

TO: ICCL

Visit Feng Chia University, Late Autumn, in Taiwan

~Where there is number, there is beauty ~

~ Proclus ~

--- 科普夢 ---

Academic Tour 手札

(Story setting off on Middle Nov. 15, 2007, finished in early March, 2008)

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Nov. 15 (Thursday), 2007

在高鐵站，休休地速度由板橋直奔台中，0851 的出發，0931 的到達，再次感受 HSR(High Speed Rail)的快感。 It is a special invitation from Prof. C.C. Chang who was my major professor when I was mater learning student。藉此，想著一件事，我們往往邀請重量級人物來增添氣勢，抬高氣焰，拉高能見度，也累積更多的能量。記得 CPU (Central Police University)前些日子邀請了 NTU (National Taiwan University), principal, Prof. Lee 來校演講，想透過此種邀約，拉拔點 CPU 的知名度。這是人際關係交往的模式之一 藉由外力的輸入來提昇自我內部的能量 藉此達到 “win-win”，邀約單位/人贏的是 所帶來的氣勢與給此處所注入新生命與正面評價 對於被邀約的 key-note speaker 則是一種研究/地位的肯定 也為

工作/生活的歷練 增添/累積新的 credit 該是值得驕傲與成就上的回饋。

台中 在 HSR 的疾駛速度裡 一眨眼間 新竹站擦身而過 過站不停 開往下個驛站 此行程的目的地 台中, **FCU** (**Feng Chia University**)。In Taichung HSR station, there are two students there. PY and CY who are my NTHU graduate students were waiting and looking for me. We made the appointment here to set off the academic visit to Prof. Chang and his “super-lab”. For PY, it is the third time to bring him visiting somewhere in my academic tour. One is in the international scenario, Juju island, in Korea, middle Oct. 2007 (See IEEE, IPC, Jeju-visit/international view at web-talks, Oct. 12 2007). The second time is in the occasion of visiting CMA (See **CMA**-visit/international view at web-talks, Oct. 17 2007) in Oct., 2007 soon after we finished the academic tour in Korea. It is kind of learning for students if they can step out to get the fresh air, own all things fresh. I believe that the dimension energy will come to the brain and deeply inject into the mind much more received in the office/meeting rooms. That is why I always visit somewhere together with my ICCL research members. As to another guy, CY, who is the PHD student studying in NTHU. In this visit of FCU with him, that is because I want him to know more about some other research team, in particular for the “super-lab” of Prof. Chang. By the way, CY can learn the deserved attitudes in the research works. If we do something with a right concept, the target can be touched down soon on the basis of the adoptions of strategies/methodologies. In the HSR station, we were meeting to each other. After that, we got a taxi on the way to FCU.

The office of Prof. Chang is the building of aeronautics. We three guys, PY, CY and me arrive there on time, 1030 and then show up in this office. Just right before we went into the office door, there is a tag shown above the door, “evacuation”. At that moment, I talked to PY and CY, saying that it is really interesting at the dubbed word right posted out of the office. We wonder that the gate is the key-door to open in case

of something emergency case happenings. But it is also the office-door of Prof. Chang. So, I want to say, the office should be intelligence key-door in the life road. Welcome to get it open to make the concerns you worried smooth down. That is the place of intellectuals. Next, we went in the office and chatted with another one guest who is Prof. Shen and is also visiting to Prof. Chang. Prof. Shen is also the past student following the guidance of Prof. Chang in the course of around 1989-1990. He was invited to join the regular meeting of “super-lab” every week. It is so relaxed in the chats with all guests here, this office, due to the achievements under the surveillance of Prof. Chang is abundant. The members who were the students over the past time now have grown up to share the educational ideas, life dreams with the guide. It is a kind of growing if the conversations are connected in the friendships, not the connection line of teacher-student.

