Q3 1)

$$
\begin{array}{c}
y_{11} \\ \vdots \\ y_{113} \\ y_{2} \\ \vdots \\ y_{219} \\ y_{31} \\ \vdots \\ y_{317}
\end{array}
\quad X = \begin{pmatrix}
1 & 0 & x_{11} \\
\vdots & 0 & \vdots \\
 & 0 & \vdots \\
\vdots & 0 & x_{113} \\
\vdots & 1 & x_{21} \\
 & \vdots & \vdots \\
\vdots & 1 & x_{219} \\
1 & 0 & x_{3,1} \\
 & \vdots & \vdots \\
1 & 0 & x_{3,17}
\end{pmatrix}
$$

Homework 6: Stat 424

Evangeline Lim

11/21/2019

Question 2

```
##
## Call:
## lm(formula = strength ~ thickness + CO + PO, data = newStarch)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -172.86 -109.12  -62.05   44.43  635.77
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   64.681    174.723   0.370 0.712974
## thickness     71.269     17.163   4.152 0.000145 ***
## CO            -5.746     12.296  -0.467 0.642536
## PO             4.286      6.005   0.714 0.479158
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 167.1 on 45 degrees of freedom
## Multiple R-squared:  0.6719, Adjusted R-squared:   0.65
## F-statistic: 30.71 on 3 and 45 DF,  p-value: 5.816e-11
```

```
##             Df  Sum Sq Mean Sq F value   Pr(>F)
## thickness    1 2553357 2553357  91.429 2.08e-12 ***
## CO           1    5718    5718   0.205    0.653
## PO           1   14221   14221   0.509    0.479
## Residuals   45 1256725   27927
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

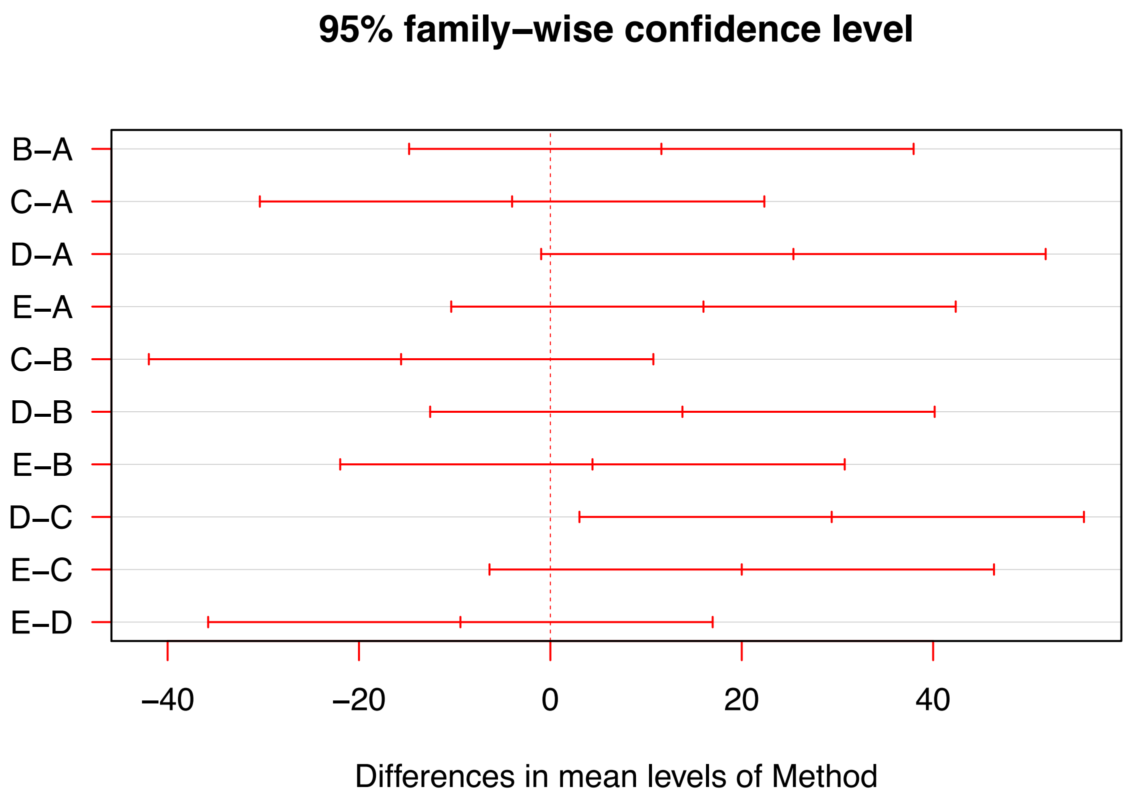
The new regresion analysis with only three intercept and slope terms show that the thickness parameter shows a greater slope coefficient compared to table 3.10 whereas for CO and PO terms, we have smaller slope coefficients. The ANOVA table for the regression analysis of three terms shows MSE and SSThickness, SSCO, SSPO, SSResiduals is more than the table 3.10.

Question 3

