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Mathbox for Steven Nguyen

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Theorem dffltz 38077

Description: Fermat's Last Theorem (FLT) for non-zero integers is equivalent to the original scope of natural numbers. The backwards direction takes $(a \uparrow n) + (b \uparrow n) = (c \uparrow n)$, and adds the negative of any negative term to both sides, thus creating the corresponding equation with only positive integers. There are six combinations of negativity, so the proof is particularly long. (Contributed by Steven Nguyen, 27-Feb-2023.)

Assertion

Ref	Expression
dffltz	$\vdash (\forall n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \forall n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$

Distinct variable group: n, a, b, c, x, y, z

Proof of Theorem dffltz

Step	Hyp	Ref	Expression
1		simp-4r 804	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow a \in (\mathbb{Z} \setminus \{0\}))$
2		eldifi 3948	$\dots \vdash (a \in (\mathbb{Z} \setminus \{0\}) \rightarrow a \in \mathbb{Z})$
3		eldifsnl 4529	$\dots \vdash (a \in (\mathbb{Z} \setminus \{0\}) \rightarrow a \neq 0)$
4	2, 3	jca 508	$\dots \vdash \text{11 } \vdash (a \in (\mathbb{Z} \setminus \{0\}) \rightarrow (a \in \mathbb{Z} \wedge a \neq 0))$
5		nnabscl 14435	$\dots \vdash \text{11 } \vdash ((a \in \mathbb{Z} \wedge a \neq 0) \rightarrow (\text{abs}^{\cdot} a) \in \mathbb{N})$
6	1, 4, 5	3syl 18	$\dots \vdash \text{10 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow (\text{abs}^{\cdot} a) \in \mathbb{N})$
7		simp-6r 812	$\dots \vdash \text{14 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b \rightarrow a \in (\mathbb{Z} \setminus \{0\}))$
8	7	eldifad 3799	$\dots \vdash \text{13 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b \rightarrow a \in \mathbb{Z})$
9		simplr 786	$\dots \vdash \text{13 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b \rightarrow 0 < a)$
10		elnnz 11703	$\dots \vdash \text{13 } \vdash (a \in \mathbb{N} \leftrightarrow (a \in \mathbb{Z} \wedge 0 < a))$
11	8, 9, 10	sylanbrc 579	$\dots \vdash \text{12 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b \rightarrow a \in \mathbb{N})$
12		eldifsnl 4529	$\dots \vdash \text{15 } \vdash (b \in (\mathbb{Z} \setminus \{0\}) \rightarrow b \neq 0)$
13	12	ad6antlr 735	$\dots \vdash \text{14 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow b \neq 0)$
14		simplr 786	$\dots \vdash \text{14 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow \neg 0 < b)$
15		eldifi 3948	$\dots \vdash \text{15 } \vdash (b \in (\mathbb{Z} \setminus \{0\}) \rightarrow b \in \mathbb{Z})$
16	15	ad6antlr 735	$\dots \vdash \text{14 } \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow b \in \mathbb{Z})$

37	<u>34, 35,</u> <u>36</u>	<u>negn0nposznnd</u> 38037 ¹² $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow -a \in \mathbb{N}$
38	<u>33, 37</u>	<u>ifclda</u> 4329 ¹¹ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg 0 < a) \rightarrow \text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \in \mathbb{N}$
39	<u>23, 38</u>	<u>ifclda</u> 4329 ¹⁰ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) \in \mathbb{N}$
40	<u>6, 39</u>	<u>ifcld</u> 4340 ⁹ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow \text{if}(n / 2 \in \mathbb{N}, (\text{abs}^{\cdot} a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) \in \mathbb{N}$
41		<u>simpllr</u> 794 ¹¹ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
42	<u>15, 12</u>	<u>jca</u> 508 ¹¹ $\vdash (b \in (\mathbb{Z} \setminus \{0\}) \rightarrow (b \in \mathbb{Z} \wedge b \neq 0))$
43		<u>nnabscl</u> 14435 ¹¹ $\vdash ((b \in \mathbb{Z} \wedge b \neq 0) \rightarrow (\text{abs}^{\cdot} b) \in \mathbb{N})$
44	<u>41, 42,</u> <u>43</u>	<u>3syl</u> 18 ¹⁰ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow (\text{abs}^{\cdot} b) \in \mathbb{N})$
45		<u>simp-5r</u> 808 ¹⁴ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
46	<u>45</u>	<u>eldifad</u> 3799 ¹³ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b) \rightarrow b \in \mathbb{Z})$
47		<u>simpr</u> 478 ¹³ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b) \rightarrow 0 < b)$
48	<u>46, 47,</u> <u>31</u>	<u>sylanbrc</u> 579 ¹² $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge 0 < b) \rightarrow b \in \mathbb{N})$
49		<u>simp-5r</u> 808 ¹⁵ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow c \in (\mathbb{Z} \setminus \{0\})$
50	<u>49</u>	<u>eldifad</u> 3799 ¹⁴ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow c \in \mathbb{Z})$
51		<u>simpr</u> 478 ¹⁴ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow 0 < c)$
52		<u>elnnz</u> 11703 ¹⁴ $\vdash (c \in \mathbb{N} \leftrightarrow (c \in \mathbb{Z} \wedge 0 < c))$
53	<u>50, 51,</u> <u>52</u>	<u>sylanbrc</u> 579 ¹³ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow c \in \mathbb{N})$
54		<u>eldifbsni</u> 4529 ¹⁵ $\vdash (c \in (\mathbb{Z} \setminus \{0\}) \rightarrow c \neq 0)$
55	<u>54</u>	<u>ad5antlr</u> 731 ¹⁴ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow c \neq 0)$
56		<u>simpr</u> 478 ¹⁴ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow \neg 0 < c)$
57		<u>eldifi</u> 3948 ¹⁵ $\vdash (c \in (\mathbb{Z} \setminus \{0\}) \rightarrow c \in \mathbb{Z})$
58	<u>57</u>	<u>ad5antlr</u> 731 ¹⁴ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow c \in \mathbb{Z})$

			$\{0\}) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \wedge (n / 2) \in \mathbb{N} \rightarrow c \in (\mathbb{Z} \setminus \{0\})$
79	78	eldifad 3799	$\dots_{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N} \rightarrow c \in \mathbb{Z})$
80	78, 54	syl 17	$\dots_{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N} \rightarrow c \neq 0)$
81		nnabsc1 14435	$\dots_{11} \vdash ((c \in \mathbb{Z} \wedge c \neq 0) \rightarrow (\text{abs}'c) \in \mathbb{N})$
82	79, 80, 81	syl2anc 580	$\dots_{10} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N} \rightarrow (\text{abs}'c) \in \mathbb{N})$
83		simp-5r 808	$\dots_{14} \vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow c \in (\mathbb{Z} \setminus \{0\})$
84	83	eldifad 3799	$\dots_{13} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow c \in \mathbb{Z})$
85		simp-7r 816	$\dots_{19} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow a \in (\mathbb{Z} \setminus \{0\})$
86	85	eldifad 3799	$\dots_{18} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow a \in \mathbb{Z})$
87	86	zred 11799	$\dots_{17} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow a \in \mathbb{R})$
88		eluzge3nn 12001	$\dots_{19} \vdash (n \in (\mathbb{Z}_{\geq}^{'3}) \rightarrow n \in \mathbb{N})$
89	88	ad7antr 737	$\dots_{18} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow n \in \mathbb{N})$
90	89	nnnn0d 11667	$\dots_{17} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow n \in \mathbb{N}_0)$
91	87, 90	reexpcl1 13308	$\dots_{16} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow (a \uparrow n) \in \mathbb{R})$
92		simp-6r 812	$\dots_{19} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
93	92	eldifad 3799	$\dots_{18} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow b \in \mathbb{Z})$
94	93	zred 11799	$\dots_{17} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow b \in \mathbb{R})$
95	94, 90	reexpcl1 13308	$\dots_{16} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow (b \uparrow n) \in \mathbb{R})$
96		simplr 786	$\dots_{17} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge 0 < b) \rightarrow 0 < a)$

