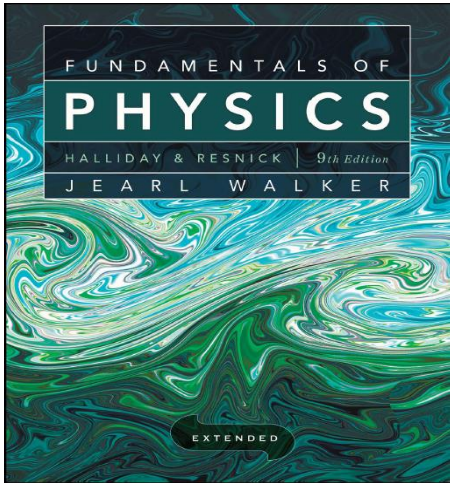
# FİZ 137- 25
# CHAPTER 2

# MOTION ALONG A STRAIGHT LINE

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***In this chapter we will study kinematics i.e. how objects move along a straight line.***

***The following parameters will be defined:***

- ***Displacement***
- ***Average velocity***
- ***Average Speed***
- ***Instantaneous velocity***
- ***Average acceleration***
- ***Instantaneous acceleration***

- For constant acceleration we will ***develope the equations of motion*** that give us the velocity and position at any time.

- We will study the motion under the ***influence of gravity*** close to the surface of the earth.

- We will study a graphical integration method that can be used to analyze the motion when the ***acceleration is not constant.***

# Mechanics

The study of ***Physics*** begins with mechanics. ***Mechanics*** is the branch of physics that focuses on the motion of objects and the forces that cause the motion to change.

**Mechanics**

***1. Kinematics*: ** deals with the concepts that are needed to describe ***motion***, without any reference to forces (Kinematics in one dimension, Kinematics in two dimensions).

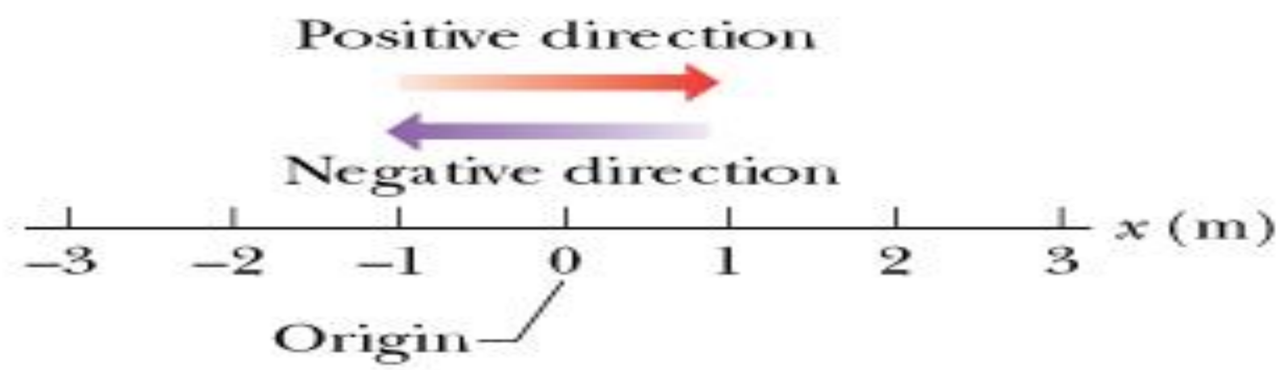
***2. Dynamics:*** deals with the effect that ***forces*** have on motion (Chapter 5, 6: Dynamics).

**<u>*Kinematics*</u> is the part of mechanics that describes the motion of physical objects.**

- We assume that the moving objects are ***"particles"***.
- We restrict our discussion to the motion of objects for which all the points move <u>*in the same way*</u>.
- The <u>*causes of the motion will not be investigated*</u>. This will be done later in Chapter 5 and Chapter 6.

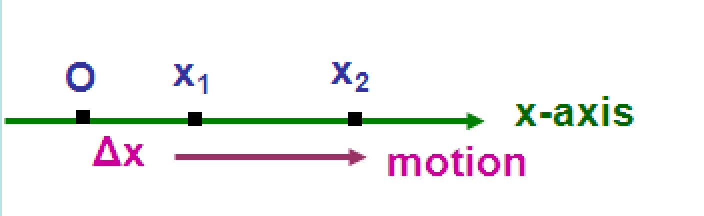
Consider an object is moving along a straight line taken to be the ***x-axis.*** ***The object's position*** at any time t is described by its coordinate **x(t)** defined with respect to the origin **O**.

The coordinate **x** can be positive or negative depending whether the object is located on the positive or the negative part of the x-axis.



# Displacement

• If an object moves from position $x_1$ to position $x_2$, the change in position is described by the '***displacement'***.



