İST 292 STATISTICS

Sections: 05-06

For Department of Computer Engineering

LESSON 4 SAMPLING DISTRIBUTIONS

Dr. Ayten Yiğiter and Dr. Esra Polat Lecture Notes

4. SAMPLING DISTRIBUTIONS

- Since sample statistics are random variables, they therefore have (possess) probability distributions that are either discrete or continuous. These probability distributions, called ***sampling distributions*** because they characterize the distribution of values of the various statistics over a very large number of samples, are the topic of this lesson.

- A ***parameter*** is a numerical descriptive measure of a population. It is calculated from the observations in the population. Since it is almost impossible to get all observations of a population because it is costly and time consuming, a sample which would be desribed well to the population is taken from a population and then ***sample statistics*** <u>are used to make inference about the parameters of a population.</u>

- A ***sample statistic*** is a numerical descriptive measure of a sample. It is calculated from the observations in the sample. Not that the term ***statistic*** refers to a sample quantity and the term ***parameter*** refers to a population quantity.

- If we want to estimate a parameter of a population- such as μ - there are a number of sample statistics that could be used for the estimation. Sample statistics for μ: the sample mean $\overline{X}$ and the sample median $\overline{X}'$. **Which of these do you think will provide a better estimate of μ?**

- **Answer:** Neither the sample mean nor the sample median will always fall closer to the population mean. Consequently, we can not compare these two sample statistics, or, in general, any two sample statistics, on the basis of their performance ***for a single sample.***

- The *sample statistics (for example,* $\bar{x}$) are themselves random variables, they must be judged and compared on the basis of their *probability distributions* i.e the collection of values and associated probabilities of each statistic that would be obtained if the sampling experiment were repeated a very large number of times.

- The *sampling distribution* of a sample statistic calculated from a sample of n measurments is the probability disribution of the statistic. A sampling distribution is the distribution of a statistic "under repeated sampling". In other words, it tell us the values that a statistic takes on, and how often it takes them on.

Example: A large tank of fish from a hatchery is being delivered to the lake. We want to know the average length of the fish in the tank. Instead of measuring all of the fish, we randomly sample 20 fish and use the sample mean $(\overline{X})$ to estimate the population mean (μ).

Denote the sample mean of the 20 fish as $\overline{X}_1$. Suppose we take a separate sample of size 20 from the same hatchery. Denote that sample mean as $\overline{X}_2$.

Would $\overline{X}_1$ equal $\overline{X}_2$? Not necessarily. What if we took another sample and found the mean? Consider now taking 1000 random samples of 20 and recording all of the sample means. We could take the 1000 sample means and create a histogram. This would give us a picture of what the distribution of the sample means looks like. The distribution of all of these sample means is the sampling distribution of the sample mean.

A ***point estimator*** of a population parameter is rule of formula that tells us how to use the sample data to calculate a single number that can be used as an estimate of the population parameter.

population parameter	***point estimate of the parameter***
μ	$\bar{x}$ (sample mean)
σ^2	s^2 (sample variance)

By examining the sampling distribution, we can determine how large the difference between an estimate and the true value of the parameter (called the error of estimation) is likely to be. **For example:** *The population mean is μ and sample mean is $\bar{x}$ than $\bar{x}-\mu$ is called as* ***the error of estimation or sampling error.***

- If the sampling distribution of a sample statistic has a mean equal to the population parameter which the statistic is intended to estimate, the statistic is said to be an unbiased estimate of the parameter. For example, sample mean is an unbiased estimator of a population mean showed like: $E(\bar{x}) = \mu$

- If the mean of the sampling distribution is not equal to the parameter, the statistic is said to be a biased estimate of the parameter.

$$E(\bar{X}) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n}\left(E(X_1) + E(X_2) + \ldots + E(X_n)\right)$$

$$= \frac{1}{n}(\mu + \mu + \ldots + \mu) = \frac{n\mu}{n} = \mu$$

- The *standard deviation of a sampling distribution* (For example standart deviation of $\bar{X}$ is showed as $S_{\bar{x}} = \frac{S}{\sqrt{n}}$ called also as standard error of the estimate) measures another important property of statistics- the spread of these estimates generated by repeated sampling.

- For example the means of the two sampling distributions are the same $E(\bar{X}_1) = 65$ and $E(\bar{X}_2) = 65$, we turn to their standard deviations (standard error of the estimate), such as $S_{\bar{x}_1} = 3$ and $S_{\bar{x}_2} = 1$ in order to decide which will provide estimates that fall closer to the unknown population parameter $(\mu = 65)$ we are estimating. Becareful, since both $E(\bar{X}_1) = E(\bar{X}_2) = \mu = 65$, these two estimates are unbiased estimates. Naturally, we will choose the sample statistic that has the smaller standart deviation. Hence choose, $\bar{X}_2$

- Properties of Good Estimators: unbiasedness and having smaller standart deviation

4.1.The Distribution of the Mean

- Since statistics (sample quantities: sample mean, variance, etc.) are random variables, their values will vary from a sample to another sample. **Sample mean has a probability distribution and also its expected value and variance can be found.**

- Let $X_1, X_2, \ldots, X_n$ be a random sample from a an infinite population with the mean μ and the variance σ^2, then

$$E(\bar{X}) = \mu, \quad V(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$$

- $\bar{X}$ is an *unbised estimator of the population mean* μ, and $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$ is called *standard error the mean*. It refers to a measure how close to the mean of a sample is the population mean μ. When the sample size n gets close to ∞, $\frac{\sigma^2}{n} \to 0$, and so we say that $\bar{X} \to \mu$.

4.1.1.The Distribution of the Mean: Finite Populations

- If an experiment consists of selecting one or more values from a finite set of numbers $\{c_1, c_2, \ldots c_N\}$, this set is referred to as *a finite population size N. Assume that we are sampling without replacement from a finite population size N.*

- If $\bar{X}$ is the mean of a random sample of size n from a finite population size N with the mean μ and the variance σ^2, then $$E(\bar{X}) = \mu, \quad V(\bar{X}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

where $\frac{N-n}{N-1}$ is called the *finite population correction factor.*

- This term is usually negligible (ihmal edilebilir); as you can write $\frac{\mathrm{N}-\mathrm{n}}{\mathrm{N}-1}$ as $\frac{1-\frac{\mathrm{n}}{\mathrm{N}}}{1-\frac{1}{\mathrm{N}}}$

you can see that $\mathrm{N} \to \infty$ then $\frac{1-\frac{\mathrm{n}}{\mathrm{N}}}{1-\frac{1}{\mathrm{N}}} \to 1$ so that this term could be negligible.

