MEEN 673

Homework 7

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Problem 1:

Table 1.1 Total displacements of node 17 (at the free end) in a cantilevered plate under uniform load; obtained with the updated Lagrangian formulation (5Q8 with iteration).

	3x3 Gauss rule for K				2x2 Gauss rule for K			
$f_0=q_0h$	x	-y	-u	-v	x	-y	-u	-v
50	9.9787	0.1145	0.0213	0.6145	9.9786	0.1163	0.0214	0.6163
100	9.9159	0.7181	0.0841	1.2181	9.9155	0.7225	0.0845	1.2225
150	9.8152	1.3010	0.1848	1.8010	9.8138	1.3091	0.1862	1.8091
200	9.6816	1.8554	0.3184	2.3554	9.6785	1.8688	0.3215	2.3688
250	9.5212	2.3758	0.4788	2.8758	9.5152	2.3960	0.4848	2.8960
300	9.3402	2.8593	0.6598	3.3593	9.3300	2.8874	0.6700	3.3874
350	9.1444	3.3046	0.8556	3.8046	9.1288	3.3417	0.8712	3.8417
400	8.9391	3.7125	1.0609	4.2125	8.9168	3.7587	1.0832	4.2587
450	8.7284	4.0846	1.2716	4.5846	8.6985	4.1398	1.3015	4.6398
500	8.5161	4.4229	1.4839	4.9229	8.4776	4.4871	1.5224	4.9871
550	8.3046	4.7302	1.6954	5.2302	8.2571	4.8028	1.7429	5.3028
600	8.0962	5.0093	1.9038	5.5093	8.0392	5.0897	1.9608	5.5897
650	7.8922	5.2628	2.1078	5.7628	7.8270	5.3480	2.1730	5.8480
700	7.6937	5.4932	2.3063	5.9932	7.6195	5.5843	2.3805	6.0843
750	7.5013	5.7031	2.4987	6.2031	7.4180	5.7995	2.5820	6.2995
800	7.3172	5.8922	2.6828	6.3922	7.2233	5.9956	2.7767	6.4956
850	7.1384	6.0668	2.8616	6.5668	7.0356	6.1746	2.9644	6.6746
900	6.9664	6.2268	3.0336	6.7268	6.8550	6.3382	3.1450	6.8382

Table 1.2 Total displacements of node 17 (at the free end) in a cantilevered plate under uniform load; obtained with the updated Lagrangian formulation (5Q8 without iteration).

	3x3 Gauss rule for K				2x2 Gauss rule for K			
$f_0=q_0h$	x	-y	-u	-v	x	-y	-u	-v
50	10.0000	0.1163	0.0000	0.6163	10.0000	0.1181	0.0000	0.6181
100	9.9690	0.2686	0.0310	0.7686	9.9689	0.2707	0.0311	0.7707
150	9.8901	1.1644	0.1099	1.6644	9.8896	1.1698	0.1104	1.6698
200	9.8272	1.2658	0.1728	1.7658	9.8264	1.2719	0.1736	1.7719
250	9.6208	2.2693	0.3792	2.7693	9.6183	2.2818	0.3817	2.7818
300	9.5421	2.3359	0.4579	2.8359	9.5387	2.3493	0.4613	2.8493
350	9.2429	3.2102	0.7571	3.7102	9.2347	3.2349	0.7653	3.7349
400	9.1659	3.2796	0.8341	3.7796	9.1561	3.3051	0.8439	3.8051
450	8.8026	4.0388	1.1974	4.5388	8.7828	4.0812	1.2172	4.5812
500	8.7266	4.1054	1.2734	4.6054	8.7047	4.1480	1.2953	4.6480
550	8.3545	4.7082	1.6455	5.2082	8.3180	4.7692	1.6820	5.2692
600	8.2768	4.7839	1.7232	5.2839	8.2380	4.8448	1.7620	5.3448
650	7.9257	5.2508	2.0743	5.7508	7.8691	5.3291	2.1309	5.8291
700	7.8407	5.3368	2.1593	5.8368	7.7821	5.4146	2.2179	5.9146
750	7.5280	5.6913	2.4720	6.1913	7.4510	5.7835	2.5490	6.2835
800	7.4303	5.7880	2.5697	6.2880	7.3514	5.8796	2.6486	6.3796
850	7.1618	6.0550	2.8382	6.5550	7.0657	6.1576	2.9343	6.6576
900	7.0488	6.1600	2.9512	6.6600	6.9505	6.2621	3.0495	6.7621

