OPTI/ASTR 528

Introduction to numerical image restoration methods

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But first… team projects

- Presentations will be next week, March 10 and 12.
- Format will be 15 minutes + 5 minutes for thunderous applause and audience questions.
- Each team member will need to present their piece of the work.

Tuesday 3/10

Team	Topic	Time slot
1	Microscopy	11:00 – 11:20
2	Remote sensing (looking down)	11:25 – 11:45
3	Coherence time (we think, but we're not entirely sure…)	11:50 – 12:10

Thursday 3/11

Team	Topic	Time slot
4	Coronography	11:00 – 11:20
5	Free-space communication	11:25 – 11:45
6	Measurements of scintillation	11:50 – 12:10

Scintillation effects on the PSF

We saw last time (in simulation) that phase variations in the wavefront have a much more disastrous effect on the PSF than do amplitude variations.

Recall our expression for the OTF:

$$\mathcal{O}(\boldsymbol{\xi}) = \int \left\langle \alpha(\mathbf{x}' + \boldsymbol{\xi}/2)\alpha(\mathbf{x}' - \boldsymbol{\xi}/2)\mathrm{e}^{i(\varphi(\mathbf{x}'+\boldsymbol{\xi}/2)-\varphi(\mathbf{x}'-\boldsymbol{\xi}/2))} \right\rangle$$

$$\Pi(\mathbf{x}' + \boldsymbol{\xi}/2)\Pi(\mathbf{x}' - \boldsymbol{\xi}/2)\mathrm{d}^2\mathbf{x}'$$

We can rewrite the wave as:

$$\alpha(\mathbf{x})\mathrm{e}^{i\varphi(\mathbf{x})} = \mathrm{e}^{i(\varphi(\mathbf{x})-il(\mathbf{x}))}$$

l is the *log-amplitude* of the wavefront (it’s found to be approximately a zero-mean gaussian random variable)

The wave structure function

Then the OTF becomes

$$\mathcal{O}(\boldsymbol{\xi}) = \mathrm{e}^{-D_u(\boldsymbol{\xi})/2} \int \Pi(\mathbf{x}' + \boldsymbol{\xi}/2)\Pi(\mathbf{x}' - \boldsymbol{\xi}/2)\mathrm{d}^2\mathbf{x}'$$

Here, D_u replaces D_ϕ as the structure function, and it now contains a piece for the wave amplitude:

$$D_u(\boldsymbol{\xi}) = \left\langle \left| \varphi(\mathbf{x}+\boldsymbol{\xi}) - \varphi(\mathbf{x}) + i(\ell(\mathbf{x}+\boldsymbol{\xi}) - \ell(\mathbf{x})) \right|^2 \right\rangle$$

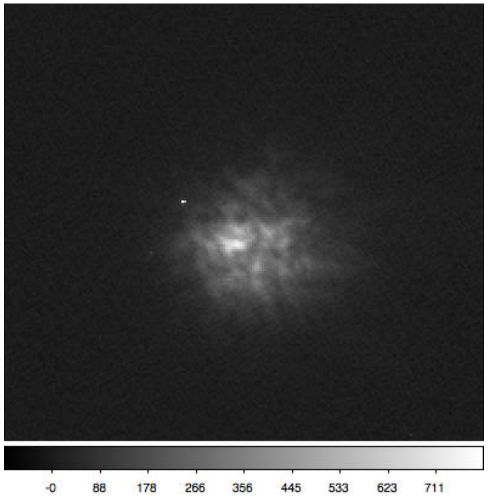
$$= \left\langle \left| \Delta\varphi(\boldsymbol{\xi}) + \Delta\ell(\boldsymbol{\xi}) \right|^2 \right\rangle$$

Now, the *phase structure function* $\left\langle \left| \Delta\varphi(\boldsymbol{\xi}) \right|^2 \right\rangle$ can grow without limit, messing up the PSF all the way.

