OPTI 330 HW8 TA : MohanXu

I. $t(x,y) = \mathrm{circ}\left(\frac{x-D/2}{d/2}, \frac{y}{d/2}\right) + \mathrm{circ}\left(\frac{x+D/2}{d/2}, \frac{y}{d/2}\right)$

normally incident.

$\lambda = 500\,nm$

observation distance for Fraunhofer diffraction

$$z_{12} >> \frac{\pi(x_1^2+y_1^2)}{\lambda} = \frac{\pi L_1^2}{\lambda} = \frac{\pi\cdot\left(\frac{d}{2}\right)^2}{\lambda} = \frac{\pi d^2}{4\lambda}$$

the diffracted field are given as

$$u(x_2,y_2) = \frac{e^{jkz_{12}}}{j\lambda z_{12}} \exp\left\{j\frac{\pi}{\lambda z_{12}}(x_2^2+y_2^2)\right\} \mathcal{F}\{u(x_1,y_1)\}\Big|_{\xi=\frac{x_2}{\lambda z_{12}}, \eta=\frac{y_2}{\lambda z_{12}}}$$

and $\boxed{I(x_2,y_2) = \left(\frac{1}{\lambda z_{12}}\right)^2 \left|\mathcal{F}\{u(x_1,y_1)\}\right|^2}$

$$\mathcal{F}\{u(x_1,y_1)\} = \mathcal{F}\{t(x,y)\} = \mathcal{F}\left\{\mathrm{circ}\left(\frac{x}{d/2}, \frac{y}{d/2}\right) \circledast \left[\delta(x-\tfrac{D}{2}, y) + \delta(x+\tfrac{D}{2}, y)\right]\right\}$$

$$= \frac{\pi}{4}(d/2)^2\, \mathrm{somb}\left(\frac{d}{2}\xi, \frac{d}{2}\eta\right) \cdot 2 \cdot \cos\left(2\pi\frac{D}{2}\cdot\xi\right)$$

$$= \frac{\pi}{8}d^2\, \mathrm{somb}\left(\frac{\frac{d}{2}r_2}{\lambda z_{12}}\right) \cdot \cos\left(\frac{\pi D x_2}{\lambda z_{12}}\right)$$

for 1) $D = 10\,cm$, $d = 1\,cm$

$$z_{12} >> \frac{\pi d^2}{4\lambda} = \frac{\pi\times10^{-4}}{4\times500\times10^{-9}} = 157\,m$$

$$I(x_2,y_2) = \left(\frac{1}{\lambda z_{12}}\right)^2 \left|\mathcal{F}\{u(x_1,y_1)\}\right|^2$$

$$= \left(\frac{1}{\lambda z_{12}}\right)^2 \left(\frac{\pi d^2}{8}\right)^2 \mathrm{somb}^2\left(\frac{d\,r_2}{2\lambda z_{12}}\right) \cdot \cos^2\left(\frac{\pi D x_2}{\lambda z_{12}}\right)$$

