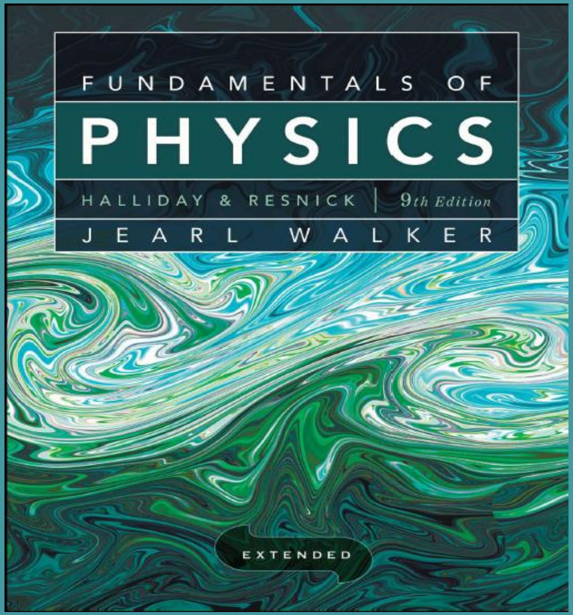
FİZ 137 - 25
CHAPTER 7

KINETIC ENERGY AND WORK



Dr. Şeyda ÇOLAK

2018 - 2019

In this chapter we will introduce the following concepts:

- In this chapter an ***alternative approach to mechanics*** will be introduced.

- It uses **scalars** such as ***work*** and ***kinetic energy*** rather than vectors. Therefore it *simpler* to apply.

Subtitles

- Kinetic energy of a moving object
- Work done by a force
- Work & Kinetic Energy Theorem
- Power

Kinetic Energy

We define a new physical parameter to describe the state of motion of an object of mass ***m*** and speed ***v***.

We define its kinetic energy *K* as: $\boxed{K = \frac{mv^2}{2}}$

$$\boxed{K = \frac{mv^2}{2}}$$

We can use the equation above to define the SI unit for work (**joule**, symbol: **J**).

An object of mass *m* = 1 kg that moves with speed *v* = 1 m/s has a kinetic energy *K* = 1 J

Work

- If a force ***F*** is applied to an object of mass ***m***, it can accelerate the object and increase its speed ***v*,** thus its kinetic energy ***K*****.**

- Similarly ***F*** can decelerate ***m*** and decrease its kinetic energy **K.**

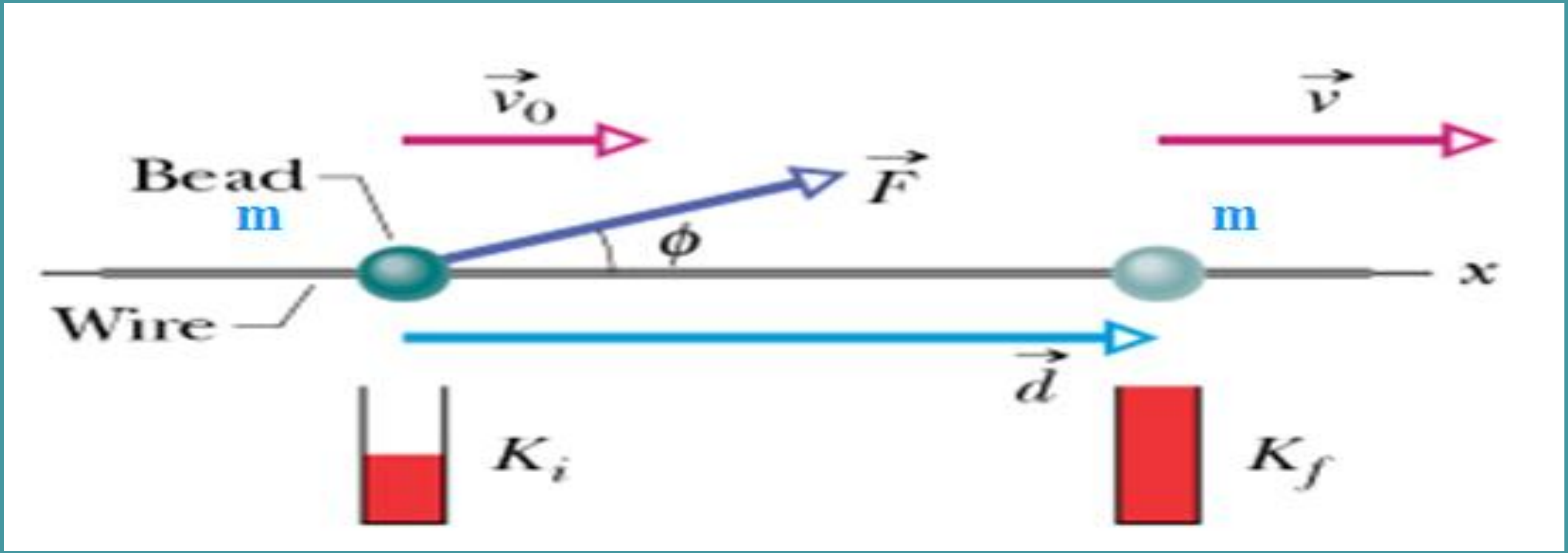
If energy is transferred to *m* (its *K* increases) $\rightarrow$ work is done by the applied force *F* on the object (*W* > 0).

If energy is transferred from the object (its *K* decreases) → work is done by *m* (*W* < 0).