```
## Analysis of Variance Table
##
## Response: Throughput
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Day        1   84.5   84.50  3.2899   0.09119 .
## Operator   1   11.5   11.52  0.4485   0.51393
## Method     4 2857.6  714.40 27.8147 1.573e-06 ***
## Machine    4 3424.8  856.20 33.3356 5.129e-07 ***
## Residuals 14  359.6   25.68
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = lm_new)
##
## $Method
##      diff         lwr      upr     p adj
## B-A  11.6 -14.7614442 37.96144 0.6843733
## C-A  -4.0 -30.3614442 22.36144 0.9905466
## D-A  25.4  -0.9614442 51.76144 0.0623764
## E-A  16.0 -10.3614442 42.36144 0.3921784
## C-B -15.6 -41.9614442 10.76144 0.4166162
## D-B  13.8 -12.5614442 40.16144 0.5341875
## E-B   4.4 -21.9614442 30.76144 0.9864805
## D-C  29.4   3.0385558 55.76144 0.0242485
## E-C  20.0  -6.3614442 46.36144 0.1956041
## E-D  -9.4 -35.7614442 16.96144 0.8209033
```



From the plot and the tukey multiple comparisons, we see that D vs C is significant at the $\alpha = 0.05$ level.

Question 6

```
##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                0.5             1450
## 2             70                0.5             1600
## 3             70                0.7             1450
```

```
## 4             70                 0.7            1600
## 5            120                 0.5            1450
## 6            120                 0.5            1600
## 7            120                 0.7            1450
## 8            120                 0.7            1600
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3                            61
## 4                            75
## 5                            59
## 6                            90
## 7                            52
## 8                            87

##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                 0.5            1450
## 2             70                 0.5            1600
## 3             70                 0.7            1450
## 4             70                 0.7            1600
## 5            120                 0.5            1450
## 6            120                 0.5            1600
## 7
## 8            120                 0.7            1600
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3                            61
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87

##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                 0.5            1450
## 2             70                 0.5            1600
## 3
## 4             70                 0.7            1600
## 5            120                 0.5            1450
## 6            120                 0.5            1600
## 7
## 8            120                 0.7            1600
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87

##   OilTemperature PercentageOfCarbon SteelTemperature
## 1
```

```
## 2             70                0.5             1600
## 3
## 4             70                0.7             1600
## 5            120                0.5             1450
## 6            120                0.5             1600
## 7
## 8            120                0.7             1600
##   PercentageofNoncrackedSprings
## 1
## 2                            79
## 3
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87
```

From the one factor at a time approach, we note that it has ommited key values in the Percentage of Noncracked Springs (90 or 87) and instead returned 79. From this, we can say that the OFAT approach is sometimes inefficient and can miss out optimal settings when used to analyze factorial effects.

Appendix

```
knitr::opts_chunk$set(echo = F)
starch <- read.csv("starch.csv", header=T)

y <-  starch$strength
x<-starch$thickness

x1.1= matrix(0,length(starch$strength))
x1.2= matrix(0,length(starch$strength))
x1.3= matrix(0,length(starch$strength))

i_1 = which(starch$starch=="CA")
i_2 = which(starch$starch=="CO")
i_3 = which(starch$starch=="PO")

x1.1 [i_1]= x[i_1]
x1.2 [i_2]= x[i_2]
x1.3 [i_3]= x[i_3]

thickness = x1.1+x1.2+x1.3

newStarch = data.frame(y,x1.1,x1.2,x1.3,thickness)
colnames(newStarch) = c("strength","CA","CO","PO","thickness")

