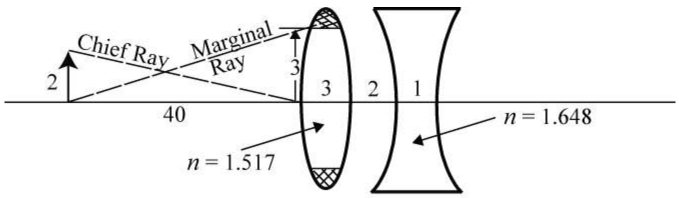
# HW #2: Due Feb. 09, 2015

## Problem 1:

*(D2 – 10.2)* Paraxial ray trace the marginal and chief rays for the lens system below:




$|R_1| = |R_2| = 10, \quad |R_3| = |R_4| = 20$

| Surface | 0 | | 1 | | 2 | | 3 | | 4 | | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| R | | | 10 | | -10 | | -20 | | 20 | | |
| C | | | 0.1 | | -0.1 | | -0.05 | | 0.05 | | |
| t | | 40 | | 3 | | 2 | | 1 | | | |
| n | | 1 | | 1.517 | | 1 | | 1.648 | | 1 | |
| φ | | | −0.0517 | | −0.0517 | | +0.0324 | | +0.0324 | | |
| t/n | | 40 | | 1.977587 | | 2 | | 0.606796 | | ? | |
| $y_p$ | 2 | | 0 | | -0.098879 | | -0.188655 | | -0.219602 | | |
| $nu_p$ | | -0.05 | | -0.05 | | -0.044888 | | -0.051 | | -0.058115 | |
| $\alpha_* = \frac{0.5\,CA_{P(A.S.)}}{Y_{P(A.S.)}}$ | | | | | | | | | | | |
| $y_m$ | 2 | | 2 | | 1.795517 | | 1.403061 | | 1.311575 | | 0 |
| $nu_m$ | | 0 | | -0.1034 | | -0.1962 | | -0.1508 | | -0.1083 | |

**a.** What is the Lagrange invariant?

$$Ж = n\bar{u}y - nu\bar{y}$$

At the object, y=0, y_bar=2, and nu=3/40 (small angle approximation):

$$Ж = -nu\bar{y} = -\frac{3}{40} \cdot 2 = -.15\ cm$$

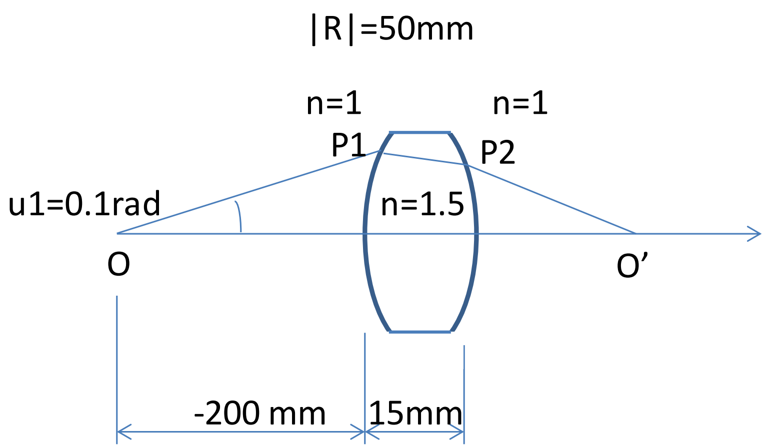
**b.** What is the *BFD*?

Tracing a ray from infinity:

$$y' = y + \frac{t}{n} \cdot nu = 0 = 1.311575 + \frac{t}{n} \cdot (-.1083)$$

$$\frac{t}{n} = BFD = 12.11\ cm$$

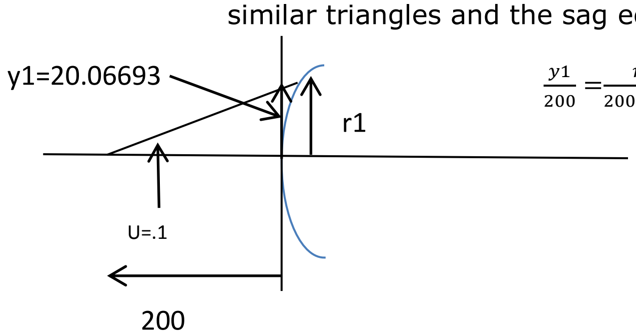
## Problem 2:




a. Carry out exact ray trace for the following optical system. Find ray height at points P1, P2 and O’.