4.1.2. The Distribution of the Mean: A Sample from Normal Distribution (Population)

- Let $X_1, X_2, \ldots, X_n$ be a random sample *from normal distribution* with the mean μ and the variance σ^2. It is shown as $X_1, X_2, \ldots, X_n$ ~ $N(\mu, \sigma^2)$, then

$$E(\bar{X}) = \mu, \quad V(\bar{X}) = \frac{\sigma^2}{n}$$

- also the distribution of a sample mean $\bar{X}$ is a normal distribution with the mean μ and the variance σ^2 / n and shown $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

- In addition, standardized random variable has the standard normal distribution:

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

4.2. The Distribution of the Ratio: A Sample from Bernoulli Distribution (Population)

Suppose that we are interested in whether each value of a population has a particular status or not- *0* or *1* value is assigned, *0* refers to *“failure”* and *1* refers to *“success”*- say that the population has Bernoulli distribution with probability *p* (each success occurrs with the probability *p*). It is shown as $X \sim Bernoulli(p)$.

Remember:

Bernoulli distribution probability function:

$$f(x) = p^x (1-p)^{1-x} \qquad x = 0,1$$

$$\mu = E(X) = p \qquad and \qquad \sigma^2 = V(X) = p(1-p)$$

Suppose a random sample of n measurements is drawn from this Bernoulli distribution, **the total number of successes and the ratio of success are calculated:**

If $X_1, X_2, \ldots, X_n \sim Bernoulli(p)$ (here, each X_i takes value 0 for failure or 1 for success) :

$\sum_{i=1}^{n} X_i$ is the *total number of successes* and $\hat{p} = \frac{\sum_{i=1}^{n} X_i}{n}$ is the *ratio of success* in the sample. These are statistics calculated from the sample, then:

$$E\left(\sum_{i=1}^{n} X_i\right) = np\,, \quad V\left(\sum_{i=1}^{n} X_i\right) = npq$$

$$E(\hat{p}) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{\sum_{i=1}^{n} E(X_i)}{n} = \frac{np}{n} = p$$

$$V(\hat{p}) = V\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{\sum_{i=1}^{n} V(X_i)}{n^2} = \frac{npq}{n^2} = \frac{pq}{n}$$

For Example: The Bernoulli random variable might be the status of single computer microchip (good or defective, **be careful for each microchip only two statuses**):

$\sum_{i=1}^{n} X_i$ is the number of n such chips that are good

$\hat{p} = \frac{\sum_{i=1}^{n} X_i}{n}$ is the fraction of good chips in a set of n

Note that $X_1, X_2, \ldots, X_n \sim Bernoulli(p)$, $\sum_{i=1}^{n} X_i$ has a **Binomial distribution with parameters (n,p).**

Remember:

Binomial distribution probability function:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \cdots, n$$

$$\mu = E(X) = np \qquad and \qquad \sigma^2 = V(X) = np(1-p)$$

4.3. The Student's t Distribution

If a random sample $\left(X_1, X_2, \ldots, X_n\right)$ is drawn from normal distribution with the mean μ and the variance σ^2 showed as $\mathrm{N}\left(\mu, \sigma^2\right)$, the random variable sample mean $\left(\bar{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}\right)$ has a normal distribution with the mean μ and the variance σ^2 / n showed as: $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ then,

$Z = \dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$ has the *standart normal distribution.*

- S^2 is the variance of random sample of size n from normal distribution with the mean μ and the variance σ^2.

- Let our interest be find the exact disribution of the random variable $\frac{\bar{X}-\mu}{S/\sqrt{n}}$, its distribution is known as ***t distribution with n-1 degrees of freedom.***

- The *t* distribution was introduced by W. S. Gosset, who published his scientific writings under pen name "Student" since the company which he worked, a brewery, did not permit publication by employees. Thus the t distribution is also known as the *Student t distribution* or *Student's t distribution*. (John E. Freund's Math. Stat. with Applications, 2004).

- If T is a Student t distribution with ν degrees of freedom, T has zero mean $E(T) = \mu = 0$ and $V(T) = \sigma^2 = \frac{v}{v-2} \; (v > 2)$ variance.

- *t distribution is a family of distributions that look almost identical to the normal distribution curve, only a bit shorter and fatter. The t distribution is used instead of the normal distribution when you have small samples. The larger the sample size, the more the t distribution looks like the normal distribution. In fact, for sample sizes larger than 30 (e.g. more degrees of freedom), the distribution is almost exactly like the normal distribution.*

Definition of Degrees of Freedom

- In <u>statistics</u>, the **degrees of freedom (df)** appears in many contexts throughout statistics *including probability distributions, hypothesis tests and regression analysis etc.*

- *Degrees of freedom are the number of independent values that a statistical analysis can estimate.* You can also think of it as the number of values that are free to vary as you estimate parameters.

- <u>Typically, the degrees of freedom equal your sample size minus the number of parameters you need to calculate during an analysis. It is a positive whole number.</u>

- Degrees of freedom is a combination of how much data you have and how many parameters you need to estimate. <u>It indicates how much independent information goes into a parameter estimate</u>. It's easy to see that you want a lot of information to go into parameter estimates to obtain more precise estimates. So, you want many degrees of freedom!

<u>For more information:</u>

https://statisticsbyjim.com/hypothesis-testing/degrees-freedom-statistics/

There is a special table for t distribution. The t distribution table values are critical values of the t distribution. **The column header are the t distribution probabilities (alpha). These probabilities are for right hand side probabilities of distribution as showed like $P(T \geq t_{\alpha,\nu}) = \alpha$ and could be seen as in Figure 1.** The random variable T having t distribution takes values $-\infty < t < +\infty$, both negative and positive values!! **The density is symmetrical about *t=0* and hence** $t_{1-\alpha,\nu} = -t_{\alpha,\nu}$ **where** $P(T \geq t_{\alpha,\nu}) = \alpha.$

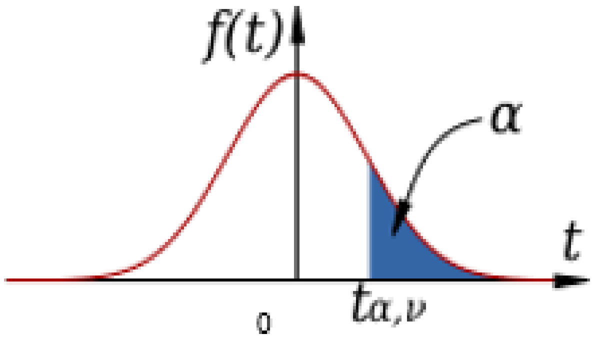


Figure 1. *t* distribution

There are two versions here, both of them are t distribution tables. The red rectangular show the alphas (the rihgt hand side probabilities) , v in first table , sd in second table shows degrees of freedoms.