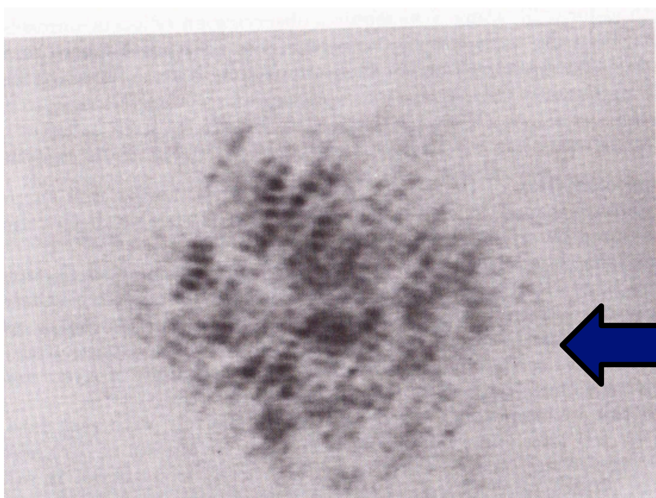
BUT – the growth of the *log-amplitude structure function* $\left\langle \left| \Delta\ell(\boldsymbol{\xi}) \right|^2 \right\rangle$ is restricted because amplitude variations naturally saturate.

So phase variations do more harm to the PSF than amplitude variations

Speckled PSFs



Short(ish) exposure J-band (1.2 μm) image of star SAO135108



Short exposure V-band (0.55 μm) image of Capella from Kitt Peak 4 m

- Recall that the speckles arise from coherent addition of the field across the pupil, so they preserve information at the diffraction limit of the aperture.
- Speckles get washed out over long integration times as the (mostly) phase aberrations shift the areas of constructive and destructive interference.
- *Short exposures* though (i.e. integration time < ~ τ_0) can be analyzed in a variety of ways to extract high resolution images of the object.

Capella is a binary star system (actually a double binary). Each of the speckles appears elongated because it's actually a wee image of both (pairs of) stars. The stars are separated by only 0.04 arc sec, whereas the seeing disk is about 1 arc sec across.

Incoherent imaging equation

Because the sources in the Capella image are not coherent with each other, the image is simply the sum of two PSFs weighted by the brightnesses of the two stars α_1 and α_2:

$$
\begin{aligned}
I(\boldsymbol{\kappa}) &= \alpha_1 \Psi(\boldsymbol{\kappa}) + \alpha_2 \Psi(\boldsymbol{\kappa} + \delta\boldsymbol{\kappa}) \\
&= \Psi(\boldsymbol{\kappa}) \bullet \left[\alpha_1 \delta(\boldsymbol{\kappa}_1) + \alpha_2 \delta(\boldsymbol{\kappa}_1 + \delta\boldsymbol{\kappa}) \right]
\end{aligned}
$$

Convolution

Angular positions of the 2 stars

For a fully resolved source *S*, this generalizes to

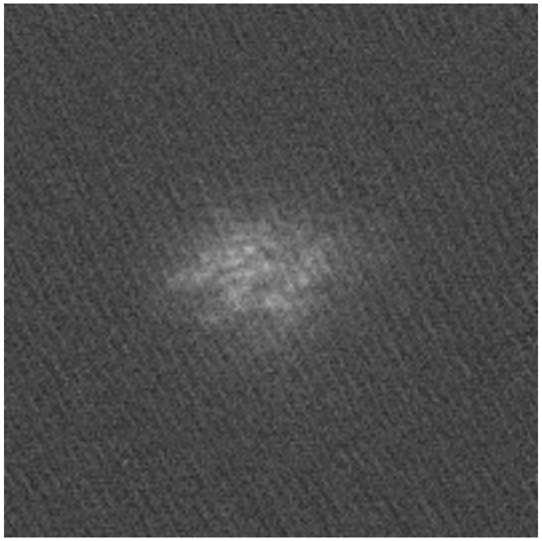
$$
I(\boldsymbol{\kappa}) = \Psi(\boldsymbol{\kappa}) \bullet S(\boldsymbol{\kappa})
$$

This equation is the basis of most numerical methods to improve image quality.

The goal is somehow to unscramble Ψ and *S* given *I*.

Simple shift-and-add

Since each speckle is a diffraction-limited image of the source, we can attempt to make an image by simply stacking lots of short exposures on top of each other, each shifted to center its brightest speckle.

