Table 9.4.1: Total displacements* of node 17 (at the free end) in a cantilevered plate under uniform load; obtained with the *updated Lagrangian formulation* (5Q8).

	3 × 3 Gauss rule for **K**				2 × 2 Gauss rule for **K**			
$f_0 = q_0 h$	x	$-y$	$-u$	$-v$	x	$-y$	$-u$	$-v$
50	9.9787	0.1145	0.0213	0.6145	9.9786	0.1163	0.0214	0.6163
	10.0000	0.1164	0.0000	0.6164	10.0000	0.1181	0.0000	0.6181
100	9.9159	0.7181	0.0841	1.2181	9.9155	0.7225	0.0845	1.2225
	9.9690	0.2686	0.0310	0.7686	9.9689	0.2707	0.0311	0.7707
150	9.8152	1.3010	0.1848	1.8010	9.8138	1.3091	0.1862	1.8091
	9.8901	1.1644	0.1099	1.6644	9.8896	1.1698	0.1104	1.6698
200	9.6816	1.8554	0.3184	2.3554	9.6785	1.8688	0.3215	2.3688
	9.8272	1.2658	0.1728	1.7658	9.8264	1.2719	0.1736	1.7719
250	9.5212	2.3758	0.4788	2.8758	9.5152	2.3960	0.4848	2.8960
	9.6208	2.2693	0.3792	2.7693	9.6183	2.2818	0.3817	2.7818
300	9.3402	2.8593	0.6598	3.3593	9.3300	2.8874	0.6699	3.3874
	9.5421	2.3359	0.4579	2.8359	9.5387	2.3493	0.4613	2.8493
350	9.1444	3.3046	0.8556	3.8046	9.1288	3.3417	0.8712	3.8417
	9.2429	3.2102	0.7571	3.7102	9.2347	3.2350	0.7653	3.7350
400	8.9391	3.7125	1.0609	4.2125	8.9167	3.7587	1.0833	4.2587
	9.1659	3.2796	0.8341	3.7796	9.1561	3.3051	0.8439	3.8051
450	8.7284	4.0846	1.2716	4.5846	8.6984	4.1398	1.3016	4.6398
	8.8026	4.0388	1.1974	4.5388	8.7828	4.0812	1.2172	4.5812
500	8.5160	4.4229	1.4840	4.9229	8.4776	4.4871	1.5224	4.9871
	8.7266	4.1054	1.2734	4.6054	8.7046	4.1480	1.2954	4.6480
550	8.3046	4.7302	1.6954	5.2302	8.2571	4.8028	1.7429	5.3028
	8.3545	4.7083	1.6455	5.2083	8.3179	4.7692	1.6821	5.2692
600	8.0962	5.0093	1.9038	5.5093	8.0392	5.0897	1.9608	5.5897
	8.2768	4.7839	1.7232	5.2839	8.2380	4.8448	1.7620	5.3448
650	7.8922	5.2628	2.1078	5.7628	7.8270	5.3480	2.1730	5.8480
	7.9257	5.2508	2.0743	5.7508	7.8691	5.3291	2.1309	5.8291
700	7.6937	5.4933	2.3063	5.9933	7.6195	5.5843	2.3805	6.0843
	7.8407	5.3368	2.1593	5.8368	7.7820	5.4146	2.2180	5.9146
750	7.5013	5.7031	2.4987	6.2031	7.4180	5.7995	2.5820	6.2995
	7.5280	5.6913	2.4720	6.1913	7.4510	5.7835	2.5490	6.2835
800	7.3171	5.8922	2.6829	6.3922	7.2233	5.9956	2.7767	6.4956
	7.4303	5.7881	2.5697	6.2881	7.3514	5.8796	2.6486	6.3796
850	7.1384	6.0668	2.8616	6.5668	7.0356	6.1746	2.9644	6.6746
	7.1618	6.0550	2.8382	6.5550	7.0657	6.1576	2.9343	6.657
900	6.9664	6.2268	3.0336	6.7268	6.8550	6.3383	3.1450	6.8383
	7.0488	6.1600	2.9512	6.6600	6.9505	6.2621	3.0495	6.7621

*The first row corresponds to solution with iteration and the second row corresponds to the solution with no iteration.

