a.)

Suppose $M_i \geq 0, m_i \geq 0$:
Then, $M_i = M_i', m_i = m_i'$, so $M_i' - m_i' = M_i - m_i$

Suppose $M_i \geq 0, m_i \leq 0$:
Then $0 \leq m_i' \leq |m_i|$ and $M_i' = \max\{M_i, |m_i|\}$. But, $M_i' - m_i' \leq M_i' - |m_i| = \max\{M_i - |m_i|, 0\} \leq M_i$. So, $M_i' - m_i' \leq M_i \leq M_i - m_i$.

Finally, $M_i \leq 0, m_i \leq 0$:
Then, $M_i' = |m_i|, m_i' = |M_i|$, and $M_i - m_i = |m_i| + M_i = |m_i| - |M_i| = m_i' - M_i'$.

So, in all cases, $M_i' - m_i' \leq M_i - m_i$. Then, from the definitions of $U(f, \mathcal{P})$ and $L(f, mathcalP)$ and the corresponding upper and lower sums on $|f|$, we have

$$\begin{aligned}
\sum_{i=1}^{n} M_i'(t_{i-1} - t_i) - \sum_{i=1}^{n} m_i'(t_{i-1} - t_i) &= \sum_{i=1}^{n}(M_i' - m_i')(t_{i-1} - t_i) \\
&\leq \sum_{i=1}^{n}(M_i - m_i)(t_{i-1} - t_i) \\
&= \sum_{i=1}^{n} M_i(t_{i-1} - t_i) - \sum_{i=1}^{n} m_i(t_{i-1} - t_i) \\
&= U(f, \mathcal{P}) - L(f, \mathcal{P})
\end{aligned}$$

b.)

If f integrable, $\exists \mathcal{P} : U(f, \mathcal{P}) - L(f, \mathcal{P}) < \varepsilon$. Since from the above, $U(|f|, \mathcal{P}) - L(|f|, \mathcal{P}) \leq U(f, \mathcal{P}) - L(f, \mathcal{P}) < \varepsilon$, $|f|$ satisfies the Riemann criterion, so it is integrable.

c.)

$f + |f| = 2f_+$, so $\frac{f}{2} + \frac{|f|}{2} = f_+$. Since we've proven that both of those functions are integrable, so is f_+. The same arugment applies to f_-, which can be written as $\frac{f}{2} - \frac{|f|}{2}$.