Homework 6

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Problem 3.8(a). Find the probability the counter is locked at time t.

Solution.
Using theorem 3.4 in Durrett the probability is

$$\frac{\tau}{\tau + 1/\lambda}$$

Problem 3.8(b). Compute the limiting fraction of particles that get registered.

Solution.
Using theorem 3.4 in Durrett the limiting fraction is

$$\frac{1/\lambda}{\tau + 1/\lambda}$$

Problem 3.21. What is the probability you get a ticket?

Solution.
The probability that you get a ticket given that you park for less than 2 hours is zero. The probability that you get a ticket given that you park for a time greater than 2 hours is the probability that you park for greater than 2 hours, $\frac{1}{2}$, times the probability the parking official catches you. This is tantamount to the parking official first arriving earlier than 2 hours before you leave. This is equal to probability of you parking for greater than 2 hours (draw a picture). Hence the probability of you getting ticketed is $\frac{1}{4}$.

Problem 4.2(a). Write the matrix for the transition rates Q_{ij} and find the stationary distribution.

Solution.

$$\begin{pmatrix} -2 & 2 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 2 & 0 & -4 & 2 \\ 0 & 2 & -2 & 0 \end{pmatrix}$$

Using Mathematica Solve[] the stationary distribution π is $(0, \frac{1}{2}, 0, \frac{1}{2})$.

Problem 4.2(b). At what rate does the store make sales?

Solution.
50% of the time the store is in the state with zero computers hence 100%-50%=50% of the time they are in a state of selling computers. In the one and only such state they sell computers at a rate of two per week.

Problem 4.9. Formulate a Markov chain representation and find the long run fraction that the molecule is in each state.

Solution.
The transition rate matrix is

$$\begin{pmatrix} -3 & 3 & 0 \\ 1 & -3 & 2 \\ 0 & 4 & -4 \end{pmatrix}$$

Using Mathematica Solve[] the stationary distribution π is $(\frac{2}{11}, \frac{6}{11}, \frac{3}{11})$.