Astronomy of a neutrino source

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1 Flux from a monochromatic source and luminosity distance

We will consider the case of a monochromatic source located at redshift z. The source emits N_0 particles (be them photons or neutrinos) at a fixed energy E_0 in a time interval Δt. The luminosity of the source is defined as:

$$L' = \frac{E_0' N_0'}{\Delta t'} \tag{1}$$

The flux measured by the observer has to take into account the energy redshift and the time dilation due to the relative motion of the source:

$$E_0 = \frac{E_0'}{1+z} \tag{2}$$

$$\Delta t = (1+z)\Delta t' \tag{3}$$

The flux measured by the observer is then:

$$\phi = \frac{EdN}{dA\,dt} = \frac{E_0 N_0}{4\pi d_c^2\,\Delta t} = \frac{E_0' N_0'}{4\pi d_c^2\,(1+z)^2\,\Delta t'} \tag{4}$$

Where we have assumed particle number conservation $N_0 = N_0'$ and d_c is the comoving distance between source and observer. We can still find a natural relation between the luminosity in the source frame and the flux in the observer frame by redefining the distance as *luminosity distance*:

$$d_L = (1+z)d_c \tag{5}$$

so that the flux can be re-defined as:

$$\phi = \frac{L'}{4\pi d_L^2} \tag{6}$$

2 General case

In practice, astronomical sources are rarely monochromatic, whereas observation instruments have limited bandwidth and non-flat response. In general, the observer measures (integrates) the source spectrum in a band $[E_0, E_1]$ that corresponds to a band $[E_0(1+z), E_1(1+z)]$ at the source. This shifting requires to apply a so-called *k-correction* to the observed flux. In astronomy, the typical definition of the k-correction refers to magnitudes. Consider a source observed in a bandpass R with a magnitude m_R:

$$m_R = M_Q + DM + K_{QR} \tag{7}$$

where M_Q is the source absolute magnitude in a bandpass Q (in general $Q \neq R$). The general derivation of the K_{QR} factor is quite complicate as it depends on the spectrum of the source and the energy-dependent (or wavelength-dependent) instrument response function.

If we assume that the instrument response does not change dramatically between E and $(1+z)E$, we just need to account for the effect of the limited integration interval (the shifting of the observed bandwidth w.r.t. the source spectrum). For a generic particle energy spectrum, we write the luminosity as:

$$L' = \frac{d}{dt'} \int_{E_0}^{E_1} E' \frac{dN'}{dE'} \, dE' \tag{8}$$

We then need to find the relation between this luminosity and the particle flux at Earth ($\frac{dN}{dA\,dE\,dt}$) when integrating an energy range $[E_0, E_1]$. We would like to define a correction function $k(E_0, E_1, z)$ so that:

$$L' = k(E_0, E_1, z) \, 4\pi d_L^2 \int_{E0}^{E_1} E\phi(E) \, dE \tag{9}$$

3 Flux from a power-law spectrum

We consider a power-law spectrum with spectral index γ:

$$\frac{dN'}{dE'} = N_0 \left(\frac{E'}{E_0}\right)^{-\gamma} \tag{10}$$

replacing:

$$L' = \frac{d}{dt'} \int_{E_0}^{E_1} E' N_0 \left(\frac{E'}{E_0}\right)^{-\gamma} \, dE' \tag{11}$$

We now consider the usual conversions between the rest frame of the source and the observer:

$$t' = \frac{t}{1+z} \; ; \; dt' = \frac{dt}{1+z} \tag{12}$$

$$E' = (1+z)E \; ; \; dE' = (1+z)dE \tag{13}$$

and replace in (3):

$$
\begin{aligned}
L' = (1+z)\frac{d}{dt}\int_{E_0}^{E_1}(1+z)EN_0\left(\frac{(1+z)E}{E_0}\right)^{-\gamma}(1+z)\,dE = \\
= (1+z)^{3-\gamma}\frac{d}{dt}\int_{E_0}^{E_1}EN_0\left(\frac{E}{E_0}\right)^{-\gamma}dE = \\
= (1+z)^{3-\gamma}\frac{d}{dt}\int_S\frac{d}{dA}\left(\int_{E_0}^{E_1}N_0E\left(\frac{E}{E_0}\right)^{-\gamma}dE\right)dA = \\
= (1+z)^{3-\gamma}\left(4\pi d_c^2\right)\frac{d}{dt\,dA}\left(\int_{E_0}^{E_1}N_0E\left(\frac{E}{E_0}\right)^{-\gamma}dE\right). \qquad (14)
\end{aligned}
$$

Where d_c is the *comoving distance* separating the source and the observer. Using the definition of *luminosity distance*:

$$
d_L = (1+z)d_c \qquad (15)
$$

we can rewrite once again:

$$
\begin{aligned}
L' = 4\pi d_L^2(1+z)^{1-\gamma}\frac{d}{dt\,dA}\int_{E_0}^{E_1}N_0E\left(\frac{E}{E_0}\right)^{-\gamma}dE = \\
= 4\pi d_L^2(1+z)^{1-\gamma}\frac{d}{dt\,dA}\int_{E_0}^{E_1}\frac{dN}{dE}\,dE = \\
= 4\pi d_L^2(1+z)^{1-\gamma}\int_{E_0}^{E_1}\frac{dN}{dt\,dA\,dE}\,dE \qquad (16)
\end{aligned}
$$

For the peculiar case of an unbroken power law, we find the k-correction depends only on the redshift and not on $[E_0, E_1]$.