2b) since $-1 \leq \sin(k) \leq 1 \quad \forall k \in \mathbb{R}$, $-x \leq \sin(\frac{1}{x}) \leq x$.

So, $|x| \geq x\sin(\frac{1}{x})$ and $-|x| \leq \sin(\frac{1}{x})$

and $\lim_{x \to 0} |x| = 0$, $\lim_{x \to 0} -|x| = 0$, so

$$\lim_{x \to 0} x\sin\left(\frac{1}{x}\right) = 0.$$

Q3) ~~Let p∈R. Let ε be provided~~

Want to show

$$\lim_{x \to p} (f(x) - g(x)) = 0 \iff \forall \varepsilon > 0 \; \exists \delta > 0 : |x - p| < \delta \Rightarrow |f(x) - g(x)| < \varepsilon$$

So, let ε be provided. Let p be an arbitrary real number. Then:

f continuous, so

$$\exists \delta_f : |p - x| < \delta_f \Rightarrow |f(p) - f(x)| < \frac{\varepsilon}{2}$$

$$\exists \delta_g : |p - x| < \delta_g \Rightarrow |g(p) - g(x)| < \frac{\varepsilon}{2}.$$

Take $\delta_A = \min\{\delta_f, \delta_g\}$

Then, since S is dense,

$$\exists q \in S : |q - x| < \delta_A \Rightarrow |f(x) - f(q)| < \frac{\varepsilon}{2}$$

and

$$|q - x| < \delta_A \Rightarrow |g(x) - g(q)| < \frac{\varepsilon}{2} \Rightarrow |g(x) - f(q)| < \frac{\varepsilon}{2}$$

So, $|f(x) - g(x)| < \varepsilon$

Since S is dense, no matter what $x \in \mathbb{R}$ you pick, there is a δ_A satisfying the above properties. Therefore, for an arbitrary choice of δ, $|f(x) - g(x)| < \varepsilon$.

So, $\lim_{x \to p} (f(x) - g(x)) = 0$

since f, g are continuous,

$f(p) = g(p)$, for an arbitrary p, so $f = g$

QED.