Project 3 - Bicycle

MEMS 1029 Pre #3

Live Recording: https://www.youtube.com/watch?v=-pEQ-IkWN5A.
Supplementary Folder: https://github.com/ice-bear-git/_MEMS1029_DesignII_onGithub/tree/main/Project/Project3.

2022/04/22

Team Member

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Puhang Cai	puc4@pitt.edu	(work2)
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* The Fifth guy is totally unreachable – never reply to meeting email.

Project Overview - Workload

Task	Time (hrs)	Members
Overall Managements		
Project Orgnization	5	ZIC
Analysis		
F, v on diff inclines	5	ALL
Chain	10	MZC, GYM
Brakes	10	ZIC
Bolted Joints	15	PHC
All Hand-drawing FBD	1	ZIC
Presentation -- Slide		
Overview	1	ZIC
F, v on diff inclines	3	GYM, ZIC
Chain	3	MZC
Brakes	3	ZIC
Bolted Joints	3	PHC
Comments	1	ZIC
Improvements	3	GYM
Summary	1	ZIC

ZIC: Ziang Cao, MZC: Mingze Cai, GYM: Yuming Gu, PHC: Puhang Cai

Project Overview -

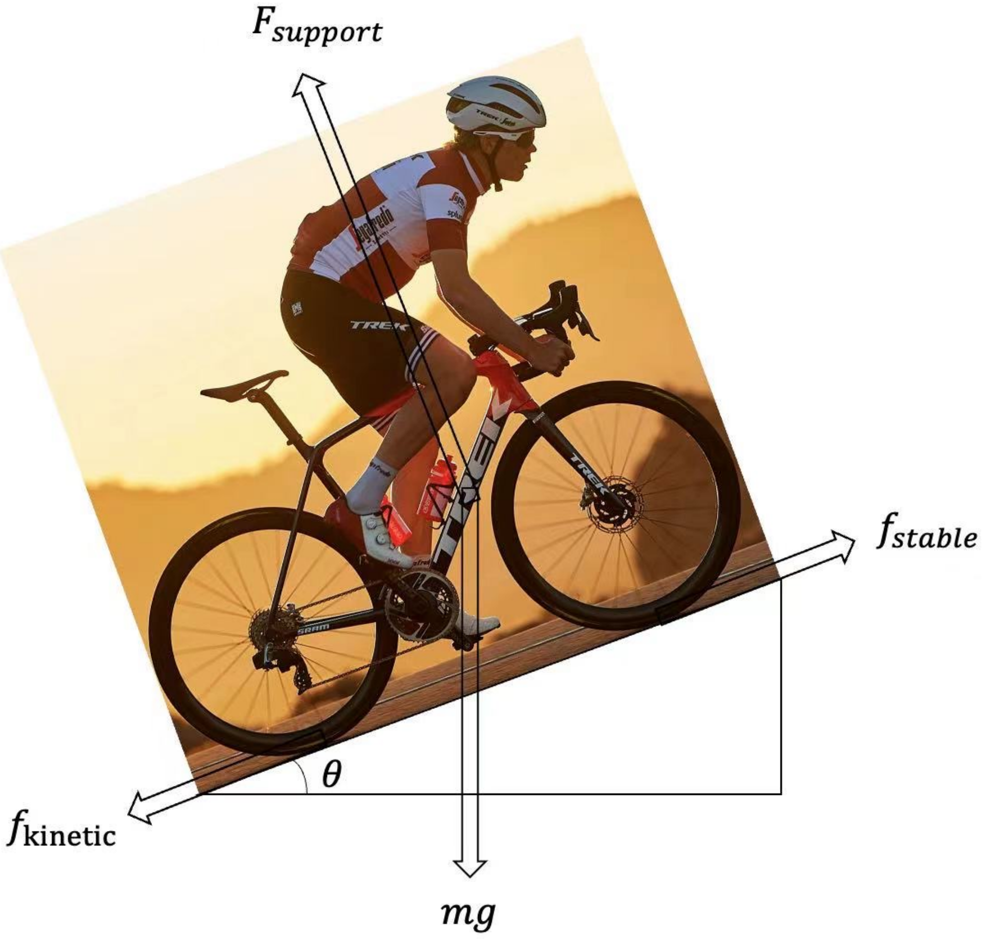
2022/04/22

Project Overview – Measurement and Reference

Measure

SHIMANO R8000

Drivetrain - Overview

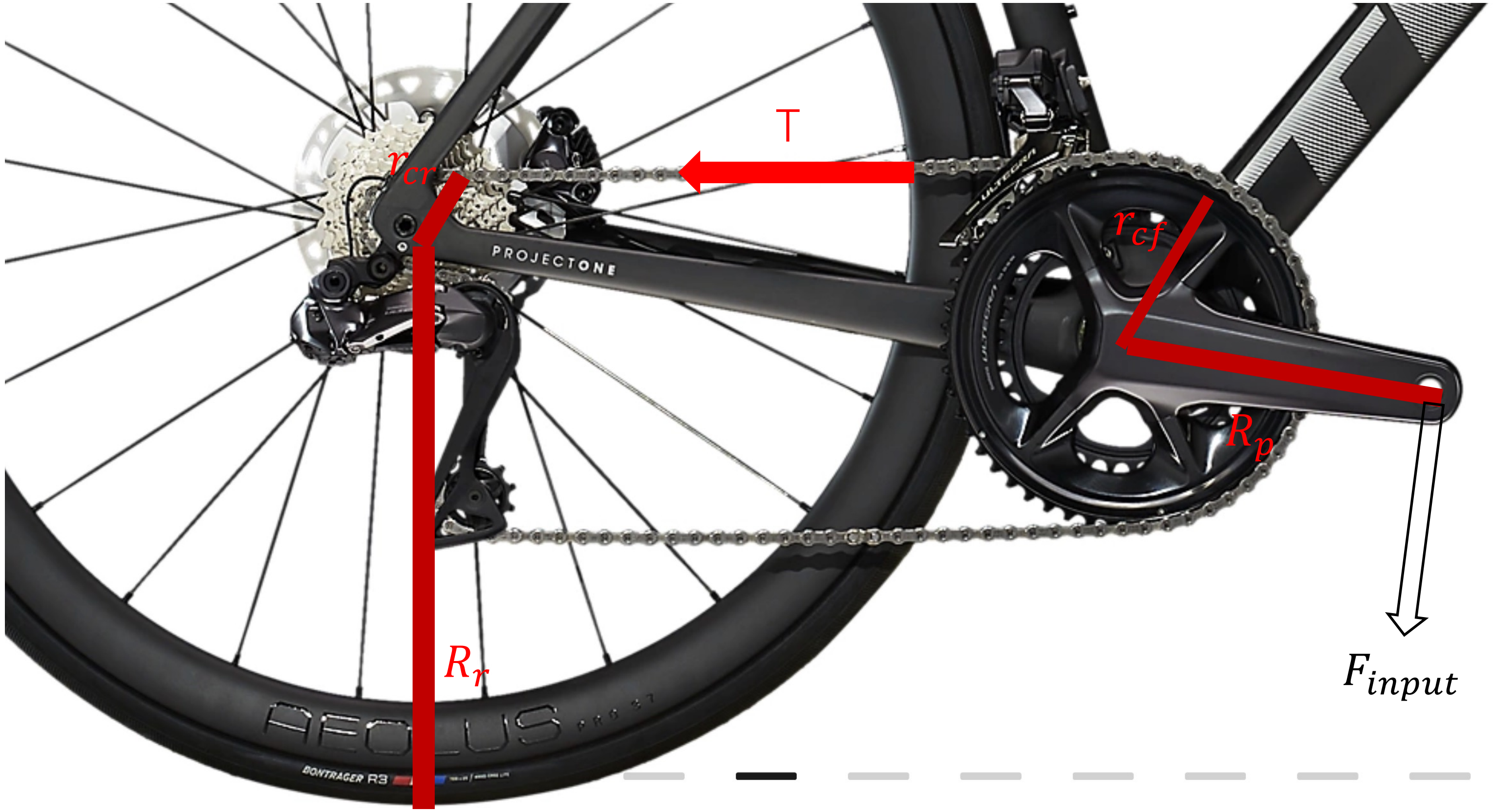


Kinematic Friction here under constant velocity is only based on rolling friction

The Stable Friction is a representation for driving force based on adhesion friction;

Here, we do not care how we convert the rotation of rear wheel to the force; We study the torque balance at rear wheel directly.

