## HW #1: Due Feb. 2, 2015

**Problem 1 (30 pts):**
Show that the ellipsoidal surface of revolution in Fig. 1 will give perfect image formation with refraction for the parallel beam at the focus *F'* of the ellipse provided that the eccentricity *e* = *c/a* satisfies *e* = n/n'. NO paraxial approximation.

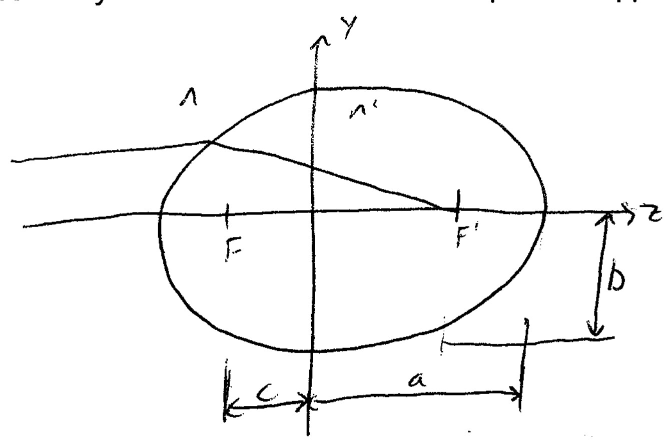



Fig. 1

~~Proof~~: using Fermat's principle states that the OPL on axis is equal to off axis OPL

$$OPL(OF') = OPL(PAF')$$

$$n'(a+c) = n(a-z) + n'(x^2+(z+c)^2)^{1/2} \quad ①$$

From analytic geometry, the equation of the ellipse is

$$\frac{x^2}{b^2} + \frac{z^2}{a^2} = 1$$

parameters a, b, and c are related by $a^2 = b^2 + c^2$

plugging these equations for b and z in ① and simplifying gives

$$1 - \frac{n}{n'} + \frac{c}{a} + \frac{n}{n'a} z = \frac{1}{a}\left[(1-b^2/a^2)z^2 + 2cz + a^2\right]^{1/2}$$

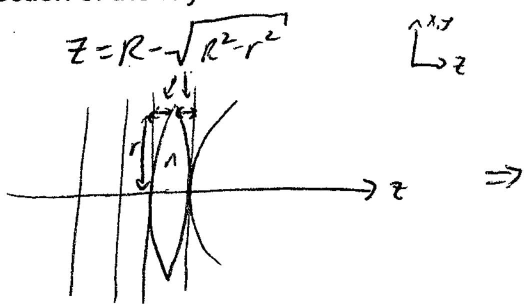
$$\hookrightarrow \; 1 - \frac{n}{n'} + \frac{c}{a} + \frac{n}{n'a} z = \frac{c}{a^2} z + 1$$

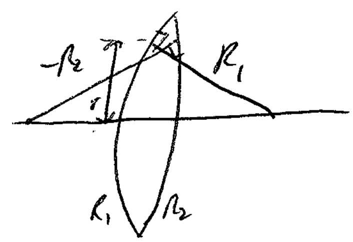
this equation is true for all z since a is arbitrary on ellipse. $z^1$ and $z^0$ coefficients must be equal and is satisfied only if $\frac{n}{n'} = \frac{c}{a}$ [can be solved with other methods]

**Problem 2 (30 pts):**
A plane wave is incident from air onto a bi-convex thin lens having radius |R1|=|R2| =R that has an index of refraction of *n*. Derive an expression for the wave surface after refraction by the thin lens. Express your answer in terms of the radial distance (r) from the optical axis (z) that is defined as the original propagation direction of the plane wave through the point of first contact of a wavefront with the lens.

Hint: Assume that deflection of the ray at the two surfaces are negligible.

$z = R - \sqrt{R^2 - r^2}$



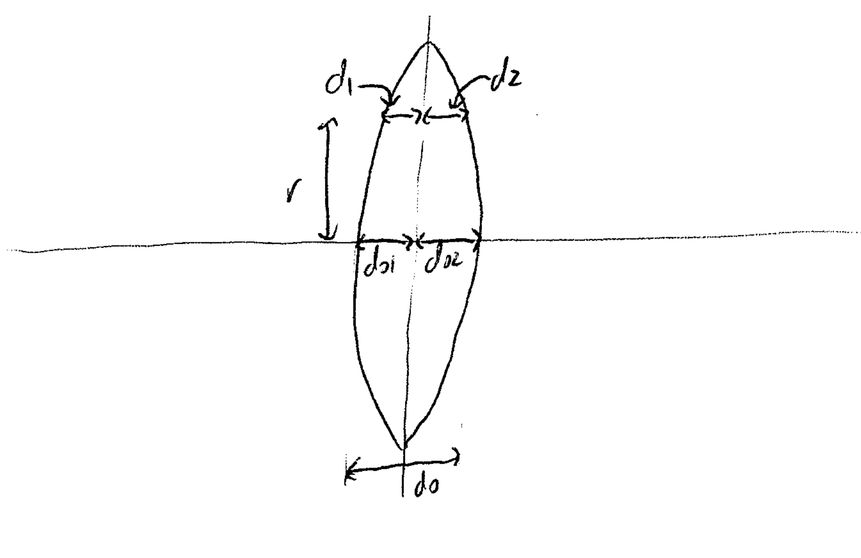

From derivation of Sag

↳ $r^2 + (R1 - z1)^2 = R1^2 \rightarrow z1 = R1 - \sqrt{R1^2 - r^2}$ & $z2 = -R2 - \sqrt{R2^2 - r^2}$

