MEEN 673

Homework 3

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Problem 1:

Boundary conditions:

$$T(0,y)=500^{\circ}K, T(\mathrm{a},y)=300^{\circ}K, \frac{\partial T}{\partial y}=0 @ y=0,b$$

Box1.1 Input file for 4×4 linear elements.

```
4    2   1   1    1   0   2    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
4    4                          NX, NY
0.0  0.045   0.045   0.045   0.045          X0, (DX(I), I=1,NX)
0.0  0.025   0.025   0.025   0.025          Y0, (DY(I), I=1,NY)
10                              NSPV and next lines ISPV, VSPV
1 1    6 1    11 1    16 1    21 1    5 1    10 1    15 1    20 1    25 1

500.0  500.0  500.0  500.0  500.0  300.0  300.0  300.0  300.0  300.0

0                               NSSV
0.2  0.0  0.0                   A10, A1X, A1Y
0.2  0.0  0.0                   A20, A2X, A2Y
0.0  0.0  0.0                   A00, A0X, A0Y
0                               ICONV
0.0  0.0  0.0                   F0,  FX,  FY
4.0E-4  0.0  0.0  4.0E-4  0.0  0.0  A1U,A1UX,A1UY, A2U,A2UX,A2UY
1    10     0.001  0.0          NLS, ITMAX, EPS, GAMA
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0                    GLU(I)
```

Box1.2 Input file for 2×2 quadratic elements.

```
9    3   2   1    1   0   2    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
2    2                          NX, NY
0.0  0.09    0.09               X0, (DX(I), I=1,NX)
0.0  0.05  0.05                 Y0, (DY(I), I=1,NY)
10                              NSPV and next lines ISPV, VSPV
1 1    6 1    11 1    16 1    21 1    5 1    10 1    15 1    20 1    25 1

500.0  500.0  500.0  500.0  500.0  300.0  300.0  300.0  300.0  300.0

0                               NSSV
0.2  0.0  0.0                   A10, A1X, A1Y
0.2  0.0  0.0                   A20, A2X, A2Y
0.0  0.0  0.0                   A00, A0X, A0Y
0                               ICONV
0.0  0.0  0.0                   F0,  FX,  FY
```

```
4.0E-4   0.0   0.0   4.0E-4   0.0   0.0   A1U,A1UX,A1UY, A2U,A2UX,A2UY
1    10      0.001   0.0                  NLS, ITMAX, EPS, GAMA
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0                          GLU(I)
```

Box1.3 Input file for 8×8 linear elements.

```
4    2  1  1    1  0  2    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
8    8                      NX, NY
0.0 0.0225 0.0225 0.0225 0.0225 0.0225 0.0225 0.0225 0.0225    X0, (DX(I), I=1,NX)
0.0 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125    Y0, (DY(I), I=1,NY)
18                          NSPV and next lines ISPV, VSPV
1 1    10 1    19 1    28 1    37 1    46 1    55 1    64 1    73 1
9 1    18 1    27 1    36 1    45 1    54 1    63 1    72 1    81 1

500.0  500.0  500.0  500.0  500.0  500.0  500.0  500.0  500.0
300.0  300.0  300.0  300.0  300.0  300.0  300.0  300.0  300.0

0                           NSSV
0.2  0.0  0.0              A10, A1X, A1Y
0.2  0.0  0.0              A20, A2X, A2Y
0.0  0.0  0.0              A00, A0X, A0Y
0                           ICONV
0.0  0.0  0.0              F0,  FX,  FY
4.0E-4   0.0   0.0   4.0E-4   0.0   0.0   A1U,A1UX,A1UY, A2U,A2UX,A2UY
1    10      0.001   0.0                  NLS, ITMAX, EPS, GAMA
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0                                          GLU(I)
```