Finding an expression for Work:

Consider a bead of mass m that can move without friction along a straight wire along the x-axis. A constant force $\vec{F}$ applied at an angle ϕ to the wire is acting on the bead

$$\boxed{K = \frac{mv^2}{2}}$$



We apply Newton's second law: $\boxed{F_x = ma_x}$ We assume that the bead had an initial velocity $\vec{v}_o$ and after it has travelled a distance $\vec{d}$ its velocity is $\vec{v}$. We apply the third equation of kinematics: $\boxed{v^2 - v_o^2 = 2a_x d}$ We multiply both sides by $\underline{m/2} \rightarrow$

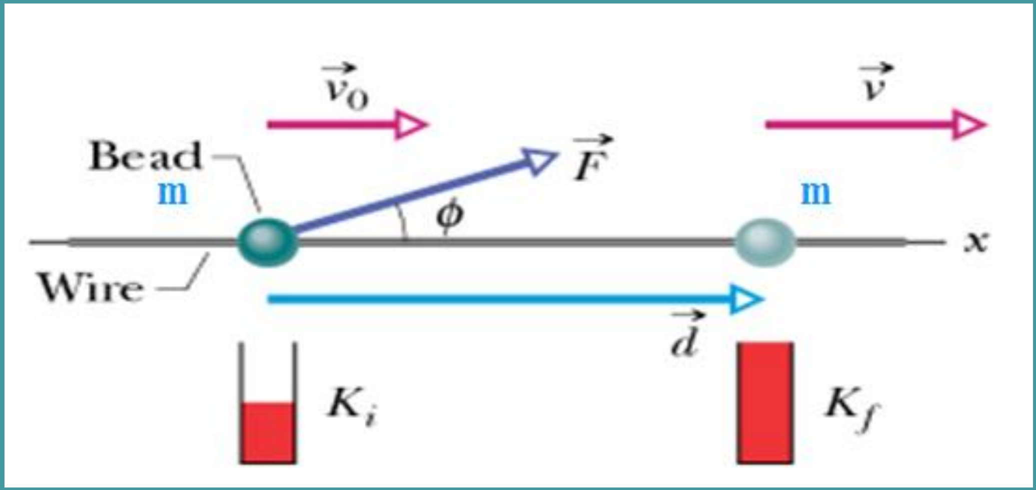
$$\frac{m}{2}v^2 - \frac{m}{2}v_o^2 = \frac{m}{2}2a_x d = \frac{m}{2}2\frac{F_x}{m}d = F_x d = F\cos\phi d \qquad K_i = \frac{m}{2}v_o^2$$

$$K_f = \frac{m}{2}v^2 \quad \rightarrow \text{ The change in kinetic energy } \boxed{K_f - K_i = Fd\cos\varphi}$$

Thus the work W done by the force on the bead is given by: $\boxed{W = F_x d = Fd\cos\varphi}$

$$\boxed{W = Fd\cos\varphi}$$

$$\boxed{W = \vec{F}\cdot\vec{d}}$$



The unit of W is the same as that of K i.e. joules

Note 1: The expressions for work we have developed apply when F is constant

Note 2: We have made the implicit assumption that the moving object is point-like

Note 3: $W > 0$ if $0 < \phi < 90°$, $W < 0$ if $90° < \phi < 180°$

$$\boxed{W = Fd\cos\varphi}$$

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

Net Work: If we have several forces acting on a body (say three as in the picture) there are two methods that can be used to calculate the net work W_{net}

Method 1: First calculate the work done by each force: W_A by force $\vec{F}_A$, W_B by force $\vec{F}_B$, and W_C by force $\vec{F}_C$. Then determine $W_{net} = W_A + W_B + W_C$

Method 2: Calculate first $\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$; Then determine $W_{net} = \vec{F} \cdot \vec{d}$

$$\boxed{W = \vec{F} \cdot \vec{d}}$$

Work-Kinetic Energy Theorem

We have seen earlier that: $K_f - K_i = W_{net}$.

We define the change in kinetic energy as:

$\Delta K = K_f - K_i$. The equation above becomes the work-kinetic energy theorem

$$\boxed{\Delta K = K_f - K_i = W_{net}}$$

$$\boxed{\Delta K = W_{net}}$$

$$\left[\begin{array}{c}\text{Change in the kinetic}\\ \text{energy of a pareticle}\end{array}\right] = \left[\begin{array}{c}\text{net work done on}\\ \text{the particle}\end{array}\right]$$

$$\boxed{\Delta K = K_f - K_i = W_{net}}$$

The work-kinetic energy theorem holds for both positive and negative values of W_{net}

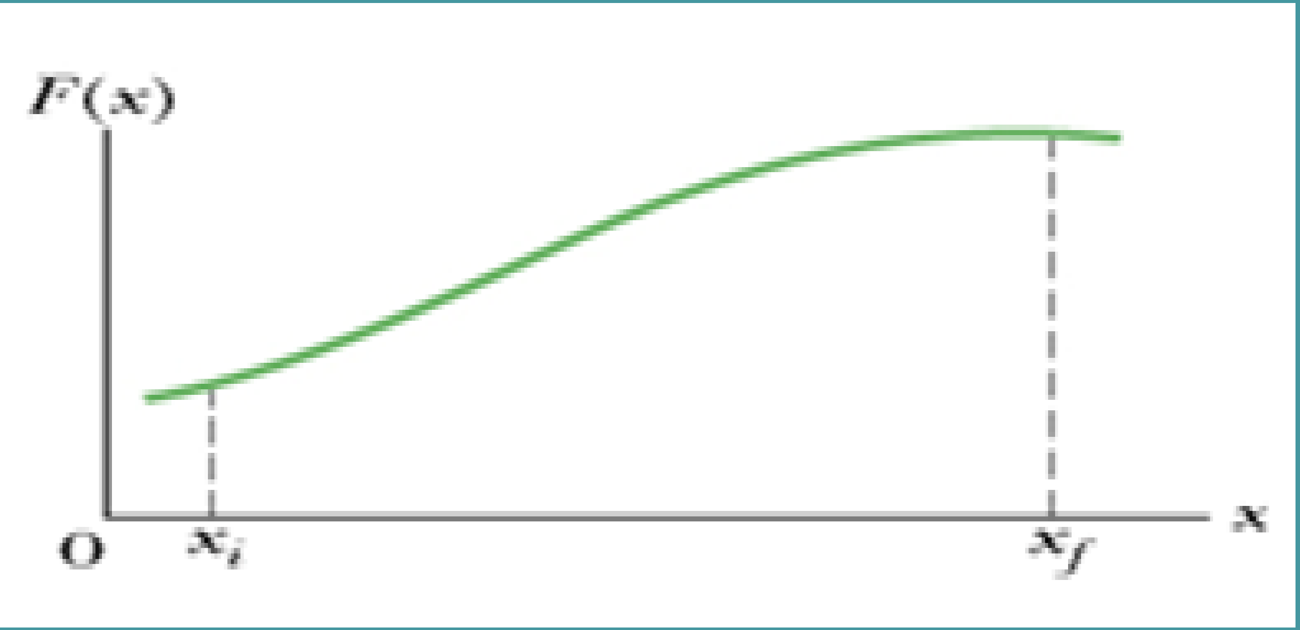
$$\boxed{\text{If} \quad W_{net} > 0} \rightarrow K_f - K_i > 0 \rightarrow \boxed{K_f > K_i}$$