since $|R1| = |R2| = R$ from Biconvex lens

$z1 = R - \sqrt{R^2 - r^2}$ & $z2 = -R - \sqrt{R^2 - r^2}$

Now examine OPL of two rays; must look at thickness of lens



Pb2

using Fermat's principle

$$OPL_{r=0} = n(do1 + do2) = n do$$

at a distance r above optical axis

$$d1(r) = do1 - (R1 - \sqrt{R1^2 - r^2}) = do1 - R1\left(1 - \sqrt{1 - r^2/R1^2}\right)$$

$$d2(r) = do2 - (-R2 - \sqrt{R2^2 - r^2}) = do2 + R2\left(1 - \sqrt{1 - r^2/R2^2}\right)$$

using series approximations due to paraxial ~~other~~ condition

$$\left(1 - \sqrt{1 - R^2/R^2}\right) \approx 1 - \frac{r^2}{2R^2}$$

thus: $d1(r) = do1 - R1\left(1 - \frac{r^2}{2R1^2}\right)$

$$d2(r) = do2 - R2\left(1 - \frac{r^2}{2R2^2}\right)$$

total thickness at r is

$$d(r) = do - \frac{r^2}{2}\left(\frac{1}{R1} - \frac{1}{R2}\right)$$

Equivalent OPL at a distance r from optical axis (using paraxial)

$$OPL_r = n\left(do - \frac{r^2}{2}\left(\frac{1}{R1} - \frac{1}{R2}\right)\right) + \frac{r^2}{2}\left(\frac{1}{R1} - \frac{1}{R2}\right) = n \cdot do$$

we want OPL for the wave surface a distance $z'$

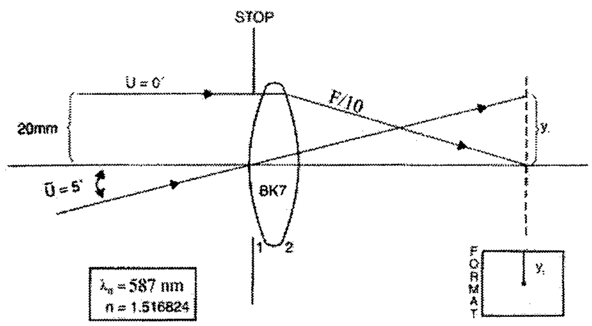
$$OPL_r + z' = OPL_{r=0} \rightarrow z' = (n-1)\frac{r^2}{2}\left(\frac{1}{R1} - \frac{1}{R2}\right)$$

and $|R_1| = |R2| = R$

then $z' = 2(n-1)\frac{r^2}{2R}$

**Problem 3 (18 pts):**
With the information provided in the figure below, find:
a. The effective focal length (EFL),
b. The lens power φ ,
c. Surface curvatures C1 and C1 (assume an equi-convex lens).
d. Radius of curvatures R1 and R2,
e. Format size (assume square), and
f. Airy disk diameter.

Note: The lens can be considered as a thin lens.




ⓐ $f/\# = \frac{EFL}{D_{EP}} \rightarrow EFL = f/\# \cdot 2r = 400\,mm$

ⓑ $\Phi = \frac{1}{EFL} = .0025\ mm^{-1} = 2.5D$

ⓒ thin Lens assumption $\Phi = (n-1)(C_1 - C_2)$

$C_1 = -C_2$ (equiconvex)

$C_1 = \frac{\Phi}{2(n-1)} = \frac{.0025\,mm^{-1}}{2(1.511824-1)} = 2.42\times10^{-3}\ mm^{-1}$

$C_2 = -2.42\times10^{-3}\ mm^{-1}$

ⓓ $R_1 = \frac{1}{C_1} = 413\ mm$ , $R_2 = -413\,mm$

ⓔ $y_1 = efl \cdot \tan\bar{U} = 400 \cdot \tan 5° = 35\,mm$

$2y_1 \times 2y_2 = 70 \times 70$ [mm]

ⓕ $D_{airy} = \frac{2.44\lambda f}{\#w} = (2.44)(.587)(10) = 14.3\,\mu m$

**Problem 4 (9 pts):**
An achromatic doublet consists of a crown glass positive lens of index 1.52 and of thickness 1 cm, cemented to a flint glass negative lens of index 1.62 and of thickness 0.5 cm. All surfaces have a radius of curvature of magnitude 20 cm. If the doublet is to be used in air, determine:

**a.** The system matrix elements for input and output planes adjacent to the lens surfaces;
**b.** The cardinal points;
**c.** The focal length of the combination, using the lensmaker's equation and the equivalent focal length ($f^*$) of two lenses in contact. Compare this calculation of $f^*$, which assumes thin lenses, with the previous value.

