Pentomino Pathfinding

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1 Introduction

[Dec24] posed the following problem: Given a rectangular $n \times m$ grid of squares, place a subset of the twelve pentominoes (see Figure 1), and endpoints A and B on the grid without overlaps such that $\#^p_{n,m}$ = the length of (the shortest nonempty path between A and B) is maximized.

The above notation is for the length of a particular path p; for the maximum such path, the notation is $\#_{n,m}$, and when $n = m$, the notation is $\#_n$.

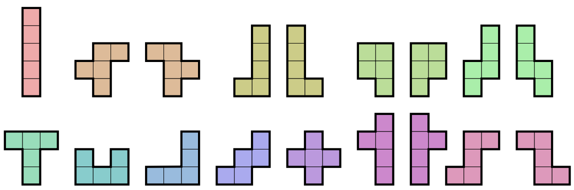

Figure 1: The twelve pentominoes and their reflections [Non08]; from left-to-right they are named I, F, L, P, N, T, U, V, W, X, Y, Z, where F, L, P, N, Y, Z are chiral and have their reflections shown.

2 Trivial solutions

2.1 No pentominoes

For $n = 1$ and 2, $n \times n < 5$, so no pentomino can fit: $\#_1 = 1, \#_2 = 3$. For $n = 3$, 9 squares minus a pentomino is 4 squares, so the length 5 path is optimal: $\#_3 = 5$.

1

	3
1	2

		5
		4
1	2	3

Similar reasoning holds for $n = 2, m <= 6$: there is a path of length $m + 1 <= 7$, while $2m - 5 <= m + 1 <= 7$, so you cannot do any better than placing nothing. It turns out there is enough room for the I piece, and so there are two solutions ignoring symmetry:

					7
1	2	3	4	5	6

					7
1	2	3	4	5	6

And finally, $\#1, n = n$:

1	$\cdots$	n

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References

[Dec24] Deckard. *Pentomino Facts*. Aug. 2, 2024. URL: https://youtu.be/LPDAzHpSyAo?t=700.

[Non08] R. A. Nonenmacher. *All 18 Pentominoes*. CC BY-SA 4.0 https://creativecommons.org/licenses/by-sa/4.0, via Wikimedia Commons; latest uploaded on day. July 21, 2008. URL: https://commons.wikimedia.org/wiki/File:All_18_Pentominoes.svg.