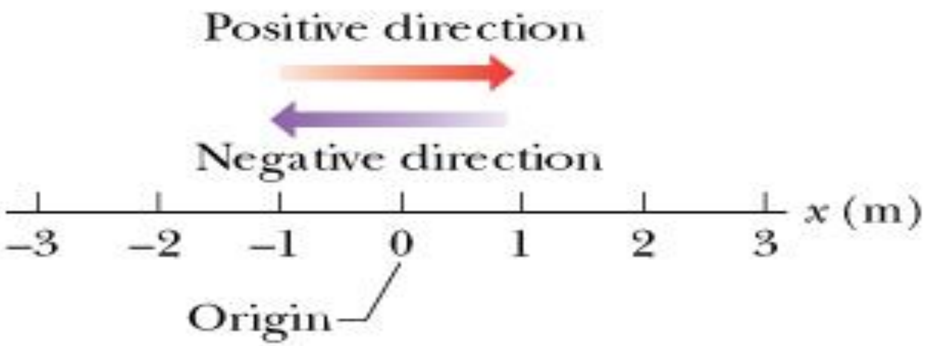

$$\boxed{\Delta x = x_2 - x_1}$$

• If $x_1$ = 5 m and $x_2$ = 12 m then $\Delta x$ = 12 – 5 = 7 m. The positive sign of $\Delta x$ indicates that the motion is along the ***positive x-direction***.

• If instead the object moves from $x_1$ = 5 m and $x_2$ = 1 m then $\Delta x$ = 1 – 5 = - 4 m. The negative sign of $\Delta x$ indicates that the motion is along the ***negative x-direction.***

***‘Displacement`*** **is a vector quantity that has both** ***magnitude*** **and** ***direction*****.**

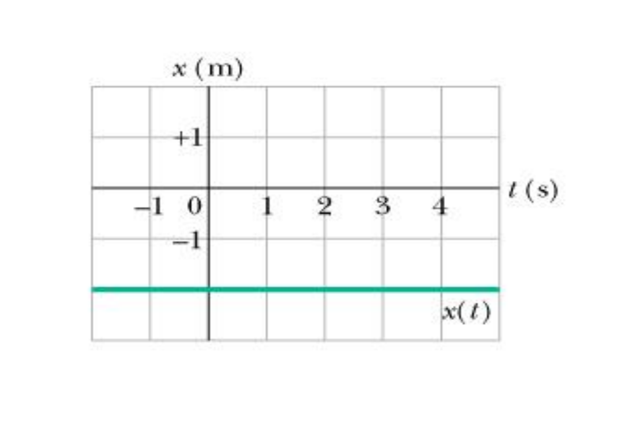
**In restricted one-dimensional motion, the direction is described by the algebraic sign of $\Delta x$.**

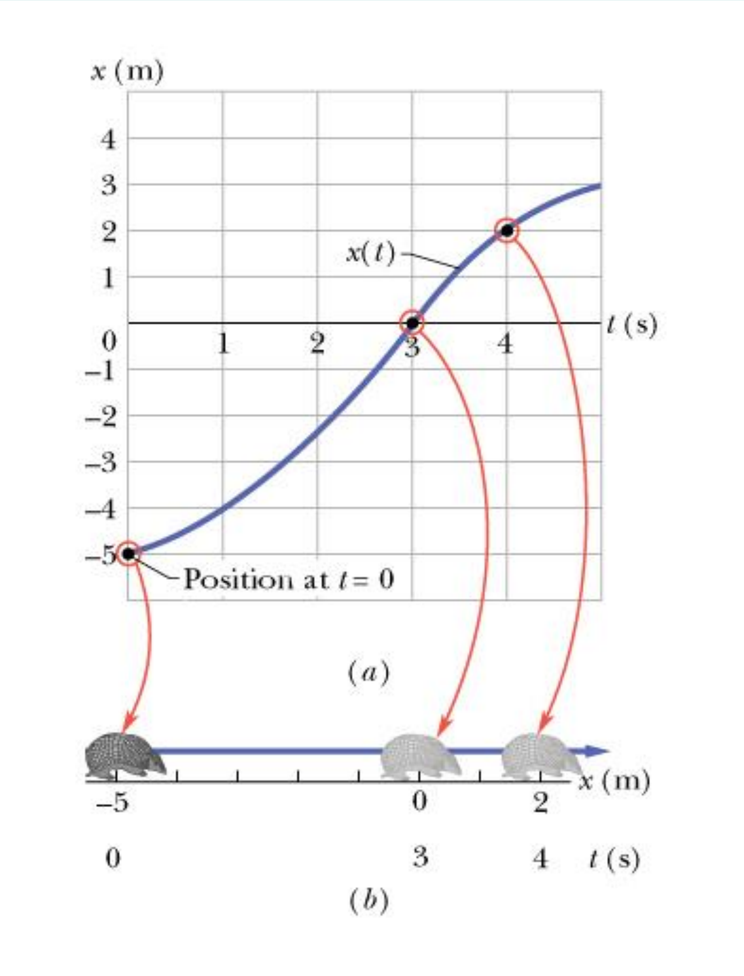



Consider as an example the motion of an object from an initial position $x_1$ = 5 m to x = 200 m and then back to $x_2$ = 5 m. Even though the total distance covered is 390 m the displacement then $\Delta x = 0$.