Drivetrain - Analysis



Directly Use the Idea that Single Equivalent Forces are balance on the chain.

$$\mathrm{F}_{input} * R_p = T * r_{cf}$$

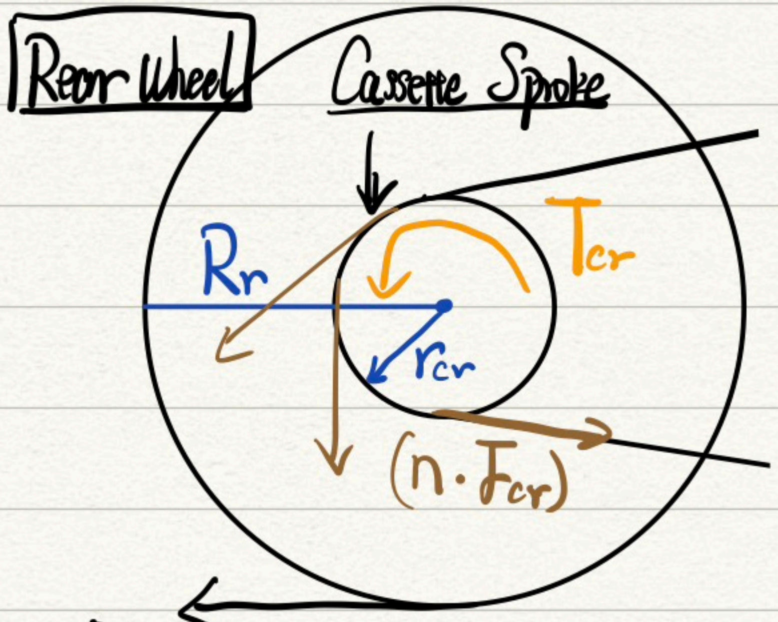
$$f * R_r = T * r_{cr}$$

$$F_{input} = T * \frac{r_{cf}}{R_p} = f_{stable} * \frac{R_r}{r_{cr}} * \frac{r_{cf}}{R_p}$$

Note: T for here is Tension, not Torque

Drivetrain - Analysis

Detail Analysis of chain transmission from the Energy view



Assume there is $\boxed{\text{no energy loss}}$ on chain

↳ $P = T \cdot \omega$

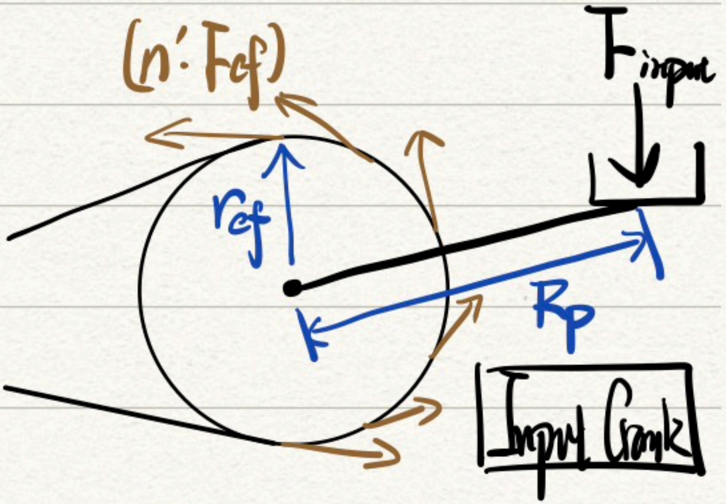
↳ $T_{cr} \cdot \omega_{cr} = T_{cf} \cdot \omega_{cf}$.

T_{cr}, T_{cf} means Torque at two place.

∵ Chain ⇒ $\omega_{cr} \cdot r_{cr} = \omega_{cf} \cdot r_{cf}$

with some module $r_{cr} \propto n_{cr}$

∴ $\boxed{T_{cf}} = \frac{\omega_{cr}}{\omega_{cf}} \cdot T_{cr} = \boxed{\frac{n_{cf}}{n_{cr}} \cdot T_{cr}}$



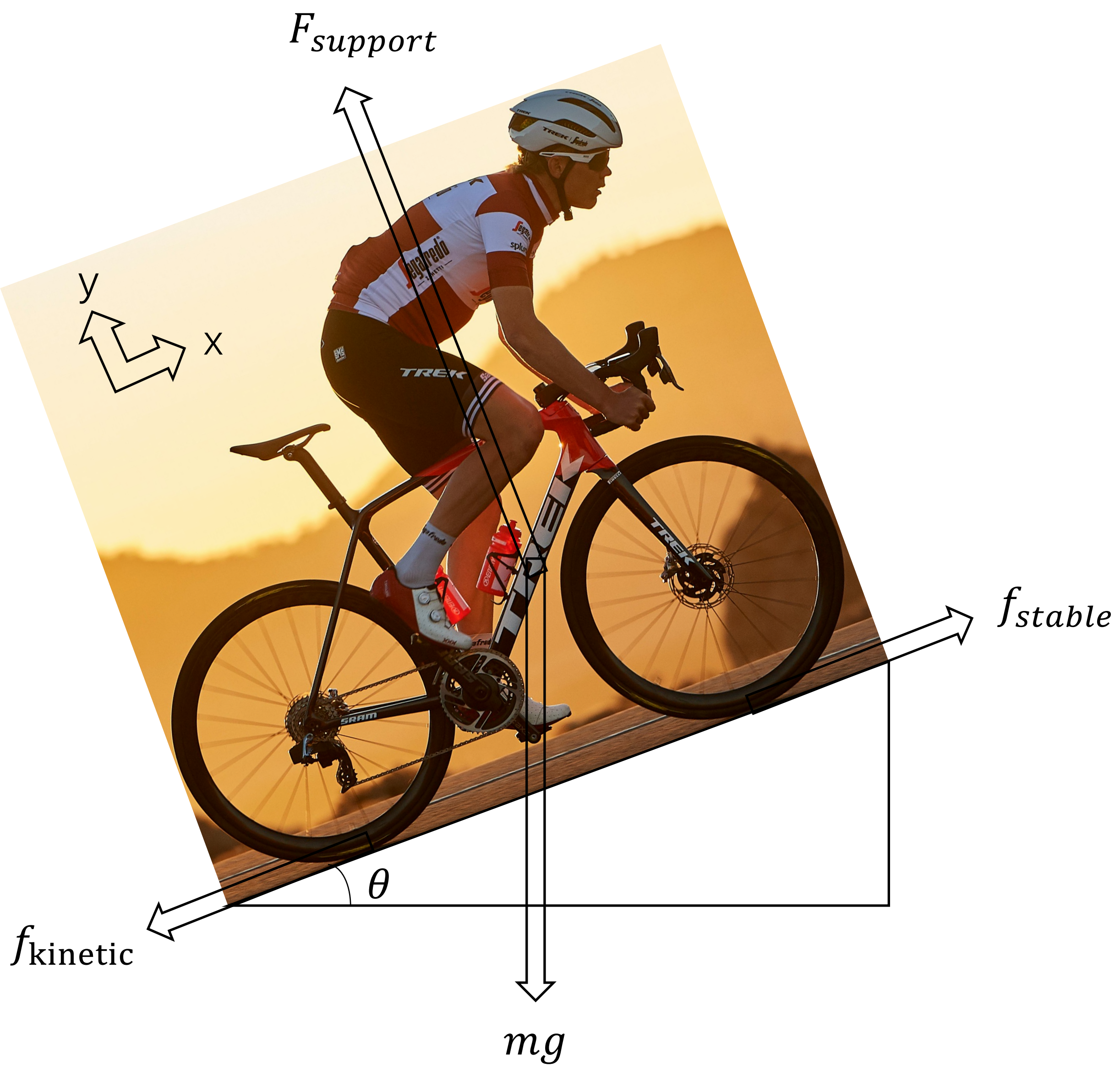
$\Sigma T = 0: \boxed{T_{cf}} = (n' \cdot F_{cf} \cdot r_{cf}) = \boxed{R_p \cdot F_{input}}$.

$F_f = Mg(\sin\theta + \mu_{a.r} \cdot \cos\theta)$

$\Sigma T = 0: \boxed{F_f \cdot R_r} = (n \cdot F_{cr} \cdot r_{cf}) = \boxed{T_{cr}}$

Combine All 3 equations ⇒ $\left[F_{input} = \frac{1}{R_p} \cdot \frac{n_{cf}}{n_{cr}} \cdot F_f \cdot R_r \right]$

Drivetrain - Overview



$$\Sigma x = 0, \Sigma y = 0$$

$$f_{stable} = (m_h + m_b) * g * \sin(\theta) + f_{kinetic}$$

$$f_{kinetic} = \mu * (m_h + m_b) * g * \cos(\theta)$$

$$F_{input} = (m_h + m_b) * g * (\sin(\theta) + \mu * \cos(\theta)) \frac{R_r}{R_p} * \frac{r_{cf}}{r_{cr}}$$

