Homework #2 OPTI 370 1/21/2015

(due date: 1/28/2015)

Problem 1:

Both, a laser beam and gasoline flowing through a gasoline pipeline carry energy. In the first two problems, compare the order of magnitude for a given set of example parameters.

Assume you have a cylindrical gasoline pipeline (or a hose, such as the one at a gas station) of diameter 2 cm. Through any given cross section (for example the terminal cross section of the hose where the gasoline comes out), we have a flux of 7.5 Gallons/minute (1 Gallon = 3.785×10^{-3} m³). Assuming the density of the gasoline to be 700 kg/m³, and usable (combustion) energy of 50 MJ/kg (megajoule/kg), how much energy comes out through the cross section per second? Give your answer in units of Watts.

Given the known area of the cross section, how many Watts per m² does this correspond to?

(10 points)

Problem 2:

Continuing problem 1, assume now that you have a 1.5 mW (milli Watts) He-Ne laser beam with a beam of circular cross section (radius 1.2 mm). Determine the intensity (often called irradiance) of this laser beam in units of Watts per m² (this is almost trivial; just assume the intensity to be constant over the entire cross section of the beam). Furthermore, determine the hypothetical radius that you would need for a beam of the given intensity (the one you just calculated for the He-Ne laser), so that the power (= intensity integrated over the cross section) would equal that of the gasoline pipe from problem 1. Is that a realistic laser beam radius?

(10 points)

Problem 3:

Consider a monochromatic plane wave

$$u(\mathbf{r},t) = a\cos(2\pi vt - \mathbf{k}\cdot\mathbf{r} + \varphi)$$

with frequency 550 THz, amplitude a=8 (units of W^{1/2}/cm) and $\varphi=0$. Assuming propagation in vacuum, determine the magnitude of the wave vector. Also, write down the complex wave function $U(\mathbf{r},t)$ as function of time and space and determine its magnitude. At $\mathbf{r}=0$, sketch the real-valued

wave $u(\mathbf{r},t)$ over at least three optical cycles. Also, sketch the real part and the imaginary part of the complex wave function over the same time interval. Which of those functions are equal?

(10 points)

Problem 4:

Using the superposition of two monochromatic plane waves propagating in z-direction,

$$u(\mathbf{r},t) = a_1 \cos(2\pi vt - k z + \varphi_1) + a_2 \cos(2\pi vt - k z + \varphi_2)$$

with the same frequency as in the previous problem, plot the intensity as function of z for the following cases:

- (i) both amplitudes equal (as in previous problem), $\varphi_1 = \pi$, $\varphi_2 = \pi/4$;
- (ii) $a_2=a_1/2$ (a_1 given in previous problem), and $\varphi_1=\pi/4$, $\varphi_2=\pi/2$.

(10 points)