			$\rightarrow 0 < a)$
113	<u>111</u> , <u>112</u> , <u>10</u>	<u>sylanbrc</u> 579	$\dots_{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow a \in \mathbb{N})$
114		<u>simp-7r</u> 816	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow b \in (\mathbb{Z} \setminus \{0\}))$
115	<u>114</u> , <u>12</u>	<u>syl</u> 17	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow b \neq 0)$
116		<u>simplr</u> 786	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow \neg 0 < b)$
117	<u>114</u>	<u>eldifad</u> 3799	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow b \in \mathbb{Z})$
118	<u>115</u> , <u>116</u> , <u>117</u>	<u>negr0nposznnd</u> 38037	$\dots_{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow -b \in \mathbb{N})$
119	<u>113</u> , <u>118</u>	<u>ifclda</u> 4329	$\dots_{12} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b) \rightarrow \text{if}(0 < c, a, -b) \in \mathbb{N})$
120	<u>109</u> , <u>119</u>	<u>ifclda</u> 4329	$\dots_{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \rightarrow \text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \in \mathbb{N})$
121		<u>simp-7r</u> 816	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow b \in (\mathbb{Z} \setminus \{0\}))$
122	<u>121</u>	<u>eldifad</u> 3799	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow b \in \mathbb{Z})$
123		<u>simplr</u> 786	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow 0 < b)$
124	<u>122</u> , <u>123</u> , <u>31</u>	<u>sylanbrc</u> 579	$\dots_{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow b \in \mathbb{N})$
125		<u>simp-8r</u> 820	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow a \in (\mathbb{Z} \setminus \{0\}))$
126	<u>125</u> , 3	<u>syl</u> 17	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow a \neq 0)$

			$\in \mathbb{R})$
142	<u>141</u> , <u>137</u>	<u>reexpcl</u> 13308	$\dots\dots\dots^{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (b \uparrow n) \in \mathbb{R})$
143	<u>138</u> , <u>142</u>	<u>readdcld</u> 10375	$\dots\dots\dots^{16} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((a \uparrow n) + (b \uparrow n)) \in \mathbb{R})$
144		<u>0red</u> 10349	$\dots\dots\dots^{16} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow 0 \in \mathbb{R})$
145	<u>3</u>	<u>neneqd</u> 2993	$\dots\dots\dots^{22} \vdash (a \in (\mathbb{Z} \setminus \{0\}) \rightarrow \neg a = 0)$
146	<u>133</u> , <u>145</u>	<u>syl</u> 17	$\dots\dots\dots^{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg a = 0)$
147		<u>zcn</u> 11698	$\dots\dots\dots^{23} \vdash (a \in \mathbb{Z} \rightarrow a \in \mathbb{C})$
148	<u>133</u> , <u>147</u> , <u>2</u> , <u>3syl</u> 18		$\dots\dots\dots^{22} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow a \in \mathbb{C})$
149		<u>expeq0</u> 13173	$\dots\dots\dots^{22} \vdash ((a \in \mathbb{C} \wedge n \in \mathbb{N}) \rightarrow ((a \uparrow n) = 0 \leftrightarrow a = 0))$
150	<u>148</u> , <u>136</u> , <u>149</u>	<u>syl2anc</u> 580	$\dots\dots\dots^{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((a \uparrow n) = 0 \leftrightarrow a = 0))$
151	<u>146</u> , <u>150</u>	<u>mtbird</u> 317	$\dots\dots\dots^{20} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg(a \uparrow n) = 0)$
152		<u>simplr</u> 786	$\dots\dots\dots^{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg 0 < a)$
153		<u>simpllr</u> 794	$\dots\dots\dots^{22} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg(n / 2) \in \mathbb{N})$
154	<u>135</u> , <u>136</u> , <u>153</u>	<u>oexprereposd</u> 38048	$\dots\dots\dots^{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (0 < a \leftrightarrow 0 < (a \uparrow n)))$
155	<u>152</u> , <u>154</u>	<u>mtbid</u> 316	$\dots\dots\dots^{20} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg 0 < (a \uparrow n))$
156		<u>ioran</u> 1007	$\dots\dots\dots^{20} \vdash (\neg((a \uparrow n) = 0 \vee 0 < (a \uparrow n)) \leftrightarrow (\neg(a \uparrow n) = 0 \wedge \neg 0 < (a \uparrow n)))$
157	<u>151</u> , <u>155</u> , <u>156</u>	<u>sylanbrc</u> 579	$\dots\dots\dots^{19} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg((a \uparrow n) = 0 \vee 0 < (a \uparrow n)))$
158	<u>138</u> , <u>144</u>	<u>lttrid</u> 10483	$\dots\dots\dots^{19} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((a \uparrow n) < 0 \leftrightarrow \neg((a \uparrow n) = 0 \vee 0 < (a \uparrow n))))$

159	<u>157</u> , <u>158</u>	<u>mpbird</u> ²⁴⁹ ¹⁸ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (a \uparrow n) < 0)$
160		<u>zcn</u> ¹¹⁶⁹⁸ ²³ $\vdash (b \in \mathbb{Z} \rightarrow b \in \mathbb{C})$
161	<u>139</u> , <u>15</u> , <u>160</u>	<u>3syl</u> ¹⁸ ²² $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow b \in \mathbb{C})$
162	<u>139</u> , <u>12</u>	<u>syl</u> ¹⁷ ²² $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow b \neq 0$
163		<u>eluzelz</u> ¹¹⁹⁶⁷ ²³ $\vdash (n \in (\mathbb{Z}_{\geq}^{'3}) \rightarrow n \in \mathbb{Z})$
164	<u>163</u>	<u>ad7antr</u> ⁷³⁷ ²² $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow n \in \mathbb{Z})$
165	<u>161</u> , <u>162</u> , <u>164</u>	<u>expne0d</u> ¹³²⁹⁷ ²¹ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (b \uparrow n) \neq 0$
166	<u>165</u>	<u>neneqd</u> ²⁹⁹³ ²⁰ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow \neg(b \uparrow n) = 0$
167		<u>simpr</u> ⁴⁷⁸ ²¹ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow \neg 0 < b$
168	<u>141</u> , <u>136</u> , <u>153</u>	<u>oexpreposd</u> ³⁸⁰⁴⁸ ²¹ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (0 < b \leftrightarrow 0 < (b \uparrow n))$
169	<u>167</u> , <u>168</u>	<u>mtbid</u> ³¹⁶ ²⁰ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow \neg 0 < (b \uparrow n)$
170		<u>ioran</u> ¹⁰⁰⁷ ²⁰ $\vdash (\neg((b \uparrow n) = 0 \vee 0 < (b \uparrow n)) \leftrightarrow (\neg(b \uparrow n) = 0 \wedge \neg 0 < (b \uparrow n)))$
171	<u>166</u> , <u>169</u> , <u>170</u>	<u>sylanbrc</u> ⁵⁷⁹ ¹⁹ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow \neg(b \uparrow n) = 0 \vee 0 < (b \uparrow n))$
172	<u>142</u> , <u>144</u>	<u>ltrrid</u> ¹⁰⁴⁸³ ¹⁹ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow ((b \uparrow n) < 0 \leftrightarrow \neg((b \uparrow n) = 0 \vee 0 < (b \uparrow n)))$
173	<u>171</u> , <u>172</u>	<u>mpbird</u> ²⁴⁹ ¹⁸ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (b \uparrow n) < 0$
174	<u>138</u> , <u>142</u> , <u>144</u> , <u>144</u>	<u>lt2addd</u> ¹⁰⁹⁶⁴ ¹⁷ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow ((a \uparrow n) + (b \uparrow n)) < (0 + 0))$

	<u>159</u> , <u>173</u>	
175	<u>00id</u> 10519 ₁₇ $\vdash (0 + 0) = 0$
176	<u>174</u> , <u>175</u> ₁₆ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((a \uparrow n) + (b \uparrow n)) < 0$
177	<u>143</u> , <u>144</u> , <u>176</u> ₁₅ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg 0 < ((a \uparrow n) + (b \uparrow n))$
178	<u>simp-4r</u> 804 ₁₇ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)$
179	<u>178</u> ₁₆ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (c \uparrow n) = ((a \uparrow n) + (b \uparrow n))$
180	<u>179</u> ₁₅ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (0 < (c \uparrow n) \leftrightarrow 0 < ((a \uparrow n) + (b \uparrow n)))$
181	<u>177</u> , <u>180</u> ₁₄ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg 0 < (c \uparrow n)$
182	<u>131</u> ₁₆ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow c \in \mathbb{Z}$
183	<u>182</u> ₁₅ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow c \in \mathbb{R}$
184	<u>183</u> , <u>136</u> , <u>153</u> ₁₄ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (0 < c \leftrightarrow 0 < (c \uparrow n))$
185	<u>181</u> , <u>184</u> ₁₃ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow \neg 0 < c$
186	<u>132</u> , <u>185</u> , <u>182</u> ₁₂ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow -c \in \mathbb{N}$
187	<u>130</u> , <u>186</u> ₁₁ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \rightarrow \text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \in \mathbb{N}$
188	<u>120</u> , <u>187</u> ₁₀ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \rightarrow \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 < c, a, -b)), \text{if}(0 < b, \text{if}(0 < c, b, -a), -c)) \in \mathbb{N}$
189	<u>82</u> , <u>188</u> ₉ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \rightarrow \text{if}((n / 2) \in \mathbb{N}, (\text{abs}^{'c}), \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 <$