# Average Velocity

***One method of describing the motion of an object is to plot its position x(t) as function of time t.*** In the left picture we plot x versus t for an object that is stationary with respect to the chosen origin O. Notice that x is constant.




In the equation given below, $x_2$ and $x_1$ are the positions $x(t_2)$ and $x(t_1)$, respectively. The **time interval** $\Delta t$ is defined as:

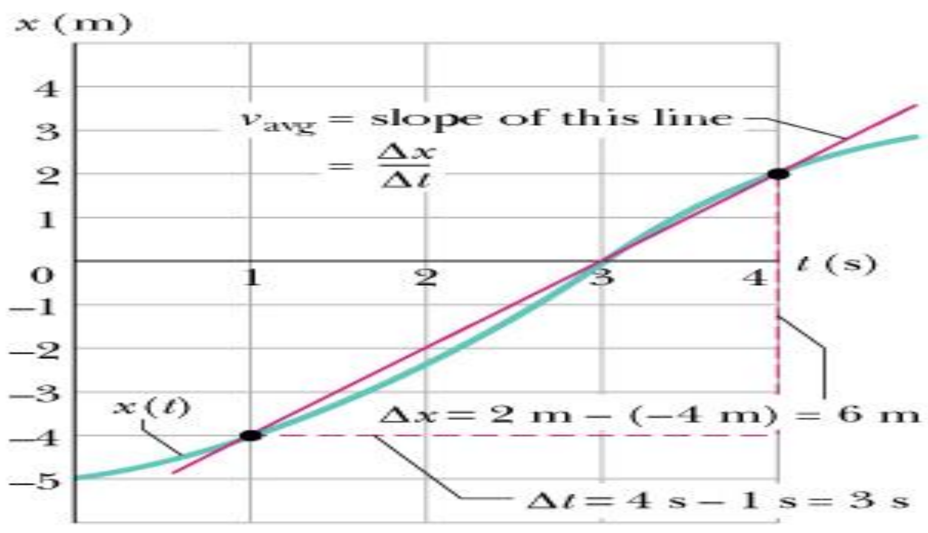
$\Delta t = t_2 - t_1$

$$\boxed{v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}}$$

**The unit of $v_{avg}$ is→ m/s**

# Graphical Determination of $v_{avg}$

On an x versus t plot we can determine $v_{avg}$ from the slope of the straight line that connects point ( $t_1$, $x_1$) with point ( $t_2$, $x_2$ ). *In the plot below $t_1 = 1$ s, and $t_2 = 4$ s. The corresponding positions are: $x_1 = -4$ m and $x_2 = 2$ m.*




$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{2-(-4)}{4-1} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}$$

# <u>Speed</u>

We define speed as the ***<u>magnitude</u>*** of an object's velocity vector.

***The <u>average velocity</u> and the <u>average speed</u> for the same time interval $\Delta t$ can be quite different.***

# <u>Average Speed</u> $s_{avg}$

The average speed is defined in terms of the ***<u>total distance</u>*** (***<u>not the displacement Δx</u>***) traveled in a time interval Δt.