$$\omega_{paddle} = \frac{P}{F_{input} * R_p}$$

$$\omega_{rear(wheel)} = \frac{r_{cf}}{r_{cr}} * \omega_{paddle}$$

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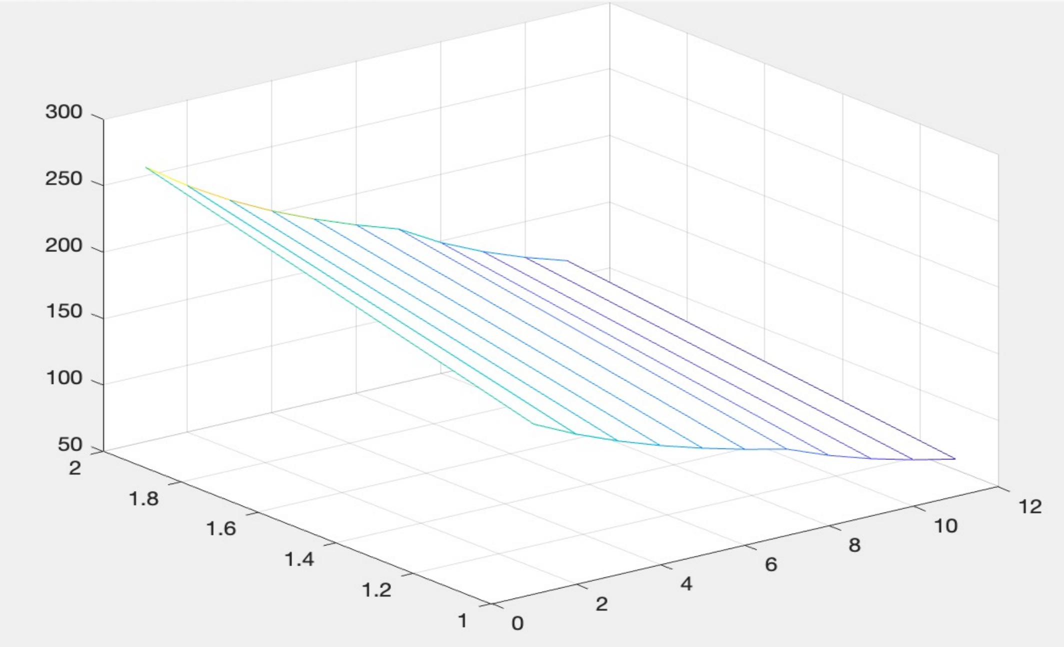
Drivetrain - Overview

```
#Section 1: Analysis the Power input on the pedal
mh=70;                                    #mh is the weight of human, in kg
mb=8;                                     #mb is the weight of bike, in kg
g=9.8;                                    #g is the gravitiy acceleration, in N/kg
theta=math.pi/9;                          #theta is degree of the incline
Rr=0.068/2;                               #Rr is the radius of the wheel (27 inches diameter)
Rp=0.17;                                  #Rp is the length of the crank, we choose 170mm
miu=0.014;
Tcf=[36,52];                              #Tcf is the number of teeth of the crankset, we choose 52-36T
Tcr=[11,12,13,14,15,16,17,19,21,23,25]    #Tcr is the number of teeth of the casette, we choose 11-25T
P=300                                     #P is average input power, in W
```

Force Input for different setting combination		
Number of Teeth on Casette	Number of Teeth at Crankset	
	36	52
11	177.7067425	256.6875169
12	162.8978473	235.2968905
13	150.3672436	217.1971297
14	139.6267262	201.683049
15	130.3182778	188.2375124
16	122.1733854	176.4726679
17	114.9867157	166.0919227
19	102.8828509	148.6085624
21	93.08448415	134.455366
23	84.99018118	122.763595
25	78.19096669	112.9425074



Drivetrain - Overview

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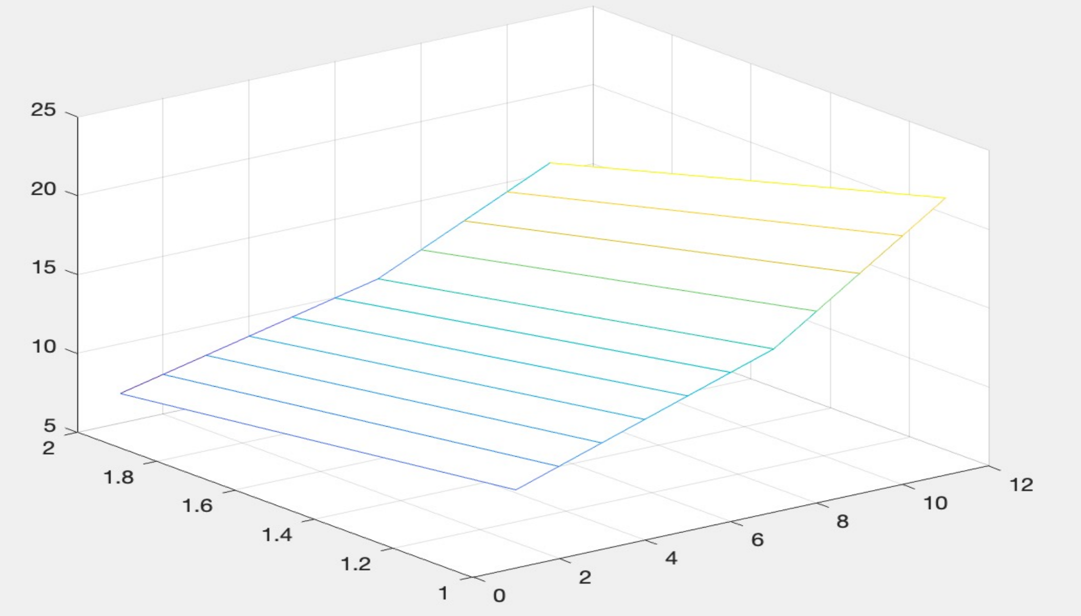
P=300                                     #P is average input power, in W
```

Paddle Rotating Speed for different setting combination		
Number of Teeth on Casette	Number of Teeth at Crankset	
	36	52
11	9.930438529	6.874918982
12	10.83320567	7.499911617
13	11.73597281	8.124904251
14	12.63873995	8.749896886
15	13.54150709	9.374889521
16	14.44427422	9.999882155
17	15.34704136	10.62487479
19	17.15257564	11.87486006
21	18.95810992	13.12484533
23	20.7636442	14.3748306
25	22.56917848	15.62481587



Drivetrain - Overview

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Tcr=[11,12,13,14,15,16,17,19,21,23,25]  #Tcr is the number of teeth of the casette, we choose 11-25T
P=300                                   #P is average input power, in W
```
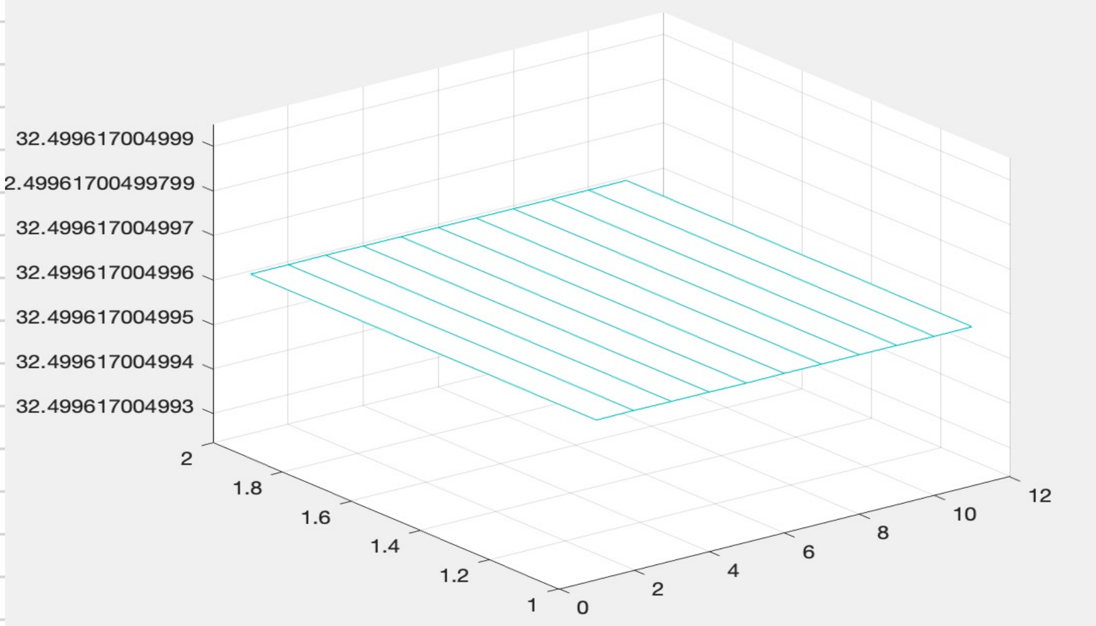
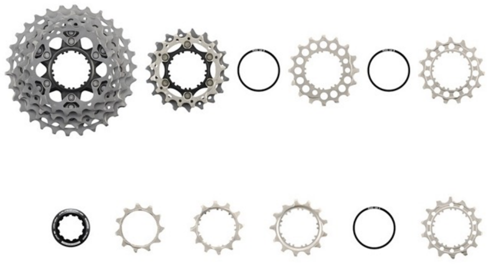

Wheel Rotating Speed for different setting combination		
Number of Teeth on Casette	Number of Teeth at Crankset	
	36	52
11	32.499617	32.499617
12	32.499617	32.499617
13	32.499617	32.499617
14	32.499617	32.499617
15	32.499617	32.499617
16	32.499617	32.499617
17	32.499617	32.499617
19	32.499617	32.499617
21	32.499617	32.499617
23	32.499617	32.499617
25	32.499617	32.499617



Drivetrain - Cassette

The number of sprockets in the Cassette is 11.

