFD
  - •Node Voltage Method itFD
  - •Mesh Current Method itFD

Sinusoidal Steady State

- Sinusoidal Voltage:

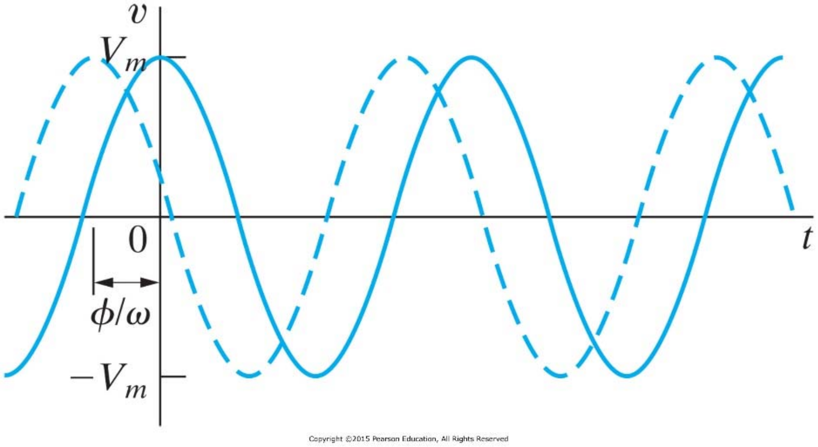


$$v(t) = V_m \cos(\omega t + \phi)\ \text{V}$$

- V_m : Amplitude
- ω : angular frequency [rad/s]
- t : time [s]
- ϕ : phase angle [deg]
- $f = \omega / 2\pi$: frequency [Hz]
- $T = 1/f$: period [s]

Sinusoidal Steady State

- A change in phase angle:



- RMS (Root Mean Square) value:

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t+\phi)dt}$$

$$= \frac{V_m}{\sqrt{2}}$$

Sinusoidal Steady State

- Qn. 1: A sinusoidal voltage is given as

  $$v=300\cos(120\pi t+30^\circ)$$

  Define the period?
  Define the frequency?
  What is v for t=2.778ms?
  What is the rms value of v?
  Define v as a sine function.

Sinusoidal Steady State

- Qn. 1 (cont.): $v = 300\cos\left(120\pi t + 30^{\circ}\right)$

$$\omega = 120\pi = \frac{2\pi}{T}$$

$$\Rightarrow T = 16.667\,\text{ms}$$

$$f = \frac{1}{T}$$

$$\Rightarrow f = 60\,\text{Hz}$$

$$v = 300\cos\left(120\pi t + 30^{\circ}\right) = 300\cos\left(120\pi t \times \frac{180}{\pi} + 30^{\circ}\right)$$

$$v = 300\cos\left(21600t + 30^{\circ}\right)$$

$$\Rightarrow v\Big|_{t=2.778ms} = 300\cos\left(60^{\circ} + 30^{\circ}\right) = 0\text{V}$$

Sinusoidal Steady State

- Qn. 1 (cont.): $v = 300\cos\left(120\pi t + 30^\circ\right)$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = \frac{300}{\sqrt{2}} = 212.13\text{V}$$

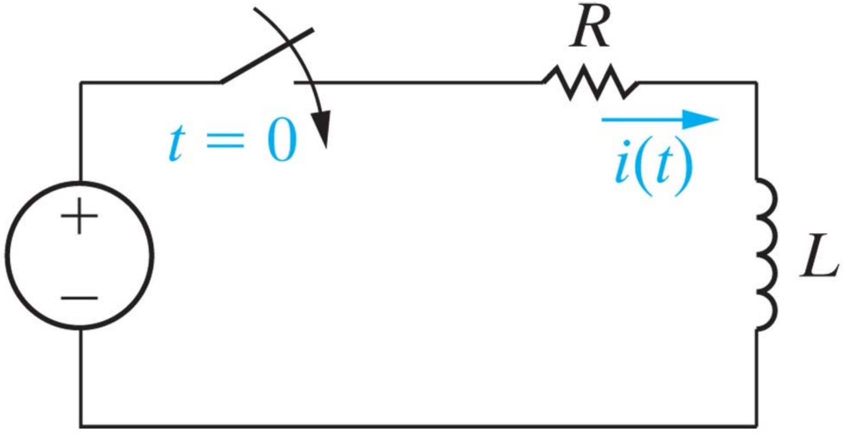
$$\sin(x) = \cos\left(x - 90^\circ\right)$$
$$\cos(x) = \sin\left(90^\circ - x\right)$$
$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$

$$v = 300\cos\left(120\pi t + 30^\circ\right) = 300\cos\left(-120\pi t - 30^\circ\right)$$
$$= \sin\left(90^\circ - \left(-120\pi t - 30^\circ\right)\right)$$
$$\Rightarrow v = \sin\left(120\pi t + 120^\circ\right)$$

Sinusoidal Steady State

- Sinusoidal Response:

$$V_m \cos(\omega t + \phi) = v_s$$



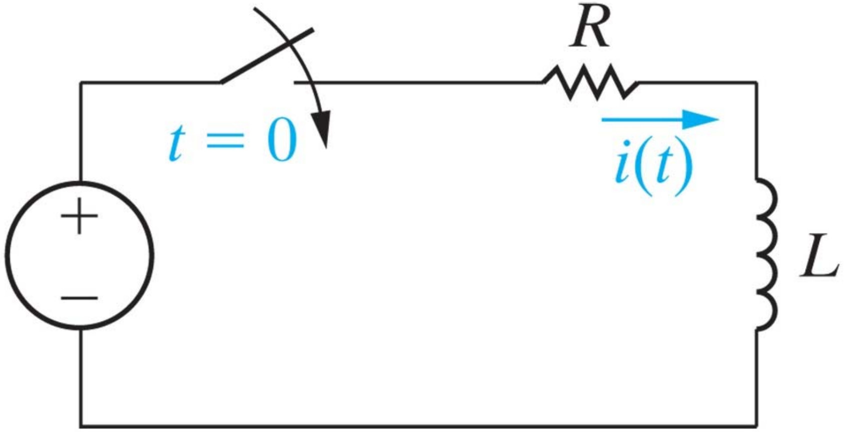
$$KVL: -v_s + v_R + v_L = -V_m \cos(\omega t + \phi) + iR + L\frac{di}{dt} = 0$$

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta), \quad \theta = \frac{\omega L}{R}$$

Sinusoidal Steady State

- Sinusoidal Response :

$$V_m \cos(\omega t + \phi) = v_s$$



Transient Response

Steady-State Response

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta), \quad \theta = \frac{\omega L}{R}$$

Sinusoidal Steady State

- Steady–State Response:

  - It is a sinusoidal function,
  - The frequency of the response and the source are the same.
  - The maximum amplitude of the response and the source are generally different.
  - The phase of the response and the source are generally different.

$$v_s = V_m \cos(\omega t + \phi) \qquad i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

Sinusoidal Steady State

- Phasor:

  - It is a complex number that carries the amplitude and phase information.

  - Euler Equation:

$$e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$$

$$\cos\theta = \mathrm{Re}\left\{e^{j\theta}\right\}$$

$$\sin\theta = \mathrm{Im}\left\{e^{j\theta}\right\}$$

  - Sinusoidal Source:

$$\begin{aligned} v &= V_m \cos(\omega t + \phi) \\ &= V_m \mathrm{Re}\left\{e^{j(\omega t + \phi)}\right\} = V_m \mathrm{Re}\left\{e^{j\omega t} e^{j\phi}\right\} \\ &= \mathrm{Re}\left\{V_m e^{j\phi} e^{j\omega t}\right\} \end{aligned}$$

Phasor representation including the amplitude and phase!!!

Sinusoidal Steady State

- Phasor (cont.):
  - Phasor Transformation:

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi = V_m \cos\phi + jV_m \sin\phi$$
$$= \mathbf{P}\{V_m \cos(\omega t + \phi)\}$$

  - Phasor transformation transforms the function from time domain to complex number domain.
  - Also called **Frequency Domain.**
  - Inverse Phasor Transform:

$$\mathbf{P}^{-1}\{V_m e^{j\phi}\} = V_m \cos(\omega t + \phi)$$

ω can not be known

Sinusoidal Steady State

- Qn. 2: $y_1 = 20\cos(\omega t - 30^\circ),\ y_2 = 40\cos(\omega t + 60^\circ)$
  define $y_1 + y_2$ using phasor consept.

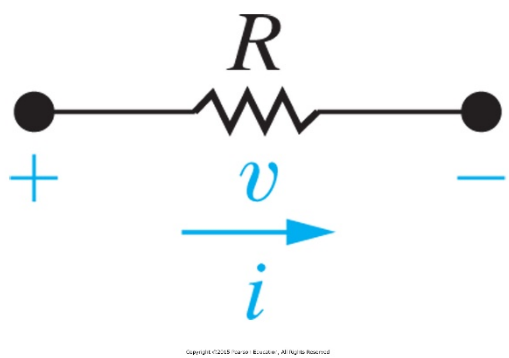
$$y_1 = 20\cos(\omega t - 30^\circ) \Rightarrow \mathbf{Y}_1 = 20\angle -30^\circ$$
$$y_2 = 40\cos(\omega t + 60^\circ) \Rightarrow \mathbf{Y}_2 = 40\angle 60^\circ$$

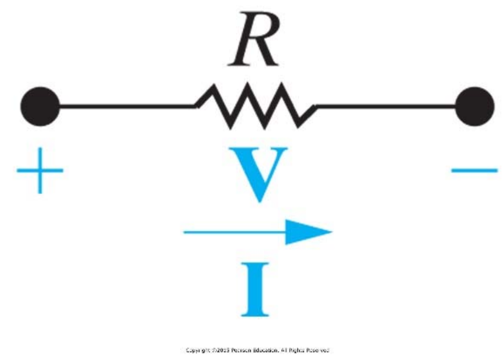
$$\begin{aligned}\mathbf{Y}_1 + \mathbf{Y}_2 &= 20\angle -30^\circ + 40\angle 60^\circ = 20e^{-j30^\circ} + 40e^{j60^\circ} \\ &= (17.32 - j10) + (20 + j34.64) \\ &= 37.32 + j24.64 \\ &= 44.72\angle 33.43^\circ\end{aligned}$$

$$\boxed{\begin{aligned}y_1 + y_2 &= \mathbf{P}^{-1}\{44.72\angle 33.43^\circ\} = \mathrm{Re}\{44.72e^{j33.43^\circ}e^{j\omega t}\} \\ &= 44.72\cos(\omega t + 33.43^\circ)\end{aligned}}$$

Sinusoidal Steady State

- V-I Relationship for a Resistor:





$$v = Ri$$

$$i = I_m \cos(\omega t + \phi_i)$$

$$v = RI_m \cos(\omega t + \phi_i)$$

$$\mathbf{I} = I_m e^{j\phi_i} = I_m \angle \phi_i$$

$$\mathbf{V} = RI_m e^{j\phi_i} = RI_m \angle \phi_i$$

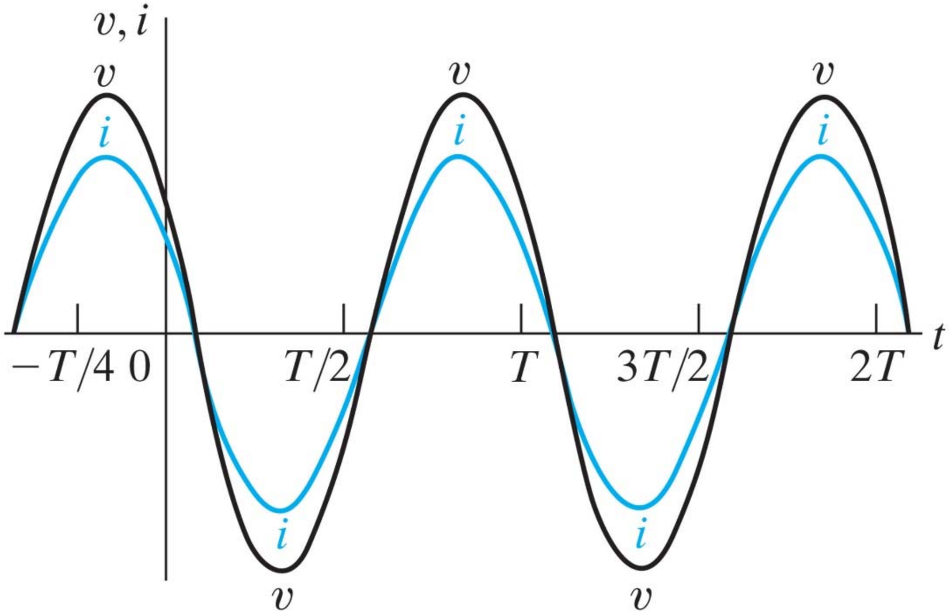
$$\boxed{\mathbf{V} = R\mathbf{I}}$$

- The voltage and current are called in-phase since the resistor does not cause a phase difference.