			$((\text{abs}^{\cdot} \text{a}) \uparrow 2) \uparrow (\text{n} / 2))$
216	<u>208</u> , <u>212</u> , <u>214</u>	<u>expmuld</u> 13294	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((a \uparrow 2 \cdot (n / 2))) = ((a \uparrow 2) \uparrow (n / 2)))$
217	<u>207</u> , <u>215</u> , <u>216</u>	<u>3eqtr4d</u> 2860	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} a) \uparrow (2 \cdot (n / 2))) = ((a \uparrow (2 \cdot (n / 2))))$
218		<u>simp-5l</u> 806	$\dots_{18} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow n \in (\mathbb{Z}_{\geq}^{\cdot} 3))$
219		<u>nncn</u> 11348	$\dots_{18} \vdash (n \in \mathbb{N} \rightarrow n \in \mathbb{C})$
220	<u>218</u> , <u>88</u> , <u>219</u>	<u>3syl</u> 18	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow n \in \mathbb{C})$
221		<u>2cnd</u> 11418	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow 2 \in \mathbb{C})$
222		<u>2ne0</u> 11451	$\dots_{18} \vdash 2 \neq 0$
223	<u>222</u>	<u>a1i</u> 11	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow 2 \neq 0)$
224	<u>220</u> , <u>221</u> , <u>223</u>	<u>divcan2d</u> 11118	$\dots_{16} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (2 \cdot (n / 2)) = n$
225	<u>224</u>	<u>eqcomd</u> 2820	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow n = (2 \cdot (n / 2)))$
226	<u>225</u>	<u>oveq2d</u> 6910	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} a) \uparrow n) = ((\text{abs}^{\cdot} a) \uparrow (2 \cdot (n / 2)))$
227	<u>225</u>	<u>oveq2d</u> 6910	$\dots_{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (a \uparrow n) = (a \uparrow (2 \cdot (n / 2)))$
228	<u>217</u> , <u>226</u> , <u>227</u>	<u>3eqtr4d</u> 2860	$\dots_{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} a) \uparrow n) = (a \uparrow n))$
229		<u>simp-4r</u> 804	$\dots_{19} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
230	<u>229</u>	<u>eldifad</u> 3799	$\dots_{18} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow b \in \mathbb{Z})$
231	<u>230</u>	<u>zred</u> 11799	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow b \in \mathbb{R})$
232		<u>absresq</u> 14412	$\dots_{17} \vdash (b \in \mathbb{R} \rightarrow ((\text{abs}^{\cdot} b) \uparrow 2) = (b \uparrow 2))$
233	<u>231</u> , <u>232</u>	<u>syl</u> 17	$\dots_{16} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} b) \uparrow 2) = (b \uparrow 2))$
234	<u>233</u>	<u>oveq1d</u> 6909	$\dots_{15} \vdash (((((n \in (\mathbb{Z}_{\geq}^{\cdot} 3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (((\text{abs}^{\cdot} b) \uparrow 2) \uparrow (n / 2)) = ((b \uparrow 2) \uparrow (n / 2)))$

235	229, 15, 160	3syl ¹⁸ ¹⁸ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow b \in \mathbb{C})$
236	235	abscl ¹⁴⁵⁴⁵ ¹⁷ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (\text{abs}^{\cdot} b) \in \mathbb{R})$
237	236	recnd ¹⁰³⁷⁴ ¹⁶ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (\text{abs}^{\cdot} b) \in \mathbb{C})$
238	237, 212, 214	expmul ¹³²⁹⁴ ¹⁵ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} b) \uparrow (2 \cdot (n / 2))) = (((\text{abs}^{\cdot} b) \uparrow 2) \uparrow (n / 2)))$
239	235, 212, 214	expmul ¹³²⁹⁴ ¹⁵ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (b \uparrow (2 \cdot (n / 2))) = ((b \uparrow 2) \uparrow (n / 2)))$
240	234, 238, 239	3eqtr4d ²⁸⁶⁰ ¹⁴ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} b) \uparrow (2 \cdot (n / 2))) = (b \uparrow (2 \cdot (n / 2)))$
241	225	oveq2d ⁶⁹¹⁰ ¹⁴ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} b) \uparrow n) = ((\text{abs}^{\cdot} b) \uparrow (2 \cdot (n / 2)))$
242	225	oveq2d ⁶⁹¹⁰ ¹⁴ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (b \uparrow n) = (b \uparrow (2 \cdot (n / 2)))$
243	240, 241, 242	3eqtr4d ²⁸⁶⁰ ¹³ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} b) \uparrow n) = (b \uparrow n)$
244	228, 243	oveq12d ⁶⁹¹² ¹² $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (((\text{abs}^{\cdot} a) \uparrow n) + ((\text{abs}^{\cdot} b) \uparrow n)) = ((a \uparrow n) + (b \uparrow n))$
245	79	zred ¹¹⁷⁹⁹ ¹⁶ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow c \in \mathbb{R})$
246		absresq ¹⁴⁴¹² ¹⁶ $\vdash (c \in \mathbb{R} \rightarrow ((\text{abs}^{\cdot} c) \uparrow 2) = (c \uparrow 2))$
247	245, 246	syl ¹⁷ ¹⁵ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} c) \uparrow 2) = (c \uparrow 2))$
248	247	oveq1d ⁶⁹⁰⁹ ¹⁴ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (((\text{abs}^{\cdot} c) \uparrow 2) \uparrow (n / 2)) = ((c \uparrow 2) \uparrow (n / 2))$
249		zcn ¹¹⁶⁹⁸ ¹⁸ $\vdash (c \in \mathbb{Z} \rightarrow c \in \mathbb{C})$
250	78, 57, 249	3syl ¹⁸ ¹⁷ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow c \in \mathbb{C})$
251	250	abscl ¹⁴⁵⁴⁵ ¹⁶ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (\text{abs}^{\cdot} c) \in \mathbb{R})$
252	251	recnd ¹⁰³⁷⁴ ¹⁵ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (\text{abs}^{\cdot} c) \in \mathbb{C})$
253	252, 212,	expmul ¹³²⁹⁴ ¹⁴ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^{\cdot} c) \uparrow (2 \cdot (n / 2))) = ((\text{abs}^{\cdot} c) \uparrow 2) \uparrow (n / 2))$

	214		$((\text{abs}^c n) \uparrow 2) \uparrow (n / 2))$
254	250 , 212 , 214	expmuld 13294	$\dots^{14} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((c \uparrow 2 \cdot (n / 2))) = ((c \uparrow 2) \uparrow (n / 2)))$
255	248 , 253 , 254	3eqtr4d 2860	$\dots^{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^c n) \uparrow (2 \cdot (n / 2))) = ((c \uparrow (2 \cdot (n / 2))))$
256	225	oveq2d 6910	$\dots^{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^c n) \uparrow n) = ((\text{abs}^c n) \uparrow (2 \cdot (n / 2)))$
257	225	oveq2d 6910	$\dots^{13} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (c \uparrow n) = (c \uparrow (2 \cdot (n / 2)))$
258	255 , 256 , 257	3eqtr4d 2860	$\dots^{12} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{abs}^c n) \uparrow n) = (c \uparrow n)$
259	201 , 244 , 258	3eqtr4d 2860	$\dots^{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (((\text{abs}^a n) \uparrow n) + ((\text{abs}^b n) \uparrow n)) = ((\text{abs}^c n) \uparrow n)$
260		iftrue 4301	$\dots^{14} \vdash ((n / 2) \in \mathbb{N} \rightarrow \text{if}((n / 2) \in \mathbb{N}, (\text{abs}^a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) = (\text{abs}^a))$
261	260	oveq1d 6909	$\dots^{13} \vdash ((n / 2) \in \mathbb{N} \rightarrow (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) \uparrow n) = ((\text{abs}^a n) \uparrow n))$
262		iftrue 4301	$\dots^{14} \vdash ((n / 2) \in \mathbb{N} \rightarrow \text{if}((n / 2) \in \mathbb{N}, (\text{abs}^b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, c, -c), -b))) = (\text{abs}^b))$
263	262	oveq1d 6909	$\dots^{13} \vdash ((n / 2) \in \mathbb{N} \rightarrow (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, c, -c), -b))) \uparrow n) = ((\text{abs}^b n) \uparrow n))$
264	261 , 263	oveq12d 6912	$\dots^{12} \vdash ((n / 2) \in \mathbb{N} \rightarrow ((\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) \uparrow n) + (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, c, -c), -b))) \uparrow n)) = (((\text{abs}^a n) \uparrow n) + ((\text{abs}^b n) \uparrow n))$
265	264	adantl 474	$\dots^{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) \uparrow n) + (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, c, -c), -b))) \uparrow n)) = (((\text{abs}^a n) \uparrow n) + ((\text{abs}^b n) \uparrow n))$
266		iftrue 4301	$\dots^{13} \vdash ((n / 2) \in \mathbb{N} \rightarrow \text{if}((n / 2) \in \mathbb{N}, (\text{abs}^c), \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 < c, a, -b)), \text{if}(0 < b, \text{if}(0 < c, b, -a), -c))) = (\text{abs}^c))$
267	266	oveq1d 6909	$\dots^{12} \vdash ((n / 2) \in \mathbb{N} \rightarrow (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^c), \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 < c, a, -b)), \text{if}(0 < b, \text{if}(0 < c, b, -a), -c))) \uparrow n) = ((\text{abs}^c n) \uparrow n))$
268	267	adantl 474	$\dots^{11} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^c), \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 < c, a, -b)), \text{if}(0 < b, \text{if}(0 < c, b, -a), -c))) \uparrow n) = (((\text{abs}^c n) \uparrow n))$
269	259 , 265 , 268	3eqtr4d 2860	$\dots^{10} \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (n / 2) \in \mathbb{N}) \rightarrow ((\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^a), \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) \uparrow n) + (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, -c, b)), \text{if}(0 < b, \text{if}(0 < c, c, -b), -c))) \uparrow n)) = (((\text{abs}^a n) \uparrow n) + ((\text{abs}^b n) \uparrow n))$