Time is up, it is around 1030, The show to lecture my research is now to set off. We all move to the meeting room located in the same floor of the office. When we went into the room, a lot of students over there, they are all the members who follow the guide, Prof. Chang in the academic researches, it is a big team. That is why I want to say this Lab as “super-lab”. Besides, I also saw some senior research members in the Lab. I thought they are some professors who also join the meeting to contribute the ideas so as to make more academic credits in the areas. Before my study talks to all the members in the “super-lab”, Prof. Chang said something story with me to the audiences. It was since 1989...(See **NCCU** -visit/international view at web-talks, Mid-July, 2007), 20-min later, go back to my time about the topic “Digital Forensics and Evidence in Computer and Internet Systems”, it is a particular issue, something fresh if compared to the areas of CHANG-team (“super-lab”). They paid a lot of works in computer security and digital image processing to get avalanche attention received from worldwide academic scholars. I thought that my studies must be a

subset of the “super-lab” in the first moment, except for FORENSICS when I received the invitation of CHANG-team. The topic, “FORENSICS” is therefore here to begin my invited lecture in this visit tour. As a matter of fact, whenever I meet Prof. Chang, he always hints me the mathematical story due to his great contributions in mathematical applications in computer science studies. That is why the story is entitled as, **“Dream of Mathematical Science”** in the tour-report。

故事其實該從現在開始的…， the tour of visiting FCU, is just to push me getting it written down on this paper…。 …， 總偶而利用研究討論之餘，溫故哲學、數學、科學的書堆世界。近些時候，再讀了 George Polya（喬治 坡里雅）的數學書一遍，有了更多的感覺在其中，迫不急待地想整理出徘徊在心中許久的一些規律性。想著若再不留下痕跡，或許隨著塵囂的生活瑣事又將淹沒在那每次難得的衝動裡…，說到就做到吧！ Here, I would like to show the words, by the way, in life/work determinations as saying **3-C**: “我說到 做到 賞罰必信 in mission running, subject to Conquer”；“你說了 算 是信任 in work assignments, subject to Confidence”；“他說的 是嗎 做確認 in source destination, subject to Confirmation”。 It is really interesting with my determinations, authorities for you and suspects to the anonymous third parties。

數學裡有著歸納法(Induction, so-called mathematical induction)，自然界亦有著歸納法(Induction)，有何不同與差異。數學的發展在早期人類文明史上是自然界萬物一種形與數的規律整理之一。那是實體的世界的觀察與測量的規律性痕跡；而後有了空間的變化，思維的轉換，因而有了抽象的函數表達。這使得自然界綜合現象的解釋 人們生活的問題解決，經濟/工程的應用，有了軌跡可循。這樣的成就使得數學的概觀不再只有形的，那是來自三大數學哲學家之一的 Newton（牛頓）在計算自然界天體運行下所帶來的成就。人類思維的跳躍進展，也使得數字的規律有了有形與無形的相依共生，補足了具體世界裡的想像世界的數學工程規律性。藉由形與數的互動，發展關係變化的代數學與形象變化的幾何

學。形成與數兩者的相得益彰、相互為用、共生共存、截長補短。數少了形的表達，似少了觸感，形少了數的組織，也少了靈性！在 $1+2+\dots+n = n(n+1)/2$ 裡，我們看到了數與形故事的開始，然後呢……在地上撿起一些小石子，在不經意地排列下成了一些三角形狀，諸如：



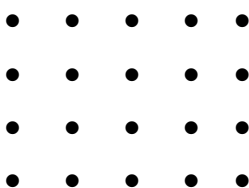
與



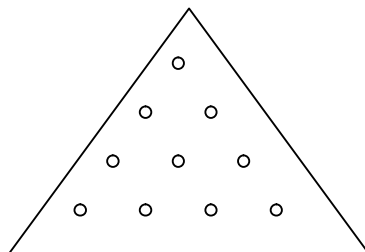
算算其個數，恰有 3 個與 6 個。是否想像下個三角形的形狀為何？個數又為何？在形的堆砌下，數形出來了：



仔細一算，共有 10 個。所以從數形中，我們看到了小的三角形裡， $1+2=3$ ，3 個石頭。中的三角形裡， $1+2+3=6$ ，共 6 個石頭。大三角形裡， $1+2+3+4=10$ ，共 10 個石頭。再大一點的三角形的石頭，將可能是 $1+2+3+4+5=15$ 。藉此，我們似乎想知道當一直排下來去的時候，亦將形成三角形，如同 $1+2+3+4+5+\dots+n$ 。這個三角形會有幾顆石頭在裡面？既然是三角形，那就是矩形的一半囉，所以就是一個矩形石頭數的一半囉。如果有