Sinusoidal Steady State

- V–I Relationship for a Resistor (cont.):

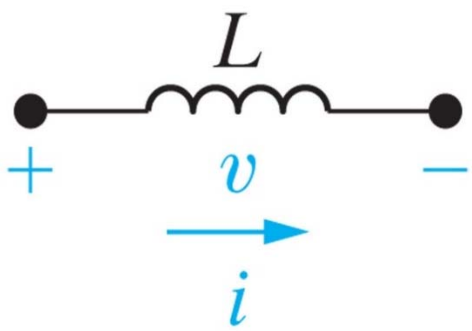
$$\boxed{\mathbf{V} = R\mathbf{I}}$$

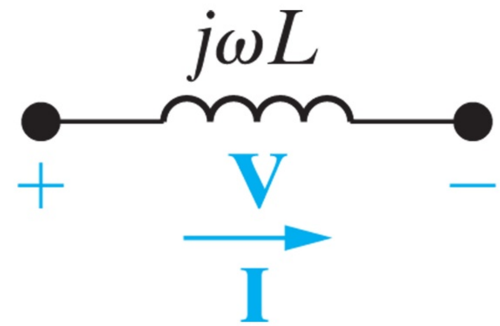


- The resistor does not cause a phase difference.

Sinusoidal Steady State

- V–I Relationship for an Inductor:





$$\boxed{\mathbf{V} = j\omega L\mathbf{I}}$$

$$i = I_m \cos(\omega t + \phi_i)$$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi_i)$$

$$= -\omega L I_m \cos(\omega t + \phi_i - 90^\circ)$$

$$\mathbf{I} = I_m e^{j\phi_i} = I_m \angle \phi_i$$

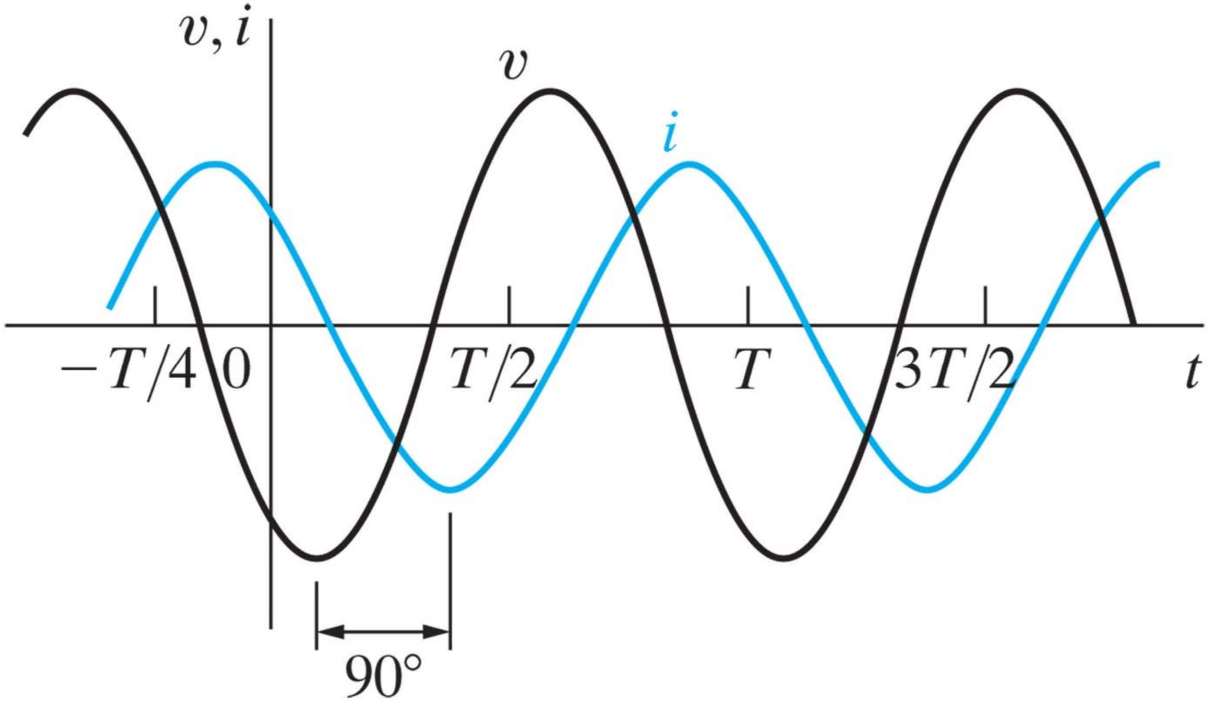
$$\mathbf{V} = -\omega L I_m e^{j(\phi_i - 90^\circ)} = -\omega L I_m e^{j\phi_i} e^{-j90^\circ}$$

$$= j\omega L I_m e^{j\phi_i} = j\omega L\mathbf{I} = \omega L I_m \angle(\phi_i + 90^\circ)$$

- Voltage leads current by 90°.

Sinusoidal Steady State

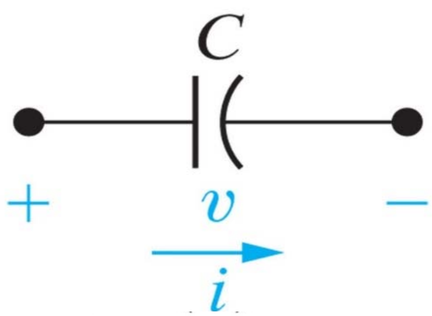
- V–I Relationship for an Inductor (cont.):

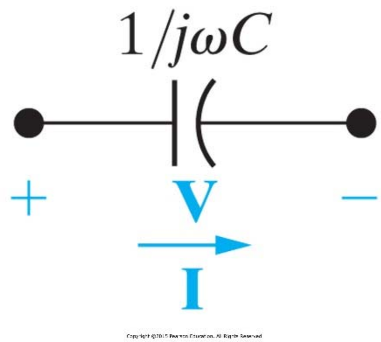


- Voltage leads current by 90°.

Sinusoidal Steady State

- V–I Relationship for a Capacitor :





$$\boxed{\mathbf{I} = j\omega C\mathbf{V}}$$

$$\boxed{\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}}$$

$$v = V_m \cos(\omega t + \phi_v)$$

$$i = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t + \phi_v)$$

$$= -\omega C V_m \cos(\omega t + \phi_v - 90^\circ)$$

$$\mathbf{V} = V_m e^{j\phi_v} = V_m \angle \phi_v$$

$$\mathbf{I} = -\omega C V_m e^{j(\phi_v - 90^\circ)} = -\omega C V_m e^{j\phi_v} e^{-j90^\circ}$$

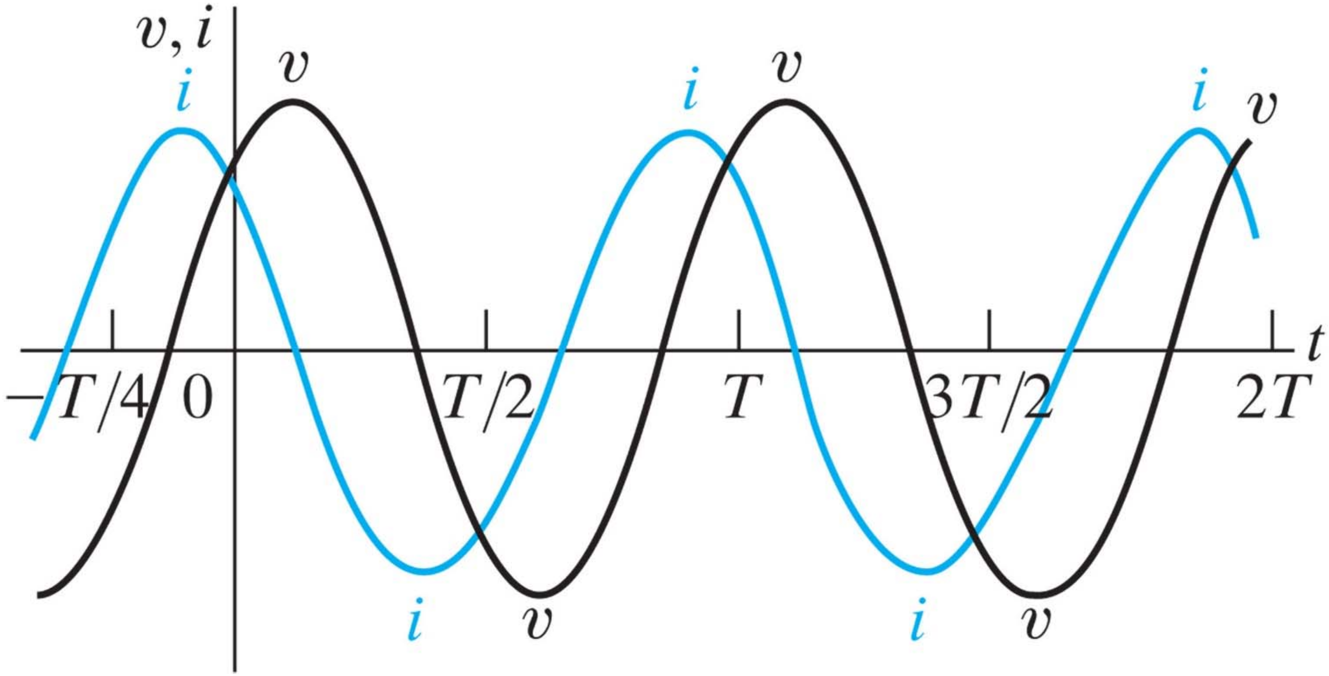
$$= j\omega C V_m e^{j\phi_i} = j\omega C\mathbf{V} = \omega C V_m \angle(\phi_v + 90^\circ)$$

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = -\frac{j}{\omega C}\mathbf{I} = \frac{1}{\omega C} I_m \angle(\phi_i - 90^\circ)$$

- Current leads voltage by 90°.

Sinusoidal Steady State

- V-I Relationship for a Capacitor (cont.):



- Current leads voltage by 90°.

Sinusoidal Steady State

- Impedance and Reactance:

$$\mathbf{V} = R\mathbf{I} \qquad \mathbf{V} = j\omega L\mathbf{I} \qquad \mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

$$\boxed{\mathbf{V} = Z\mathbf{I}}$$

- Z is the impedance of the circuit element.
- Unit for impedance is Ohm.
- It is a complex number but NOT A PHASOR.
- Impedance is defined in the frequency domain.
- The imaginary part of the impedance is called Reactance.

Sinusoidal Steady State

- Qn. 3: The current on a 20mH inductor is given as 10cos(10000t+30°) mA.

What is the reactance of the inductor?
What is the impedance of the inductor?
What is the phasor voltage V on the inductor?
Find the steady state voltage v(t) on the inductor.

Sinusoidal Steady State

- Qn. 3 (cont.): $i(t) = 10\cos(10000t + 30^\circ)\,\text{mA}$

$$\mathbf{V} = j\omega L\mathbf{I}$$

$$\Rightarrow \text{Reactance}: \omega L = 10000 \times 20 \times 10^{-3} = 200\Omega$$

$$\Rightarrow \text{Impedance}: Z = j\omega L = j10000 \times 20 \times 10^{-3} = j200\Omega$$

$$\Rightarrow \mathbf{V} = j\omega L\mathbf{I} = j200 \times 10 \times 10^{-3}\angle 30^\circ = 2\angle 120^\circ\,\text{V}$$

$$\Rightarrow v(t) = 2\cos(10000t + 120^\circ)\text{V}$$

Sinusoidal Steady State

- Qn. 4: The voltage on a 5μF capacitor is given as $30\cos(4000t+25°)$ V.

  What is the reactance of the capacitor?
  What is the impedance of the capacitor?
  What is the phasor current I on the capacitor?
  Find the steady state current i(t) on the capacitor.

