And assume by contradiction

$x > 0 \Rightarrow \frac{7}{x} - \frac{1}{x^2} + \frac{1}{x^3} \leq 0.$

so,

$$\frac{7x^2 - x + 1}{x^3} \leq 0$$

But $x > 0$ so

$7x^2 - x + 1 > 0 - 0 + 1$

$7x^2 - x + 1 \geq 1$ and

$x^3 > 0$

so $\frac{7x^2 - x + 1}{x^3} > 0,$

contradiction.

Finally,

$$\lim_{x \to \infty} \frac{7}{x} - \frac{1}{x^2} + \frac{1}{x^3}$$

$$= \lim_{x \to 0^+} (x - x^2 + x^3)$$

$$= 0$$

Therefore, the theorem proven at the beginning applies, and so $\lim_{x \to \infty} \frac{x^3 + 4x - 7}{7x^2 - x + 1} = \infty.$

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