For if, assume $v \in span(e_1, ...e_n)$. Then,

$$\begin{aligned}
||v||^2 &= \langle v, v\rangle \\
&= \langle v, a_1 e_1 + ... + a_m e_m\rangle \\
\\
&= \sum_{j=1}^{m}\sum_{i=1}^{m}\langle a_j e_j, a_i e_i\rangle \\
&= \sum_{i=1}^{m}\langle a_i e_i, a_i e_i\rangle \\
&= \sum_{i=1}^{m}|a_i|^2\langle e_i, e_i\rangle \\
&= \sum_{i=1}^{m}|a_i\langle e_i, e_i\rangle|^2 && \textit{since } \langle e_i, e_i\rangle = 1 \\
&= \sum_{i=1}^{m}|\langle a_i e_i, e_i\rangle|^2 \\
&= \sum_{i=1}^{m}\langle a_i e_i, a_i e_i\rangle \\
&= \sum_{i=1}^{m}\langle v, a_i e_i\rangle && \textit{since all other terms of the inner product go to 0}
\end{aligned}$$

As desired. Now, for only if: We know V has an orthonormal basis, $e_1, ...e_k$ with $k \geq m$. Then, if $v \notin \text{span}(e_1, ...e_m)$, $v \in$span$(e_1, ...e_n)$ with $k \geq n > m$. So, $v = a_1 e_1 + ... + a_n e_n$ with at least 1 of $e_i \neq 0, i > m$. Then, we have:

$$\begin{aligned}
||v||^2 &= \langle v, v\rangle \\
&= \langle a_1 e_1 + ... + a_n e_n, a_1 e_1 + ... + a_n e_n\rangle \\
&= \sum_{i=1}^{n}|a_i|^2\langle e_i, e_i\rangle \\
&= \sum_{i=1}^{n}|a_i|^2 \\
&\textit{and} \\
\sum_{i=1}^{m}|\langle v, e_i\rangle|^2 &= \sum_{i=1}^{m}|a_i|^2 \\
&\textit{So, since set these equal, and we get} \\
\sum_{i=1}^{m}|a_i|^2 - \sum_{i=1}^{n}|a_i|^2 &= 0 \\
&= -\sum_{i=m+1}^{n}|a_i|^2
\end{aligned}$$

So, we get:

$$-\sum_{i=m+1}^{n}|a_i|^2 = 0$$

Which is only possible if each $a_i = 0$, which contradicts our assumption above. Therefore, we need $v \in \text{span}(e_1, ..., e_m)$.
QED.