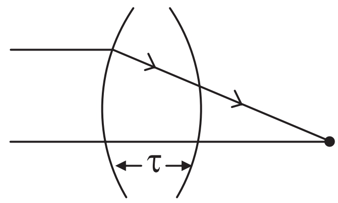
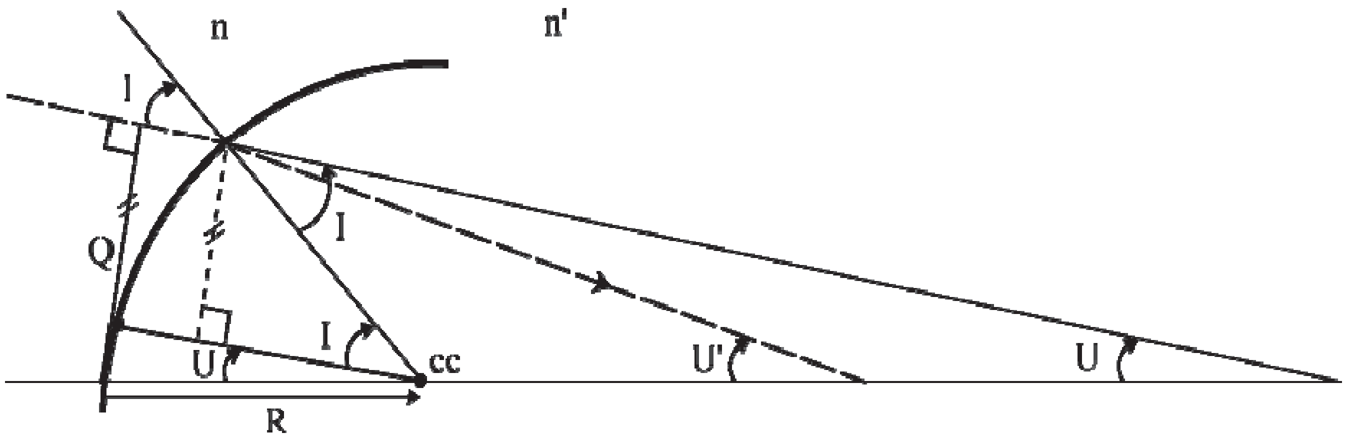
Exact Ray Trace

Analytical/exact ray tracing





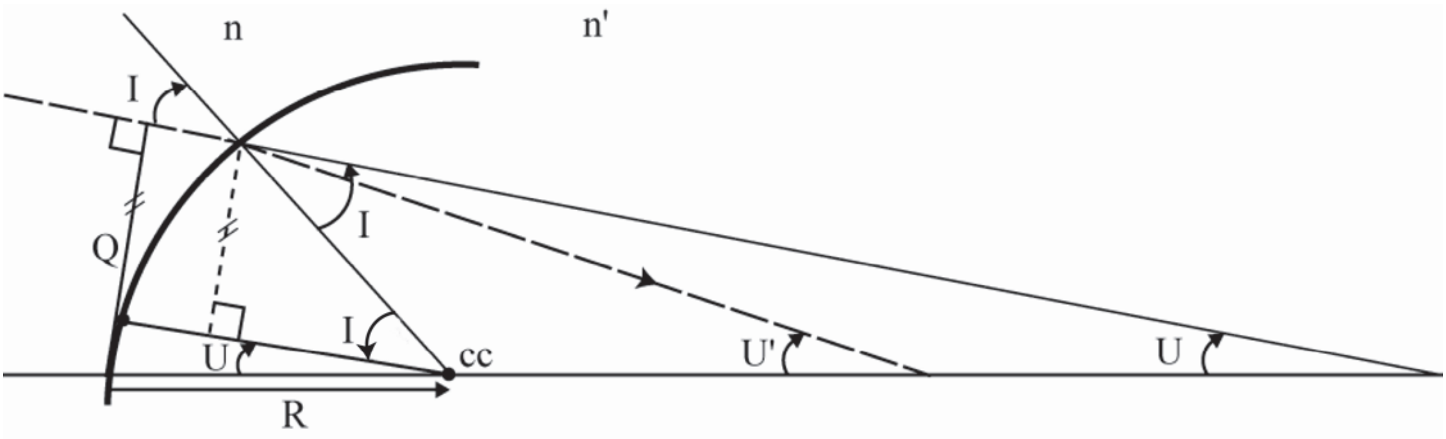
<u>Q – U Method</u>

- ray has height *Q* and ∡ U

<u>Givens:</u>

1. Surface radius – R
2. n ′
3. n
4. Q,U – ray description

* O’Shea, Elements of Modern Optics Design, Wiley (1985)