Table 1.3 Stress (x10^{-5}) evaluated at the left-most Gauss point nearest to the top of element 1 in a cantilevered plate under uniform load; obtained with the updated Lagrangian formulation with 5Q8 mesh (X=0.4227, Y=0.7887 when evaluate the Piola-Kirchhoff stress)

3x3 Gauss rule for K			Cauchy stress			Piola-Kirchhoff stress		
$f_0=q_0h$	x	y	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$-\bar{\sigma}_{xy}$	$\bar{S}_{xx}$	$\bar{S}_{yy}$	$-\bar{S}_{xy}$
50	0.4253	0.7861	0.7776	0.1819	0.0539	0.7885	0.1840	0.0487
100	0.4280	0.7836	1.5457	0.3579	0.1198	1.5894	0.3661	0.0994
150	0.4308	0.7810	2.2962	0.5268	0.1975	2.3950	0.5448	0.1521
200	0.4335	0.7784	3.0226	0.6880	0.2861	3.1978	0.7191	0.2070
250	0.4362	0.7757	3.7200	0.8411	0.3845	3.9918	0.8880	0.2640
300	0.4389	0.7732	4.3854	0.9863	0.4915	4.7721	1.0509	0.3230
350	0.4415	0.7706	5.0171	1.1237	0.6057	5.5354	1.2076	0.3838
400	0.4441	0.7680	5.6151	1.2537	0.7258	6.2795	1.3579	0.4462
450	0.4466	0.7655	6.1793	1.3767	0.8507	7.0025	1.5018	0.5098
500	0.4490	0.7630	6.7118	1.4932	0.9793	7.7050	1.6396	0.5746
550	0.4514	0.7605	7.2141	1.6039	1.1108	8.3870	1.7716	0.6402
600	0.4536	0.7580	7.6879	1.7092	1.2444	9.0489	1.8980	0.7064
650	0.4558	0.7556	8.1352	1.8096	1.3795	9.6916	2.0193	0.7731
700	0.4579	0.7532	8.5579	1.9055	1.5157	10.3160	2.1356	0.8402
750	0.4600	0.7508	8.9577	1.9973	1.6525	10.9230	2.2474	0.9074
800	0.4619	0.7486	9.3271	2.0816	1.7862	11.5000	2.3511	0.9736
850	0.4638	0.7463	9.6857	2.1660	1.9230	12.0740	2.4544	1.0408
900	0.4657	0.7440	10.0260	2.2473	2.0598	12.6350	2.5542	1.1080

2x2 Gauss rule for K			Cauchy stress			Piola-Kirchhoff stress		
$f_0=q_0h$	x	y	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$-\bar{\sigma}_{xy}$	$\bar{S}_{xx}$	$\bar{S}_{yy}$	$-\bar{S}_{xy}$
50	0.4254	0.7860	0.7929	0.1840	0.0545	0.8042	0.1862	0.0491
100	0.4282	0.7833	1.5779	0.3584	0.1220	1.6235	0.3670	0.1004
150	0.4310	0.7806	2.3476	0.5225	0.2022	2.4514	0.5416	0.1540
200	0.4339	0.7779	3.0961	0.6761	0.2943	3.2812	0.7093	0.2097
250	0.4367	0.7752	3.8185	0.8193	0.3970	4.1075	0.8699	0.2671
300	0.4396	0.7724	4.5117	0.9527	0.5092	4.9254	1.0232	0.3261
350	0.4424	0.7697	5.1731	1.0766	0.6293	5.7309	1.1693	0.3864
400	0.4452	0.7670	5.8027	1.1921	0.7562	6.5221	1.3086	0.4476
450	0.4479	0.7643	6.4003	1.2997	0.8886	7.2970	1.4414	0.5096
500	0.4505	0.7617	6.9668	1.4004	1.0254	8.0547	1.5682	0.5721
550	0.4531	0.7590	7.5032	1.4948	1.1657	8.7948	1.6894	0.6349
600	0.4556	0.7564	8.0111	1.5836	1.3088	9.5172	1.8055	0.6978
650	0.4580	0.7540	8.4794	1.6630	1.4502	10.2040	1.9118	0.7597
700	0.4603	0.7514	8.9337	1.7422	1.5961	10.8900	2.0184	0.8225
750	0.4626	0.7489	9.3650	1.8176	1.7431	11.5600	2.1212	0.8851
800	0.4649	0.7464	9.7742	1.8897	1.8907	12.2150	2.2205	0.9474
850	0.4670	0.7440	10.1630	1.9588	2.0385	12.8560	2.3166	1.0094
900	0.4692	0.7416	10.5320	2.0253	2.1864	13.4820	2.4097	1.0710