		$(\text{abs}^c b), \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, c, -c), -b))) \uparrow n))$ $= (\text{if}((n / 2) \in \mathbb{N}, (\text{abs}^c c), \text{if}(0 < a, \text{if}(0 < b, c, \text{if}(0 < c, a, -b)), \text{if}(0 < b, \text{if}(0 < c, b, -a), -c))) \uparrow n))$
270	iftrue 4301	$\dots_{18} \vdash (0 < b \rightarrow \text{if}(0 < b, a, \text{if}(0 < c, -b, a)) = a)$
271	oveq1d 6909	$\dots_{17} \vdash (0 < b \rightarrow (\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) = (a \uparrow n))$
272	iftrue 4301	$\dots_{18} \vdash (0 < b \rightarrow \text{if}(0 < b, b, \text{if}(0 < c, c, -c)) = b)$
273	oveq1d 6909	$\dots_{17} \vdash (0 < b \rightarrow (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n) = (b \uparrow n))$
274	oveq12d 6912	$\dots_{16} \vdash (0 < b \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = ((a \uparrow n) + (b \uparrow n)))$
275	adantl 474	$\dots_{15} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge 0 < b) \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = ((a \uparrow n) + (b \uparrow n)))$
276	iftrue 4301	$\dots_{17} \vdash (0 < b \rightarrow \text{if}(0 < b, c, \text{if}(0 < c, a, -b)) = c)$
277	oveq1d 6909	$\dots_{16} \vdash (0 < b \rightarrow (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n) = (c \uparrow n))$
278	adantl 474	$\dots_{15} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge 0 < b) \rightarrow (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n) = (c \uparrow n))$
279	104, 275, 278 , 3eqtr4d 2860	$\dots_{14} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge 0 < b) \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n))$
280	simp-7r 816	$\dots_{21} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
281	280, 15, 160 , 3syl 18	$\dots_{20} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow b \in \mathbb{C})$
282	simp-8l 818	$\dots_{21} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow n \in (\mathbb{Z}_{\geq}^{'3})$
283	282, 88 , syl 17	$\dots_{20} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow n \in \mathbb{N})$
284	simp-4r 804	$\dots_{21} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow \neg(n / 2) \in \mathbb{N})$
285	2nn 11413	$\dots_{22} \vdash 2 \in \mathbb{N}$
286	nndivdvds 15359	$\dots_{22} \vdash ((n \in \mathbb{N} \wedge 2 \in \mathbb{N}) \rightarrow (2 \parallel n \leftrightarrow (n / 2) \in \mathbb{N}))$
287	283, 285, 286 , sylanc1 581	$\dots_{21} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow (2 \parallel n \leftrightarrow (n / 2) \in \mathbb{N}))$
288	284, 287 , mtbird 317	$\dots_{20} \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \wedge 0 < c) \rightarrow \neg(n / 2) \in \mathbb{N})$

			$\langle c \rangle \rightarrow \neg 2 \parallel n)$
289	oexpneg 15436		$\dots \dots \dots \dots \dots 20 \vdash ((b \in \mathbb{C} \wedge n \in \mathbb{N} \wedge \neg 2 \parallel n) \rightarrow (\neg b \uparrow n) = \neg(b \uparrow n))$
290	<u>281, 283, 288, 289</u> , syl3anc 1491		$\dots \dots \dots \dots \dots 19 \vdash (((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (\neg b \uparrow n) = \neg(b \uparrow n))$
291	<u>290</u> oveq1d 6909		$\dots \dots \dots \dots \dots 18 \vdash ((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow ((\neg b \uparrow n) + (c \uparrow n)) = (\neg(b \uparrow n) + (c \uparrow n)))$
292	nnnn0 11615		$\dots \dots \dots \dots \dots 23 \vdash (n \in \mathbb{N} \rightarrow n \in \mathbb{N}_0)$
293	<u>282, 88, 292</u> , 3syl 18		$\dots \dots \dots \dots \dots 22 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow n \in \mathbb{N}_0)$
294	<u>281, 293</u> , expcll 13291		$\dots \dots \dots \dots \dots 21 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (b \uparrow n) \in \mathbb{C})$
295	<u>294</u> negcll 10689		$\dots \dots \dots \dots \dots 20 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (\neg b \uparrow n) \in \mathbb{C})$
296	simp-6r 812		$\dots \dots \dots \dots \dots 22 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow c \in (\mathbb{Z} \setminus \{0\}))$
297	<u>296, 57, 249</u> , 3syl 18		$\dots \dots \dots \dots \dots 21 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow c \in \mathbb{C})$
298	<u>297, 293</u> , expcll 13291		$\dots \dots \dots \dots \dots 20 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (c \uparrow n) \in \mathbb{C})$
299	<u>295, 298</u> , addcomd 10546		$\dots \dots \dots \dots \dots 19 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (\neg b \uparrow n) + (c \uparrow n) = ((c \uparrow n) + (\neg b \uparrow n)))$
300	<u>298, 294</u> , negsubd 10708		$\dots \dots \dots \dots \dots 19 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow ((c \uparrow n) + (\neg b \uparrow n)) = ((c \uparrow n) - (b \uparrow n)))$
301	<u>299, 300</u> , eqtrd 2850		$\dots \dots \dots \dots \dots 18 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (\neg b \uparrow n) + (c \uparrow n) = ((c \uparrow n) - (b \uparrow n)))$
302	<u>110, 2, 147</u> , 3syl 18		$\dots \dots \dots \dots \dots 20 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow a \in \mathbb{C})$
303	<u>302, 293</u> , expcll 13291		$\dots \dots \dots \dots \dots 19 \vdash (((((((n \in (\mathbb{Z}_{\geq}^3) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c \rightarrow (a \uparrow n) \in \mathbb{C})$

304		simp-5r 808 ²⁰ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n))$
305	304	eqcomd 2820 ¹⁹ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)))$
306	303 , 294 , 305	mvrraddd 10755 ¹⁸ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((c \uparrow n) - (b \uparrow n)) = (a \uparrow n))$
307	291 , 301 , 306	3eqtrd 2854 ¹⁷ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((-b \uparrow n) + (c \uparrow n)) = (a \uparrow n))$
308		iftrue 4301 ²⁰ $\vdash (0 < c \rightarrow \text{if}(0 < c, -b, a) = -b)$
309	308	oveq1d 6909 ¹⁹ $\vdash (0 < c \rightarrow (\text{if}(0 < c, -b, a) \uparrow n) = (-b \uparrow n))$
310		iftrue 4301 ²⁰ $\vdash (0 < c \rightarrow \text{if}(0 < c, c, -c) = c)$
311	310	oveq1d 6909 ¹⁹ $\vdash (0 < c \rightarrow (\text{if}(0 < c, c, -c) \uparrow n) = (c \uparrow n))$
312	309 , 311	oveq12d 6912 ¹⁸ $\vdash (0 < c \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((-b \uparrow n) + (c \uparrow n)))$
313	312	adantl 474 ¹⁷ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((-b \uparrow n) + (c \uparrow n)))$
314		iftrue 4301 ¹⁹ $\vdash (0 < c \rightarrow \text{if}(0 < c, a, -b) = a)$
315	314	oveq1d 6909 ¹⁸ $\vdash (0 < c \rightarrow (\text{if}(0 < c, a, -b) \uparrow n) = (a \uparrow n))$
316	315	adantl 474 ¹⁷ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow (\text{if}(0 < c, a, -b) \uparrow n) = (a \uparrow n))$
317	307 , 313 , 316	3eqtr4d 2860 ¹⁶ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge 0 < c) \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = (\text{if}(0 < c, a, -b) \uparrow n))$
318		simp-8r 820 ²² $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow a \in (\mathbb{Z} \setminus \{0\}))$
319	318 , 2 , 147	3syl 18 ²¹ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow a \in \mathbb{C})$
320	88	ad8antr 741 ²² $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow n \in \mathbb{N})$
321	320	nnnn0d 11667 ²¹ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow n \in \mathbb{N}_0)$
322	319 , 321	expcll 13291 ²⁰ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow$

