

Hacettepe University – Computer Engineering Dept.  
BBM 231 Logic Design  
2018-2019 Fall – MIDTERM II

Date : December 24, 2018

Duration : 120 Minutes

Full Name :

Signature:

Student ID :

BBM231 Section:

Ques.	1	2	3	4	Total
Points	20	25	35	20	100

**Question 1 (20 points):**

Implement the following function using nothing but a single 4x1 multiplexer.

$$F(a,b,c,d) = ab' + a'b'cd' + a'cd + bd$$

Show your solution in detail for full credit.

The handwritten solution for Question 1 is as follows:

**Truth Table:**

a	b	c	d	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

**Karnaugh Map:**

ab \ cd	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	1	1	1	1
10	1	1	1	1

**Simplified Boolean Expression:**

$$F(a,b,c,d) = ab' + a'b'cd' + a'cd + bd$$

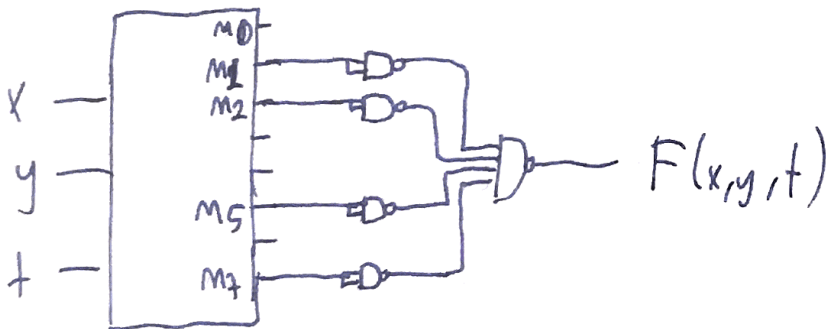
**4x1 Multiplexer Circuit Diagram:**

The circuit diagram shows a 4x1 multiplexer with inputs  $a$  and  $b$  at the bottom, and inputs  $c$  and  $d$  at the top. The output is  $F$ . The inputs are connected to the select lines of the multiplexer. The output  $F$  is the result of the function  $F(a,b,c,d)$ .

**Question 2 (25 points):**

Implement  $F(x, y, t) = (x \oplus t)'y + (x'y) \oplus t$  with one 3x8 decoder and NAND gates only. You must minimize the number of NAND gates as much as possible. You may use NAND gates with 2 or more inputs. Show your solution in detail for full credit.

$$\begin{aligned} F(x, y, t) &= (x \oplus t)'y + (x'y) \oplus t \\ &= (xt + x't')y + x'y t' + (x + y')t \\ &= \underbrace{(xyt)}_{\text{m}_1} + \underbrace{(x'y t')}_{\text{m}_2} + \cancel{x'y t} + \cancel{xyt} + \underbrace{(xy't)}_{\text{m}_5} + \cancel{xy't'} + \underbrace{(x'y't)}_{\text{m}_7} \\ &= \sum(1, 2, 5, 7) \end{aligned}$$



### Question 3 (35 points):

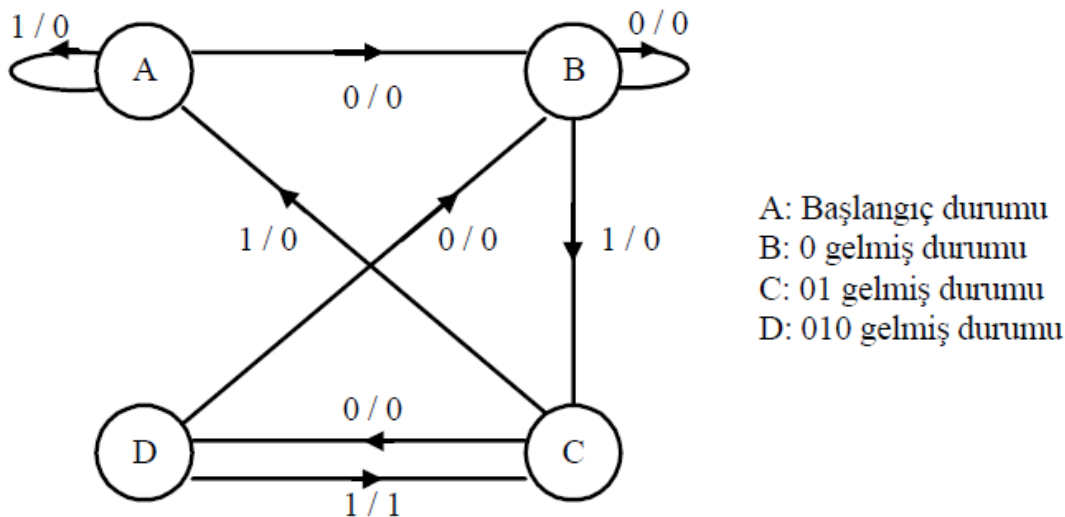
Design a one input one output circuit such that as the input is evaluated at every clock, when the input 0101 is detected, output becomes 1, otherwise it is 0. An example input-output sequence is given below.

