a)

For all $x > 1$, $\frac{sqrtx}{x^2} > \frac{\sqrt{x}}{1+x^2}$, we have

$$\int_1^\infty \frac{\sqrt{x}}{x^2} > \int_1^\infty \frac{\sqrt{x}}{x^2+1} \tag{1}$$

and we have

$$\begin{aligned}\int_1^\infty \frac{\sqrt{x}}{x^2} &= \int_1^\infty \frac{1}{x\sqrt{x}} \\ &= \int_1^\infty x^{-\frac{3}{2}} \\ &= \lim_{x\to\infty}\left(\frac{-2}{\sqrt{x}} + 2\right) \\ &= 2\end{aligned}$$

And since this converges, so does $\frac{\sqrt{x}}{1+x^2}$.

b)

Note that $sin(x) = x$ at $x = 0$ and $\frac{d}{dx}x = 1 \geq cos(x) = \frac{d}{dx}sin(x)$. So, we have on $[0, \frac{\pi}{4}]x \geq sin(x) \Rightarrow \frac{1}{x} \leq \frac{1}{sin(x}$. Therefore,

$$\int_0^{\frac{pi}{4}} \frac{1}{x} \leq \int_0^{\frac{pi}{4}} \frac{1}{sin(x)}$$

But:

$$\begin{aligned}\int_0^{\frac{pi}{4}} \frac{1}{x} &= log(\frac{\pi}{4}) - \log(0) \\ &= log(\frac{\pi}{0}) \\ &= \infty\end{aligned}$$

Since this diverges, so does $\int_0^{\frac{pi}{4}} \frac{1}{sin(x)}$

c)

Since $\sqrt{log(x)} \leq \sqrt{x}, x \geq 1$ then $\frac{\sqrt{x}}{x^2} \geq \frac{\sqrt{log(x)}}{x^2}$. And we have from a) that

$$\int_1^\infty \frac{\sqrt{x}}{x^2}$$

converges, it follows that

$$\int_1^\infty \frac{\sqrt{log(x)}}{x^2}$$

also converges.