337	326, 336	eqtrd 2850	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow ((a \uparrow n) + (-c \uparrow n)) = -(b \uparrow n))$
338		simp-4r 804	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow \neg(n / 2) \in \mathbb{N})$
339	320, 285, 286	sylanc1 581	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow (2 \parallel n \leftrightarrow (n / 2) \in \mathbb{N}))$
340	338, 339	mtbird 317	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow \neg 2 \parallel n)$
341		oexpneg 15436	$\ldots \vdash ((c \in \mathbb{C} \wedge n \in \mathbb{N} \wedge \neg 2 \parallel n) \rightarrow (-c \uparrow n) = -(c \uparrow n))$
342	324, 320, 340, 341	syl3anc 1491	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow (-c \uparrow n) = -(c \uparrow n))$
343	342	oveq2d 6910	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow ((a \uparrow n) + (-c \uparrow n)) = ((a \uparrow n) + (c \uparrow n)))$
344	328, 320, 340, 289	syl3anc 1491	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow (-b \uparrow n) = -(b \uparrow n))$
345	337, 343, 344	3eqtr4d 2860	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow ((a \uparrow n) + (-c \uparrow n)) = (-b \uparrow n))$
346		iffalse 4304	$\ldots \vdash (\neg 0 < c \rightarrow \text{if}(0 < c, -b, a) = a)$
347	346	oveq1d 6909	$\ldots \vdash (\neg 0 < c \rightarrow (\text{if}(0 < c, -b, a) \uparrow n) = (a \uparrow n))$
348		iffalse 4304	$\ldots \vdash (\neg 0 < c \rightarrow \text{if}(0 < c, c, -c) = -c)$
349	348	oveq1d 6909	$\ldots \vdash (\neg 0 < c \rightarrow (\text{if}(0 < c, c, -c) \uparrow n) = (-c \uparrow n))$
350	347, 349	oveq12d 6912	$\ldots \vdash (\neg 0 < c \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((a \uparrow n) + (-c \uparrow n)))$
351	350	adantl 474	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((a \uparrow n) + (-c \uparrow n)))$
352		iffalse 4304	$\ldots \vdash (\neg 0 < c \rightarrow \text{if}(0 < c, a, -b) = -b)$
353	352	oveq1d 6909	$\ldots \vdash (\neg 0 < c \rightarrow (\text{if}(0 < c, a, -b) \uparrow n) = (-b \uparrow n))$
354	353	adantl 474	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow (\text{if}(0 < c, a, -b) \uparrow n) = (-b \uparrow n))$
355	345, 351, 354	3eqtr4d 2860	$\ldots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge 0 < a \wedge \neg 0 < b \wedge \neg 0 < c) \rightarrow ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = (\text{if}(0 < c, a, -b) \uparrow n))$

356	317, 355	pm2.61dan 848	$\dots \vdash (((((\text{if}(0 < \text{c}, -\text{b}, \text{a}) \uparrow n) + (\text{if}(0 < \text{c}, \text{c}, -\text{c}) \uparrow n)) = (\text{if}(0 < \text{c}, \text{a}, -\text{b}) \uparrow n))) \rightarrow ((\text{if}(0 < \text{c}, -\text{b}, \text{a}) \uparrow n) + (\text{if}(0 < \text{c}, \text{c}, -\text{c}) \uparrow n)) = (\text{if}(0 < \text{c}, \text{a}, -\text{b}) \uparrow n))$
357		iffalse 4304	$\dots \vdash (\neg 0 < b \rightarrow \text{if}(0 < b, a, \text{if}(0 < c, -b, a))) = \text{if}(0 < c, -b, a))$
358	357	oveq1d 6909	$\dots \vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n)) = (\text{if}(0 < c, -b, a) \uparrow n))$
359		iffalse 4304	$\dots \vdash (\neg 0 < b \rightarrow \text{if}(0 < b, b, \text{if}(0 < c, c, -c))) = \text{if}(0 < c, c, -c))$
360	359	oveq1d 6909	$\dots \vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = (\text{if}(0 < c, c, -c) \uparrow n))$
361	358, 360	oveq12d 6912	$\dots \vdash (\neg 0 < b \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n))) = ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)))$
362	361	adantl 474	$\dots \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (\neg (n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = ((\text{if}(0 < c, -b, a) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)))$
363		iffalse 4304	$\dots \vdash (\neg 0 < b \rightarrow \text{if}(0 < b, c, \text{if}(0 < c, a, -b))) = \text{if}(0 < c, a, -b))$
364	363	oveq1d 6909	$\dots \vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n)) = (\text{if}(0 < c, a, -b) \uparrow n))$
365	364	adantl 474	$\dots \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (\neg (n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \rightarrow (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n) = (\text{if}(0 < c, a, -b) \uparrow n))$
366	356, 362, 365	3eqtr4d 2860	$\dots \vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (\neg (n / 2) \in \mathbb{N}) \wedge 0 < a) \wedge \neg 0 < b) \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n))$
367	279, 366	pm2.61dan 848	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (\neg (n / 2) \in \mathbb{N}) \wedge 0 < a) \rightarrow ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)) = (\text{if}(0 < b, c, \text{if}(0 < c, a, -b)) \uparrow n))$
368		iftrue 4301	$\dots \vdash (0 < a \rightarrow \text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a))) = \text{if}(0 < b, a, \text{if}(0 < c, -b, a)))$
369	368	oveq1d 6909	$\dots \vdash (0 < a \rightarrow (\text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) \uparrow n)) = (\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n))$
370		iftrue 4301	$\dots \vdash (0 < a \rightarrow \text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, -c, -b))) = \text{if}(0 < b, b, \text{if}(0 < c, c, -c)))$
371	370	oveq1d 6909	$\dots \vdash (0 < a \rightarrow (\text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, -c, -b)) \uparrow n)) = (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n))$
372	369, 371	oveq12d 6912	$\dots \vdash (0 < a \rightarrow ((\text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) \uparrow n) + (\text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, -c, -b)) \uparrow n))) = ((\text{if}(0 < b, a, \text{if}(0 < c, -b, a)) \uparrow n) + (\text{if}(0 < b, b, \text{if}(0 < c, c, -c)) \uparrow n)))$
373	372	adantl 474	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge (\neg (n / 2) \in \mathbb{N}) \wedge 0 < a) \rightarrow ((\text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) \uparrow n) + (\text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, -c, -b)) \uparrow n))) = ((\text{if}(0 < a, \text{if}(0 < b, a, \text{if}(0 < c, -b, a)), \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) \uparrow n) + (\text{if}(0 < a, \text{if}(0 < b, b, \text{if}(0 < c, c, -c)), \text{if}(0 < b, \text{if}(0 < c, -c, -b)) \uparrow n)))$

			$< c \rightarrow ((a \uparrow n) + (-a \uparrow n) + (c \uparrow n)) = ((a \uparrow n) + (b \uparrow n))$
403	<u>388</u> , <u>393</u> , <u>395</u> , <u>402</u>	<u>addcanad</u> 10549	$\dots\dots\dots^{18} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow (-a \uparrow n) + (c \uparrow n) = (b \uparrow n))$
404	<u>386</u> , <u>403</u>	<u>eqtrd</u> 2850	$\dots\dots\dots^{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow (-a \uparrow n) + (c \uparrow n) = (b \uparrow n))$
405		<u>iftrue</u> 4301	$\dots\dots\dots^{20} \vdash (0 < c \rightarrow \text{if}(0 < c, -a, b) = -a)$
406	<u>405</u>	<u>oveq1d</u> 6909	$\dots\dots\dots^{19} \vdash (0 < c \rightarrow (\text{if}(0 < c, -a, b) \uparrow n) = (-a \uparrow n))$
407	<u>406</u> , <u>311</u>	<u>oveq12d</u> 6912	$\dots\dots\dots^{18} \vdash (0 < c \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((-a \uparrow n) + (c \uparrow n)))$
408	<u>407</u>	<u>adantl</u> 474	$\dots\dots\dots^{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((-a \uparrow n) + (c \uparrow n)))$
409		<u>iftrue</u> 4301	$\dots\dots\dots^{19} \vdash (0 < c \rightarrow \text{if}(0 < c, b, -a) = b)$
410	<u>409</u>	<u>oveq1d</u> 6909	$\dots\dots\dots^{18} \vdash (0 < c \rightarrow (\text{if}(0 < c, b, -a) \uparrow n) = (b \uparrow n))$
411	<u>410</u>	<u>adantl</u> 474	$\dots\dots\dots^{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow (\text{if}(0 < c, b, -a) \uparrow n) = (b \uparrow n))$
412	<u>404</u> , <u>408</u> , <u>411</u>	<u>3eqtr4d</u> 2860	$\dots\dots\dots^{16} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge 0 < c) \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = (\text{if}(0 < c, b, -a) \uparrow n))$
413		<u>simp-7r</u> 816	$\dots\dots\dots^{24} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow b \in (\mathbb{Z} \setminus \{0\})$
414	<u>413</u> , <u>15</u> , <u>160</u>	<u>3syl</u> 18	$\dots\dots\dots^{23} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow b \in \mathbb{C})$
415		<u>simp-8l</u> 818	$\dots\dots\dots^{24} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow n \in (\mathbb{Z}_{\geq}^{'3})$
416	<u>415</u> , <u>88</u> , <u>292</u>	<u>3syl</u> 18	$\dots\dots\dots^{23} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow n \in \mathbb{N}_0$
417	<u>414</u> , <u>416</u>	<u>expcll</u> 13291	$\dots\dots\dots^{22} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow (b \uparrow n) \in \mathbb{C})$
418	<u>417</u>	<u>negcld</u> 10689	$\dots\dots\dots^{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow -(b \uparrow n) \in \mathbb{C})$
419		<u>simp-6r</u> 812	$\dots\dots\dots^{23} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N} \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow c \in (\mathbb{Z} \setminus \{0\})$

