# Roteiro 5

## Modelagem e Análise de Sistemas Lineares

Ícaro Bicalho Quintão
EC - 14.1.8083

**1.1)** Potenciômetro de entrada:

$$\frac{V_{en}(s)}{\theta_{es}(s)} = \frac{10}{s \cdot 2\pi} = 0,318 \rightarrow K_{pot} = \frac{1}{\pi}$$

**1.2)** Pré amplificador:

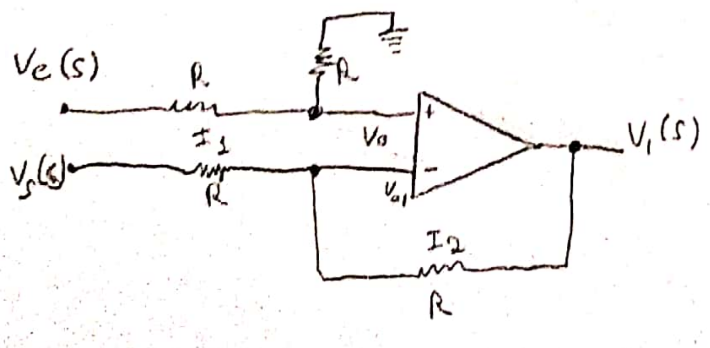
Entrada → $V_e(t) = V_{en}(t) - \cancel{V_s(t)}$ 0

$V_e(t) = V_{en}(t)$ → $V_e(s) = V_{en}(s)$

Saída → $V_p(t) \rightarrow V_p(s)$ $\quad \frac{V_p(s)}{V_e(s)} = K$

Subtrator: Entrada → $V_p(t)$ [tensão pré-amplificador]
Saída → $E_a(t)$ [tensão para o motor]

$$\frac{E_a(s)}{V_p(s)} = \frac{K_1}{s+a} \rightarrow \frac{100}{s+100} \quad \begin{cases} K_1 = 100 \\ a = 100 \end{cases}$$




$$\frac{V_e - V_o}{R} = \frac{V_o}{R}$$

$$V_e = 2V_o \rightarrow V_o = \frac{V_e}{2}$$

$$\frac{V_s - V_o}{R} = \frac{V_o - V_1}{R} \rightarrow V_s = 2V_o - V_1$$

$$V_s = V_e - V_1$$

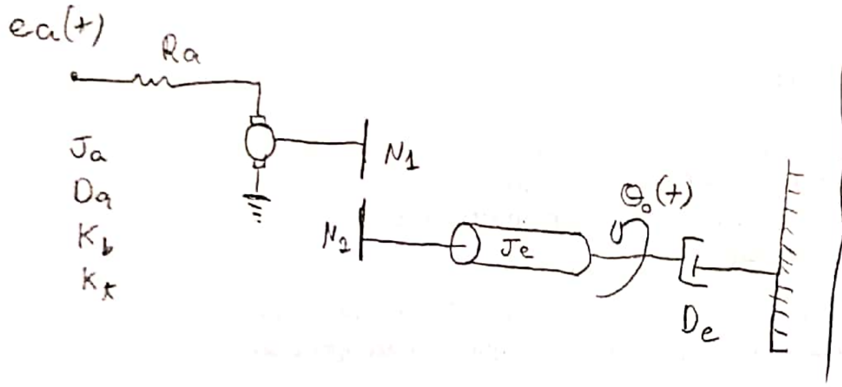
$$\boxed{V_1(s) = V_e(s) - V_s(s)}$$

1.3) Motor e carga:

$J_a = 0{,}02\ Kg\ m^2$ ; $N_1 = 25$; $D_a = 0{,}01\ N.m.seg/rad$

$J_c = 1\ Kg.m^2$; $N_2 = 250$; $D_c = 1\ N.m.seg/rad$

$K_t = 0{,}5\ N.m/A$ ; $K_b = 0{,}5\ V.seg/rad$; $R_a = 8\Omega$




$$J_m = J_a + J_e\left(\frac{N_1}{N_2}\right)^2 = 0{,}03$$

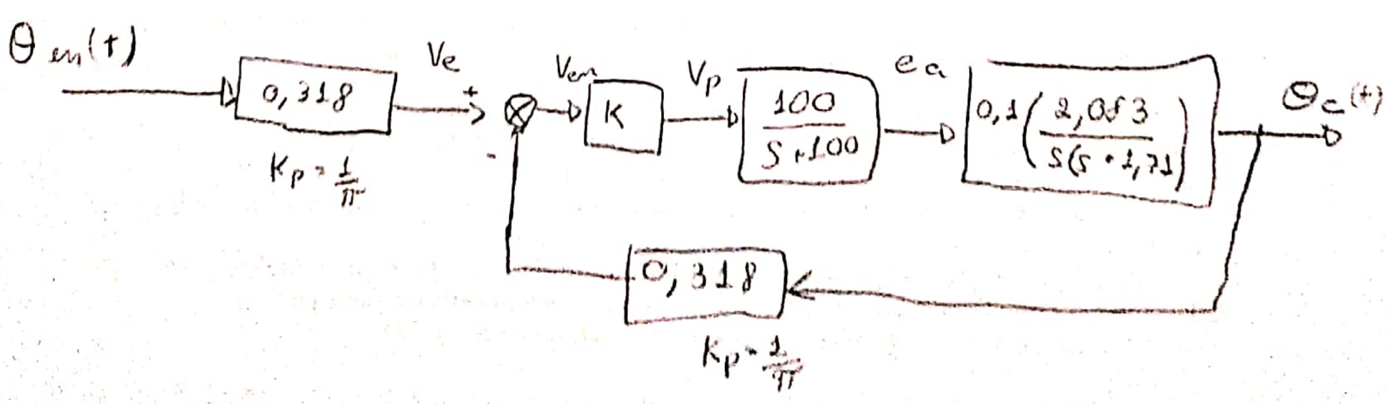
$$D_m = D_a + D_e\left(\frac{N_1}{N_2}\right)^2 = 0{,}02$$

$$\frac{\theta_m}{\theta_c} = \frac{250}{25} \rightarrow \theta_c = 0{,}1\,\theta_m$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{R_a J_m}}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \rightarrow \frac{0{,}5/(8 \cdot 0{,}03)}{s\left[s + \frac{1}{0{,}03}\cdot\left(0{,}02 + \frac{0{,}5\ 0{,}5}{8}\right)\right]}$$

$$\boxed{\frac{\theta_c(s)}{E_a(s)} = 0{,}1\left(\frac{2{,}083}{s(s+1{,}71)}\right)}$$

1.4) Diagrama de Blocos:

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2.1) $\frac{E_a(s)}{V_p(s)} = \frac{100}{s+100} \rightarrow E_a(s)\,s + E_a(s).100 = V_p(s).100$

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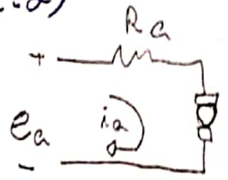
$\dot{e}_a + 100\,e_a = V_p\,100$

$$[\dot{e}_a] = [-100][e_a] + [100][V_p]$$

2.2)



- $R_a\,I_a + L_a S\,I_a + V_{ce} = E_a$ (1)
- $V_{ce} = K_{ce}\cdot S\cdot \theta_m$ (2)
- $t_m = (J_m S^2 + D_m S)\,\theta_m$ (4)
- $K_T . I_a = t_m$ (3)

$I_a = \frac{t_m}{K_T}$

(2) e (3) em (1):

$$\frac{t_m}{K_T}\cdot R_a + L_a . S . \frac{t_m}{K_T} + K_{ce}\cdot S . \theta_m = E_a$$

$$\frac{t_m}{K_T}(R_a + L_a . S) + K_{ce} . S . \theta_m = E_a \quad (5)$$

(4) em (5) e assumindo $R_a >> L_a$:

$$\left[\frac{R_a}{K_T}\cdot(J_m . S + D_m) + K_{ce}\right]. S . \theta_m = E_a$$

$$\underbrace{\frac{R_a . J_m}{K_T}}_{0,48}\cdot\ddot{\theta}_m + \underbrace{\left(\frac{R_a}{K_T}. D_m + K_{ce}\right)}_{0,82}\cdot\dot{\theta}_m = E_a$$

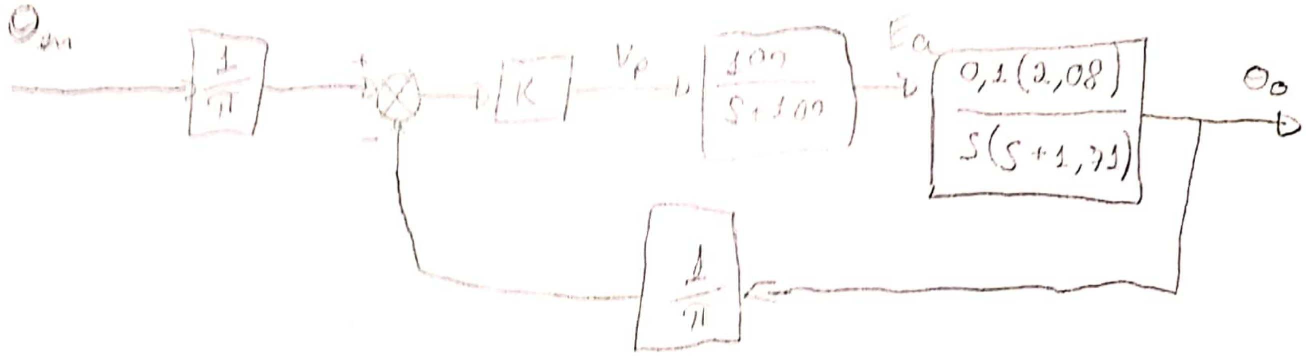
$$\left.\begin{array}{ll} x_1 = \theta_m & \rightarrow \dot{x}_1 = x_2 \\ x_2 = \dot{\theta}_m & \rightarrow \dot{x}_2 = \ddot{\theta}_m \end{array}\right\} 0,48\,\dot{x}_2 + 0,82\,x_2 = E_a$$

