Chapter 3 - modeling in the Time Domain

• The frequency domain analysis is limited in terms of application, It can be only applied to linear, time-invariant systems or systems ~~that~~ can be approximated such.

Advantage

- Stability
- Transient response

} we can see immediately the effects of varying system parameters until an acceptable design is met.

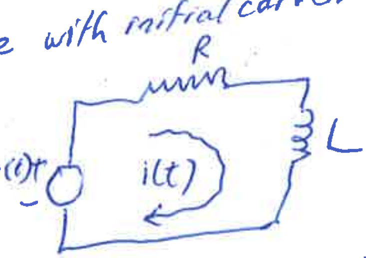
Modern Control Systems (state-space) or time domain approach

MIMO systems can be compactly represented as in SISO systems. Attractive -or more compatible- to use with the digital systems (computer ~~hardware~~ and software).

We will use a system similar to what we had in frequency domain analysis. We need to use differential equations only to solve for a selected subset of system variables because all other remaining variables can be evaluated algebraically from the variables in the subset. Our examples take the following approach:

1. We select particular subset of all possible system variables and call the variables in this subset state variables.
2. For an nth-order-system, we write n simultaneous, first order differential equations aka state equations.
3. If we know the initial condition of all ~~of~~ the state variables at t_0 as well as the system input for $t \geq t_0$, we can solve the simultaneous differential equations for the state variables for $t \geq t_0$.
4. We algebraically combine the state variables with the system's input and find all of the other system variables for $t \geq t_0$. We call this algebraic equation the output equation.
5. We consider the state equations and the output equations a viable representation of the system. We call this representation of the system a state-space representation.

1) Consider the RL network in Figure with initial current $i(0)$.



2) We write the loop equation,

$$L\frac{di}{dt} + \underbrace{R\,i}_{\text{(state var)}} = v(t)$$ } state equation

3) Taking Laplace transform using Table 2.2 item 7 and include all initial conditions, yields;

$$L\left[sI(s) - i(0)\right] + RI(s) = V(s)$$

Assuming the input $v(t)$ to be unit step, $u(t)$, whose Laplace Transform is $V(s) = 1/s$ we solve for $I(s)$ and get,

$$I(s) = \frac{1}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right) + \frac{i(0)}{s+\frac{R}{L}}$$

$$i(t) = \frac{1}{R}\left(1 - e^{-(R/L)t}\right) + i(0) \cdot e^{-(R/L)t}$$

4) $v_R(t) = R \cdot i(t)$ } output equation

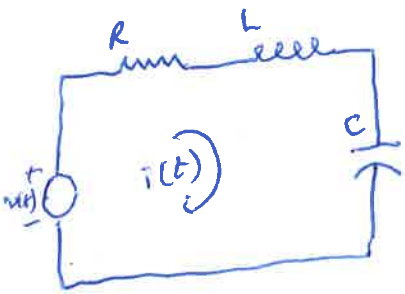
$v_L = v(t) - R\,i(t)$ } output eqation

The derivative of the current is $\frac{di}{dt} = \frac{1}{L}[\underbrace{v(t)}_{\text{input}} - R\,\underbrace{i(t)}_{\text{state var}}]$ } output equation

5) From 1 to 4, the variables of interest are completely described and these form a viable representation of network, which we call a state-space representation.

The state equation is described by $i(t)$ and it can be described by other network variables. for example $i(t) = v_R/R$

$$\frac{L}{R}\frac{dv_R}{dt} + v_R = v_t \quad \text{where } v_R(0) = R \cdot i(0) \ \& \ v(t)$$



1. The network is second order, two simultaneous, first-order differential eqnations are needed to solve for two state variables. we pick $i(t)$ and $q(t)$, the charge on the capacitor, as the two state variables.
2. writing loop equations yields,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v(t) \qquad i = dq/dt$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = v(t)$$

nth order differential equation can be converted to n simultaneous first-order differential equations, with each equation of the form

$$\frac{dx_i}{dt} = \underbrace{a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n}_{\text{Linear Combination of the state variables}} + b_i f(n)$$

x_i : state variable
a_{ij}s & b_i are constants for linear, time invariant systems
$f(t)$: input

