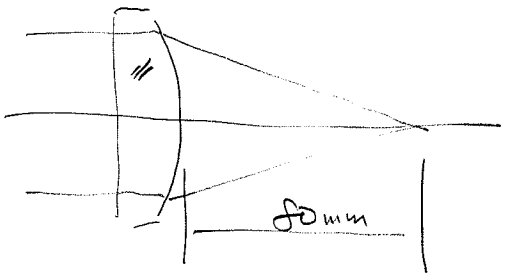
OPTI 340 Midterm #1, Solutions. 2012

Pb 1 Part A

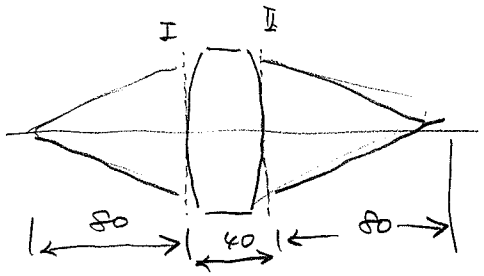


$$\frac{1-n}{-R} = \frac{1}{f}$$

$R = -40\,\text{mm}$ (5pt)
$\uparrow$
$n = 1.5$

$f = 80\,\text{mm}$ (5pt)

Thick lens



HO. 2012/1/19.

$$M_{sys}_{I\to II} = \begin{bmatrix} 1-40\frac{0.5}{1.5}\frac{1}{R_1} & \frac{40}{1.5} \\ -\phi_{sys} & -\frac{-0.5\times 40}{1.5(-R_1)}+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{40}{3}C_1 & \frac{40}{1.5} \\ -\phi_{sys} & 1-\frac{40}{3}C_1 \end{bmatrix} \quad \text{where } C_1=\frac{1}{R_1}$$

obj → Img $\begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix} M_{sys} \begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}$ //

$$= \begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1-\frac{40}{3}C_1 & \frac{40}{1.5} \\ -\phi_{sys} & 1-\frac{40}{3}C_1 \end{bmatrix}\begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 80 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1-\frac{40}{3}C_1 & (1-\frac{40}{3}C_1)80+\frac{40}{1.5} \\ -\phi_{sys} & -80\phi_{sys}+1-\frac{40}{3}C_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{40}{3}C_1-80\phi_{sys} & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\phi_{sys} = 2(n-1)C_1 + \frac{40}{1.5}(n-1)^2 C_1^2$$

$$= C_1 + \frac{40}{1.5} 0.5^2 C_1^2$$

$$\therefore A = 1 - \frac{40}{3}C_1 - 80\left(C_1 - \frac{20}{3}C_1^2\right) = -1 \quad (M = -1)$$

solve for C_1,

$$2 - \frac{40}{3}C_1 - 80C_1 + \frac{1600}{3}C_1^2 = 0$$

$$2 - \frac{280}{3}C_1 + \frac{1600}{3}C_1^2 = 0$$

$$C_1^2 - \frac{280}{1600}C_1 + \frac{3}{800} = 0$$

$$C_1^2 - \frac{7}{40}C_1 + \frac{3}{800} = 0 \qquad \frac{28}{160} = \frac{7}{40}$$

$$C_1 = \frac{+\frac{7}{40} \pm \sqrt{\left(\frac{7}{40}\right)^2 - 4\frac{3}{800}}}{2}$$

$$\sqrt{\quad} = \sqrt{\frac{49}{1600} - \frac{24}{1600}} = \sqrt{\frac{25}{1600}} = \frac{5}{40}$$

$$= \frac{1}{2}\left(+\frac{7}{40} - \frac{5}{40}\right) = \frac{1}{40}$$

$$R_1 = 40 \text{ mm} //$$

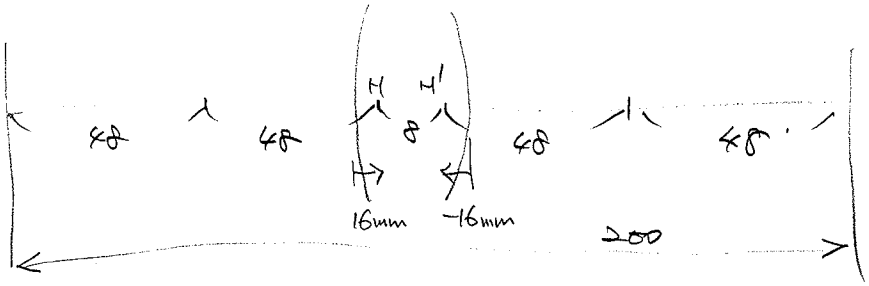
$$\phi = \phi_1 + \phi_2 - \frac{z}{n}\phi_1\phi_2$$

