HW #3: Due Feb. 23, 2015

Problem 1 (5pt):

Which are valid wavefront coefficients for 10th order wavefront aberration of a rotational symmetric optical system?

a. W_{555}	**b.** W_{551}	**c.** W_{372}	**d.** W_{193}	**e.** W_{464}
f. W_{642}	**g.** W_{730}	**h.** W_{822}	**i.** W_{821}	**j.** W_{911}

$$W = W_{klm} H^{2j} \rho^{2n} (H\rho \cos(\theta))^m$$

$$k = 2j + m, \qquad l = 2n + m, \qquad j = \frac{k-m}{2}$$

$$order = 2 \cdot (m + n + j)$$

a. W_{555}

$$k = l = m = 5, \qquad so:$$
$$j = 0, n = 0$$
$$order = 2 \cdot (0 + 0 + 0) = 10 \surd\ valid$$

b. W_{551}

$$k = l = 5, \qquad m = 1\ \ so:$$
$$j = 2, \qquad n = 2$$
$$order = 2 \cdot (1 + 2 + 2) = 10 \surd\ valid$$

c. W_{372}

$$k = 3\ \ l = 7\ \ m = 2\ \ so:$$
$$j = 1/2, \qquad n = 5/2$$
$$X\ not\ \ valid\ (no\ fractions)$$

d. W_{193}

$$k = 1\ \ l = 9\ \ m = 3\ \ so:$$

$$j = -2, \qquad n = 3$$
$$X\ not\ valid\ (negative\ value)$$

e. W_{464}

$$k = 4\ \ l = 6\ \ m = 4\ \ so:$$

$$j = 0, \qquad n = 1$$
$$order = 2 \cdot (0 + 1 + 4) = 10 \surd\ valid$$

f. W_{642}

$$k = 6\ \ l = 4\ \ m = 2\ \ so:$$

$$j = 2, \qquad n = 1$$
$$order = 2 \cdot (2 + 1 + 2) = 10 \surd\ valid$$

g. W_{730}

$$k = 7\ \ l = 3\ \ m = 0\ \ so:$$

$$j = 7/2, \qquad n = 3/2$$
$$X\ not\ valid\ (fractions)$$

h. W_{822}

$$k = 8 \quad l = 2 \quad m = 2 \;\; so:$$

$$j = 3, \qquad n = 0$$
$$order = 2 \cdot (3 + 0 + 2) = 10 \surd \;\; valid$$

i. W_{821}

$$k = 8 \quad l = 2 \quad m = 1 \;\; so:$$

$$j = 7/2, \qquad n = 1/2$$
$$X \; not \;\; valid \; (fractions)$$

j. W_{911}

$$k = 9 \quad l = 1 \quad m = 1 \;\; so:$$

$$j = 4, \qquad n = 0$$
$$order = 2 \cdot (4 + 0 + 1) = 10 \surd \;\; valid$$

Problem 2 (15pts):

Assume an F/3 system is used for a wavelength of 0.5 μm, with a wavefront defined as W_{351} = 2 μm.

a. Write the expression for wavefront aberration.

$$W = W_{351} H^3 \rho^5 \cos\theta = W_{351} \cdot H^3 \cdot y_p \cdot ({x_p}^2 + {y_p}^2)^2$$

b. Derive the expression for tangential ray aberration (ε_y).

$$Set \; x_p = 0 \;\; and \; take \; the \; derivative \; of \; the \; wavefront \; with \; respect \; to \; y_p:$$

Multiply by -2*F/#, and:

$$\varepsilon_y = -10 \cdot H^3 \cdot F/\# \cdot W_{351} \cdot {y_p}^4$$

c. What is the value of ray aberration at the edge of the field, and at y_p=1?

$$\varepsilon_y = -10 \cdot 1^3 \cdot 3 \cdot 2 \cdot 1 = -60 \; \mu m = -120 \; waves$$

