OPTI 330 Homework 3 Spring 2015

ASSIGNMENT DUE DATE: February 24, 2015 (by 5pm)
LATE DUE DATE: February 25, 2015 (by 1pm in class)

We will use the following definitions in this homework:
Convolution: $f(x) \otimes g(x) = \int f(\alpha) g(x-\alpha) d\alpha$
Cross-correlation: $f(x) \star g(x) = \int f(\alpha) g^*(\alpha - x) d\alpha$

1. Consider a system defined by the following operator:

$$g(y) = \mathcal{S}\ \{f(x)\} = \int_{-\infty}^{+\infty} f(\alpha) \mathrm{rect}(y-\alpha) d\alpha$$

a. Is the system linear? Is it shift-invariant? Show your proof.
b. Calculate and sketch the output $g(y)$ for $f(x) = \mathrm{rect}\,(x)$.
c. Calculate the output $g(y)$ for $f(x) = \mathrm{rect}\,(x+2)$.

[40 points]

2. Assume the system is characterized by the following operator:

$$\mathcal{S}\{f(x)\} = \left[a\left(\frac{d^2}{dx^2}\right) + b\right] f(x)$$

where a and b are arbitrary constants.
a. Is the system linear ? Shift invariant? Show your proof.
b. Calculate and sketch the output for $a = (2\pi)^{-1}$, $b = 0$, and $f(x) = \mathrm{Gaus}(x)$.

[30 points]

3. If the system operator $\mathcal{S}$ is *linear* and *shift-invariant* and we know that $\mathcal{S}\{f_1(x)\} = g_1(x)$ and $\mathcal{S}\{f_2(x)\} = g_2(x)$ then what would be the system output for the input:
a. $f(x) = \alpha \cdot f_1(x) + \beta \cdot f_2(x - x_o)$? Show your steps and reasoning.
b. $f(x) = \delta(x)$? Is there a name of this particular output?
c. If you know the output from (b) how would you use it to express the system output for a general input function $k(x)$ i.e. $\mathcal{S}\{k(x)\} =?$

[30 points]

4. For an arbitrary function$f(x)$ and b, x_o, and x_1 real constants, do the following convolutions:
a. $\delta(bx + x_o) \otimes f(x)$
b. $\delta(x - x_o) \otimes f(x + x_1)$
c. $[\delta\,(x - x_o) - \delta\,(x + x_o)] \otimes f\,(x)$
d. $\delta^{(1)}(x) \otimes f(x)$

[45 points]

5. Perform the following convolutions and sketch the output:
a. $\mathrm{rect}\,(x) \otimes \mathrm{rect}\left(\frac{x}{2}\right)$
b. $\mathrm{rect}\,(x) \otimes \mathrm{tri}\,(x - 1)$
c. $\mathrm{rect}(x) \otimes [\delta(x - 2) + \delta(x + 2)]$

[45 points]

6. Sketch each of the following functions, then compute and sketch the auto-correlation (i.e. $f(x) \star f(x)$) of each:
a. $f(x) = \mathrm{tri}(x)$
b. $f(x) = \mathrm{ramp}\,(x) \cdot \mathrm{rect}(x)$

[30 points]

7. Show that if $\gamma_{ff}(x) = f(x) \star f(x)$ and $f(x)$ is real and even function, then : $\gamma_{ff}(x) = \gamma_{ff}(-x)$ and $\gamma_{ff}(x) = \gamma^*_{ff}(x)$

[10 points]