Modelagem
Roteiro 3

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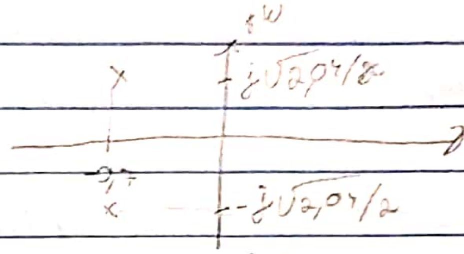
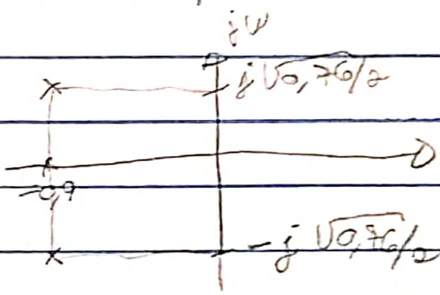
a) • 0,7 → $\dfrac{Y(S)}{R(S)} = \dfrac{1}{S^2+1,4S+1}$



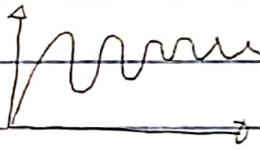
$S_1 e S_2 = -0,7 \pm j\sqrt{2,04}/2$

• 0,9 → $\dfrac{Y(S)}{R(S)} = \dfrac{1}{S^2+1,8S+1}$



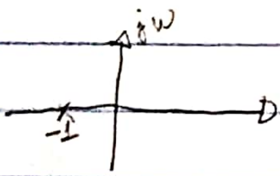
$S_1 e S_2 = -0,9 \pm j\sqrt{0,76}/2$

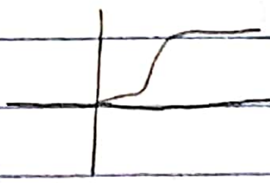
$0 < \xi < 1$ → Sistema subamortecido → 2 raízes complexas com parte real $= \xi$.



• P/ $\xi = 1$ $\dfrac{Y(S)}{R(S)} = \dfrac{1}{S^2+2S+1}$ $S^2+2S+1=0$ $\dfrac{-2}{2}$

$\Delta = 0$ $S_1 = S_2 = -1$





2 raízes iguais

Sistema criticamente amortecido

b) Sistemas subamortecidos $0 < \xi < 1$, 0 à 100%

t_r (Tempo de subida)

$$c(t) = 1 - e^{-\xi \omega_n t}\left(\cos \omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen} \omega_d t\right)$$

p/ $c(t) = 1$

$$c(t_r) = 1 - e^{-\xi \omega_n t_r}\left(\cos \omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen} \omega_d t_r\right)$$

$$0 = -e^{-\xi \omega_n t_r}\left(\cos \omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen} \omega_d t_r\right)$$, como $e^{-\xi\omega_n t_r} \neq 0$

e $\omega_d = \omega_n\sqrt{1-\xi^2}$

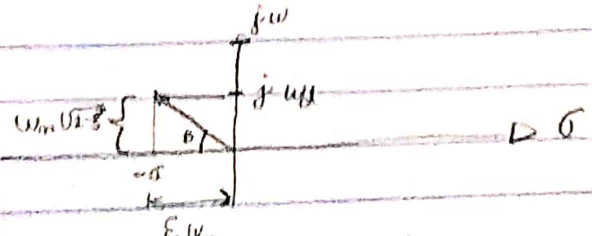
e $\xi \omega_n = \sigma$

$$\cos \omega_d t_r = -\frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen} \omega_d t_r$$

$$\operatorname{sen} \omega_d t_r = -\cos \omega_d t_r \cdot \frac{\sqrt{1-\xi^2}}{\xi} \quad (\div \cos \omega_d t_r)$$

$$tg(\omega_d t_r) = -\frac{\sqrt{1-\xi^2}}{\xi} \cdot \frac{\omega_d}{\omega_d} \rightarrow \frac{\omega_d}{\sigma}$$

$$t_r = \frac{1}{\omega_d}\, tg^{-1}\left(\frac{-\omega_d}{\sigma}\right) \rightarrow t_r = \frac{\pi - \beta}{\omega_d}$$