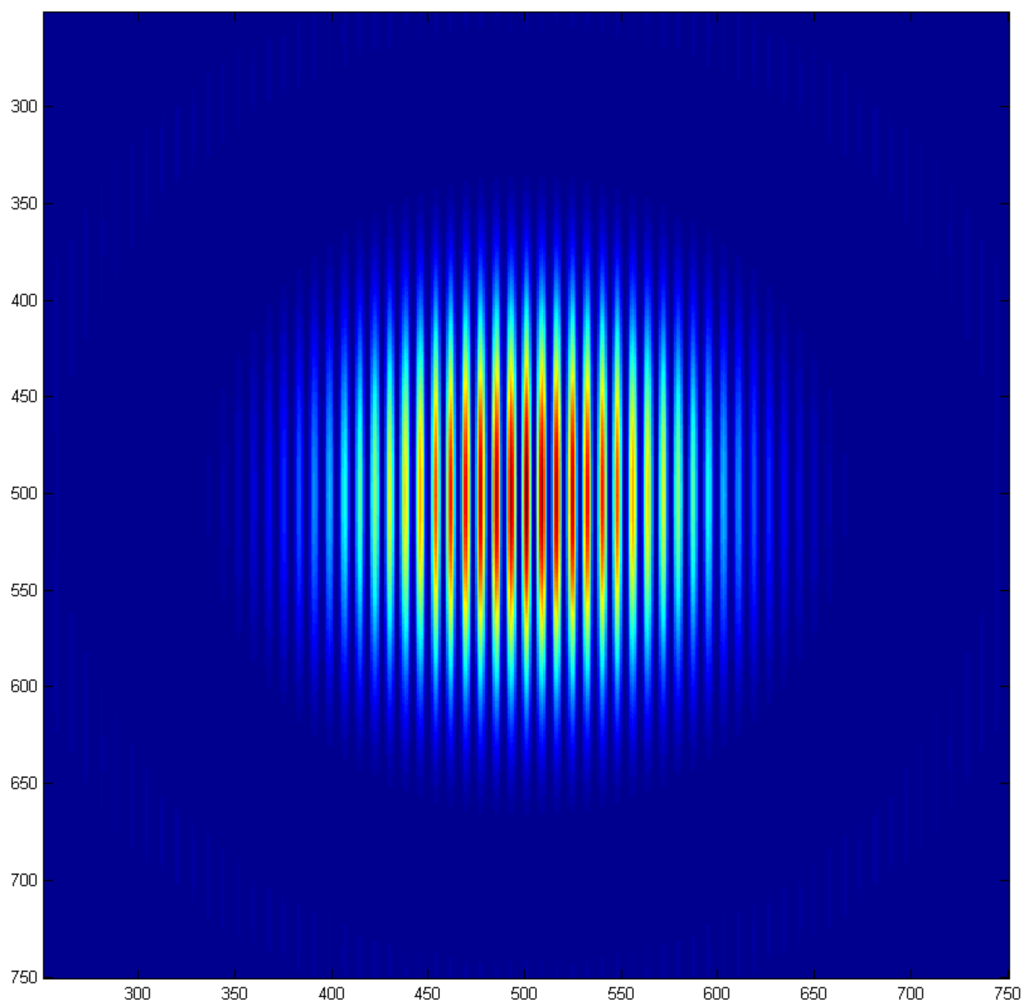
$$= \left(\frac{\pi d^2}{8\lambda z_{12}}\right)^2 \mathrm{somb}^2\left(\frac{d\,r_2}{2\lambda z_{12}}\right) \cdot \cos^2\left(\frac{\pi D x_2}{\lambda z_{12}}\right)$$

2) $D = 10\,cm$, $d = 2\,cm$

$$z_{12} >> \frac{\pi d^2}{4\lambda} = \frac{4\pi\times10^{-4}}{4\times500\times10^{-9}} = 628\,m$$

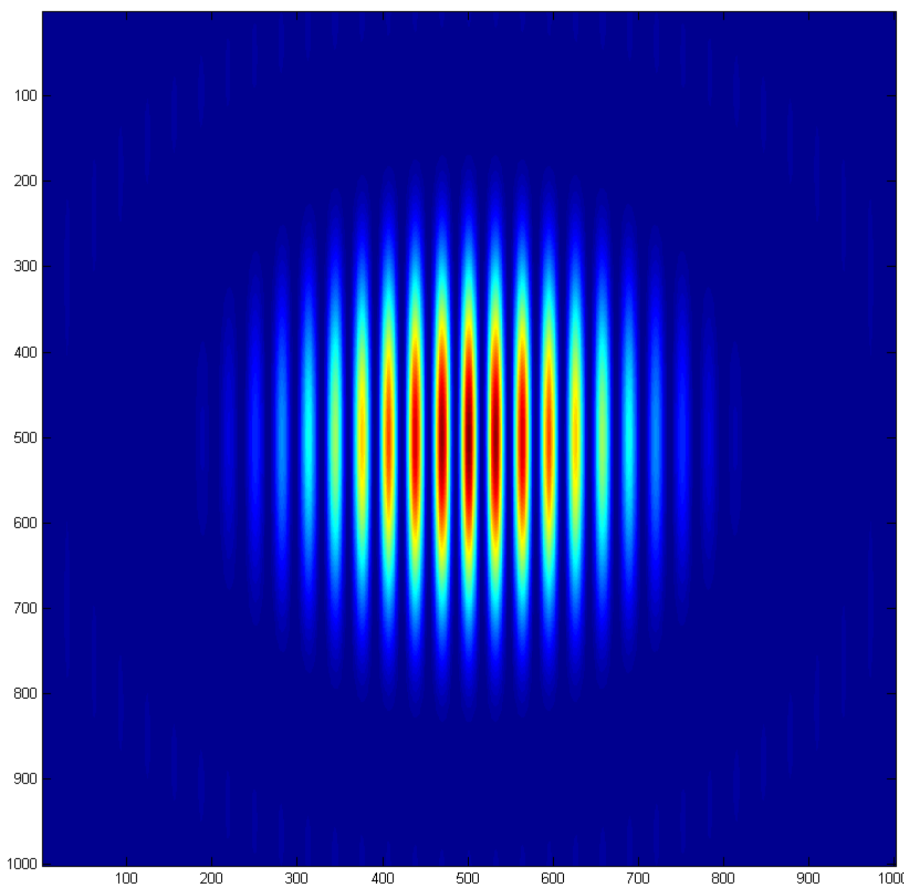
1)

```
D=0.1;%m
lambda=500e-9;%m
x=-0.05:.0001:0.05;
y=x;
[X,Y]=meshgrid(x,y);
r=sqrt(X.^2+Y.^2);
d=0.01;
z12=157;
Z=somb1(d*r/(2*lambda*z12));
I=(pi*(d^2)/(8*lambda*z12))^2.*(Z.^2).*(cos(pi*D*X./(lambda*z12)).^2);
figure();
imagesc(I);
axis equal;
```