$$\boxed{\text{If} \quad W_{net} < 0} \rightarrow K_f - K_i < 0 \rightarrow \boxed{K_f < K_i}$$

Work Done by a Variable Force

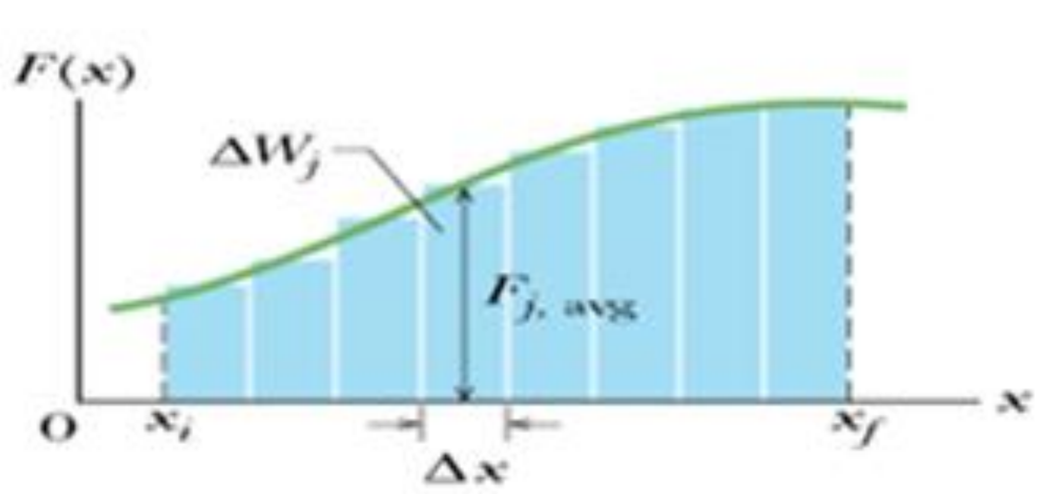
Work done by a variable force $F(x)$ acting along the x-axis:

A force F that is not constant but instead varies as function of x is shown in fig.a. We wish to calculate the work W that F does on an object it moves from position x_i to position x_f.



