Functional Programming Languages

BBM 301 – Programming Languages

Introduction

- The design of the imperative languages is based directly on the *von Neumann architecture*
  – Efficiency is the primary concern, rather than the suitability of the language for software development
- The design of the functional languages is based on *mathematical functions*
  – A solid theoretical basis that is also closer to the user, but relatively unconcerned with the architecture of the machines on which programs will run

Mathematical Functions

- A mathematical function is a *mapping* of members of one set, called the *domain set*, to another set, called the *range set*
- In math functions, the evaluation order is controlled by recursion
- They don’t have side effects: same value given the same arguments

Lambda Expressions

- Lambda expressions describe nameless functions
- A *lambda expression* specifies the parameter(s) and the mapping of a function in the following form

  `λ(x) x * x * x`

  for the function `cube (x) = x * x * x`
- Lambda expressions are applied to parameter(s) by placing the parameter(s) after the expression

  e.g., `(λ(x) x * x * x)(2)`

  which evaluates to 8

Functional Forms

- A higher-order function, or *functional form*, is one that either takes functions as parameters or yields a function as its result, or both

Function Composition

- A functional form that takes two functions as parameters and yields a function whose value is the first actual parameter function applied to the application of the second

  Form: $h \equiv f \circ g$

  which means $h(x) \equiv f(g(x))$

  For $f(x) \equiv x + 2$ and $g(x) \equiv 3 * x$,

  $h \equiv f \circ g$ yields $(3 * x) + 2$

Apply-to-all

- A functional form that takes a single function as a parameter and yields a list of values obtained by applying the given function to each element of a list of parameters

  Form: α

  For `h(x) ≡ x * x`

  α`( h, (2, 3, 4))` yields `(4, 9, 16)`

Fundamentals of Functional Programming Languages

- The objective of the design of a FPL is to mimic mathematical functions to the greatest extent possible
- The basic process of computation is fundamentally different in a FPL than in an imperative language
  - In an imperative language, operations are done and the results are stored in variables for later use
  - Management of variables is a constant concern and source of complexity for imperative programming
- In an FPL, variables are not necessary, as is the case in mathematics

Fundamentals of Functional Programming Languages (cont'd.)

- *Referential Transparency* - In an FPL, the evaluation of a function always produces the same result given the same parameters
- *Tail Recursion* – Writing recursive functions that can be automatically converted to iteration

