# **Numeral Semantics** — Monday

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# Numeral Semantics

$[\![\text{twelve}]\!]$

# Numeral Semantics

$$[\![\text{twelve}]\!]$$

$$[\![\text{Twelve students came to the party}]\!]$$

# Overview

**Monday**: Theories of numeral semantics

**Tuesday**: Continued

**Wednesday**: Continued

**Thursday**: (Im)precision

**Friday**: Beyond semantics

# Numerals as determiners

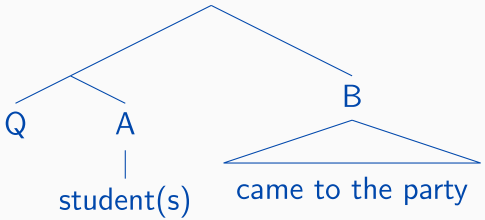
**Twelve** students came to the party.

# Numerals as determiners

**Twelve** students came to the party.
**Several** students came to the party.
**Some** students came to the party.
**Most** students came to the party.
**No** students came to the party.

# Generalized Quantifier Theory

Q student(s) came to the party




Barwise & Cooper, 1980; Keenan & Stavi 1984

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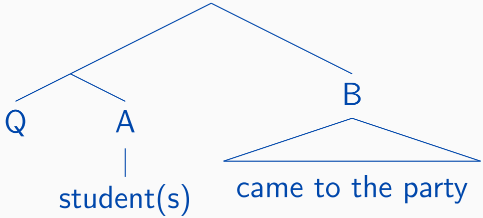


every(A)(B) iff $A \subseteq B$

no(A)(B) iff $A \cap B = \emptyset$

some(A)(B) iff $A \cap B \neq \emptyset$

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Barwise & Cooper, 1980; Keenan & Stavi 1984

# Generalized Quantifier Theory

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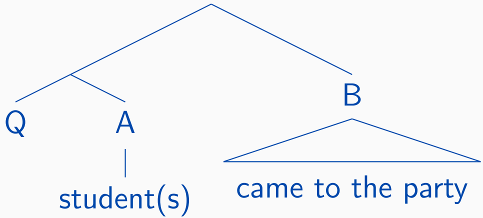


**Big research question:**

Which of these are realised in NL? What properties do all realised quantifiers share?

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Barwise & Cooper, 1980; Keenan & Stavi 1984

# Constraints on quantifiers

QUANT: $Q(A)(B) \Leftrightarrow Q(F(A))(F(B))$ for any bijection $F$

van Benthem, 1984

# Constraints on quantifiers

QUANT: $Q(A)(B) \Leftrightarrow Q(F(A))(F(B))$ for any bijection $F$

Upshot: only cardinalities ever matter, not the actual content of a set

every(A)(B) iff $|A \cap B| = |A|$
no(A)(B) iff $|A \cap B| = 0$
some(A)(B) iff $|A \cap B— \neq 0$

The ultimate quantifier: twelve(A)(B) iff $|A \cap B| = 12$

van Benthem, 1984

# Numerals are not quantifiers

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(1) **Twelve** students came to the party.

(2) **Twelve** people can fit in the lift.

# Numerals are not quantifiers

(3) **Twelve** of my students built a boat together.

(4) ***Every** student of mine built a boat together.

(5) ***Most** of my students built a boat together.

# Numerals are not quantifiers

(6) In this picture, **twelve** dots surround the triangle.

(7) ??In this picture, **every** dot surrounds the triangle.

# Numerals are not quantifiers

(8) This house has **twelve** windows.

(9) *This house has **every / most** window(s).

# Numerals are not quantifiers

(10) The meeting lasted **twelve** hours.

(11) *The meeting lasted **most / every** hour(s).

# Numerals are not quantifiers

(12) The girls in this class are **twelve** of our most promising students.

(13) *The girls in this class are **all / every one** of our most promising students.

# Numerals are not quantifiers

(14) Every **two** houses come with one parking space.

# Numerals are not quantifiers

(15) Rod A is **three** times longer than rod B.