2)

```
D=0.1;%m
lambda=500e-9;%m
x=-0.05:.0001:0.05;
y=x;
[X,Y]=meshgrid(x,y);
r=sqrt(X.^2+Y.^2);
d=0.02;
z12=628;
Z=somb1(d*r/(2*lambda*z12));
I=(pi*(d^2)/(8*lambda*z12))^2.*(Z.^2).*(cos(pi*D*X./(lambda*z12)).^2);
figure();
imagesc(I);
axis equal;
```



2. $P(x,y) = \text{rect}\left(\frac{x}{L_x}, \frac{y}{L_y}\right)$ $L_x = 1cm$, $L_y = 2cm$, $\lambda = 1\mu m$, $d_i = d_o = 1m$ and $f = 0.5m$

a) CTF is the exit pupil in scaled coordinates

$$\boxed{H(\xi,\eta) = P(-\lambda d_i \xi, -\lambda d_i \eta) = \text{rect}\left(\frac{\lambda d_i \xi}{L_x}, \frac{\lambda d_i \eta}{L_y}\right)} = \text{rect}(10^{-4}\xi, 0.5\times10^{-4}\eta)$$

$$\tilde{h}(x_i, y_i) = \frac{L_x \cdot L_y}{(\lambda d_i)^2} \text{sinc}\left(\frac{L_x x_i}{\lambda d_i}\right) \cdot \text{sinc}\left(\frac{L_y y_i}{\lambda d_i}\right)$$

b). OTF:

$$\mathcal{H}(\xi,\eta) = \frac{\mathcal{F}\{|\tilde{h}|^2\}}{\mathcal{F}\{|\tilde{h}|^2\}_{\xi=0,\eta=0}}$$

$$|\tilde{h}(x_i, y_i)|^2 = \left[\frac{L_x L_y}{(\lambda d_i)^2}\right]^2 \text{sinc}^2\left(\frac{L_x x_i}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right)$$

$$\mathcal{F}\{|\tilde{h}|^2\} = \left(\frac{L_x \cdot L_y}{\lambda^2 d_i^2}\right)^2 \frac{\lambda d_i}{L_x} \cdot \frac{\lambda d_i}{L_y} \text{tri}\left(\frac{\lambda d_i \xi}{L_x}, \frac{\lambda d_i \eta}{L_y}\right)$$

$$= \frac{L_x \cdot L_y}{\lambda^2 d_i^2} \text{tri}\left(\frac{\lambda d_i \xi}{L_x}, \frac{\lambda d_i \eta}{L_y}\right)$$

$$\boxed{\mathcal{H}(\xi,\eta) = \text{tri}\left(\frac{\lambda d_i \xi}{L_x}, \frac{\lambda d_i \eta}{L_y}\right)}$$

c). this pupil is a low-pass filter with cutoff frequency for CTF

$$\xi = \frac{L_x}{2\lambda d_i} = \frac{10^{-2}}{2\times10^{-6}\times 1} = 5000 \text{ cyl/m}$$

