OPTI 330 Homework 1 Spring 2015

ASSIGNMENT DATE: January 26, 2015
DUE DATE: February 2 (Groundhog day), 2015 (by 5pm)
LATE DUE DATE: February 3, 2015 (by 5pm)

NOTE: Please drop-off your homework in my mailbox located in the mail-room on 4th floor (East end).

1. Given the complex number $u = Ae^{j\phi}$, show that:
a. Re{u} $= A\ cos(\phi)$
b. Im{u} $= A\ sin(\phi)$ [10 points]

2. Given the complex number $u = v + jw$, show that:
a. Re{u} $= \frac{1}{2}$(u+u*)
b. Im{u} $= \frac{1}{2j}$(u-u*) [10 points]

3. Using the result in problem 2 and Euler's identity, show that:
a. $cos(2\pi\xi_o x) = \frac{e^{j2\pi\xi_o x}+e^{-j2\pi\xi_o x}}{2}$
a. $sin(2\pi\xi_o x) = \frac{e^{j2\pi\xi_o x}-e^{-j2\pi\xi_o x}}{2j}$ [10 points]

4. Find all of the roots of the following equations, and show the locations of these roots in the complex plane.
a. $x^4 = 1$
b. $x^3 = 8e^{j\pi}$

[20 points]

5. Let $u(x) = A\ exp\{j2\pi\xi_o x\}$, where A and ξ_o are real positive constants. Find, and sketch as functions of x, the following:
a. $u(x)u^*(x)$
b. $u(x) + u^*(x)$
c. $|u(x) + u^*(x)|^2$

[20 points]

6. Consider two plane waves: $u_1(\vec{r}) = e^{j\vec{k}_1\cdot\vec{r}}$ in direction $\vec{k}_1$ and $u_2(\vec{r}) = e^{j\vec{k}_2\cdot\vec{r}}$ in direction $\vec{k}_2$. The interference between these plane waves produces a fringe pattern in the observed intensity. Simplify the intensity $I(\vec{r}) = |u_1(\vec{r}) + u_2(\vec{r})|^2$ to express the fringe direction $\vec{K}$ in terms of $\vec{k}_1$ and $\vec{k}_2$.

[30 points]