

Q4) a) Take  $\delta = \varepsilon \Rightarrow \delta = \frac{\varepsilon}{2}$

Then,  $|x-a| < \delta \Rightarrow |f(x) - f(a)| < \delta = \varepsilon$ .

So  $|f(x) - f(a)| < \varepsilon$ .

Q.E.D.

b)  $\forall c > 0 \exists x \in X \exists a \in X : |f(x) - f(a)| > c|x-a|$

c) choose  $x=0, a < \frac{1}{c^2}$ .

Then, note that  $|f(x) - f(a)| = |-f(a)| = f(a) = \sqrt{a}$

and  $|x-a| = |a| = a$

So if  $0 < a < \frac{1}{c^2}$

$$ac^2 < 1$$

$$\sqrt{a} < 1$$

$$c < \frac{1}{\sqrt{a}}$$

$$c < \frac{\sqrt{a}}{a}$$

$$ca < \sqrt{a}$$

$$c|x-a| < |f(x) - f(a)|$$

so  $|f(x) - f(a)| > c|x-a|$

Q.E.D.