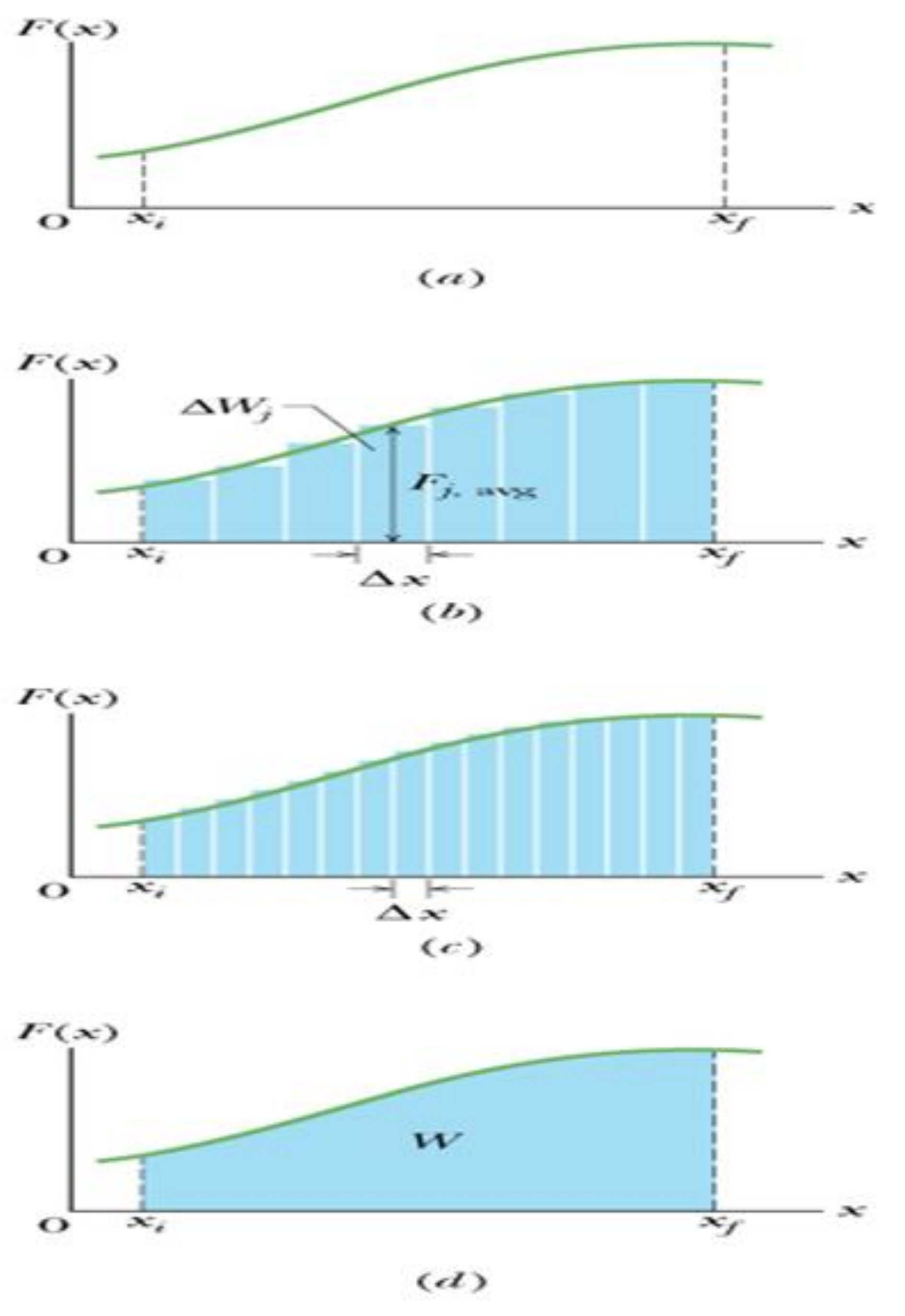
$$\vec{W} = \vec{F} \cdot \vec{d}$$

We partition the interval (x_i, x_f) into $\underline{N \text{ "elements"}}$ of length Δx each as is shown in fig.b. The work done by F in the j-th interval is: $\Delta W_j = F_{j,avg}\Delta x$ Where $F_{j,avg}$ is the average value of F over the j-th element. $\boxed{W = \sum_{j=1}^{N} F_{j,avg}\Delta x}$ We then take the limit of the sum as $\Delta x \to 0$, (or equivalently $N \to \infty$)

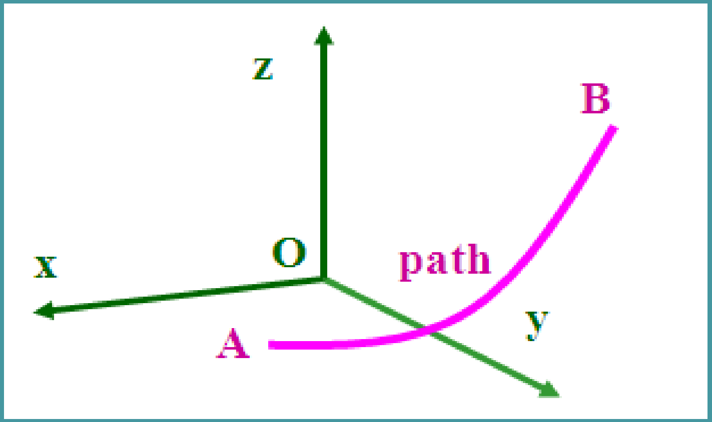


$W = \lim_{N \to \infty} \sum_{j=1}^{N} F_{j,avg} \Delta x = \int_{x_i}^{x_f} F(x)dx$ Geometrically, W is the area between $F(x)$ curve and the x-axis, between x_i and x_f (shaded blue in fig.d)

$$\boxed{W = \int_{x_i}^{x_f} F(x)dx}$$



$$W = \int_{x_i}^{x_f} F(x)\,dx$$



Three dimensional Analysis:

In the general case the force $\vec{F}$ acts in three dimensional space and moves an object on a three dimensional path from an initial point A to a final point B

The force has the form: $\vec{F} = F_x(x,y,z)\hat{i} + F_y(x,y,z)\hat{j} + F_z(x,y,z)\hat{k}$

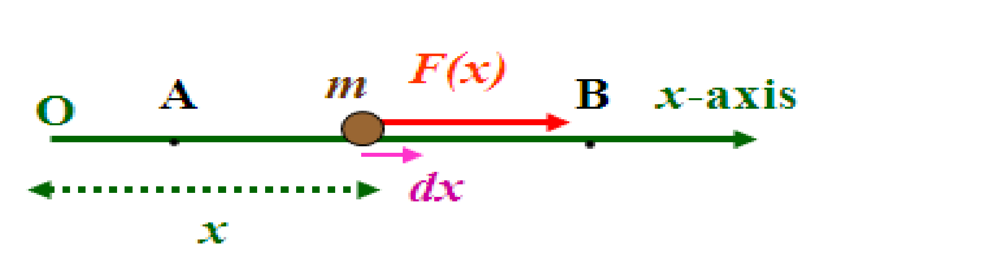
Points A and B have coordinates (x_i, y_i, z_i) and (x_f, y_f, z_f), respectively

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$W = \int_A^B dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

$$\boxed{W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz}$$

Work-Kinetic Energy Theorem with a Variable Force:



Conside a variable force F(x) which moves an object of mass m from point A($x = x_i$) to point B($x = x_f$). We apply Newton's second law: $\boxed{F = ma = m\frac{dv}{dt}}$ We then multiply both sides of the last equation with dx and get: $F dx = m\frac{dv}{dt}dx$