lm <- lm(strength~ thickness+CO+PO, newStarch)
summary(lm)
```

```
##
## Call:
## lm(formula = strength ~ thickness + CO + PO, data = newStarch)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -172.86 -109.12  -62.05   44.43  635.77
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   64.681    174.723   0.370 0.712974
## thickness     71.269     17.163   4.152 0.000145 ***
## CO            -5.746     12.296  -0.467 0.642536
## PO             4.286      6.005   0.714 0.479158
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 167.1 on 45 degrees of freedom
## Multiple R-squared:  0.6719, Adjusted R-squared:   0.65
## F-statistic: 30.71 on 3 and 45 DF,  p-value: 5.816e-11
```

```
lm_aov <- aov(strength~thickness+CO+PO, newStarch)
summary(lm_aov)
```

```
##             Df  Sum Sq Mean Sq F value   Pr(>F)
## thickness    1 2553357 2553357  91.429 2.08e-12 ***
## CO           1    5718    5718   0.205    0.653
## PO           1   14221   14221   0.509    0.479
## Residuals   45 1256725   27927
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
data <- read.table("throughput.txt", header=T)

lm <- lm(Throughput~ Day+Operator+Method+Machine, data)
anova(lm)
```

```
## Analysis of Variance Table
##
## Response: Throughput
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Day        1   84.5   84.50  3.2899   0.09119 .
## Operator   1   11.5   11.52  0.4485   0.51393
## Method     4 2857.6  714.40 27.8147 1.573e-06 ***
## Machine    4 3424.8  856.20 33.3356 5.129e-07 ***
## Residuals 14  359.6   25.68
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

