MATCHED FILTERING
EEE 5502 SUMMER '15

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1. Introduction

The task is to simulate a Code Division Multiple Access (CMDA) system using spreading spectrum codes.

1.1. CDMA. Code Division Multiple Access is a multiple access scheme - it employs distinct modulating codes assigned to distinct transmitters in order that these several transmitters may send simultaneously over a single channel. Furthermore spreading codes are used for purposes of privacy preservation: a receiver cannot demodulate the transmitted signal without prior knowledge of the modulating sequence. To effect this the codes are typically pseudo-random generated and therefore, for those not in possession of the chipping sequence, mask the signals as noise.

To be specific let the messages

$$\begin{aligned} m[l] &= (m_1, \ldots, m_j) \\ n[l] &= (n_1, \ldots, n_j) \end{aligned}$$

where the symbols $n_i, m_i \in \{-1, 1\}$. Then the spreading sequences

$$\begin{aligned} s_1[l] &= (s_{1,1}, \ldots, s_{1,k}) \\ s_2[l] &= (s_{2,1}, \ldots, s_{2,k}) \end{aligned}$$

where k is the processing gain. This produces modulated transmission sequences

$$\begin{aligned} S_1[l] &= (m_1 \cdot s_{1,1}, \ldots, m_1 \cdot s_{1,k}, m_2 \cdot s_{1,1}, \ldots, m_2 \cdot s_{1,k}, \ldots, m_j \cdot s_{1,1}, \ldots, m_j \cdot s_{1,k}) \\ &:= (m_1 \cdot s_1, m_2 \cdot s_1, \ldots, m_j \cdot s_1) \\ S_2[l] &= (n_1 \cdot s_2, n_2 \cdot s_2, \ldots, n_j \cdot s_2) \end{aligned}$$

Then if S_1 and S_2 are transmitted, on the receiving end of the channel recovery of either m or n is done by first convolving with a filter matching the appropriate spreading sequence:

$$\begin{aligned} h_1[l] &= S_1[l] \star s_1[j-l] \\ h_2[l] &= S_2[l] \star s_2[j-l] \end{aligned}$$

Note that the filter must be inverted because in the convolution it's inverted again. With these configuration parameters[1] the highest amplitude responses in h_1 and h_2 are where the filter correlates or anti-correlates perfectly with transmitted signal, and at these points $h_i[l]$ is either k or $-k$, depending on whether the symbol was 1 or -1.

2. Analysis

Three experiments were performed:

(1) A 20 symbol mesage m was generated, with $m_i \in \{-1, 1\}$ and encoded using the spreading spectrum $s_1 = (-1, 1, -1, 1, 1)$ to produce S_1. Then S_1 was convolved with the matching filter s_1 reflected $\reflectbox{$s$}_1 = (1, 1, -1, 1, -1)$, i.e. $h_1 = S_1 \star \reflectbox{s}_1$. h_1 was then thresholded to entries $|h_i| \geq 5$ to recover the original m. Error rates were computed.

(2) The same S_1 from part 1 was convolved with the matching filter

$$\reflectbox{s}_2 = (1, -1, -1, -1, -1)$$

to attempt to recover the same message m. Error rates were computed.

(3) Additive White Gaussian Noise was used to perturb m to an SNR level of 10dB and error rates were computed. The exact process was

   (a) Compute the signal power of m in Watts

$$P_m = \frac{1}{100} \sum_{i=1}^{100} |m_i|^2$$

   (b) The decibel watts of P_m was computed

$$\mathrm{dBW}_m = 10 \cdot \log_{10}\left(\frac{P_m}{1W}\right)$$

   (c) The noise power in dbW was computed

$$\mathrm{ndBW} = \mathrm{dBW}_m - 10$$

   (d) The power in Watts of ndBW was computed

$$P_n = 10^{\mathrm{ndBW}/10}$$

[1] And no noise in the channel.