Convert two simultaneous first-order diff. eqs in $i(t)$ and $q(t)$.

i/ $\frac{dq}{dt} = i$ $\quad \int i\,dt = q$

ii/ $\frac{di}{dt} = -\frac{1}{LC}q - \frac{R}{L}i + \frac{1}{L}v(t)$

3. The state equations can be solved simultaneously. The laplace transform can be used with the known input $v(t)$ and the initial conditions for $q(t)$ and $i(t)$

4. $v_L(t) = -\frac{1}{C} q(t) - Ri(t) + v(t)$ Output equation

Linear combination of the state variables

5. The result of #2 and #4 form a viable state-space representation.

* In general, the order of the differential equation will result in the minimum number of state variables required to describe a system.
* We can define more state variables than the minimal set; however, within this minimal set the state variables must be linearly independent.

$$\frac{dv_R}{dt} = -\frac{R}{L} v_R - \frac{R}{L} v_C + \frac{R}{L} v(t)$$

$$\frac{dv_C}{dt} = \frac{1}{RC} \cdot v_R$$

$V_R(t)$ picked
$i(t)$ cannot because we can write $V_R(t)$ in terms of $i(t)$.

$\dot{x} = Ax + Bu$ where $\dot{x} = \begin{bmatrix} dq/dt \\ di/dt \end{bmatrix}$; $x = \begin{bmatrix} q \\ i \end{bmatrix}$;

$A = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$; $u = v(t)$

$y = Cx + Du$

where:

$y = v_L(t)$

$C = [-1/C \quad -R]$

$x = \begin{bmatrix} q \\ i \end{bmatrix}$

$D = 1$

$u = v(t)$

These two equations represent the state space representation which consists of
- Simultaneous first-order differential equations from which the state variables can be solved.
- The algebraic output equation from which all other system variables can be found.

* A state-space representation is not unique, since different choice of state variables leads to different representation of the same system

(First this one) General State Space Representation

Linear Combination: A linear combination of n variables, x_i, for $i = 1$ to n, is given by the following sum, S. where each K_i is constant

$$S = K_n x_n + K_{n-1} x_{n-1} + \cdots + K_1 x_1$$

Linear independence: A set of variables is said to be linearly independent if none of the variables can be written as linear combination of others.

$x_1, x_2, x_3 \longrightarrow x_2 = 5x_1 + 6x_3$ then the variables are NOT linearly independent

$K_2 x = K_1 x_1 + K_3 x_3 \rightarrow \forall K_i = 0$ and no $x_i = 0$ for all $t \geq 0$.

~ week 6 ~

System Variables: Any variable that responds to an input or initial conditions in a system.

State Variables: The smallest set of linearly independent system variables such as the values of members of the set at time t_0 along with known forcing functions completely determine the value of all system variables for all $t \geq t_0$.

State Vector: A vector whose elements are the state variables.

State Space: The n-dimensional space whose axes are the state variables.

State Equations: A set of n simultaneous, first-order differential equations with n vars. where the n variables to be solved are the state variables.

Output Equations: The algebraic equation that expresses the output variables of a system as linear combinations of a state variables and the inputs.

the initial conditions are $x(t_0)$ for $t \geq t_0$ and the initial

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x = state vector

$\dot{x}$ = derivative of the state vector with respect to time

y = output vector

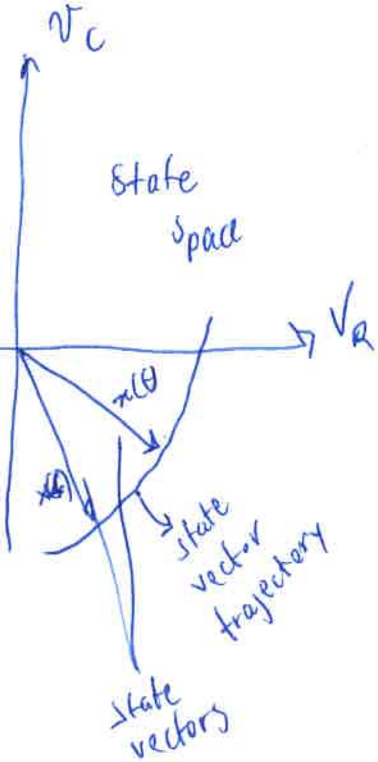
u = input or control vector

A = system matrix

B = input matrix

C = output matrix

D = feed forward matrix



$$\frac{dx_1}{dt} = a_{11}\underbrace{x_1 + a_{12}x_2}_{\text{state vars}} + b_1 v(t)$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2 v(t)$$