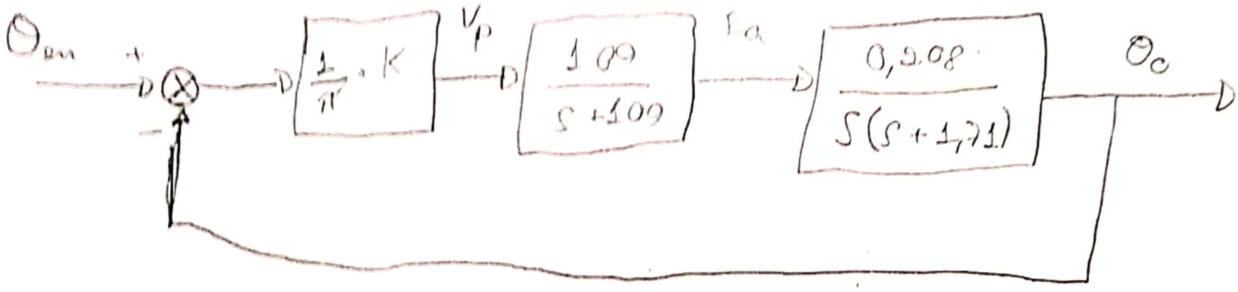
$$\left\{\begin{array}{l} \dot{x}_2 = \frac{E_a - 0,82\,x_2}{0,48} \\ \theta_m = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -1,7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 2,08\end{bmatrix} E_a \end{array}\right.$$

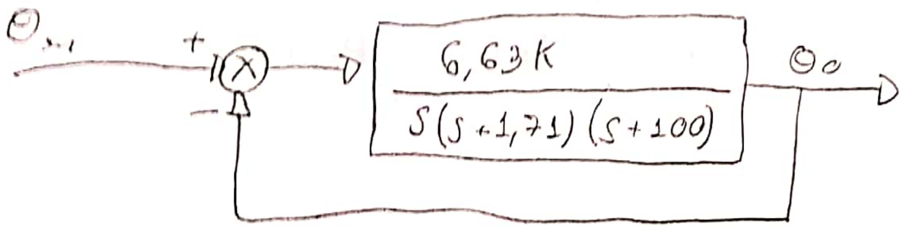
$$\dot{\theta}_S = \frac{N_2}{N_1}\,x_2$$

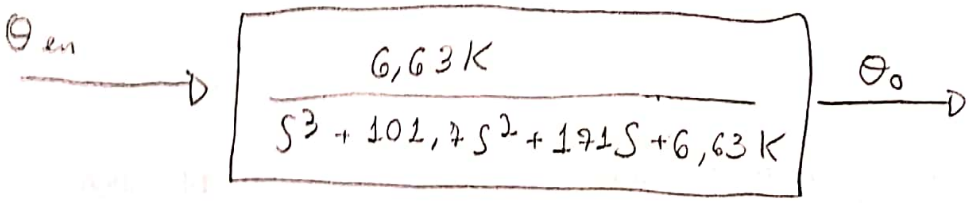
$$\theta_S \Rightarrow \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 10\\ 0 & -1,7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 2,08\end{bmatrix} E_a$$

3.1)