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

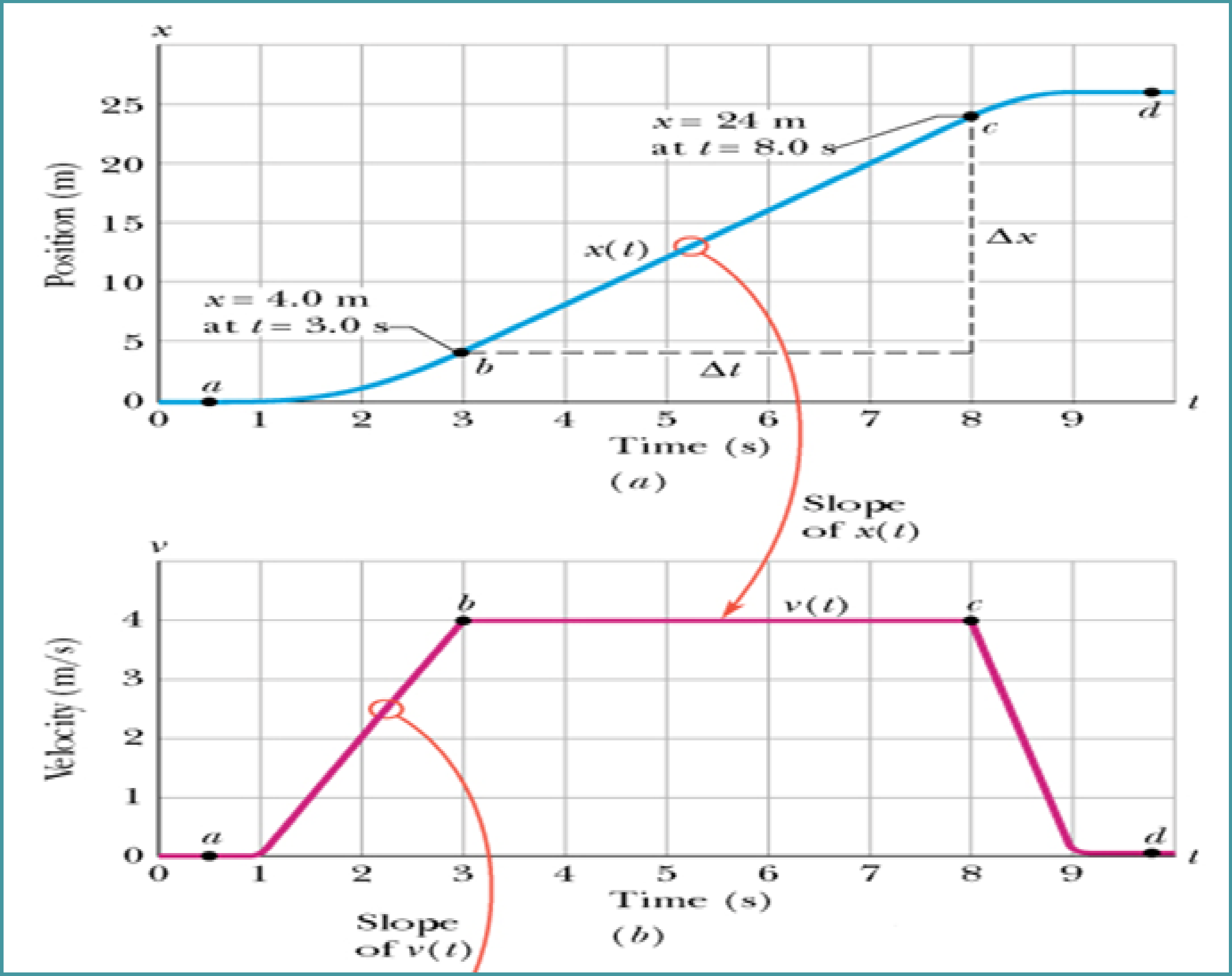
# Instantaneous Velocity

The average velocity $v_{avg}$ determined between times $t_1$ and $t_2$ provide a useful description on **"how fast"** an object is moving between two selected times.

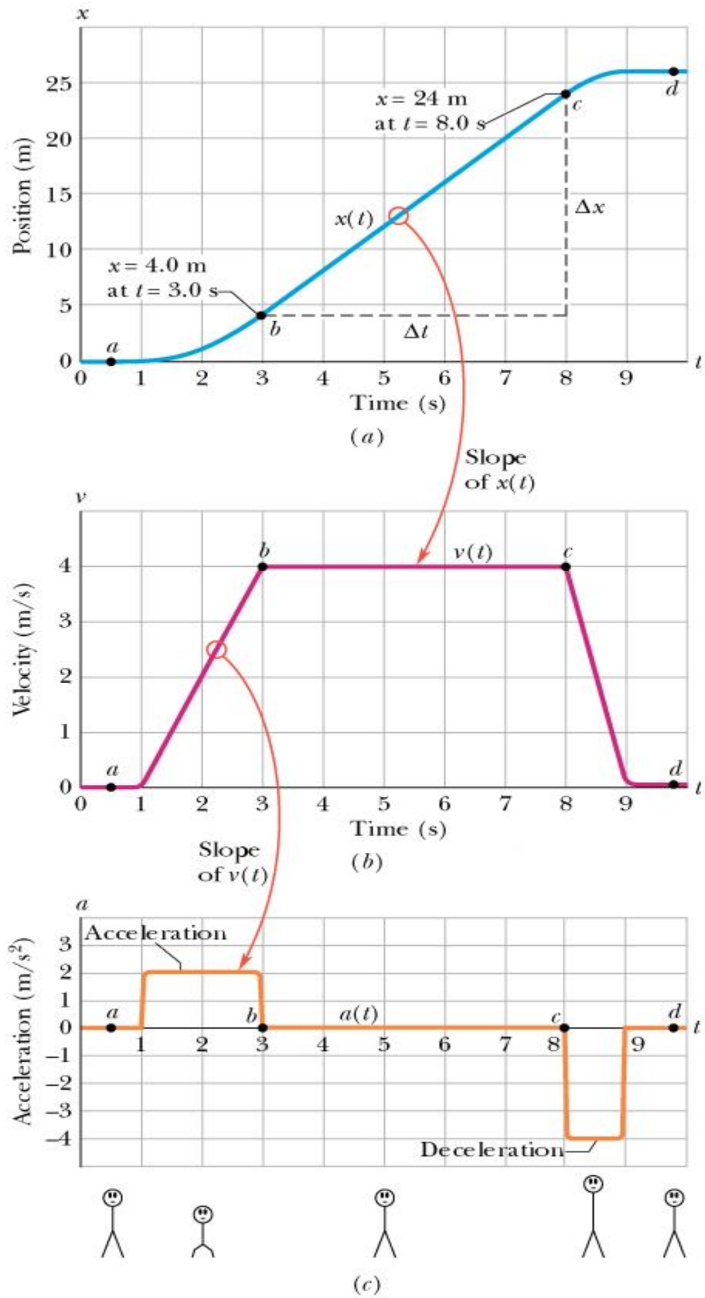
***Instantaneous velocity is defined as the limit of the average velocity determined for a time interval $\Delta t$ as we let $\Delta t \to 0$.***

