ASTR/OPTI 428/528 Part 1
Wave Propagation in Random Media
Lecture 5: Imaging Theory

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Interaction: Pupils and Phase Screens

Consider our wave passing through a transmission and phase mask

$$\psi_0(\mathbf{x}) = \psi(\mathbf{x}, z=0) \longrightarrow \Pi(\mathbf{x})e^{i\varphi(\mathbf{x})} \longrightarrow \psi(\mathbf{x}, z=\epsilon)$$

Write this by multiplying the complex amplitude by the mask phasors

$$\psi(\mathbf{x}, z=\epsilon) = \Pi(\mathbf{x})e^{i\varphi(\mathbf{x})}\psi_0(\mathbf{x})$$

We will generically refer to the “size” of a pupil as D.
The characteristic diffraction angle is

$$\theta_d = \lambda/D$$

Fresnel Scale and Angle

For simplicity, we will define our reference Fresnel Scale without the $\sqrt{2}$, or simply

$$R_f = \sqrt{\lambda z}$$

From the distance z, this subtends an angle of

$$\theta_f = R_f/z = \sqrt{\lambda/z}$$

When the distance from a pupil edge or feature is large compared with R_f, we can pretend the edge is not there without greatly changing the result. With a pupil of size D, when $R_f > D$, we cannot ignore the pupil and it is very important.

Scale Ratios

Compare the Fresnel Scale to the pupil size:

$$R_f/D = \frac{\sqrt{\lambda z}}{D}$$

$$R_f/D = \frac{\sqrt{\lambda z}/\lambda}{D/\lambda} = \frac{\sqrt{z/\lambda}}{D/\lambda}$$

Therefore

$$\frac{R_f}{D} = \frac{\theta_d}{\theta_f}$$

Fourier Optics

Starting with a field in the $z = 0$ plane, consider what happens in to the field as $z \to \infty$. Since the distant field will expand linearly with range, define an appropriate scaled coordinate $\mathbf{x} = z\boldsymbol{\theta}$ and take the limit...

$$\psi(\mathbf{x}, z) = \lim_{z\to\infty} \frac{1}{i\lambda z} \int \mathrm{d}^2x' \exp\left\{ \frac{-i\pi\left(z\boldsymbol{\theta} - \mathbf{x}'\right)^2}{\lambda z} \right\} \psi_0(\mathbf{x}')$$

$$\psi(\mathbf{x}, z) = \lim_{z\to\infty} \frac{1}{i\lambda z} \int \mathrm{d}^2x' \exp\left\{ \frac{-i\pi}{\lambda z} \left(z^2\theta^2 - 2z\boldsymbol{\theta}\cdot\mathbf{x}' + \mathbf{x}'\cdot\mathbf{x}'\right) \right\} \psi_0(\mathbf{x}')$$

$$\psi(\mathbf{x}, z) = \lim_{z\to\infty} \frac{1}{i\lambda z} \int \mathrm{d}^2x' \underbrace{\exp\left\{ \frac{-i\pi z\theta^2}{\lambda} \right\}}_{spherical\,wave} \underbrace{\exp\left\{ \frac{i2\pi\boldsymbol{\theta}\cdot\mathbf{x}'}{\lambda} \right\}}_{indep\,of\,z} \underbrace{\exp\left\{ \frac{-i\pi}{\lambda z}\mathbf{x}'\cdot\mathbf{x}' \right\}}_{\to 1}$$

$$\psi(\mathbf{x}, z) = \lim_{z\to\infty} \frac{1}{i\lambda z} \mathrm{e}^{-i\pi z\theta^2/\lambda} \int \mathrm{d}^2x' \, \mathrm{e}^{ik\boldsymbol{\theta}\cdot\mathbf{x}'} \psi_0(\mathbf{x}')$$

Typical scales in lensless Fourier optics

$$\psi(\mathbf{x}, z) = \frac{1}{i\lambda z} \mathrm{e}^{-i\pi\theta^2/\theta_f^2} \int \mathrm{d}^2x' \, \mathrm{e}^{ik\boldsymbol{\theta}\cdot\mathbf{x}'} \psi_0(\mathbf{x}')$$

The spherical wave phase pattern varies rapidly for $\theta > \theta_f$.
We care up to angles of tens of $\theta_d = \lambda/D$, so we need to compare θ_f to θ_d

$$\frac{\theta_f}{\theta_d} = \frac{\sqrt{\lambda/z}}{\lambda/D} = \frac{D}{R_f}$$

Therefore we cannot ignore the wavefront phase across the PSF.
If we are only interested in the irradiance, we may not care.
This also happens if you put a lens in the pupil plane and make an image at a finite distance.
Why???

Telecentricity

Because it is not *telecentric*.
To make the wavefront flat in the image plane, you need to place the imaging lens halfway between the pupil and the image plane. This can be worked out using two propagation integrals and is a recommended “homework assignment.”

[Geogebra Telecentricity Demo]

Note

When we just use a Fourier transform without correcting for the spherical wavefront, we are implicitly assuming a telecentric image plane.