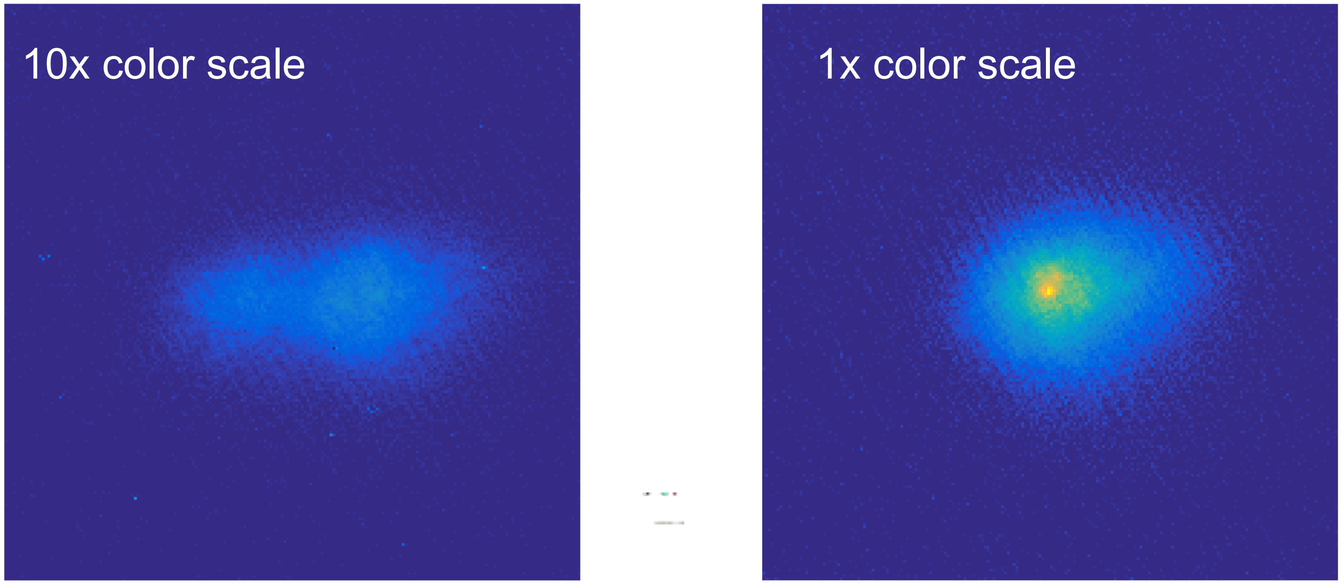


Star data from the MMT a couple of months ago

An excellent reference for early work on this and similar techniques is:

E. K. Hege, (1989) "First Order imaging Methods: An Introduction," Diffraction-Limited Imaging with Very Large Telescopes, (Dordrecht: Kluwer)

Simple shift-and-add



Pros:

- SAA works reasonably well for point sources (i.e. stars or star clusters).
- It's best when the seeing isn't too bad (D/r_0<3 where the number of speckles is low).

Cons:

- Generates a halo with lots of energy – leads to a low-contrast image.
- Very wasteful of information – only uses one speckle per frame.

Speckle interferometry

In 1970, Labyerie realized the key point that speckle structure retains diffraction-limited information and started making measurements of binary stars whose separation was < λ/r_0 (seeing disk size).

You can estimate the *average modulus squared* of the PSF $\left\langle |\Psi|^2 \right\rangle$ from observations of a star.