| Parameter | Object Space | Surface 1 | Lens Space | Surface 2 | Image Space | |
|---|---|---|---|---|---|---|
| R | | 50 | | -50 | | |
| C | | | | | | |
| t | 200 | | 15 | | 45.71363246 | **BFD** |
| n | 1 | | 1.5 | | 1 | |
| U | 0.1 | | | | -0.083389263 | |
| sin(U) | 0.099833417 | | | | -0.083292652 | |
| Q | | 19.966683 | | | | |
| sin(I) | 0.499167083 | | | | -0.474375564 | |
| I | 0.522637273 | | | | -0.494254569 | |
| sin(I') | | | 0.332778055 | | -0.711563346 | |
| I' | | | 0.33924801 | | -0.791720727 | |
| U' | | | -0.08338926 | | -0.380855421 | |
| sin(U') | | | -0.08329265 | | -0.371714732 | |
| Q' | | | 20.80353537 | | 16.99243066 | |
| Q2 | | | | 19.554146 | | 0 |
| y | | 20.066934 | | | 18.30396458 | 0 |

The highlighted values are for the height of the incoming ray in a plane tangential to the vertex. To find the height at the surface, we use similar triangles and the sag equation:




$$\frac{y1}{200} = \frac{r1}{200+sag} = \frac{r1}{200+50-\sqrt{50^2-r1^2}}$$

Solving for r, we get r1=20.5084 mm. Applying the same method but for the second surface, yields r2=19.9701 mm.

$$\frac{y2}{BFD} = \frac{r_2}{BFD + sag} \rightarrow r_2 = 19.9701$$

b. Compare the result from (a) with the results from single ray trace by using CodeV. (Tip, use RSI or SIN command)

| | X | Y | Z | TANX | TANY | LENGTH |
|---|---|---|---|---|---|---|
| OBJ | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.10033 | |
| 1 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.10033 | 0.00000 |
| 2 | 0.00000 | 20.06693 | 0.00000 | 0.00000 | 0.10033 | 201.00418 |
| STO | 0.00000 | 20.50835 | 4.39948 | 0.00000 | -0.08358 | 4.42157 |
| 4 | 0.00000 | 19.97014 | -4.16122 | 0.00000 | -0.40040 | 6.46175 |
| 5 | 0.00000 | 18.30396 | 0.00000 | 0.00000 | -0.40040 | 4.48240 |
| 6 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -0.40040 | 49.24196 |
| IMG | 0.00000 | 0.00000 | 0.00000 | 0.00000 | -0.40040 | 0.00000 |

Examining the Y height values, it is clear the results from both methods are nearly identical.

## Problem 3:

A CMOS array has a pixel size of 4μm, with 1024 x 1024 pixels, (assume 100% fill factor) used to record a visible image (400-700 nm wavelength). The objective lens is an F/2, 100 mm efl, used with magnification of −0.5 ($M_T$ = −0.5).

**a.** What is the depth of field, or determine the near and far object distance.

$$F/\# = \frac{f}{D_{EP}} \rightarrow D_{EP} = 50\ mm$$

$$M_T = \frac{L_0'}{L_0} \rightarrow L'_0 = -.5L_0$$

$$\frac{1}{L'_0} = \frac{1}{L_0} + \frac{1}{100}$$

$$L_0 = -300\ mm,\ L'_0 = 150\ mm$$

$$L_{near} = \frac{L_0 \cdot f \cdot D}{f \cdot D - L_0 \cdot B'} = \frac{-300 \cdot 100 \cdot 50}{100 \cdot 50 - (-300).004} = -299.928\ mm$$

$$L_{far} = \frac{L_0 \cdot f \cdot D}{f \cdot D + L_0 \cdot B'} = \frac{-300 \cdot 100 \cdot 50}{100 \cdot 50 + (-300).004} = -300.072\ mm$$