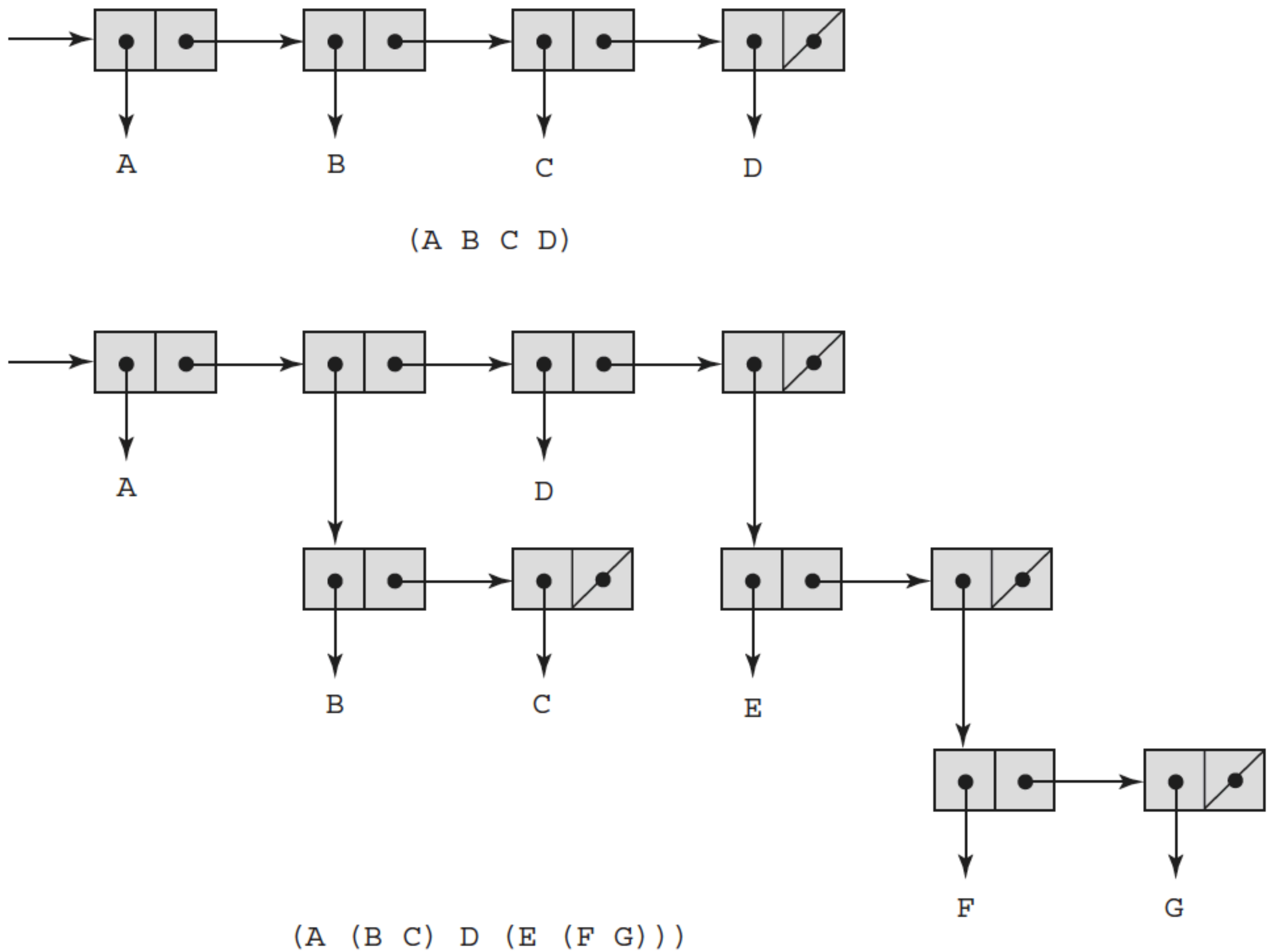
LISP Data Types and Structures

- LISP was developed by John McCarthy at MIT in 1959
- *Data object types*: originally only atoms and lists
- *List form*: parenthesized collections of sublists and/or atoms

  e.g., `(A B (C D) E)`
- Originally, LISP was a typeless language
- LISP lists are stored internally as single-linked lists

Figure 15.1

Internal representation of two LISP lists



LISP Interpretation

- Lambda notation is used to specify functions and function definitions.
- **Function applications and data have the same form**.

  e.g., If the list `(A B C)` is interpreted as data it is

  a simple list of three atoms, `A`, `B`, and `C`
- *If it is interpreted as a function application, it means that the function named `A` is applied to the two parameters, `B` and `C`*

Origins of Scheme

- A mid-1970s dialect of LISP, designed to be a cleaner, more modern, and simpler version than the contemporary dialects of LISP
- Uses only static scoping
- Functions are first-class entities
  - They can be the values of expressions and elements of lists
  - They can be assigned to variables, passed as parameters, and returned from functions

The Scheme Interpreter

- In interactive mode, the Scheme interpreter is an infinite read-evaluate-print loop (REPL)
  - This form of interpreter is also used by Python and Ruby
- Expressions are interpreted by the function `EVAL`
- Literals evaluate to themselves

Primitive Function Evaluation

- Parameters are evaluated, in no particular order
- The values of the parameters are substituted into the function body
- The function body is evaluated
- The value of the last expression in the body is the value of the function