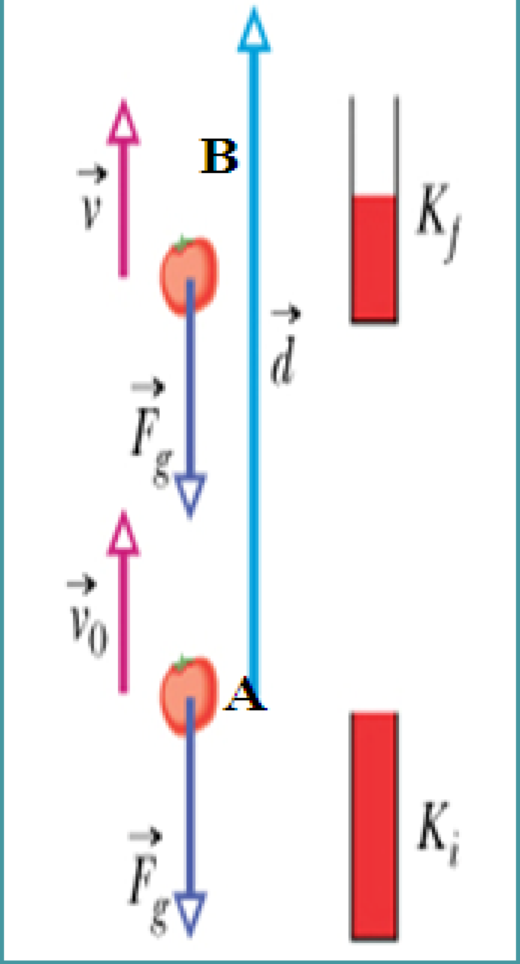
We integrate both sides over dx from x_i to x_f : $\int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} m\frac{dv}{dt}dx$

$\boxed{\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}}$ $\boxed{\frac{dv}{dx}\frac{dx}{dt}dx = v dv}$ Thus the integral becomes:

$$W = m\int_{x_i}^{x_f} v dv = \frac{m}{2}\left[v^2\right]_{x_i}^{x_f} = \boxed{\frac{mv_f^2}{2} - \frac{mv_i^2}{2} = K_f - K_i = \Delta K}$$

Note: The work-kinetic energy theorem has exactly the same form as in the case when F is constant!

Work Done by Gravitational Force



Work Done by the Gravitational Force:

Consider a tomato of mass m that is thrown upwards at point A with initial speed v_o. As the tomato rises, it slows down by the gravitational force F_g so that at point B its has a smaller speed v.

The work $W_g(A \rightarrow B)$ done by the gravitational force on the tomato as it travels from point A to point B is:

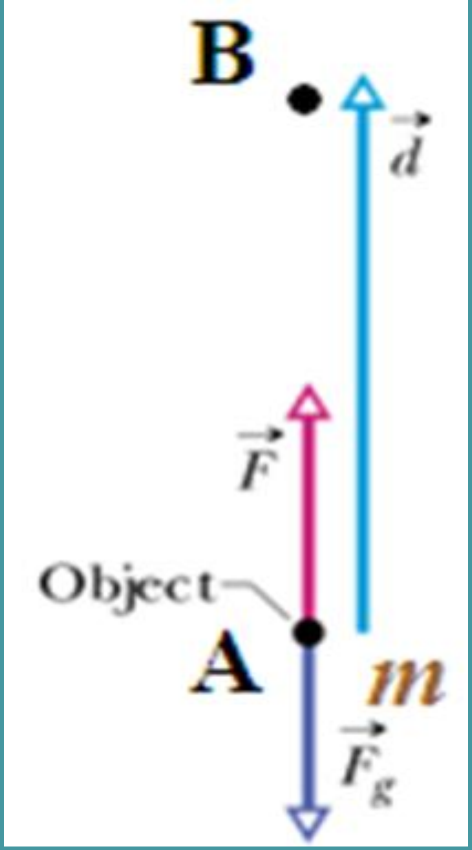
$$\boxed{W_g(A \rightarrow B) = mgd\cos 180° = -mgd}$$

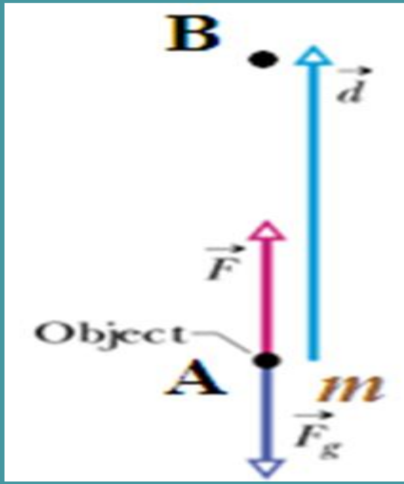
The work $W_g(B \rightarrow A)$ done by the gravitational force on the tomato as it travels from point B to point A is:

$$\boxed{W_g(B \rightarrow A) = mgd\cos 0° = mgd}$$

Work done by a force in Lifting an object:

Consider an object of mass m that is lifted by a force F form point A to point B. The object starts from rest at A and arrives at B with zero speed. The force F is not necessarily constant during the trip.



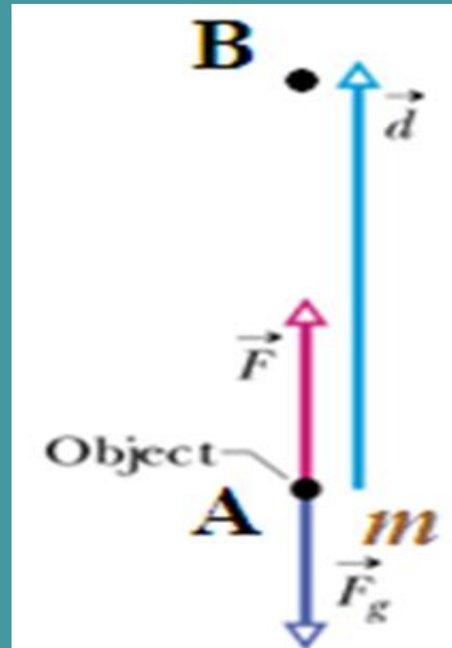


The work-kinetic energy theorem states that: $\boxed{\Delta K = K_f - K_i = W_{net}}$

