Outline of psdmul

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We have

$$(FG)(B) = \sum_{b \le B} F(b)G(B-b) \tag{mul}$$

$$\frac{\partial F}{\partial X} = (k \mapsto (k(X)+1)F(k+X)) \tag{psd}$$

We prove

$$\frac{\partial(FG)}{\partial X} = \left(\frac{\partial F}{\partial X} G\right) + \left(F \frac{\partial G}{\partial X}\right)$$

from

$$\frac{\partial(FG)}{\partial X}(k) = \left(\frac{\partial F}{\partial X} G\right)(k) + \left(F \frac{\partial G}{\partial X}\right)(k) \tag{1 = 2 + 3}$$

$$\begin{aligned}
\frac{\partial(FG)}{\partial X}(k) &= (k(X)+1)FG(k+X) \\
&= (k(X)+1) \sum_{b \le k+X} F(b)G((k+X)-b) \\
&= \sum_{b \le k+X} (k(X)+1)F(b)G((k+X)-b)
\end{aligned} \tag{1}$$

$$\begin{aligned}
\left(\frac{\partial F}{\partial X} G\right)(k) &= \sum_{b \le k} \left(\frac{\partial F}{\partial X}\right)(b)G(k-b) \\
\xrightarrow[b=b'-X]{} & \sum_{X \le b' \le k+X} \left(\frac{\partial F}{\partial X}\right)(b'-X)G(k-(b'-X)) \\
&= \sum_{X \le b' \le k+X} ((b'-X)(X)+1)F((b'-X)+X)G(k-(b'-X)) \\
&= \sum_{X \le b \le k+X} b(X)F(b)G((k+X)-b)
\end{aligned} \tag{2}$$

$$\begin{aligned}
\left(F \frac{\partial G}{\partial X}\right)(k) &= \sum_{b \le k} F(b) \left(\frac{\partial G}{\partial X}\right)(k-b) \\
&= \sum_{b \le k} F(b)((k-b)(X)+1)G((k-b)+X) \\
&= \sum_{b \le k} ((k-b)(X)+1)F(b)G((k-b)+X) \\
&= \sum_{b \le k} ((k-b)(X)+1)F(b)G((k+X)-b)
\end{aligned} \tag{3}$$

Reference	$b < X$*	$X \leq b \leq k$	$k < b \leq k + X$
(2)		b(X) F(b) G((k+X)−b)	b(X) F(b) G((k+X)−b)
(3)	((k−b)(X) + 1) F(b) G((k+X)−b)	((k-b)(X) + 1) F(b) G((k+X)−b)	
(1)	(k(X) + 1) F(b) G((k+X)−b)	(k(X) + 1) F(b) G((k+X)−b)	(k(X) + 1) F(b) G((k+X)−b)

* bags without the variable X

For the first column,

$$b < X$$

implies

$$b(X) = 0$$

, so

$$(k - b)(X) = k(X) - b(X) = k(X)$$

.

For the last column,

$$k < b \leq k + X$$

implies

$$b(X) = (k + X)(X) = k(X) + 1$$

.