這樣矩形/正方形排列石頭的再一半石頭，答案會是 2 個三角形的合成，即共有 $4*5$ 個石頭在這個合成的矩形中。所以一半的矩形為三角形，此三角形的數形如下：



矩形的一半即有 $(4*5)/2=1/2*4*(1+4)=1+2+3+4=10$ 個石頭。將之延伸至 $1+2+3+4+5$ 的三角形中，在類推下可得到 $1/2*5*(1+5)=15$ ，當有 $1+2+3+\dots+n$ 的石頭排列呈三角形時，在數形的輔助下，石頭顆粒數隨即原形畢露無遺了：

$$1/2*n*(n+1)=n*(n+1)/2。$$

這個自然下的規律，不覺得神奇而奧妙嗎？這樣觀察在古希臘時代已經由 Pythagoras (畢達哥拉斯) 提出，藉此証實古人對萬物數形觀察之細微。除此外，古人對於天地/陰陽/日月/卜爻的具觀精微，尚還存留著資訊科技的 0/1 世界蘊於其中等著探究呢(More, see **TKU** -visit/international view at web-talks, Late June, 2007)!

歸納法即是形成數與形規律的前置作業，它是觀察自然萬物的現象並整理出具有規律性行為的一種方式。換言之，那即是科學性方法沿用於所有的自然界、生命/現象/科學界探討/研究。當然數學的出現/發展/研究/亦是科學發展的一部份，而在數學裡亦有利用歸納法的精髓來佐證所附予的數與形的具體關連，那即是數學歸納法的來由故事。在數學的世界裡，有許多的有趣/驚訝的結果，皆是由觀察後加以整理而來， it is, therefore, subject to a kind of induction mentioned above。

在數學的數字(Number)世界裡，觀察的本身可從嘗試/實驗而得的，諸如：

$$1^3+2^3+3^3+\dots+n^3=[n(n+1)/2]^2$$

$$1^3=1=1^2=(1)^3$$

$$1^3+2^3=9=3^2=(1+2)^2$$

$$1^3+2^3+3^3=36=6^2=(1+2+3)^2$$

$$1^3+2^3+3^3+4^3=100=10^2=(1+2+3+4)^2$$

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$$1^3+2^3+3^3+\dots+n^3=? \quad (1+2+3+\dots+n)^2,$$

其中 $1+2+3+\dots+n=n(n+1)/2$ ，已在先前數與形的聊述(See **TKU** -visit/international view at web-talks, Late June, 2007)! 在形的排列裡，成就了數的規律。

若我們將 $1^3+2^3+3^3+\dots+n^3=\{n(n+1)/2\}^2$ ，做如此大膽假定，並對更多的數據做推敲，例如：

$$1^3+2^3+3^3+4^3+5^3=225=15^2=(1+2+3+4+5)^2,$$

與

$$1^3+2^3+3^3+4^3+5^3+6^3$$

$$=441$$

$$=21^2$$

$$=(1+2+3+4+5+6)^2。$$

藉此有限的範例，若可由 n 推至 $(n+1)$ 的論證，亦或是在次一數字的依然成立，我們將之命之為數學歸納法，如此罷了。其實是那麼自然地，對於所給的證明/討論在自然的歸納法下多加了些數學上的邏輯推敲，就成了數學歸納法的起源。而究竟如此地推敲合理性值得信賴嗎？

回顧 $1^3+2^3+3^3+\dots+n^3$ 的數字遊戲吧！

依據 $n=1,2,3,4,5,6$ ，

$$1^3+2^3+3^3+\dots+n^3=(1+2+3+\dots+n)^2 \text{ 的事實似乎是正確的。}$$

但再進一個數字的 $(n+1)$ ，諸如

$$1^3+2^3+3^3+\dots+n^3+(n+1)^3=[1+2+3+\dots+n+(n+1)]^2$$

是否依然 ok？就是有趣的詭論。當假定在數字 n 下的可能性，無法有確切的肯定，怎可以再說/猜測未來的(下一個數字， $n+1$)的定論。換言之，若無法肯定這一觀察為 true 時，如何再膽敢進行下個連續而有關連敘述？談到這裡，聯想到人生兩三事。on the basis of induction (就生活哲理而言…)，若未能先有基本溫飽，如何再能“廣施善惠”，照顧更需要關心的民胞物予。

