**İST299 Probability Exercises**

**Exercise 1:** A California license plate consists of a sequence of seven symbols: number, letter, letter, letter, number, number, number, where a letter is any one of 26 letters and a number is one among  . Assume that all license plates are equally likely.

**(a)** What is the probability that all symbols are diﬀerent?

**(b)** What is the probability that all symbols are different and the ﬁrst number is the largest among the numbers?

**Solution:**

1. ****
2. ****

**Exercise 2:** A couple are planning to have a family. They decide to stop having children *either* when they have two boys or when they have four children. Suppose that they are successful in their plan.

1. Write down the sample space.
2. Assume that, each time that they have a child, the probability that it is a boy is 1/2, independent of all other times. Find P(E) and P(F) where E =“there are at least two girls”, F =“there are more girls than boys”.

**Solution:**

1. ****
2. ****

****

Now we have  and similarly for other outcomes. So .

**Exercise 3:** What is the probability the sum is 9 in three rolls of a die?

Write the event A: “the sum of the rolls is 9” in three rolls of a die;

**Solution:**

*The 25 outcomes of the event “the sum of the rolls is 9”are*

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The probability that the sum of the rolls is 9 is .

**Exercise 4.** Five integers are taken at random and without replacement from {1, 2,…,100}.Let X be the random variable counting how many, among the five, are divisible by 3. Find the probability of X random variable.

**Solution:** the number of the points in the sample space: . There are 3\*10+3=33 numbers divisible by 3 and left of them is not divided by 3.

The random variable *X* takes values 0, 1, 2, 3, 4, 5.

 

 

 

**Exercise 5.** Five integers are taken at random and without replacement from {1, 2,…,10}.Let *X* be the random variable determining their maximum. Find the probability mass function of *X* and expected value of *X*.

**Solution:** The number of the points in the sample space: 

The random variable *X* takes the values 5, 6, 7, 8, 9, 10.

How many values being less than or equal 5 are there? For P(X=5), there are 4 values in 10 number since the maximum of chosen 5 numbers would be 5. For P(X=6), there are 5 values in 10 number since the maximum of chosen 5 numbers would be 6, then go forth.

  

  



**Exercise 6.** Suppose that a player starts with a fortune of $8. A fair coin is tossed three times. If the coin comes up heads, the player fortune is doubled, otherwise it is halved. What is the player’s expected fortune?

**Solution.** Sample space for a fair coin tossed three times:







**Exercise 7:** A tennis tournament has 2n participants, n Swedes and n Norwegians. First, n people are chosen at random from the 2n (with no regard to nationality) and then paired randomly with the other n people. Each pair proceeds to play one match. An outcome is a set of n (ordered) pairs, giving the winner and the loser in each of the n matches.

1. Determine the number of outcomes.
2. Under the assumption that all outcomes are equally likely, compute the probability that all Swedes are the winners.

**Solution:**

1. Divide into two groups (winners and losers), then pair them.
2. The number of good events is  , the choice of a Norwegian paired with each Swede. The probability will be 

**Exercise 8:** Adam is taking a multiple choice exam in which each question has five possible answers, exactly one of which is correct. If Adam knows the answer, he selects the correct answer. Otherwise, he selects one answer at random from the five possible answers. Suppose that, for each question, there is a 70% chance that Adam knows the answer.

1. Compute the probability that, on a randomly chosen question, Adam gets the correct answer.
2. Compute the probability that Adam knows the answer to a question given that he has answered the question correctly.

**Solution:**

C is an event that Adam answers the question correctly.

N is an event that Adam knows correct answer.

1. 
2. 

**Exercise 9:** Three cards are taken from at random and without replacement from standard deck of 52 cards. Find the probability that all are jacks given that at least one of them is a face card (J, Q or K).

**Solution:**

J3 denote the event of choosing three jacks

F denote the event of obtaining at least one face in a drawing of three cards.



In each of the 4 groups there are 3 faces, then there is no face in 52-3×4=40 cards.

 

**Exercise 10:** Events A and B are independent, events A and C are mutually exclusive, and events B and C are independent. If P(A)=1/2, P(B)=1/3, P(C)=1/4, find .