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

From its definition, instantaneous velocity is the first derivative of the position coordinate x with respect to time or equal to the slope of the x versus t plot.




$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$




## Average Acceleration

We define as the average acceleration $a_{avg}$ between $t_1$ and $t_2$ as:

$$\boxed{a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}}$$

**The unit of $a_{avg}$ is $\rightarrow$ m/s$^2$**

# Instantaneous Acceleration

If we take the limit of $a_{avg}$ as $\Delta t \to 0$ we get the instantaneous acceleration $a_{ins}$ which describes how fast the velocity is changing at any time t.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad , \quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The acceleration is the slope of the v versus t plot.

***Note:*** The human body does not react to velocity but it <u>does react to acceleration</u>.

# Deceleration

• An object speeds up when the acceleration and velocity vectors point in the same direction.

• Whenever the acceleration and velocity vectors have opposite directions, the object slows down and is said to be ***"decelerating."***

**Example 4:** A drag racer crosses the finish line, and the driver deploys a parachute and applies the brakes to slow down. The driver begins slowing down when $t_0$ = 9.0 s and the car's velocity is $\mathbf{v_0}$ = +28 m/s. When $t$ = 12.0 s, the velocity has been reduced to $\mathbf{v}$ = +13 m/s. What is the average acceleration of the dragster?

Sample Problem

## Acceleration and $dv/dt$

A particle's position on the $x$ axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with $x$ in meters and $t$ in seconds.

(a) Because position $x$ depends on time $t$, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

**KEY IDEAS**

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

***Calculations:*** Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with $v$ in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with $a$ in meters per second squared.

(b) Is there ever a time when $v = 0$?

***Calculation:*** Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

***Reasoning:*** We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the $x$ axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing.

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing.

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m. Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.

# Motion with Constant Acceleration

Motion with a = 0 is a special case but it is rather common so we will develop the equations that describe it.

$a = \dfrac{dv}{dt} \rightarrow dv = adt$ If we intergrate both sides of the equation we get:

$\int dv = \int adt = a\int dt \rightarrow \boxed{v = at + C}$ Here C is the integration constant

C can be determined if we know the velocity $v_o = v(0)$ at t = 0

$v(0) = v_o = (a)(0) + C \rightarrow C = v_o \rightarrow \quad v = v_0 + at \quad$ (eqs.1)

$$v = v_0 + at$$

$$v = \frac{dx}{dt} \rightarrow dx = vdt = (v_0 + at)dt = v_0 dt + atdt$$ If we integrate both sides we get:

$$\int dx = \int v_0 dt + a\int tdt \rightarrow x = v_o t + \frac{at^2}{2} + C'$$ Here $C'$ is the integration constant

$C'$ can be determined if we know the position $x_o = x(0)$ at t = 0

$$x(0) = x_o = (v_o)(0) + \frac{a}{2}(0) + C' \rightarrow C' = x_o$$

$$\boxed{x(t) = x_o + v_o t + \frac{at^2}{2}} \quad \text{(eqs.2)}$$

$v = v_0 + at$ (eqs.1) ; $x = x_o + v_o t + \frac{at^2}{2}$ (eqs.2)

If we eliminate the time t between equation 1 and equation 2 we get:

$\boxed{v^2 - v_o^2 = 2a(x - x_o)}$ (eqs.3)