We also have that $K_i = K_f \rightarrow \Delta K = 0 \rightarrow \boxed{W_{net} = 0}$ There are two forces acting on the object: The gravitational force F_g and the applied force F that lifts the object. $W_{net} = W_a(\mathrm{A} \rightarrow \mathrm{B}) + W_g(\mathrm{A} \rightarrow \mathrm{B}) = 0 \rightarrow$

$W_a(\mathrm{A} \rightarrow \mathrm{B}) = -W_g(\mathrm{A} \rightarrow \mathrm{B})$

$\underline{W_g}\,\underline{(\mathrm{A} \rightarrow \mathrm{B}) = mgd\cos 180° = \text{-}mgd}$ $\rightarrow \underline{W_a(\mathrm{A} \rightarrow \mathrm{B}) = mgd}$

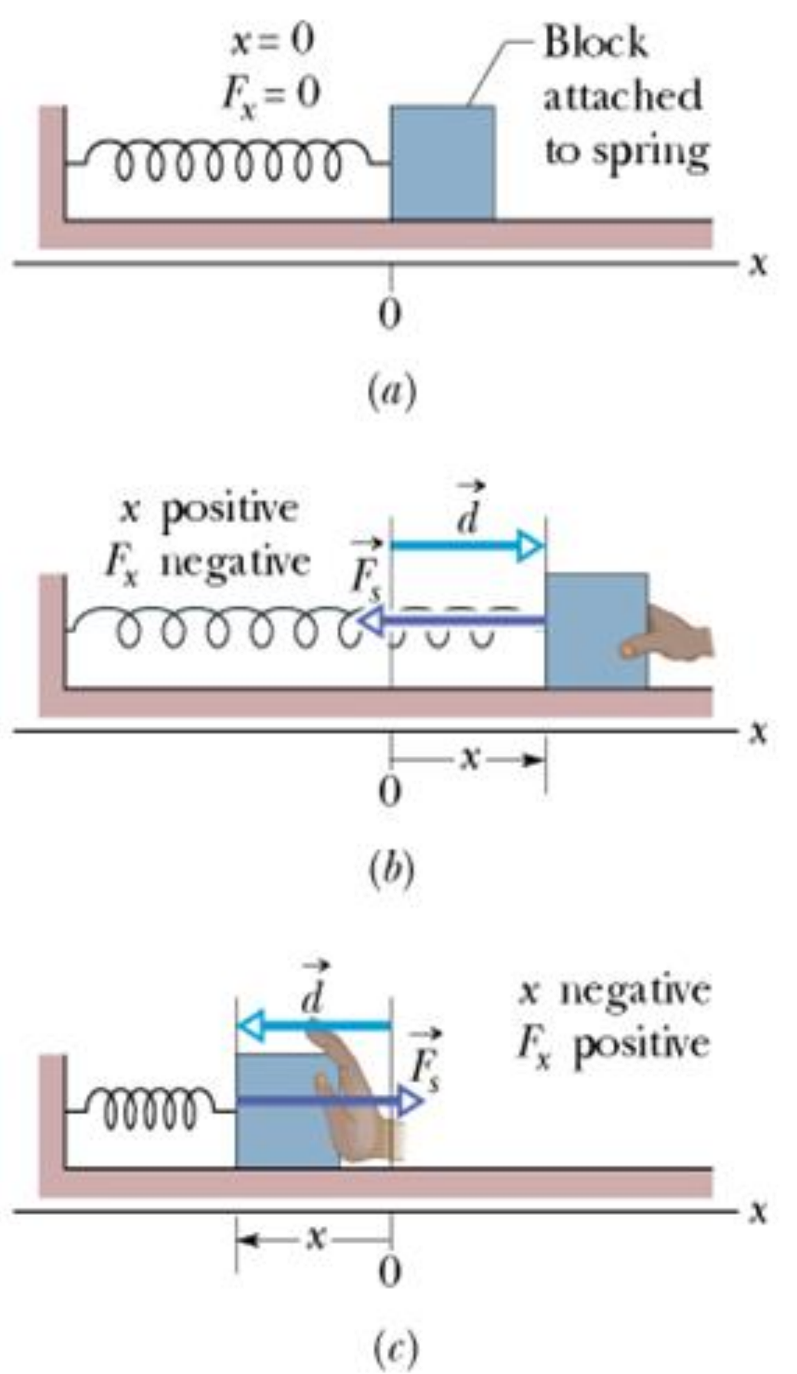


Work done by a force in Lowering an object:

In this case the object moves from B to A

$$W_g(\mathrm{B} \to \mathrm{A}) = mgd\cos 0^\circ = mgd \qquad W_a(\mathrm{B} \to \mathrm{A}) = -W_g(\mathrm{B} \to \mathrm{A}) = -mgd$$

Work Done by Spring Force



$$F = -kx$$

Hooke's Law

The Spring Force:

Fig.a shows a spring in its relaxed state.

In fig.b we pull one end of the spring and stretch it by an amount d. The spring resits by exerting a force F on our hand in the opposite direction.

In fig.c we push one end of the spring and compress it by an amount d. Again the spring resists by exerting a force F on our hand in the opposite direction