VALUES OF $t_{\alpha,v}$

v	.10	.05	.025	.02	.015	.01	.005	v
1	3.078	6.314	12.706	15.895	21.205	31.821	63.657	1
2	1.886	2.920	4.303	4.849	5.643	6.965	9.925	2
3	1.638	2.353	3.182	3.482	3.896	4.541	5.841	3
4	1.533	2.132	2.776	2.999	3.298	3.747	4.604	4
5	1.476	2.015	2.571	2.757	3.003	3.365	4.032	5
6	1.440	1.943	2.447	2.612	2.829	3.143	3.707	6
7	1.415	1.895	2.365	2.517	2.715	2.998	3.499	7
8	1.397	1.860	2.306	2.449	2.634	2.896	3.355	8
9	1.383	1.833	2.262	2.398	2.574	2.821	3.250	9
10	1.372	1.812	2.228	2.359	2.527	2.764	3.169	10
11	1.363	1.796	2.201	2.328	2.491	2.718	3.106	11
12	1.356	1.782	2.179	2.303	2.461	2.681	3.055	12
13	1.350	1.771	2.160	2.282	2.436	2.650	3.012	13
14	1.345	1.761	2.145	2.264	2.415	2.624	2.977	14
15	1.341	1.753	2.131	2.249	2.397	2.602	2.947	15
16	1.337	1.746	2.120	2.235	2.381	2.583	2.921	16
17	1.333	1.740	2.110	2.224	2.368	2.567	2.898	17
18	1.330	1.734	2.101	2.214	2.356	2.552	2.878	18
19	1.328	1.729	2.093	2.205	2.346	2.539	2.861	19
20	1.325	1.725	2.086	2.197	2.336	2.528	2.845	20
21	1.323	1.721	2.080	2.189	2.328	2.518	2.831	21
22	1.321	1.717	2.074	2.183	2.320	2.508	2.819	22
23	1.319	1.714	2.069	2.177	2.313	2.500	2.807	23
24	1.318	1.711	2.064	2.172	2.307	2.492	2.797	24
25	1.316	1.708	2.060	2.167	2.301	2.485	2.787	25
26	1.315	1.706	2.056	2.162	2.296	2.479	2.779	26
27	1.314	1.703	2.052	2.150	2.291	2.473	2.771	27
28	1.313	1.701	2.048	2.154	2.286	2.467	2.763	28
29	1.311	1.699	2.045	2.150	2.282	2.462	2.756	29
∞	1.282	1.645	1.960	2.054	2.278	2.326	2.576	∞

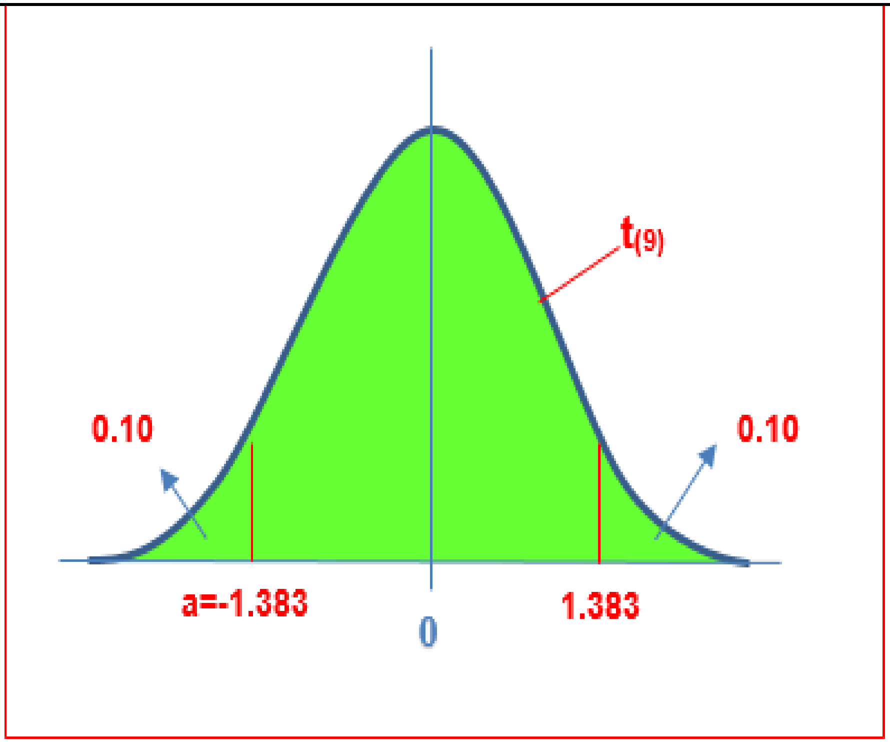
	TEK YÖNLÜ (BİR YANLI) TEST İÇİN α											
	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
	İKİ YÖNLÜ (İKİ YANLI) TEST İÇİN α											
	0.50	0.40	0.30	0.20	0.10	0.05	0.04	0.02	0.01	0.005	0.002	0.001
sd												
1	1.000	1.376	1.963	3.078	6.314	12.710	15.890	31.820	63.660	127.300	318.300	636.600
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.090	22.330	31.600
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.210	12.920
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1,093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0,697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0,663	0.859	1,063	1.323	1.721	2,080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.150	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.614
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.056	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.631	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.295	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	0.674	0.841	1.036	1.282	1.640	1.960	2.054	2.326	2.576	2.807	3.091	3.291

