Dry Adiabatic Lapse Rate

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A *lapse rate*, Γ, is defined as the rate at which temperature *decreases* with height.

$$\Gamma = -\frac{dT}{dz} \sim \text{ K km}^{-1} \tag{1}$$

In order to derive the *dry adiabatic lapse rate* we must first recall an application of the 2nd law of thermodynamics. That is,

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} + H \tag{2}$$

Recall that H represents diabatic processes diabatic processes, specifically, $H = Q - (\nabla \cdot \vec{F}) + \epsilon$, that is, heating related to chemical reactions and phase changes (latent heating), radiative heating and the conversion of kinetic energy to internal energy (thermalization).

By definition, $H = 0$ in adiabatic processes. This allows a substantial simplification of equation (2).

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt} \tag{3}$$

$$dt \rho c_p \frac{dT}{dt} = \frac{dp}{dt} dt \tag{4}$$

$$\rho c_p dT = dp \tag{5}$$

$$\frac{dT}{dp} = \frac{1}{\rho c_p} \tag{6}$$

Equation (6) defines the derivative of temperature with resepct to temperature, however we would like to derive temperature with resepect to height. Recall hydrostatic balance,

$$\frac{dp}{dz} = -\rho g \tag{7}$$

$$dp = -\rho g \, dz \tag{8}$$

Substituting (8) into the denominator of the LHS in (6),

$$\begin{aligned}
\frac{dT}{-\rho g\, dz} &= \frac{1}{\rho c_p} \\
\frac{dT}{dz} &= \frac{-\rho g}{\rho c_p} \\
\frac{dT}{dz} &= -\frac{g}{c_p}
\end{aligned}$$

Thus, the dry adiabatic lapse rate is,

$$\boxed{\Gamma_d \approx 9.8 \text{ K km}^{-1}}$$