Q2a).

Suppose $\sqrt{210} = \frac{n}{m}$, for minimal $n, m \in \mathbb{Z}$. Then:

$$201m^2 = n^2$$
$$(2)(105)m^2 = n^2$$

So n^2 is even $\Rightarrow n$ is even $\Rightarrow n = 2k, k \in \mathbb{Z}$. So,

$$(2)(105)m^2 = 4k^2$$
$$(105)m^2 = 2k^2$$

So $(105)m^2$ is even $\Rightarrow$ 105 is even or m^2 is even. Since 105 is not even, m^2 is even $\Rightarrow m$ is even, so both m and n are even, contradicting the assumption that m, n are minimal. $\sqrt{210}$ is therefore irrational.

Q2b).

First, show that a is irrational $\Rightarrow \sqrt{a}$ is irrational. Assume by contradiction that $\sqrt{a} = \frac{m}{n}, m, n \in \mathbb{Z} \Rightarrow a = \frac{m^2}{n^3}$, but since a is irrational, this is a contradiction, completing the proof. So, use this to prove that $\sqrt{7} - \sqrt{2}$ is irrational.

$$(\sqrt{7} - \sqrt{2})^2 = 9 - 2\sqrt{14}$$

By the above, if $(\sqrt{7} - \sqrt{2})^2$ is irrational, $\sqrt{7} - \sqrt{2}$ is also irrational. Assume by contradiction that:

$$(\sqrt{7} - \sqrt{2})^2 = \frac{m}{n}, m, n \in \mathbb{Z}$$
$$\Rightarrow \sqrt{14} - \frac{9n - m}{2n}$$
$$\Rightarrow \sqrt{14}\text{is rational. If that's the case, you should be able to pick minimal } a, b \in \mathbb{Z} \text{ so that } \frac{a}{b} = \sqrt{14}.$$
$$\Rightarrow a^2 = 14b^2$$
$$\Rightarrow a^2 \text{ is even, } \Rightarrow a \text{ is even, } \Rightarrow a = 2p, p \in \mathbb{Z}.$$
$$\Rightarrow 4k^2 = 14b^2$$
$$\Rightarrow 2k^2 = 7b^2$$
$$\Rightarrow 7b^2 \text{ is even} \Rightarrow 7 \text{ is even or } b^2 \text{ is even. 7 is not even, so } b^2 \text{ is even } \Rightarrow b \text{ is even.}$$

That means a, b are both even, contradicting the assumption that they were minimal. Thus, $\sqrt{14}$ is irrational $\Rightarrow (\sqrt{7} - \sqrt{2})^2$ is irrational $\Rightarrow \sqrt{7} - \sqrt{2}$ is irrational, completing the proof.

Q2c).

let $\sqrt[3]{7} = \frac{a}{b}$ by contradiction, with minimal $a, b \in \mathbb{Z}$.Then,

$$7b^3 = a^3$$
$$Mult_7(7b^3) = Mult_7(a^3)$$
$$1 + 3Mult_7(b) = 3Mult_7(a)$$

Since a, b are in lowest terms, $Mult_7(a)$ or $Mult_7(b)$ must be 0. If $Mult_7(b) = 0$, either $Mult_7(a) = 0$ (Which is a contradiction, because only one can be 0), or $3Mult_7(a) \geq 3$, which is also a contradiction because it must be equal to $3Mult_7(b) + 1 = 1$. If $Mult_7(a) = 0$, that is also a contradiction, because it must be equal to $3Mult_7(b) + 1$, which is minimum 4. Therefore, $\sqrt[3]{7}$ is irrational.