$$\eta = \frac{L_y}{2\lambda d_i} = \frac{2\times10^{-2}}{2\times10^{-6}\times1} = 10000 \text{ cyl/m}$$

d). for OTF. cuttoff frequency

$$\xi = \frac{L_x}{\lambda d_i} = 10000 \text{ cyl/m}$$

$$\eta = \frac{L_y}{\lambda d_i} = 20000 \text{ cyl/m}$$

e). $U_i(\xi,\eta) = H(\xi,\eta) \cdot U_g(\xi,\eta)$

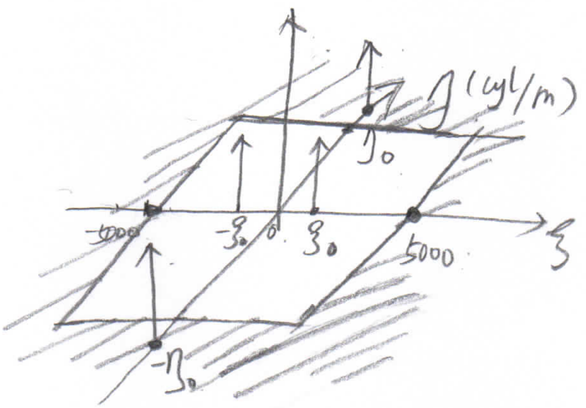
$\xi_0 = 500$ cyl/m, $\eta_0 = 15000$ cyl/m

$$U_g(\xi,\eta) = \mathcal{F}\{u_o(x_o,y_o)\} = \mathcal{F}\{\cos(2\pi\xi_0 x_o) + \cos(2\pi\eta_0 y_o)\}$$

$$= \frac{1}{2}\delta(\xi-\xi_0,\eta) + \frac{1}{2}\delta(\xi+\xi_0,\eta) + \frac{1}{2}\delta(\xi,\eta-\eta_0) + \frac{1}{2}\delta(\xi,\eta+\eta_0)$$

so $U_i(\xi,\eta) = H(\xi,\eta)\cdot U_g(\xi,\eta)$

$$= \frac{1}{2}\mathrm{rect}(10^{-4}\xi,\ 0.5\times10^{-4}\eta)\left[\delta(\xi-\xi_0,\eta)+\delta(\xi+\xi_0,\eta)+\delta(\xi,\eta-\eta_0)+\delta(\xi,\eta+\eta_0)\right]$$



in this sense

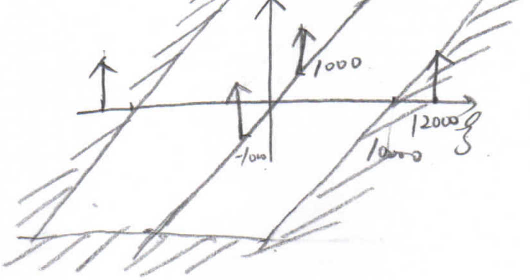
$$U_i(\xi,\eta) = \frac{1}{2}\delta(\xi-\xi_0,\eta) + \frac{1}{2}\delta(\xi+\xi_0,\eta)$$

so $u_i(x_i,y_i) = \cos(2\pi\xi_0 x_i) = \cos(1000\pi x_i)$

f) $\boxed{\text{Change the question, } I_o(x_o,y_o) = 1+\cos(2\pi\xi_0 x_o)+\cos(2\pi\eta_0 y_o)}$

$$G_g(\xi,\eta) = \frac{\mathcal{F}\{I\}}{\mathcal{F}\{I\}|_{\xi=0,\eta=0}} = \delta(\xi,\eta) + \frac{1}{2}\delta(\xi-\xi_0,\eta) + \frac{1}{2}\delta(\xi+\xi_0,\eta) + \frac{1}{2}\delta(\xi,\eta-\eta_0) + \frac{1}{2}\delta(\xi,\eta+\eta_0)$$

$$\mathcal{H}(\xi,\eta) = \mathrm{tri}\left(\frac{\lambda d_i \xi}{L_x},\ \frac{\lambda d_i \eta}{L_y}\right)$$



so. $G_i(\xi,\eta) = \delta(\xi,\eta) + \frac{1}{2}\delta(\xi,\eta-\eta_0) + \frac{1}{2}\delta(\xi,\eta+\eta_0)$

so $I_i(x_i,y_i) = 1 + \cos(2\pi\eta_0 y_i)$

$\eta_0 = 1000$ cyl/m

3. $L_x = 2cm$, $L_y = 2cm$, $\lambda = 1 \mu m$, $d_i = d_o = 1m$, and $f = 0.5m$.