Box1.4 Input file for 4×4 quadratic elements.

```
9    3  2  1    1  0  2    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
4    4                      NX, NY
0.0  0.045   0.045   0.045   0.045        X0, (DX(I), I=1,NX)
0.0  0.025   0.025   0.025   0.025        Y0, (DY(I), I=1,NY)
18                          NSPV and next lines ISPV, VSPV
1 1    10 1    19 1    28 1    37 1    46 1    55 1    64 1    73 1
9 1    18 1    27 1    36 1    45 1    54 1    63 1    72 1    81 1
```

```
500.0  500.0  500.0  500.0  500.0  500.0  500.0  500.0  500.0
300.0  300.0  300.0  300.0  300.0  300.0  300.0  300.0  300.0

0                                          NSSV
0.2   0.0   0.0                       A10, A1X, A1Y
0.2   0.0   0.0                       A20, A2X, A2Y
0.0   0.0   0.0                       A00, A0X, A0Y
0                                          ICONV
0.0   0.0   0.0                       F0,   FX,   FY
4.0E-4   0.0   0.0   4.0E-4   0.0   0.0   A1U,A1UX,A1UY, A2U,A2UX,A2UY
1    10     0.001   0.0                    NLS, ITMAX, EPS, GAMA
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0                                                        GLU(I)
```

The numerical results of Problem 1 are given in Table 1.1. Both direct iteration and Newton iteration give essentially the same results. Note that the solution is independent of the mesh in the y-direction.

Table 1.1. Numerical results of Problem 1 (Primary variable).

X(m)	Linear	Direct Iteration				Newton Iteration			
		4×4L4	2×2Q9	8×8L4	4×4Q9	4×4L4	2×2Q9	8×8L4	4×4Q9
0.0000	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
0.0225	475.00	-	-	477.24	477.24	-	-	477.24	477.24
0.0450	450.00	453.94	453.94	453.94	453.94	453.94	453.94	453.94	453.94
0.0675	425.00	-	-	430.06	430.06	-	-	430.05	430.05
0.0900	400.00	405.54	405.54	405.54	405.54	405.54	405.54	405.54	405.54
0.1125	375.00	-	-	380.35	380.35	-	-	380.34	380.34
0.1350	350.00	354.40	354.40	354.40	354.40	354.40	354.40	354.40	354.40
0.1575	325.00	-	-	327.65	327.65	-	-	327.65	327.65
0.1800	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00

Table 1.2. Numerical results of Problem 1 (Secondary variable, 4×4 linear elements).

The orientation of gradient vector is measured from the positive x-axis

x-coord.	y-coord.	-a11(du/dx)	-a22(du/dy)	Flux Mgntd	Orientation
0.2250E-01	0.1250E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.6750E-01	0.1250E-01	0.4000E+03	-0.6765E-12	0.4000E+03	-0.00
0.1125E+00	0.1250E-01	0.4000E+03	-0.3201E-12	0.4000E+03	-0.00
0.1575E+00	0.1250E-01	0.4000E+03	-0.3009E-12	0.4000E+03	-0.00
0.2250E-01	0.3750E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.6750E-01	0.3750E-01	0.4000E+03	0.6765E-12	0.4000E+03	0.00
0.1125E+00	0.3750E-01	0.4000E+03	-0.3201E-12	0.4000E+03	-0.00
0.1575E+00	0.3750E-01	0.4000E+03	-0.3009E-12	0.4000E+03	-0.00
0.2250E-01	0.6250E-01	0.4000E+03	-0.7108E-12	0.4000E+03	-0.00
0.6750E-01	0.6250E-01	0.4000E+03	-0.6765E-12	0.4000E+03	-0.00
0.1125E+00	0.6250E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.1575E+00	0.6250E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.2250E-01	0.8750E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.6750E-01	0.8750E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.1125E+00	0.8750E-01	0.4000E+03	-0.6403E-12	0.4000E+03	-0.00
0.1575E+00	0.8750E-01	0.4000E+03	-0.3009E-12	0.4000E+03	-0.00

Table 1.3. Numerical results of Problem 1 (Secondary variable, 2×2 quadratic elements).