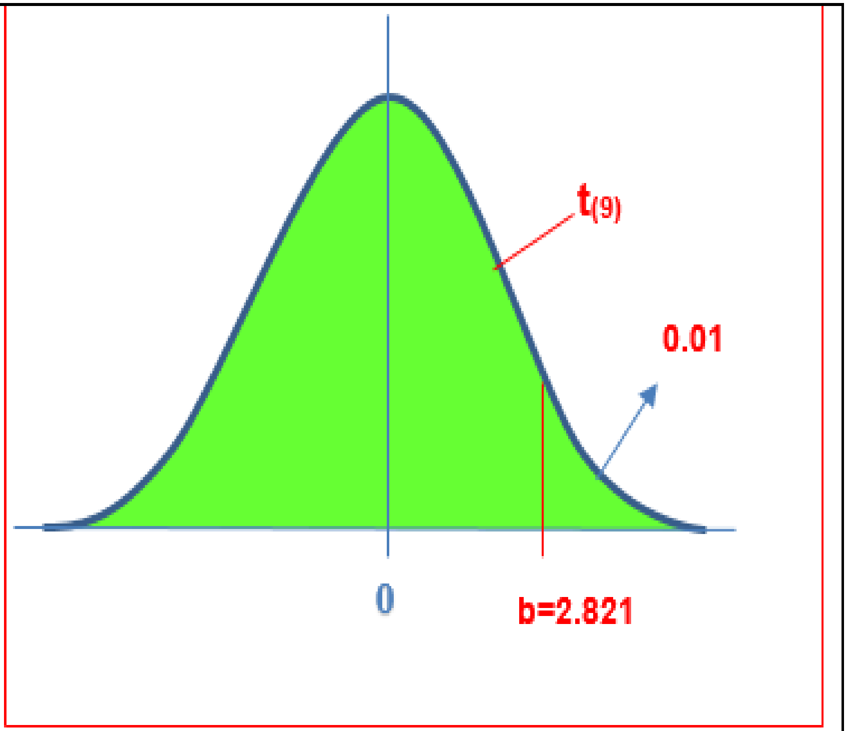
Examples 1: If $T \sim t_{(9)}$, find a, b and c values given in $P(T < a) = 0.10$, $P(T > b) = 0.01$, $P(-c < T < c) = 0.96$. ***Becareful degrees of freedom=9.***

Solution:

$P(T < a) = 0.10$ **a=-1.383** ($t_{0.10;9}$=1.383)

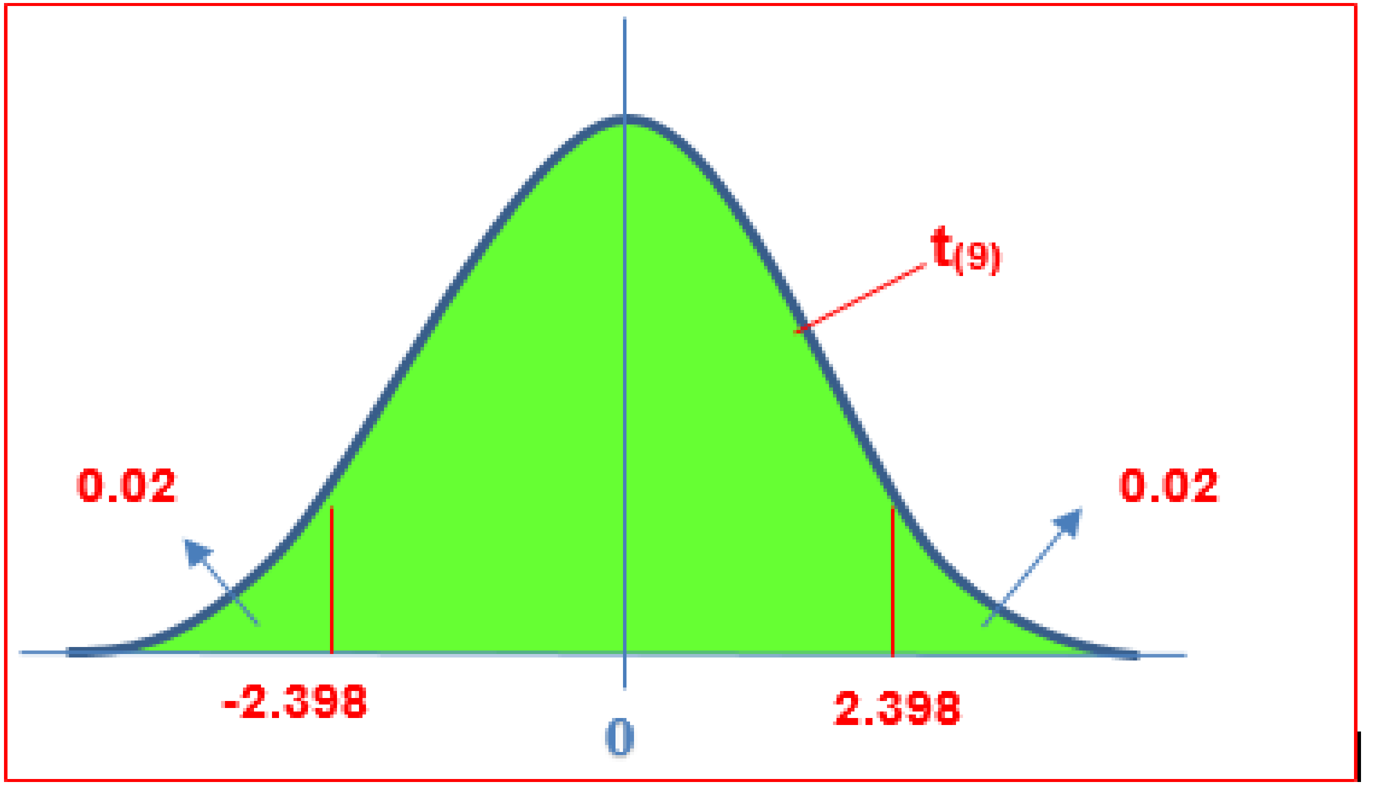
$P(T > b) = 0.01$ **b=2.821** ($t_{0.01;9}$=2.821)