			$0 < c \rightarrow ((b \uparrow n) + -(c \uparrow n)) = (-(b \uparrow n) + -(c \uparrow n)))$
435	415 , 88	syl 17	$\dots_{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow n \in \mathbb{N})$
436		simp-4r 804	$\dots_{22} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow \neg(n / 2) \in \mathbb{N})$
437	435 , 285 , 286	sylanc1 581	$\dots_{22} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow (2 \parallel n \leftrightarrow (n / 2) \in \mathbb{N}))$
438	436 , 437	mtbird 317	$\dots_{21} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow \neg 2 \parallel n)$
439	420 , 435 , 438 , 341	syl3anc 1491	$\dots_{20} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow (-c \uparrow n) = -(c \uparrow n))$
440	439	oveq2d 6910	$\dots_{19} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow ((b \uparrow n) + (-c \uparrow n)) = ((b \uparrow n) + -(c \uparrow n)))$
441	418 , 421	negdid 10715	$\dots_{19} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow \neg(-(b \uparrow n) + (c \uparrow n)) = \neg(-(b \uparrow n) + -(c \uparrow n)))$
442	434 , 440 , 441	3eqtr4d 2860	$\dots_{18} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow ((b \uparrow n) + (-c \uparrow n)) = \neg(-(b \uparrow n) + (c \uparrow n)))$
443	426 , 435 , 438 , 384	syl3anc 1491	$\dots_{18} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow (-a \uparrow n) = -(a \uparrow n))$
444	431 , 442 , 443	3eqtr4d 2860	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow ((b \uparrow n) + (-c \uparrow n)) = (-a \uparrow n))$
445		iffalse 4304	$\dots_{20} \vdash (\neg 0 < c \rightarrow \text{if}(0 < c, -a, b) = b)$
446	445	oveq1d 6909	$\dots_{19} \vdash (\neg 0 < c \rightarrow (\text{if}(0 < c, -a, b) \uparrow n) = (b \uparrow n))$
447	446 , 349	oveq12d 6912	$\dots_{18} \vdash (\neg 0 < c \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((b \uparrow n) + (-c \uparrow n)))$
448	447	adantl 474	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = ((b \uparrow n) + (-c \uparrow n)))$
449		iffalse 4304	$\dots_{19} \vdash (\neg 0 < c \rightarrow \text{if}(0 < c, b, -a) = -a)$
450	449	oveq1d 6909	$\dots_{18} \vdash (\neg 0 < c \rightarrow (\text{if}(0 < c, b, -a) \uparrow n) = (-a \uparrow n))$
451	450	adantl 474	$\dots_{17} \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c \rightarrow (\text{if}(0 < c, b, -a) \uparrow n) = (-a \uparrow n))$

452	<u>444</u> , <u>448</u> , <u>451</u>	<u>3eqtr4d</u> 2860	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b \wedge \neg 0 < c) \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = (\text{if}(0 < c, b, -a) \uparrow n))$
453	<u>412</u> , <u>452</u>	<u>pm2.61dan</u> 848	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b) \rightarrow ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)) = (\text{if}(0 < c, b, -a) \uparrow n))$
454		<u>iftrue</u> 4301	$\dots \vdash (0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, -a, b), -a)) = \text{if}(0 < c, -a, b)$
455	<u>454</u>	<u>oveq1d</u> 6909	$\dots \vdash (0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n)) = (\text{if}(0 < c, -a, b) \uparrow n))$
456		<u>iftrue</u> 4301	$\dots \vdash (0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, c, -c), -b)) = \text{if}(0 < c, c, -c))$
457	<u>456</u>	<u>oveq1d</u> 6909	$\dots \vdash (0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n)) = (\text{if}(0 < c, c, -c) \uparrow n))$
458	<u>455</u> , <u>457</u>	<u>oveq12d</u> 6912	$\dots \vdash (0 < b \rightarrow ((\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) + (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n))) = ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)))$
459	<u>458</u>	<u>adantl</u> 474	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b) \rightarrow ((\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) + (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n)) = ((\text{if}(0 < c, -a, b) \uparrow n) + (\text{if}(0 < c, c, -c) \uparrow n)))$
460		<u>iftrue</u> 4301	$\dots \vdash (0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, b, -a), -c)) = \text{if}(0 < c, b, -a))$
461	<u>460</u>	<u>oveq1d</u> 6909	$\dots \vdash (0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \uparrow n)) = (\text{if}(0 < c, b, -a) \uparrow n))$
462	<u>461</u>	<u>adantl</u> 474	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b) \rightarrow (\text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \uparrow n) = (\text{if}(0 < c, b, -a) \uparrow n))$
463	<u>453</u> , <u>459</u> , <u>462</u>	<u>3eqtr4d</u> 2860	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge 0 < b) \rightarrow ((\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) + (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n)) = (\text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \uparrow n))$
464	<u>178</u>	<u>negeqd</u> 10584	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow -((a \uparrow n) + (b \uparrow n)) = -(c \uparrow n))$
465	<u>136</u> , <u>285</u> , <u>286</u>	<u>sylanc1</u> 581	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (2 \parallel n \leftrightarrow (n / 2) \in \mathbb{N}))$
466	<u>153</u> , <u>465</u>	<u>mtbird</u> 317	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow \neg 2 \parallel n)$
467	<u>148</u> , <u>136</u> , <u>466</u> , <u>384</u>	<u>syl3anc</u> 1491	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (-a \uparrow n) = -(a \uparrow n))$
468	<u>161</u> , <u>136</u> , <u>466</u> , <u>289</u>	<u>syl3anc</u> 1491	$\dots \vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b) \rightarrow (-b \uparrow n) = -(b \uparrow n))$

