Problem Set 3 - CUS715

Order of Functions.

1. What is the time complexity $T(n)$ of the of the nested loops below? For simplicity , you may assume that n is a power of 2. That is, $n = 2^k$ for some positive integer k.

   Answer (counts when the loop condition fails):

   $$\mathrm{T(n)} = 1+1+(\log(\mathrm{n})+2)+(\log(\mathrm{n})+1)+(\frac{(\log(n))^2+5\log(n)+4}{2})+((\log(n))^2+3\log(\mathrm{n})+2)+(\log(\mathrm{n})+1)$$

   $$=\frac{3}{2}(\log(n))^2+\frac{17}{2}(\log(n))+10$$

```
int k = 0;
//1st term

int i = n;
//2nd term

while (i >= 1) {
//3rd term

int j = i;
//4th term

while (j <= n) {
//5th term

k++;
//half of 6th term

j = 2 * j;
//half of 6th term

}
i = floor( i/2.0);
//7th term


}
```

2. Consider the following algorithm:

```
int add_them (int n, int[] A) {
```

```
int i,j,k;
k = 0;
for (i = 1;  i <= n; i++) {
     j = j + A[i];
}

k = 1;
for (i=1; i <= n; i++) {
     k = k + k;
}

return j + k


}
```

(a) if n=5 and the array A contains 2,5,3,7, and 8, what is the output of the above function?

Technically, there is an out of bounds error, but assuming the code should implement i=0, the loops' conditions both set to i<n and j initialized to 0, then…

Answer: 57

(b) What is the time complexity T(n) of the algorithm?

Answer (counts when the loop fails):

$T(n) = 4n+6$

(c) Try to improve the efficiency of the algorithm.

Answer:

```
int add_them (int n, int[] A) {
int i,j,k;
k = 1;
j = 0; //initialize j to 0

for (i = 0;  i <= n; i++) {
j = j + A[i];
k = k+k;
}
 return j + k
}
```

Changes made:

-j was initialized to 0.

-k was just initialized to 1. It was unnecessary to initialize it as 0 at first.

-rather than use two for-loops, I just combined their inner statements into one for-loop (since the original two loops had the same run conditions).

T(n) of the algorithm after the changes is 3n + 5.

3. Show directly that $f(n) = n^2 + 3n^3 \in \Theta(n^3)$. That is, use the definitions of Ω and O to show that $f(n)$ is in both $O(n^3)$ and $\Omega(n^3)$.

Answer:

Proof that $f(n) = n^2 + 3n^3 \in O(n^3)$:

$3n^3 + n^2 \leq cn^3$

$3n^3 + n^2 \leq 3n^3 + n^3$

$3n^3 + n^2 \leq 4n^3$

c can be 4

Proof that $f(n) = n^2 + 3n^3 \in \Omega(n^3)$:

$3n^3 + n^2 \geq cn^3$

$3n^3 + n^2 \geq n^3$

c can be 1

Since $f(n)$ is in both $O(n^3)$ and $\Omega(n^3)$, it is also in $\Theta(n^3)$!