The orientation of gradient vector is measured from the positive x-axis

x-coord.	y-coord.	-a11(du/dx)	-a22(du/dy)	Flux Mgntd	Orientation
0.1902E-01	0.1057E-01	0.4000E+03	0.1427E-11	0.4000E+03	0.00
0.1902E-01	0.3943E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.7098E-01	0.1057E-01	0.4000E+03	0.1348E-11	0.4000E+03	0.00
0.7098E-01	0.3943E-01	0.4000E+03	0.1348E-11	0.4000E+03	0.00
0.1090E+00	0.1057E-01	0.4000E+03	0.0000E+00	0.4000E+03	0.00
0.1090E+00	0.3943E-01	0.4000E+03	0.6434E-12	0.4000E+03	0.00
0.1610E+00	0.1057E-01	0.4000E+03	0.1797E-11	0.4000E+03	0.00
0.1610E+00	0.3943E-01	0.4000E+03	-0.1797E-11	0.4000E+03	-0.00
0.1902E-01	0.6057E-01	0.4000E+03	0.2855E-11	0.4000E+03	0.00
0.1902E-01	0.8943E-01	0.4000E+03	0.7136E-12	0.4000E+03	0.00
0.7098E-01	0.6057E-01	0.4000E+03	0.2696E-11	0.4000E+03	0.00
0.7098E-01	0.8943E-01	0.4000E+03	0.6740E-12	0.4000E+03	0.00
0.1090E+00	0.6057E-01	0.4000E+03	0.1287E-11	0.4000E+03	0.00
0.1090E+00	0.8943E-01	0.4000E+03	-0.1287E-11	0.4000E+03	-0.00
0.1610E+00	0.6057E-01	0.4000E+03	0.1797E-11	0.4000E+03	0.00
0.1610E+00	0.8943E-01	0.4000E+03	-0.5991E-12	0.4000E+03	-0.00

Note: the other results with 8×8 linear elements and 4×4 quadratic elements are presented in the output files.

Problem 2:

Boundary conditions:

$$T(0,y)=500^{\circ}K, T(\mathrm{a},y)=300^{\circ}K, \frac{\partial T}{\partial y}=0 @ y=0, T(x,b)=500-1000x^{\circ}K$$

Box2.1 Input file for 4×4 linear elements.

```
4    2   1   1    1   0   1    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
4    4                         NX, NY
0.0  0.05   0.05   0.05   0.05          X0, (DX(I), I=1,NX)
0.0  0.025  0.025  0.025  0.025         Y0, (DY(I), I=1,NY)
13                                NSPV and next lines ISPV, VSPV
1 1   6 1   11 1   16 1   21 1   5 1   10 1   15 1   20 1   25 1
22 1   23 1   24 1

500.0  500.0  500.0  500.0  500.0  300.0  300.0  300.0  300.0  300.0
450.0  400.0  350.0

0                                 NSSV
0.2  0.0  0.0                     A10, A1X, A1Y
0.2  0.0  0.0                     A20, A2X, A2Y
0.0  0.0  0.0                     A00, A0X, A0Y
0                                 ICONV
0.0  0.0  0.0                     F0,  FX,  FY
4.0E-2  0.0  0.0  4.0E-2  0.0  0.0  A1U,A1UX,A1UY, A2U,A2UX,A2UY
1    10     0.001  0.0            NLS, ITMAX, EPS, GAMA
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0                       GLU(I)
```

Box2.2 Input file for 2×2 quadratic elements.

```
9    3  2  1    1   0   1    NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
2    2                        NX, NY
0.0  0.1   0.1                    X0, (DX(I), I=1,NX)
0.0  0.05  0.05                   Y0, (DY(I), I=1,NY)
13                          NSPV and next lines ISPV, VSPV
1 1   6 1   11 1   16 1   21 1   5 1   10 1   15 1   20 1   25 1
22 1   23 1   24 1

500.0  500.0  500.0  500.0  500.0  300.0  300.0  300.0  300.0  300.0
450.0  400.0  350.0

0                           NSSV
0.2  0.0  0.0               A10, A1X, A1Y
```

```
0.2    0.0    0.0                                A20, A2X, A2Y
0.0    0.0    0.0                                A00, A0X, A0Y
0                                                       ICONV
0.0    0.0    0.0                                F0,    FX,    FY
4.0E-2    0.0    0.0    4.0E-2    0.0    0.0    A1U,A1UX,A1UY, A2U,A2UX,A2UY
1      10       0.001    0.0                           NLS, ITMAX, EPS, GAMA
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0                                       GLU(I)
```

