Q2a

Let $y = arctanx$, then $x = tan(y) = \frac{siny}{cosy} = \frac{siny}{\sqrt{1-sin^2y}}$

$$\begin{aligned} x &= \frac{siny}{\sqrt{1-sin^2y}} \\ x^2 - x^2sin^2y &= sin^2y \\ x^2 = sin^2(y)(1+x^2 & \\ sin^2y &= \frac{x^2}{1+x^2} \\ sin(y) &= \frac{x}{\sqrt{1+x^2}} \end{aligned}$$

For $cos(y)$:

$$\begin{aligned} cos(y) &= \sqrt{1-sin^2y} \\ &= \sqrt{1-\frac{x^2}{1+x^2}} \\ &= \sqrt{\frac{1+x^2-x^2}{1+x^2}} \\ &= \frac{1}{\sqrt{1+x^2}} \end{aligned}$$

Q2b

$$\begin{aligned} sin(u) &= sin(2\frac{u}{2}) \\ &= sin(2arctan(x)) \\ &= 2sin(arctan(x))cos(arctan(x)) \\ &= 2\frac{x}{\sqrt{1+x^2}}\frac{1}{\sqrt{1+x^2}} \\ &= \frac{2x}{1+x^2} \end{aligned}$$

$$\begin{aligned} cos(u) &= cos(2arctan(x)) \\ &= 2cos^2(arctan(x)) - 1 \\ &= \frac{2}{1+x^2} - 1 \\ &= \frac{1-x^2}{1+x^2} \end{aligned}$$