469	<u>467</u> , <u>468</u>	<u>oveq12d</u> 6912 ¹⁷ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((-a \uparrow n) + (-b \uparrow n)) = (-a \uparrow n) + (-b \uparrow n)))$
470	<u>133</u> , <u>3</u>	<u>syl</u> ¹⁷ ¹⁹ $\vdash (((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow a \neq 0)$
471	<u>148</u> , <u>470</u> , <u>164</u>	<u>expclzd</u> 13296 ¹⁸ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (a \uparrow n) \in \mathbb{C})$
472	<u>161</u> , <u>162</u> , <u>164</u>	<u>expclzd</u> 13296 ¹⁸ $\vdash ((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (b \uparrow n) \in \mathbb{C})$
473	<u>471</u> , <u>472</u>	<u>negdid</u> 10715 ¹⁷ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow -((a \uparrow n) + (b \uparrow n)) = (-a \uparrow n) + (-b \uparrow n)))$
474	<u>469</u> , <u>473</u>	<u>eqtr4d</u> 2853 ¹⁶ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((-a \uparrow n) + (-b \uparrow n)) = -((a \uparrow n) + (b \uparrow n)))$
475	<u>131</u> , <u>57</u> , <u>249</u>	<u>3syl</u> ¹⁸ ¹⁷ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow c \in \mathbb{C})$
476	<u>475</u> , <u>136</u> , <u>466</u> , <u>341</u>	<u>syl3anc</u> 1491 ¹⁶ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (-c \uparrow n) = -(c \uparrow n))$
477	<u>464</u> , <u>474</u> , <u>476</u>	<u>3eqtr4d</u> 2860 ¹⁵ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((-a \uparrow n) + (-b \uparrow n)) = (-c \uparrow n))$
478		<u>iffalse</u> 4304 ¹⁸ $\vdash (\neg 0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, -a, b), -a) = -a)$
479	<u>478</u>	<u>oveq1d</u> 6909 ¹⁷ $\vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) = (-a \uparrow n))$
480		<u>iffalse</u> 4304 ¹⁸ $\vdash (\neg 0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, c, -c), -b) = -b)$
481	<u>480</u>	<u>oveq1d</u> 6909 ¹⁷ $\vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n) = (-b \uparrow n))$
482	<u>479</u> , <u>481</u>	<u>oveq12d</u> 6912 ¹⁶ $\vdash (\neg 0 < b \rightarrow ((\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) + (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n)) = ((-a \uparrow n) + (-b \uparrow n)))$
483	<u>482</u>	<u>adantl</u> ⁴⁷⁴ ¹⁵ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow ((\text{if}(0 < b, \text{if}(0 < c, -a, b), -a) \uparrow n) + (\text{if}(0 < b, \text{if}(0 < c, c, -c), -b) \uparrow n)) = ((-a \uparrow n) + (-b \uparrow n)))$
484		<u>iffalse</u> 4304 ¹⁷ $\vdash (\neg 0 < b \rightarrow \text{if}(0 < b, \text{if}(0 < c, b, -a), -c) = -c)$
485	<u>484</u>	<u>oveq1d</u> 6909 ¹⁶ $\vdash (\neg 0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \uparrow n) = (-c \uparrow n))$
486	<u>485</u>	<u>adantl</u> ⁴⁷⁴ ¹⁵ $\vdash (((((((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \wedge c \in (\mathbb{Z} \setminus \{0\})) \wedge ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n)) \wedge \neg(n / 2) \in \mathbb{N}) \wedge \neg 0 < a \wedge \neg 0 < b \rightarrow (\text{if}(0 < b, \text{if}(0 < c, b, -a), -c) \uparrow n) = (-c \uparrow n))$

	<u>193</u> , <u>197</u> , <u>200</u> , <u>510</u>		
512	<u>511</u> rexlimdva2 32327 $\vdash (((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \wedge b \in (\mathbb{Z} \setminus \{0\})) \rightarrow (\exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \rightarrow \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n)))$	
513	<u>512</u> rexlimdva 32296 $\vdash ((n \in (\mathbb{Z}_{\geq}^{'3}) \wedge a \in (\mathbb{Z} \setminus \{0\})) \rightarrow (\exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \rightarrow \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n)))$	
514	<u>513</u> rexlimdva 32295 $\vdash (n \in (\mathbb{Z}_{\geq}^{'3}) \rightarrow (\exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \rightarrow \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n)))$	
515	<u>514</u> reximia 3206	...4 $\vdash (\exists n \in (\mathbb{Z}_{\geq}^{'3}) \exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \rightarrow \exists n \in (\mathbb{Z}_{\geq}^{'3}) \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n))$	
516	nne 299213 $\vdash (\neg ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n))$	
517	<u>516</u> bicom1 21612 $\vdash (((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \neg ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
518	<u>517</u> rexbiai 324011 $\vdash (\exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \exists c \in (\mathbb{Z} \setminus \{0\}) \neg ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
519	rexnal 319211 $\vdash (\exists c \in (\mathbb{Z} \setminus \{0\}) \neg ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow \neg \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
520	<u>518</u> , <u>519</u> bitri 26710 $\vdash (\exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \neg \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
521	<u>520</u> rexbiai 32409 $\vdash (\exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \exists b \in (\mathbb{Z} \setminus \{0\}) \neg \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
522	rexnal 31929 $\vdash (\exists b \in (\mathbb{Z} \setminus \{0\}) \neg \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow \neg \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
523	<u>521</u> , <u>522</u> bitri 2678 $\vdash (\exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \neg \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
524	<u>523</u> rexbiai 32407 $\vdash (\exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \exists a \in (\mathbb{Z} \setminus \{0\}) \neg \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
525	rexnal 31927 $\vdash (\exists a \in (\mathbb{Z} \setminus \{0\}) \neg \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow \neg \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
526	<u>524</u> , <u>525</u> bitri 2676 $\vdash (\exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \neg \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
527	<u>526</u> rexbiai 32405 $\vdash (\exists n \in (\mathbb{Z}_{\geq}^{'3}) \exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \exists n \in (\mathbb{Z}_{\geq}^{'3}) \neg \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
528	rexnal 31925 $\vdash (\exists n \in (\mathbb{Z}_{\geq}^{'3}) \neg \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow \neg \forall n \in (\mathbb{Z}_{\geq}^{'3}) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
529	<u>527</u> , <u>528</u> bitri 267	...4 $\vdash (\exists n \in (\mathbb{Z}_{\geq}^{'3}) \exists a \in (\mathbb{Z} \setminus \{0\}) \exists b \in (\mathbb{Z} \setminus \{0\}) \exists c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) = (c \uparrow n) \leftrightarrow \neg \forall n \in (\mathbb{Z}_{\geq}^{'3}) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$	
530	nne 299213 $\vdash (\neg ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n))$	

531	530	bicomi 21612 $\vdash (((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \neg ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
532	531	rexbiai 324011 $\vdash (\exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \exists z \in \mathbb{N} \neg ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
533		rexnal 319211 $\vdash (\exists z \in \mathbb{N} \neg ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \neg \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
534	532 , 533	bitri 26710 $\vdash (\exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \neg \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
535	534	rexbiai 32409 $\vdash (\exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \exists y \in \mathbb{N} \neg \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
536		rexnal 31929 $\vdash (\exists y \in \mathbb{N} \neg \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \neg \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
537	535 , 536	bitri 2678 $\vdash (\exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \neg \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
538	537	rexbiai 32407 $\vdash (\exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \exists x \in \mathbb{N} \neg \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
539		rexnal 31927 $\vdash (\exists x \in \mathbb{N} \neg \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \neg \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
540	538 , 539	bitri 2676 $\vdash (\exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \neg \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
541	540	rexbiai 32405 $\vdash (\exists n \in (\mathbb{Z}_{\geq} \setminus 3) \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \exists n \in (\mathbb{Z}_{\geq} \setminus 3) \neg \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
542		rexnal 31925 $\vdash (\exists n \in (\mathbb{Z}_{\geq} \setminus 3) \neg \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \neg \forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
543	541 , 542	bitri 2674 $\vdash (\exists n \in (\mathbb{Z}_{\geq} \setminus 3) \exists x \in \mathbb{N} \exists y \in \mathbb{N} \exists z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) = (z \uparrow n) \leftrightarrow \neg \forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
544	515 , 529 , 543	3imtr3i 2833 $\vdash (\neg \forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)) \rightarrow \neg \forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
545	544	con4i 1142 $\vdash (\forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n)) \rightarrow \forall n \in (\mathbb{Z}_{\geq} \setminus 3) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$
546		0nnn 113768 $\vdash \neg 0 \in \mathbb{N}$
547		disjsn 44548 $\vdash ((\mathbb{N} \cap \{0\}) = \emptyset \leftrightarrow \neg 0 \in \mathbb{N})$
548	546 , 547	mpbir 2237 $\vdash (\mathbb{N} \cap \{0\}) = \emptyset$
549		disj3 42347 $\vdash ((\mathbb{N} \cap \{0\}) = \emptyset \leftrightarrow \mathbb{N} = (\mathbb{N} \setminus \{0\}))$
550	548 , 549	mpbi 2226 $\vdash \mathbb{N} = (\mathbb{N} \setminus \{0\})$
551		nnssz 117137 $\vdash \mathbb{N} \subseteq \mathbb{Z}$
552		ssdif 39617 $\vdash (\mathbb{N} \subseteq \mathbb{Z} \rightarrow (\mathbb{N} \setminus \{0\}) \subseteq (\mathbb{Z} \setminus \{0\}))$
553	551 , 552	ax-mp 56 $\vdash (\mathbb{N} \setminus \{0\}) \subseteq (\mathbb{Z} \setminus \{0\})$
554	550 , 553	eqsstri 38495 $\vdash \mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\})$

