EXERCISES FOR SECTION 1 AND SECTION 2

1) Find the number of ways in which one A, three B's, two C's, and one F can be distributed among seven students taking course in statistics.

   Solution: $\frac{7!}{3!2!} = \frac{4.5.6.7}{2} = 420$

2) If someone takes three shots at a target and we care only whether each shot is a hit or a miss, describe a suitable sample spaces that constitute event M that the person will miss the target three times in a row, and the elements of event N that the person will hit the target once and miss it twice. Find probabilities of M, N events if he hits the target with p ($0<p<1$) probability.

   Solution: S={ (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1) }

   M={ (0,0,0) } and N={ (1,0,0), (0,1,0), (0,0,1) }

   $P(M)= (1-p)^3$ $\quad$ $P(N)=3(1-p)^2p$

3) The bowl contains 3 Blue, 5 White, 4 Red idential balls.
   a) Construct the sample space when 3 balls drawn without replacement. Find the probability of an event that 2 Red, 1 White balls drawn.
   b) Construct the sample space when 3 balls drawn consecutively without replacement. Find the probability of an event that 2 Red, 1 White balls drawn.

   Solution:

   a) S {(3 Red Balls), (3 White Balls), (3 Blue Balls),
   (1 Blue and 2 White Balls), (1 Blue and 2 Red Balls), (1 Red and 2 White Balls), (1 Red and 2 Blue Balls), (1 White and 2 Red Balls),
   (1 White and 2 Blue Balls) (1 Blue 1 White 1 Red Balls) }

   $$P(\{2Red, 1White\}) = \frac{4}{12}.\frac{3}{11}.\frac{5}{10}.\frac{3!}{2!} = \frac{3}{22}$$

   or

   $$P(\{2Red, 1White\}) = \frac{\binom{4}{2}\binom{5}{1}\binom{3}{0}}{\binom{12}{3}} = \frac{\frac{3\times4}{2!}(5)}{\frac{12!}{3!9!}} = (3)\frac{3\times4\times5}{10\times11\times12} = \frac{3}{22}$$

   b)

S={ (RRR), (RRB), (RRW), (RBR), (RWR), (BRR), (WRR), (RBW), (RWB)

(BBB), (BBR), (BBW), (BRB), (BWB), (RBB), (WBB), (BRW), (BWR)

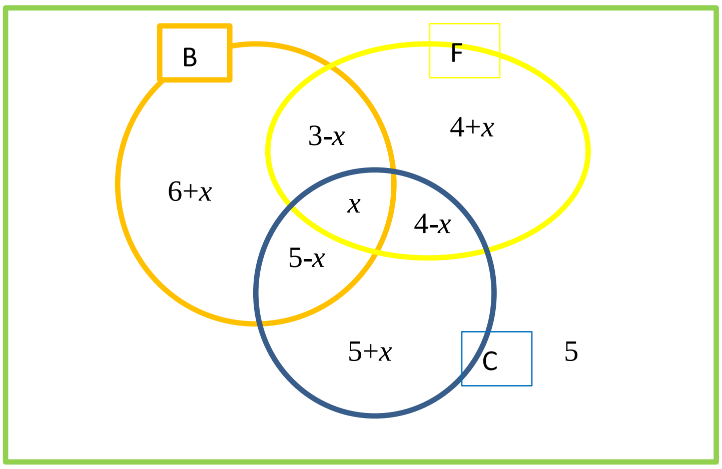
(WWW), (WWR), (WWB), (WRW), (WBW), (RWW), (BWW), (WRB), (WBR) }

$$P\left(\{2Red,1White\}\right)=P\left(\left(RRW\right)or\ \left(RWR\right)or\ \left(WRR\right)\right)$$
$$=\frac{4}{12}.\frac{3}{11}.\frac{5}{10}+\frac{4}{12}.\frac{5}{11}.\frac{3}{10}+\frac{5}{12}.\frac{4}{11}.\frac{3}{10}$$
$$=\frac{4}{12}.\frac{3}{11}.\frac{5}{10}.3=\frac{3}{22}$$

4) If Ms.Brown buys one of the 35 houses advertised for sale in a Seattle newspaper, B is the event that the house has three or more baths, F is the event that it has a fireplace, C is the event that it costs more than \$ 100,000. Find the value of intersection of these three events. $B\cap F\cap C$.