Table 9.4.2: Stresses* ($\times 10^{-5}$) evaluated at the left-most Gauss point nearest to the top of element 1 in a cantilevered plate under uniform load; obtained with the *updated Lagrangian formulation* and 5Q8 mesh ($X = 0.42265$ and $Y = 0.788675$).

			Cauchy stress components			Piola–Kirchhoff stress components		
$f_0 = q_0 h$	x	y	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$-\bar{\sigma}_{xy}$	$\bar{S}_{XX}$	$\bar{S}_{YY}$	$-\bar{S}_{XY}$
50	0.4253	0.7861	0.7776	0.1819	0.0539	0.7885	0.1840	0.0487
	0.4254	0.7860	0.7929	0.1840	0.0545	0.8042	0.1862	0.0491
100	0.4280	0.7835	1.5457	0.3578	0.1198	1.5894	0.3661	0.0994
	0.4282	0.7833	1.5779	0.3584	0.1220	1.6236	0.3670	0.1004
150	0.4308	0.7810	2.2962	0.5268	0.1975	2.3950	0.5448	0.1521
	0.4310	0.7806	2.3476	0.5225	0.2022	2.4514	0.5416	0.1540
200	0.4335	0.7783	3.0226	0.6880	0.2861	3.1978	0.7191	0.2070
	0.4339	0.7779	3.0961	0.6761	0.2943	3.2812	0.7093	0.2097
250	0.4362	0.7757	3.7200	0.8411	0.3845	3.9918	0.8880	0.2640
	0.4367	0.7752	3.8185	0.8193	0.3970	4.1075	0.8700	0.2671
300	0.4389	0.7731	4.3854	0.9863	0.4915	4.7721	1.0509	0.3230
	0.4396	0.7724	4.5117	0.9527	0.5092	4.9255	1.0232	0.3261
350	0.4415	0.7706	5.0172	1.1237	0.6057	5.5354	1.2076	0.3838
	0.4424	0.7697	5.1731	1.0766	0.6293	5.7310	1.1693	0.3864
400	0.4441	0.7680	5.6151	1.2537	0.7258	6.2795	1.3579	0.4461
	0.4452	0.7670	5.8027	1.1921	0.7562	6.5221	1.3086	0.4476
450	0.4466	0.7655	6.1793	1.3767	0.8507	7.0025	1.5018	0.5098
	0.4478	0.7643	6.400	1.3000	0.8886	7.2971	1.4414	0.5096
500	0.4491	0.7630	6.7118	1.4933	0.9793	7.7050	1.6396	0.5746
	0.4505	0.7617	6.9668	1.4004	1.0254	8.0548	1.5682	0.5721
550	0.4514	0.7605	7.2141	1.6040	1.1108	8.3870	1.7716	0.6402
	0.4531	0.7590	7.5032	1.4948	1.1658	8.7948	1.6894	0.6349
600	0.4536	0.7580	7.6879	1.7093	1.2444	9.0489	1.8981	0.7064
	0.4556	0.7564	8.0111	1.5836	1.3088	9.5172	1.8055	0.6978
650	0.4558	0.7556	8.1352	1.8096	1.3795	9.6916	2.0193	0.7731
	0.4580	0.7540	8.4794	1.6631	1.4502	10.2038	1.9118	0.7597
700	0.4579	0.7532	8.5579	1.9055	1.5157	10.3161	2.1356	0.8402
	0.4603	0.7514	8.9337	1.7422	1.5961	10.8897	2.0184	0.8225
750	0.4600	0.7508	8.9577	1.9973	1.6525	10.9233	2.2474	0.9074
	0.4626	0.7489	9.3650	1.8176	1.7431	11.5602	2.1212	0.8851
800	0.4619	0.7486	9.3272	2.0817	1.7862	11.4997	2.3511	0.9736
	0.4649	0.7464	9.7742	1.8897	1.8907	12.2153	2.2205	0.9474
850	0.4638	0.7463	9.6857	2.1660	1.9230	12.0742	2.4544	1.0408
	0.4670	0.7440	10.1628	1.9588	2.0385	12.8557	2.3166	1.0094
900	0.4657	0.7440	10.0265	2.2473	2.0598	12.6351	2.5542	1.1080
	0.4692	0.7416	10.5321	2.0253	2.1864	13.4821	2.4097	1.0710