Single output

$$y = c_1 x_1 + c_2 x_2 + d_1 v(t)$$

As an example for a linear time invariant, second order system with a single input $v(t)$ the state equations could take on the form on the left.

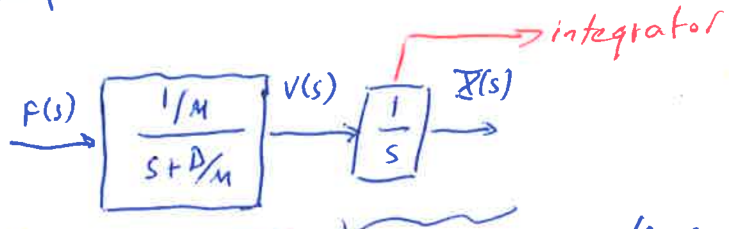
Applying State Space Equations.

Linearly Independent State Variables:

Minimum Number of State Variables: A third order differential equation describes the system, then three simultaneous first-order differential equations are required along with three state variables. From the perspective of the transfer function, the denominator's order will highlight the order.

Another way of thinking that is the number of independent energy-storage elements in the system. That will result in the order of the differential equation and number of state vars.

The first block is the transfer function equivalent to $M\frac{dv(t)}{dt} + D\,v(t) = f(t)$.



Sufficient to define the system.

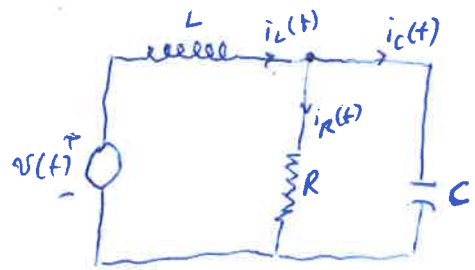
To define in terms of the position.

This increment in the order is because the type of output requested.

$$G(s) = \frac{V(s)}{F(s)} \quad \text{(first order)}$$

$$G(s) = \frac{X(s)}{F(s)} \quad \text{(second order)}$$

Ex Representing an Electrical Network (3.1)



Find the state-space representation if the output is the current through resistor.

1) Label all of the branch currents.
2) Select the state variables by writing the derivative equation for all energy storage elements, here an inductor and capacitor

$$\underbrace{C\cdot\frac{dV_c}{dt}}_{\text{state var 1}} = i_c \quad [1] \qquad \underbrace{L\cdot\frac{di_L}{dt}}_{\text{state var 2}} = V_L \quad [2]$$

* The representation will be completed if we can write the rhs of the equations in linear combinations of the state vars and the input.

$$i_C = -i_R + i_L$$
$$= -\frac{1}{R}V_c + i_L \quad [3]$$
$$V_L = -V_c + v(t) \quad [4]$$

Plug-in to [1] and [2],

$$C\cdot\frac{dV_c}{dt} = -\frac{1}{R}\cdot V_c + i_L \Rightarrow \frac{dV_c}{dt} = -\frac{1}{RC}V_c + i_L\cdot\frac{1}{C} \quad [5]$$

$$L\cdot\frac{di_L}{dt} = -V_c + v(t) \Rightarrow \frac{di_L}{dt} = -\frac{V_c}{L} + v(t)\cdot\frac{1}{L} \quad [6]$$

output $\rightarrow i_R(t) \Rightarrow i_R = \frac{1}{R}V_c \quad [7]$

from [5], [6], and [7]

