

Let  $f(x) = x^k$ ,  $\mathcal{P} = \{0, 1, \dots, n\}$ . Then,  $L(f, \mathcal{P}) = \sum_{i=1}^n m_i = \sum_{i=1}^n (n-1)^k = S_k(n-1)$  and  $U(f, \mathcal{P}) = \sum_{i=1}^n M_i = \sum_{i=1}^n n^k = S_k(n)$ .  
 So, know

$$L(f, \mathcal{P}) \leq \int_0^n f(x) dx \leq M(f, \mathcal{P})$$

and

$$\int_0^n x^k dx = \frac{n^{k+1}}{n+1}$$

And so,

$$S_k(n-1) = L(f, \mathcal{P}) \leq \int_0^n x^k dx = \frac{n^{k+1}}{n+1} \leq M(f, \mathcal{P}) = S_k(n)$$

So

$$S_k(n-1) \leq \frac{n^{k+1}}{n+1} \leq S_k(n)$$

As desired.