The number of the teeth is 11-12-13-14-15-16-17-19-21-23-25



Drivetrain - Chainrings

The number of sprockets in the front is 2

The number of the teeth is 36-52

Ref:

Drivetrain – Gear ratio

1. Gear Ratio = Number of Teeth in Chainring ÷ Number of Teeth in Cassette

The number of teeth in Cassette

The number of teeth in Chainrings		11	12	13	14	15	16	17	19	21	23	25
	36	36/11	36/12	36/13	36/14	36/15	36/16	36/17	36/19	36/21	36/23	36/25
	52	52/11	52/12	52/13	52/14	52/15	52/16	52/17	52/19	52/21	52/23	52/25

Table of available gear ratios

2. average spacing between possible gear ratios

$$\boldsymbol{gear\ ratio = R_i}$$

$$average\ spacing\ netween\ gear\ ratio = \frac{R_{max} - R_{min}}{n-1} \quad (n\ is\ the\ number\ of\ possible\ gear\ ratio)$$

The average spacing between possible gears is $\frac{\frac{52}{11}-\frac{36}{25}}{2\times 11-1}$=0.1565

Ref:

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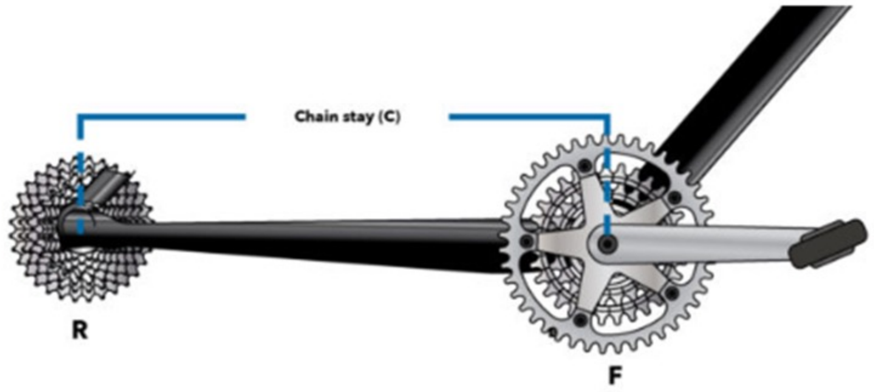
Drivetrain – Chain Analysis

Chain Length L_p

$$L = 2(C) + (F/4 + R/4 + 1)$$



L = Length – your chain length in inches (rounded to the nearest inch).

C = Chain stay – the distance between the middle of the crank to the rear axle. Measure to the closest ⅛ and convert this to decimal form.

F = Front chainring – The number of teeth on the largest (front) chainring. This number is often printed on the sprockets and cogs, but if not you'll have to count.

R = Rear cog – The number of teeth on largest rear cog. Again, this number is usually printed on the sprocket.

Ref:

Drivetrain – Chain Analysis

Tension on Chain

$$F = (m_h + m_b) * g * (sin(\theta) + \mu * cos(\theta)) * (\frac{R_r}{R_p}) * (\frac{i}{j})$$

m_h is the weight of human, in kg
m_b is the weight of bike, in kg
g is the gravitiy acceleration, in N/kg
θ is degree of the incline
Rr is the radius of the wheel (27 inches diameter)
Rp is the length of the crank, we choose 170mm
Tcf is the number of teeth of the crankset, we choose 52-36T
Tcr is the number of teeth of the casette, we choose 11-25T
P is average input power, in W
i is the constant from Tcf
j is the constant from Tcr

Ref:

Drivetrain – Chain Analysis

Chain Power

$$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}} \quad \text{hp} \tag{17–33}$$

where N_1 = number of teeth in the smaller sprocket

n_1 = sprocket speed, rev/min

p = pitch of the chain, in

K_r = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

Drivetrain – Chain Analysis

The chain life span is limited by the chain power due to impact fatigue

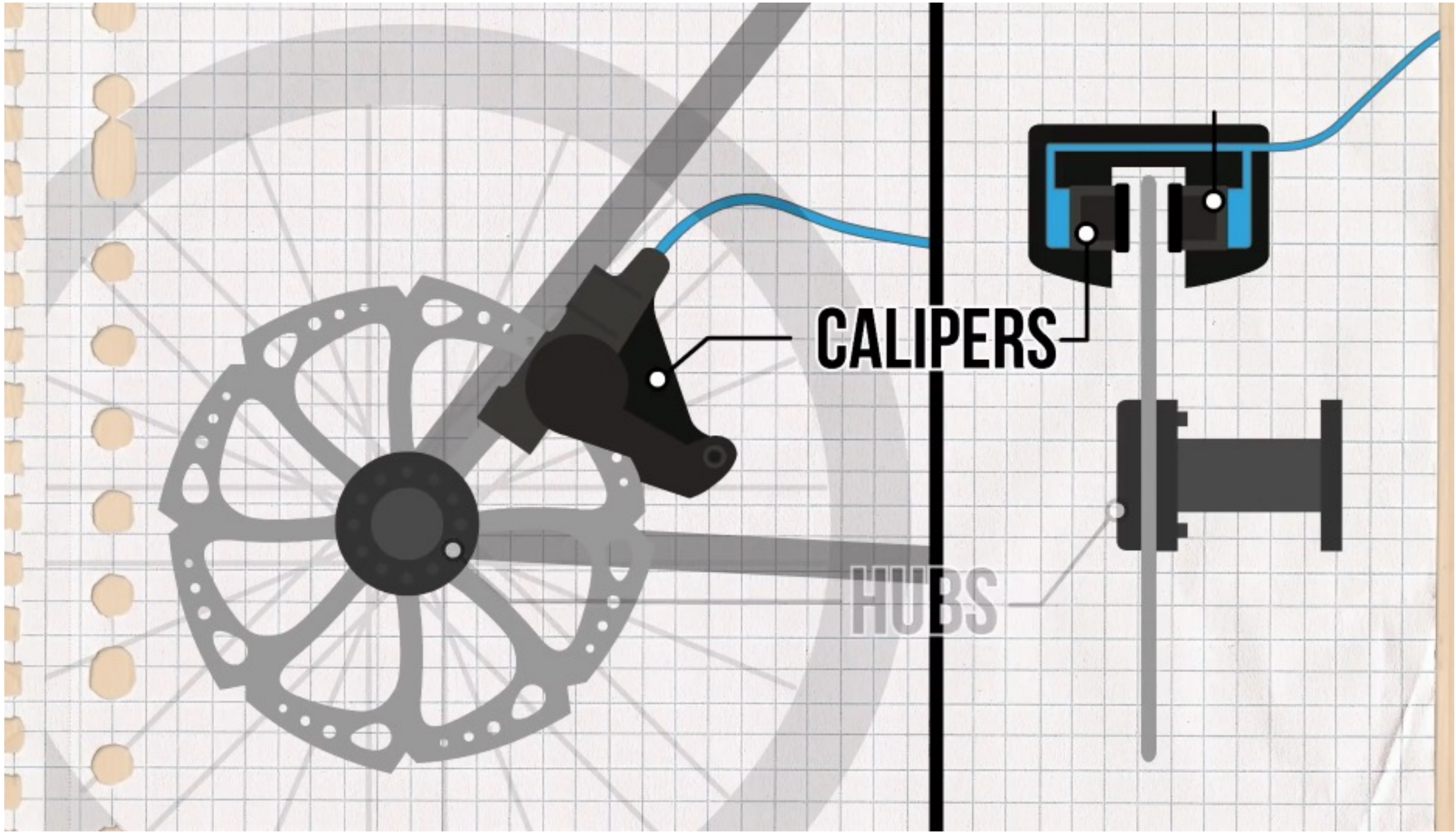
$$H_2 = 1000\left[K_r\left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8}\left(\frac{L_p}{100}\right)^{0.4}\left(\frac{15\,000}{h}\right)^{0.4}\right]$$

the Chain life span h

According to the estimation, the chain is able to experience 7500-hour use before the failure occurs