$P(-c < T < c) = 0.96$ **then** $P(T > c) = 0.02$

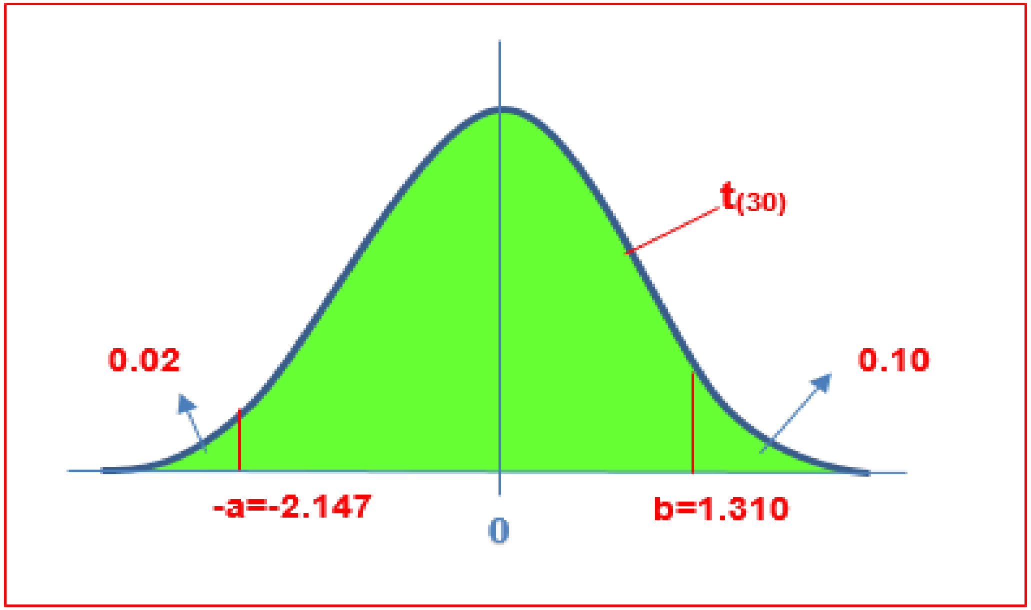
so c=2.398 ($t_{0.02;9}$=2.398)



Example 2: If $T \sim t_{(30)}$, find a and b values satisfing to $P(-a < T < b) = 0.88$, $P(T > -a) = 0.98$. ***Becareful degrees of freedom=30.***

Solution: Since $P(T > a) = 0.02 \Rightarrow a = 2.147$ **($t_{0.02;30}$=2.147)** then because of symmetry of t distribution $P(T > -a) = 0.98 \Rightarrow -a = -2.147$

and so $P(T > b) = 0.10 \Rightarrow b = 1.310$ **($t_{0.10;30}$=1.310)**



4.4. The Chi-square Distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from normal distribution with the mean μ and the variance σ^2. For the sample mean and variance, $\bar{X}$ and S^2, then,

- $\bar{X}$ and S^2 are independent random variables (statistics)

- The random variable $\bar{X}$ has a normal distribution with the mean μ and the variance σ^2 / n, and shown $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

- The random variable $\frac{(n-1)S^2}{\sigma^2}$ has a chi-square distribution with *n-1* degrees of freedom, and shown $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$.

If the random variable X has a chi-square distribution with v degrees of freedom $\left(X \sim \chi_v^2\right)$, it takes positive values $\left(x > 0\right)$ and $\text{Mean}: E\left(X\right) = v,\ \text{Variance}: V\left(X\right) = 2v \quad \text{where} \left(v > 0\right)$. The density is positively (right) skewed. The Figure 2 shows its density as follow:

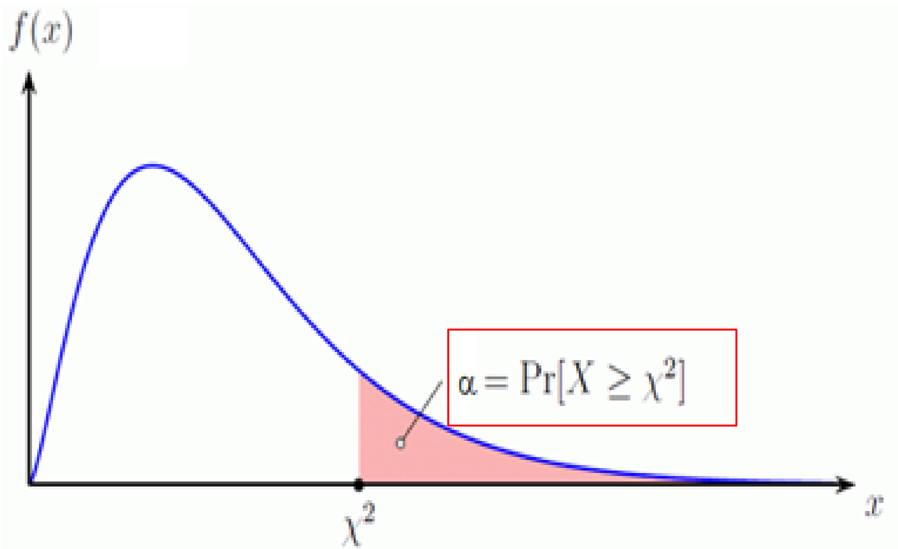
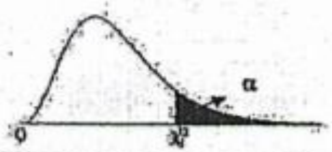


Figure 2. *Chi-square* distribution.

The probabilities in *chi-square* table show areas to its right under the chi-square curve with degrees of freedom v. $P(X \geq \chi^2_{\alpha,v}) = \alpha$, as seen as in Figure 2. *When v (degrees of freedom) is greater than 30, chi-square distributions are usually appoximated with normal distributions.*

There are two versions here, both of them are Chi-Square tables. The Chi-Square table gives χ2 values for selected levels of significance. All of the levels of significance shown represent areas in the right tail of the chi square distribution. Here, sd shows degrees of freedom.

KI-KARE (χ^2) TABLOSU



sd	α 0.995	0.990	0.975	0.950	0.900	0.750	0.500	0.250	0.100	0.05	0.025	0.010	0.005
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33705	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03726	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84391	15.37916	17.29189	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74941	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

The other version of Chi-Square table. Here first column in the left hand side shows degrees of freedom. The red rectangular show the alphas (the rihgt hand side probabilities) .

VALUES OF $\chi^2_{\alpha,\nu}$

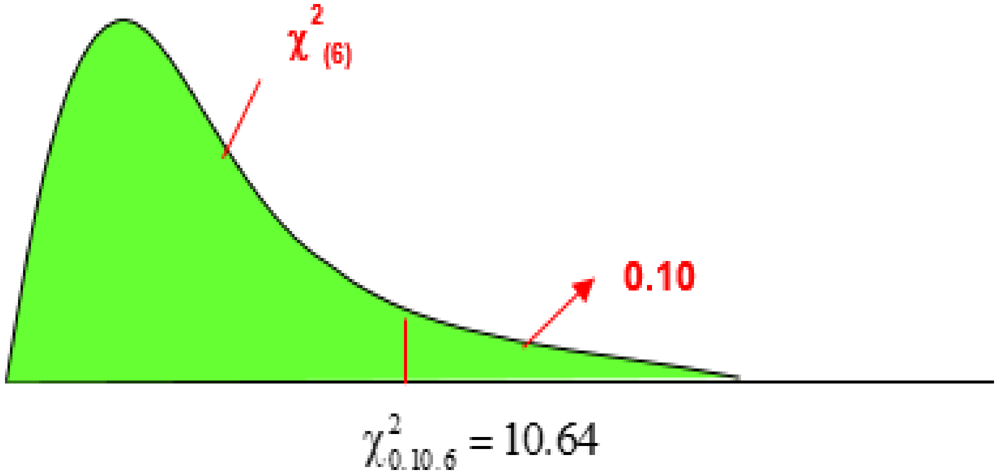
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	.00004	.00016	.00098	.00393	.01579	2.7055	3.8415	5.024	6.635	7.879
2	.0100	.0201	.0506	.1026	.2107	4.605	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	.584	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	.676	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672