c) Sistemas subamortecidos ~ tempo de pico (t_p)

$$t_p: \quad \frac{dc}{dt} = -(-\xi\omega_n)\, e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen}\omega_d t\right) + e^{-\xi\omega_n t}\left(\omega_d \operatorname{sen}\omega_d t - \frac{\xi\omega_d}{\sqrt{1-\xi^2}} \cos\omega_d t\right)$$

com $t = t_p$

$$\left.\frac{dc}{dt}\right|_{t=t_p} = \operatorname{sen}(\omega_d t_p) \frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t_p} = 0, \quad \operatorname{sen}\omega_d t_p = 0 \text{ ou}$$

$$\omega_d t_p = 0, \pi, 2\pi \ldots$$

Como t_p é o 1º pico, $\omega_d t_p = \pi$. Então: $\boxed{t_p = \frac{\pi}{\omega_d}}$

O tempo de pico (t_p) corresponde a meio ciclo da frequencia de oscilação amortecida.

d) Tempo de Acomodação (t_s)

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \operatorname{sen}\left(\omega_d t + tg^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right), \text{ com } \tau = \frac{1}{\xi\omega_n}$$

• 2%

$$t_s = 4\tau = 4\,\frac{1}{\xi\omega_n}, \quad \xi\omega_n = \sigma$$

$$t_s = \frac{4}{\sigma}$$

$$e^{-\sigma t} = 0{,}02 \Rightarrow -\sigma t = \ln(0{,}02)$$

$$t = \frac{4}{\sigma} = \frac{4}{\xi\omega_n}$$

• 5%

$$t_s = 3\tau = \frac{3}{\sigma}$$

$$t_s = \frac{3}{\xi\omega_n}$$

$$e^{-\sigma t} = 0{,}05$$

$$-\sigma t = \ln(0{,}05)$$

$$t = \frac{3}{\sigma} \Rightarrow \frac{3}{\xi\omega_n}$$

e) Máximo Sobressinal (M_p)

M_p ocorre quando $t = t_p = \frac{\pi}{\omega_d}$

Supondo valor final de saída unitário

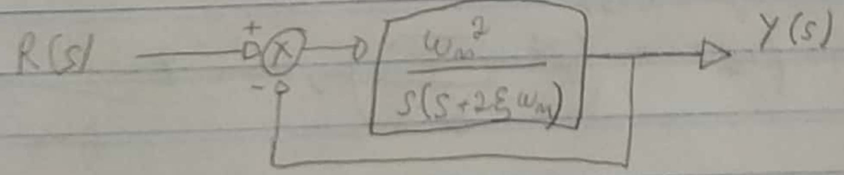
$M_p = c(t_p) - 1$

$$M_p = 1 - e^{-\xi \omega_n (\pi/\omega_d)} \left(\cos \omega_d \frac{\pi}{\omega_d} + \frac{\xi}{\sqrt{1-\xi^2}} \operatorname{sen} \omega_d \frac{\pi}{\omega_d} \right) - 1$$

$$M_p = e^{-(\sigma/\omega_d)\pi}$$

$$\boxed{M_p = e^{-(\xi/\sqrt{1-\xi^2})\pi}}$$

f) $\omega_n = 0{,}5$, $\xi = 0{,}6$



$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\omega_d = \omega_n \sqrt{1-\xi^2}$

$\omega_d = 0{,}5\sqrt{1-0{,}6^2} = 0{,}4$

$\sigma = -\xi\omega_n \to -0{,}3$

$$t_r = \frac{1}{\omega_d} tg^{-1}\left(\frac{-\omega_d}{\sigma}\right)$$

$$t_r = \frac{1}{0{,}4} tg^{-1}\left(\frac{-0{,}4}{-0{,}3}\right)$$

$$\boxed{t_r = 132{,}8_a}$$

$t_p = \frac{\pi}{\omega d} \rightarrow \frac{\pi}{0,4} = \boxed{7,85\ s}$

t_s: 2%

$t_s = \frac{4}{(0,6)(0,5)} = 13,3$

5%

$t_s = \frac{3}{(0,6)(0,5)} = 10$

$M_p = e^{-(0,3/0,4)\pi}$

$\boxed{M_p = 10,55\ s}$