Box2.3 Input file for 8×8 linear elements.

```
4      2    1    1      1    0    1      NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
8      8                                       NX, NY
0.0 0.025    0.025    0.025    0.025    0.025    0.025    0.025    0.025      X0, (DX(I), I=1,NX)
0.0 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125 0.0125      Y0, (DY(I), I=1,NY)
25                                                 NSPV and next lines ISPV, VSPV
1 1      10 1      19 1      28 1      37 1      46 1      55 1      64 1      73 1
9 1      18 1      27 1      36 1      45 1      54 1      63 1      72 1      81 1
74 1    75 1      76 1      77 1      78 1      79 1      80 1

500.0    500.0    500.0    500.0    500.0    500.0    500.0    500.0    500.0
300.0    300.0    300.0    300.0    300.0    300.0    300.0    300.0    300.0
475.0    450.0    425.0    400.0    375.0    350.0    325.0

0                                                       NSSV
0.2    0.0    0.0                                A10, A1X, A1Y
0.2    0.0    0.0                                A20, A2X, A2Y
0.0    0.0    0.0                                A00, A0X, A0Y
0                                                       ICONV
0.0    0.0    0.0                                F0,    FX,    FY
4.0E-2    0.0    0.0    4.0E-2    0.0    0.0    A1U,A1UX,A1UY, A2U,A2UX,A2UY
1      10       0.001    0.0                           NLS, ITMAX, EPS, GAMA
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0                                                              GLU(I)
```

Box2.4 Input file for 4×4 quadratic elements.

```
9    3   2   1     1   0   1      NPE,NGPF,NGPR,MESH,NPRNT,IGRAD,NONLIN
4    4                                  NX, NY
0.0   0.05     0.05     0.05     0.05              X0, (DX(I), I=1,NX)
0.0   0.025   0.025   0.025   0.025            Y0, (DY(I), I=1,NY)
25                                            NSPV and next lines ISPV, VSPV
1 1     10 1     19 1     28 1     37 1     46 1     55 1     64 1     73 1
9 1     18 1     27 1     36 1     45 1     54 1     63 1     72 1     81 1
74 1   75 1     76 1     77 1     78 1     79 1     80 1

500.0   500.0   500.0   500.0   500.0   500.0   500.0   500.0   500.0
300.0   300.0   300.0   300.0   300.0   300.0   300.0   300.0   300.0
475.0   450.0   425.0   400.0   375.0   350.0   325.0

0                                                     NSSV
0.2   0.0   0.0                              A10, A1X, A1Y
0.2   0.0   0.0                              A20, A2X, A2Y
0.0   0.0   0.0                              A00, A0X, A0Y
0                                                     ICONV
0.0   0.0   0.0                              F0,    FX,    FY
4.0E-2   0.0   0.0   4.0E-2   0.0   0.0   A1U,A1UX,A1UY, A2U,A2UX,A2UY
1     10       0.001   0.0                           NLS, ITMAX, EPS, GAMA
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
0.0                                                                GLU(I)
```

Table 2.1. Numerical results of Problem 2 (Primary variable).

X(m)	Y(m)	Direct Iteration				Newton Iteration			
		4×4L4	2×2Q9	8×8L4	4×4Q9	4×4L4	2×2Q9	8×8L4	4×4Q9
0.0000	0.0000	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
0.0250	0.0000	-	-	477.92	477.90	-	-	477.92	477.90
0.0500	0.0000	455.16	455.01	455.05	455.02	455.15	455.01	455.05	455.01
0.0750	0.0000	-	-	431.49	431.45	-	-	431.48	431.44
0.1000	0.0000	407.45	407.21	407.28	407.22	407.44	407.19	407.27	407.21
0.1250	0.0000	-	-	382.34	382.29	-	-	382.33	382.27
0.1500	0.0000	356.63	356.41	356.47	356.42	356.62	356.40	356.46	356.40
0.1750	0.0000	-	-	329.25	329.21	-	-	329.24	329.21

