Prova 01

MASL

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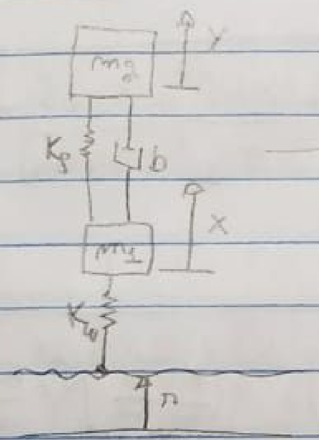
EC-14.18083

①



$\longrightarrow V_{con}$ $\quad \dfrac{Y(s)}{R(s)}$

$F_{x_1} = R K_w$

$$F = (m_1 s^2 + k_w + k_s + bs)\, X_1(s) - (k_s + bs)\, Y_1(s)$$

$$0 = -(k_s + bs)\, X_1(s) + (k_s + bs + m_2 s^2)\, Y_1(s)$$

$$\Delta = \begin{vmatrix} m_1 s^2 + k_w + k_s + bs & -k_s - bs \\ -k_s - bs & k_s + bs + m_2 s^2 \end{vmatrix}$$

$$\underline{m_1 s^2 k_s} + \underline{m_1 b s^3} + \underline{m_1 m_2 s^4} + \underline{k_w k_s} + \underline{k_w b s} + \underline{m_2 k_w s^2} + \cancel{k_s^2} + \cancel{k_s b s} +$$
$$\underline{m_2 k_s s^2} + \cancel{b k_s s} + \cancel{b^2 s^2} + \underline{m_2 b s^3}$$
$$-(\cancel{k_s^2} + \cancel{b k_s s} + \cancel{b k_s s} + \cancel{b^2 s^2})$$

$$\Delta_y = \begin{vmatrix} m_1 s^2 + k_w + k_s + bs & R(s) k_w \\ -k_s - bs & 0 \end{vmatrix}$$

$$0 - (-k_s R k_w - b s R k_w) \sim k_s R k_w + b R k_w s$$

$$Y(s) = \frac{R(s) \cdot (k_s k_w + b k_w s)}{m_1 m_2 s^4 + m_1 b s^3 + m_2 b s^3 + m_1 k_s s^2 + m_2 k_w s^2 + m_2 k_s s^2 + k_w b s + k_w}$$

• isolando S^4 e passando $R(s)$ p/ o outro lado temos:

$$\frac{Y(s)}{R(s)} = \frac{\left(\frac{K_S K_W}{m_1 m_2}\right) + \left(\frac{b\,K_W S}{m_1 m_2}\right)}{S^4 + \left(\frac{bS^3}{m_2}\right) + \left(\frac{bS^3}{m_1}\right) + \left(\frac{K_S S^2}{m_2}\right) + \left(\frac{K_W S^2}{m_1}\right) + \left(\frac{K_S S^2}{m_1}\right) + \left(\frac{K_W b S}{m_1 m_2}\right) + \left(\frac{K_W K_S}{m_1 m_2}\right)}$$

$$\frac{Y(s)}{R(s)} = \frac{\left(\frac{K_S K_W}{m_1 m_2}\right) + \left(\frac{b K_W}{m_1 m_2}\right) S}{S^4 + \left(\frac{b}{m_2} + \frac{b}{m_1}\right) S^3 + \left(\frac{K_S}{m_2} + \frac{K_W}{m_1} + \frac{K_S}{m_1}\right) S^2 + \left(\frac{K_W b}{m_1 m_2}\right) S + \frac{K_W K_S}{m_1 m_2}}$$

② a) $Q(\alpha x_1 + \beta x_2) = \alpha Q(x_1) + \beta Q(x_2)$ $\quad Q = K\sqrt{H}$

$$K\sqrt{\alpha x_1 + \beta x_2} \neq \alpha K\sqrt{x_1} + \beta K\sqrt{x_2}$$

Como as equações são diferentes o sistema é não linear.

b) $$q(t) = q(t)\Big|_{P.O.} + \frac{\partial q(t)}{\partial t}\Big|_{P.O.} \delta H$$

derivada de $K\sqrt{H_0}$ $\rightarrow \frac{K}{2\sqrt{H_0}}$

$$q(t) = K\sqrt{H_0} + \frac{K}{2\sqrt{H_0}}\,\delta H$$

$$\delta q(t) = \frac{K}{2\sqrt{H_0}}\,\delta H$$

c) $q(t) = K\sqrt{H}$

$q(t) = q_0(t) + \frac{h}{R}$ $\quad R = \frac{2\sqrt{\bar{h}_0}}{K} \approx \frac{2\sqrt{4}}{0,5} = 8$

$q(t) = 0,5\sqrt{h}$

p/ $q(t) = 1$ $\quad 2 = \sqrt{h}$ $\quad q(t) = 0,5\sqrt{h}$

$1 = 0,5\sqrt{h}$ $\quad \boxed{\bar{h} = 4}$ $\quad q_L(t) = 1 + \frac{h}{8} - \frac{\bar{h}}{8}$

$h(t)$	0,25	0,75	1,5	2,25	3,0	3,75	4,0	4,75	5,5
$q(t)$	0,25	0,433	0,612	0,75	0,866	0,968	1	1,089	1,172
$q_L(t)$	0,531	0,593	0,687	0,781	0,875	0,968	1	1,093	1,187

