MAP4102, Spring 2014, Homework 4 *due Wed, February 19*

These problems are due at the beginning of the class next Wednesday.

1. **Snakes and ladders** A simple game of "snakes and ladders" is played on a board of nine squares. At each turn a player tosses a fair coin and advances two places if it is heads, one if it is tails. If you land at the foot at a ladder, you immediately climb to the top. If you land at the head of a snake you immediately slide down to his tail.

   (a) How many turns on average does it take to complete the game?

   (b) What is the probability that a player who has reached the middle square will complete the game without slipping back to square 1?

   (c) Suppose now that after you make it to the end, you start over. (If you're on square 9, flipping a tail means you move to square 1. Flipping a head means you move to square 7, since square 2 sends you there automatically.) Does this Markov chain have a unique stationary distribution? Why or why not? In the long-run, in which square will we spend the most time, and what fraction of the time will we be there?

   (d) In this recurrent model, what is the expected return time for square 1? Why is this the same/different as question (a)?
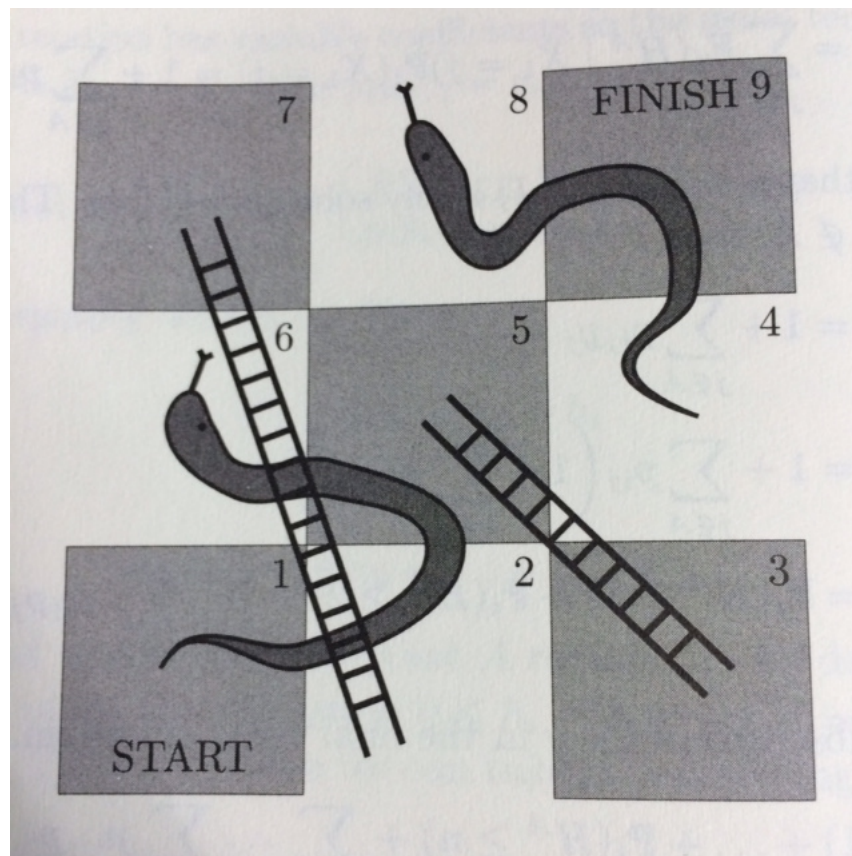
Credit J.R. Norris, "Markov Chains" for the idea for this problem.