(16) **Two** is a Fibonacci number.

# Three strands of thought

1. the number view

$[\![\text{twelve}]\!] \;=\; 12$

2. the modifier view

$[\![\text{twelve}]\!] \;=\; \lambda x.\#x = 12$

3. the quantifier view (revisited)

$[\![\text{twelve}]\!] \;=\;$ the set of intervals that end in 12

# The number view

$[\![\text{twelve}]\!] = 12$ whatever that means

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(16) **Two** is a Fibonacci number.

$[\![\text{is a Fibonacci number}]\!] = \{0, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$

# The number view

$[\![\text{twelve}]\!] = 12$ whatever that means

(15') Rod A is longer than rod B.

means: the length of rod A $>$ the length of rod B

(15) Rod A is **three** times longer than rod B.

means: the length of rod A $= 3 \times$ the length of rod B

# What could a number be?

Basic semantic ontology:

- Entities; type $e$
- Truth-values; type $t$

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- Entities; type $e$
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We are going to add **degrees** to this picture: type $d$

- Degrees are like entities, but ordered

$$\text{John} < \text{Mary} \qquad 2 < 3$$

- Numbers are a special case of degrees

# What could a degree be?

(17) John is taller than Mary

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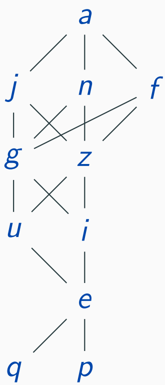
John's height $>$ Mary's height

(18) John is 2cm taller than Mary
John's height $=$ Mary's height $+$ 2cm

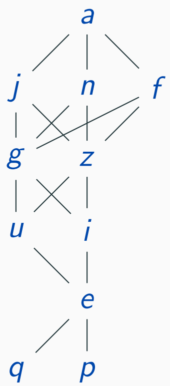
(19) John is this tall
this demonstrates a degree

# What could a degree be?

- We seem to be committed to a domain of abstract entities
- We don't have to be:
  - Cresswell 1977: tall involves an ability to decide that $j < m$
  - We can see degrees as equivalence classes of individuals



# Would could a degree be?




- This is an ordinal scale
- No distances, no zero, no multiplication
- Height etc. would need to be added to the equivalence classes (Bale 2011)
- Back to square one

# The number view: challenges

How does this meaning connect with nouns, to give us sets with particular cardinalities?

(1) Twelve students came to the party.

(2) Twelve people can fit in the lift.

(20) Every two houses come with one parking space.

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**Preview of options:**

1. Keep this meaning of numerals, add something that connects it with nouns (lecture 2)
2. Assume a different meaning of numerals (in this position)

# The modifier view

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$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

$[\![\text{American}]\!] = \lambda x.\mathit{american}(x)$ (the set of American entities)

$$\begin{array}{cc} \lambda x.\mathit{american}(x) & \lambda x.\mathit{student}(x) \\ | & | \\ \text{American} & \text{student} \end{array}$$

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$\lambda x.\mathit{american}(x) \wedge \mathit{student}(x)$

$\lambda x.\mathit{american}(x)$ $\lambda x.\mathit{student}(x)$

American student

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The numeral expresses cardinality, not quantificational force

# The modifier view

In the absence of a determiner, it's parallel to a bare plural:

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(21) American students came to the party

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We will return to this.

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Maybe it's not a problem:

$[\![$is a Fibonacci number$]\!] =$
$\{\lambda x.\#x = 1,\ \lambda x.\#x = 2,\ \lambda x.\#x = 3, \lambda x.\#x = 5, \ldots\}$

# Type landscape

| $e$ entity | $\langle e, t\rangle$ property | $\langle\langle e, t\rangle, t\rangle$ quantifier | $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ quantifier |
|---|---|---|---|
| $d$ degree | $\langle d, t\rangle$ degree property | $\langle\langle d, t\rangle, t\rangle$ degree quantifier | |

# Type landscape

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See you tomorrow!