Examples 3: If $X \sim \chi^2_{(6)}$, find the probabilities $P(X < 10.64)$, $P(10.65 < X < 14.45)$, $P(X \geq 2.2)$. ***Becareful degrees of freedom=6.***

Solution:

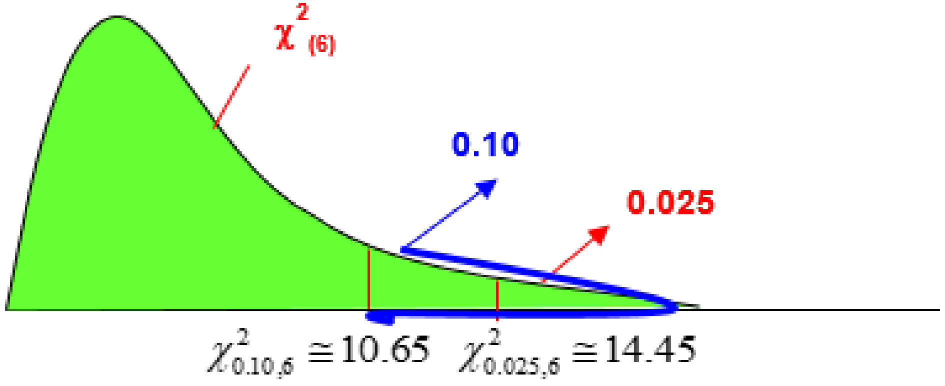
χ^2 **Table gives you for df=6,** $P(X > 10.64) = 0.10$ **then you find**

$P(X < 10.64) = 0.90$

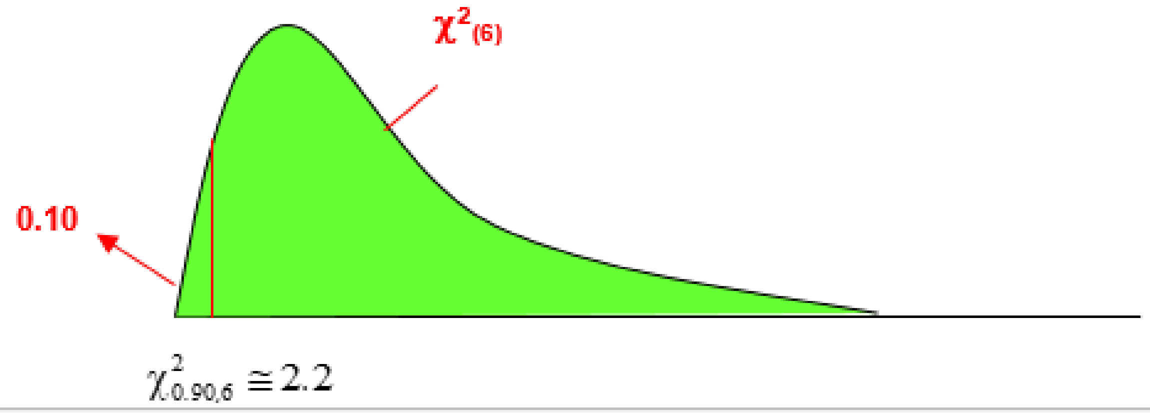


χ^2 **Table gives you for df=6,** $P(X > 10.65) = 0.10$ **and** $P(X > 14.45) = 0.025$ **then you find**

$P(10.65 < X < 14.45) = 0.10 - 0.025 = 0.075$

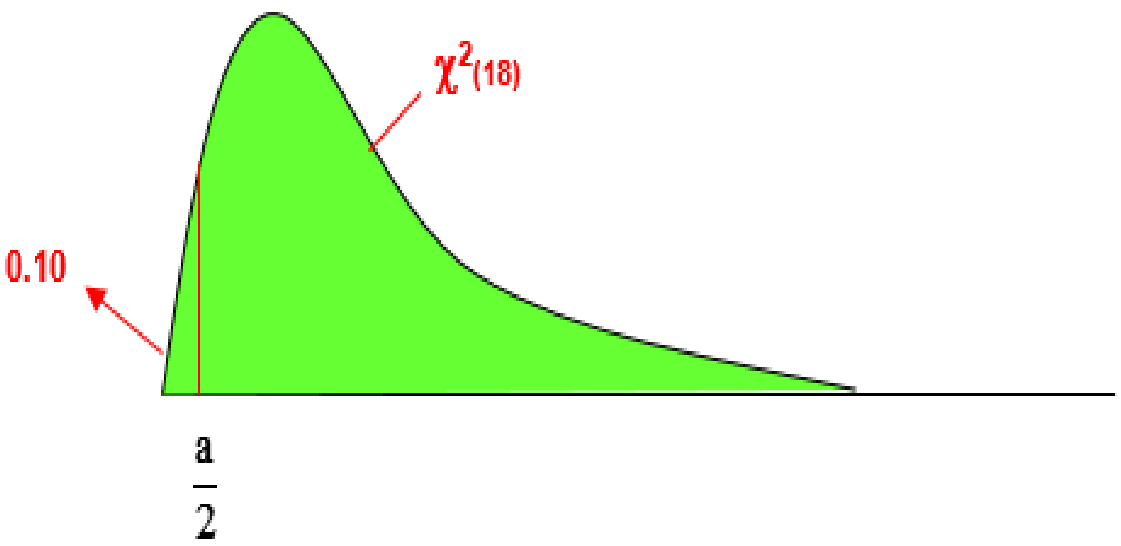


$P(X \geq 2.2) = 0.90$



Examples 4: If $X \sim \chi^2_{(18)}$, find a and b values given in $P(2X < a) = 0.10$, $P(X - 1 < b) = 0.25$. ***Becareful degrees of freedom=18.***

