Mathematical Equations for PT-Quaternionic Cosmology

1 Cosmological Model Equations

The modified Friedmann equation for a flat universe ($\Omega_k = 0$) in our model is:

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{\text{eff}}} \tag{1}$$

where:

- H_0 is the Hubble constant,
- Ω_m is the matter density parameter,
- Ω_{eff} is the effective dark energy density.

The luminosity distance is given by:

$$d_L(z) = (1+z) \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} \tag{2}$$

where $E(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_{\text{eff}}}$.

2 Distance Modulus

The observed distance modulus for Type Ia supernovae is:

$$\mu_{\text{obs}} = 5 \log_{10} \left(\frac{d_L}{\text{Mpc}} \right) + 25 + M_{\text{offset}} \tag{3}$$

where M_{offset} is the absolute magnitude offset.

3 Gravitational Wave Speed Deviation

For the gravitational-wave analysis, the relative deviation in speed is:

$$\frac{\delta c_g}{c} = \frac{\Delta t}{d/c} \tag{4}$$

where:

- Δt is the observed time delay between GW170817 and GRB 170817A,
- d is the distance to the source.

The predicted deviation based on PT-symmetric quaternionic geometry is:

$$\frac{\delta c_g}{c} \sim \gamma \epsilon \left(\frac{\int \tilde{\alpha}(t) dt}{H_0^{-1}} \right). \tag{5}$$