*The first row corresponds to the 3×3 Gauss rule for the evaluation of **K** and the second row corresponds to the 2×2 Gauss rule for the evaluation of **K**.

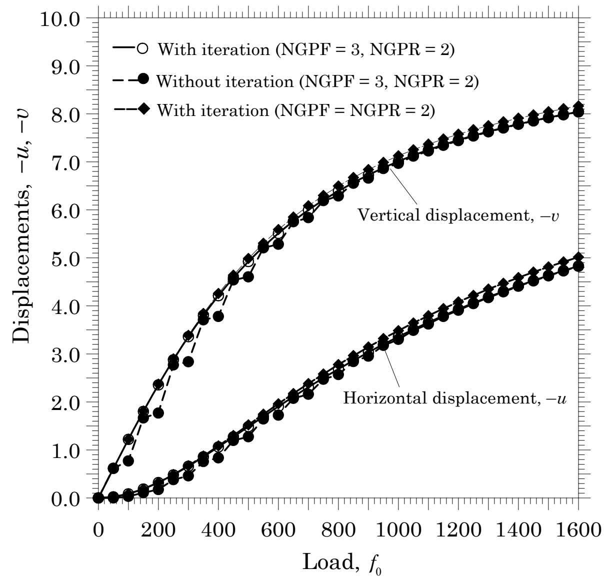

Fig. 9.4.2: Node 17 displacements $-v$ and $-u$ versus load $f_0 = q_0h$ (obtained with the UL formulation).

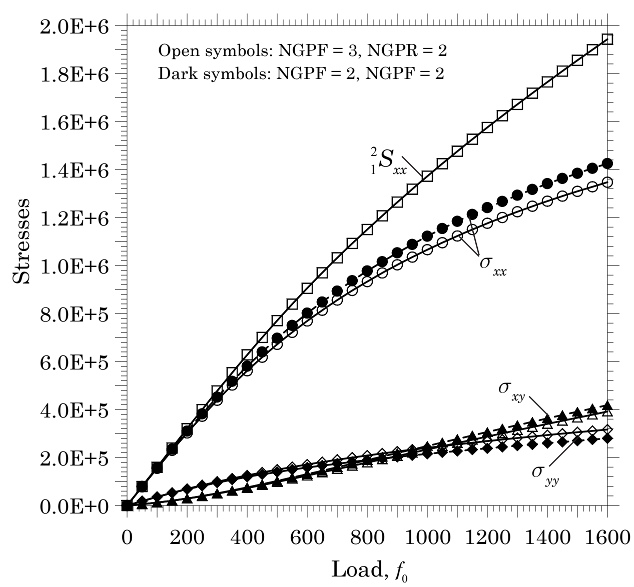

Fig. 9.4.3: Stresses versus load $f_0 = q_0h$ (obtained with the UL formulation).

The mesh of five nine-node quadratic elements also gives almost identical results for displacements (u, v) and stresses $(\sigma_{xx},\ \sigma_{xy})$, as can be seen from Table 9.4.3. The stiffness coefficients were computed using the 3×3 Gauss rule while stresses were calculated at the 2×2 Gauss point locations.

Table 9.4.3: Total displacements of node 22 and stresses* ($\times 10^{-5}$) in a cantilevered plate under uniform load; obtained with the *updated Lagrangian formulation* and the mesh of 5Q9 elements (3×3 Gauss rule for $\mathbf{K}$ and 2×2 Gauss rule for stresses).