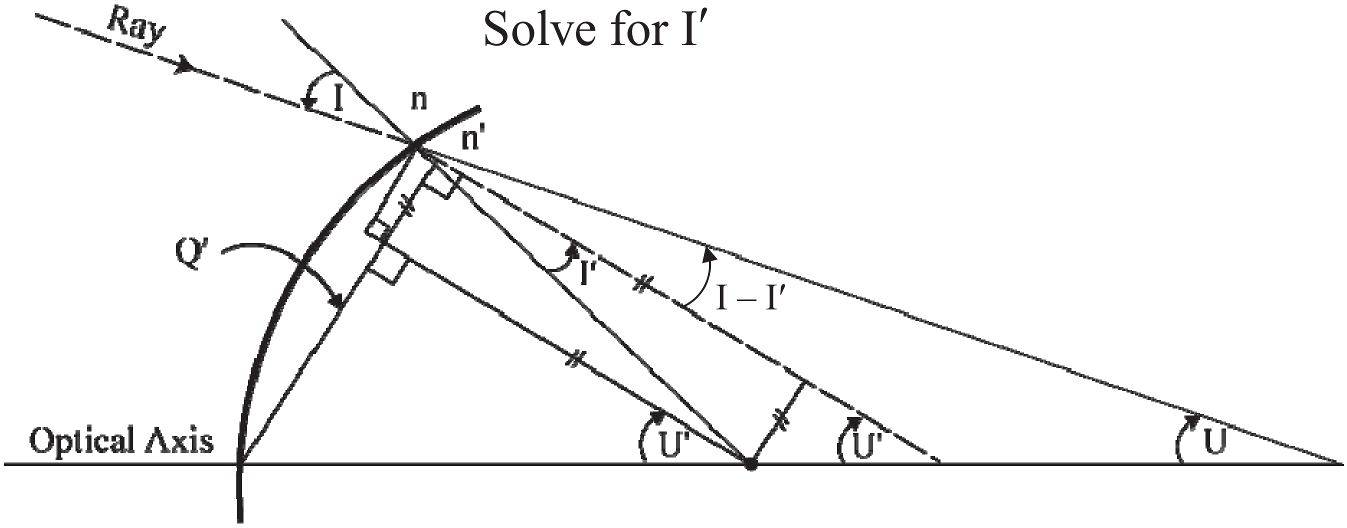


$$Q = R\sin I + R\sin(-U) = R\sin I - R\sin U$$

solve for I;

$$n'\sin I' = n\sin I$$

Solve for I'