0.2000	0.0000	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0000	0.0125	-	-	500.00	500.00	-	-	500.00	500.00
0.0250	0.0125	-	-	477.89	477.87	-	-	477.89	477.87
0.0500	0.0125	-		454.99	454.95	-	-	454.99	454.95
0.0750	0.0125	-	-	431.41	431.36	-	-	431.40	431.35
0.1000	0.0125	-	-	407.18	407.13	-	-	407.17	407.12
0.1250	0.0125	-	-	382.25	382.19	-	-	382.23	382.17
0.1500	0.0125	-	-	356.39	356.34	-	-	356.38	356.32
0.1750	0.0125	-	-	329.20	329.17	-	-	329.20	329.16
0.2000	0.0125	-	-	300.00	300.00	-	-	300.00	300.00
0.0000	0.0250	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
0.0250	0.0250	-	-	477.79	477.77	-	-	477.79	477.77
0.0500	0.0250	454.90	454.76	454.80	454.76	454.90	454.76	454.80	454.76
0.0750	0.0250	-	-	431.15	431.11	-	-	431.14	431.10
0.1000	0.0250	407.06	406.82	406.89	406.84	407.05	406.80	406.88	406.83
0.1250	0.0250	-	-	381.96	381.90	-	-	381.94	381.89
0.1500	0.0250	356.31	356.09	356.15	356.10	356.30	356.08	356.14	356.08
0.1750	0.0250	-	-	329.06	329.02	-	-	329.05	329.02
0.2000	0.0250	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0000	0.0375	-	-	500.00	500.00	-	-	500.00	500.00
0.0250	0.0375	-	-	477.62	477.60	-	-	477.62	477.60
0.0500	0.0375	-	-	454.47	454.44	-	-	454.47	454.44
0.0750	0.0375	-	-	430.72	430.67	-	-	430.71	430.67
0.1000	0.0375	-		406.40	406.35	-	-	406.38	406.33
0.1250	0.0375	-	-	381.47	381.41	-	-	381.45	381.40
0.1500	0.0375	-	-	355.73	355.68	-	-	355.72	355.67
0.1750	0.0375	-	-	328.81	328.77	-	-	328.80	328.77
0.2000	0.0375	-	-	300.00	300.00	-	-	300.00	300.00
0.0000	0.0500	500.00	50000	500.00	500.00	500.00	500.00	500.00	500.00
0.0250	0.0500	-	-	477.36	477.34	-	-	477.36	477.34
0.0500	0.0500	454.10	453.97	454.00	453.97	454.10	453.96	454.00	453.96
0.0750	0.0500	-	-	430.08	430.04	-	-	430.07	430.03
0.1000	0.0500	405.81	405.60	405.68	405.63	405.80	405.58	405.67	405.62
0.1250	0.0500	-	-	380.75	380.70	-	-	380.74	380.69
0.1500	0.0500	355.28	355.08	355.13	355.07	355.28	355.07	355.12	355.06
0.1750	0.0500	-	-	328.44	328.40	-	-	328.44	328.40
0.2000	0.0500	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0000	0.0625	-	-	500.00	500.00	-	-	500.00	500.00
0.0250	0.0625	-	-	477.01	476.99	-	-	477.01	476.99
0.0500	0.0625	-	-	453.35	453.32	-	-	453.35	453.32
0.0750	0.0625	-	-	429.22	429.19	-	-	429.22	429.19
0.1000	0.0625	-	-	404.71	404.67	-	-	404.70	404.66
0.1250	0.0625	-	-	379.78	379.74	-	-	379.77	379.73