Brakes – Overview

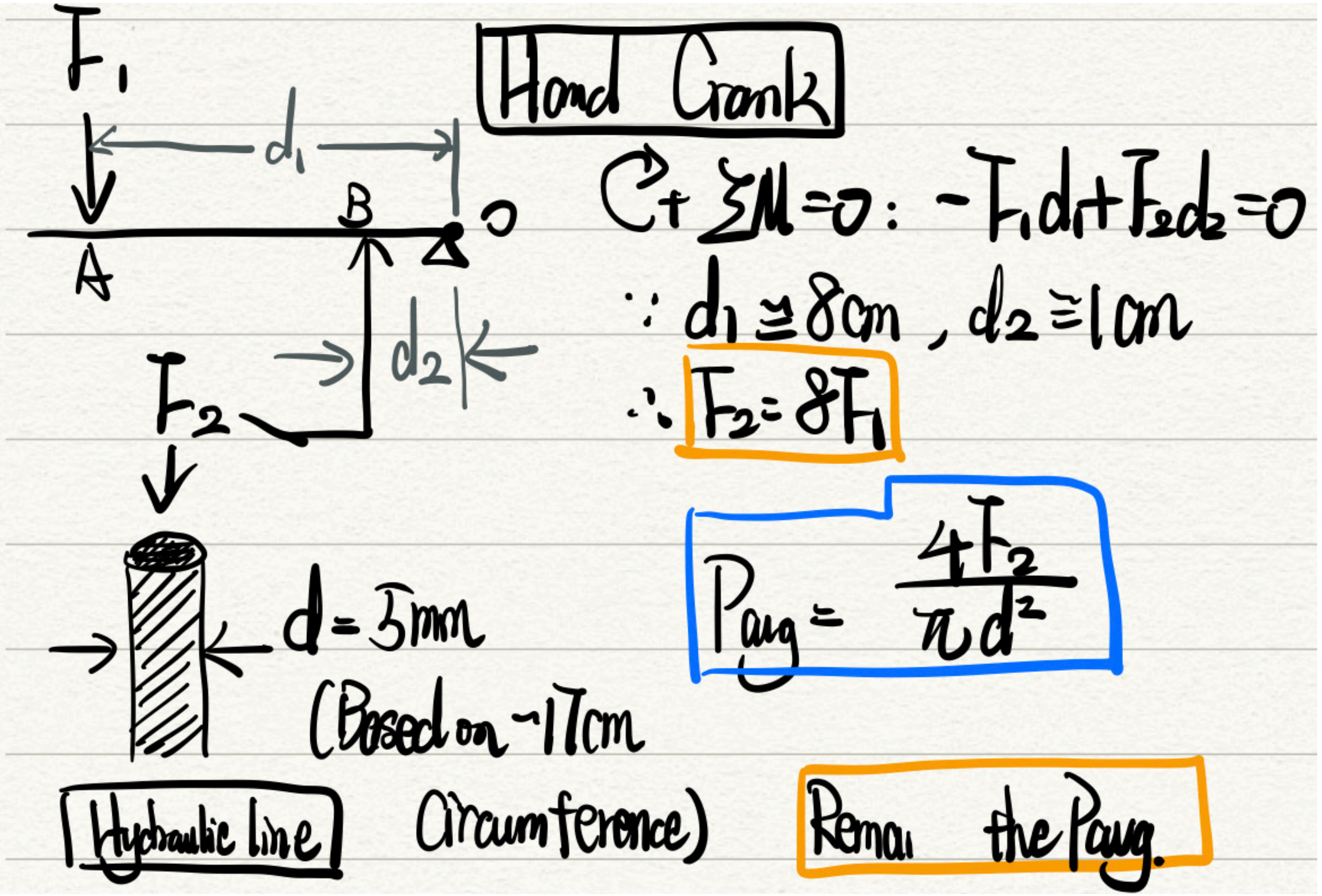
Fundamentals



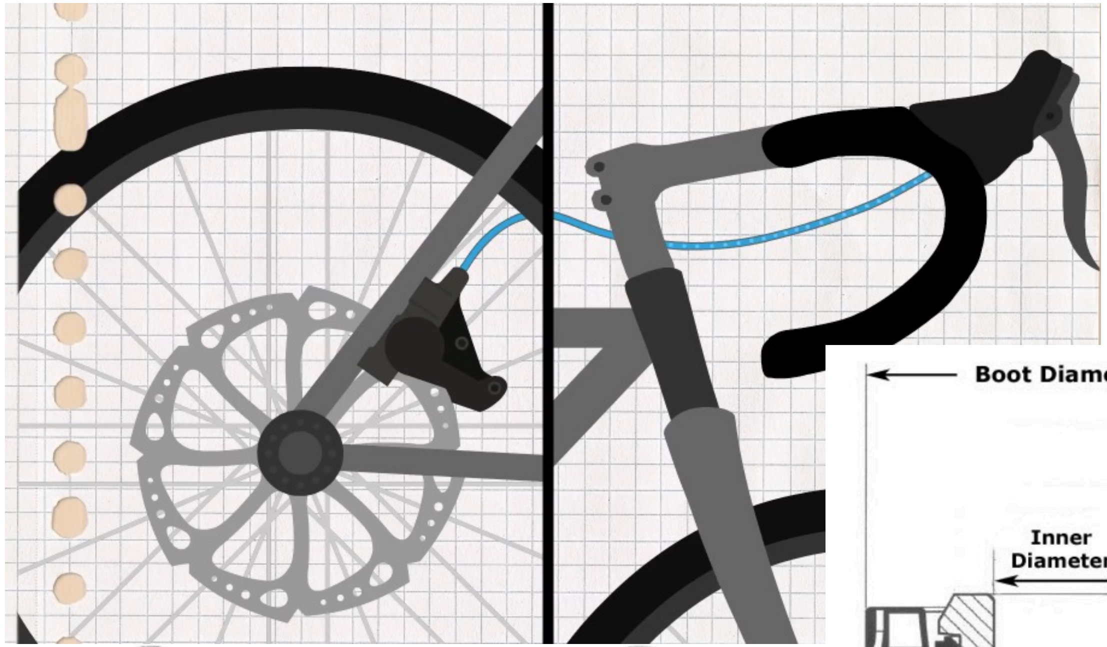
Ref: https://www.youtube.com/watch?v=U_Rnr_flVq8, https://bike.shimano.com/en-AU/product/component/ultegra-r8000/ST-R8020-L.html

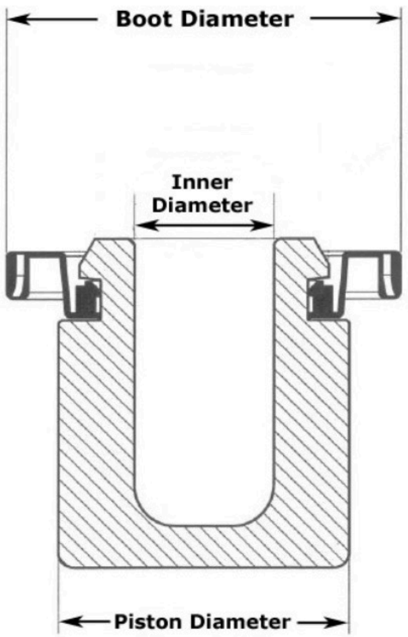
Brakes – Hand Crank (lever)

FBD + Calculation

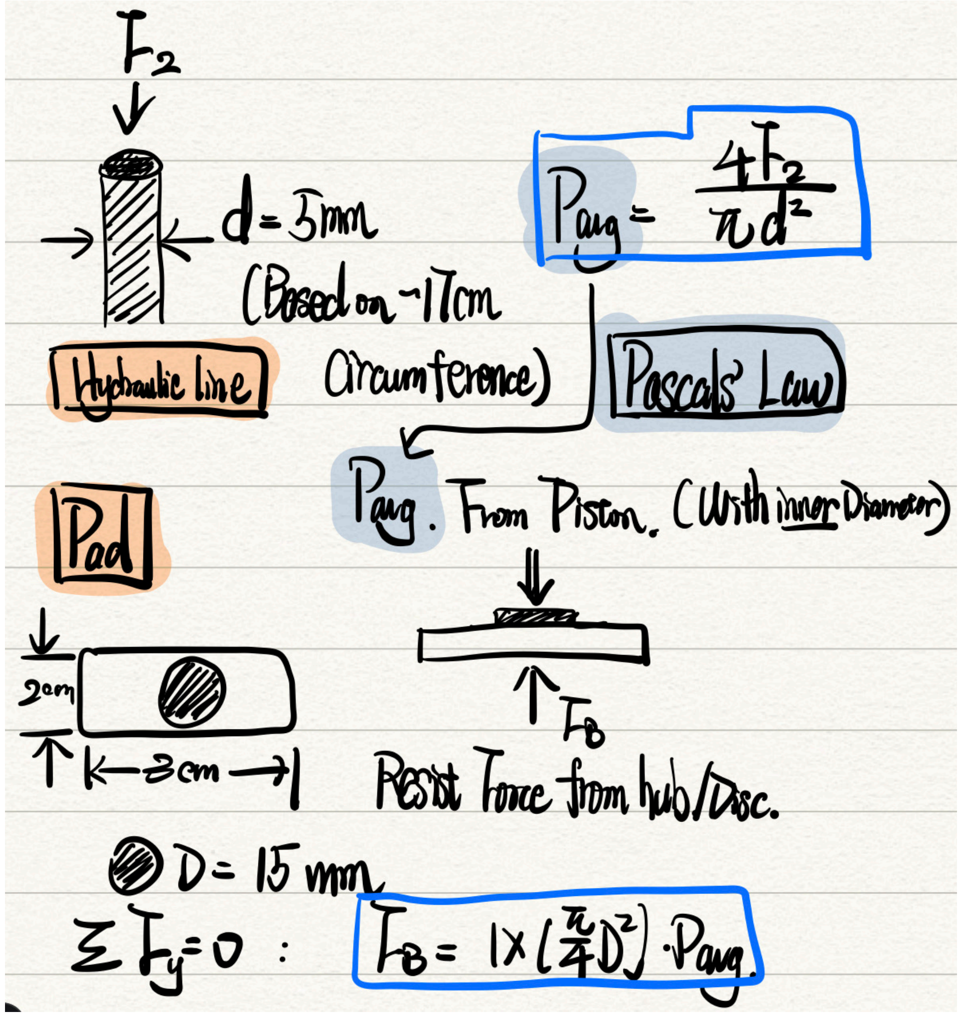


Brakes – Pascals' Law





FBD + Calculation



Ref: https://www.youtube.com/watch?v=U_Rnr_flVq8, https://hypertextbook.com/facts/2005/steel.shtml

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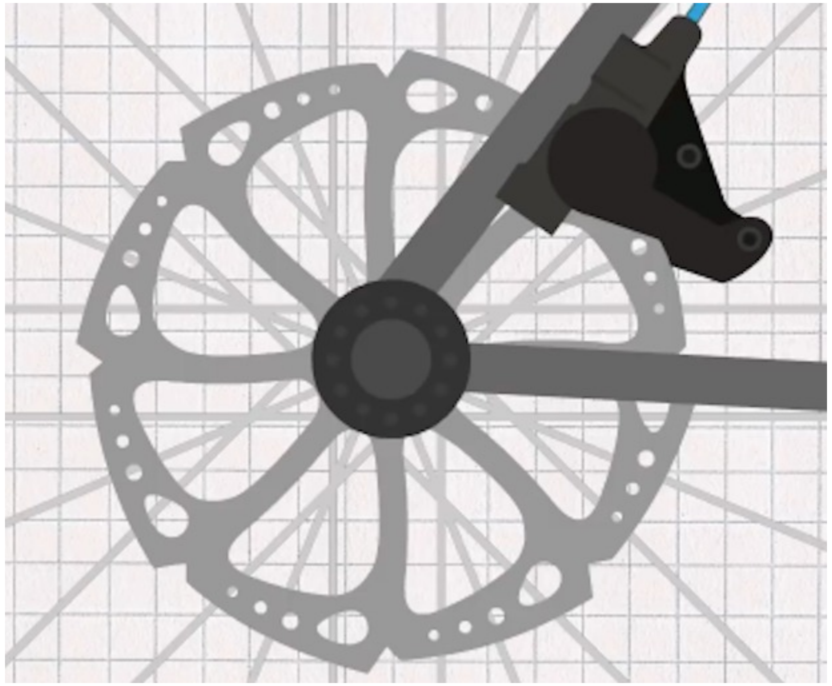
Brakes – Brake Surface

