ASTR/OPTI 428/528

Lecture 10: Spatial, Angular, and Temporal Scales

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Spatial scales

Although we will usually use Fried's length r_0, defined as the diameter of a circular aperture of size with an rms wavefront aberration of r_0, we could also talk about the average distance between two points with an rms phase difference of 1 rad.

$$D_\phi(\ell_0) = \left\langle (\phi_1 - \phi_2)^2 \right\rangle = 1\,\mathrm{rad}^2.$$

Since the MCF for a phase-only field (i.e. no scintillation) is

$$\Gamma(\mathbf{s}) = \langle \psi(\mathbf{x}+\mathbf{s})\psi^*(\mathbf{x})\rangle = \Gamma(0)\mathrm{e}^{-D_\phi(\mathbf{s})/2}$$

then our definition is

$$\Gamma(\ell_0)/\Gamma(0) = \mathrm{e}^{-1/2} \approx 0.6.$$

Another perfectly reasonable definition might be

$$\Gamma(\ell_0)/\Gamma(0) = 1/2.$$

Spatial Scales

Compare this to the MCF at Fried's length using $D_\phi(r) = 6.88(r/r_0)^{5/3}$:

$$\Gamma(r_0)/\Gamma(0) = e^{-6.88/2} \approx 0.032.$$

We can compare ℓ_0 and r_0 by using

$$D_\phi(\ell_0) = 1\,\text{rad}^2 = 6.88(\ell_0/r_0)^{5/3}.$$

$$\ell_0/r_0 = 6.88^{-3/5} = 0.314$$

Or, the Fried length is about 3 times bigger than the simple linear coherence length.

Taylor's Hypothesis

Consider a field that has passed through a moving phase screen.

$$\phi(\mathbf{x}, t) = \phi(\mathbf{x} - \mathbf{v}t)$$

Assuming that the turbulence is advected (i.e. *translated*) by the wind is called the "Taylor Hypothesis."
If we just consider the resulting field after passing through the screen (i.e. no propagation or scintillation), we get

$$\psi(\mathbf{x}, t) = \mathrm{e}^{i\phi(\mathbf{x} - \mathbf{v}t)}.$$

If we compare the field coherence at a single point but at two times, we can write

$$\Gamma(t_1, t_2) = \langle \psi(\mathbf{x}, t_1) \psi^*(\mathbf{x}, t_2) \rangle = \left\langle \mathrm{e}^{i\phi(\mathbf{x} - \mathbf{v}t_1) - i\phi(\mathbf{x} - \mathbf{v}t_2)} \right\rangle$$

Coherence over Time

Making the usual assumptions (Gaussian phase, stationary stats, isotropic, no scintillation)

$$\Gamma(t_1,t_2) = \langle \psi(\mathbf{x},t_1)\psi^*(\mathbf{x},t_2)\rangle = \left\langle e^{iD_\phi(\mathbf{v}(t_1-t_2))/2} \right\rangle$$

If we call the time difference between the measurements $\tau = t_1 - t_2$, we can write

$$\Gamma(\tau) = e^{-D_\phi(\mathbf{v}\tau)/2}.$$

Coherence Time Scale τ_0

Since the wind moving the field past our sensor is much like the linear coherence problem, we can define a similar coherence time τ_0 where the phase has wandered off from the starting phase by 1 rad rms

$$\Gamma(\tau_0) = \Gamma(0)e^{-1/2}.$$

This is the same as saying

$$D_\phi(v\tau_0) = 1$$

or

$$v\tau_0/r_0 = 6.88^{-3/5} = 0.314$$

$$\tau_0 = 0.314 r_0/v$$

Greenwood Time Delay and Greenwood Frequency

$$\tau_0 = 0.314 r_0 / v$$

This is sometimes called the *Greenwood time delay*.
It is the inverse of the *Greenwood Frequency* (we will learn more in adaptive optics).

$$f_g = 1/\tau_0 = 3.18 v / r_0$$

It is an important time scale for adaptive optics, because it tells us how fast a correcting element must move in order to correct the changing wavefront.

Example

Suppose $\lambda = 632$ nm, $r_0 = 15$ cm, and wind $v = 15$ m/s (33.5 mi/hr).
Then

$$\tau_0 = 0.314(0.15\,\mathrm{m})/(15\,\mathrm{m/s}) = 3.14\,\mathrm{ms}.$$

Isoplanatic Patch

If we make the crude statement that the wavefront cannot be distinguished from a plane wave if the rms deviation from the plane wave does not differ by more than 1 radian, then we can think of r_0 as the size of what is effectively a "plane wave patch." That is, two rays pass through the same planer patch ("iso-planatic") if they are separated by less than r_0.
If we are standing off from a phase screen by a distance z, the isoplanatic patch subtends an angle of size (in radians)

$$\theta_0 = r_0/z.$$

Isoplanatic Patch

Example: Suppose our phase screen is 1 km away and r_0 is 15 cm. Then rays appear to have passed through the same isoplanatic patch if they are closer than

$$\theta_0 = 15\,\mathrm{cm}/1000\,\mathrm{m} = 1.5 \times 10^{-4}\,\mathrm{rad} \approx 31\,\mathrm{arcsec}$$

Scattering Angle

The isoplanatic patch is an angle, but may not be directly apparent. What will be more apparent is the angular extent of the scattered light.

Just as the angular diffraction width of a telescope's PSF is

$$\theta_d = \lambda/D,$$

the angular size of the "scattering disk" (the "speckle halo") is

$$\theta_s = \lambda/r_0.$$

Example: For $\lambda = 500$ nm and $r_0 = 15$ cm,

$$\theta_s = 5 \times 10^{-4}/0.15 \approx 0.7\,\text{arcsec}.$$

Note that this is much smaller than our isoplanatic patch size.

Isoplanatic Patch

Remember that $r_0 = r_0(\lambda) \propto \lambda^{6/5}$ *is a function of wavelength. Use the correct* r_0 *value for the observed wavelength.*
Example: For $r_0 = 15$ cm at $\lambda = 500$ nm, what is r_0 and θ_s at $\lambda = 5$ microns?

$$r_0(5\,\text{microns}) = r_0(500\,\text{nm})(5000\,\text{nm}/500\,\text{nm})^{6/5} = 0.15(15.85) = 2.38\,\text{m}$$

$$\theta_s = 206265(5\,\text{microns})/2.38\,\text{m} = 0.43\,\text{arcsec}$$