0.1500	0.0625	-	-	354.30	354.25	-	-	354.29	354.24
0.1750	0.0625	-	-	327.93	327.89	-	-	327.93	327.89
0.2000	0.0625	-	-	300.00	300.00	-	-	300.00	300.00
0.0000	0.0750	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
0.0250	0.0750	-	-	476.54	476.52	-	-	476.54	476.52
0.0500	0.0750	452.59	452.49	452.50	452.47	452.59	452.49	452.49	452.47
0.0750	0.0750	-	-	428.12	428.09	-	-	428.11	428.09
0.1000	0.0750	403.53	403.38	403.47	403.44	403.53	403.37	403.46	403.43
0.1250	0.0750	-	-	378.53	378.50	-	-	378.52	378.49
0.1500	0.0750	353.34	353.19	353.20	353.16	353.34	353.18	353.20	353.16
0.1750	0.0750	-	-	327.24	327.20	-	-	327.24	327.20
0.2000	0.0750	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
0.0000	0.0875	-	-	500.00	500.00	-	-	500.00	500.00
0.0250	0.0875	-	-	475.90	475.88	-	-	475.90	475.88
0.0500	0.0875	-	-	451.39	451.38	-	-	451.39	451.37
0.0750	0.0875	-	-	426.72	426.71	-	-	426.72	426.71
0.1000	0.0875	-	-	401.91	401.89	-		401.91	401.89
0.1250	0.0875	-	-	376.95	376.93	-	-	376.95	376.93
0.1500	0.0875	-	-	351.79	351.76	-	-	351.79	351.76
0.1750	0.0875	-	-	326.31	326.28	-	-	326.31	326.28
0.2000	0.0875	-	-	300.00	300.00	-	-	300.00	300.00
0.0000	0.1000	500.00	500.00	500.00	500.00	500.00	500.00	500.00	500.00
0.0250	0.1000	-	-	475.00	475.00	-	-	475.00	475.00
0.0500	0.1000	450.00	450.00	450.00	450.00	450.00	450.00	450.00	450.00
0.0750	0.1000	-	-	425.00	425.00	-	-	425.00	425.00
0.1000	0.1000	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
0.1250	0.1000	-	-	375.00	375.00	-	-	375.00	375.00
0.1500	0.1000	350.00	350.00	350.00	350.00	350.00	350.00	350.00	350.00
0.1750	0.1000	-	-	325.00	325.00	-	-	325.00	325.00
0.2000	0.1000	300.00	300.00	300.00	300.00	300.00	300.00	30000	300.00

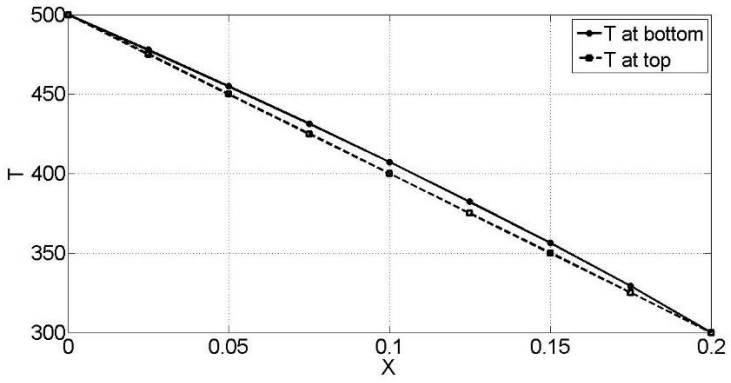

Figure 2.1 Temperature vs X with 4×4 quadratic 9-node element mesh and Newton Iteration.

Table 2.2. Numerical results of Problem 2 (Secondary variable, 4×4 linear elements).

The orientation of gradient vector is measured from the positive x-axis					
x-coord.	y-coord.	-a11(du/dx)	-a22(du/dy)	Flux Mgntd	Orientation
0.2500E-01	0.1250E-01	0.1736E+05	0.9751E+02	0.1736E+05	0.32
0.7500E-01	0.1250E-01	0.1667E+05	0.2256E+03	0.1667E+05	0.78
0.1250E+00	0.1250E-01	0.1572E+05	0.2212E+03	0.1572E+05	0.81
0.1750E+00	0.1250E-01	0.1505E+05	0.8554E+02	0.1505E+05	0.33
0.2500E-01	0.3750E-01	0.1755E+05	0.3101E+03	0.1756E+05	1.01
0.7500E-01	0.3750E-01	0.1675E+05	0.7135E+03	0.1676E+05	2.44
0.1250E+00	0.3750E-01	0.1564E+05	0.7005E+03	0.1566E+05	2.56
0.1750E+00	0.3750E-01	0.1486E+05	0.2727E+03	0.1486E+05	1.05
0.2500E-01	0.6250E-01	0.1798E+05	0.5827E+03	0.1799E+05	1.86
0.7500E-01	0.6250E-01	0.1690E+05	0.1315E+04	0.1695E+05	4.45
0.1250E+00	0.6250E-01	0.1549E+05	0.1295E+04	0.1554E+05	4.78
0.1750E+00	0.6250E-01	0.1443E+05	0.5147E+03	0.1444E+05	2.04
0.2500E-01	0.8750E-01	0.1873E+05	0.9940E+03	0.1876E+05	3.04
0.7500E-01	0.8750E-01	0.1710E+05	0.2111E+04	0.1723E+05	7.04
0.1250E+00	0.8750E-01	0.1530E+05	0.2097E+04	0.1544E+05	7.81
0.1750E+00	0.8750E-01	0.1368E+05	0.8836E+03	0.1370E+05	3.70