Sinusoidal Steady State

- Qn. 4 (cont.): $v(t) = 30\cos\left(4000t + 25^\circ\right)\mathrm{V}$

$$\mathbf{V} = -\frac{j}{\omega C}\mathbf{I} \Leftrightarrow \mathbf{I} = j\omega C\mathbf{V}$$

$$\Rightarrow \text{Reactance}: -\frac{1}{\omega C} = -\frac{1}{4000 \times 5 \times 10^{-6}} = -50\Omega$$

$$\Rightarrow \text{Impedance}: Z = -\frac{j}{\omega C} = -\frac{j}{4000 \times 5 \times 10^{-6}} = -j50\Omega$$

$$\Rightarrow \mathbf{I} = j\omega C\mathbf{V} = j0.02 \times 30\angle 25^\circ = 0.6\angle 115^\circ\,\mathrm{A}$$

$$\Rightarrow i(t) = 0.6\cos\left(4000t + 115^\circ\right)\mathrm{A}$$

Sinusoidal Steady State

- Kirchoff Laws in Frequency Domain:

  Kirchoff Voltage Law

$$v_1 + v_2 + \cdots + v_n = 0$$

$$V_{m_1}\cos(\omega t + \phi_1) + V_{m_2}\cos(\omega t + \phi_2) + \cdots + V_{m_n}\cos(\omega t + \phi_n) = 0$$

$$\mathrm{Re}\{V_{m_1}e^{j\phi_1}e^{j\omega t}\} + \mathrm{Re}\{V_{m_2}e^{j\phi_2}e^{j\omega t}\} + \cdots + \mathrm{Re}\{V_{m_n}e^{j\phi_n}e^{j\omega t}\} = 0$$

$$\mathrm{Re}\{V_{m_1}e^{j\phi_1}e^{j\omega t} + V_{m_2}e^{j\phi_2}e^{j\omega t} + \cdots + V_{m_n}e^{j\phi_n}e^{j\omega t}\} = 0$$

$$\mathrm{Re}\{(V_{m_1}e^{j\phi_1} + V_{m_2}e^{j\phi_2} + \cdots + V_{m_n}e^{j\phi_n})e^{j\omega t}\} = 0$$

$$\mathrm{Re}\{(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}\} = 0$$

$$\boxed{\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0}$$

Sinusoidal Steady State

- Kirchoff Laws in Frequency Domain:

Kirchoff Current Law

$$i_1 + i_2 + \cdots + i_n = 0$$

$$I_{m_1}\cos(\omega t + \phi_1) + I_{m_2}\cos(\omega t + \phi_2) + \cdots + I_{m_n}\cos(\omega t + \phi_n) = 0$$

$$\mathrm{Re}\left\{I_{m_1} e^{j\phi_1} e^{j\omega t}\right\} + \mathrm{Re}\left\{I_{m_2} e^{j\phi_2} e^{j\omega t}\right\} + \cdots + \mathrm{Re}\left\{I_{m_n} e^{j\phi_n} e^{j\omega t}\right\} = 0$$

$$\mathrm{Re}\left\{I_{m_1} e^{j\phi_1} e^{j\omega t} + I_{m_2} e^{j\phi_2} e^{j\omega t} + \cdots + I_{m_n} e^{j\phi_n} e^{j\omega t}\right\} = 0$$

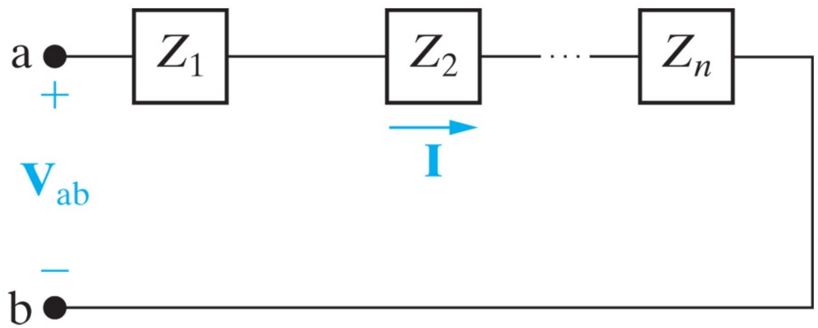
$$\mathrm{Re}\left\{\left(I_{m_1} e^{j\phi_1} + I_{m_2} e^{j\phi_2} + \cdots + I_{m_n} e^{j\phi_n}\right) e^{j\omega t}\right\} = 0$$

$$\mathrm{Re}\left\{\left(\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n\right) e^{j\omega t}\right\} = 0$$

$$\boxed{\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0}$$

Sinusoidal Steady State

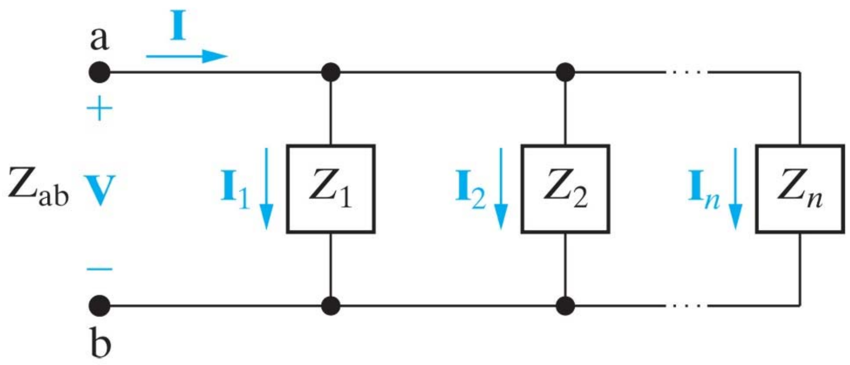
- Series and Parallel Impedances:



$$KVL: \mathbf{V}_{ab} = Z_1\mathbf{I} + Z_2\mathbf{I} + \cdots + Z_n\mathbf{I}$$
$$= \left(Z_1 + Z_2 + \cdots + Z_n\right)\mathbf{I}$$
$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}} = Z_1 + Z_2 + \cdots + Z_n$$

Sinusoidal Steady State

- Series and Parallel Impedances (cont.):



$$KCL: \mathbf{I} = \frac{\mathbf{V}}{Z_1} + \frac{\mathbf{V}}{Z_2} + \cdots + \frac{\mathbf{V}}{Z_3}$$

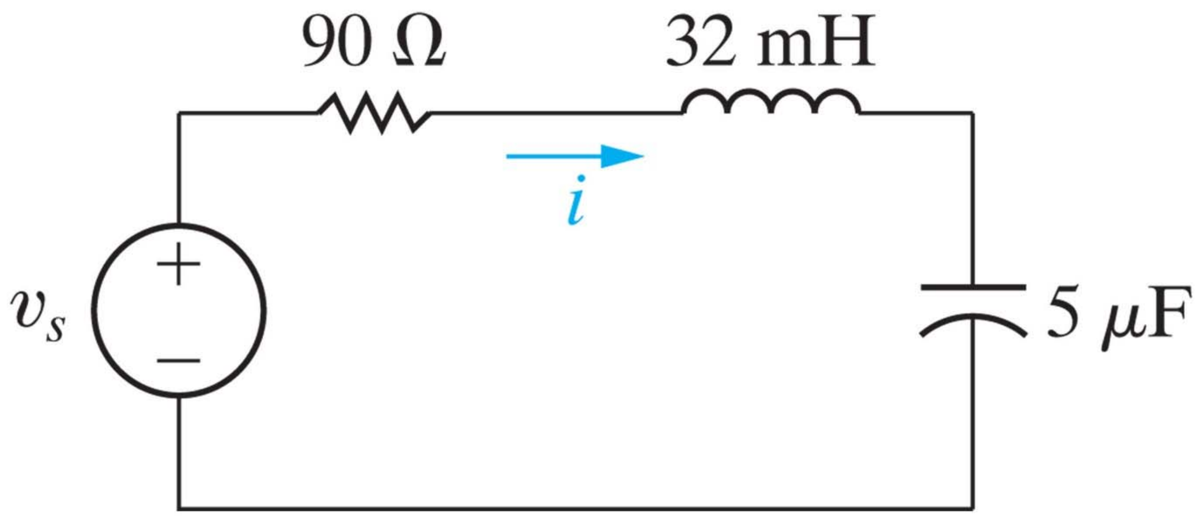
$$= \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_3} \right) \mathbf{V}$$

$$\frac{1}{Z_{ab}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_3}$$

Sinusoidal Steady State

- Qn. 5: $v_s = 750\cos(5000t + 30°)$ V.

  Draw the circuit in frequency domain. Find the steady state current i(t) using phasor concept.



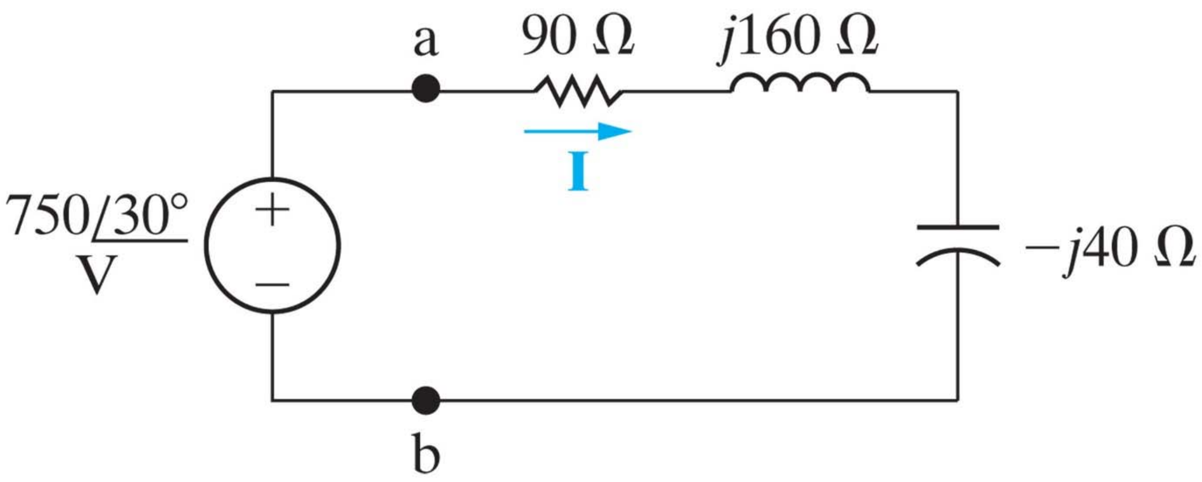
Sinusoidal Steady State

- Qn. 5 (cont.): $v_s(t) = 750\cos\left(5000t + 30^\circ\right)\text{V} \Rightarrow \mathbf{V} = 750\angle 30^\circ\ \text{V}$

$$Z_R = R = 90\Omega$$

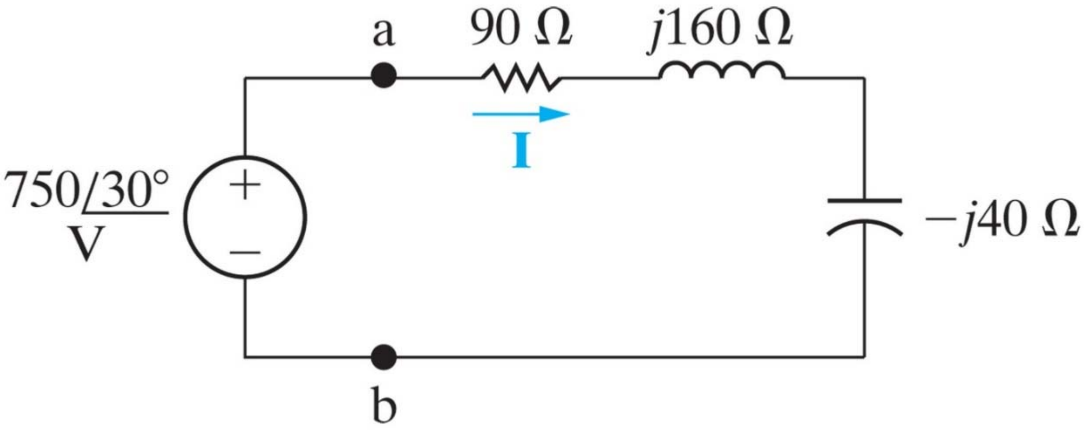
$$Z_L = j\omega L = j5000 \times 32 \times 10^{-3} = j160\Omega$$

$$Z_C = -\frac{j}{\omega C} = -\frac{j}{5000 \times 5 \times 10^{-6}} = -j40\Omega$$



