Let

$$g(x) = \begin{cases} 0 & x = 0 \\ x^2 sin\left(\frac{1}{x}\right) & x \neq 0 \end{cases}$$

and

$$h(x) = \begin{cases} 0 & x = 0 \\ 2xsin\left(\frac{1}{x}\right) & x \neq 0 \end{cases}$$

We know

$$g'(x) = \begin{cases} 0 & x = 0 \\ 2xsin\left(\frac{1}{x}\right) - cos\left(\frac{1}{x}\right) & x \neq 0 \end{cases}$$

So $f(x) = h(x) - g'(x)$. Then,

$$\begin{aligned} F(x) &= \int_0^x f(t)dt \\ &= \int_0^x (h(t) - g'(t))dt \\ &= \int_0^x h(t)dt - \int_0^x g'(t)dt \\ &= \int_0^x h(t)dt - g(x) && \text{from FTC 2} \\ F'(x) &= \frac{d}{dx}(\int_0^x h(t)dt - g(x)) \\ &= h(x) - g'(x) && \text{By FTC 1, since } h(x) \text{ is continuous} \end{aligned}$$

So,

$$F'(x) = \begin{cases} 0 & x = 0 \\ cos\left(\frac{1}{x}\right) & x \neq 0 \end{cases}$$

Meaning $F(x)$ is differentable.