Modelagem e Análise de Sistemas Lineares

Correção Prova 2

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① $y(t) = KA\left[1 - e^{-\sigma t}\cdot\left(\cos(w_d t) + \frac{\sigma}{w_d}\,\mathrm{sen}(w_d t)\right)\right]$

$y(t) = KA - KA\cdot e^{-\sigma t}\left[\cos w_d t + \frac{\sigma}{w_d}\cdot \mathrm{sen}(w_d t)\right]$

* t_r : $y(t_r) = KA$

$$KA - KA\,e^{-\sigma t_r}\left(\cos w_d t_r + \frac{\sigma}{w_d}\cdot \mathrm{sen}\, w_d t_r\right) = \cancel{KA}$$

$$\underbrace{-KA\,e^{-\sigma t_r}}_{\neq 0}\underbrace{\left(\cos w_d t_r + \frac{\sigma}{w_d}\cdot \mathrm{sen}\, w_d t_r\right)}_{\text{será igual a } 0} = 0$$

$$-\cos(w_d t_r)\cdot\frac{w_d}{\sigma} = \mathrm{sen}(w_d t_r) \qquad (\div \cos(w_d t_r))$$

$$-\frac{w_d}{\sigma} = tg(w_d t_r)$$

$$t_r = \frac{1}{w_n}\, tg^{-1}\left(-\frac{w_d}{\sigma}\right)$$

$$\boxed{t_r = \frac{\pi - \beta}{w_d}}$$

* t_p : para obtermos o tempo de pico, diferenciamos $y(t)$ e igualamos a derivada igual a zero.

$$y(t) = KA - KAe^{-\sigma t} \cdot \cos(\omega_d t) - KAe^{-\sigma t} \cdot \frac{\sigma}{\omega_d} \operatorname{sen}(\omega_d t)$$

$$\frac{\partial y(t)}{\partial t} = 0$$

$$\cancel{KA\sigma . e^{-\sigma t} \cos(\omega_d t)} - KAe^{-\sigma t} \cdot \omega_d \cdot \operatorname{sen}(\omega_d t) + KA \cdot \frac{\sigma}{\omega_d} \cdot \sigma . e^{-\sigma t} \cdot \operatorname{sen}(\omega_d t) \cancel{- KA \frac{\sigma}{\omega_d} \cdot e^{-\sigma t} \omega_d \cos(\omega_d t)} = 0$$

$$-KA . e^{-\sigma t} \cdot \omega_d . \operatorname{sen}(\omega_d t) + KA \frac{\sigma^2}{\omega_d} . e^{-\sigma t} . \operatorname{sen}(\omega_d t) = 0$$

Como o tempo do pico corresponde ao primeiro pico do sobressinal:

$$\omega_d t_p = \pi \leadsto \boxed{t_p = \frac{\pi}{\omega_d}}$$

$* T_s$:

$$y(t) = KA \left[1 - e^{-\sigma t} \cdot \frac{\sigma}{\omega_d} \cdot \operatorname{sen}(\omega_d t) + tg\left(\frac{\omega_d}{\sigma}\right) \right]$$

P/ 2%: $e^{-\sigma_d T} = 0{,}02, \; -\sigma_d t_s = -3{,}91 \leadsto \boxed{t_s = \frac{3{,}91}{\sigma_d}}$

$* M_p(\%)$:

$$M_p = c(T_p) - 1$$

$$M_p = -e^{\frac{\sigma}{\omega_d}} \cos(\pi) + \frac{\sigma}{\omega_d} \cdot \operatorname{sen}(\pi)$$

$$M_p = e^{-\left(\frac{\sigma}{\omega_d}\right)\pi}$$

$$\boxed{M_p(\%) = (100\%)\left(e^{-\left(\frac{\sigma}{\omega_d}\right)\pi}\right)}$$