(a) $$M_{sys} = \begin{bmatrix} 0.9764 & 9.6755 \\ 0.0009 & 1.0333 \end{bmatrix}$$

(b) $$\overline{V_1 P} = \frac{D - \frac{n_3}{n_1'}}{C} = \frac{(1.0333-1)}{0.000918} = 36.3\,mm$$

$$\overline{V_k P^*} = \frac{1-A}{C} = \frac{(1-0.9764)}{0.000918} = 25.7\,mm$$

$$f = -f^* = \frac{1}{C} = 1089\,mm$$

(c) $$\Phi_1 = \frac{n_1 - 1}{R} = \frac{(1.52-1)}{200} = 2.6\times10^{-3}\,mm^{-1}$$

$$\Phi_2 = \frac{n_2 - n_1}{-R} = \frac{(1.62-1.52)}{-200} = -0.5\times10^{-3}\,mm^{-1}$$

$$\Phi_3 = \frac{1-n_2}{R} = \frac{(1-1.62)}{200} = -3.1\times10^{-3}\,mm^{-1}$$

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 = -0.001\,mm^{-1}$$

$$f^* = \frac{1}{\Phi} = -1000\,mm$$

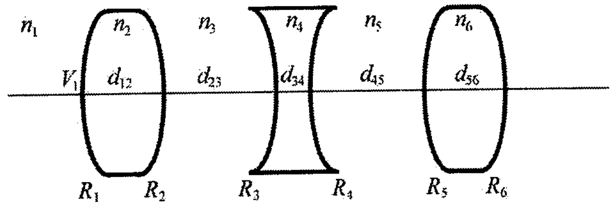
~~**Problem 5 (9 pts):**~~

Problem 5 (9pts)

*(D2 – 10.7)* Using a ray from infinity, ray trace this marginal ray (*y* = 1) through the lens below.

Determine:

**a.** The back focal distance (*BFD*).
**b.** The effective focal length (*efl*).
**c.** The location of the principal planes (*P* and *P**) relative to $V_1$.




$|R_1| = 70$ cm $\quad d_{12} = 5$cm $\quad n_1 = n_3 = n_5 = n_7 = 1.0$
$|R_2| = 70$ cm $\quad d_{23} = 10$cm $\quad n_2 \rightarrow$ N-BK7 (1.517)
$|R_3| = 50$ cm $\quad d_{34} = 2$cm $\quad n_4 \rightarrow$ N-KF9 (1.523)
$|R_4| = 60$ cm $\quad d_{45} = 12$cm $\quad n_6 \rightarrow$ N-BK7 (1.517)
$|R_5| = 60$ cm $\quad d_{56} = 6$cm
$|R_6| = 60$ cm

## Paraxial Ray Tracing Table

Unit: Cm

| Surface | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| R | | | 70 | -70 | -50 | 60 | 60 | -60 | | |
| -φ | | | -0.007386 | -0.007386 | +0.01046 | +0.008717 | -0.008617 | -0.008617 | | |
| $y_{pn}$ | | 1 | 1 | 0.9757 | 0.8297 | 0.82198 | 0.8370 | 0.8134 | | |
| Backward $y_m$ | | | 0.6988 | 0.7246 | 0.7491 | 0.7626 | 0.9659 | 1 | 1 | |

| Space | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| t | | | 5 | 10 | 2 | 12 | 6 | | |
| n | | 1 | 1.517 | 1 | 1.523 | 1 | 1.517 | 1 | |
| t/n | | FFD 53.9 | 3.296 | 10 | 1.313 | 12 | 3.955 | 62.7 | BFD |
| $nu_{pn}$ | | 0 | -0.007386 | -0.01459 | -0.005913 | 0.001252 | -0.005980 | -0.01297 | |
| $nu_m$ | | 0.01297 | 0.007808 | 0.02456 | 0.01029 | 0.01694 | 0.008617 | 0 | |

$u_a = \frac{0.5\,CA_{p(Astop)}}{y_{p(Astop)}}$

(a) $EFL = \frac{y_1}{nu_{last}} = \frac{1}{.01297} = 77.1\text{cm}$

(b) $\overline{V_1P^*} = \sum_{i=2}^{6} t_i + BFD - efl = 5+10+2+12+6+62.7-77.1 = 20.6\text{cm}$

(c) $\overline{V_1P} = EFL - FFD = 77.1\text{cm} - \frac{.6988}{.01297} = 23.22\text{cm}$