$$\boxed{F = -kx}$$

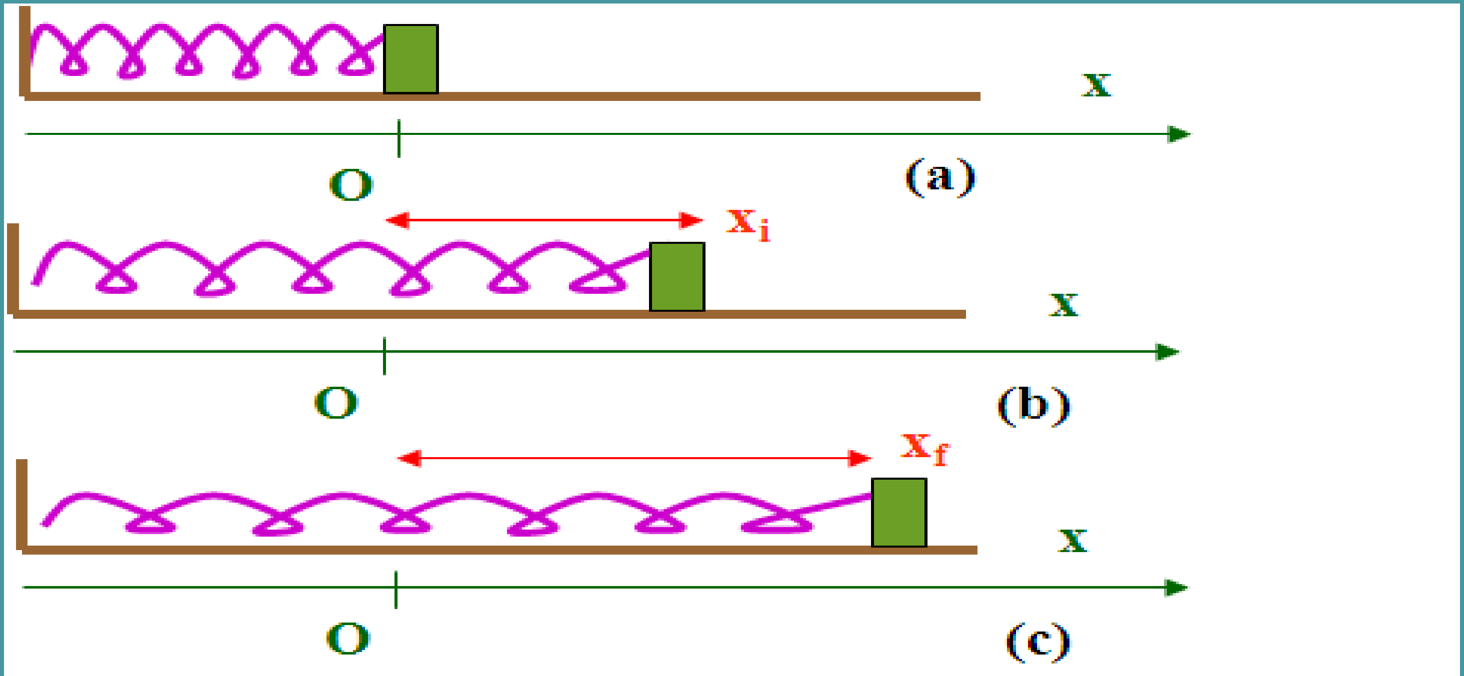
The force F exerted by the spring on whatever agent (in the picture our hand) is trying to change its natural length either by extending or by compressing it is given by the equation: $\boxed{F = -kx}$ Here x is the amount by which the spring has been extended or compressed. This equation is known as "Hookes law"

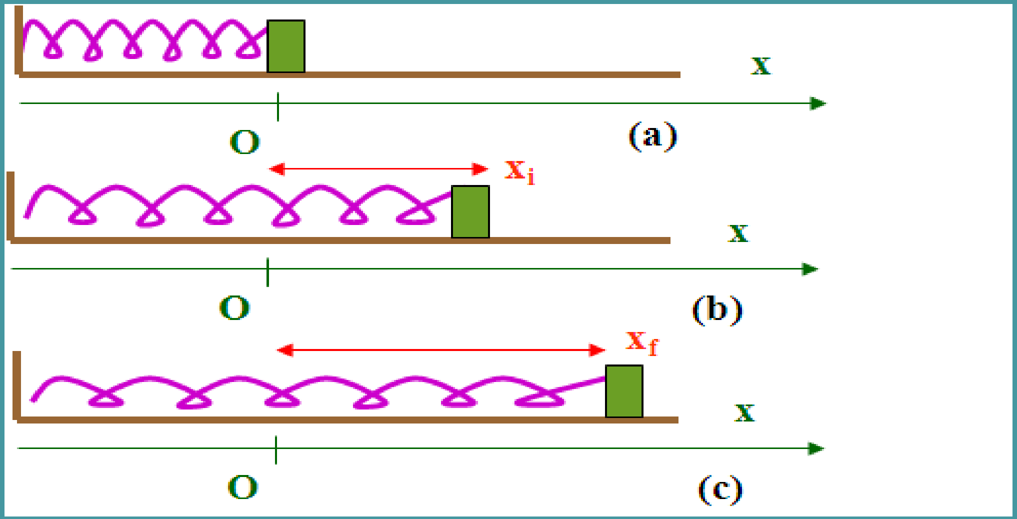
k is known as "spring constant"

Work Done by a Spring Force

Consider the relaxed spring of spring constant k shown in (a)

By applying an external force we change the spring's length from x_i (see b) to x_f (see c). We will calculate the work W_s done by the spring on the external agent (in this case our hand) that changed the spring length. We assume that the spring is massless and that it obeys Hooke's law





$$F = -kx$$

We will use the expression: $W_s = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx = -k\int_{x_i}^{x_f} xdx$

$$W_s = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f} = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$$

Quite often we start we a relaxed spring ($x_i = 0$) and we either stretch or compress the spring by an amount x ($x_f = \pm x$). In this case $\boxed{W_s = -\frac{kx^2}{2}}$