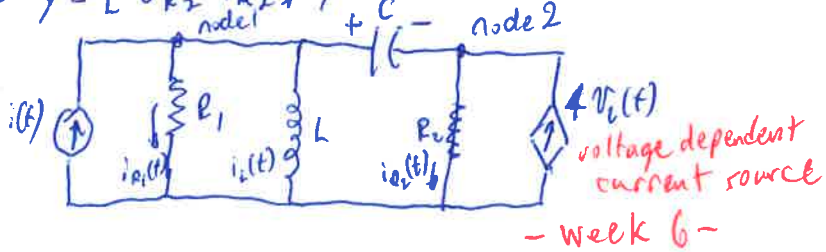
$$\begin{bmatrix}\dot{V}_c \\ \dot{i}_L\end{bmatrix} = \begin{bmatrix}-1/(RC) & 1/C \\ -1/L & 0\end{bmatrix}\begin{bmatrix}V_c \\ V_L\end{bmatrix} + \begin{bmatrix}0 \\ 1/L\end{bmatrix} v(t)$$

$$i_R = \begin{bmatrix}1/R & 0\end{bmatrix}\begin{bmatrix}V_c \\ i_L\end{bmatrix}$$

where $\dot{x}$ is the derivative with respect to time.

Component	V-I	I-V	V-q	Imp
capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\cdot\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$1/Cs$
resistor	$v(t) = R\cdot i(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R
inductor	$v(t) = L\cdot\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls

Ex: Representing an electrical network with a dependent source Find the state and output equations for the electrical network shown in Figure if the output vector is $y = [V_{R_2} \; i_{R_2}]^T$, where T means transpose.



$$L\cdot\frac{di_L}{dt} = V_L \quad [1]$$

$$C\cdot\frac{dV_c}{dt} = i_C \quad [2]$$

Step 1: Label the currents at the branches.

Step 2: select state vars

$L \cdot \frac{di_L}{dt} = V_L$ $\quad x_1 = i_L$

$C \cdot \frac{dV_C}{dt} = i_C$ $\quad x_2 = V_C$

Step 3: Remember the form of state equation

$\dot{x} = Ax + Bu$

$V_L = V_C + V_{R_2} = V_C + i_{R_2} R_2$

$= V_C + \underbrace{(i_C + 4V_L)}_{i_{R_2} \to node2} R_2$ [1]

Solving for $V_L \to V_L = \frac{1}{1-4R_2}(V_C + i_C R_2)$

$(1-4R_2)V_L - R_2 i_C = V_C$

$-\frac{1}{R_1} V_L - i_C = i_L - i(t)$

$i_C = i(t) - i_{R_1} - i_L$

$= i(t) - \frac{V_{R_1}}{R_1} - i_L$

$= i(t) - \frac{V_L}{R_1} - i_L$ [2]

$$V_L = \frac{\begin{vmatrix} V_C & -R_2 \\ i_L - i(t) & -1 \end{vmatrix}}{\begin{vmatrix} 1-4R_2 & -R_2 \\ -\frac{1}{R_1} & -i_C \end{vmatrix}} = \frac{1}{\Delta}\left[R_2 i_L - V_C - R_2 i(t)\right]$$

$$\frac{di_L}{dt} = \frac{V_L}{L}$$

$$i_C = \frac{\begin{vmatrix} 1-4R_2 & V_C \\ -\frac{1}{R_1} & i_L - i(t) \end{vmatrix}}{\begin{vmatrix} 1-4R_2 & -R_2 \\ -\frac{1}{R_1} & -i_C \end{vmatrix}} = \frac{1}{\Delta} \cdot \left[(1-4R_2)i_L + \frac{1}{R_1}V_C - (1-4R_2)i(t)\right]$$

$$\frac{dV_C}{dt} = \frac{i_C}{C}$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dV_C}{dt} \end{bmatrix} = \begin{bmatrix} \frac{R_2}{L\Delta} & -1/(L\Delta) \\ \frac{(1-4R_2)}{C\Delta} & 1/(R_1 C\Delta) \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{-R_2}{L\Delta} \\ -\frac{(1-4R_2)}{C\Delta} \end{bmatrix} i(t)$$