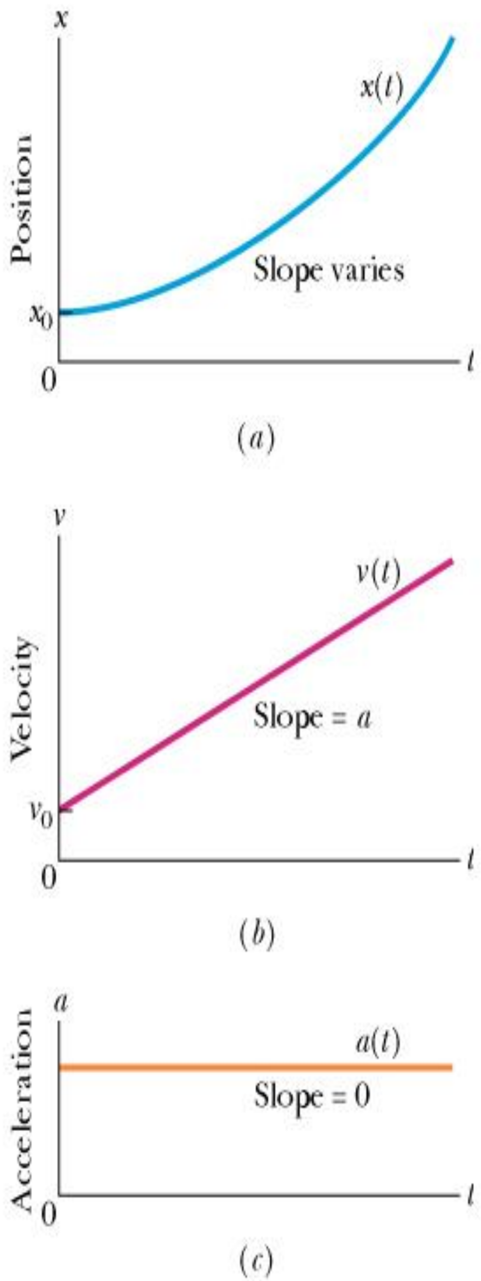
Below we plot the position x(t), the velocity v(t) and the acceleration a versus time t

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{xo} t + \frac{1}{2} a_x t^2$$

$$(v_x)^2 = (v_{xo})^2 + 2a(x\text{-}x_o)$$

$$x = x_0 + \frac{1}{2}(v_{x0} + v_x)t$$




# The acceleration a is a constant

The x(t) versus t plot is a parabola that intercepts the vertical axis at x = $x_o$

