HW2 Solution

1. Simplify the following Boolean expressions (do not use K-map) to a minimum number of literals. After simplification, draw the logic diagrams of the circuits that implement the original and simplified expressions, respectively.
   (a) x’yz+xy’z+xyz+xyz’
   (b) (x’y’+xw’)(x’w+yz).

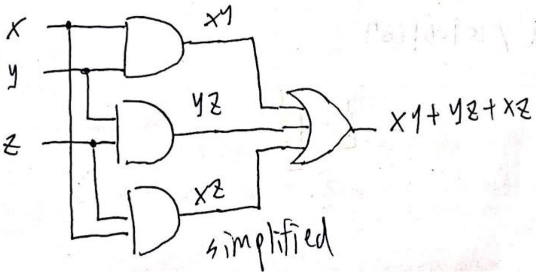
1.

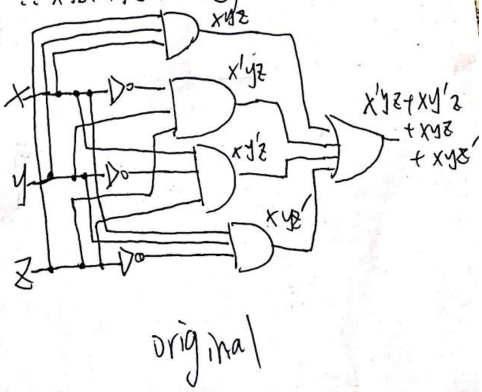
(a) $x'yz+xy'z+xyz+xyz'$

$= x'yz+xy'z+xyz'+xyz+xyz+xyz$ ($\because x+x=x \therefore xyz+xyz=xyz$)

$=(x'+x)yz+x(y'+y)z+xy(z'+z)$

$= yz+xz+xy$ ($\because x'+x=1$)



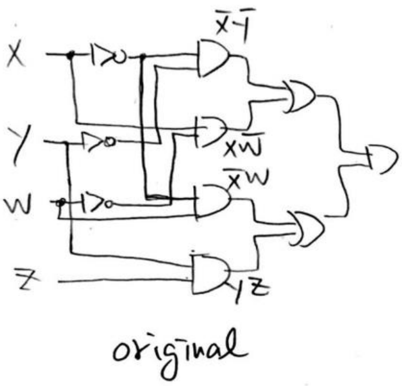


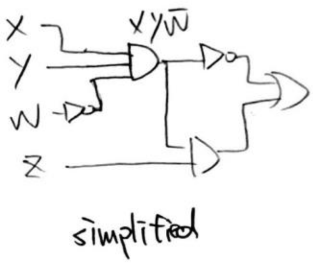
(b)

$(\bar{x}\bar{y}+x\bar{w})(\bar{x}w+yz)$

$= \bar{x}\bar{y}\bar{x}w + \bar{x}\underbrace{\bar{y}y}_{=0}z + \underbrace{x\bar{w}\bar{x}w}_{=0} + x\bar{w}yz$

$= \bar{x}\bar{y}w + xy\bar{w}z$





2. Use DeMorgan's theorem to remove the complement outside the braces:
   (a) ((x'+w)y+wyz+x'z(x+y))',
   (b) (x(y'+z)+y'z(x+w))',
   (c) (x(y+y'(z+w)))',
   (d) (xy'+y(x+z))'.

2. (a) $((x'+w)y+wyz+x'z(x+y))'$

$= ((x'+w)y)'(wyz)'(x'z(x+y))'$

$= ((x'+w)'+y')(w'+y'+z')(xz'+(x+y)')$

$= (\square xw'+y')(w'+y'+z')(xz'+x'y')$

(b) $(x(y'+z)+y'z(x+w))'$

$= (x(y'+z))'(y'z(x+w))'$

$= (x'+(y'+z)')((y'z)'+(x+w)')$

$= (x'+yz')(y+z'+x'w')$

(c) $(x(y+y'(z+w)))'$

$= x'+(y+y'(z+w))'$

$= x'+y'(y'(z+w))'$

$= x'+y'(y+(z+w)')$

$= x'+y'(y+z'w')$

(d) $(xy'+y(x+z))'$

$= (xy')'(y(x+z))'$

$= (x'+y)(y'+(x+z)')$

$= (x'+y)(y'+x'z')$

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3. We can perform logical operations on strings of bits by considering each pair of corresponding bits separately (called ***bitwise*** operation). Given two eight-bit strings A=11010101 and B=01110001, evaluate the eight-bit result after the following logical operations:
   (a) AND,
   (b) XNOR,
   (c) NOT A.

