Q2

$$\begin{aligned} f((0,1,0),(0,1,0)) &= 0 \cdot 0 + 0 \cdot 0 \\ &= 0 \end{aligned}$$

However, $(0,1,0) \neq 0$, so f does not satisfy the property of definiteness.

Q3

Let S be the set of inner products with the positivity condition. Let S' be the set of inner products with the new condition. To start, let $f \in S$. Then, $f(v,v) > 0 \; \forall v \in V$ with $v \neq 0$, so the new condition is satisfied since $\exists v \in V$ with $f(v,v) > 0$. So, $f \in S'$, so $S \subseteq S'$. INCMPLETE

Q8

Let $u, v \in V$, $||u|| = ||v|| = 1$. Then, $|\rangle u, v \langle| = |1| = 1 = ||u||||v||$. So, by Cauchy-Schwartz, u and v are collinear, and have the same magnitude, so $u = -v or u = v$. However, if $u = -v$, $\langle u, v \rangle = \langle -v, v \rangle = -\langle v, v \rangle < 0$. But $\langle u, v \rangle = 1$, which is a contradiction, so $u = v$.

Q11

Q13

Add image here.

$$\begin{aligned} ||u-v||^2 &= ||u||^2 + ||v||^2 - 2||u||||v|| \cos\theta \\ \rangle u-v, u-v \langle &= \rangle u, u \langle + \langle v, v \rangle - 2||u||||v|| \cos\theta \\ \langle u, u \rangle - 2\langle u, v \rangle + \langle v, v \rangle &= \rangle u, u \langle + \langle v, v \rangle - 2||u||||v|| \cos\theta \\ -2\langle u, v \rangle &= -2||u||||v|| \cos\theta \\ \langle u, v \rangle = ||u||||v|| \cos\theta \end{aligned}$$