a) incoherent PSF:

$$|\tilde{h}(x_i, y_i)|^2 = \tilde{h}(x_i, y_i)\tilde{h}^*(x_i, y_i) = \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \text{sinc}^2\left(\frac{L_x x_i}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right)$$

b) $\Delta = 0.1mm$

$$I_i(x_i, y_i) = |\tilde{h}(x_i, y_i)|^2 \circledast I_g(x_i, y_i)$$

$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \text{sinc}^2\left(\frac{L_x x_i}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right) \circledast [\delta(x_i - \Delta, y_i) + \delta(x_i + \Delta, y_i)]$$

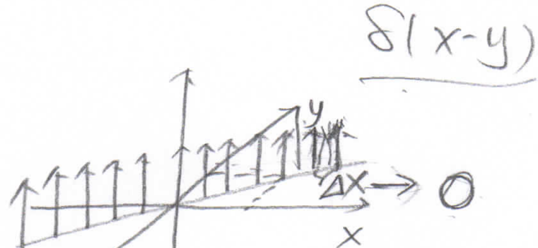
$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \left[\text{sinc}^2\left(\frac{L_x(x_i - \Delta)}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right) + \text{sinc}^2\left(\frac{L_x(x_i + \Delta)}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right)\right]$$

$$I_i(x_i, y_i) = 4 \times 10^{-8} \left[\text{sinc}^2(2\times10^4(x_i - 0.1\times10^{-3}), 2\times10^4 y_i) + \text{sinc}^2(2\times10^4(x_i + 0.1\times10^{-3}), 2\times10^4 y_i)\right]$$

c) $$I_i(x_i, y_i) = 4\times10^{-8}\left[\text{sinc}^2(2\times10^4(x_i - 0.5\times10^{-3}), 2\times10^4 y_i) + \text{sinc}^2(2\times10^4(x_i + 0.5\times10^{-3}), 2\times10^4 y_i)\right]$$

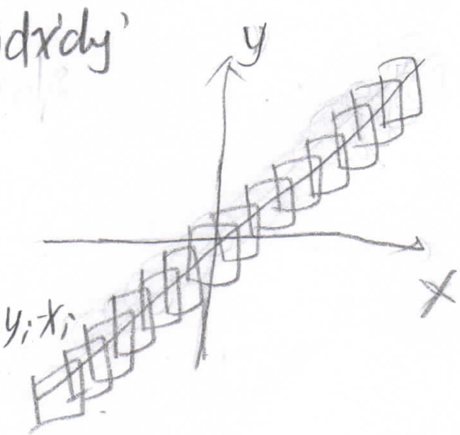
d) please check next page

e) $$I_i(x_i, y_i) = |\tilde{h}(x_i, y_i)|^2 \circledast I_g(x_i, y_i)$$



$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \text{sinc}^2\left(\frac{L_x x_i}{\lambda d_i}, \frac{L_y y_i}{\lambda d_i}\right) \circledast \delta(x_i - y_i)$$

$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \iint \text{sinc}^2\left(\frac{L_x x'}{\lambda d_i}, \frac{L_y y'}{\lambda d_i}\right) \delta(x_i - x' - (y_i - y'))\, dx'dy'$$



$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \int \text{sinc}^2\left(\frac{L_x(x_i - y_i + y')}{\lambda d_i}, \frac{L_y y'}{\lambda d_i}\right) dy'$$

$$= \left(\frac{L_x L_y}{\lambda^2 d_i^2}\right)^2 \int \text{sinc}^2\left(\frac{L_x(\tau - y')}{\lambda d_i}, \frac{L_y y'}{\lambda d_i}\right) dy', \quad \tau = y_i - x_i$$

$$= \frac{L_x L_y}{\lambda^2 d_i^2} \mathcal{F}^{-1}\left\{ \text{tri}\left(\frac{\lambda d_i \xi}{L}\right) \text{tri}\left(\frac{\lambda d_i \xi}{L}\right)\right\}$$