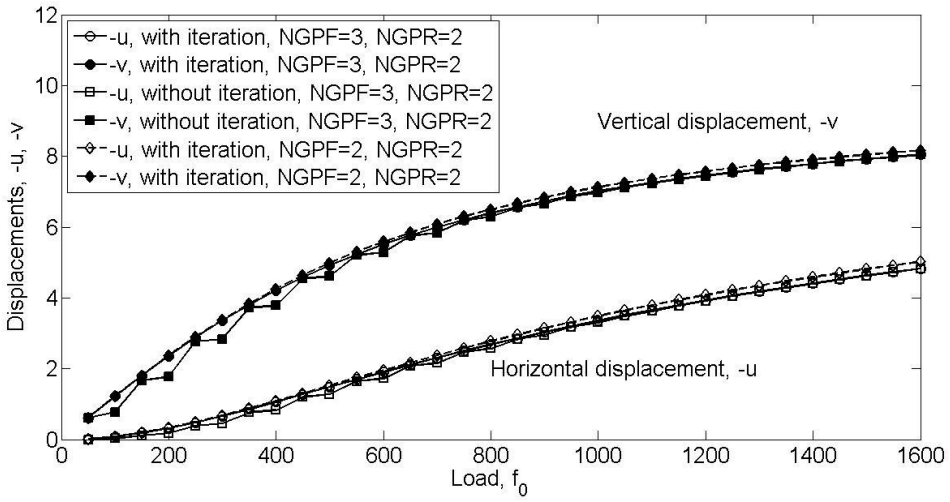

Figure 1.1 Node 17 displacements –u, and -v versus load (obtained with UL formulation).

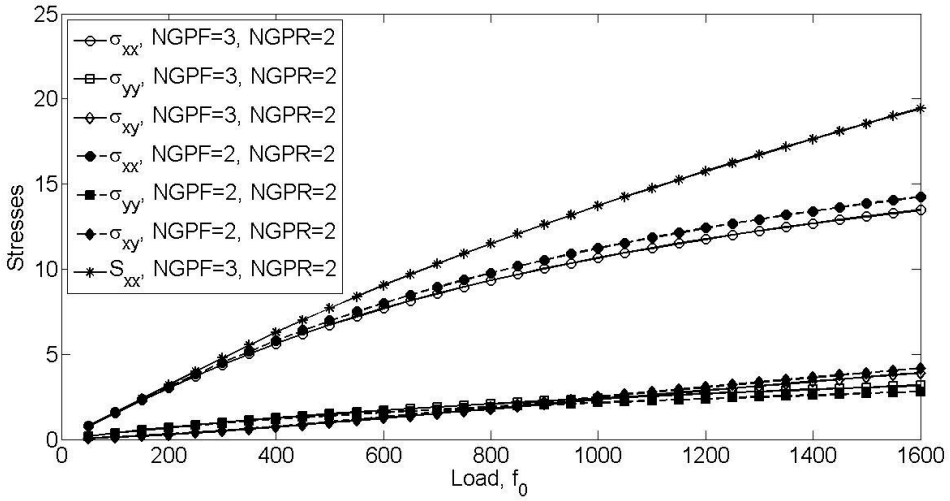

Figure 1.2 Stresses versus load (obtained with UL formulation).