We know I, I', and U. From the geometry we can derive

$$Q' = R\sin(-U') + R\sin I'$$

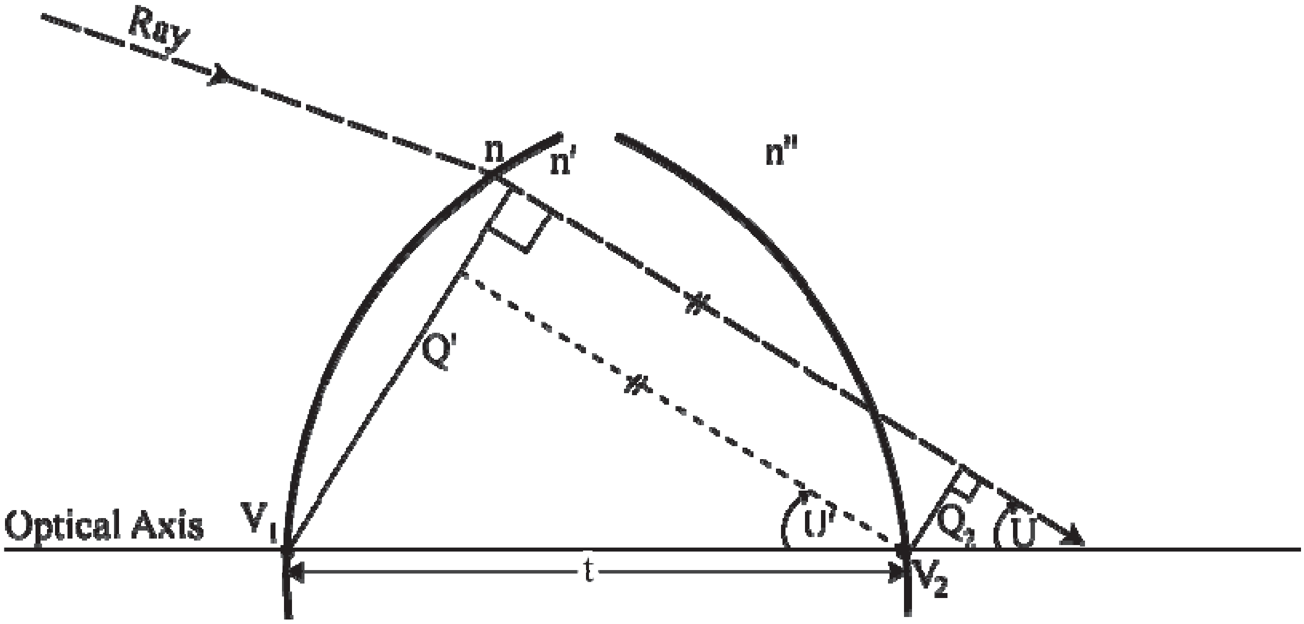
$$Q' = R(\sin I' - \sin U')$$

Recall that $U' = U - I + I'$

Therefore we now have a new ray, characterized by $Q' - U'$ of refracted ray just inside surface in space n' angle

Axial Transfer

Axial transfer of an exact ray to next surface, thickness (τ) away, where τ is vertex distances.



Surface$_1$ → Q_1, Q_1'

Surface$_2$ → Q_2, Q_2'

$$Q_2 = Q' - t\sin(-U')$$

U' is negative in sign convention

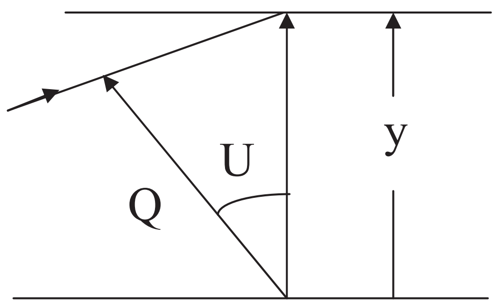
$$Q_2 = Q' + t\sin U'$$

Now we have $Q_2 U_2$ at surface$_2$, since $U_2 = U'$

Plane Surface ($R = \infty$) problem

y = Exact ray height at surface, $R = \infty$

$$y = \frac{Q}{\cos U} = \frac{Q'}{\cos U'}$$



$$\sin U' = \frac{n}{n'} \sin U$$

$$Q' = Q\left(\frac{\cos U'}{\cos U}\right)$$

Q-U Ray Trace

From geometry of Q – U at the surface of a known input ray:

1) $Q = R\sin(-U) + R\sin I$

find I from knowing Q and U

$$\sin I = \frac{Q}{R} - \sin(-U)$$

2) $\sin I' = \dfrac{n}{n'}\sin I$

find I' from I via Snell's Law

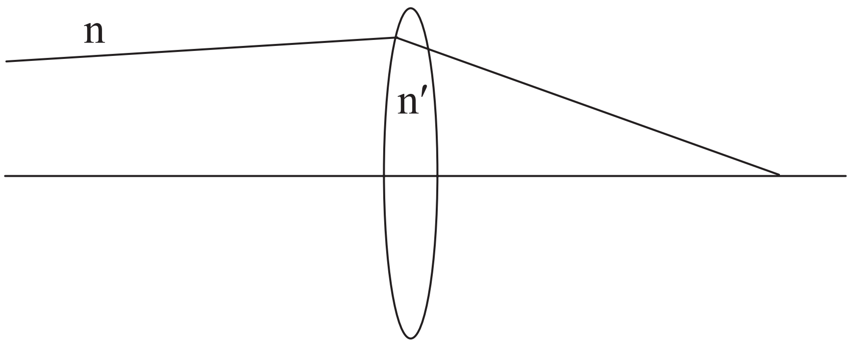
3) $U' = U + I' - I$ [Geometry]

4) $Q' = R\left(\sin I' - \sin U'\right)$

find Q'

5) $Q_2 = Q' + \tau \sin U'$ at next surface

Exact Ray Trace (Q U Trace)



space surface$_1$ space surface$_2$ space

r c t n	n	r_1 c_1	t n'	r_2 c_2
U	U			
sinU	sinU			
Q		Q		Q_2
sinI		sinI		
I		I		
sinI′		sinI'		
I′		I′		
U′			U'	
y		y		
z		z		