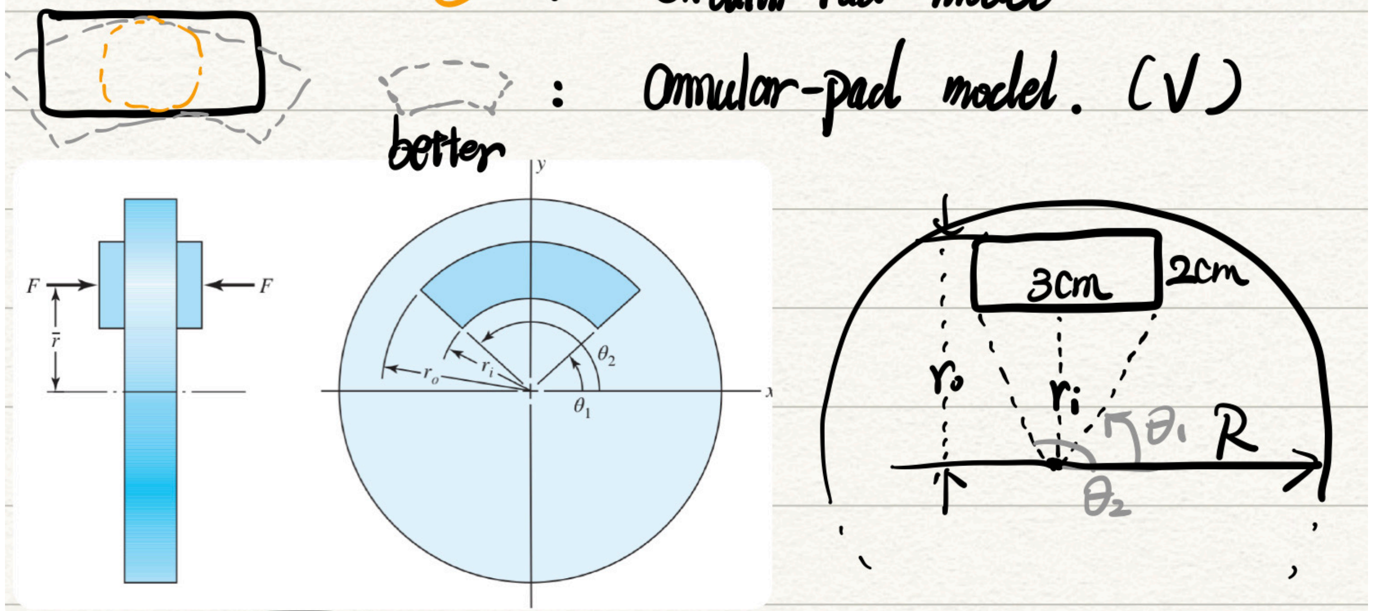


FBD + Calculation

○ : Circular Pad model

: Omnular-pad model. (V)

better



$\boxed{\text{Uniform Wear Mode}}$

Notes: $R = 80\ mm$, $r_i = 55mm$, $r_o = 75mm$

$F = (\theta_2 - \theta_1) P_a \cdot r_i (r_o - r_i)$

$(\theta_2 - \theta_1) = 2 \times \tan^{-1}\left(\frac{15mm}{55mm}\right) \approx 30°$

$T = \frac{1}{2}(\theta_2 - \theta_1) \cdot f p_a \cdot r_i (r_o^2 - r_i^2)$, where f is μ_B

Ref: https://www.youtube.com/watch?v=U_Rnr_flVq8, https://bike.shimano.com/en-AU/product/component/ultegra-r8000/ST-R8020-L.html

Brakes – Friction coefficient in Rainy Day

Method 1

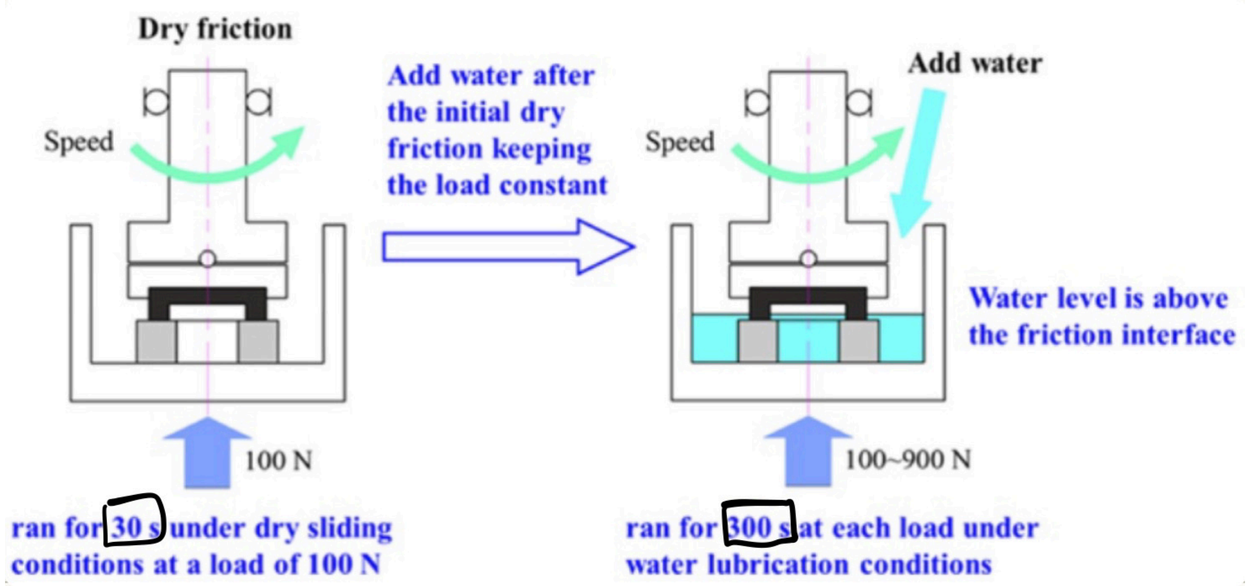
Surface	Adhesion coefficient
Dry Concrete	0.85
Wet Concrete	0.55
Sand	0.35
Ice	0.1

Sand → Ice // $K = \frac{0.1}{0.35} = \frac{1}{3.5}$

0.35 → 0.1 // For the Brake Surface

Dry : η_B ⇒ Wet: $(\frac{1}{3.5})\eta_B$

Method 2



• From Lagrange (Energy) view : $\propto K$: $1 \to \frac{1}{10}$

⇓

• From Newton (Force) view : $\propto \sqrt{K}$: $1 \to \frac{1}{\sqrt{10}} \approx \frac{1}{3.5}$

Conclusion: The friction changing **Trend/Factor** from Ideal/Dry to Rainy scenario will be **1/3.5**

Ref: https://hypertextbook.com/facts/2005/steel.shtml, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5428033/

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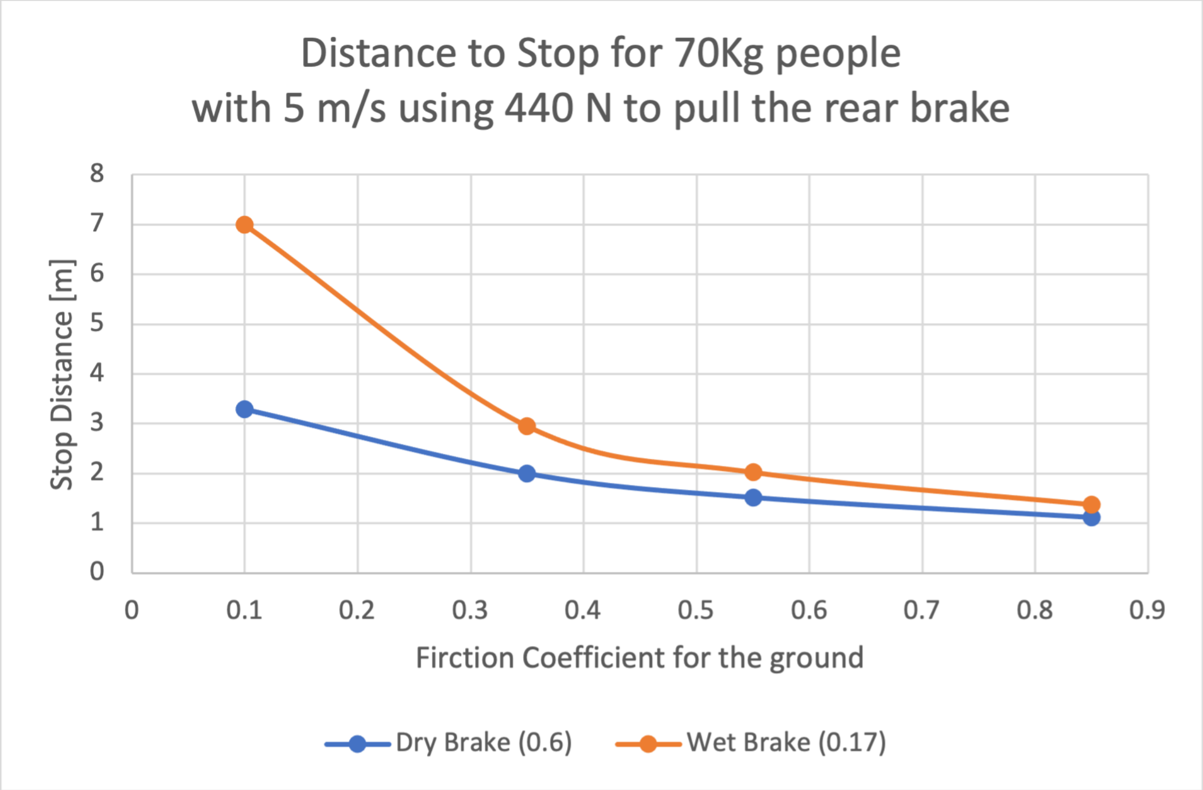
Brakes – Performances in Multiple Case