B: the house has three or more baths	14
F: the house has a fireplace	11
C: the house cost's is more than \$ 100,000	14
$B\cap F$	3
$B\cap C$	5
$F\cap C$	4
$(B\cup F\cup C)'$	5

Solution:



$$s(B\cup F\cup C)=s(B)+s(F)+s(C)-s(B\cap F)-s(B\cap C)-s(F\cap C)+s(B\cap F\cap C)$$
$$30=14+11+14-3-5-4+x\Rightarrow x=30-27=3$$

5) A hat contains twenty white slips of paper numbered from 1 through 20, ten red slips of paper numbered from 1 through 10, forty yellow slips of paper numbered from 1 through 40, and ten blue slips of paper numbered 1 through 10. If these 80 slips of paper are thoroughly shuffled so that each slip has the same probability of being draw, find the probabilities of drawing a slip of paper that is

a) Blue or white;
b) Numbered 1,2,3,4, or 5;
c) Red or yellow and numbered 1, 2, 3, or 4;
d) Numbered 5, 15, 25, or 35;
e) White and numbered higher than 12 or yellow and numbered higher than 26.

Solution:

White 20, Red 10, Yellow 40, Blue 10 and total 80 slips of paper

a) $A = \{drawn\, a\ paper\, is\, "Blue"\}$ $\quad B = \{drawn\, a\ paper\, is\, "White"\}$

$$P(A \cup B) = P(A) + P(B) = \frac{10}{80} + \frac{20}{80} = \frac{3}{8}$$

b) $C = \{numbered\, 1, 2, 3, 4\, or\, 5\}$

$$P(C) = P("numbered\, 1") + P("numbered\, 2") + P("numbered\, 3") + P("numbered\, 4") + P("numbered\, 5")$$

$$= \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} = \frac{20}{80} = \frac{1}{4}$$

c) $D = \{Red\ or\ yellow\ and\ numbered\ 1,\ 2,\ 3,\ or\ 4\}$

$$P(D) = P(\mathrm{Re}d \cap "1") + P(\mathrm{Re}d \cap "2") + P(\mathrm{Re}d \cap "3") + P(\mathrm{Re}d \cap "4") + P(Yellow \cap "1") + P(Yellow \cap "2") + P(Yellow \cap "3") + P(Yellow \cap "4")$$

$$= \frac{1}{80} + \frac{1}{80} + \frac{1}{80} + \frac{1}{80} + \frac{1}{80} + \frac{1}{80} + \frac{1}{80} + \frac{1}{80} = \frac{1}{10}$$

d)

$$P(E) = P("5") + P("15") + P("25") + P("35")$$

$$= \frac{4}{80} + \frac{2}{80} + \frac{1}{80} + \frac{1}{80} = \frac{1}{10}$$

e)

$$P(F) = P("White\, and\ higher\ than\, 12") + P("Yellow\, and\ higher\ than\, 26")$$

$$= \frac{8}{80} + \frac{14}{80} = \frac{22}{80} = \frac{11}{40}$$

6) On a Friday morning, the pro shop of a tennis club has 14 identical cans of tennis balls.

a) If they are all sold by Sunday night and we are interested only in how many were sold on each day, <u>in how many different ways</u> could the tennis balls have been sold on Friday, Saturday, and Sunday?

$$\binom{16}{14} = \frac{16!}{14!2!} = \frac{16.15.14!}{14!2} = 120$$

b) Given that at least two of the cans of tennis balls were sold on each of three days. <u>In how many different ways</u> could the tennis balls have been sold on Friday, Saturday, and Sunday?

$$\binom{10}{8} = \frac{10!}{8!2!} = \frac{10.9.8!}{8!2} = 45$$