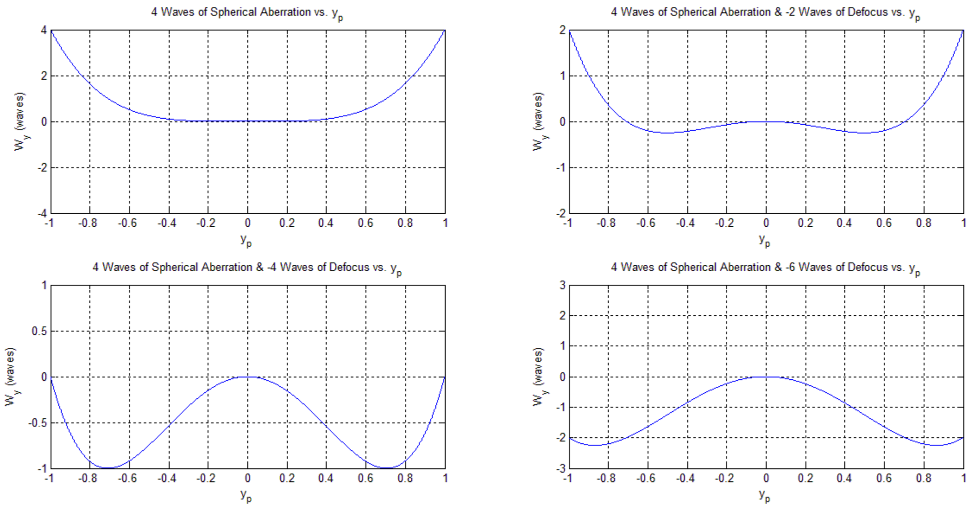
Problem 3 (40 pts):

Assume an optical system having a spherical aberration W_{040} is used for a wavelength λ. Also, assume defocus W_{020} exists.

a. Write the general expression for wavefront aberration.

$$W = W_{040}\rho^4 + W_{020}\rho^2$$

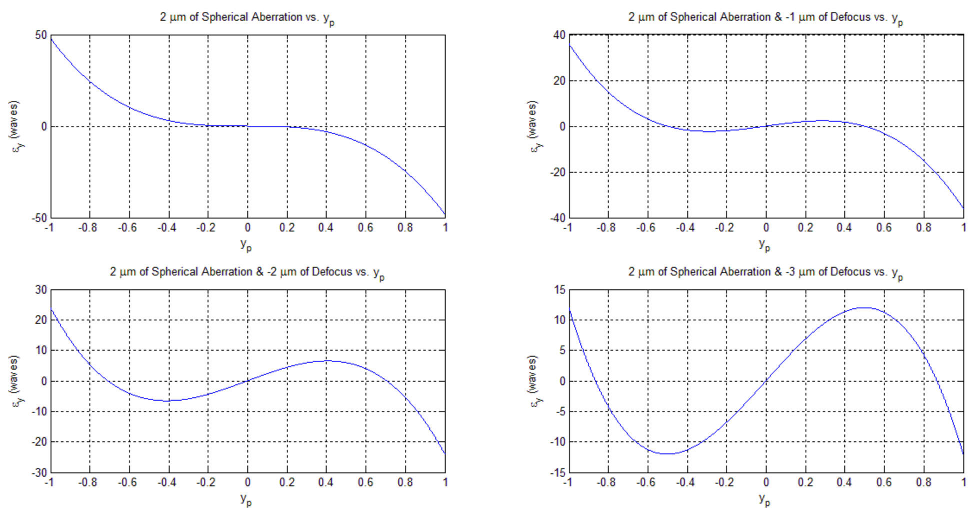
b. Plot wavefront aberration for (F/#)=3, λ = 0.5 μm, W_{040} = 2 μm and W_{020} = 0, -1, -2, and -3 μm.



c. Derive the expression for tangential ray aberration (εy).