Sinusoidal Steady State

- Qn. 5 (cont.):



$$KVL: \mathbf{V} = Z_1\mathbf{I} + Z_2\mathbf{I} + Z_3\mathbf{I} = (90 + j160 - j40)\mathbf{I}$$

$$\mathbf{V} = (90 + j120)\mathbf{I}$$

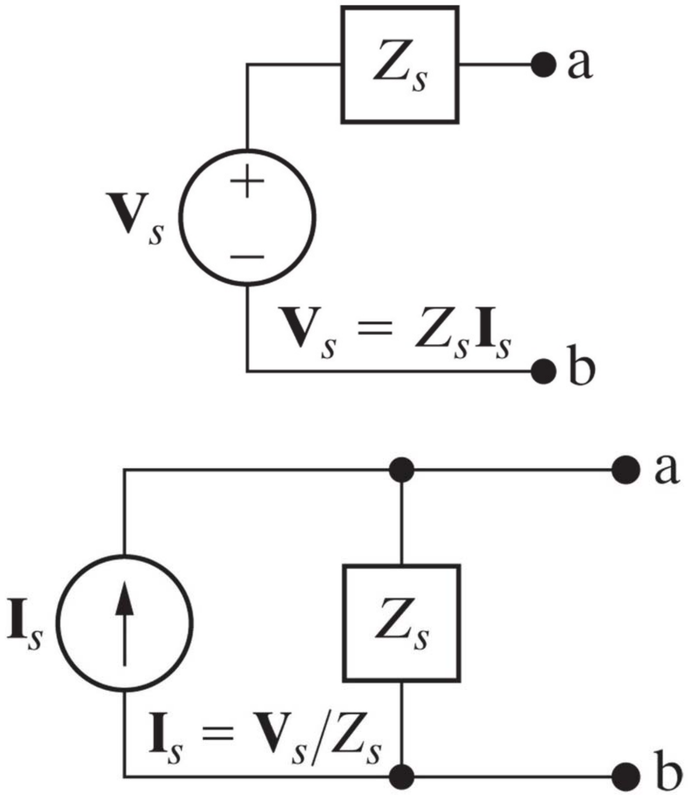
$$750\angle 30^\circ = 150\angle 53.13^\circ\,\mathbf{I}$$

$$\Rightarrow \mathbf{I} = \frac{750\angle 30^\circ}{150\angle 53.13^\circ} = 5\angle -23.13^\circ \text{ A}$$

$$\boxed{\Rightarrow i(t) = 5\cos(5000t - 23.13^\circ)\text{A}}$$

Sinusoidal Steady State

- Source Transformation:



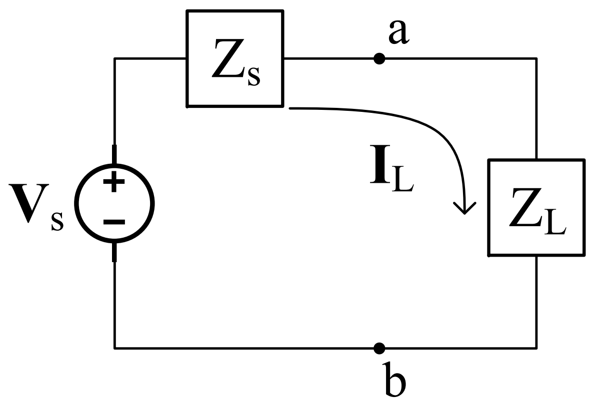
The circuits on the left are equivalent in time domain.

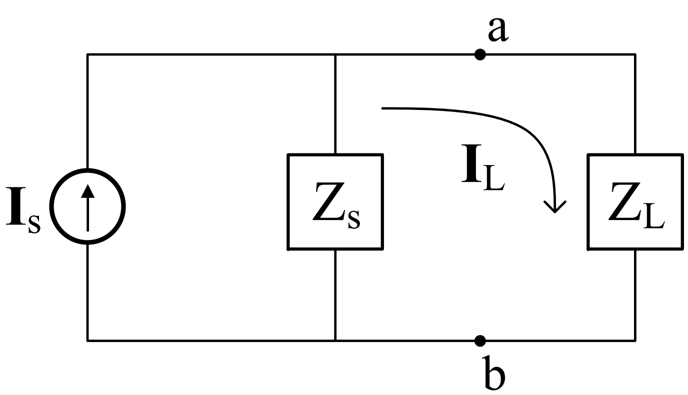
It can also be shown that they are equivalent in the frequency domain also.

The voltage and current in the terminals ab must be equal in order to say that these circuits are equivalent!!!

Sinusoidal Steady State

- Source Transformation (cont.):





$$\mathbf{I}_L = \frac{\mathbf{V}_S}{Z_s + Z_L}$$

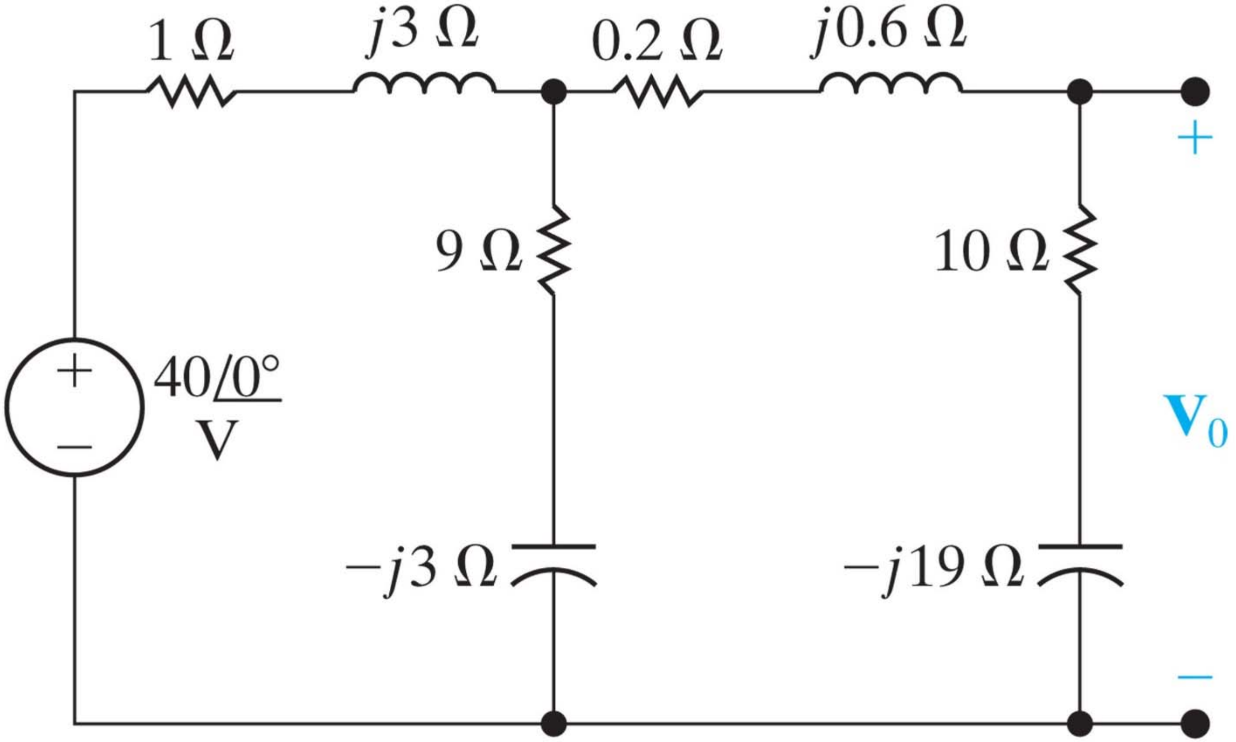
$$\mathbf{I}_L = \frac{Z_s}{Z_s + Z_L}\mathbf{I}_S$$

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{Z_s + Z_L} = \frac{Z_s}{Z_s + Z_L}\mathbf{I}_S$$

$$\boxed{\mathbf{I}_S = \frac{\mathbf{V}_S}{Z_s}}$$

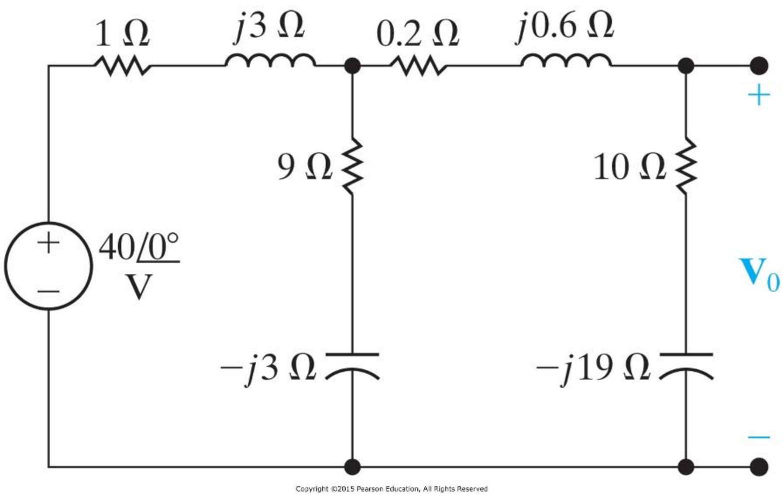
Sinusoidal Steady State

- Qn. 6: Find V_0 using source transformation.



Sinusoidal Steady State

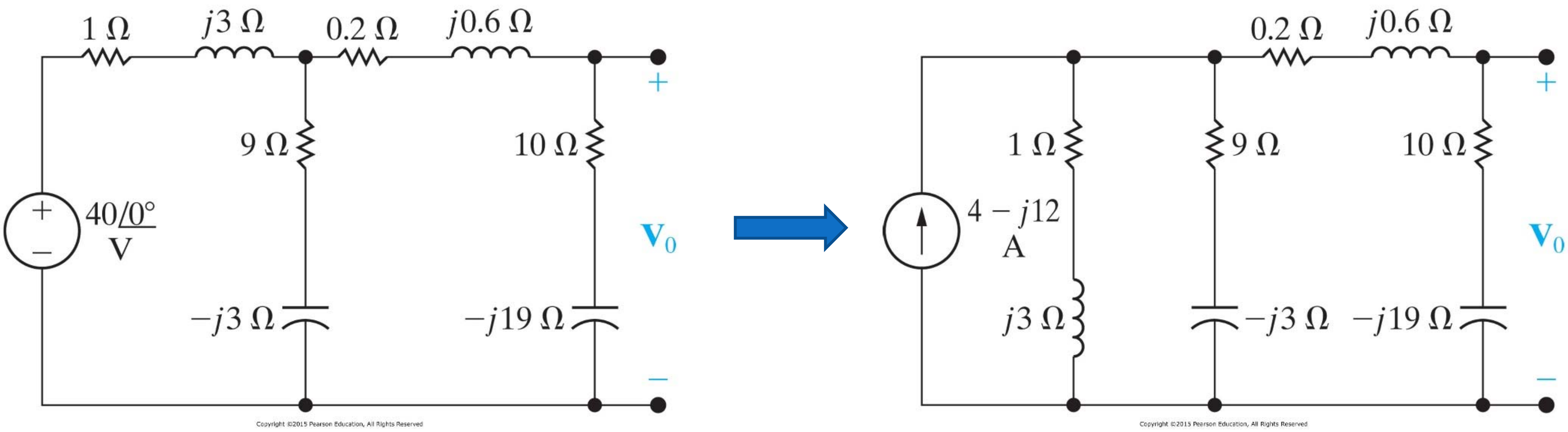
- Qn. 6 (cont.):



$$\mathbf{I} = \frac{40\angle 0^\circ}{1+j3} = \frac{40}{(1+j3)(1-j3)}(1-j3) = 4 - j12\text{A}$$