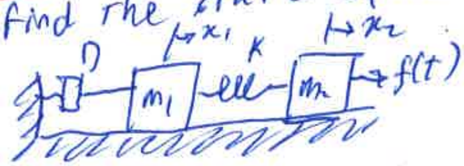
$V_{R_2} = -V_C + V_L$

$i_{R_2} = i_C + 4V_L$

$$\begin{bmatrix} V_{R_2} \\ i_{R_2} \end{bmatrix} = \begin{bmatrix} R_2/\Delta & -(1+1/\Delta) \\ 1/\Delta & (1-4R_1)/(\Delta R_1) \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} -R_2/\Delta \\ -1/\Delta \end{bmatrix} i(t)$$

Mechanical Systems: In electrical systems, it was convenient to pick the energy storing elements. However, in mechanical systems picking energy storing elements doesn't result in the differential that we can use in the representation. Instead, we use position and velocity of each point or linearly independent motion

Ex 3.3: Representing a translational Mechanical System
Find the state equations for the translational mechanical system shown



$(M_1 s^2 + Ds + K)X_1(s) - KX_2(s) = 0$ [1]

$-KX_1(s) + (M_2 s^2 + K)X_2(s) = F(s)$ [2]

$M_1 \frac{d^2 x_1}{dt^2} + D\frac{dx}{dt} + Kx_1 - Kx_2 = 0$ [3]

$-Kx_1 + M_2 d^2x_2/dt^2 + Kx_2 = f(t)$ [4]

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$$\frac{d^2x_1}{dt^2} = \frac{dv_1}{dt} \ \& \ \frac{d^2x_2}{dt^2} = \frac{dv_2}{dt}$$ then $x_1, v_1, x_2,$ and v_2 are state variables.

$$\frac{dx_1}{dt} = v_1$$

$$\frac{dv_1}{dt} = -\frac{K}{M_1}x_1 - \frac{D}{M}v_1 + \frac{K}{M_1}x_2$$

$$\frac{dx_2}{dt} = v_2$$

$$\frac{dv_2}{dt} = +\frac{K}{M_2}x_1 - \frac{K}{M_2}x_2 + \frac{1}{M_2}f(t)$$

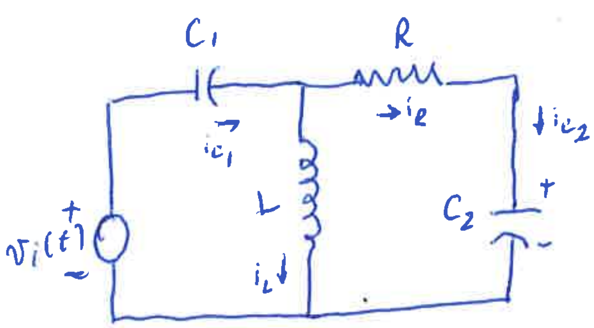
In vector form

output equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t)$$

Exercise (skill assessment 3.1): Find the state-space representation of the electrical network shown in Figure 3.8. The output is $v_o(t)$

KVL & KCL



$$C_1 \cdot \frac{dv_{C_1}}{dt} = i_{C_1}$$

$$L \cdot \frac{di_L}{dt} = v_L$$

$$C_2 \frac{dv_{C_2}}{dt} = i_{C_2}$$

$$\begin{cases} i_{C_1} = i_L + i_R = i_L + \frac{1}{R}(v_L - v_{C_2}) \\ v_L = -v_{C_1} + v_i \\ i_{C_2} = i_R = \frac{1}{R} \cdot (v_L - v_{C_2}) \end{cases}$$

$$\frac{dv_{C_1}}{dt} = -\frac{1}{RC_1}v_{C_1} + \frac{1}{C_1}i_L - \frac{1}{RC_1}v_{C_2} + \frac{1}{RC_1}v_i$$

$$\frac{di_L}{dt} = -\frac{1}{L}v_{C_1} + \frac{1}{L}v_i$$

$$\frac{dv_{C_2}}{dt} = -\frac{1}{RC_2}v_{C_1} - \frac{1}{RC_2}v_{C_2} + \frac{1}{RC_2}v_i$$

$$\dot{x} = \begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{C_1} & -\frac{1}{RC_1} \\ -\frac{1}{L} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} x + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{L} \\ \frac{1}{RC_2} \end{bmatrix} v_i(t)$$

$$y = [0 \ 0 \ 1] x$$

Finally, the solution to the differential equation is $y(t)$ or x_1.