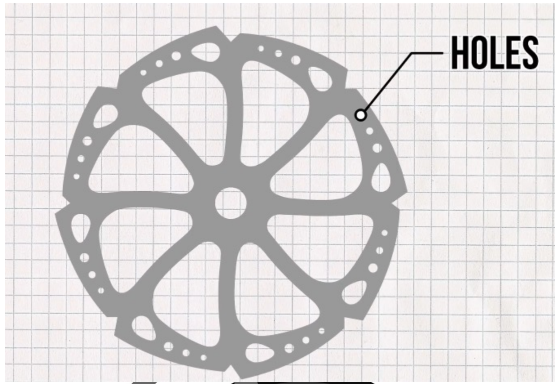
Distance to Stop

Distance to Stop [m]		
Ground Condition	**Dry Brake (0.6)**	**Wet Brake (0.17)**
Dry Concrete (0.85)	1.12	1.37
Wet Concrete (0.55)	1.52	2.02
Sand (0.35)	2	2.95
Ice (0.1)	3.29	7

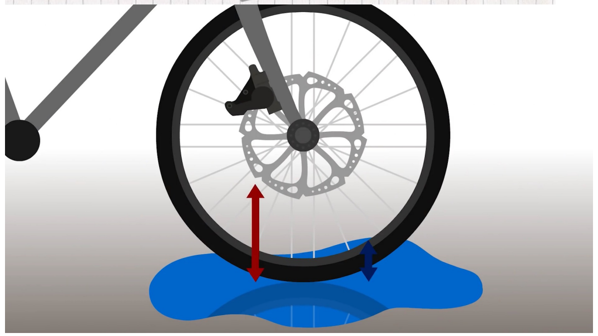


Conclusion

- Ground Scenario dominants the distance
- **Brakes coefficient becomes matter** for more critical/dangerous case -- on **Ice**
- Want shrink the difference



allows stuffs to leave from the braking surface



out of the small water puddle

Ref: https://www.youtube.com/watch?v=U_Rnr_flVq8

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Bolted Joints - Overview

Figure bolt1. Overview of bolt size [1].

Table bolt1. table for bolt size [2].

Name	Diameter	Head/nut width	Nut height	Thread Cross-sectional areas	Thread lengths	Total length
Seat Post Bolt	M7, M8	12 mm	6.5 mm	36.6 mm^2	30 mm	45, 50 mm
Seat Post Binder	M5, M6, M8	7 mm	5 mm			22, 30 or 40 mm
Bottle Cage Bolt	M5	14 mm	4 mm	14.2 mm^2	12 mm	16 mm
Star Nut Bolt	M6					18, 23, 26, 40 mm
Stem Bolt	M5	12 mm	4 mm	14.2 mm^2	14 mm	16, 18, 22 mm
Disc Brake Bolt	M5, M6	10 mm	6 mm	14.2 mm^2	16 mm	16, 20 mm

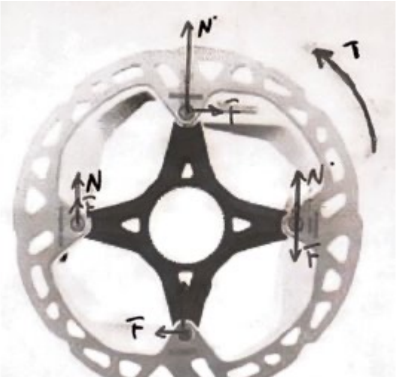
Bolted Joints - Analysis

Chainring bolt

Table bolt2. table for bolt specifications .

Figure bolt2. Chainring for R8000[3].

Disc mounting bolt



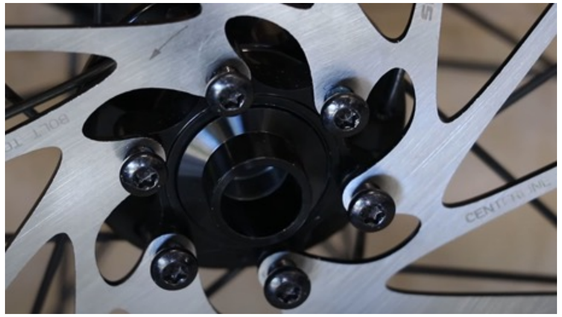


Figure bolt3. Disc for R8000[4].

Name	Diameter	Total Length	Thread Length	Torque	Bolt Stiffness	Member Stiffness	Modulus of Elasticity	Tensile Stress Area	Preload for Each Bolt	Joint Stiffness Constant
Disc Mounting Bolt	M5	18 mm	15 mm	2-4NM	795.6N/μm	1827N/ μm	200 GPa	14.2mm^2	4856.4N	0.3033
Chainring Bolt	M8	10.1 mm	8 mm	7.9-10.7NM	917.53N/μm	1180.24N/ μm	200 GPa	36.6mm^2	8219.5N	0.4374

Equations used for analysis:

Grip length:

$$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$

Fastener stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

Fastener stiffness:

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_i}$$

$$k_m = \frac{0.5774\pi E d}{2 \ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)}$$

Load Factor:

$$n_L = \frac{S_p A_t - F_i}{CP}$$

Torque:

$$T = K \times F_i \times d$$

Preload:

$$F_i = \begin{cases} 0.75F_p & \text{for nonpermanent connections, reused fasteners} \\ 0.90F_p & \text{for permanent connections} \end{cases}$$

To avoid Fatigue:

$$F_i \leq (1 - C) S_{ut} A_t$$

Factor of Safety Against Joint Separation:

$$n_0 = \frac{F_i}{P(1 - C)}$$

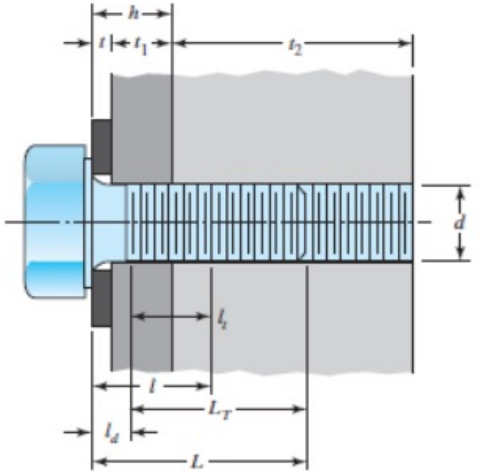


Figure bolt4. Analysis length for the bike[5].

Bolted Joints - Analysis

Equations used for analysis:

Load Factor of Safety:

$$n_L = \frac{S_p A_t - F_i}{CP}$$

Bearing in bolts, all bolts loaded:

$$F = \frac{2tdS_y}{n_d}$$

Bearing in members, all bolts active:

$$F = \frac{2td(S_y)mem}{n_d}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$F = \frac{0.577 \times n \times A_d S_y}{n_d}$$

If the bolt threads extend into a shear plane:

$$F = \frac{0.577 \times n \times A_t S_y}{n_d}$$

Edge shearing of member at two margin bolts:

$$F = \frac{4at0.577(S_y)_{mem}}{n_d}$$

Where nd is the design factor, and we do not consider tensile yielding of members across bolt holes. Because the area is much larger, and failure is not likely to happen here.

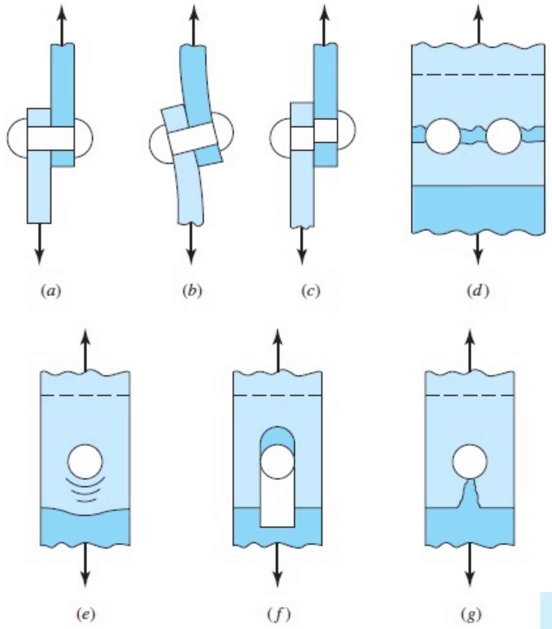


Figure 8–25

Modes of failure in shear loading of a bolted or riveted connection: (*a*) shear loading; (*b*) bending of rivet; (*c*) shear of rivet; (*d*) tensile failure of members; (*e*) bearing of rivet on members or bearing of members on rivet; (*f*) shear tear-out; (*g*) tensile tear-out.