$f_0 = q_0 h$	x	$-y$	u	v	x	y	σ_{xx}	$-\sigma_{xy}$
50	9.9782	0.1218	0.0218	0.6218	0.4254	0.7860	0.7759	0.0549
					0.4226	0.7887	0.7877	0.0485
100	9.9142	0.7323	0.0858	1.2323	0.4282	0.7833	1.5429	0.1239
					0.4226	0.7887	1.5904	0.0986
150	9.8115	1.3214	0.1885	1.8214	0.4311	0.7806	2.2924	0.2065
					0.4226	0.7887	2.3997	0.1504
200	9.6754	1.8810	0.3246	2.3810	0.4340	0.7780	3.0175	0.3018
					0.4226	0.7887	3.2081	0.2041
250	9.5123	2.4057	0.4877	2.9057	0.4368	0.7753	3.7131	0.4084
					0.4226	0.7887	4.0087	0.2597
300	9.3286	2.8923	0.6714	3.3923	0.4396	0.7726	4.3758	0.5249
					0.4226	0.7887	4.7966	0.3171
350	9.1301	3.3400	0.8699	3.8400	0.4424	0.7699	5.0042	0.6497
					0.4226	0.7887	5.5681	0.3764
400	8.9224	3.7494	1.0776	4.2494	0.4450	0.7673	5.5980	0.7814
					0.4226	0.7887	6.3207	0.4372
450	8.7095	4.1224	1.2905	4.6224	0.4476	0.7646	6.1572	0.9185
					0.4226	0.7887	7.0524	0.4994
500	8.4953	4.4611	1.5047	4.9611	0.4501	0.7621	6.6840	1.0600
					0.4226	0.7887	7.7638	0.5628
550	8.2823	4.7684	1.7177	5.2684	0.4525	0.7595	7.1800	1.2047
					0.4226	0.7887	8.4548	0.6272
600	8.0725	5.0472	1.9275	5.5472	0.4549	0.7570	7.6470	1.3521
					0.4226	0.7887	9.1257	0.6925
650	7.8675	5.3002	2.1325	5.8002	0.4572	0.7545	8.0870	1.5012
					0.4226	0.7887	9.7774	0.7586
700	7.6681	5.5301	2.3319	6.0301	0.4594	0.7520	8.5019	1.6517
					0.4226	0.7887	10.4110	0.8252
750	7.4750	5.7392	2.5250	6.2392	0.4615	0.7496	8.8936	1.8030
					0.4226	0.7887	11.0270	0.8923
800	7.2886	5.9299	2.7114	6.4299	0.4635	0.7472	9.2638	1.9547
					0.4226	0.7887	11.6270	0.9598
850	7.1114	6.1013	2.8886	6.6013	0.4655	0.7449	9.6050	2.1020
					0.4226	0.7887	12.1960	1.0260
900	6.9392	6.2604	3.0608	6.7604	0.4674	0.7426	9.9366	2.2534
					0.4226	0.7887	12.7650	1.0939
950	6.7738	6.4064	3.2262	6.9064	0.4693	0.7403	10.2510	2.4048
					0.4226	0.7887	13.3220	1.1620
1,000	6.6150	6.5407	3.3850	7.0407	0.4711	0.7380	10.5510	2.5558
					0.4226	0.7887	13.8660	1.2303
1,050	6.4627	6.6644	3.5373	7.1644	0.4728	0.7357	10.8350	2.7064
					0.4226	0.7887	14.3990	1.2986
1,100	6.3167	6.7787	3.6833	7.2787	0.4745	0.7334	11.1060	2.8564
					0.4226	0.7887	14.9210	1.3669

*The second row corresponds to the second Piola–Kirchhoff stress components.

Table 9.4.4 contains the displacements and stresses obtained with the total Lagrangian formulation and 5Q8 and 5Q9 meshes. The results are not in close agreement with those obtained with the updated Lagrangian formulation because the material properties are not updated.

Table 9.4.4: Total displacements and stresses* in a cantilevered plate under uniform load (*total Lagrangian formulation* is used; $X = 0.422650$ and $Y = 0.788675$).