$$\Delta L = L_{near} - L_{far} = -299.928 - (-300.072) = 144\ \mu m$$

**b.** If used for a distance object, what is the hyperfocal distance ($M_T$ = 0)?

$$L_H = -\frac{f \cdot D}{B'} = \frac{-100 \cdot 50}{.004} = -1.25\ km$$

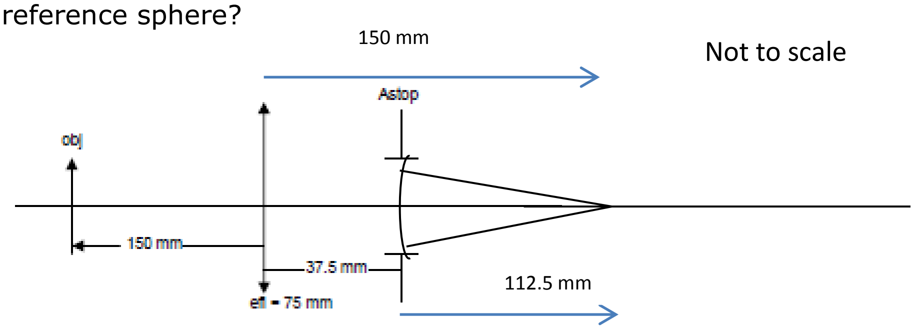
**c.** Where should the CMOS array be placed relative to the lens (BFD) for maximum depth of field ($M_T$=0)?

The hyperfocal distance found in (b) was in object space, so we simply image that distance to find the CMOS array location for maximum depth of field; Since the distance is very large, the location is approximately the focal length:

$$\frac{1}{L'_H} = \frac{1}{L_H} + \frac{1}{100} \rightarrow L'_H = 100.008\ mm$$

## Problem 4:

For the optical system shown in the figure, what is the radius of curvature of the reference sphere?




The aperture stop is located in image space; therefore, it is the exit pupil. The Gaussian reference sphere is the reference wavefront radius of curvature at the exit pupil, so we find the where the spherical wavefront forms an image. The Gaussian reference sphere has its center of curvature at the Gaussian image point (no aberrations).

With z=-150 & f=75:
$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \rightarrow \frac{1}{z'} = \frac{1}{-150} + \frac{1}{75} \rightarrow z' = 150\ mm$

The Gaussian image point from the exit pupil:

$$z' - 37.5 = 150 - 37.5 = 112.5\ mm$$

## Problem 5:

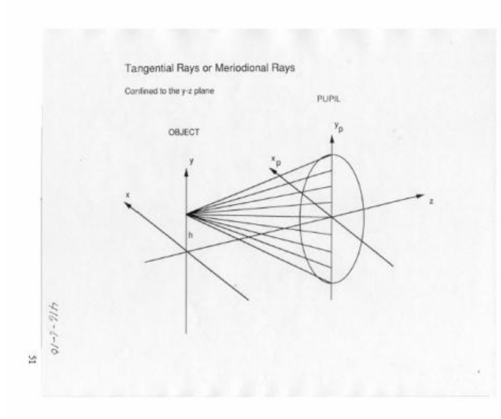
Why is the ray aberration not anti-symmetrical for the tangential plane ray fan for field angles of one (H=1)? Why are the sagittal (skew) ray fans anti-symmetric?

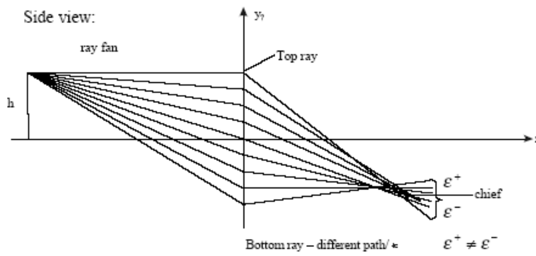
An anti-symmetric function is odd:

$$f(x) = -\mathrm{f}(-\mathrm{x})$$

Note: Not anti-symmetric does not mean that a curve is symmetric.

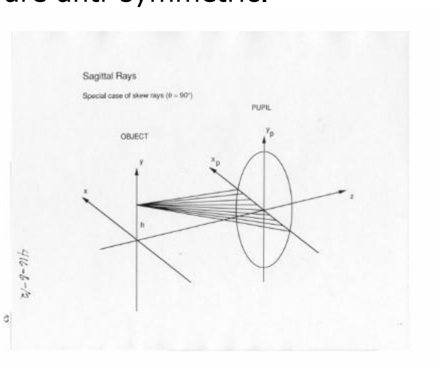
The ray aberration is not anti-symmetrical for the tangential plane ray fan @H=1 because the rays that enter the entrance pupil are not anti-symmetric.