Sinusoidal Steady State

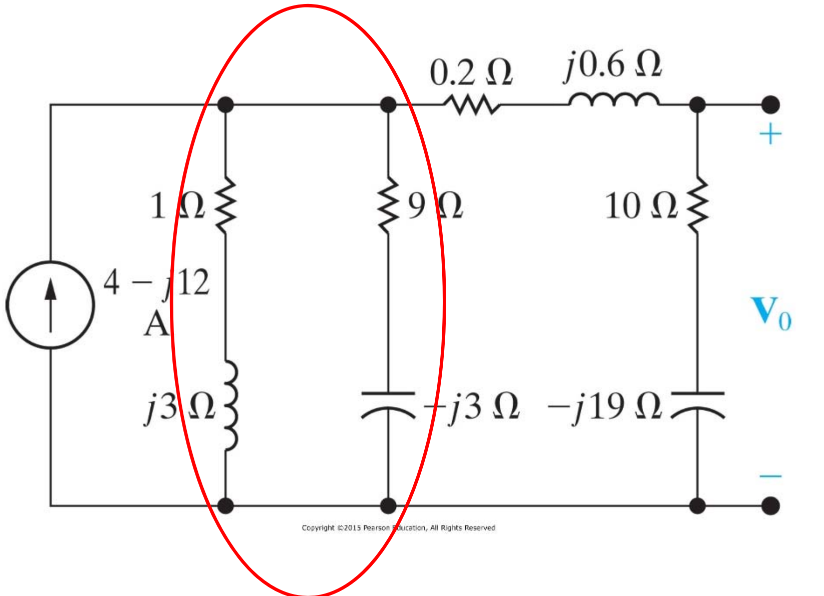
- Qn. 6 (cont.):



$$\mathbf{I} = \frac{40\angle 0^\circ}{1+j3} = \frac{40}{(1+j3)(1-j3)}(1-j3) = 4 - j12\text{A}$$

Sinusoidal Steady State

- Qn. 6 (cont.):

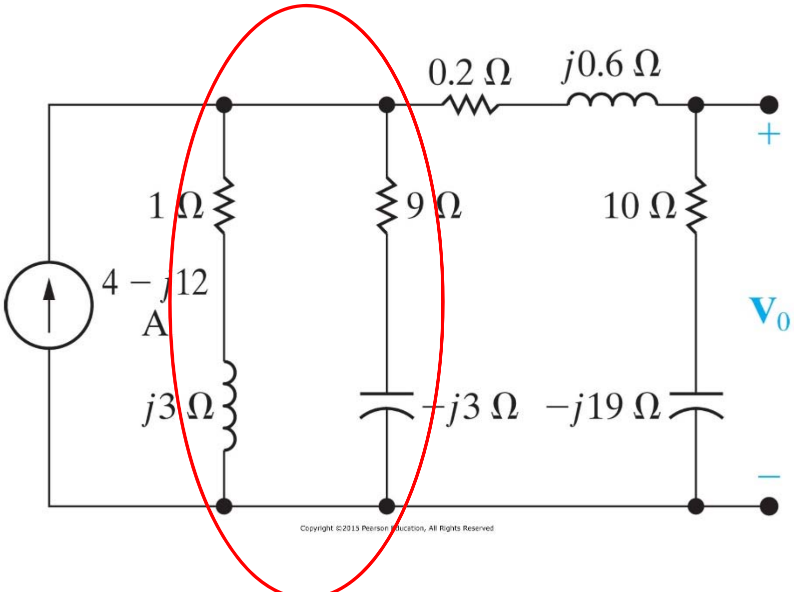


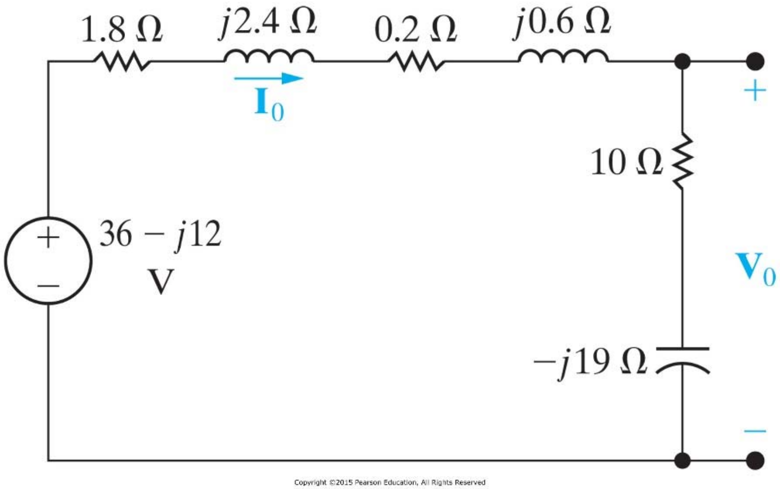
$$(1+j3)\|(9-j3)=\frac{(1+j3)(9-j3)}{10}=1.8+j2.4\Omega$$

$$\mathbf{V}=(4-j12\text{A})(1.8+j2.4\Omega)=36-12j\text{V}$$

Sinusoidal Steady State

- Qn. 6 (cont.):





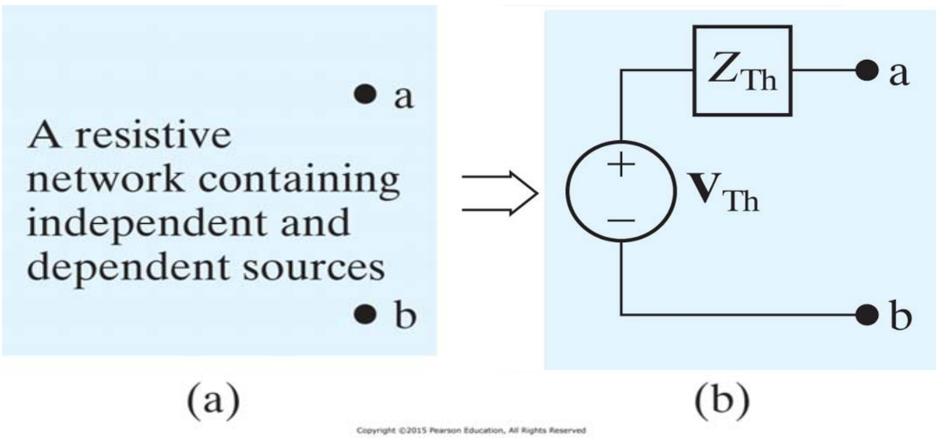
$$(1+j3)\|(9-j3)=\frac{(1+j3)(9-j3)}{10}=1.8+j2.4\Omega$$

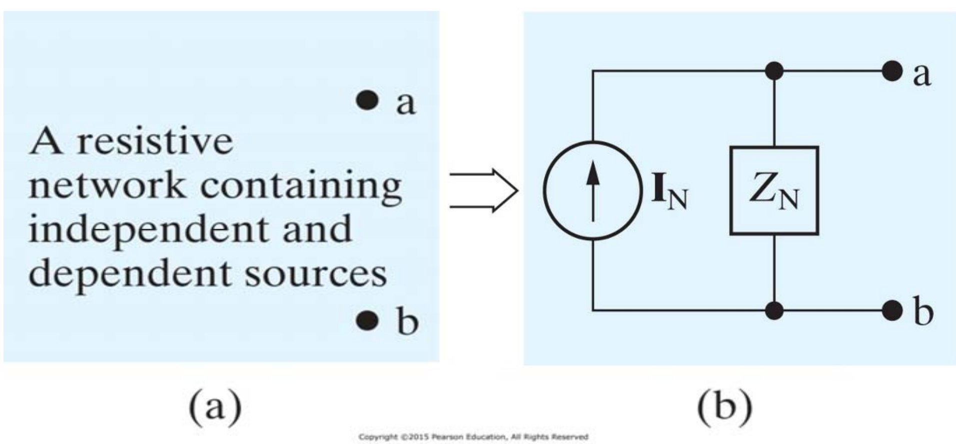
$$\mathbf{V}=(4-j12\text{A})(1.8+j2.4\Omega)=36-12j\text{V}$$

$$\boxed{\begin{aligned}\mathbf{V}_0&=(36-12j\text{V})\frac{10-j19\Omega}{12-j16\Omega}\\&=36.12-18.84\text{V}\\&\equiv 40.74\angle-27.55^\circ\end{aligned}}$$

Sinusoidal Steady State

- Thévenin–Norton Equivalent Circuits:





•1st Step: Find open circuit voltage (V_{th}).

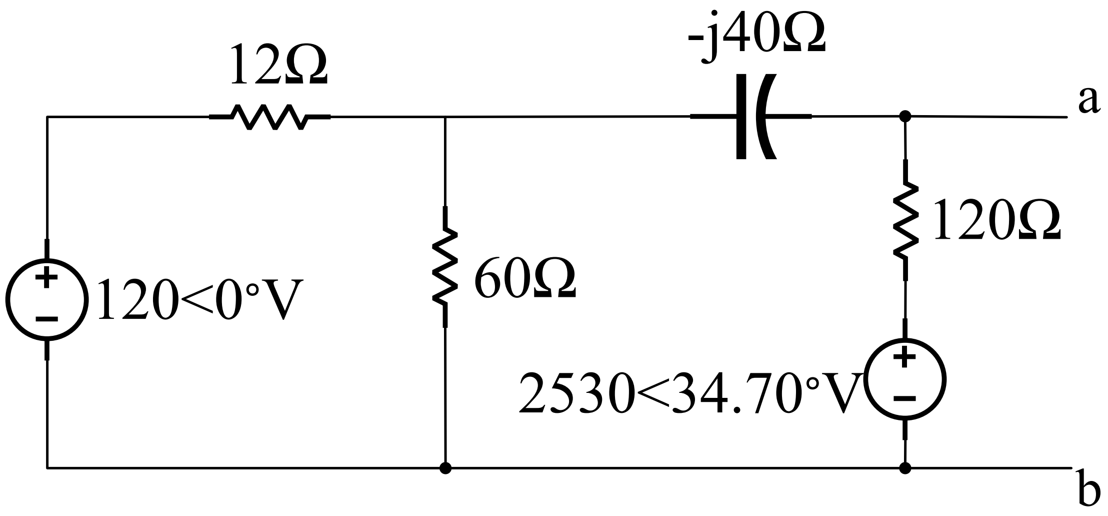
•2nd Step: Find short circuit current (I_{sc}).

•3rd Step: $R_{th} = V_{th} / I_{sc}$

•4th Step (Optional): Find Norton equivalent using source transformation.

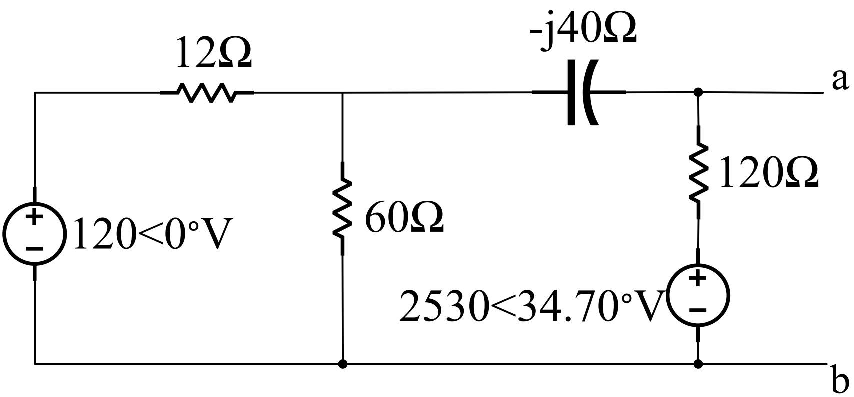
Sinusoidal Steady State

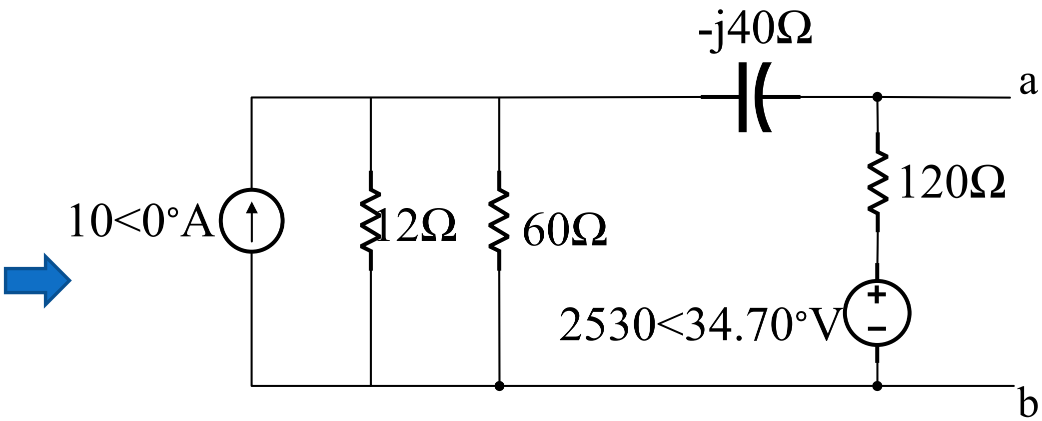
- Qn. 7: Find Thévenin equivalent w.r.t. terminals ab.