$$y = [1 \; 0 \; 0 \; \dots \; 0] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

In summary,

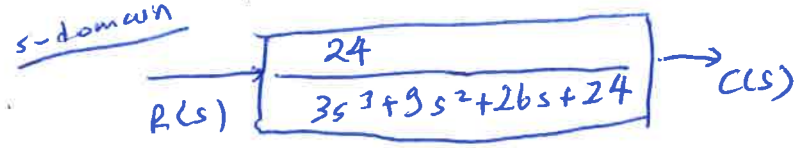
- Convert transfer function to a differential equations in phase variable form. (Cross multiply and take inverse Laplace transform with z.i.t)
- Then set the differential equation in state-space.

Ex: Find the state space representation in phase-variable form.

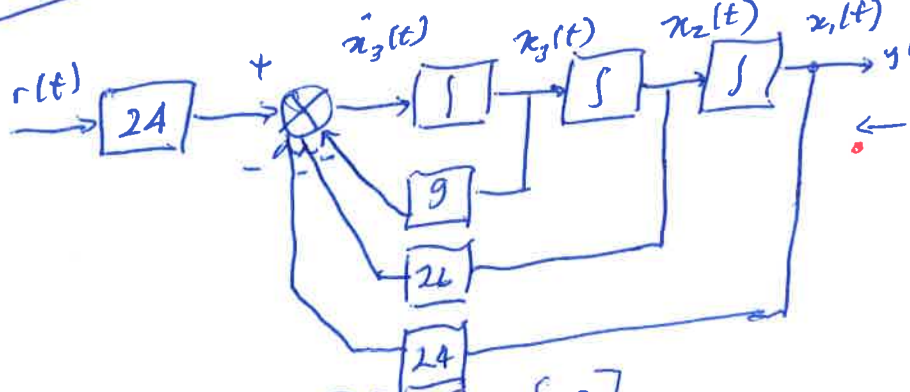
$$\frac{C(s)}{R(s)} = \frac{24}{(s^3+9s^2+26s+24)}$$

in block diagram

s-domain



time domain



$(s^3+9s^2+26s+24)C(s) = 24R(s)$

$\dddot{c} + 9\ddot{c} + 26\dot{c} + 24c = 24r$

$x_1 = c \qquad \dot{x}_1 = x_2$

$x_2 = \dot{c} \qquad \dot{x}_2 = x_3$

$x_3 = \ddot{c} \qquad \dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r$

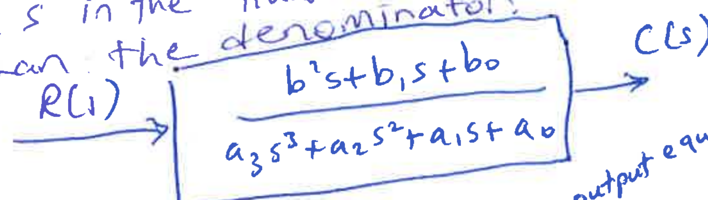
$y = c = x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r$$

- From here we can create the block diagram of the system.