Power

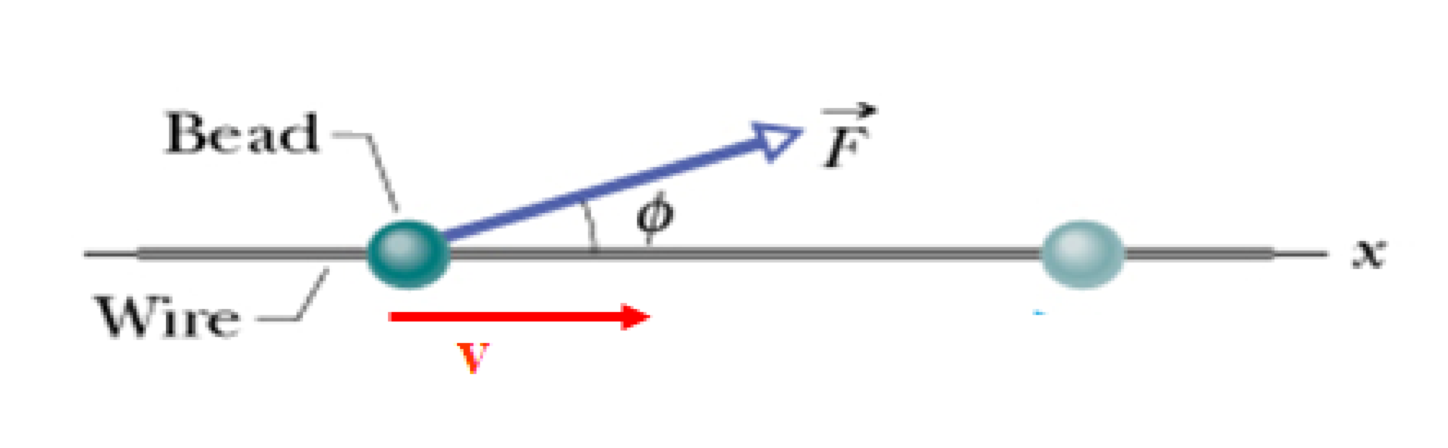
We define "power" P as the rate at which work is done by a force F.

If F does work W in a time interval Δt then we define as the average power as:

$$\boxed{P_{avg} = \frac{W}{\Delta t}}$$

The instantaneous power is defined as:

$$\boxed{P = \frac{dW}{dt}}$$



Consider a force F acting on a particle at an angle ϕ to the motion. The rate at which F does work is given by: $P = \dfrac{dW}{dt} = \dfrac{F\cos\phi\, dx}{dt} = F\cos\phi\dfrac{dx}{dt} = Fv\cos\phi$

$$\boxed{P = Fv\cos\phi = \vec{F}\cdot\vec{v}}$$

Unit of P: The SI unit of power is the watt. It is defined as the power of an engine that does work $W = 1$ J in a time $t = 1$ second

A commonly used non-SI power unit is the horsepower (hp) defined as:

$$1 \text{ hp} = 746 \text{ W}$$

The kilowatt-hour The kilowatt-hour (kWh) is a unit of work. It is defined as the work performed by an engine of power $P = 1000$ W in a time $t = 1$ hour

$$W = Pt = 1000 \times 3600 = 3.60 \times 10^{6} \text{ J}$$

5. An object moves in a circle at constant speed. The work done by the centripetal force is zero because:

A. the displacement for each revolution is zero

B. the average force for each revolution is zero

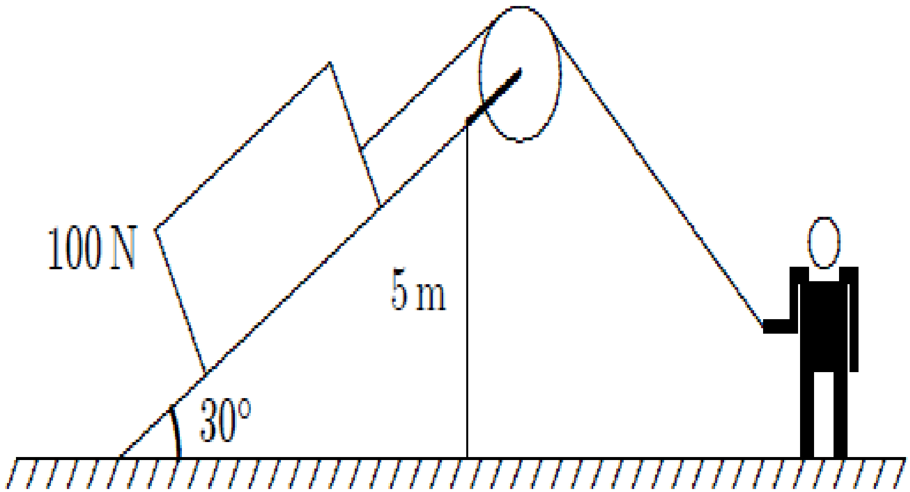
C. there is no friction

D. the magnitude of the acceleration is zero

E. the centripetal force is perpendicular to the velocity

ans: E

16. A man pulls a 100-N crate up a frictionless 30° slope 5 m high, as shown. Assuming that the crate moves at constant speed, the work done by the man is:



A. −500 J
B. −250 J
C. 0
D. 250 J
E. 500 J

ans: E

22. A particle moves 5 m in the positive x direction while being acted upon by a constant force $\vec{F} = (4\,\mathrm{N})\hat{\mathrm{i}} + (2\,\mathrm{N})\hat{\mathrm{j}} - (4\,\mathrm{N})\hat{\mathrm{k}}$. The work done on the particle by this force is:

A. 20 J

B. 10 J

C. −20 J

D. 30 J

E. is impossible to calculate without knowing other forces

ans: A

34. An object is constrained by a cord to move in a circular path of radius 0.5 m on a horizontal frictionless surface. The cord will break if its tension exceeds 16 N. The maximum kinetic energy the object can have is:

A. 4 J

B. 8 J

C. 16 J

D. 32 J

E. 64 J

ans: A

46. A 2-kg block is attached to a horizonal ideal spring with a spring constant of 200 N/m. When the spring has its equilibrium length the block is given a speed of 5 m/s. What is the maximum elongation of the spring?

A. 0
B. 0.05 m
C. 5 m
D. 10 m
E. 100 m

ans: C