Sinusoidal Steady State

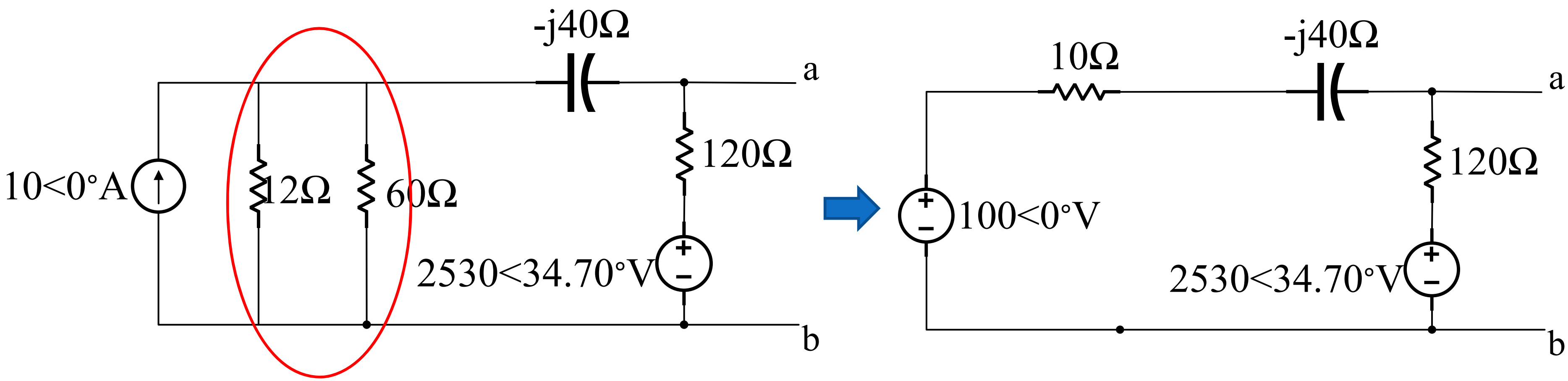
- Qn. 7 (cont.):





Sinusoidal Steady State

- Qn. 7 (cont.):



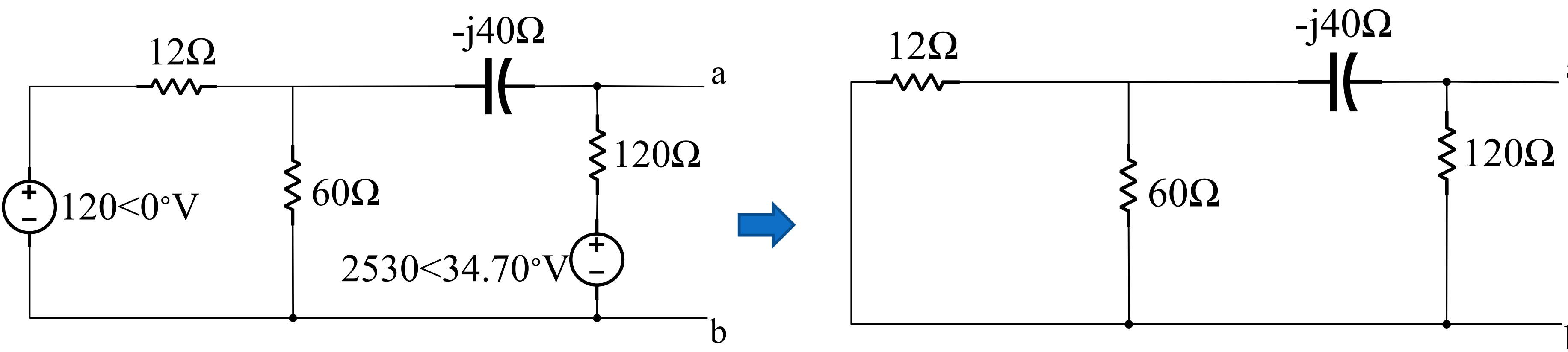
$12 \| 60 = 10\Omega$

$$\mathbf{I} = \frac{100\angle 0^\circ - 2530\angle 34.70^\circ}{130 - j40} = -10.8 - j14.4\text{A}$$

$$\boxed{\begin{aligned}\mathbf{V}_{ab} = \mathbf{V}_{Th} &= 2350\angle 34.70^\circ + 120(-10.8 - j14.4) \\ &= 835.22\angle -20.17^\circ\end{aligned}}$$

Sinusoidal Steady State

- Qn. 7 (cont.):



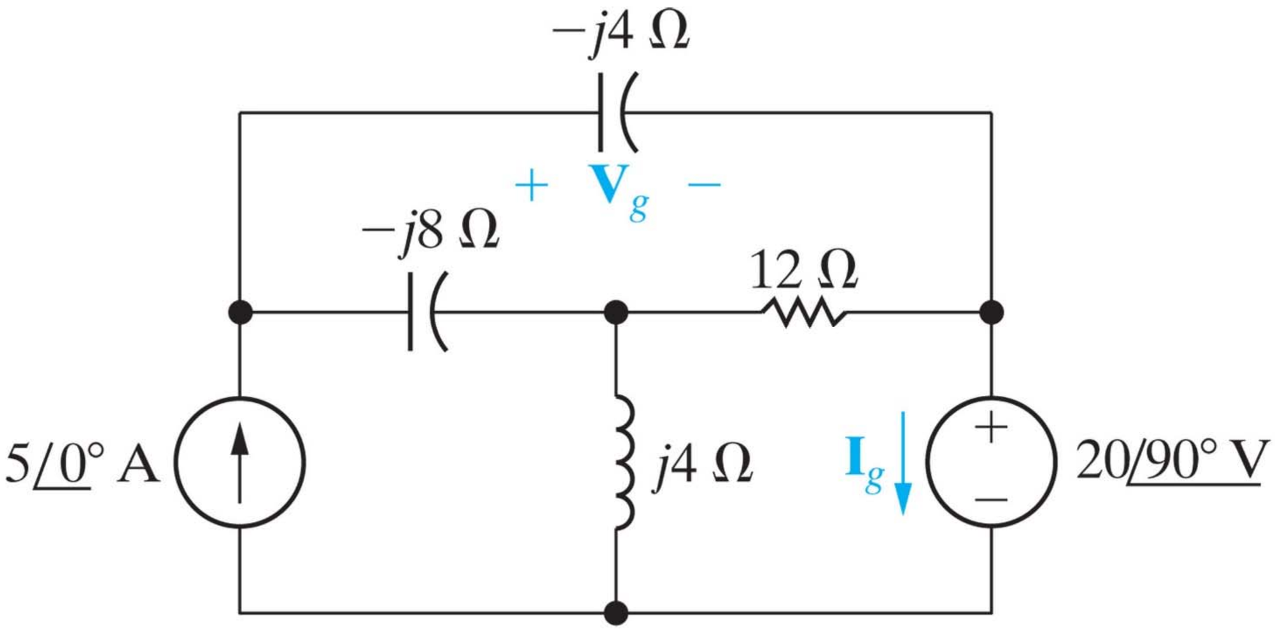
$$Z_{ab} = Z_{Th} = \left(12 \| 60 + j40\right) \| 120 = 18.81 - j31.14\Omega$$

Sinusoidal Steady State

- Node Voltage Method:
    1. Define essential nodes
    2. Pick a reference node
    3. Give names to remaining essential nodes
    4. Solve KCL for each essential node
    5. Check the power balance

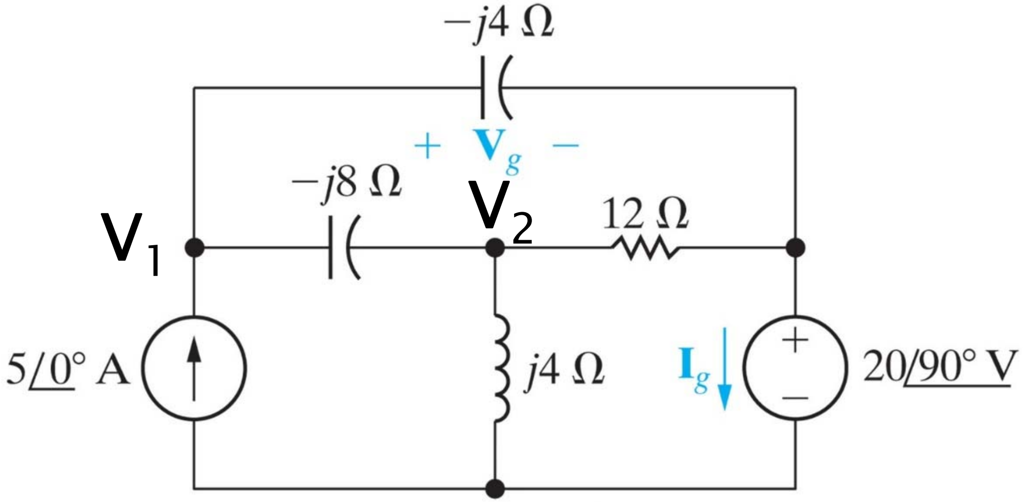
Sinusoidal Steady State

- Qn. 8: Find the phasor V_g using node voltage method.



Sinusoidal Steady State

- Qn. 8 (cont.):

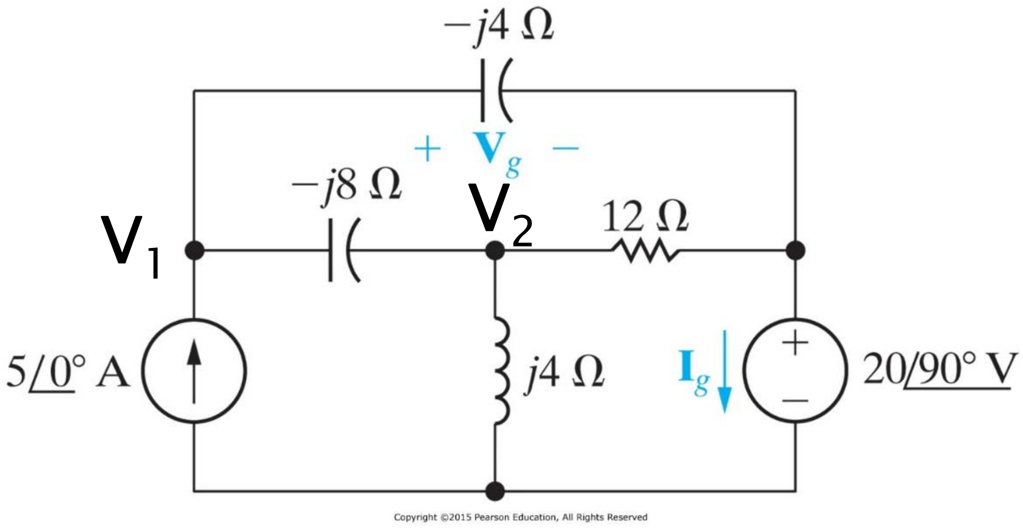


$$-5\angle 0^\circ + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\angle 90^\circ}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\angle 90^\circ}{12} = 0$$

Sinusoidal Steady State

- Qn. 8 (cont.):



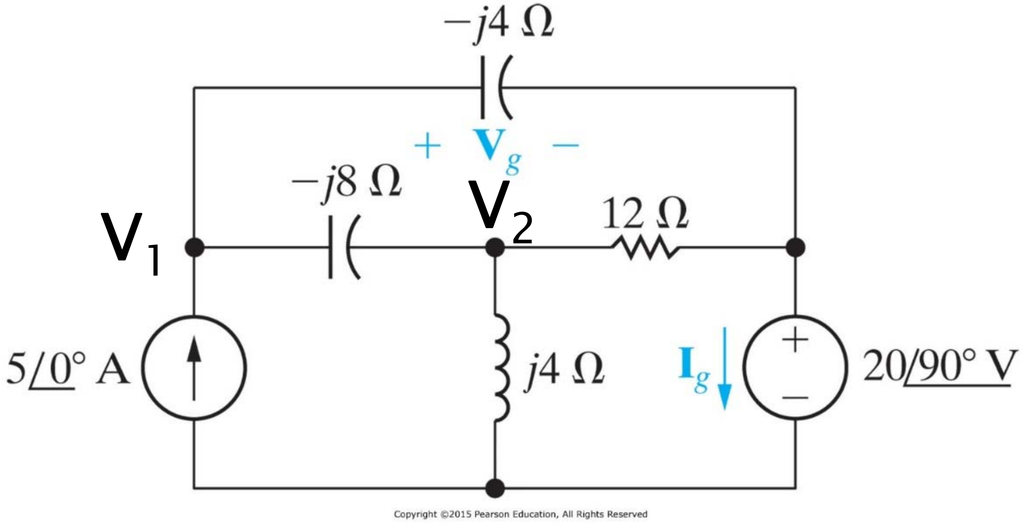
$$-5\angle 0^\circ + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\angle 90^\circ}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\angle 90^\circ}{12} = 0$$

$$\Rightarrow \begin{aligned} \mathbf{V}_1 &= -2.67 + j1.33\text{V} \\ \mathbf{V}_2 &= -8 + j4\text{V} \end{aligned}$$