Table 2.3. Numerical results of Problem 2 (Secondary variable, 2×2 quadratic elements).

The orientation of gradient vector is measured from the positive x-axis					
x-coord.	y-coord.	-a11(du/dx)	-a22(du/dy)	Flux Mgntd	Orientation
0.2113E-01	0.1057E-01	0.1743E+05	0.7668E+02	0.1743E+05	0.25
0.2113E-01	0.3943E-01	0.1767E+05	0.3129E+03	0.1767E+05	1.01
0.7887E-01	0.1057E-01	0.1664E+05	0.1976E+03	0.1664E+05	0.68
0.7887E-01	0.3943E-01	0.1671E+05	0.7903E+03	0.1673E+05	2.71
0.1211E+00	0.1057E-01	0.1576E+05	0.2023E+03	0.1576E+05	0.74
0.1211E+00	0.3943E-01	0.1569E+05	0.8103E+03	0.1571E+05	2.96
0.1789E+00	0.1057E-01	0.1497E+05	0.7183E+02	0.1497E+05	0.27
0.1789E+00	0.3943E-01	0.1473E+05	0.2927E+03	0.1474E+05	1.14
0.2113E-01	0.6057E-01	0.1801E+05	0.5238E+03	0.1802E+05	1.67
0.2113E-01	0.8943E-01	0.1890E+05	0.9980E+03	0.1893E+05	3.02
0.7887E-01	0.6057E-01	0.1683E+05	0.1310E+04	0.1688E+05	4.45
0.7887E-01	0.8943E-01	0.1699E+05	0.2247E+04	0.1714E+05	7.53
0.1211E+00	0.6057E-01	0.1557E+05	0.1344E+04	0.1563E+05	4.93
0.1211E+00	0.8943E-01	0.1541E+05	0.2321E+04	0.1558E+05	8.57
0.1789E+00	0.6057E-01	0.1439E+05	0.4897E+03	0.1439E+05	1.95
0.1789E+00	0.8943E-01	0.1350E+05	0.9237E+03	0.1353E+05	3.91

Note: the other results with 8×8 linear elements and 4×4 quadratic elements are presented in the output files.