$q(t) = 0,5\sqrt{0,25} = 0,25$

$q_L(t) = 1 + \frac{0,25}{8} - \frac{4}{8} = 0,531$

$q(t) = 0,5\sqrt{0,75} = 0,433$

$q_L(t) = 1 + \frac{0,75}{8} - \frac{4}{8} = 0,593$

$q(t) = 0,5\sqrt{1,5} = 0,6123$

$q_L(t) = 1 + \frac{1,5}{8} - \frac{4}{8} = 0,6875$

$q(t) = 0,5\sqrt{2,25} = 0,75$

$q_L(t) = 1 + \frac{2,25}{8} - \frac{4}{8} = 0,7812$

$q(t) = 0,5\sqrt{3} = 0,866$

$q_L(t) = 1 + \frac{3}{8} - \frac{4}{8} = 0,875$

$q(t) = 0,5\sqrt{3,75} = 0,96824$

$q_L(t) = 1 + \frac{3,75}{8} - \frac{4}{8} = 0,96875$

$q(t) = 0,5\sqrt{4} = 1$

$q_L(t) = 1 + \frac{4}{8} - \frac{4}{8} = 1$

$q(t) = 0,5\sqrt{4,75} = 1,0897$

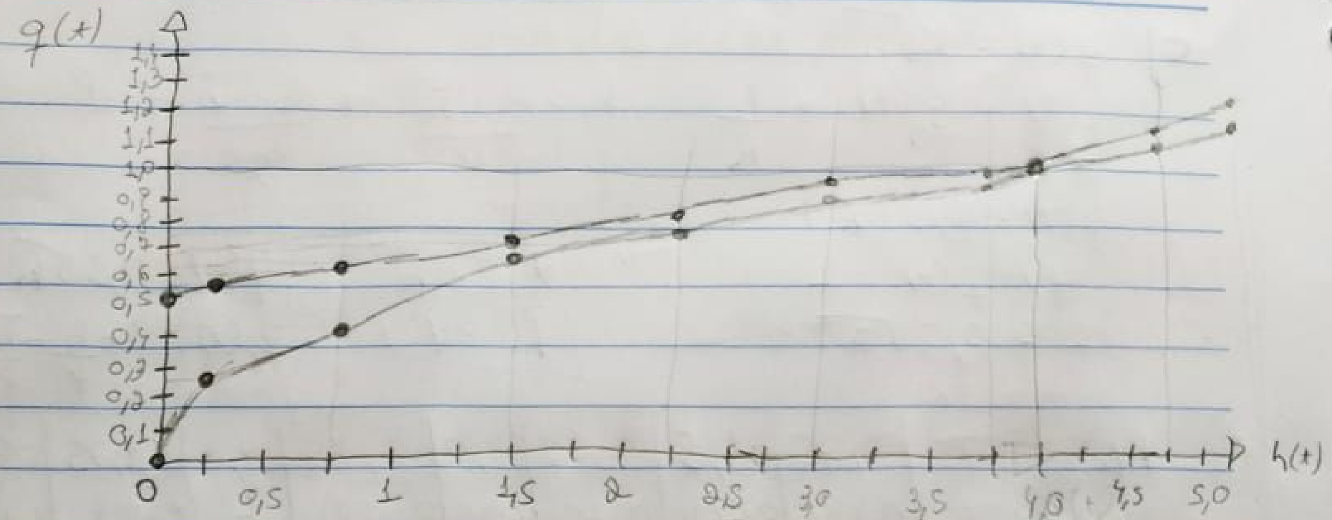
$q_L(t) = 1 + \frac{4,75}{8} - \frac{4}{8} = 1,09375$

$q(t) = 0,5\sqrt{5,5} = 1,1726$

$q_L(t) = 1 + \frac{5,5}{8} - \frac{4}{8} = 1,1875$

$q(t) = 0,5\sqrt{0} = 0$

$q_L(t) = 1 - \frac{4}{8} = 0,5$



d) tanque 1

$$C\dot{H}_1 = qc_{(t)} - q_1(t)$$

$$C\dot{H}_1 = qc_{(t)} - \frac{H_1(t)}{R}$$

$$\dot{H}_1 = \frac{qc(t)}{C} - \frac{H_1(t)}{RC}$$

Tanque 2

$$C\dot{H}_2 = \frac{H_1(t)}{R} - \frac{H_2(t)}{R}$$

$$\dot{H}_2 = \frac{H_1(t)}{RC} - \frac{H_2(t)}{RC}$$

tanque 3

$$C\dot{H}_3 = \frac{H_2(t)}{R} - \frac{H_3(t)}{R}$$

$$\dot{H}_3 = \frac{H_2(t)}{RC} - \frac{H_3(t)}{RC}$$

Vazão $\rightsquigarrow V = C\dot{H}$

$$R = \frac{2\sqrt{H}}{K}$$

$$\begin{bmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 & 0 \\ \frac{1}{RC} & -\frac{1}{RC} & 0 \\ 0 & \frac{1}{RC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} + \begin{bmatrix} 1/C \\ 0 \\ 0 \end{bmatrix} qc_{(t)}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

e) $C\dfrac{dh_3}{dt} = q_2(t) - q_3(t)$

$$C\dot{h}_3 = q_2(t) - \frac{h_3}{R_3}$$

$$CSH_3(s) = Q_2(s) - \frac{H_3(s)}{R_3}$$

$$H_3(s)(R_3CS+1) = R_3Q_2(s)$$

$$\frac{H_3(s)}{Q_2(s)} = \frac{R_3}{R_3CS+1} = \boxed{\frac{1/C}{S+1/R_3C}}$$