Outline of psdmul

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We have

$$(FG)(B) = \sum_{b \le B} F(b)G(B - b) \tag{mul}$$

$$\frac{\partial F}{\partial X} = (k \mapsto (k(X) + 1)F(k + X)) \tag{psd}$$

We prove

$$\frac{\partial (FG)}{\partial X} = \left(\frac{\partial F}{\partial X} G\right) + \left(F \frac{\partial G}{\partial X}\right)$$

from

$$\frac{\partial (FG)}{\partial X}(k) = \left(\frac{\partial F}{\partial X} G\right)(k) + \left(F \frac{\partial G}{\partial X}\right)(k) \tag{1 = 2 + 3}$$

$$\begin{aligned}
\frac{\partial (FG)}{\partial X}(k) &= (k(X) + 1)FG(k + X) \\
&= (k(X) + 1) \sum_{b \le k + X} F(b)G((k + X) - b) \\
&= \sum_{b \le k + X} (k(X) + 1)F(b)G((k + X) - b) \qquad (1)
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\partial F}{\partial X} G\right)(k) &= \sum_{b \le k} \left(\frac{\partial F}{\partial X}\right)(b)G(k - b) \\
&= \sum_{b \le k} (b(X) + 1)F(b + X)G(k - b) \\
\underset{b = b' - X}{\longrightarrow} &\sum_{X \le b' \le k + X} ((b' - X)(X) + 1)F((b' - X) + X)G(k - (b' - X)) \\
= &\sum_{X \le b \le k + X} b(X)F(b)G((k + X) - b) \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\left(F \frac{\partial G}{\partial X}\right)(k) &= \sum_{b \le k} F(b) \left(\frac{\partial G}{\partial X}\right)(k - b) \\
&= \sum_{b \le k} F(b)((k - b)(X) + 1)G((k - b) + X) \\
&= \sum_{b \le k} ((k - b)(X) + 1)F(b)G((k - b) + X) \\
&= \sum_{b \le k} ((k - b)(X) + 1)F(b)G((k + X) - b) \qquad (3)
\end{aligned}$$

Reference	$b < X^*$	$X \leq b \leq k$	$k < b \leq k + X$
(2)		b(X) F(b) G((k+X)−b)	b(X) F(b) G((k+X)−b)
(3)	((k−b)(X) + 1) F(b) G((k+X)−b)	((k-b)(X) + 1) F(b) G((k+X)−b)	
(1)	(k(X) + 1) F(b) G((k+X)−b)	(k(X) + 1) F(b) G((k+X)−b)	(k(X) + 1) F(b) G((k+X)−b)

* bags without the variable X

For the first column,

$$b < X$$

implies

$$b(X) = 0$$

, so

$$(k - b)(X) = k(X) - b(X) = k(X)$$

.

For the last column,

$$k < b \leq k + X$$

implies

$$b(X) = (k + X)(X) = k(X) + 1$$

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