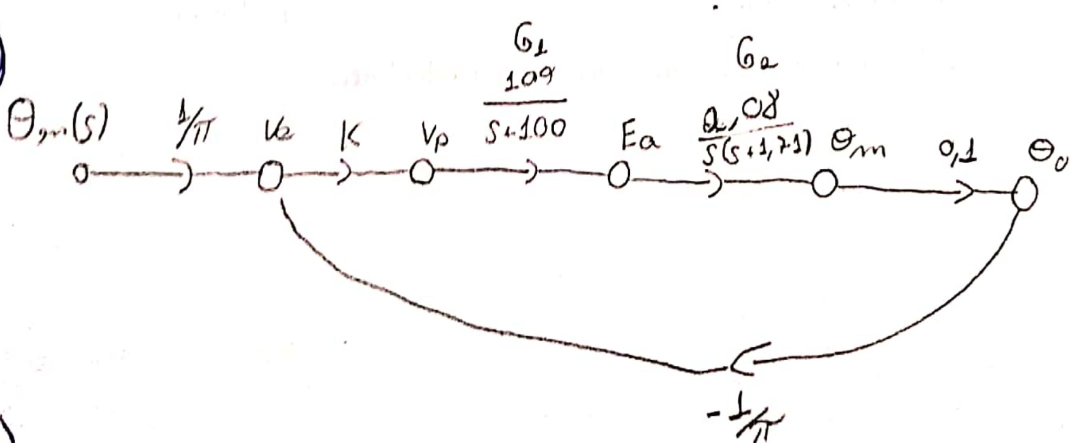



3.2)




3.3) $K=1$, $t_1 = \frac{K}{\pi}\cdot\frac{100}{S+100}\cdot\frac{2,08}{S(S+1,07)}$ ; $\Delta = 1-\frac{K}{\pi}\cdot G_1\cdot G_2$ , $\Delta_1 = 1$

$$\frac{\Theta_m(S)}{\Theta_{en}(S)} = \frac{\frac{K}{\pi}G_1G_2}{1-\frac{K}{\pi}G_1G_2} \rightsquigarrow \boxed{\frac{\Theta_m(S)}{\Theta_{en}(S)} = \frac{K\,G_1G_2}{\pi-KG_1G_2}}$$

$$\begin{cases} G_1 = \frac{100}{S+100} \\ G_2 = \frac{2,08}{S(S+1,71)} \end{cases}$$

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4.1) $T_{w_1} = \left(\frac{100}{S+100}\right) \cdot 0{,}1 \cdot \frac{2{,}08}{S(S+1{,}71)} \cdot S$

$T_{w_1} = \frac{20{,}8}{(S+100)(S+1{,}71)} \rightarrow \frac{20{,}8}{S^2 + 101{,}71 S + 171}$

$w_n = \sqrt{171}$ ; $101{,}71 = 2 \xi w_n$

$\boxed{w_n = 13{,}08}$

$\xi = \frac{101{,}71}{2\sqrt{171}} \rightarrow \boxed{\xi = 3{,}89}$

4.2) $Y(s) = w_s(S) \rightarrow \frac{20{,}83}{S(S+100)(S+1{,}71)} = \frac{A}{S} + \frac{B}{S+100} + \frac{C}{S+1{,}71}$

$A = \left.\frac{20{,}83\, S}{S(S+100)(S+1{,}71)}\right|_{S \to 0} \rightarrow A = 0{,}122$

$B = \left.\frac{(20{,}83)(S+100)}{S(S+100)(S+1{,}71)}\right|_{S \to -100} \rightarrow B = 2{,}12 \cdot 10^{-3}$

$C = \left.\frac{(20{,}83)(S+1{,}71)}{S(S+100)(S+1{,}71)}\right|_{S \to -1{,}71} \rightarrow C = -0{,}124$

$W(s) = \frac{0{,}122}{S} + \frac{2{,}12 \cdot 10^{-3}}{S+100} - \frac{0{,}124}{S+1{,}71}$

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$w_s(t) = 0{,}122 + (2{,}12 \cdot 10^{-3})\, e^{-100t} - 0{,}124 \cdot e^{-1{,}71 t}$

4.3) $\frac{W_s(s)}{V_p(s)} = \frac{20,83}{s^2+101,7s+1,71}$

$$W_s(s)\cdot(s^2+101,7s+1,71) = V_p(s)\cdot 20,83$$

$$s^2 W_s(s) + 101,7\cdot s\cdot W_s(s) + 1,71\, W_s(s) = 20,83\, V_p(s)$$

$$\ddot{w}_s + 101,7\,\dot{w}_s + 1,71\, w_s = 20,83\, V_p$$

$$\left.\begin{array}{ll} x_1 = w_s & \rightarrow \dot{x}_1 = x_2 \\ x_2 = \dot{w}_s & \rightarrow \dot{x}_2 = \ddot{w}_s \end{array}\right\} \quad \dot{x}_2 + 101,7\, x_2 + 1,71\, x_1 = 20,83\, V_p$$

$$\dot{x}_2 = -101,7\, x_2 - 1,71\, x_1 + 20,83\, V_p$$

$$\ddot{w}_s \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1,71 \\ 1 & 101,7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 20,83 \\ 0 \end{bmatrix} V_p$$