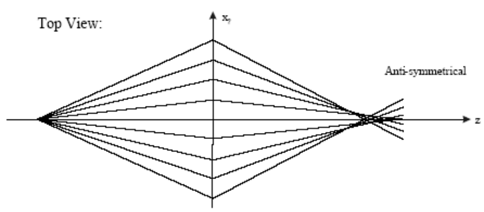



The ray angels are not symmetric (or anti-symmetric) for $\pm y_?$ values

Sagittal Ray fans are anti-symmetric because the rays that enter the entrance pupil are anti-symmetric.




Note the symmetry: The ray deviations are anti-symmetric for $\pm x_?$.
The OPD or Wavefront error must be symmetric.

## Problem 6:

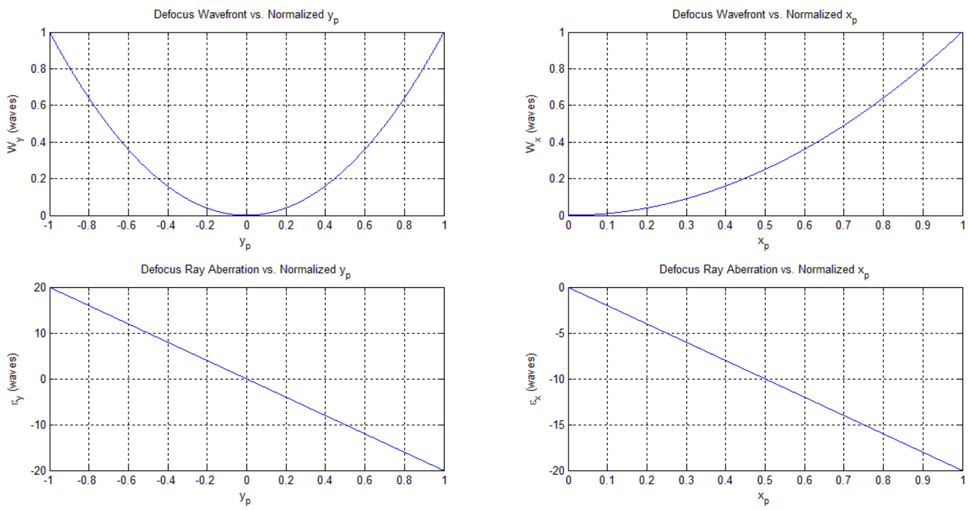
An **F/5** system has one wave of defocus (W = $W_{020}\,\rho^2$; λ = 500 nm; $W_{020}$ = 1λ = 500 nm).

**a.** Plot the wavefront as a function of normalized x or y pupil coordinates.

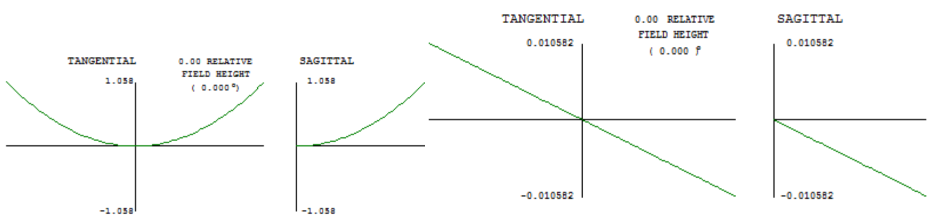
$$W = W_{020}(x_p^2 + y_p^2)$$

**b.** Plot the ray aberration at paraxial focus versus normalized x or y.

$$\epsilon_x = -4 \cdot F_{\#} \cdot W_{020} \cdot x_p$$
$$\epsilon_y = -4 \cdot F_{\#} \cdot W_{020} \cdot y_p$$




**c.** Compare your results with CodeV's ones, by using Lens Module (MOD) option.




Wavefront (in waves) and Ray Aberration Plots (in mm)

The plots from Code V and the derived plots match as the derived plots are in waves while the Code V Wavefront Aberration plot is in waves and the ray aberration is in mm.