Table 1.4 Total displacements of node 22 in a cantilevered plate under uniform load; obtained with the updated Lagrangian formulation and the 5Q9 mesh (3x3 Gauss rule for K and 2x2 Gauss rule for stresses)

$f_0=q_0h$	x	-y	-u	-v
50	9.9782	0.1218	0.0218	0.6218
100	9.9142	0.7323	0.0858	1.2323
150	9.8115	1.3214	0.1885	1.8214
200	9.6754	1.8810	0.3246	2.3810
250	9.5123	2.4057	0.4877	2.9057
300	9.3286	2.8923	0.6714	3.3923
350	9.1301	3.3400	0.8699	3.8400
400	8.9224	3.7494	1.0776	4.2494
450	8.7095	4.1224	1.2905	4.6224
500	8.4953	4.4611	1.5047	4.9611
550	8.2823	4.7684	1.7177	5.2684
600	8.0725	5.0472	1.9275	5.5472
650	7.8675	5.3002	2.1325	5.8002

700	7.6681	5.5301	2.3319	6.0301
750	7.4750	5.7392	2.5250	6.2392
800	7.2887	5.9299	2.7113	6.4299
850	7.1114	6.1013	2.8886	6.6013
900	6.9392	6.2604	3.0608	6.7604
950	6.7738	6.4064	3.2262	6.9064
1000	6.6150	6.5407	3.3850	7.0407
1050	6.4627	6.6644	3.5373	7.1644
1100	6.3167	6.7787	3.6833	7.2787

Table 1.5 Stresses (x10^{-5}) in a cantilevered plate under uniform load; obtained with the updated Lagrangian formulation and the 5Q9 mesh (3x3 Gauss rule for K and 2x2 Gauss rule for stresses)

$f_0=q_0h$	x	y	$\bar{\sigma}_{xx}$	$-\bar{\sigma}_{xy}$	x	y	$\bar{S}_{xx}$	$-\bar{S}_{xy}$
50	0.4254	0.7860	0.7760	0.0549	0.4227	0.7887	0.7877	0.0485
100	0.4282	0.7834	1.5429	0.1239	0.4227	0.7887	1.5904	0.0986
150	0.4311	0.7807	2.2924	0.2065	0.4227	0.7887	2.3997	0.1504
200	0.4340	0.7780	3.0175	0.3018	0.4227	0.7887	3.2080	0.2041
250	0.4368	0.7753	3.7131	0.4084	0.4227	0.7887	4.0087	0.2597
300	0.4396	0.7726	4.3758	0.5249	0.4227	0.7887	4.7966	0.3171
350	0.4424	0.7699	5.0042	0.6497	0.4227	0.7887	5.5681	0.3764
400	0.4450	0.7673	5.5980	0.7814	0.4227	0.7887	6.3207	0.4372
450	0.4476	0.7647	6.1572	0.9185	0.4227	0.7887	7.0524	0.4994
500	0.4501	0.7621	6.6840	1.0600	0.4227	0.7887	7.7638	0.5628
550	0.4526	0.7595	7.1800	1.2047	0.4227	0.7887	8.4547	0.6272
600	0.4549	0.7570	7.6470	1.3521	0.4227	0.7887	9.1256	0.6925
650	0.4572	0.7545	8.0869	1.5012	0.4227	0.7887	9.7773	0.7586
700	0.4594	0.7520	8.5018	1.6517	0.4227	0.7887	10.4110	0.8252
750	0.4615	0.7496	8.8935	1.8030	0.4227	0.7887	11.0270	0.8923
800	0.4636	0.7472	9.2638	1.9547	0.4227	0.7887	11.6270	0.9598
850	0.4655	0.7449	9.6050	2.1020	0.4227	0.7887	12.1960	1.0260
900	0.4674	0.7426	9.9366	2.2534	0.4227	0.7887	12.7650	1.0939
950	0.4693	0.7403	10.2510	2.4048	0.4227	0.7887	13.3220	1.1620
1000	0.4711	0.7380	10.5510	2.5558	0.4227	0.7887	13.8660	1.2303
1050	0.4728	0.7357	10.8350	2.7064	0.4227	0.7887	14.3990	1.2986
1100	0.4746	0.7334	11.1060	2.8563	0.4227	0.7887	14.9210	1.3669