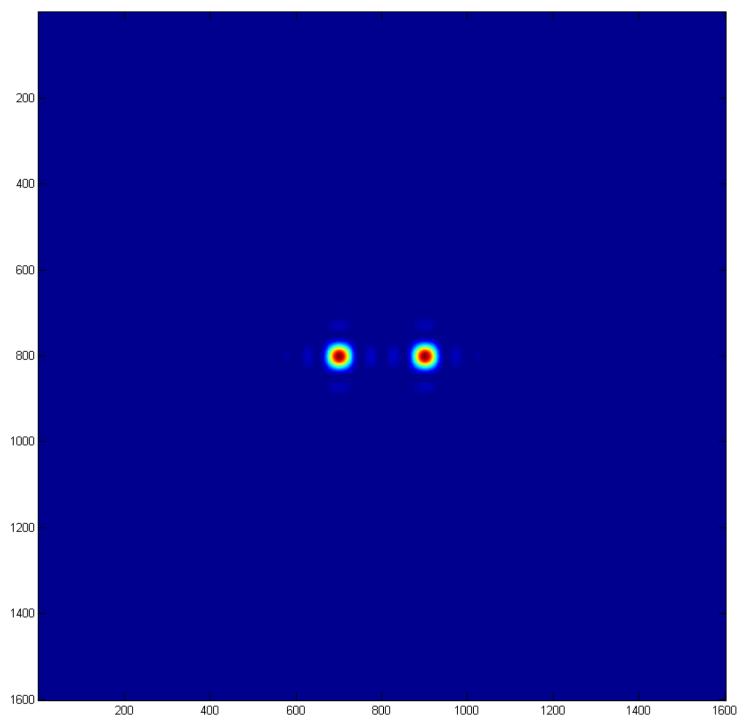
assume function $Q = \mathcal{F}^{-1}\{\text{tri}^2\}$

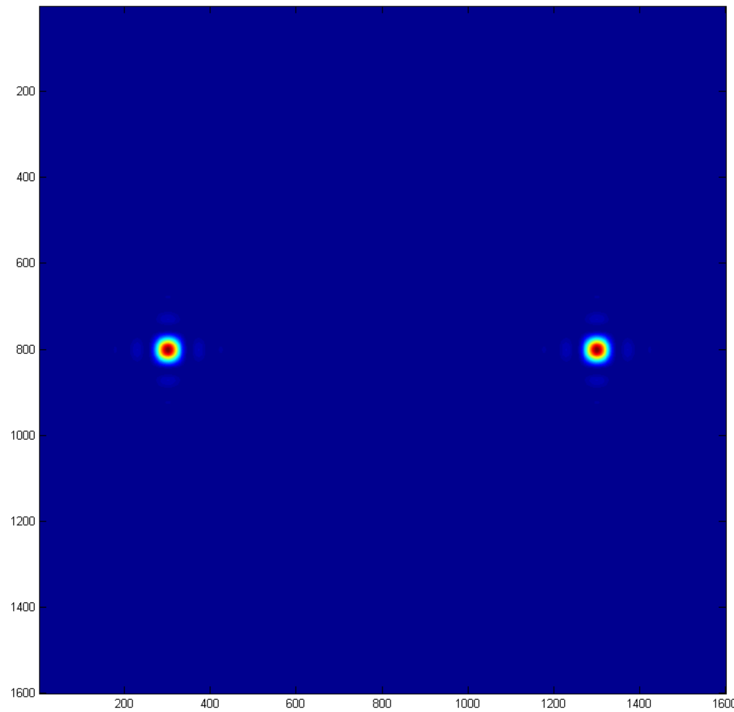
so $$I_i(y_i - x_i) \propto \mathcal{F}^{-1}\left\{\text{tri}^2\left(\frac{\lambda d_i \xi}{L}\right)\right\} \propto Q\left(\frac{L\tau}{\lambda d_i}\right)$$

d)

```
%%b)
Lx=2e-2;
Ly=2e-2;
lambda=1e-6;
di=1;
delta=1e-4;
x=-0.0008:.000001:0.0008;
y=x;
[X,Y]=meshgrid(x,y);
I=(Lx*Ly/(lambda^2*di^2)).^2*((sinc(Lx*(X-
delta)./(lambda*di)).^2.*sinc(Ly*Y./((lambda*di))).^2)+(sinc(Lx*(X+delta).
/(lambda*di)).^2.*sinc(Ly*Y./((lambda*di))).^2));
figure();
imagesc(I);
axis equal;
%%c)
delta=5e-4;
I=(Lx*Ly/(lambda^2*di^2)).^2*((sinc(Lx*(X-
delta)./(lambda*di)).^2.*sinc(Ly*Y./((lambda*di))).^2)+(sinc(Lx*(X+delta).
/(lambda*di)).^2.*sinc(Ly*Y./((lambda*di))).^2));
figure();
imagesc(I);
axis equal;
```

