Homework #5
OPTI 370
2/11/2015
(due date: 2/18/2015)

Problem 1:

Consider a Fabry-Perot resonator with a mirror separation of $d = 11.5\text{cm}$ and mirror reflectivities $\mathscr{R}_1 = 1,\ \mathscr{R}_2 = 0.982$. Assume the resonator is filled with a medium that has an absorption coefficient of $\alpha_s = 0.0185\text{cm}^{-1}$ and refractive index $n(\nu) = 1.25$ (here assumed to be independent of frequency). Calculate the total roundtrip loss coefficient (in the book called "distributed loss coefficient"), finesse, mode spacing (free spectral range), and linewidth . Are the lines narrow in the sense that their width is much less than the mode spacing (in other words, is this a good resonator? How large is the ratio of the linewidth over the mode spacing? On which side of the resonator would light come out of the resonator?

(10 points)

Problem 2:

Consider electromagnetic waves in vacuum. Write down the four Maxwell equations, derive the resulting propagation equation

$$\nabla \times \left(\nabla \times \mathfrak{E} \right) = -\frac{1}{c_0^2} \frac{\partial^2 \mathfrak{E}}{\partial t^2}$$

and show that this equation is equivalent to the wave equation

$$\nabla^2 \mathfrak{E} - \frac{1}{c_0^2} \frac{\partial^2 \mathfrak{E}}{\partial t^2} = 0$$

which was the starting point in OPTI 370.

Hint: Note that ∇ is a vector operator and use the vector identity $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$.

Problem 3:

Solve Exercise 10.2-1 on p. 381 of the book.

(10 points)

Problem 4:

Revisiting our original definition of irradiance (intensity) in terms of the real-valued wave $u(\vec{r},t)$, show that for a monochromatic plane wave at $z = 0$, the time-average of $u^2(0,t)$ over one optical cycle, $\langle u^2 \rangle$, eliminates the fast (2ω) oscillation and yields, for the definition of I used in class, $I = a^2$.

(10 points)