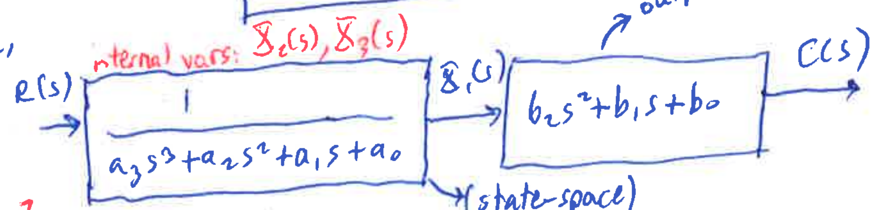
$$y = [1 \; 0 \; 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

If a transfer function has a polynomial in s in the numerator that is of order less than the denominator



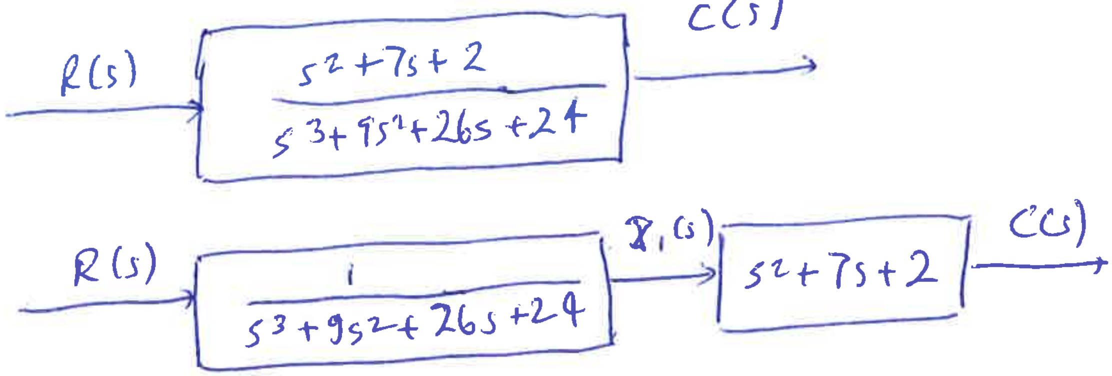
$Y(s) = C(s) = (b_2s^2+b_1s+b_0)X_1(s)$

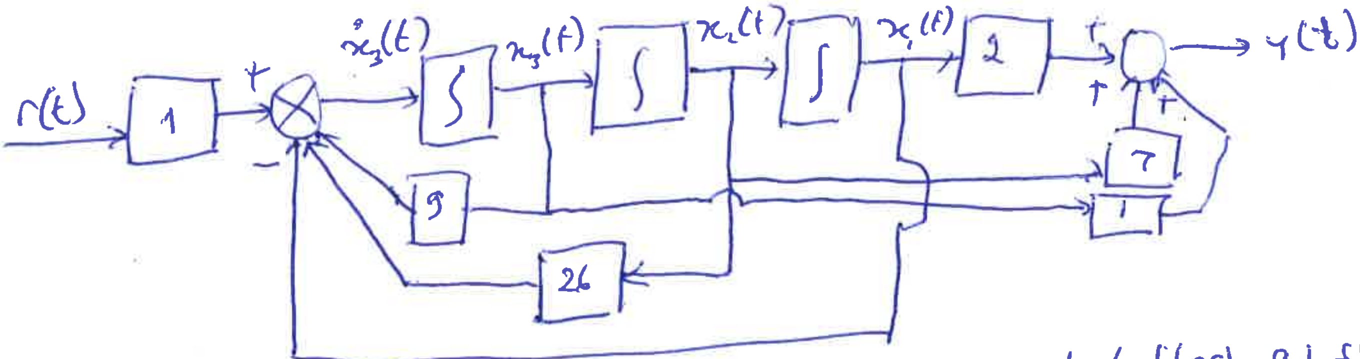
$$y(t) = b_2 \cdot \frac{d^2x_1}{dt^2} + b_1\frac{dx_1}{dt} + b_0x_1$$



-Week 7-

Ex 3.5 : find the state-space representation of the transfer function shown in Figure 3.12(a).





step1: Separate the system into two cascaded block, as show in Fig 3.12 b. First block contains the denominator and the second block contains the numerator.