Primitive Functions

- Primitive Arithmetic Functions: `+, -, *, /, ABS, SQRT, REMAINDER, MIN, MAX`

  e.g., `(+ 5 2) yields 7`

- `QUOTE` - takes one parameter; returns the parameter without evaluation
  - `QUOTE` is required because the Scheme interpreter, named `EVAL`, always evaluates parameters to function applications before applying the function. `QUOTE` is used to avoid parameter evaluation when it is not appropriate
  - `QUOTE` can be abbreviated with the apostrophe prefix operator

    `'(A B) is equivalent to (QUOTE (A B))`

Examples

Expression	Value
`42`	42
`(* 3 7)`	21
`(+ 5 7 8)`	20
`(– 5 6)`	–1
`(– 15 7 2)`	6
`(– 24 (* 4 3))`	12

Function Definition: LAMBDA

- Lambda Expressions
  - Form is based on λ notation

  e.g., `(LAMBDA (x) (* x x))`

  x is called a bound variable
- Lambda expressions can be applied to parameters

  e.g., `((LAMBDA (x) (* x x)) 7)`
- `LAMBDA` expressions can have any number of parameters

  `(LAMBDA (a b x) (+ (* a x x) (* b x)))`

`(LAMBDA (a b c x) (+ (* a x x) (* b x) c))`

Special Form Function: `DEFINE`

- A Function for constructing functions: `DEFINE` - Two forms:
  1. To bind a symbol to an expression

     e.g., `(DEFINE pi 3.141593)`

     Example use: `(DEFINE two_pi (* 2 pi))`

  - The evaluation process for `DEFINE` is different! The first parameter is never evaluated. The second parameter is evaluated and bound to the first parameter.

Special Form Function: `DEFINE`

- 2. The second use of the DEFINE function is to bind a lambda expression to a name.
- To bind a name to a lambda expression, DEFINE takes two lists as parameters. The first parameter is the prototype of a function call, with the function name followed by the formal parameters, together in a list. The second list contains an expression to which the name is to be bound.
- The general form of such a DEFINE is

Special Form Function: `DEFINE`

```
(DEFINE (function_name parameters)
(expression)
)
```

e.g., `(DEFINE (square x) (* x x))`

Example use: `(square 5)`

```
(DEFINE (hypotenuse side1 side2)
  (SQRT(+(square side1)(square side2)))
  )
```

Output Functions

- `(DISPLAY expression)`
- `(NEWLINE)`

Numeric Predicate Functions

- `#T` (or `#t`) is true and `#F` (or `#f`) is false (sometimes `()` is used for false)
- `=, <>, >, <, >=, <=`
- `EVEN?, ODD?, ZERO?, NEGATIVE?`
- The `NOT` function inverts the logic of a Boolean expression

Control Flow: `IF`

- Selection- the special form, `IF`

```
(IF predicate then_exp else_exp)
```

e.g.,

```
(IF (<> count 0)
    (/ sum count)
    0)
```

Control Flow: `COND`

- Multiple Selection - the special form, `COND`

  General form:

```
(COND
    (predicate_1  expr  {expr})
    (predicate_1  expr  {expr})
    ...
    (predicate_1  expr  {expr})
    (ELSE expr  {expr}))
```

- Returns the value of the last expression in the first pair whose predicate evaluates to true

Example of COND

```
(DEFINE (compare x y)
    (COND
      ((> x y) "x is greater than y")
      ((< x y) "y is greater than x")
      (ELSE "x and y are equal")
    )
 )
```

Example of `COND`

Ex: Leap year: Every **year** that is exactly divisible by four is a **leap year**, except for years that are exactly divisible by 100, but these centurial years are **leap** years if they are exactly divisible by 400.

```
(DEFINE (leap? year)
(COND
((ZERO? (MODULO year 400)) #T)
((ZERO? (MODULO year 100)) #F)
(ELSE (ZERO? (MODULO year 4)))
))
```

Example

```
(DEFINE (factorial n)
  (IF (<= n 1)
    1
    (* n (factorial (- n 1)))
))
```