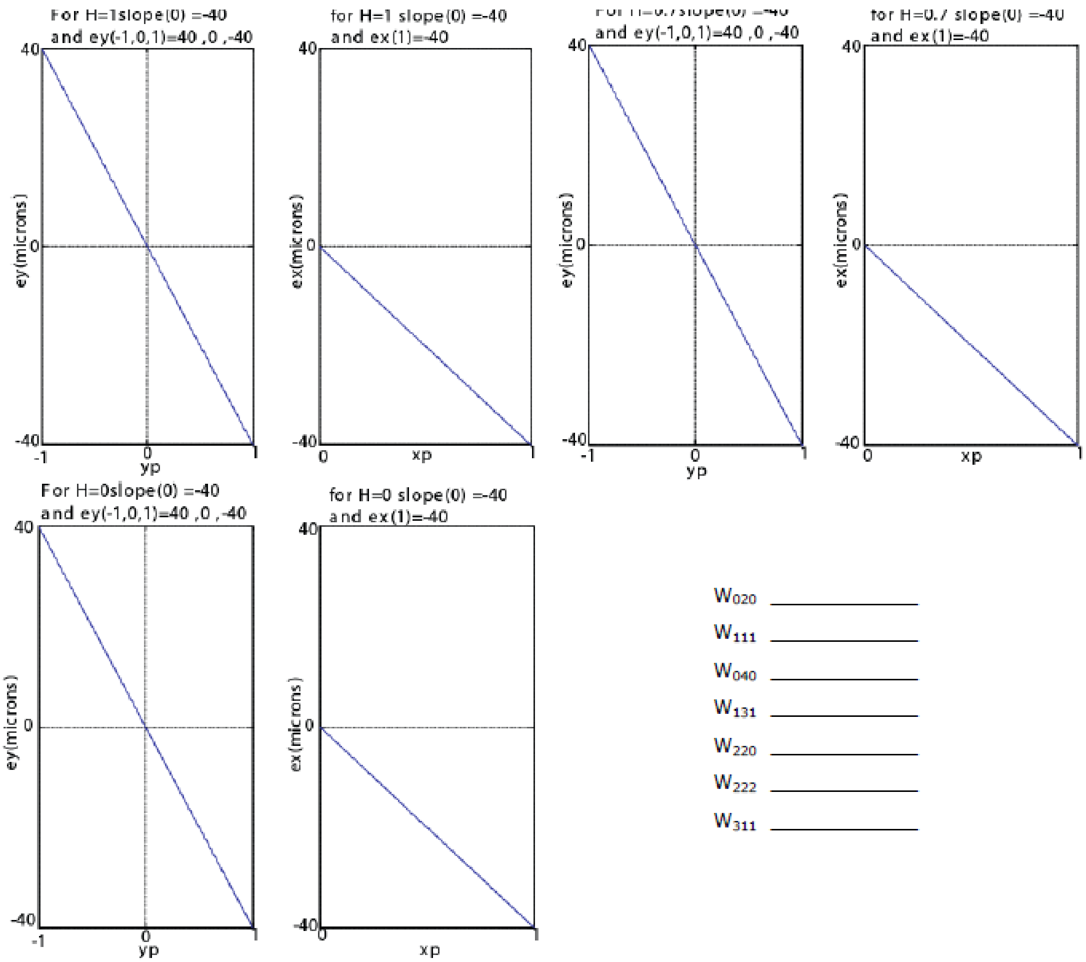
$$\varepsilon_y = -2 \cdot F/\# \cdot (4 \cdot W_{040} \cdot {y_p}^3 + 2 \cdot W_{020} \cdot y_p)$$

d. Plot εy for (F/#)=3, λ = 0.5 μm, W_{040} = 2 μm and W_{020} = 0, -1, -2, and -3 μm.



Problem 4 (20 pts):
For an F/10, λ = 0.5 μm wavelength, the following ray fan plots were obtained of various field angles.

Among the aberrations listed below, what is the wavefront coefficient present and what is its value?



W_{020} ____________

W_{111} ____________

W_{040} ____________

W_{131} ____________

W_{220} ____________

W_{222} ____________

W_{311} ____________

The same aberration amount stays constant at all fields, so there are no field dependent aberrations:

$$W_{111}, W_{222}, W_{220}, W_{311}, W_{131}$$

Out of the field independent aberrations, W_{040} , W_{020}, the only one that could be present is W_{020} because it is linearly dependent as a ray aberration.

$$\varepsilon_y(1) = -2 \cdot F/\# \cdot (2 \cdot W_{020} \cdot y_p) = -4 \cdot F/\# \cdot W_{020} = -40 \rightarrow W_{020} = 1\ \mu m$$
$$= 2 waves$$

Problem 5 (20 pts):

According to Load Rayleigh, wave aberration of $\lambda/4$ in peak-to-valley has a negligible effect on imaging quality (Rayleigh criteria). Using the criteria, obtain tolerance of defocus (maximum allowable amount of defocus) +-dz for a perfect optical system. Assume $\lambda = 0.5$ μm, and (F/#) = 10.

$$\delta z = -8 \cdot F/\#^2 \cdot W_{020} = -8 \cdot 10^2 \cdot \frac{\lambda}{4} \rightarrow \pm 200 \cdot \lambda = \pm 100\ \mu m$$