Sinusoidal Steady State

- Qn. 8 (cont.):



$$-5\angle 0^\circ + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\angle 90^\circ}{-j4} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\angle 90^\circ}{12} = 0$$

$$\Rightarrow \begin{aligned} \mathbf{V}_1 &= -2.67 + j1.33\text{V} \\ \mathbf{V}_2 &= -8 + j4\text{V} \end{aligned}$$

$$\boxed{\begin{aligned} \mathbf{V}_g &= \mathbf{V}_1 - 20\angle 90^\circ \\ &= -2.67 - j18.67\text{V} \\ &= 18.86\angle -98.14^\circ\ \text{V} \end{aligned}}$$

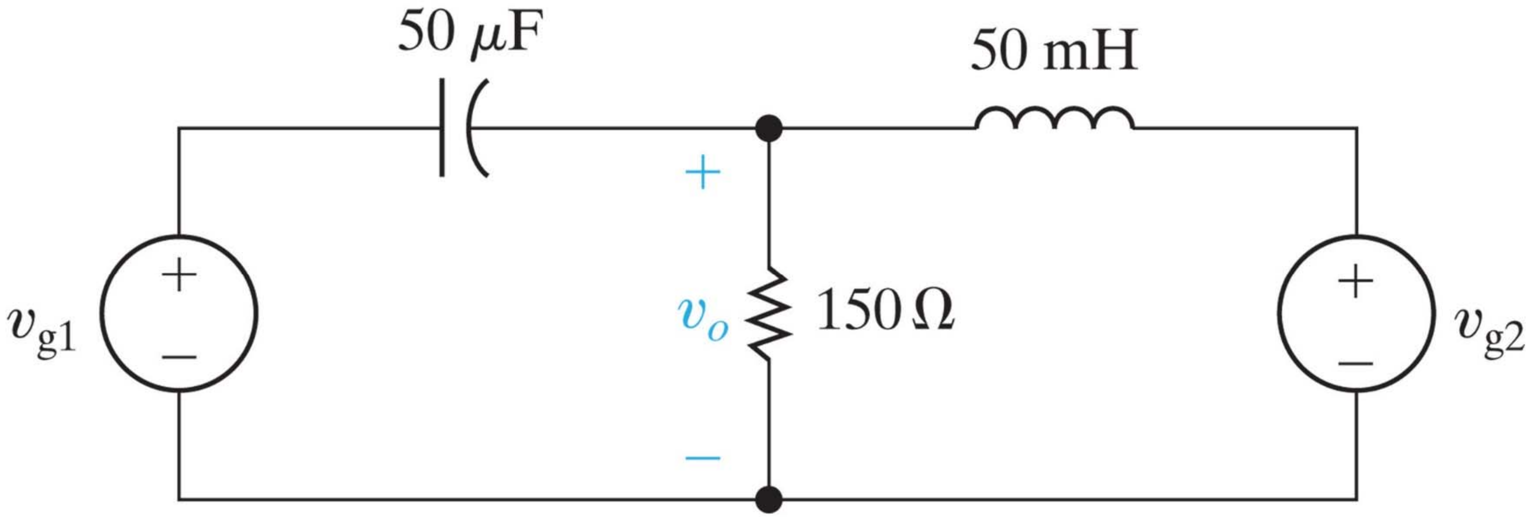
why −98.14 but not 81.86 degrees?

Sinusoidal Steady State

- Mesh Current Method:
  1. Define meshes
  2. Define currents to each mesh (son't forget to define the directions)
  3. Solve KVL equations for each mesh
  4. Check the power balance

Sinusoidal Steady State

- Qn. 9: Find the steady state value $v_o(t)$ using mesh current method.



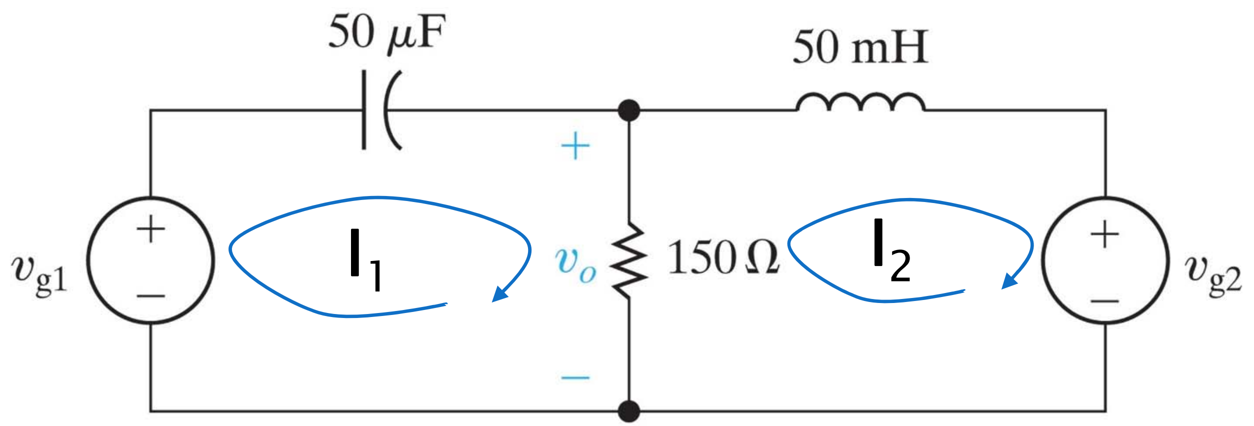
$$v_{g1}(t) = 25\sin\left(400t + 143.13^\circ\right)V$$
$$v_{g2}(t) = 18.03\cos\left(400t + 33.69^\circ\right)V$$

Sinusoidal Steady State

- Qn. 9 (cont.):

$$v_{g1}(t) = 25\sin(400t + 143.13^\circ)\text{V} \qquad \mathbf{V}_{g1} = 25\angle 53.13^\circ\,\text{V}$$

$$v_{g2}(t) = 18.03\cos(400t + 33.69^\circ)\text{V} \qquad \mathbf{V}_{g2} = 18.03\angle 33.69^\circ\,\text{V}$$



$$-25\angle 53.13^\circ + (-j50)\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

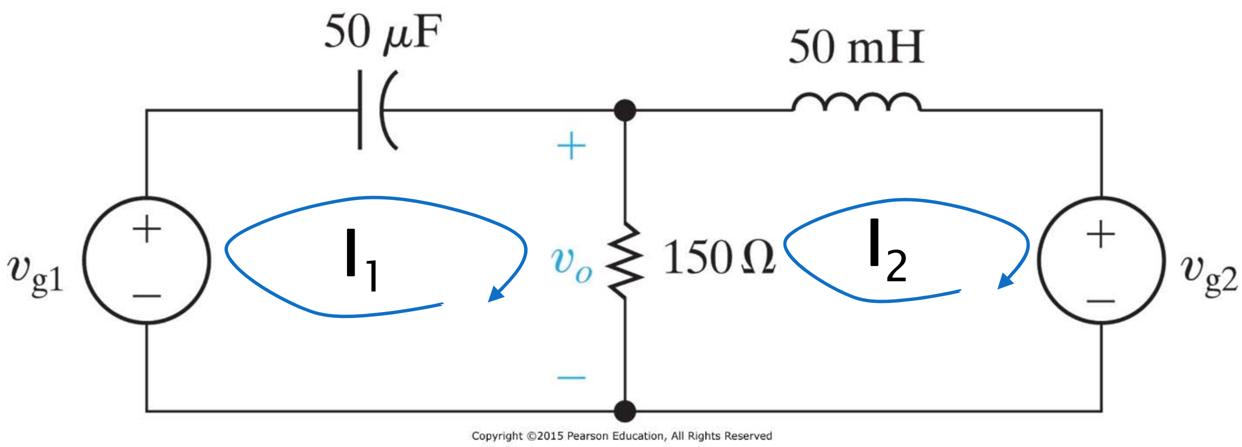
$$18.03\angle 33.69^\circ + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$$

Sinusoidal Steady State

- Qn. 9 (cont.):

$$v_{g1}(t) = 25\sin(400t + 143.13^\circ)\text{V} \qquad \mathbf{V}_{g1} = 25\angle 53.13^\circ\text{ V}$$

$$v_{g2}(t) = 18.03\cos(400t + 33.69^\circ)\text{V} \qquad \mathbf{V}_{g2} = 18.03\angle 33.69^\circ\text{ V}$$



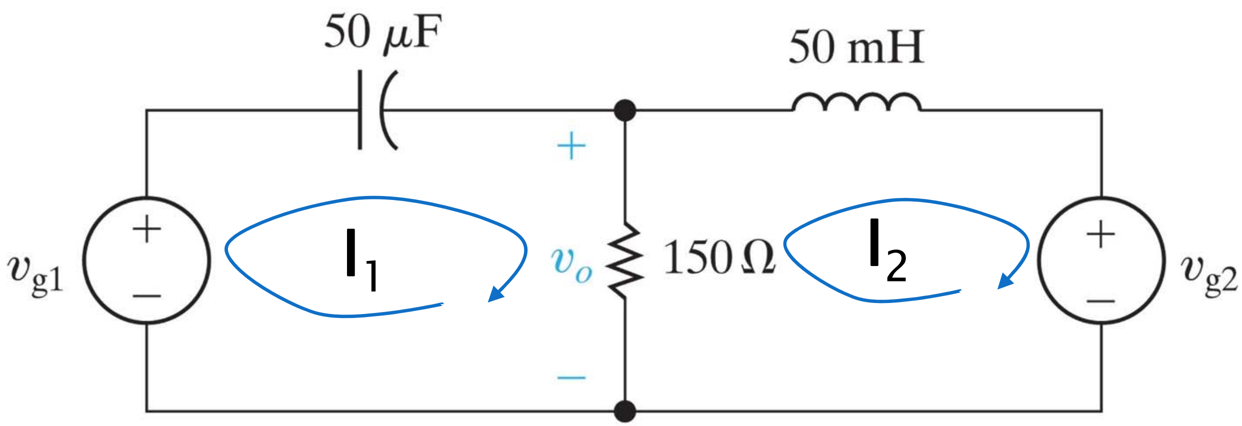
$$-25\angle 53.13^\circ + (-j50)\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$18.03\angle 33.69^\circ + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$$
$$\Rightarrow \quad \mathbf{I}_1 = -0.4\text{A} \qquad \mathbf{I}_2 = -0.5\text{A}$$

Sinusoidal Steady State

- Qn. 9 (cont.):

$$v_{g1}(t) = 25\sin(400t + 143.13^\circ)\text{V} \qquad \mathbf{V}_{g1} = 25\angle 53.13^\circ\text{ V}$$

$$v_{g2}(t) = 18.03\cos(400t + 33.69^\circ)\text{V} \qquad \mathbf{V}_{g2} = 18.03\angle 33.69^\circ\text{ V}$$



$$-25\angle 53.13^\circ + (-j50)\mathbf{I}_1 + 150(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$18.03\angle 33.69^\circ + 150(\mathbf{I}_2 - \mathbf{I}_1) + j20\mathbf{I}_2 = 0$$
$$\Rightarrow \mathbf{I}_1 = -0.4\text{A} \qquad \mathbf{I}_2 = -0.5\text{A}$$

$$\boxed{\begin{aligned}\mathbf{V}_o &= 150(\mathbf{I}_1 - \mathbf{I}_2) = 15\text{V}\\ v_o(t) &= 15\cos(400t)\text{V}\end{aligned}}$$

Sinusoidal Steady State

- We can use each method that we learned in time domain in the frequency domain:
  - Kirchoff Laws.
  - Source Transformation.
  - Thévenin–Norton Equivalents.
  - Node Voltage and Mesh Current Methods.

  Moreover, the methods below are also applicable. You are responsible to solve questions for the following methods in the frequency domain.

