OPTI 330 Homework 4 Spring 2015

ASSIGNMENT DATE: February 26, 2014
DUE DATE: March 9, 2014 (due in class)

NOTE: This homework has no late due date.

We will use the following definition in this homework:
Convolution: $f(x) \otimes g(x) = \int f(\alpha)g(x-\alpha)d\alpha$
Cross-correlation: $f(x) \star g(x) = \int f(\alpha)g^*(\alpha - x)d\alpha$

1. Compute the Fourier series coefficients (i.e. c_n) for the following periodic functions (i.e. $f(x) = f(x+T)$) and plot the coefficients as a function of n. Note that the functions are defined over one period: $x \in [-T/2, T/2]$ where T is the period length.
a. $f(x) = rect(x)$, $T = 2$
b. $f(x) = x \times rect(x/2)$, $T = 2$

[30 points]

2. Show that:
a. Fourier transform of a *real* and *even* function f(x) is real and symmetric: $F(\xi) = F^*(\xi)$ and $F(\xi) = F(-\xi)$
b. Show that Fourier transform of $g(x) = f(x) \times cos^2(2\pi\xi_o x)$ is given by: $G(\xi) = \frac{1}{4}F(\xi - 2\xi_o) + \frac{1}{2}F(\xi) + \frac{1}{4}F(\xi + 2\xi_o)$, where $F(\xi)$ and $G(\xi)$ are the Fourier transforms of $f(x)$ and $g(x)$ respectively.

[20 points]

3. Compute the Fourier transform of the following functions and sketch them.
a. $f(x) = rect(x)$
b. $f(x) = x \times rect(x/2)$
Note the similarity with Fourier series coefficients in 1(a) and 1(b).

[30 points]

4. Compute and sketch the Fourier transform of the following functions:
a. $f(x) = tri(\beta x)$
b. $f(x) = cos^2(2\pi\xi_o x)$
c. $f(x) = sinc(x) \otimes sinc(x)$
d. $f(x) = \delta(x - x_o) + \delta(x + x_o)$
e. $f(x) = tri(x) \times cos(2\pi\xi_o x)$
f. $f(x) = tri(x) \times comb(x)$
Hint: Use convolution theorem in spatial/Fourier domain as necessary.

[60 points]

5. Consider the superposition of an electric field $e(x)$ and a periodic electric field $cos^2(2\pi\xi_o x)$ with spatial frequency ξ_o. Derive and sketch the Fourier transform of the **intensity** pattern of the superposition of these two fields i.e. $I(x) = |e(x) + cos^2(2\pi\xi_o x)|^2$. For the sketch you may assume an arbitrary shape for $E(\xi)$. Hint: The result in 2(b) may be helpful here.

[30 points]