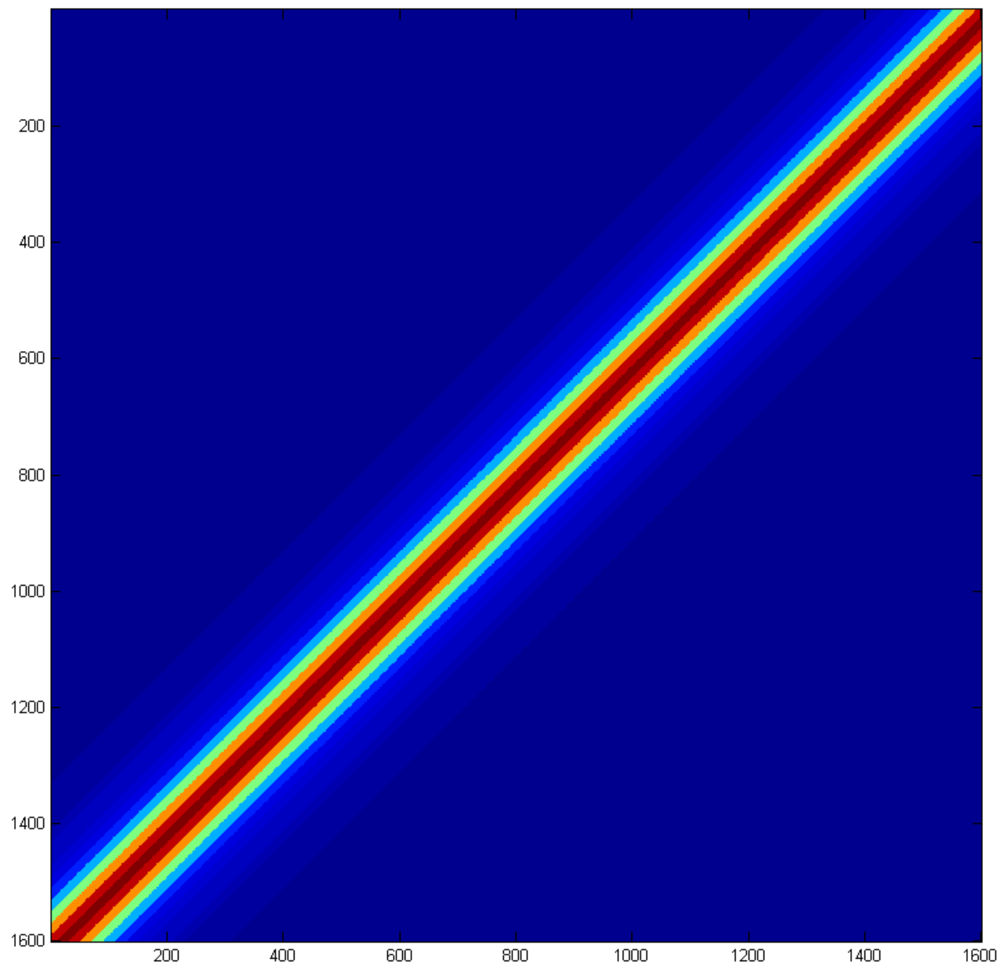
The plot for b)





f)

```
%%%%%%%%%%%Problem3f
lambda=1e-6;
di=1;
L=2e-2;
xi=-5e4:1000:5e4;
delta_xi=xi(2)-xi(1);
size_xi=length(xi);
Temp=tri(lambda*di*xi/L).^2;
plot(xi,Temp);
I=abs(ifftshift(ifft(Temp)));
time=linspace(-1/delta_xi,1/delta_xi,size_xi);
%%%
x=-0.0008:.000001:0.0008;
y=x;
[X,Y]=meshgrid(x,y);
I2d=zeros(length(x),length(y));
%%%
for i=1:1:length(x)
    for j=1:1:length(y)
I2d(i,j)=interp1(time,I,(y(j)-x(i)),'nearest');
    end;
end;
figure();
imagesc(flip(I2d));
axis equal;
```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