List Functions: `CONS` and `LIST`

- `CONS` takes two parameters, the first of which can be either an atom or a list and the second of which is a list; returns a new list that includes the first parameter as its first element and the second parameter as the remainder of its result

  e.g., `(CONS 'A '(B C))` returns `(A B C)`

- `LIST` takes any number of parameters; returns a list with the parameters as elements

  e.g. `(LIST ’apple ’orange ’grape)` returns `(apple orange grape)`

List Functions: `CAR` and `CDR`

- `CAR` takes a list parameter; returns the first element of that list
  e.g., `(CAR '(A B C))` yields `A`
  `(CAR '((A B) C D))` yields `(A B)`

- `CDR` takes a list parameter; returns the list after removing its first element
  e.g., `(CDR '(A B C))` yields `(B C)`
  `(CDR '((A B) C D))` yields `(C D)`

List Functions: `CAR` and `CDR`

- `(DEFINE (second a_list) (CAR (CDR a_list)))`

Once this function is evaluated, it can be used, as in

```
(second '(A B C))   = returns B
```

- Some of the most commonly used functional compositions in Scheme are built in as single functions.

```
(CAAR x)   = (CAR(CAR x))
(CADR x) = (CAR (CDR x))
(CADDAR x)    = (CAR (CDR (CDR (CAR x)))).


(CADDAR '((A B (C) D) E)) = (C)
```

Predicate Function: `EQ?`

- `EQ?` takes two symbolic parameters; it returns `#T` if both parameters are atoms and the two are the same; otherwise `#F`

  e.g., `(EQ? 'A 'A)` yields `#T`

  `(EQ? 'A 'B)` yields `#F`

  - Note that if `EQ?` is called with list parameters, the result is not reliable
  - Also `EQ?` does not work for numeric atoms

Predicate Function: `EQV?`

- `EQV?` is like `EQ?`, except that it works for both symbolic and numeric atoms; it is a value comparison, not a pointer comparison

`(EQV? 3 3)` yields `#T`

`(EQV? 'A 3)` yields `#F`

`(EQV? 3.4 (+ 3 0.4))` yields `#T`

`(EQV? 3.0 3)` yields `#F` (floats and integers are different)

Predicate Functions: `LIST?` and `NULL?`

- `LIST?` takes one parameter; it returns `#T` if the parameter is a list; otherwise `#F`

  `(LIST? '())` yields `#T`

- `NULL?` takes one parameter; it returns `#T` if the parameter is the empty list; otherwise `#F`
  - Note that `NULL?` returns `#T` if the parameter is `()`
  - e.g. `(NULL? '(()))` yields `#F`

Example Scheme Function: **`member`**

- `member` takes an atom and a simple list; returns `#T` if the atom is in the list; `#F` otherwise

```
(DEFINE (member atm lis)
(COND
   ((NULL? lis) #F)
   ((EQ? atm (CAR lis)) #T)
   ((ELSE (member atm (CDR lis)))
))
```

Example Scheme Function: `equalsimp`

- `equalsimp` takes two simple lists as parameters; returns `#T` if the two simple lists are equal; `#F` otherwise

```
(DEFINE (equalsimp lis1 lis2)
(COND
    ((NULL? lis1) (NULL? lis2))
    ((NULL? lis2) #F)
    ((EQ? (CAR lis1) (CAR lis2))
         (equalsimp(CDR lis1)(CDR lis2)))
    (ELSE #F)
))
```

Example Scheme Function: `equal`

- `equal` takes two general lists as parameters; returns `#T` if the two lists are equal; `#F` otherwise

```
(DEFINE (equal lis1 lis2)
  (COND
    ((NOT (LIST? lis1))(EQ? lis1 lis2))
    ((NOT (LIST? lis2)) #F)
    ((NULL? lis1) (NULL? lis2))
    ((NULL? lis2) #F)
    ((equal (CAR lis1) (CAR lis2))
         (equal (CDR lis1) (CDR lis2)))
    (ELSE #F)
))
```

Example Scheme Function: `append`

- `append` takes two lists as parameters; returns the first parameter list with the elements of the second parameter list appended at the end

```
(DEFINE (append lis1 lis2)
  (COND
    ((NULL? lis1) lis2)
    (ELSE (CONS (CAR lis1)
                (append (CDR lis1) lis2)))
))
```

