With $A \sim B \sim C \sim \text{uniform}\,(0,1)$ (i.e. each distributed uniformly on $(0,1)$) what is the probability that the polynomial equation $Ax^2 + Bx + C = 0$ has real roots? The discriminant should be positive:

$$B^2 - 4AC \geq 0 \iff B^2 \geq 4AC$$

Taking the log of both sides preserves the inequality since log is strictly increasing. Then

$$2\log(B) \geq \log(4) + \log(A) + \log(C).$$

Multiplying everything by a negative sign flips the inequality

$$-2\log(B) \leq -\log(4) - \log(A) - \log(C)$$

But

$$-\log(A) \sim -\log(C) \sim \text{exponential}(1)$$

and

$$-2\log(B) \sim \text{exponential}\,(2)$$

and

$$-\log(A) + -\log(C) \sim \text{Gamma}\,(2,1)$$

Let $X \sim \text{exponential}\,(2)$ and $Y \sim \text{Gamma}\,(2,1)$ then question becomes what is $P(X < Y + \log(1/4))$. By the fact that $P(A) = E(1_A) = E(E(1_A|X))$ and $P(A|X) = E(1_A|X)$ and then $P(A) = E(P(A|X))$ it's the case that

$$\begin{aligned}
P(X < Y + \log(1/4)) &= E(P(X - \log(1/4) < Y|X)) \\
&= \int_{\log(4)}^{\infty} P(x - \log(1/4) < Y)\left(\frac{1}{2}e^{-x/2}\right)dx \\
&= \int_{0}^{\infty}\left(\int_{x-\log(4)}^{\infty}\frac{y}{\Gamma(2)}e^{-y}dy\right)\left(\frac{1}{2}e^{-x/2}\right)dx \\
&= \int_{0}^{\infty}\left(\frac{1}{4}e^{-x}(1 + x + \log(4))\right)\left(\frac{1}{2}e^{-x/2}\right)dx \\
&= \frac{1}{36}(5 + \log(64)) \approx .254413
\end{aligned}$$

This extends to cubics $Ax^3 + Bx^2 + Cx + D$ where the discriminant is $\Delta = B^2C^2 - 4AC^3 - 4B^3D - 27A^2D^2 + 18ABCD$ and other distributions for the coefficients should in principle make for a tractable problem as well.