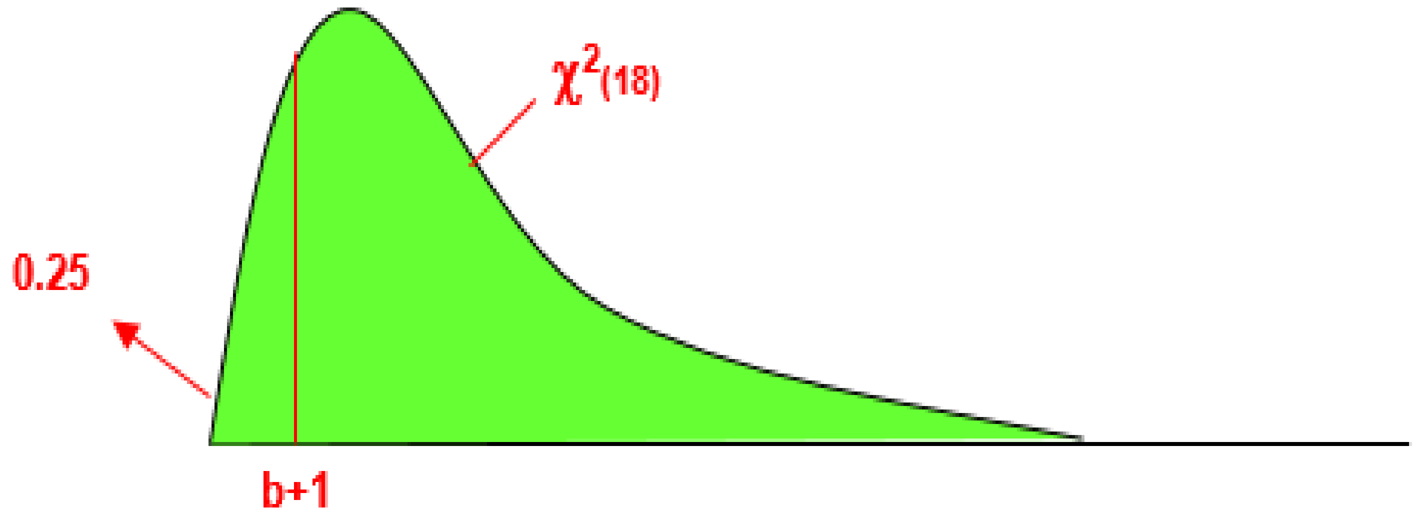
Solution: $P(X > \frac{a}{2}) = 0.90$ then since df=18, $\frac{a}{2} = 10.865 \Rightarrow a = 21.73$ $\left(\chi^2_{0.90,18} = 10.865\right)$



$P(X-1<b)=0.25$ then $P(X>b+1)=0.75$ since df=18,

$$b+1=13.67529 \Rightarrow b=12.67529$$

$$\left(\chi^2_{0.75,18}=13.67529\right)$$

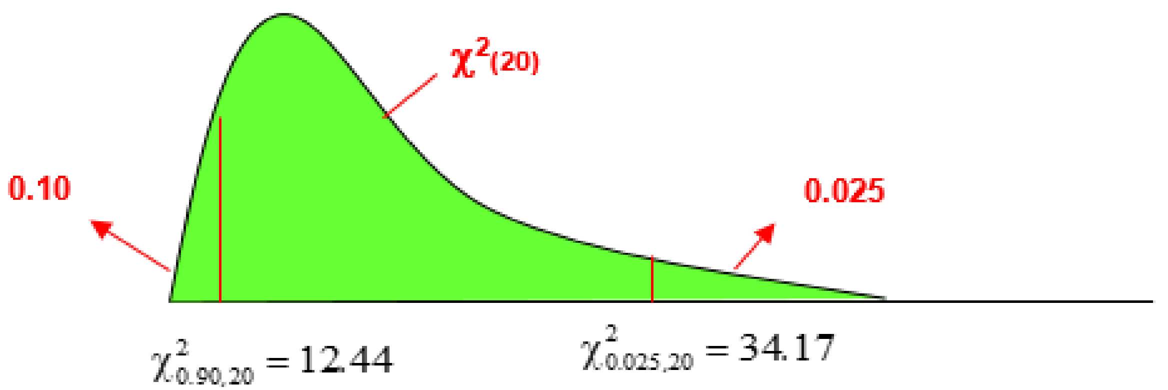


Examples 5: A random sample of size n=21 was drawn a normal population with variance 10, $\sigma^2 = 10$. ***Find probability of the sample variance being less than 17.085 and greater than 6.22.***

Solution: $P(6.22 < S^2 < 17.085) = ?$ $\qquad \dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

$$P(6.22 < S^2 < 17.085) = P\left(\frac{20\times 6.22}{10} < \frac{(n-1)S^2}{\sigma^2} < \frac{20\times 17.085}{10}\right)$$
$$= P(12.44 < \chi^2_{(n-1)} < 34.17) = 0.90 - 0.025 = 0.875$$

Becareful degrees of freedom=n-1=21-1=20



4.5. The F Distribution

- Another sampling distribution with related to normal populations is the F distribution. *Originally, it was studied as the sampling distribution of the ratio of two independent variables with chi-square distributions, each divided by its respective degrees of freedom.*

- For Example, the random variable U has a chi-square distribution with m degrees of freedom, and shown as $\left(U \sim \chi^2_m\right)$; the random variable V has a chi-square distribution with *n* degrees of freedom, and shown as $\left(V \sim \chi^2_n\right)$. The random variable defined as $X = \dfrac{U/m}{V/n} \sim F_{m,n}$ has a F distribution with degrees of freedom m and n. There are two parameters of F distribution: the degrees of freedom of numerator m and degrees of freedom of denominator. The random variable having F distribution takes positive values $(x > 0)$.

For a sample, the random variable $\frac{(n_1-1)S_1^2}{\sigma_1^2}$ has a chi-square distribution with n_1-1 degrees of freedom, and shown $\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{(n_1-1)}$. For another sample, the random variable $\frac{(n_2-1)S_2^2}{\sigma_2^2}$ has a chi-square distribution with n_2-1 degrees of freedom, and shown $\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{(n_2-1)}$.