(append '(A B) '(C D R)) returns (A B C D R)

(append '((A B) C) '(D (E F))) returns ((A B) C D (E F))

Example Scheme Function: `LET`

- General form:

```
(LET (
    (name_1 expression_1)
    (name_2 expression_2)
    ...
    (name_n expression_n))
   body
)
```

- Evaluate all expressions, then bind the values to the names; evaluate the body

LET Example

```
(DEFINE (quadratic_roots a b c)
  (LET (
    (root_part_over_2a
     (/ (SQRT (- (* b b) (* 4 a c)))(* 2 a)))
    (minus_b_over_2a (/ (- 0 b) (* 2 a))))
  (DISPLAY (+ minus_b_over_2a root_part_over_2a))
  (NEWLINE)
  (DISPLAY (- minus_b_over_2a root_part_over_2a))
))
```

Tail Recursion in Scheme

- Definition: A function is *tail recursive* if its recursive call is the last operation in the function
- A tail recursive function can be automatically converted by a compiler to use iteration, making it faster
- Scheme language definition requires that its language systems convert all tail recursive functions to use iteration

Tail Recursion in Scheme (cont'd.)

- Example of rewriting a function to make it tail recursive, using helper a function

Original:

```
(DEFINE (factorial n)
   (IF (= n 0)
      1
      (* n (factorial (- n 1)))
 ))
```

Tail recursive:

```
(DEFINE (facthelper n factpartial)
    (IF (= n 0)
       factpartial
       facthelper((- n 1) (* n factpartial)))
  ))
 (DEFINE (factorial n)
    (facthelper n 1))
```

Functional Form - Composition

– If `h` is the composition of `f` and `g`, `h(x) = f(g(x))`

```
(DEFINE (g x) (* 3 x))
(DEFINE (f x) (+ 2 x))
(DEFINE h x) (+ 2 (* 3 x)))
```
(The composition)

– In Scheme, the functional composition function `compose` can be written:

```
(DEFINE (compose f g) (LAMBDA (x) (f (g x))))
```

`((compose CAR CDR) '((a b) c d))` yields `c`

```
(DEFINE (third a_list)
   ((compose CAR (compose CDR CDR)) a_list))
```

is equivalent to `CADDR`

Functional Form – Apply-to-All

- Apply to All - one form in Scheme is `map`
  - Applies the given function to all elements of the given list;

```
(DEFINE (map fun lis)
  (COND
    ((NULL? lis) ())
    (ELSE (CONS (fun (CAR lis))
                (map fun (CDR lis))))
))
```

```
(map (LAMBDA (num) (* num num num)) '(3 4 2 6))
```

yields `(27 64 8 216)`

Functions That Build Code

- It is possible in Scheme to define a function that builds Scheme code and requests its interpretation

- This is possible because the interpreter is a user-available function, `EVAL`

Adding a List of Numbers

```
((DEFINE (adder lis)
  (COND
    ((NULL? lis) 0)
    (ELSE (EVAL (CONS '+ lis)))
))
```

- The parameter is a list of numbers to be added; `adder` inserts a + operator and evaluates the resulting list
  - Use `CONS` to insert the atom `+` into the list of numbers.
  - Be sure that `+` is quoted to prevent evaluation
  - Submit the new list to `EVAL` for evaluation

Applications of Functional Languages

- LISP is used for artificial intelligence
  - Knowledge representation
  - Machine learning
  - Natural language processing
  - Modeling of speech and vision
- Scheme is used to teach introductory programming at some universities
- Support for functional programming is increasingly creeping into imperative languages

Comparing Functional and Imperative Languages

- Imperative Languages:
  - Efficient execution
  - Complex semantics
  - Complex syntax
  - Concurrency is programmer designed
- Functional Languages:
  - Simple semantics
  - Simple syntax
  - Inefficient execution
  - Programs can automatically be made concurrent