$f_0 = q_0 h$	u	v	x	y	σ_{xx}	$-\sigma_{xy}$	S_{XX}	$-S_{XY}$
50	0.0216	0.6144	0.4253	0.7861	0.7631	0.0534	0.7736	0.0484
	0.0221	0.6218	0.4254	0.7860	0.7598	0.0544	0.7711	0.0482
100	0.0853	1.2177	0.4278	0.7836	1.489	0.1176	1.5299	0.0979
	0.0871	1.2320	0.4280	0.7834	1.4801	0.1216	1.5243	0.0971
150	0.1874	1.7997	0.4303	0.7810	2.1729	0.1916	2.2619	0.1487
	0.1912	1.8203	0.4306	0.7807	2.1557	0.2006	2.2521	0.1470
200	0.3225	2.3524	0.4327	0.7784	2.8113	0.2744	2.9642	0.2009
	0.3290	2.3786	0.4331	0.7780	2.7837	0.2900	2.9493	0.1980
250	0.4846	2.8706	0.4350	0.7759	3.4033	0.3646	3.6333	0.2545
	0.4940	2.9013	0.4355	0.7754	3.3630	0.3884	3.6121	0.2502
300	0.6671	3.3510	0.4372	0.7734	3.9494	0.4610	4.2672	0.3094
	0.6795	3.3854	0.4380	0.7727	3.8945	0.4943	4.2387	0.3036
350	0.8641	3.7927	0.4393	0.7709	4.4512	0.5622	4.8656	0.3656
	0.8795	3.8300	0.4400	0.7702	4.3802	0.6062	4.8287	0.3581
400	1.0702	4.1965	0.4413	0.7684	4.9113	0.6672	5.4290	0.4228
	1.0886	4.2358	0.4420	0.7676	4.8229	0.7228	5.3830	0.4136
450	1.2812	4.5640	0.4432	0.7660	5.3328	0.7750	5.9588	0.4809
	1.3023	4.6048	0.4439	0.7651	5.2259	0.8430	5.9032	0.4701
500	1.4936	4.8977	0.4450	0.7636	5.7187	0.8847	6.4570	0.5398
	1.5171	4.9394	0.4457	0.7626	5.5925	0.9657	6.3911	0.5274
550	1.7045	5.2003	0.4466	0.7613	6.0721	0.9957	6.9256	0.5994
	1.7302	5.2426	0.4474	0.7602	5.9260	1.0902	6.8491	0.5853
600	1.9122	5.4746	0.4482	0.7590	6.3962	1.1075	7.3668	0.6594
	1.9398	5.5171	0.4490	0.7578	6.2297	1.2159	7.2794	0.6438
650	2.1150	5.7234	0.4497	0.7567	6.6935	1.2197	7.7827	0.7198
	2.1444	5.7659	0.4505	0.7555	6.5064	1.3423	7.6842	0.7027
700	2.3121	5.9493	0.4511	0.7545	6.9667	1.3318	8.1754	0.7805
	2.343	5.9916	0.4520	0.7532	6.7587	1.4690	8.0656	0.7620
750	2.5028	6.1547	0.4524	0.7523	7.2180	1.4438	8.5467	0.8414
	2.5350	6.1967	0.4533	0.7509	6.9890	1.5956	8.4257	0.8216
800	2.6868	6.3418	0.4536	0.7501	7.4494	1.5554	8.8985	0.9025
	2.7201	6.3833	0.4546	0.7487	7.1993	1.7220	8.7662	0.8814
850	2.8654	6.5138	0.4548	0.7480	7.6637	1.6666	9.2337	0.9637
	2.8997	6.5548	0.4557	0.7465	7.3924	1.8483	9.0900	0.9415
900	3.0353	6.6697	0.4559	0.7459	7.8606	1.7769	9.5508	1.0248
	3.0704	6.7101	0.4569	0.7444	7.5682	1.9736	9.3960	1.0016

*The first row corresponds to the mesh of five 5Q8 elements and the second row corresponds to the mesh of 5Q9 elements.