- Need to assume that the seeing conditions remain the same

Procedure:

- Make a bunch of short exposure observations of the source *S*

$$I(\boldsymbol{\kappa},t)=\Psi(\boldsymbol{\kappa},t)\bullet S(\boldsymbol{\kappa})$$

- Make a bunch more observations of a **PSF calibrator** star

$$I_{\mathrm{star}}(\boldsymbol{\kappa},t')=\Psi(\boldsymbol{\kappa},t')\bullet\delta(\boldsymbol{\kappa}=0)=\Psi(\boldsymbol{\kappa},t')$$

In Fourier transform space:

$$\left\langle |\mathcal{I}|^2 \right\rangle=\left\langle |O|^2 \right\rangle |S|^2 \quad \text{and} \quad \left\langle |\mathcal{I}_{star}|^2 \right\rangle=\left\langle |O|^2 \right\rangle \quad \Rightarrow \quad \left\langle |\mathcal{I}|^2 \right\rangle \Big/ \left\langle |\mathcal{I}_{star}|^2 \right\rangle=|S|^2$$

Speckle interferometry

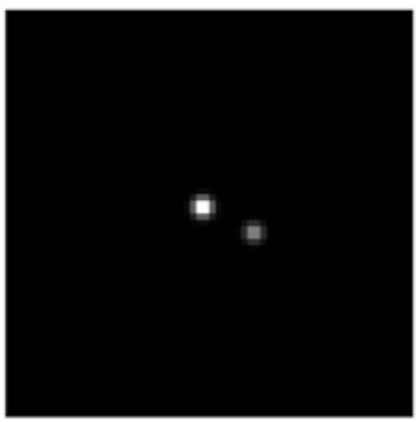
But

$$\mathrm{F}^{-1}\left(\left|\mathcal{S}\right|^2\right) = \mathrm{F}^{-1}\left(\mathcal{S}\mathcal{S}^*\right) = \mathcal{S} * \mathcal{S}$$

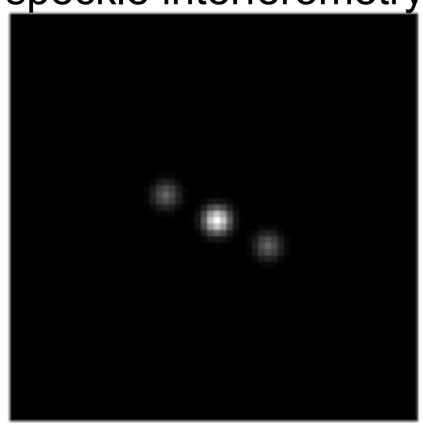
That is, by taking the inverse Fourier transform of the speckle interferometry ratio, one arrives at the *autocorrelation of the object*.

This is not an image, because the phases of F($\mathcal{S}$) have been lost, but a lot can be learned about an object from its autocorrelation (indeed, if the object is a multiple star system, you can usually figure out everything: i.e. relative positions and brightnesses to within symmetry).

Binary star



Autocorrelation given by speckle interferometry



Speckle holography

If the PSF calibrator can be seen in the same set of images as the object, then one can be a little more sophisticated and make a real image, starting from:

$$\mathcal{S} = \mathcal{I} / \mathcal{O}$$

Then multiply top and bottom by $\mathcal{O}^*$ (=$\mathcal{I}_{star}{}^*$) and average over many short exposures:

$$\mathcal{S} = \frac{\left\langle \mathcal{I}\mathcal{O}^* \right\rangle}{\left\langle \left|\mathcal{O}\right|^2 \right\rangle}$$

If the average speckle transfer function $\left\langle \left|\mathcal{O}\right|^2 \right\rangle$ has zeros in it though, the equation for $\mathcal{S}$ blows up in an ugly way.

- This leads to noise amplification

Fudge this using a small *regularization parameter* γ: $\mathcal{S} = \dfrac{\left\langle \mathcal{I}\mathcal{O}^* \right\rangle}{\left\langle \left|\mathcal{O}\right|^2 \right\rangle + \gamma}$

Bispectrum

Another clever technique (invented by Gerd Weigelt, 1977) uses a triple correlation to estimate the phases of the object FT with higher SNR:

$$\left\langle \mathcal{I}(\xi_1)\mathcal{I}(\xi_2)\mathcal{I}^*(\xi_1+\xi_2)\right\rangle = \mathcal{S}(\xi_1)\mathcal{S}(\xi_2)\mathcal{S}^*(\xi_1+\xi_2)\left\langle \mathcal{O}(\xi_1)\mathcal{O}(\xi_2)\mathcal{O}^*(\xi_1+\xi_2)\right\rangle$$

