Q2

Assume by contradiction that $S^2 = T$. Note also that $T^3 = S^6 = 0$ and $T^2 = S^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then S is necessarily nilpotent, since $S^6 = 0$. $\dim v = 3$, and for all nilpotent operators N, $N^{\dim V} = 0$. However, $S^4 \neq 0$, which is a contradiction. Therefore, no such operator S exists.