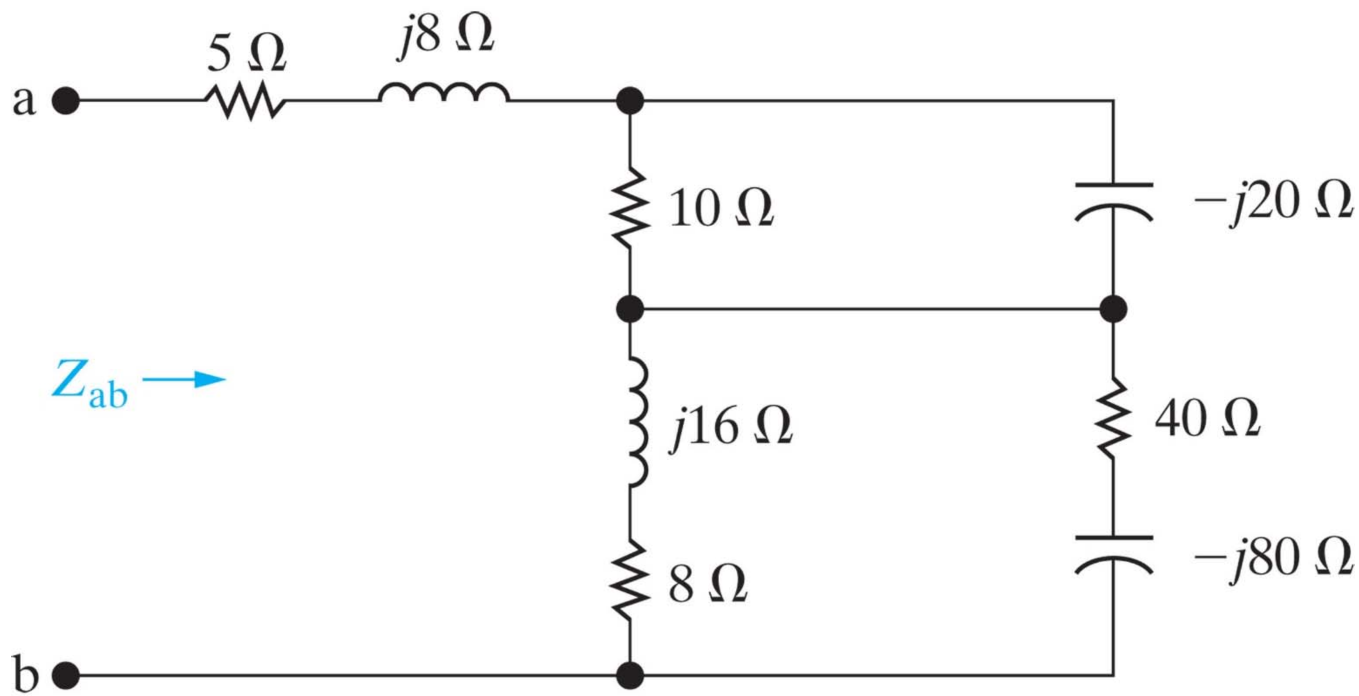
  - Superposition.
  - Current and Voltage Divisions.

Course Content

- ~~Circuit Components and Variables, Ohm’s Law and Kirchoff’s Laws~~
- ~~Circuit Analysis Techniques I - Node Voltage & Mesh Current Methods~~
- ~~Circuit Analysis Techniques II - Thevenin and Norton Equivalent Circuits~~
- ~~First Order RL and RC Circuits~~
- ~~Phasor Concept and AC Analysis of Circuits~~
- Introduction to Semiconductors
- Midterm
- BJT DC Analysis
- BJT AC Analysis
- FET Analysis
- Operational Amplifier
- Logic Gates
- Make-up Exams
- Selected Circuits

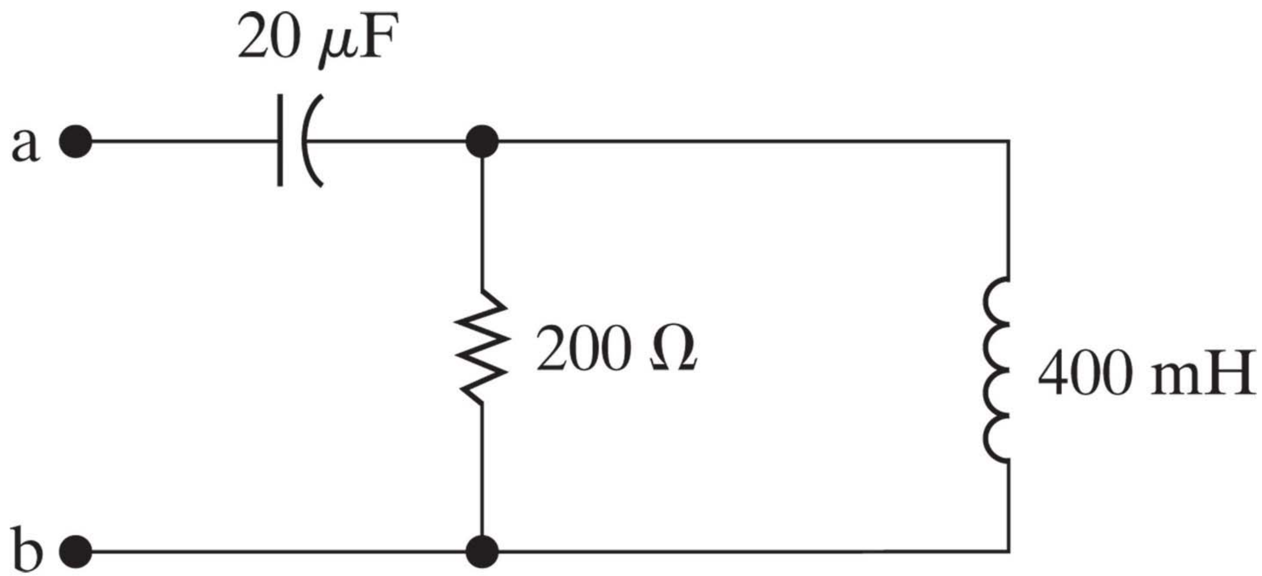
Questions

- Qn. 10: Find the equivalent impedance w.r.t. terminals ab.



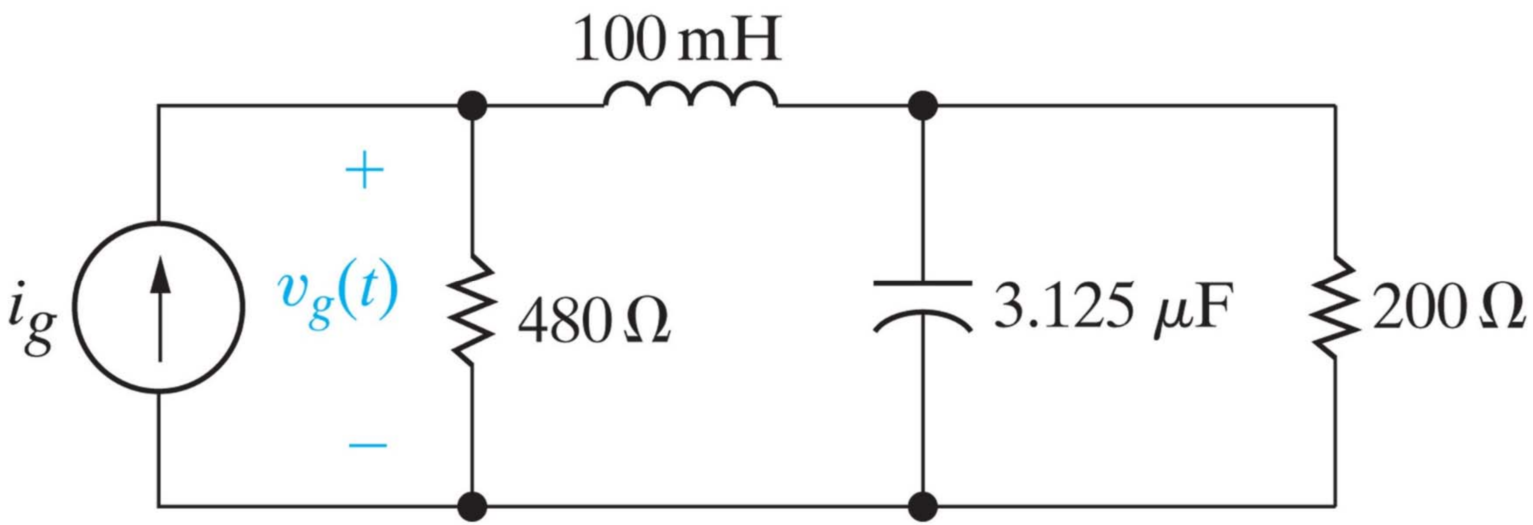
Questions

- Qn. 11: What should be the frequency for the given circuit to be fully resistive?



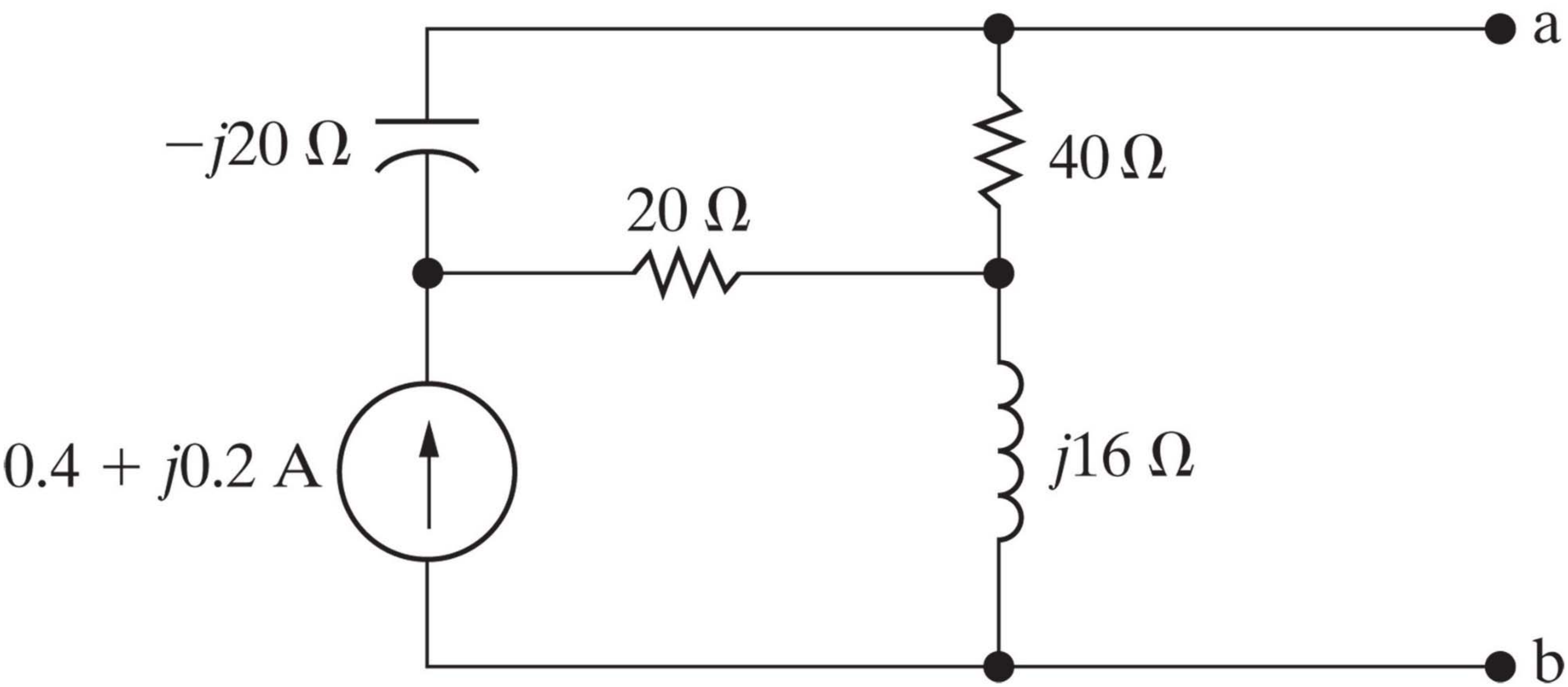
Questions

- Qn. 12: What should be the frequency for the voltage v_g and the current i_g to be in-phase?



Questions

- Qn. 13: Find the Thevenin equivalent w.r.t. terminals ab.



Questions

- Qn. 14: Find the shown currents using mesh current method.