$$x = x_o + v_o t + \frac{at^2}{2}$$

The v(t) versus t plot is a straight line with Slope = a and Intercept = $v_o$

$$v = v_0 + at$$

**a → constant**

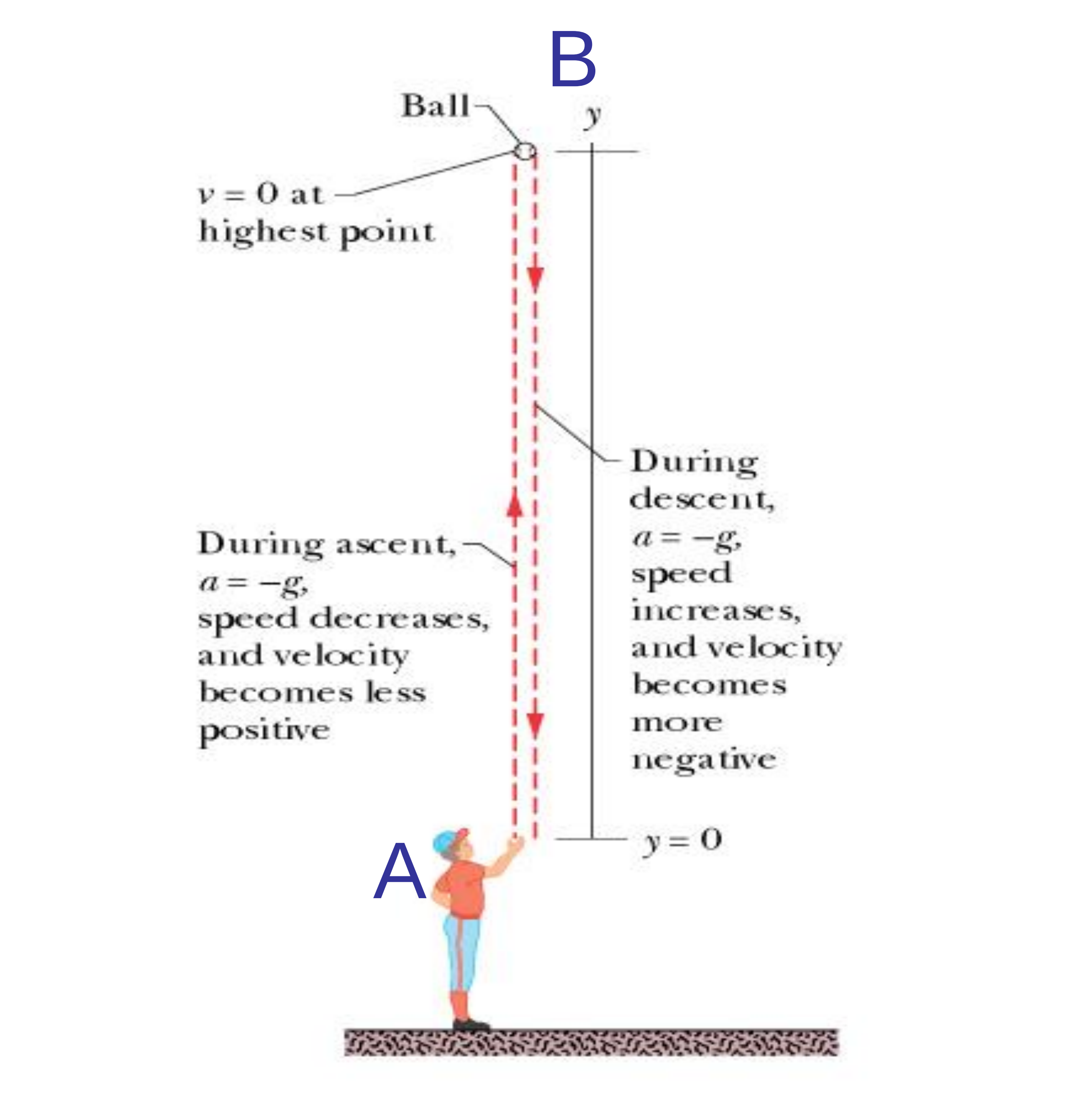
# <u>Free Fall</u>

Close to the surface of the earth all objects move towards the <u>*center of the earth*</u> with an acceleration whose magnitude is constant and equal to **9.8 m/s$^2$**.

We use the symbol ***g*** (***g*** **= 9.8 m/s$^2$**) to indicate the acceleration of an object in free fall.

$a = -g$

y



B
Ball
y
v = 0 at
highest point
During
descent,
a = −g,
speed
increases,
and velocity
becomes
more
negative
During ascent,
a = −g,
speed decreases,
and velocity
becomes less
positive
A
y = 0

If we take the y-axis to point upwards then the acceleration of an object in free fall ***a = - g*** and the equations for free fall take the form (Vo = 0 for free fall):

$$v_x = v_{x0} + a_x t$$
$$x = x_0 + v_{xo}t + \frac{1}{2}a_x t^2$$
$$(v_x)^2 = (v_{xo})^2 + 2a(x-x_o)$$
$$x = x_0 + \frac{1}{2}(v_{x0} + v_x)t$$

→

$$v = v_0 - gt \qquad \text{(eqs.1)} \quad ;$$

$$x = x_o + v_o t - \frac{gt^2}{2} \qquad \text{(eqs.2)}$$

$$v^2 - v_o^2 = -2g(x - x_o) \qquad \text{(eqs.3)}$$

**Note:** Even though with this choice of axes a < 0, the velocity can be positive (upward motion from point A to point B). It is momentarily zero at point B. The *velocity becomes negative* on the downward motion from point B to point A.

# Vertical Motion (Up)

*Initial velocity:* $v_{yo}$

$a_y = -g$

$$v_y = v_{yo} - gt$$

$$y = h = (v_{yo})t - \tfrac{1}{2} gt^2$$

$$(v_y)^2 = (v_{yo})^2 - 2g\,(y - y_o)$$

# Vertical Motion (Down)

*Initial velocity:* $-v_{yo}$

$a_y = -g$

$$v_y = v_{yo} + gt$$

$$y = h = (v_{yo})t + \tfrac{1}{2} gt^2$$

$$(v_y)^2 = (v_{yo})^2 + 2g(y - y_o)$$

## Sample Problem

### Time for full up-down flight, baseball toss

In Fig. 2-11, a pitcher tosses a baseball up along a $y$ axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

**KEY IDEAS**

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity $v$ at the maximum height must be 0.

***Calculation:*** Knowing $v$, $a$, and the initial velocity $v_0 = 12$ m/s, and seeking $t$, we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\ \text{m/s}}{-9.8\ \text{m/s}^2} = 1.2\ \text{s}. \qquad \text{(Answer)}$$

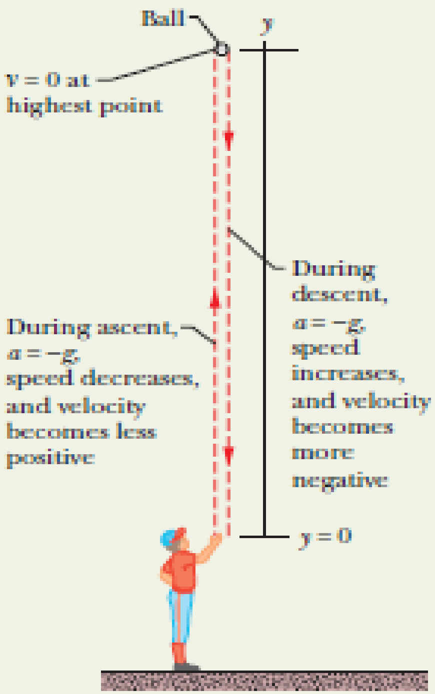



**Fig. 2-11** A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

(b) What is the ball's maximum height above its release point?