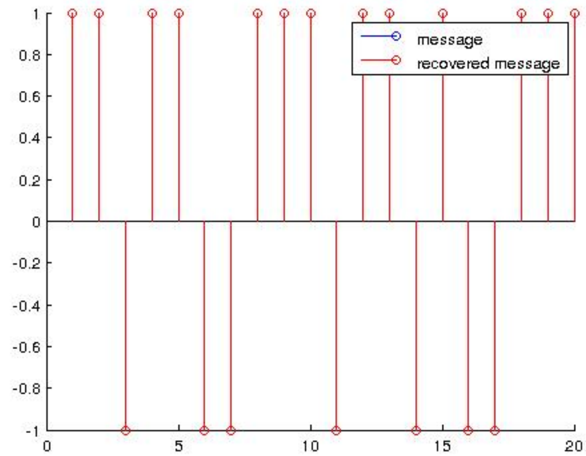

FIGURE 3.1. Experiment 1: 0% error rate.

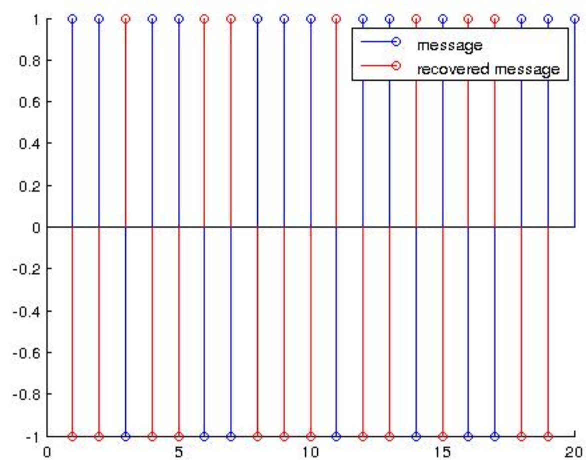

FIGURE 3.2. Experiment 2: 100% error rate.

(e) 100 n $(0,1)$ random variables were generated and scaled by $\sqrt{P_n}$, since that generates n $(0, P_n)$ random variables

3. Conclusions

The computed error rates were

(1) For the mentioned threshold of 5 the result is displayed in figure 3.1. The error rate was 0%.
(2) For a threshold of 3 the signal return was 38 which is unintelligible. Even for arbitrarily trimming to every other entry of the return the error rate was 100%. The result is display in figure 3.2.
(3) With a threshold of 4 on the first try there was 100% recovery rate. The result is on display in figure 3.3. Out of curiosity I ran it a second time and got an error rate of 30% . The result is on display in figure 3.4.

In conclusion typically this method of signal transmission works well with the right threshold, but is still subject to low probability events of heavy loss.

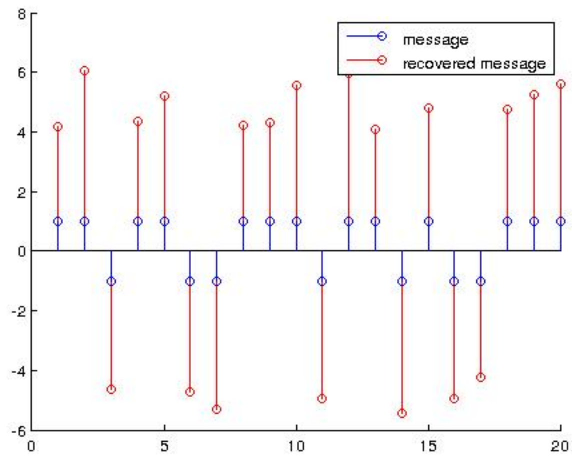

FIGURE 3.3. Experiment 3a: 0% error rate.

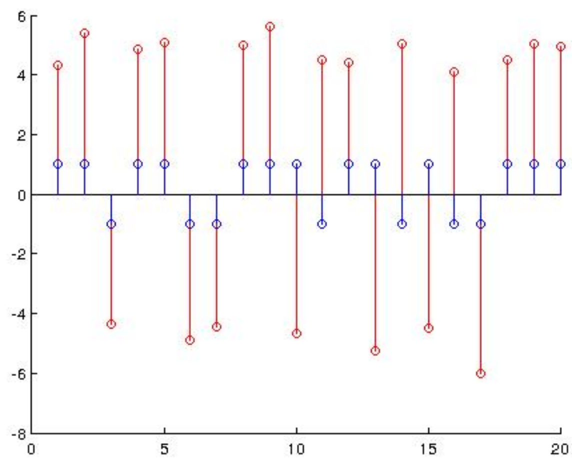

FIGURE 3.4. Experiment 3b: 30% error rate.