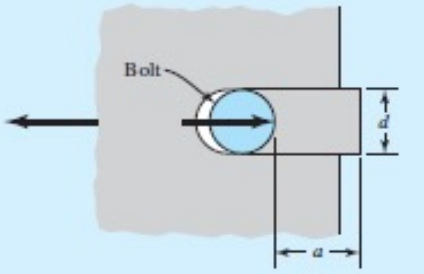


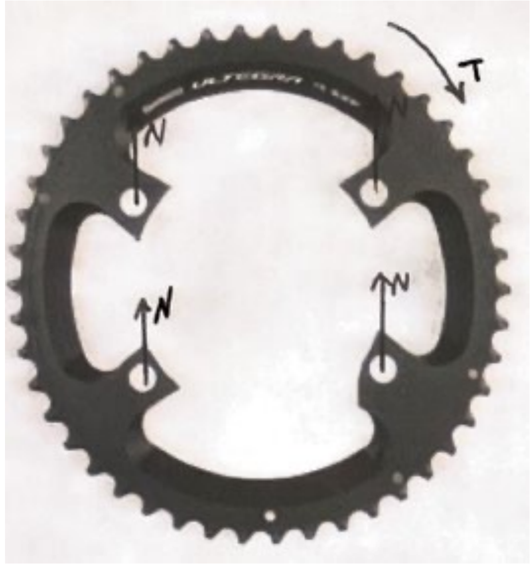
Figure 8–27

Edge shearing of member.

Figure bolt5. Shearing and bearing on bolts and members

Bolted Joints - Analysis

Free Body Diagram for Bolts on Chainring and Disc Brake.



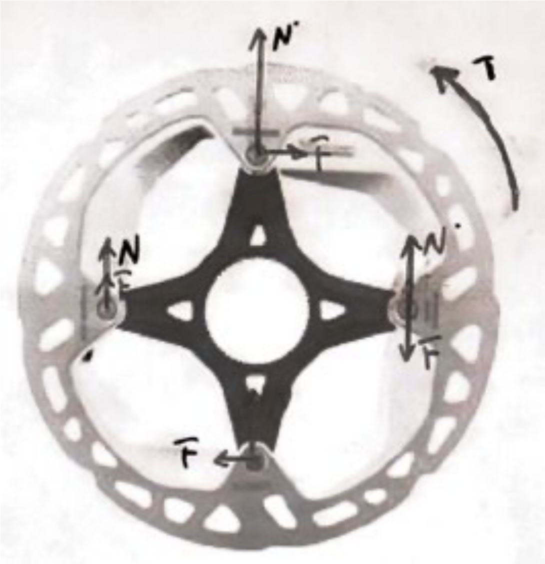


Figure bolt6. FBD of bolts

Maximum shear force and maximum load for bolts and members

Table bolt3. table for maximum forces .

Name	Maximum Load When Load Factor of Safety Equal to 1	Maximum Load When Factor of Safety Against Joint Separation Equal to 1	Maximum Bearing in the bolt	Maximum Bearing in the Member	Maximum Shear of Bolt	Maximum Shear of Bolt If Thread Extend into Shear Plane	Maximum Edge Shearing of Member
Disc Mounting Bolt	7116.8 N	27881.44 N	15960 N	13300 N	19033.34 N	13764.9 N	40390 N
Chainring Bolt	8352.19 N	58437.7 N	13440 N	3520 N	36544 N	23287.24 N	10155.2 N

Discussions - Comments

Insufficient

- Drivetrain:
  1. Only care the torque balance under constant velocity scenario.
  2. Focus on the fatigue failure for chains

- Brakes:
  1. Simulate the friction between disc brake surface by the kinetic friction for Steel
  2. Gain friction change trend from comparison
  3. Overlook the rolling friction when determine the distance to stop.

- Bolted Joints:
  1. Gain specifications from online
  2. Pick one type of Al(aluminum) among a few of candidates.

Argument

- Drivetrain:
  1. Constant Velocity is a typical assumption
  2. No need to convert the rotation of rear wheel to the corresponding driving force – based on adhesion friction.
  3. Roller Chain seldom fail due to intense shear

- Brakes:
  1. Exact info for Resin and Metal disc brake are all confidential.
  2. Rolling friction coefficient is usually low than 5% of the one for adhesion friction.

- Bolted Joints:
  1. Measurement require to disassemble.
  2. Multiple-Source verified
  3. Choose the most common aluminum

Ref:

Discussions - Improvements

1. **Belt Drivetrain**

Currently Chain Drivetrain have problems such as:

(1) cannot withstand corrosion in various environment

(2) chain fracture due to high power input

(3) the dimension of the chain component would be elongated with the lifetime increase

(4) high wear on components of the chain

Therefore, Gates Carbon Drive company release the belt drivetrain, it use high anti-corrosion rubber belt instead of current chain. What's more, experiments show that belt drivetrain system is more efficient than chain drive train when the input power is larger than 208 W. Also Belt Drivetrain is quieter than Chain Drivetrain.

Ref: https://www.gatescarbondrive.com/

Discussions - Improvements

2. Integrated Hand bar

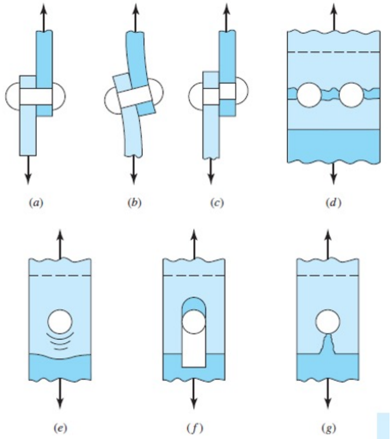
Most Steering system of road bikes composites a hand bar and a stem, in which the hand bar was fixed on the stem with one or two bolts. However, the pre-load pressure applied by the bolts may damage the internal structure of carbon fiber hands bar, result in invisible fracture.

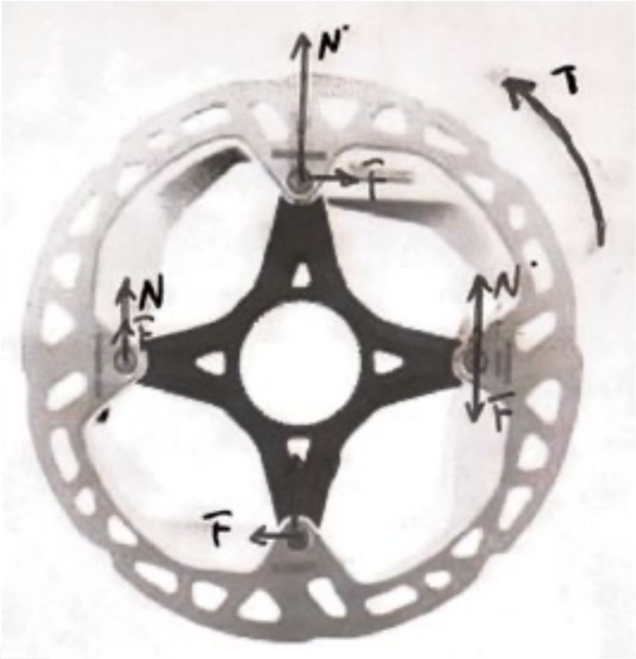
Currently most manufacturer started to develop integrated hand bar, which integrated both hand bar and the stem into a whole body. In this way, the carbon fiber distribution are calculated to make sure it could provide higher support and safety for the riders. Besides, the application of hand bar enables road bike manufacturers to hide the brake line, which increase the aerodynamic performance.

Discussions - Summary

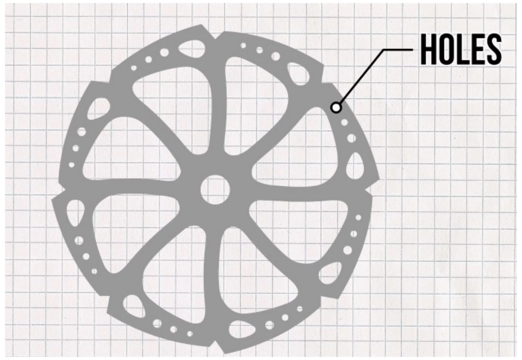
- Drivetrain (Chain):
  1. Life-Time: 7500 h
     - 2h/day → 10 years

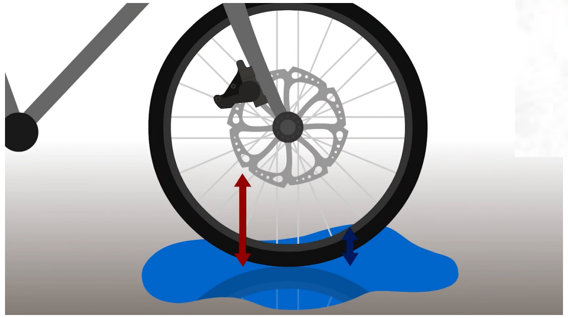
- Bolted Joints:
  1. FS=5 for the joint at paddle for a 75kg people
  2. FS=7-10 for 4 bolts shear the forces at Disc





- Brakes:
  1. Holes on Disc
  2. Position on Disc





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Thanks