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Fourier Imaging

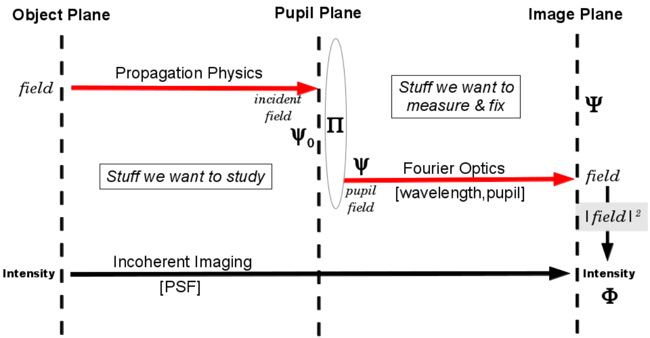
For both telecentric and non-telecentric cases, we can write the irradiance as the magnitude-squared Fourier transform of the pupil field.

This is how we can find the image plane irradiance from an object plane radiance.

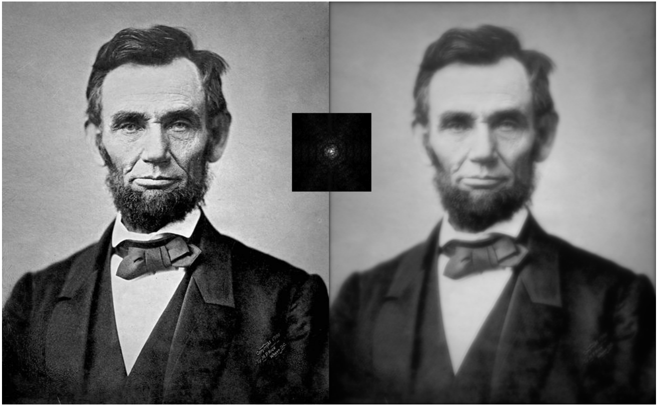
This technique is a reasonable description of a camera, and it will work when light from different parts of the object are coherent with each other. In that case, light from different parts of a coherent object field can interfere, leading to a *speckled* image.

With care, we can also work with partially-coherent and incoherent object fields.

Two Simple Imaging Theories



Incoherent Imaging



Lincoln as seen by a poorly-phased JWST.

Incoherent Imaging

- If the PSF does not change its shape as the point source is moved, it is *translation invariant*.
- This allows a simple relationship between *object plane* and *image plane* intensities:

$$I_{image} = \Phi_{psf} * I_{obj}$$

- *Convolution* is a *product* in the Fourier domain

$$\mathcal{F}\left\{I_{image}\right\} = \underbrace{\mathcal{F}\left\{\Phi_{psf}\right\}}_{OTF} \mathcal{F}\left\{I_{obj}\right\}$$

- We call the incoherent imaging spatial filter the *Optical Transfer Function* (OTF).

The Optical Transfer Function

- The OTF is the Fourier transform of the PSF

$$\mathcal{O}(\boldsymbol{\xi}) = \mathcal{F}\left\{\Phi_{psf}\right\} = \int e^{i\boldsymbol{\kappa}\cdot\boldsymbol{\xi}}\,\Phi(\boldsymbol{\kappa})\mathrm{d}^2\kappa$$

- The OTF is complex but contains no more information than the PSF.
- The *Modulation Transfer Function*: MTF $= |\mathcal{O}(\boldsymbol{\xi})|$
- The *Phase Transfer Function*: PTF $= \arg\{\mathcal{O}(\boldsymbol{\xi})\}$

The PSF|OTF carries the *intensity (irradiance)* from the object to image plane.

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Fourier Optics

- Fourier optics is an extremely simple optical model for an imaging system
- The only specifiable part in the imaging system is the pupil mask: $\Pi(\mathbf{x}) \in \mathbb{C}$.
- Fourier optics relates a general pupil plane field (ψ) to the resulting image plane field (Ψ).

Fourier Optics

- Even simpler... use spatial frequency $\boldsymbol{\kappa}$ and just talk about the *angular spectrum* of plane waves

$$\Psi(\boldsymbol{\kappa}) = \int \mathrm{e}^{i\boldsymbol{\kappa}\cdot\mathbf{x}} \psi(\mathbf{x})\, \mathrm{d}^2 x.$$

The image plane is just a scaled version of the spatial frequency.

- The pupil field is the incident field times the complex pupil mask: $\psi(\mathbf{x}) = \Pi(\mathbf{x})\psi_0(\mathbf{x})$

Image Intensity

- The intensity (*angular power spectrum* of the pupil field)

$$I_{image}(\boldsymbol{\kappa}) = |\Psi(\boldsymbol{\kappa})|^2 = \Psi(\boldsymbol{\kappa})\Psi^*(\boldsymbol{\kappa})$$

- The inverse Fourier transform is easily written in terms of the pupil field

$$\mathcal{F}^{-1}\left\{I_{image}\right\} = \int \psi(\mathbf{x}' + \boldsymbol{\xi}/2)\psi^*(\mathbf{x}' - \boldsymbol{\xi}/2)\mathrm{d}^2x'$$

A general Fourier optics result

This is true for any source illuminating the pupil—not just point sources.

The OTF and the pupil field

If ψ is the field that arises from a point source, then this is the OTF

$$\mathcal{O}(\boldsymbol{\xi}) = \int \psi(\mathbf{x}' + \boldsymbol{\xi}/2)\psi^*(\mathbf{x}' - \boldsymbol{\xi}/2)\mathrm{d}^2x'$$

where

$$\psi(\mathbf{x}) = \Pi(\mathbf{x})\psi_0(\mathbf{x})$$