Input (x)	0	0	1	0	1	0	1	1	1
Output (z)	0	0	0	0	1	0	1	0	0

In other words, design a Mealy type sequential circuit that detects the sequence 0101 using D-type flip flops and logic gates. Your circuit can have a maximum of 2 D-type flip flops.

- (10) Draw the state diagram of the circuit
- (5) Derive the complete state table from the state diagram
- (10) Using Karnaugh-maps, find the most simplified Boolean expressions for flip-flop outputs and the circuit output
- (10) Draw the complete circuit

#### State diagram:



#### First assignment type:

Let A=00, B=01, C=10, and D=11.

Let y1 and y2 be present state and Y1 and Y2 be next state variables. We can fill the state table as follows:

x	y1	y2	Y1	Y2	z
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	1	0	1

$$Z = xy_1y_2$$

$$Y1 = xy_2 + x'y_1y_2'$$

$$Y2 = x'$$

## Second assignment type:

Let A=00, B=01, C=11, and D=10.

Let y1 and y2 be present state and Y1 and Y2 be next state variables. We can fill the state table as follows:

x	y1	y2	Y1	Y2	z
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

$y_1 y_2$	00	01	11	10
x				
0			1	
1		1		1

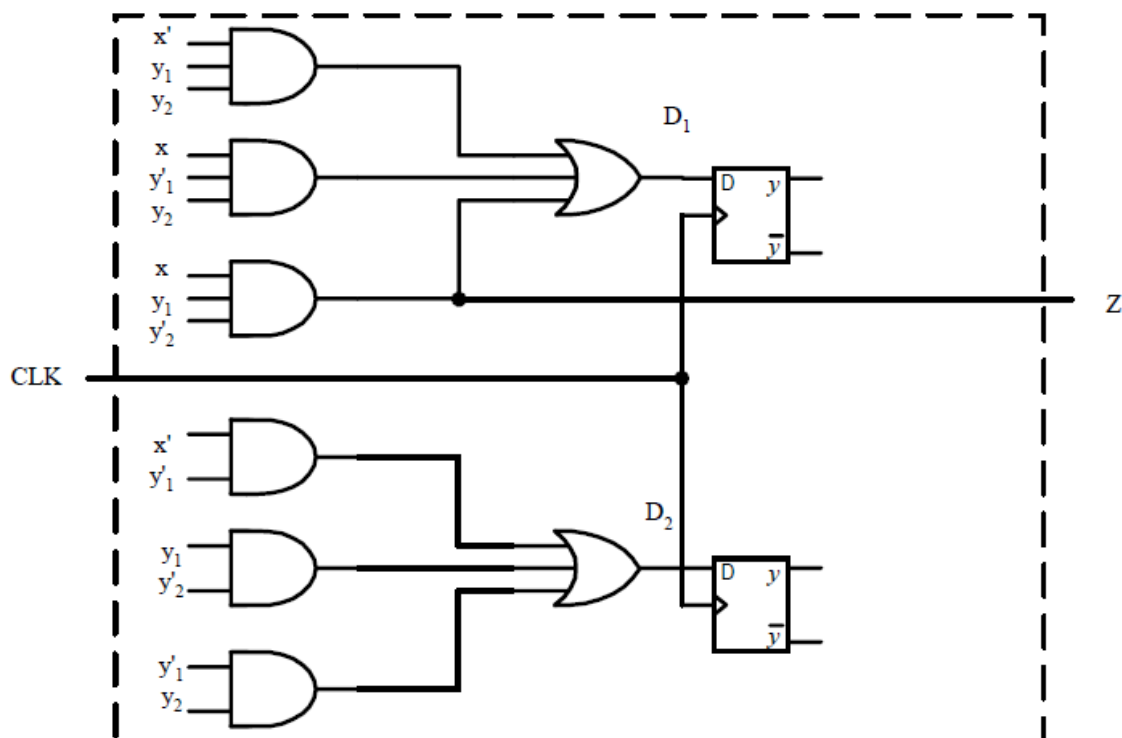
$y_1 y_2$	00	01	11	10
x				
0	1	1		1
1		1		1

$y_1 y_2$	00	01	11	10
x				
0				
1				1

$$D_1 = Y_1 = x' y_1 y_2 + x y_1' y_2 + x y_1 y_2'$$

$$D_2 = Y_2 = x' y_1' + y_1' y_2 + y_1 y_2'$$

$$Z = x y_1 y_2'$$



#### Question 4 (20 points):

For the circuit given below, at the beginning the clock signal is at 0 level and  $y_1=0$ ,  $y_2=0$ . MSB: most significant bit. LSB: least significant bit. A rise of the clock signal to 1 level and a subsequent fall back to 0 level together constitute a **pulse**. Fill the rest of the table below for  $y_1$ ,  $y_2$  (which can be 0 or 1) and Z output (which can be one of the given 8 letters). Show your solution below the figure in detail for full credit.