***Calculation:*** We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in $y$ notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for $y$. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 7.3\ \text{m}. \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

***Calculations:*** We know $v_0$, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want $t$, so we choose Eq. 2-15. Rewriting it for $y$ and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0\ \text{m} = (12\ \text{m/s})t - (\tfrac{1}{2})(9.8\ \text{m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for $t$ yields

$$t = 0.53\ \text{s} \quad \text{and} \quad t = 1.9\ \text{s}. \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

Additional examples, video, and practice available at *WileyPLUS*

# Non-Constant Accelaration

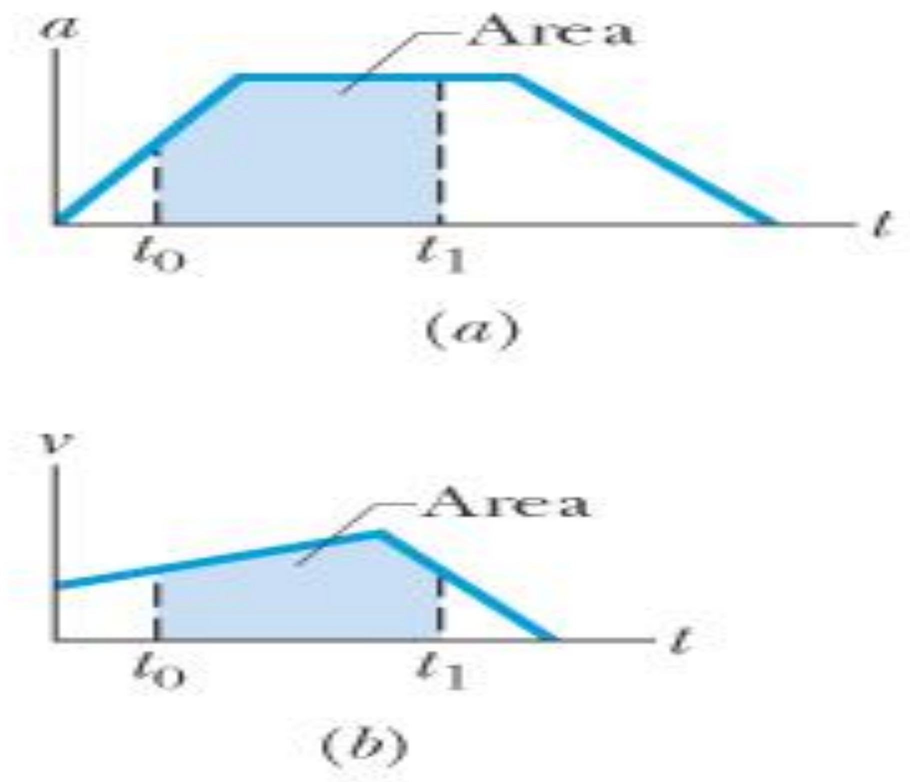
Graphical Integration in Motion Analysis (non-constant acceleration)

When the acceleration of a moving object is not constant we must use integration to determine the velocity v(t) and the position x(t) of the object.

The integration can be done either using the analytic or the graphical approach

$$a = \frac{dv}{dt} \rightarrow dv = adt \rightarrow \int_{t_o}^{t_1} dv = \int_{t_o}^{t_1} adt \rightarrow v_1 - v_o = \int_{t_o}^{t_1} adt \rightarrow v_1 = v_o + \int_{t_o}^{t_1} adt$$

$$\int_{t_o}^{t_1} adt = \left[\text{Area under the } a \text{ versus } t \text{ curve between } t_o \text{ and } t_1\right]$$




$$a=\frac{dv}{dt}\rightarrow dv=adt\rightarrow\int_{t_o}^{t_1}dv=\int_{t_o}^{t_1}adt\rightarrow v_1-v_o=\int_{t_o}^{t_1}adt\rightarrow v_1=v_o+\int_{t_o}^{t_1}adt$$

$$v=\frac{dx}{dt}\rightarrow dx=vdt\rightarrow\int_{t_o}^{t_1}dx=\int_{t_o}^{t_1}vdt\rightarrow x_1-x_o=\int_{t_o}^{t_1}vdt\rightarrow\boxed{x_1=x_o+\int_{t_o}^{t_1}vdt}$$

$$\int_{t_o}^{t_1}vdt=\left[\text{Area under the v versus t curve between } t_o \text{ and } t_1\right]$$