4a

$$\begin{aligned}
\int_a^b sin(\lambda x)dx &= -\frac{cos(\lambda x}{\lambda}|_a^b \\
&= \frac{-cos\lambda b}{\lambda} + \frac{cos\lambda a}{\lambda} \\
\lim_{\lambda\to\infty}\int_a^b sin(\lambda x)dx &= \lim_{\lambda\to\infty}(\frac{-cos\lambda b}{\lambda} - \frac{cos\lambda a}{\lambda}) \\
&= \lim_{\lambda\to\infty}\frac{1}{\lambda}(cos\lambda a - cos\lambda b) \\
&\text{Since } -1 \leq cos\lambda x \leq 1 \\
\frac{-2}{\lambda} \leq \frac{1}{\lambda}(cos\lambda a - cos\lambda b) &\leq \frac{-2}{\lambda} \\
\lim_{\lambda\to\infty}\frac{-2}{\lambda} &= \lim_{\lambda\to\infty}\frac{2}{\lambda}
\end{aligned}$$

So by the squeeze theorem, $\lim \lambda \to \infty\frac{1}{\lambda}(cos\lambda a - cos\lambda b) = \lim_{\lambda\to\infty}\int_a^b sin(\lambda x)dx = 0$.

4b

$$\begin{aligned}
lim_{\lambda\to\infty}\int_a^b s(x)sin\lambda xdx &= lim_{\lambda\to\infty}\sum_{i=1}^n\int_{t_{i-1}}^{t_i} s(x)sin\lambda xdx \\
&= lim_{\lambda\to\infty}\sum_{i=1}^n\int_{t_{i-1}}^{t_i} c_i sin\lambda xdx \qquad \textit{Since s is a step function on } [t_{i-1}, t_i] \\
&= \sum_{i=1}^n c_i lim_{\lambda\to\infty}\int_{t_{i-1}}^{t_i} sin\lambda xdx \\
&= \sum_{i=1}^n c_i(0) \\
&= 0
\end{aligned}$$

4c

Let $S_{\mathcal{P}} = x \in [t_{i-1}, t_i] : m_i$. A bit of an abuse of notation but it's just a step function defined on the partition $\mathcal{P}$ equal to the infimum of f on each interval in the partition. Then, $S_{\mathcal{P}}$ is integrable, since $U(S_{\mathcal{P}}, \mathcal{P}) = L(S_{\mathcal{P}}, \mathcal{P})$. So, we know