Repeat

$$\sin I = QC + \sin U$$

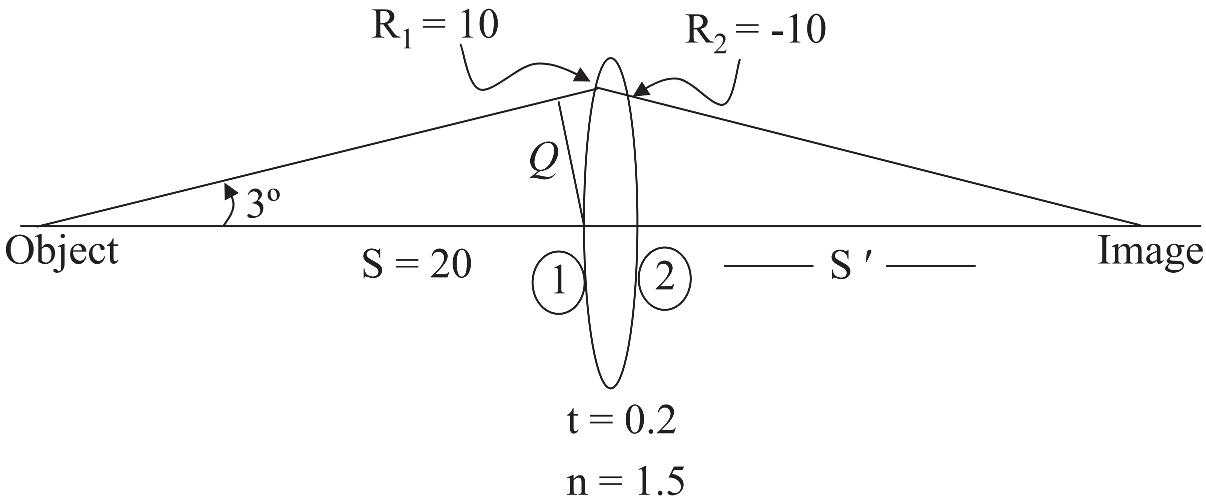
$$\sin I' = \frac{n}{n'} \sin I$$

$$U' = U - I + I'$$

$$Q' = r(\sin I' - \sin U')$$

$$Q_2 = Q' + t \sin U' \quad \text{(Q for next surface)}$$

Example of Exact Ray Trace



Open/initial knowledge

$Q = t \sin U = +20 \sin 3$

$Q = 1.046719$

		surface$_1$		surface$_2$	
r		10		-10	
c		0.1		-0.1	
t			0.2		
n	1.0		1.5		1.0
U	3				
sinU	0.52336				
Q		1.046719			
sinI					
I					
sinI′					
I′					
U′					
sinU′					
Q′					

Exact Ray Tracing Table

Parameter	Object space	$Surface_1$	Lens space	$Surface_2$	Image space
r					
c					
t					
n					
U					
sinU					
Q					
sinI					
I					
sinI′					
I′					
U′					
sinU′					
Q′					
Q_2					

Exact Ray Trace (Givens)

Parameter	Object space	Surface$_1$	Lens space	Surface$_2$	Image space
r		10		-10	
c		0.1		-0.1	
t	20		0.2		BFD = ??
n	1		1.5		1
U	3º				
sinU	0.52336				
Q		1.046719			
sinI					
I					
sinI′					
I′					
U′					
sinU′					
Q′					0
Q_2					

Exact Ray Trace (Calculations)

Parameter	Object space	Surface$_1$	Lens space	Surface$_2$	Image space
r		10		-10	
c		0.1		-0.1	
t	20		0.2		BFD = ??
n	1		1.5		1
U	3º				
sinU	0.052336				
Q		1.046719			
sinI	0.157008				
I	9.033258º				
sinI′			0.104672		
I′			6.00826º		
U′			-0.025001		
sinU′			-0.000436		
Q′		1.04235			
Q_2				1.050995	

Exact Ray Trace (Results)

Parameter	Object space	$Surface_1$	Lens space	$Surface_2$	Image space
r		1.0		-10	
c		0.1		-0.1	
t	20		0.2		**19.506692**
n	1		1.5		1
U	3º		√		
sinU	0.52336		√		
Q		1.046719		√	
sinI	0.157008				-0.1055388
I	9.033258º				-0.1057327
sinI′			0.104672		-0.1583037
I′			6.00826º		-0.1589725
U′			-0.025001		-0.0536761
sinU′			-0.000436		-0.0536503
Q′		1.04235			1.04653436
Q_2				1.05099523	

$$BFD = t = \frac{Q'}{\sin U'} = \frac{1.04653436}{-0.0536503}$$

$$BFD = 19.506692$$

Aberrations of the Rotationally Symmetric Optical System

- Paraxial systems are perfect.
- Aberrations describe the deviation of real systems from this perfection.
- An aberrated system will have image locations and magnifications approximately the same as those predicted by the paraxial or Gaussian analysis.
- The paraxial image is often used as a reference for the measurement of aberrations.
- In geometrical optics, the object is considered to be a collection of independently radiating point sources – δelta fn weighted by radiant flux.
- The image is the sum of the images of all of the point sources (independent irradiance patterns). There is no interference.