Homework #7
OPTI 370
3/4/2015
(due date: 3/11/2015)

This version corrects the width d in Problem 2 to be 0.6nm.

Problem 1:

This problem is just a reminder of material covered earlier in class. It is meant as a preparation for a problem on quantum mechanics, Problem 2 below. Consider the Helmholtz equation (5.3-16)

$$\frac{\partial^2}{\partial z^2}U(z)+\frac{(2\pi\nu)^2}{c^2}U(z)=0$$

for the case of an ideal (lossless) planar mirror resonator with vacuum between two metallic mirrors. Assume the mirrors to be located at -d/2 and +d/2, and the wave amplitude to be zero at the boundaries. Determine all possible solutions , wave functions $U_q(z)$ and frequencies ν_q . Note that the functions $U(z)$ are either cos functions (even as function of z) or sin functions (odd as function of z). Sketch $U_q(z)$ for the lowest four frequency levels.

Assuming the width to be 1.188 micron, which level (mode number q) is closest to frequency of $\hbar\omega = 5.22$ eV ?

(10 points)

Problem 2:

The simplest 1-dimensional model for an electron in an atom is given by the square-well potential V(z) which is zero for z between -d/2 and +d/2, and infinite outside that interval. The time-independent Schrödinger equation reads

$$\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V(z)\right\}\varphi(z)=E\varphi(z)$$

with m assumed to be the electron mass in vacuum. For the interval $-d/2 \le z \le d/2$ it is very similar to the Helmholtz equation in Problem 1, namely

$$\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}-E\right\}\varphi(z)=0$$

Since the electron can only be inside the potential well, the boundary condition is that of vanishing wave function at the boundaries -d/2 and +d/2, again just as in the case of the Helmholtz equation in Problem 1. Solve the Schrödinger equation and find all energies E_n. Sketch the wave function for the lowest four levels as function of *z*. In this problem, you don't need to determine the pre-factor of the wave functions. (Just as in Problem 1, the wave functions are either cos or sin functions.)

Assuming the width to be $d = 0.6$ nm, which energy difference $E_{n+1} - E_n$ corresponds to $\hbar\omega = 5.22$ eV ? In other words, which transition in the quantum well corresponds to light in the Fabry-Perot resonator mode discussed in Problem 1? You find the answer to this simply by numerically evaluating a few of the lowest energies E_n.

(10 points)

Problem 3:

Use the book (Saleh/Teich) to find the laser transition energies for the following transitions:

Ne, $2p^5\ 5s \rightarrow 2p^5\ 4p$;
CO_2 $(001) \rightarrow (020)$;
$Nd^{3+}: YAG\ \ {}^4F_{3/2} \rightarrow {}^4I_{11/2}$;
$Er^{3+}:$ Silica Fiber $\ {}^4I_{13/2} \rightarrow {}^4I_{15/2}$;
$Yb^{3+}: YAG\ \ {}^2F_{5/2} \rightarrow {}^4F_{7/2}$.

Give all transition energies in units of eV, Hz (frequency), and cm^{-1} (inverse wavenumber $1/\lambda$). Also, indicate whether the transitions are electronic, vibronic, or rotational.

(10 points)

Problem 4:

The thermal occupation of energy levels is given by $\mathrm{P}(E_m) \sim e^{-E_m/kT}$. Consider two energy levels E_1 (ground state) and E_2 (excited state) separated by 2.6 eV. Determine the temperature at which the occupation of the excited state is one half that of the ground state. Is the transition energy in the visible optical spectrum? Is the required temperature very high or very low?

(10 points)