The ratio of two independent variables with chi-square distributions, each divided by its respective degrees of freedom is defined as:

$$\frac{\dfrac{\left(\dfrac{(n_1-1)S_1^2}{\sigma_1^2}\right)}{(n_1-1)}}{\dfrac{\left(\dfrac{(n_2-1)S_2^2}{\sigma_2^2}\right)}{(n_2-1)}} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{\sigma_2^2}{\sigma_1^2}\frac{S_1^2}{S_2^2} \sim f_{\alpha,(n_1-1)(n_2-1)}$$

Like chi-square distribution, *F* **distribution is positively (right) skewed**, but it has two degrees of freedoms (ν_1, ν_2). $P(F \geq f_{\alpha,\nu_1,\nu_2}) = \alpha$. The F distribution table is also shows the area (probability) under right hand side, known as α. **Do not Forget for different α values there are different F distribution tables. In this lesson we will use F distribution table for α=0.05.**

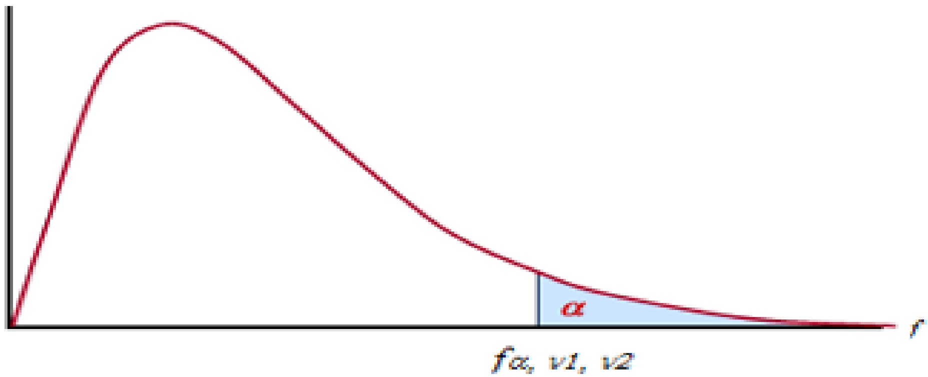


Figure 3. *F* distribution.

For obtaining values $f_{(1-\alpha),\nu_1,\nu_2}$ we will use the this relationship:

$$f_{(1-\alpha),\, \nu_1,\nu_2} = \frac{1}{f_{\alpha,\, \nu_2,\nu_1}}$$

This F distribution table for alpha=0.05, do not forget there are different tables for different alpha values like 0.01, 0.10 etc. v1 shows the degrees of freedom of numerator and v2 shows the degrees of freedom of denominator.

F TABLOSU (α= 0.05)

v_2 (Payda serbestlik derecesi)	v_1 (Pay serbestlik derecesi)																			
	1	2	3	4	5	6	7	8	9	10	12	15	18	20	25	30	40	60	100	200
1	161.45	199.60	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	247.32	248.01	249.26	250.10	251.14	252.20	253.04	254.68
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.44	19.45	19.46	19.46	19.47	19.48	19.49	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.67	8.66	8.63	8.62	8.59	8.57	8.55	8.54
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.82	5.80	5.77	5.75	5.72	5.69	5.66	5.65
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.58	4.56	4.52	4.50	4.46	4.43	4.41	4.39
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.90	3.87	3.83	3.81	3.77	3.74	3.71	3.69
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.47	3.44	3.40	3.38	3.34	3.30	3.27	3.25
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.17	3.15	3.11	3.08	3.04	3.01	2.97	2.95
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.96	2.94	2.89	2.86	2.83	2.79	2.76	2.73
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.80	2.77	2.73	2.70	2.66	2.62	2.59	2.56
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.67	2.65	2.60	2.57	2.53	2.49	2.46	2.43
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.57	2.54	2.50	2.47	2.43	2.38	2.35	2.32
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.48	2.46	2.41	2.38	2.34	2.30	2.26	2.23
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.41	2.39	2.34	2.31	2.27	2.22	2.19	2.16
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.35	2.33	2.28	2.25	2.20	2.16	2.12	2.10
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.30	2.28	2.23	2.19	2.15	2.11	2.07	2.04
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.26	2.23	2.18	2.15	2.10	2.06	2.02	1.99
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.22	2.19	2.14	2.11	2.06	2.02	1.98	1.95
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.18	2.16	2.11	2.07	2.03	1.98	1.94	1.91
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.15	2.12	2.07	2.04	1.99	1.95	1.91	1.88
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.12	2.10	2.05	2.01	1.96	1.92	1.88	1.84
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.10	2.07	2.02	1.98	1.94	1.89	1.85	1.82
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.08	2.05	2.00	1.96	1.91	1.86	1.82	1.79
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.05	2.03	1.98	1.94	1.89	1.84	1.80	1.77
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.04	2.01	1.96	1.92	1.87	1.82	1.78	1.75
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	2.02	1.99	1.94	1.90	1.85	1.80	1.76	1.73
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	2.00	1.97	1.92	1.88	1.84	1.79	1.74	1.71
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.99	1.96	1.91	1.87	1.82	1.77	1.73	1.69
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.97	1.94	1.89	1.85	1.81	1.75	1.71	1.67
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.96	1.93	1.88	1.84	1.79	1.74	1.70	1.66
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.87	1.84	1.78	1.74	1.69	1.64	1.59	1.55
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.87	1.81	1.78	1.73	1.69	1.63	1.58	1.52	1.48
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.78	1.75	1.69	1.65	1.59	1.53	1.48	1.44
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.81	1.75	1.72	1.66	1.62	1.57	1.50	1.45	1.40
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.79	1.73	1.70	1.64	1.60	1.54	1.48	1.43	1.38
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.78	1.72	1.69	1.63	1.59	1.53	1.46	1.41	1.36
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.77	1.71	1.68	1.62	1.57	1.52	1.45	1.39	1.34
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.72	1.66	1.62	1.56	1.52	1.46	1.39	1.32	1.26

This F distribution table for alpha=0.05, similar to previous one. v1 shows the degrees of freedom of numerator and v2 shows the degrees of freedom of denominator.

v_1: Degrees of freedom for numerator

v_2: Degrees of freedom for denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

VALUES OF $F_{0.05;v1,v2}$

Examples 6: If $X \sim f_{(2,9)}$, find a and b values satisfing to $P(X > a) = 0.05$, $P(X > b) = 0.95$.

Becareful degrees of freedoms 2 and 9.

Solution: if $P(X > a) = 0.05$, then **a=4.26**

Using properties of f distribution $f_{(1-\alpha),(\nu_1,\nu_2)} = \dfrac{1}{f_{\alpha,(\nu_2,\nu_1)}}$

$$f_{0.95,(2,9)} = \frac{1}{f_{(0.05),(9,2)}} = \frac{1}{19.38} \cong 0.05$$