Ensemble averaged over time, $\arg\left\{\left\langle \mathcal{O}(\xi_1)\mathcal{O}(\xi_2)\mathcal{O}^*(\xi_1+\xi_2)\right\rangle\right\} = 0$

So:

$$\mathcal{S}(\xi_1)\mathcal{S}(\xi_2)\mathcal{S}^*(\xi_1+\xi_2) = \left\langle \mathcal{I}(\xi_1)\mathcal{I}(\xi_2)\mathcal{I}^*(\xi_1+\xi_2)\right\rangle \Big/ \left\langle \mathcal{O}(\xi_1)\mathcal{O}(\xi_2)\mathcal{O}^*(\xi_1+\xi_2)\right\rangle$$

$$= \mathcal{C}(\xi_1,\xi_2)$$

Get this from the star observations again

Then one uses recursive phase closure relations of the form

$$\arg\left\{\mathcal{C}(\xi_1,\xi_2)\right\} = \arg\left\{\mathcal{S}(\xi_1)\right\} + \arg\left\{\mathcal{S}(\xi_1)\right\} - \arg\left\{\mathcal{S}(\xi_1+\xi_2)\right\}$$

to calculate the phases of $\mathcal{S}$, assuming (since the absolute phase is arbitrary) that $\mathcal{S}$ (0,0) = 0.

Phase diversity

A different class of technique, put forward by Bob Gonsalves (1979), recognizes that aberrations can be described as a sum of orthonormal basis functions (e.g. for a circular aperture, Zernike polynomials, Seidel, disk harmonics, Karhunen-Loève modes,…)

$$\varphi(\mathbf{x}) = \sum a_i Z_i(\mathbf{x})$$

Odd modes (like coma) make unique patterns in the PSF.

Even modes (like focus) make patterns that are *not quite* unique: you can tell the magnitude, but not the sign.

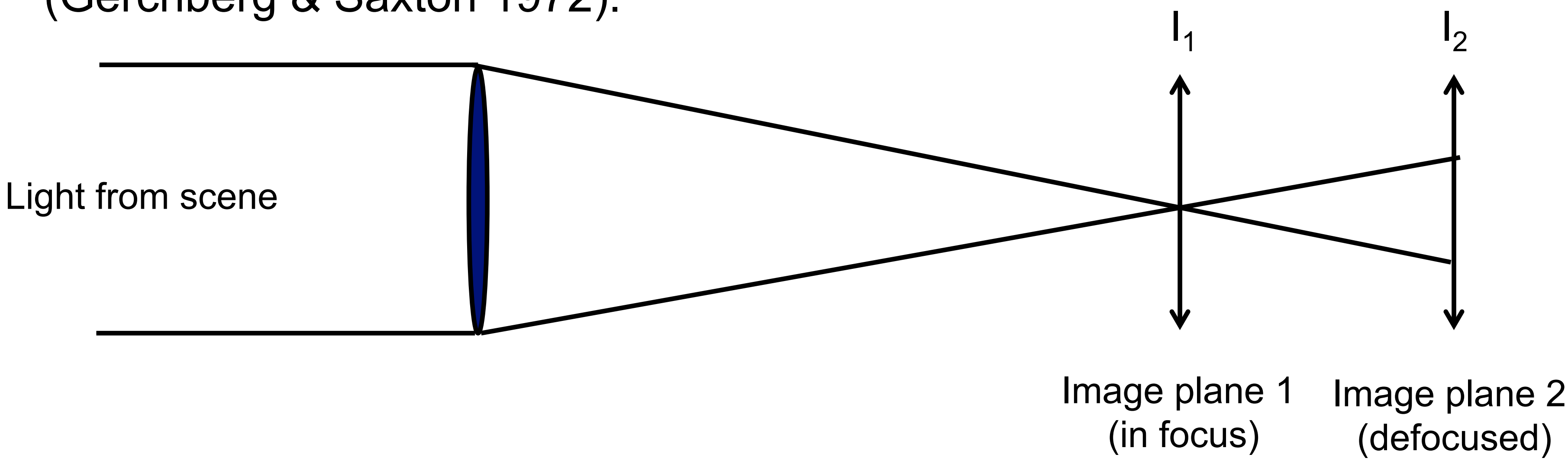
The insight here is that if you take **two** images of the same scene with a known defocus between them, you can break that ambiguity.

