Icebarf's Math Notes

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Abstract

This document contains small snippets of mathematical notes that I have remembered from school about Vectors. This may include notes for matrices as well and thus should be found here.

1 Vector Products

1.1 Dot Product of two vectors

Let there be a $\overrightarrow{A} = a\hat{i} + b\hat{j} + c\hat{k}$ and a $\overrightarrow{B} = m\hat{i} + n\hat{j} + o\hat{k}$. The dot product of there two vectors as X can be represented as follows.

$$X = \overrightarrow{A} \cdot \overrightarrow{B}$$

$$X = a \times m + b \times n + c \times o$$

From above observations, it is obvious that a dot product is a scalar quantity and as such it is also known as the scalar product of two vectors. Before we continue to the vector product, let us have a look at a simple example.

$$\overrightarrow{M} = 2\hat{i} + 12\hat{j} + 6\hat{k}$$
$$\overrightarrow{N} = 7\hat{i} + 9\hat{j} + 13\hat{k}$$

Now, the dot product of $\overrightarrow{M}$ and $\overrightarrow{N}$ is:

$$X = 2 \times 7 + 12 \times 9 + 6 \times 13$$
$$X = 14 + 108 + 78$$
$$X = 200$$

1.2 Cross Product of two vectors

As defined, above let there be two vectors $\overrightarrow{A}$ and $\overrightarrow{B}$. But unlike the previous quantity, this operation results in a vector quantity and as such is also known as the vector product. Now let us define how the operation is performed. This is done by representing the two vectors as a matrix and calculating its determinant which is represented by $|X|$ or $det(X)$ for some matrix X.

Let a matrix X be defined as follows $X = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ m & n & o \end{bmatrix}$

The cross product for vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ will be the determinant of matrix X.

$$|X| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ m & n & o \end{vmatrix}$$

$$|X| = (b \times o - c \times n)\hat{i} - (a \times o - m \times c)\hat{j} + (a \times n - b \times m)\hat{k}$$

$$|X| = (bo - cn)\hat{i} - (ao - mc)\hat{j} + (an - nb)\hat{k}$$

Since, we know that the cross product of two vectors is the same as the determinant of the matrix representing the two, we can write above equation as:

$$\overrightarrow{A} \times \overrightarrow{B} = |X| = (bo - cn)\hat{i} - (ao - mc)\hat{j} + (an - nb)\hat{k}$$