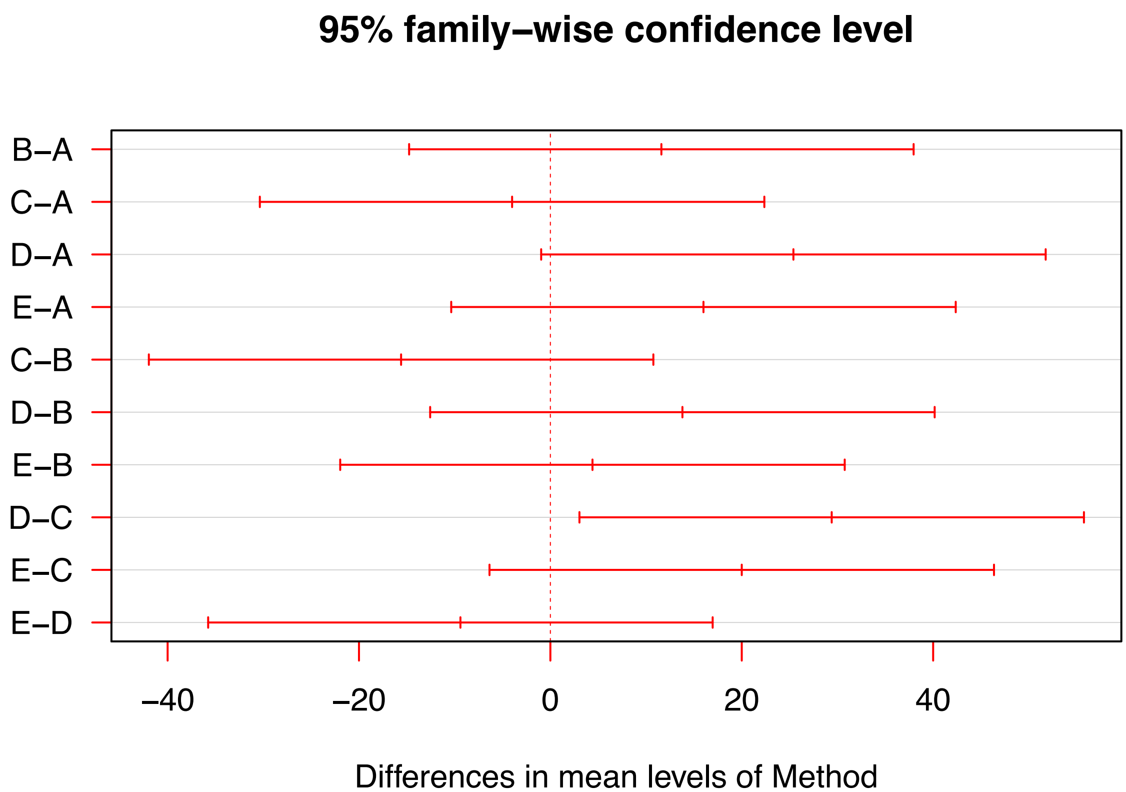
```
data$Method <-as.factor(data$Method) ## set as factor
lm_new <- lm(Throughput~Method, data)
anova = aov(lm_new)

##multiple comparisons
tukey <- TukeyHSD(x = anova, 'Method', conf.level = 0.95)
tukey
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
```

```
## Fit: aov(formula = lm_new)
##
## $Method
##      diff         lwr      upr     p adj
## B-A  11.6 -14.7614442 37.96144 0.6843733
## C-A  -4.0 -30.3614442 22.36144 0.9905466
## D-A  25.4  -0.9614442 51.76144 0.0623764
## E-A  16.0 -10.3614442 42.36144 0.3921784
## C-B -15.6 -41.9614442 10.76144 0.4166162
## D-B  13.8 -12.5614442 40.16144 0.5341875
## E-B   4.4 -21.9614442 30.76144 0.9864805
## D-C  29.4   3.0385558 55.76144 0.0242485
## E-C  20.0  -6.3614442 46.36144 0.1956041
## E-D  -9.4 -35.7614442 16.96144 0.8209033
```

```r
plot(tukey, las=1, col="red")
```




```r
spring <- read.csv("Spring.csv", header=T)
##first step
spring
```

```
##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                0.5             1450
## 2             70                0.5             1600
## 3             70                0.7             1450
## 4             70                0.7             1600
## 5            120                0.5             1450
## 6            120                0.5             1600
## 7            120                0.7             1450
## 8            120                0.7             1600
```

```
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3                            61
## 4                            75
## 5                            59
## 6                            90
## 7                            52
## 8                            87
```

```
##step-by-step replacement
spring[7,]<-""
spring
```

```
##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                0.5             1450
## 2             70                0.5             1600
## 3             70                0.7             1450
## 4             70                0.7             1600
## 5            120                0.5             1450
## 6            120                0.5             1600
## 7
## 8            120                0.7             1600
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3                            61
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87
```

```
spring[3,]<-""
spring
```

```
##   OilTemperature PercentageOfCarbon SteelTemperature
## 1             70                0.5             1450
## 2             70                0.5             1600
## 3
## 4             70                0.7             1600
## 5            120                0.5             1450
## 6            120                0.5             1600
## 7
## 8            120                0.7             1600
##   PercentageofNoncrackedSprings
## 1                            67
## 2                            79
## 3
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87
```

```r
spring[1,]<-""
spring
```

```
##   OilTemperature PercentageOfCarbon SteelTemperature
## 1
## 2             70                0.5             1600
## 3
## 4             70                0.7             1600
## 5            120                0.5             1450
## 6            120                0.5             1600
## 7
## 8            120                0.7             1600
##   PercentageofNoncrackedSprings
## 1
## 2                            79
## 3
## 4                            75
## 5                            59
## 6                            90
## 7
## 8                            87
```

Q2.) For a plot to be synergistic,

$$mE(B|A+)\, mE(B|A-) > 0$$

$\therefore$ and we know

$$mE(B|A+) = \bar{z}(B+|A+) - \bar{z}(B-|A+)$$

and

$$mE(B|A-) = \bar{z}(B+|A-) - \bar{z}(B-|A-)$$

$\vdots$

$$\bar{z}(B+|A+)\,\bar{z}(B+|A-) - \bar{z}(B-|A+)\,\bar{z}(B+|A-)$$

$$- \bar{z}(B-|A-)\,\bar{z}(B+|A+) + \bar{z}(B-|A+)\,\bar{z}(B-|A-)$$

$$> 0.$$

Q5) LSE $\hat{\beta}_j$ $\hat{\beta} = (X^T X)^{-1} X^T y$

Half fractional effect

$$\beta_j = \frac{1}{2} \{ \bar{z}(x_{ij} = +1) - \bar{z}(x_{ij} = -1) \}$$