(在靈感下，有話要說…)在 ICCL 的研究之餘，與學生閒敘著生活瑣事與生活情事。在生活瑣事裡，有著易與價值觀的故事：“鳥為食亡，人為財死”，這或許是個自私的社會價值觀：“人不自私，天誅地滅”，然如何營造“win-win”，利人且利己，可有趣了！自私是種表象，是人性的基本，保護自己，保護與個體有關的群體結構，是人們生存的基本論觀，任何可能性的攻擊，下意識本能的抵禦，似乎是與生俱來的反射意識。然智慧的錘鍊是讓人們異於一般生命體最大的生命資產，那或許即是萬物之靈的精義之一吧！

當再說著生活情事時，那是易與人/色的故事：“窈窕淑女，君子好求”，這是個天地/陰陽/男女生存的生活空間。在 易裡的“上經”，三十卦裡談著自然天地萬物的共生共存的牽引關鍵，順勢而行；在“易”裡的“下經”，有著三十四卦，則談著男女與人事的交合互動，形成人們社會諸多生活面觀，順性而為。古智者亦云：“食色性也”，原意乃鼓勵人們欣賞美的諸人/事/物，那是情感的交融，可提昇生命的戰鬥力。然如何做好“mind-mind”（心心相印），安人且安己，那有著家和事興的精髓在其中啊！

正脫離螞蟻世界的紛爭，沈浸在儒佛道的層次…！… 魂回來吧(偶而提升就好了，還是得回到生活現實面啊，哈)！讓我們再回到那邏輯世界的數學吧！若無法肯定這一觀察為 true 時，如何再膽敢進行下個連續而有關連敘述？藉此再引述：

$$\text{Part A: } 1^3 + 2^3 + 3^3 + \dots + n^3 = [n(n+1)/2]^2$$

$$\text{Part B: } 1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3$$

$$= [(n+1)(n+1+1)/2]^2$$

$$= [(n+1)(n+2)/2]^2$$

若為已存在的事實，是否 B 仍可成立呢？我們存著沒有把握的不確定性。無論如何，若將 Part B - Part A，將得到

$$B-A \Rightarrow [1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3] - [1^3 + 2^3 + 3^3 + \dots + n^3] \text{ (前者)}$$

$$= [(n+1)(n+2)/2]^2 - [n(n+1)/2]^2 \text{ (後者)}。$$

前者的化簡值 $(n+1)^3$ 將等於後者的化簡值 $(n+1)^3$ 。

換言之，這個 $(n+1)^3$ 可等於此二個式子

$[(n+1)(n+2)/2]^2$ 與 $[n(n+1)/2]^2$ 的差值。

此時令 Part C 為上述的推導事實：

$$(n+1)^3 = [(n+1)(n+2)/2]^2 - [n(n+1)/2]^2。$$

那麼“若已存在的事實” Part A + “論證推得的事實” Part C。

$$\text{Part A : } [1^3 + 2^3 + 3^3 + \dots + n^3] = [n(n+1)/2]^2。$$

$$\text{Part C : } (n+1)^3 = [(n+1)(n+2)/2]^2 - [n(n+1)/2]^2。$$

兩者合併可表示為

$$[1^3 + 2^3 + 3^3 + \dots + n^3] + (n+1)^3 = [(n+1)(n+2)/2]^2。$$

以此類推，對於數值 n 成立時，數值 $n+1$ 成立時亦成立的論述得以確認。自然的歸納法觀察，在透過導引的數學遞移推敲，藉由 Part A 為事實、Part C 亦為事實，使得 A+C 亦為事實，成就了所謂的數學邏輯歸納法，即眾所皆知的數學歸納法。

這個故事，穿梭於訪學與數學哲理間，為了是藉遊訪之間情雅致，將所瞭解的知識能得以整理，也藉此留下痕跡。偶而哲理；時而數理，那皆生命/生活的精華與依託。2007年，端出了一些生活菜色的篇幅，說著螞蟻王國的故事，藉以聊述/自警與勉語 to ICCL。2008，一個新的開始，想延續這個哲學/數學/科學的故事…。那麼就叫做科學普讀的現代術語：“**科普夢**”！將隨著研究/工作，在內修/外訪/上行/下效裡有著生活之道 (Tao)。就拭目以待，好吧！“有規劃的等待，夢才是美的”，是吧！