$$\phi_1 = \phi_2 = \frac{1.5-1}{R} = \frac{0.5}{40} = \frac{1}{80}$$

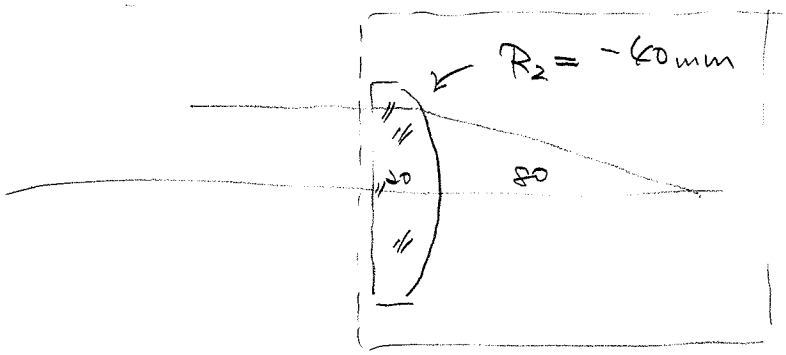
$$\therefore \phi = \frac{1}{80} + \frac{1}{80} - \frac{40}{1.5}\frac{1}{80}\frac{1}{80}$$

$$\boxed{f = 48\ \text{mm}}$$

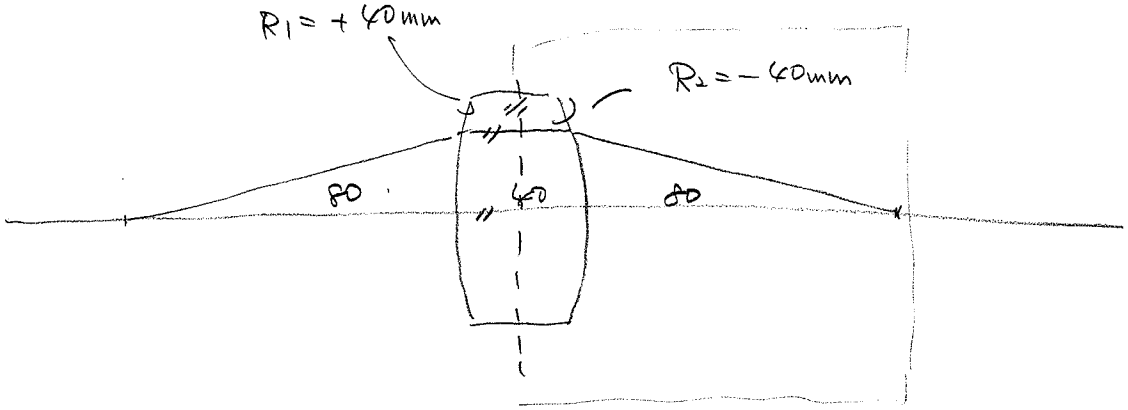
$2f - 2f$ imaging



PART: A



PART B



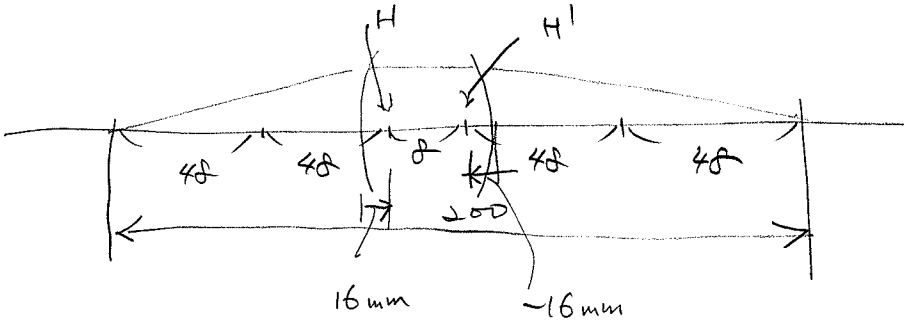
$$\phi_1 = \frac{1.5-1}{R} = \frac{0.5}{60} = \frac{1}{80} = \phi_2$$

$$\phi_{sys} = \phi_1 + \phi_2 - \frac{t}{n}\phi_1\phi_2$$

$$= 2\phi_1 - \frac{t}{n}\phi_1^2$$

$$= \frac{1}{48} \qquad (f = 48\,mm)$$

(2f - 2f imaging)



