4a

$$\int_a^b sin(\lambda x)dx = -\frac{cos(\lambda x}{\lambda}|_a^b$$
$$= \frac{-cos\lambda b}{\lambda} + \frac{cos\lambda a}{\lambda}$$
$$\lim_{\lambda\to\infty}\int_a^b sin(\lambda x)dx = \lim_{\lambda\to\infty}(\frac{-cos\lambda b}{\lambda} - \frac{cos\lambda a}{\lambda})$$
$$= \lim_{\lambda\to\infty}\frac{1}{\lambda}(cos\lambda a - cos\lambda b)$$
$$\text{Since } -1 \leq cos\lambda x \leq 1$$
$$\frac{-2}{\lambda} \leq \frac{1}{\lambda}(cos\lambda a - cos\lambda b) \leq \frac{-2}{\lambda}$$
$$\lim_{\lambda\to\infty}\frac{-2}{\lambda} = \lim_{\lambda\to\infty}\frac{2}{\lambda}$$

So by the squeeze theorem, $\lim \lambda \to \infty \frac{1}{\lambda}(cos\lambda a - cos\lambda b) = \lim_{\lambda\to\infty}\int_a^b sin(\lambda x)dx = 0$.

4b

$$lim_{\lambda\to\infty}\int_a^b s(x)sin\lambda x dx = lim_{\lambda\to\infty}\sum_{i=1}^{n}\int_{t_{i-1}}^{t_i} s(x)sin\lambda x dx$$
$$= lim_{\lambda\to\infty}\sum_{i=1}^{n}\int_{t_{i-1}}^{t_i} c_i sin\lambda x dx \qquad \textit{Since s is a step function on } [t_{i-1}, t_i]$$
$$= \sum_{i=1}^{n} c_i lim_{\lambda\to\infty}\int_{t_{i-1}}^{t_i} sin\lambda x dx$$
$$= \sum_{i=1}^{n} c_i(0)$$
$$= 0$$

4c

Let $S_{\mathcal{P}} = (x \in [t_{i-1}, t_i] : m_i)$. Forgive the abuse of notation but it's just a step function defined on the partition $\mathcal{P}$ equal to the infimum of f on each interval in the partition. Then, $S_{\mathcal{P}}$ is integrable, since $U(S_{\mathcal{P}}, \mathcal{P}) = L(S_{\mathcal{P}}, \mathcal{P})$. So, we know:

$$\forall\varepsilon > 0, \exists\mathcal{P}_1 : U(S_{\mathcal{P}_1}, \mathcal{P}_1) - \int_a^b S_{\mathcal{P}_1} < \varepsilon$$
$$\forall\varepsilon > 0, \exists\mathcal{P}_2 : U(f, \mathcal{P}_2) - \int_a^b f < \varepsilon$$
$$\forall\varepsilon > 0, \exists\mathcal{P}_3 : U(f, \mathcal{P}_3) - U(S_{\mathcal{P}_3}) < \varepsilon$$

We can rearrange these to get

$$\forall\varepsilon, \exists\mathcal{P} : \int_a^b f - \int_a^b S_{\mathcal{P}} < \varepsilon$$

Now, consider the equation:

$$\begin{aligned}
\left|\int_a^b fsin\lambda x - \int_a^b S_{\mathcal{P}} sin\lambda x\right| &= \left|\int_a^b (f - S_{\mathcal{P}}) sin\lambda x\right| \\
&< \left|\int_a^b \varepsilon sin\lambda x\right| \\
&\leq \int_a^b \varepsilon |sin\lambda x| \\
&\leq \int_a^b \varepsilon \\
&= (b-a)\varepsilon \\
&= \varepsilon && \text{Since } \varepsilon \text{ is arbitrary}
\end{aligned}$$

So, we know that for all ε there exists a $\mathcal{P}$ such that

$$\begin{aligned}
\left|\int_a^b fsin\lambda x - \int_a^b S_{\mathcal{P}} sin\lambda x\right| &< \varepsilon \\
lim_{\lambda\to\infty}\left|\int_a^b fsin\lambda x - \int_a^b S_{\mathcal{P}} sin\lambda x\right| &< lim_{\lambda\to\infty}\varepsilon \\
\left|lim_{\lambda\to\infty}\int_a^b fsin\lambda x - 0\right| &< \varepsilon \\
-\varepsilon \leq lim_{\lambda\to\infty}\int_a^b fsin\lambda x &\leq \varepsilon
\end{aligned}$$

Since this is true for all ε, we know that the limit must be 0.