② a) Gráfico a

$$\frac{y(\infty) - y(0)}{U(\infty) - U(0)} = K$$

$y(\infty) = 1,3 . 0,25 = 1,05$
$y(0) = 0,25$
$U(\infty) = 1$
$U(0) = 0,2$

$$K = \frac{0,8}{0,8} \approx 1$$

$\tau = 0,63 . K$

$$\boxed{\tau = 0,63}$$

$$\boxed{G(s) = \frac{1/0,63}{s + 1/0,63}}$$

Gráfico b

$y(\infty) = 0,6 - 1,2 = -0,6$
$y(0) = -1,2$
$U(\infty) = 1$
$U(0) = 0,4$

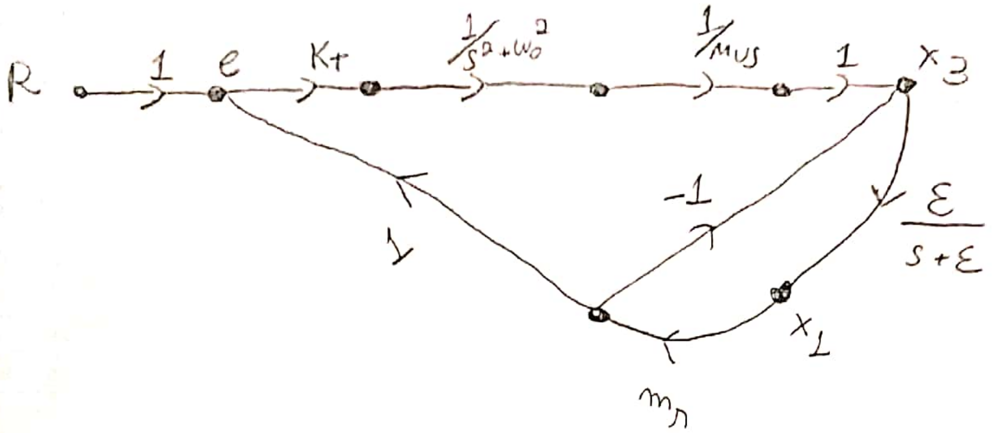
$$K = \frac{-0,6 - (-1,2)}{1 - 0,4} \approx 1$$

$$\boxed{\tau = 0,63}$$

$$\boxed{G(s) = \frac{1/0,63}{s + 1/0,63}}$$

b) Ambos os gráficos podem ser modelados pela equação característica do 1º grau. Por isso, só foi necessário o cálculo da constante de tempo τ.

③ $w_0^2 = \frac{K_T}{M_{US}}$



a) $\frac{X_3(s)}{R(s)}$: $t_1 = \overset{w_0^2}{\frac{K_t}{M_{US}}} \cdot \frac{1}{s^2+w_0^2}$

$L_1 = -m_r \frac{\varepsilon}{s+\varepsilon}$

$L_2 = \frac{\varepsilon}{s+\varepsilon} \cdot m_r \cdot K_t \cdot \frac{1}{s^2+w_0^2} \cdot \frac{1}{M_{US}}$

$\Delta = 1 - (L_1 + L_2)$

$\Delta_1 = 1$

$$\frac{X_3(s)}{R(s)} = \frac{\frac{w_0^2}{s^2+w_0^2} \cdot 1}{1 + \frac{\varepsilon}{s+\varepsilon} \cdot m_r\left(1 - \frac{w_0^2}{s^2+w_0^2}\right)} \rightsquigarrow \frac{w_0^2}{\cancel{s^2+w_0^2}} \cdot \frac{(s+\varepsilon)\cancel{(s^2+w_0^2)}}{(s+\varepsilon)(s^2+w_0^2) + (s^2+\cancel{w_0^2})\varepsilon m_r - \cancel{w_0^2 \varepsilon m_r}}$$

$$\frac{w_0^2(s+\varepsilon)}{(s+\varepsilon)(s^2+w_0^2) + s^2 \varepsilon m_r} \rightsquigarrow \boxed{\frac{w_0^2 s + \varepsilon w_0^2}{s^3 + s^2(\varepsilon + \varepsilon m_r) + s w_0^2 + \varepsilon w_0^2}}$$

b) $\frac{X_1(s)}{R(s)}$; $t_1 = K_t \cdot \frac{1}{s^2+w_0^2} \cdot \frac{1}{M_{US}} \cdot \frac{\varepsilon}{s+\varepsilon} \rightsquigarrow \frac{w_0^2}{s^2+w_0^2} \cdot \frac{\varepsilon}{s+\varepsilon}$

L_1, L_2, Δ e Δ_1 se mantém com os mesmos valores da letra a.

Logo:

$$\frac{w_0^2\,\varepsilon}{\cancel{(s^2+w_0^2)}\cancel{(s+\varepsilon)}} \cdot \frac{\cancel{(s+\varepsilon)}\cancel{(s^2+w_0^2)}}{(s+\varepsilon)(s^2+w_0^2)+(s^2+\cancel{w_0^2})\varepsilon_{mn}-\cancel{w_0^2}\varepsilon_{mn}}$$

$$\boxed{\frac{w_0^2\,\varepsilon}{s^3+s^2(\varepsilon+\varepsilon_{mn})+s\,w_0^2+\varepsilon\,w_0^2}}$$