step2: Find the state equations for the block containing the denominator, we notice that the first block's numerator is 1/24 that in the other examber,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

step 3: Introduce the effect of the block with the numerator. The second block where $b_2 = 1$ $b_1 = 7$ and $b_0 = 2$

$$C(s) = (b_2 s^2 + b_1 s + b_0) X_1(s) = (s^2 + 7s + 2) X_1(s)$$

$$c = \ddot{x}_1 + 7\dot{x}_1 + 2x_1 \qquad x_1 = x_1 \qquad \dot{x}_1 = x_2 \qquad \ddot{x}_1 = x_3$$

$$y = c(t) = x_3 + 7x_2 + 2x_1 \qquad y = [b_0 \; b_1 \; b_2] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [2 \; 7 \; 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Ex SA 3.3: Find the state equations and output equation for the phase variable representation of the transfer function $G(s) = \frac{2s+1}{s^2+7s+9}$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \qquad y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$$

Converting from State-Space to a Transfer Function

$\dot{x} = Ax + Bu$ $\qquad$ $Y(s)$ output vector

$y = Cx + Du$ $\qquad$ $U(s)$ input vector

$$sX(s) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D$$

$$(sI-A)X(s) = BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

$$Y(s) = C(sI-A)^{-1}BU(s) + DU(s)$$
$$= [\underbrace{C(sI-A)^{-1}B + D}_{\text{Transfer function matrix}}]U(s)$$

$U(s)$ = input $\qquad$ $Y(s)$ = output

Ex 7.6: $T(s) = Y(s)/U(s)$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

$$(sI-A) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2s & s+3 \end{bmatrix}$$

$B = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$

now $(sI-A)^{-1}$

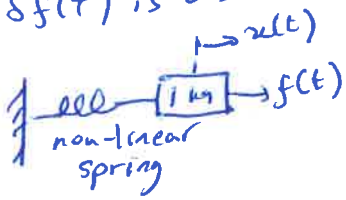
$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

$$(sI-A)^{-1} = \frac{\operatorname{adj}(sI-A)}{\det(sI-A)} = \frac{\begin{bmatrix} s^2+3s+2 & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -(2s+1) & s^2 \end{bmatrix}}{s^3+3s^2+2s+1}$$

$D = 0$

$$T(s) = \frac{10(s^2+3s+2)}{s^3+3s^2+2s+1}$$

Ex (SA 3.5): Represent the translational mechanical system in Figure in state space about the equilibrium displacement. The spring is non-linear, where the relationship between the spring force, $f_s(t)$, and the spring displacement $x_s(t)$ is $f_s(t) = 2x_s^2(t)$. The applied force is $f(t) = 10 + \delta f(t)$ where $\delta f(t)$ is a small force about the 10 N constant value



$$\frac{d^2x}{dt^2} + 2x^2 = 10 + \delta f(t)$$

$$\frac{d^2(x_0 + \delta x)}{dt^2} + 2(x_0 + \delta x)^2 = 10 + \delta f(t)$$

Now we linearise x^2

$$(x_0 + \delta x)^2 - x_0^2 = \frac{dx^2}{dx}\Big|_{x_0} \delta x = 2x_0 \delta x$$

$$(x_0 + \delta x)^2 = x_0^2 + 2x_0 \delta x$$

$$\frac{d^2\delta x}{dt^2} + 4x_0 \delta x = -2x_0^2 + 10 + \delta f(t) \qquad f = 2x^2$$

$10 = 2x_0^2$

$x_0 = \sqrt{5}$

$$\frac{d^2\delta x}{dt^2} + 4\sqrt{5}\,\delta x = \delta f(t) \qquad x_1 = \delta x \quad x_2 = \dot{\delta x}$$

$\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{\delta x} = -4\sqrt{5}x_1 + \delta f(t)$

$y = x_1$

$$\left[\begin{array}{l} \dot{x} = \begin{bmatrix} 0 & 1 \\ -4\sqrt{5} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta f(t) \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{array}\right]$$

Chapter 4 - Time Response

- Use poles and zeros of transfer functions to determine the time response of a control system.
- Describe quantitatively the transient response of first-order systems
- Write the general response of second-order systems given the pole location
- Find the damping ratio and natural frequency of second-order system.
- Find the settling time, peak time, percent overshoot, and rise time for an underdamped second order system.
- Approximate HOT and systems with zeros as first or second order systems.
- Describe the effect of non-linearities of the time response

— week 8 —