Pb 2

$$W = W_{020}\rho^2 + W_{040}\rho^4$$

$$= W_{020}(x_p^2+y_p^2) + W_{040}(x_p^2+y_p^2)^2$$

$$2W_{040}\cdot 2y_p^2$$

$$\xi_y = -2(F/\#)\frac{\partial W}{\partial y_p} = -2(F/\#)\underbrace{}_{10}\left(2W_{020}y_p + 4W_{040}y_p^3\right) \quad \cdots (1)$$

$$\frac{\partial \xi_y}{\partial y_p} = -4(F/\#)W_{020} + 12W_{040}y_p^2 \quad \cdots (2)$$

$$\therefore \left.\frac{\partial \xi_y}{\partial y_p}\right|_{y_p=0} = -4\underbrace{(F/\#)}_{10}W_{020} = -80$$

$$\boxed{W_{020} = +2\mu m}$$

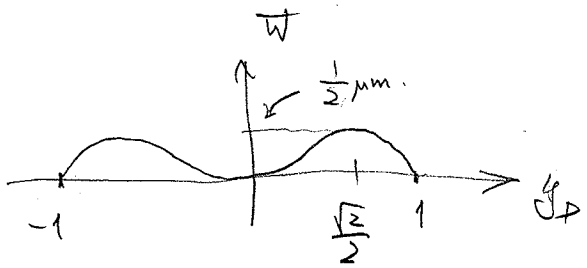
$$\xi_y\Big|_{y_p=1} = -40W_{020} - 80W_{040}$$

$$= -80 \quad \therefore -80W_{040} = +80$$

$$\uparrow$$

$$W_{020} = +2\mu m$$

$$\therefore \boxed{W_{040} = -2\mu m}$$



$$W = 2\rho^2 - 2\rho^4$$

$$= 2\cdot\frac{2}{4} - 2\cdot\frac{1}{4} = \frac{1}{2}$$

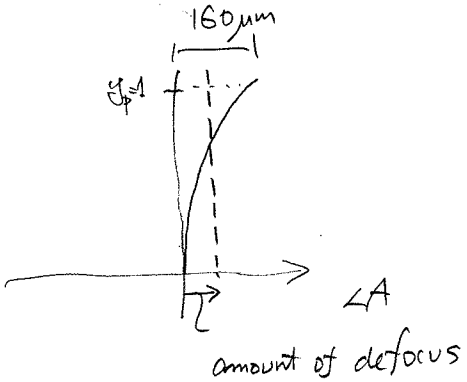
$\uparrow$ $\sqrt{2}$ 1

$$LA = +2(F/\#)\,\frac{TA}{y_p} =$$

10

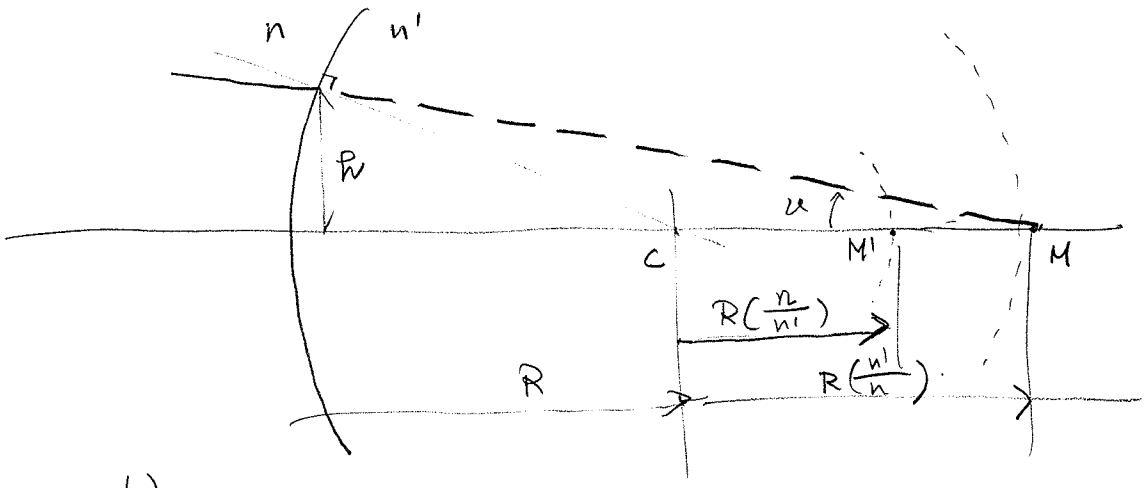
$$= 20\,\frac{1}{y_p}\left(4y_p - 8y_p^3\right)$$

$$= 80 - 160\,y_p^2$$



Pb 3

a) $\overline{CM'} = R\left(\frac{n}{n'}\right)$ Aplanatic Construction



b)

$$u = \frac{h}{R + R\frac{n'}{n}} \qquad \frac{u}{n} = \frac{h}{nR + n'R}$$

$$v' = \frac{h}{R + R\frac{n}{n'}} \qquad \frac{u'}{n'} = \frac{h}{n'R + nR}$$

$\therefore$ $W_{040} \propto S_I \propto \delta\left(\frac{u}{n}\right) = \frac{u'}{n'} - \frac{u}{n} = 0$ //.

$= -A^2 h \delta\left(\frac{u}{n}\right)$