3. (a) AND

11010101 A
01110001 B

Ans: 01010001 A AND B

A	B	AND
0	0	1
0	1	0
1	0	0
1	1	1

(b) XNOR

11010101 A
01110001 B

Ans: 01011011 A XNOR B

A	B	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

(c) NOT A

11010101 A

Ans: 00101010 NOT A

4. Obtain the truth table of function F=x'yz'+w'y+wyz' and express it in sum-of-minterms and product-of-maxterms forms.

w	x	y	z	x'yz'	w'y	wyz'	F	Minterms	Maxterms
0	0	0	0	0	0	0	0	w'x'y'z'	w+x+y+z
0	0	0	1	0	0	0	0	w'x'y'z	w+x+y+z'
0	0	1	0	1	1	0	1	w'x'yz'	w+x+y'+z
0	0	1	1	0	1	0	1	w'x'yz	w+x+y'+z'
0	1	0	0	0	0	0	0	w'xy'z'	w+x'+y+z
0	1	0	1	0	0	0	0	w'xy'z	w+x'+y+z'
0	1	1	0	0	1	0	1	w'xyz'	w+x'+y'+z
0	1	1	1	0	1	0	1	w'xyz	w+x'+y'+z'
1	0	0	0	0	0	0	0	wx'y'z'	w'+x+y+z
1	0	0	1	0	0	0	0	wx'y'z	w'+x+y+z'
1	0	1	0	1	0	1	1	wx'yz'	w'+x+y'+z
1	0	1	1	0	0	0	0	wx'yz	w'+x+y'+z'
1	1	0	0	0	0	0	0	wxy'z'	w'+x'+y+z
1	1	0	1	0	0	0	0	wxy'z	w'+x'+y+z'
1	1	1	0	0	0	1	1	wxyz'	w'+x'+y'+z
1	1	1	1	0	0	0	0	wxyz	w'+x'+y'+z'

Truth table:對應至上表 w, x, y, z, F 這 5 行

Sum-of-minterms:

F=w'x'yz'+w'x'yz+w'xyz'+w'xyz+wx'yz'+wxyz'

$=\Sigma(2,3,6,7,10,14)$

Product-of-maxterms:

F=(w+x+y+z)(w+x+y+z')(w+x'+y+z)(w+x'+y+z')(w'+x+y+z)(w'+x+y+z')

(w'+x+y'+z')(w'+x'+y+z)(w'+x'+y+z')(w'+x'+y'+z')

$=\pi(0,1,4,5,8,9,11,12,13,15)$

5. For the Boolean function F=x’y’z+xy’z+xyz+x’yz,
   (a) Obtain the truth table of F.
   (b) Draw the logic diagram for F.
   (c) Use Boolean algebra to simplify the function F to a new function, G, with minimum number of literals.
   (d) Obtain the truth table of G and show it is the same as that of F.
   (e) Draw the logic diagram for G and compare the number of literals and gates with those of F.

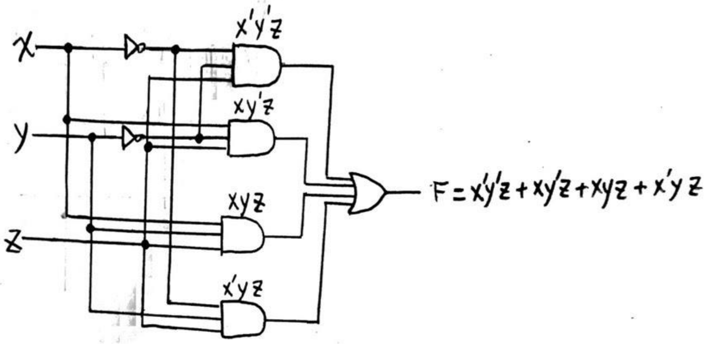
5. $F = x'y'z + xy'z + xyz + x'yz$

(a),(d)

x	y	z	F	G
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

same

(b)



(c)
$$G = x'yz + xy'z + xyz + x'y'z$$
$$= [x'y + xy' + xy + x'y]z \quad \text{(distributive)}$$
$$= [x'(y+y') + x(y'+y)]z \quad \text{(complement)} \quad (y+y')=1$$
$$= [x'+x]z \quad \text{(complement)} \quad (x+x')=1$$
$$= z$$
$G = z$ #

(e)

z —— G

F { 12 literals; 2 NOTs, 4 ANDs, 1 OR

G { 1 literal; No terms (buffer)