Consider a partition $\mathcal{P}$ on $[a, b]$. Let supremum and infimum of f on $[t_{i-1}, t_i]$ me $M_i and m_i$ as usual. Notice that the supremum and infimum of $\frac{1}{f}$ on the same interval are (since $f \geq C > 0$ and f bounded) $\frac{1}{m_i}$ and $\frac{1}{M_i}$ respectively. Therefore,

$$\begin{aligned}
U(\frac{1}{f}, \mathcal{P}) - L(\frac{1}{f}, \mathcal{P}) &= \sum_{i=1}^{n} \frac{1}{m_i}(t_{i-1} - t_i) - \sum_{i=1}^{n} \frac{1}{M_i}(t_{i-1} - t_i) \\
&= \sum_{i=1}^{n} (\frac{1}{m_i} - \frac{1}{M_i})(t_{i-1} - t_i) \\
&= \sum_{i=1}^{n} \frac{1}{m_i M_i}(M_i - m_i)(t_{i-1} - t_i) \\
&\leq \sum_{i=1}^{n} \frac{1}{C^2}(M_i - m_i)(t_{i-1} - t_i) \\
&= \frac{1}{C^2} \sum_{i=1}^{n} (M_i - m_i)(t_{i-1} - t_i) \\
&= \frac{1}{C^2}(U(f, \mathcal{P}) - L(f, \mathcal{P}))
\end{aligned}$$

And since f is integrable, we can make the above difference as small as we want. Pick a $\mathcal{P}$ so that $(U(f, \mathcal{P}) - L(f, \mathcal{P})) < C^2\varepsilon$ and we get

$$U(\frac{1}{f}, \mathcal{P}) - L(\frac{1}{f}, \mathcal{P}) \leq \frac{1}{C^2}(U(f, \mathcal{P}) - L(f, \mathcal{P})) < \frac{C^2\varepsilon}{C^2} = \varepsilon$$

as desired.