555		ssel 38107 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow (a \in \mathbb{N} \rightarrow a \in (\mathbb{Z} \setminus \{0\})))$
556		ssel 38109 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow (b \in \mathbb{N} \rightarrow b \in (\mathbb{Z} \setminus \{0\})))$
557		ssralv 38809 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow (\forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)))$
558	556, 557	imim12d 818 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow ((b \in (\mathbb{Z} \setminus \{0\}) \rightarrow \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)) \rightarrow (b \in \mathbb{N} \rightarrow \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))))$
559	558	ralimdv2 31597 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow (\forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall b \in \mathbb{N} \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)))$
560	555, 559	imim12d 816 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow ((a \in (\mathbb{Z} \setminus \{0\}) \rightarrow \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)) \rightarrow (a \in \mathbb{N} \rightarrow \forall b \in \mathbb{N} \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))))$
561	560	ralimdv2 31595 $\vdash (\mathbb{N} \subseteq (\mathbb{Z} \setminus \{0\}) \rightarrow (\forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall a \in \mathbb{N} \forall b \in \mathbb{N} \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)))$
562	554, 561	ax-mp 5	...4 $\vdash (\forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall a \in \mathbb{N} \forall b \in \mathbb{N} \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$
563		oveq1 69017 $\vdash (a = x \rightarrow (a \uparrow n) = (x \uparrow n))$
564	563	oveq1d 69096 $\vdash (a = x \rightarrow ((a \uparrow n) + (b \uparrow n)) = ((x \uparrow n) + (b \uparrow n)))$
565	564	neeq1d 30475 $\vdash (a = x \rightarrow (((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow ((x \uparrow n) + (b \uparrow n)) \neq (c \uparrow n)))$
566		oveq1 69017 $\vdash (b = y \rightarrow (b \uparrow n) = (y \uparrow n))$
567	566	oveq2d 69106 $\vdash (b = y \rightarrow ((x \uparrow n) + (b \uparrow n)) = ((x \uparrow n) + (y \uparrow n)))$
568	567	neeq1d 30475 $\vdash (b = y \rightarrow (((x \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow ((x \uparrow n) + (y \uparrow n)) \neq (c \uparrow n)))$
569		oveq1 69016 $\vdash (c = z \rightarrow (c \uparrow n) = (z \uparrow n))$
570	569	neeq2d 30485 $\vdash (c = z \rightarrow (((x \uparrow n) + (y \uparrow n)) \neq (c \uparrow n) \leftrightarrow ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n)))$
571	565, 568, 570	cbvral3v 3382	...4 $\vdash (\forall a \in \mathbb{N} \forall b \in \mathbb{N} \forall c \in \mathbb{N} ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \leftrightarrow \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
572	562, 571	sylib 210	..3 $\vdash (\forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
573	572	ralimi 3150	.2 $\vdash (\forall n \in (\mathbb{Z}_{\geq}^{'}, 3) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n) \rightarrow \forall n \in (\mathbb{Z}_{\geq}^{'}, 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n))$
574	545, 573	impbii 201	1 $\vdash (\forall n \in (\mathbb{Z}_{\geq}^{'}, 3) \forall x \in \mathbb{N} \forall y \in \mathbb{N} \forall z \in \mathbb{N} ((x \uparrow n) + (y \uparrow n)) \neq (z \uparrow n) \leftrightarrow \forall n \in (\mathbb{Z}_{\geq}^{'}, 3) \forall a \in (\mathbb{Z} \setminus \{0\}) \forall b \in (\mathbb{Z} \setminus \{0\}) \forall c \in (\mathbb{Z} \setminus \{0\}) ((a \uparrow n) + (b \uparrow n)) \neq (c \uparrow n))$

Colors of variables: wff setvar class

Syntax hints: \neg [wn 3](#) \rightarrow [wi 4](#) \leftrightarrow [wb 198](#) \wedge [wa 385](#) \vee [wo 874](#) = [wceq 1653](#) \in [wcel 2158](#) \neq [wne 2988](#) \forall [wral 3106](#) \exists [wrex 3107](#) \ [cdif 3784](#) \cap [cin 3786](#) \subseteq [wss 3787](#) \emptyset [c0 4133](#) if [cif 4295](#) { [csn 4386](#) class class class [wbr 4862](#) ‘ [cfv 6112](#) (class class class) [co 6894](#) Ccc 10239 Rcr 10240 Occ0 10241 + [caddc 10244](#) · [cmul 10246](#) < [clt 10380](#) – [cmin 10574](#) - [cneg 10575](#) / [cdiv 10998](#) Ncn 11339 2c2 11395 3c3 11396 $\text{N}_0\text{cn0}$ 11607 Zcz 11693 $\text{Z}_{\geq}\text{cuZ}$ 11957 ↑ [cexp 13143](#) [abscabs 14344](#) || [cdvds 15350](#)

This theorem was proved from axioms: [ax-mp 5](#) [ax-1 6](#) [ax-2 7](#) [ax-3 8](#) [ax-gen 1891](#) [ax-4 1905](#) [ax-5 2006](#) [ax-6 2072](#) [ax-7 2107](#) [ax-8 2160](#) [ax-9 2167](#) [ax-10 2186](#) [ax-11 2201](#) [ax-12 2214](#) [ax-13 2381](#) [ax-ext 2792](#) [ax-sep 4994](#) [ax-nul 5002](#) [ax-pow 5054](#) [ax-pr 5116](#) [ax-un 7198](#) [ax-cnex 10297](#) [ax-resscn 10298](#) [ax-1cn 10299](#) [ax-icn 10300](#) [ax-addcl 10301](#) [ax-addrc1 10302](#) [ax-mulcl 10303](#) [ax-mulrc1 10304](#) [ax-mulcom 10305](#) [ax-addass 10306](#) [ax-mulass 10307](#) [ax-distr 10308](#) [ax-i2m1 10309](#) [ax-1ne0 10310](#) [ax-1rid 10311](#) [ax-rnegex 10312](#) [ax-rrecrex 10313](#) [ax-cnre 10314](#) [ax-pre-ltrri 10315](#) [ax-pre-ltrn 10316](#) [ax-pre-ltadd 10317](#) [ax-pre-mulg0 10318](#) [ax-pre-sup 10319](#)

This theorem depends on definitions: [df-bi 199](#) [df-an 386](#) [df-or 875](#) [df-3or 1109](#) [df-3an 1110](#) [df-tru 1657](#) [df-ex 1876](#) [df-nf 1880](#) [df-sb 2065](#) [df-mo 2595](#) [df-eu 2630](#) [df-clab 2801](#) [df-cleq 2807](#) [df-clel 2810](#) [df-nfc 2947](#) [df-ne 2989](#) [df-nel 3092](#) [df-ral 3111](#) [df-rex 3112](#) [df-reu 3113](#) [df-](#)

rmo 3114 [df-rab](#) 3115 [df-v](#) 3405 [df-sbc](#) 3652 [df-csb](#) 3747 [df-dif](#) 3790 [df-un](#) 3792 [df-in](#) 3794 [df-ss](#) 3801 [df-pss](#) 3803 [df-nul](#) 4134 [df-if](#) 4296 [df-pw](#) 4369 [df-sn](#) 4387 [df-pr](#) 4389 [df-tp](#) 4391 [df-op](#) 4393 [df-uni](#) 4648 [df-iun](#) 4731 [df-br](#) 4863 [df-opab](#) 4925 [df-mpt](#) 4942 [df-tr](#) 4965 [df-id](#) 5239 [df-eprel](#) 5244 [df-po](#) 5252 [df-so](#) 5253 [df-fr](#) 5290 [df-we](#) 5292 [df-xp](#) 5337 [df-rel](#) 5338 [df-cnv](#) 5339 [df-co](#) 5340 [df-dm](#) 5341 [df-rn](#) 5342 [df-res](#) 5343 [df-ima](#) 5344 [df-pred](#) 5909 [df-ord](#) 5955 [df-on](#) 5956 [df-lim](#) 5957 [df-suc](#) 5958 [df-iota](#) 6075 [df-fun](#) 6114 [df-fn](#) 6115 [df-f](#) 6116 [df-fl](#) 6117 [df-fo](#) 6118 [df-flo](#) 6119 [df-fv](#) 6120 [df-riota](#) 6855 [df-ov](#) 6897 [df-oprab](#) 6898 [df-mpt2](#) 6899 [df-om](#) 7316 [df-1st](#) 7417 [df-2nd](#) 7418 [df-wrecs](#) 7661 [df-recs](#) 7723 [df-rdg](#) 7761 [df-er](#) 7998 [df-en](#) 8212 [df-dom](#) 8213 [df-sdom](#) 8214 [df-sup](#) 8606 [df-inf](#) 8607 [df-pnf](#) 10382 [df-mnf](#) 10383 [df-xr](#) 10384 [df-ltxr](#) 10385 [df-le](#) 10386 [df-sub](#) 10576 [df-neg](#) 10577 [df-div](#) 10999 [df-nn](#) 11340 [df-2](#) 11403 [df-3](#) 11404 [df-n0](#) 11608 [df-z](#) 11694 [df-uz](#) 11958 [df-q](#) 12061 [df-rp](#) 12102 [df-fl](#) 12877 [df-mod](#) 12953 [df-seq](#) 13085 [df-exp](#) 13144 [df-cj](#) 14209 [df-re](#) 14210 [df-im](#) 14211 [df-sqrt](#) 14345 [df-abs](#) 14346 [df-dvds](#) 15351 [df-gcd](#) 15583 [df-numer](#) 15807 [df-denom](#) 15808

This theorem is referenced by: (None)

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