Let dim $v = n$. Then, suppose $v \in G(\lambda, T) = \text{null } (T - \lambda I)^n$, so $(T - \lambda I)^n v = 0$:

$$(T - \lambda I)^n = \sum_{k=0}^{n} \binom{n}{k} (T)^{n-k} (\lambda)^k$$

So:

$$\begin{aligned}
&\sum_{k=0}^{n} \binom{n}{k} (T)^{n-k} (\lambda)^k v = 0 \\
\iff\ & \lambda^{-n} T^{-n} \sum_{k=0}^{n} \binom{n}{k} (T)^{n-k} (\lambda)^k v = (\lambda^{-n} T^{-n}) 0 \\
\iff\ & \sum_{k=0}^{n} \binom{n}{k} T^{-n} (T)^{n-k} \lambda^{-n} (\lambda)^k v = 0 \\
\iff\ & (\sum_{k=0}^{n} \binom{n}{k} T^{-k} (\lambda)^{k-n}) v = 0 \\
\iff\ & (\sum_{k=0}^{n} \binom{n}{k} T^{-k} (\frac{1}{\lambda})^{n-k}) v = 0 \\
\iff\ & (T - \frac{1}{\lambda} I)^n v = 0
\end{aligned}$$

Which implies that $v \in G(\frac{1}{\lambda}, T^{-1}) \iff v \in G(\lambda, T)$, as desired.