– If there's defocus also in the wavefront, its sign will be revealed by whether the blurred PSF gets smaller or larger in the defocused image.

The second image serves the same function as the separate images of stars required by the speckle techniques: it provides another information channel to solve for phases that got lost in image formation.

Gerchberg-Saxton algorithm

It remains to extract the unknown phases at the image plane from the two images. Phase diversity usually relies on the Gerchberg-Saxton algorithm (Gerchberg & Saxton 1972).



Algorithm:

$$\hat{\Psi}_1 = \sqrt{I_1}\, e^{i\hat{\varphi}_1} \quad \text{with} \quad \hat{\varphi}_1 = 0$$

$$\hat{\Psi}_2 = \text{prop}(\hat{\Psi}_1) \quad \text{then} \quad \hat{\Psi}_2 = \sqrt{I_2}\, e^{i\arg(\hat{\Psi}_2)}$$

$$\hat{\Psi}_1 = \text{prop}^{-1}(\hat{\Psi}_2) \quad \text{then} \quad \hat{\Psi}_1 = \sqrt{I_1}\, e^{i\arg(\hat{\Psi}_1)}$$

$$\hat{I}_1 = \left|\hat{\Psi}_1\right|^2 \qquad \hat{I}_2 = \left|\hat{\Psi}_2\right|^2$$

Iterate until image estimates stop changing

Deconvolution from wavefront sensing

Yet another way to get the needed additional information: use a wavefront sensor to directly estimate the pupil-plane phases $\varphi(\mathbf{x})$.

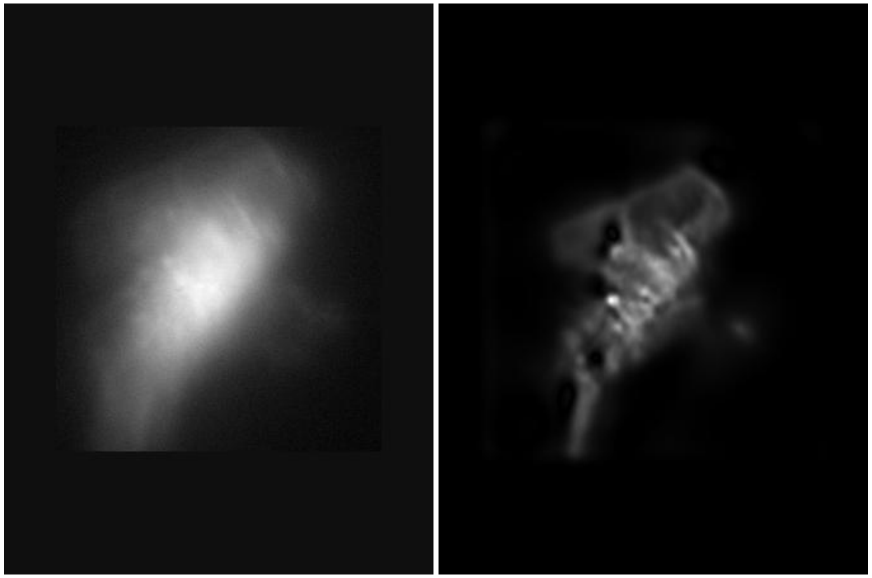
One can then estimate the PSF from (ignoring scintillation as usual)

$$\widehat{\Psi}(\boldsymbol{\kappa}) = \left| \mathrm{F}\left(\mathrm{e}^{-i\varphi(\mathbf{x})}\right) \right|^2$$

Starting from the usual incoherent imaging equation

$$I(\boldsymbol{\kappa}) = \Psi(\boldsymbol{\kappa}) \bullet S(\boldsymbol{\kappa})$$

one can deconvolve the PSF estimate.



Images of the Hubble Space Telescope, taken with another telescope on the ground, before and after deconvolution of the PSF estimated from wavefront sensor measurements.

N.B. To avoid noise amplification, this is generally done with a Wiener filter.