Modified code in the subroutine ELMATRCS2D:

```
      DO 200 NI = 1,NGPF
      DO 200 NJ = 1,NGPF
         XI    = GAUSPT(NI,NGPF)
         ETA = GAUSPT(NJ,NGPF)
         CALL INTERPLN2D(NPE,XI,ETA,DET,ELXY)
         CNST = DET*GAUSWT(NI,NGPF)*GAUSWT(NJ,NGPF)
C
C     Compute x, y, U, UX, UY needed
C
C*          ***** YOUR STATEMENTS *****
            X = 0.0
            Y = 0.0
            U = 0.0
            UX = 0.0
            UY = 0.0
            DO 140 I=1,NPE
            IF(NONLIN.GT.0)THEN
            U = U + ELU(I)*SFL(I)
            UX = UX + ELU(I)*GDSFL(1,I)
            UY = UY + ELU(I)*GDSFL(2,I)
            ENDIF
            X = X + ELXY(I,1)*SFL(I)
140         Y = Y + ELXY(I,2)*SFL(I)
C
C     Define the coefficients of the differential equation
C
C*          ***** YOUR STATEMENTS *****
            FXY = F0 + FX*X + FY*Y
            A00 = A00 + A0X*X + A0Y*Y
            A11 = A10 + A1X*X + A1Y*Y
            A22 = A20 + A2X*X + A2Y*Y
            IF(NONLIN.GT.0)THEN
            AXX = A11 + A1U*U + A1UX*UX + A1UY*UY
            AYY = A22 + A2U*U + A2UX*UX + A2UY*UY
            ENDIF
C
C     Define the element source vector and coefficient matrix
C
C*          ***** YOUR STATEMENTS ****
          DO 180 I=1,NPE
             ELF(I) = ELF(I) + FXY*SFL(I)*CNST
             DO 160 J=1,NPE
```

```
                S00 = SFL(I)*SFL(J)*CNST
                S11 = GDSFL(1,I)*GDSFL(1,J)*CNST
                S22 = GDSFL(2,I)*GDSFL(2,J)*CNST
                ELK(I,J) = ELK(I,J) + AXX*S11 + AYY*S22 + A00*S00
C
C       Define the part needed to be added to [K] in order to define [T]
C
C*          ***** YOUR STATEMENTS *****
                IF(NONLIN.GT.1)THEN
                S10 = GDSFL(1,I)*SFL(J)*CNST
                S20 = GDSFL(2,I)*SFL(J)*CNST
                S12 = GDSFL(1,I)*GDSFL(2,J)*CNST
                S21 = GDSFL(2,I)*GDSFL(1,J)*CNST
                TANG(I,J) = TANG(I,J) + UX*(A1U*S10 + A1UX*S11 + A1UY*S12)
     *                                  + UY*(A2U*S20 + A2UX*S21 + A2UY*S22)
                ENDIF
  160       CONTINUE
  180       CONTINUE

  200 CONTINUE
```

Modified code in the subroutine POSTPROCS2D:

```
DO 100 NI=1,NGPR
          DO 100 NJ=1,NGPR
          XI  = GAUSPT(NI,NGPR)
          ETA = GAUSPT(NJ,NGPR)
          CALL INTERPLN2D(NPE,XI,ETA,DET,ELXY)
C
C*          ***** YOUR STATEMENTS *****
C
          XC = 0.0
          YC = 0.0
          U = 0.0
          UX = 0.0
          UY = 0.0
          DO 60 I=1,NPE
          IF(NONLIN.GT.0)THEN
               U = U + ELU(I)*SFL(I)
               UX = UX + ELU(I)*GDSFL(1,I)
               UY = UY + ELU(I)*GDSFL(2,I)
          ENDIF
          XC = XC + ELXY(I,1)*SFL(I)
60        YC = YC + ELXY(I,2)*SFL(I)
```

```
C
C     Define the coefficients of the differential equation
C
C*         ***** YOUR STATEMENTS *****
          A11 = A10 + A1X*XC + A1Y*YC
          A22 = A20 + A2X*XC + A2Y*YC
          IF(NONLIN.GT.0)THEN
               AXX = A11 + A1U*U + A1UX*UX + A1UY*UY
               AYY = A22 + A2U*U + A2UX*UX + A2UY*UY
          ENDIF

          SX = -AXX*UX
          SY = -AYY*UY
          VALUE = DSQRT(SX*SX + SY*SY)
          IF(IGRAD.EQ.1)THEN
               QX = SX
               QY = SY
          ELSE
               QX = -SY
               QY = SX
          ENDIF
          IF(DABS(QX).LE.0.0001)THEN
               IF(QY.LT.0.0)THEN
                    ANGLE = -90.0
               ELSE
                    ANGLE = 90.0
               ENDIF
          ELSE
               ANGLE = ATAN2(QY,QX)*CONST
          ENDIF

          WRITE(IT,200) XC,YC,QX,QY,VALUE,ANGLE
  100     CONTINUE
C
  200 FORMAT(5E13.4,3X,F7.2)
  300 FORMAT(6E13.4)
  400 FORMAT(26X,2E13.4)
```