**MEEN 673**

**Assignment 1**

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**Problem 1**

Consider the nonlinear differential equation



Develop weak form and finite element model of the equation over an element. Compute the tangent coefficient matrix of the model.



(1) Weak form:



where , .

(2) Finite element model:

Let .



Thus,



(3) Tangent coefficient matrix



**Program 1**

(1) 

Direct iteration: NONLIN = 1, Newton iteration: NONLIN = 2.

Box 1. An example of input file for the validation case (Ex4.4.1)

Example 4.4.1: Nonlinear analysis of a problem (DI)

0.0 0.0 1.0 0.0 0.0 0.0 AX0,AX1,AU1,AU2,AUX1,AUX2

0.0 0.0 0.0 0.0 0.0 0.0 BX0,BX1,BU1,BU2,BUX1,BUX2

0.0 0.0 0.0 0.0 0.0 0.0 CX0,CX1,CU1,CU2,CUX1,CUX2

-1.0 0.0 0.0 0.0 0.0 0.0 FX0,FX1,FX2

0.0 1.0 X0,AL

1 2 IEL, NEM

1 0 0 NSPV, NSSV, NSMB

3 1 1.4142 ISPV(I,J),VSPV(I)

1 0 NONLIN, NPRNT

1 10 1.0E-03 0.0 NLS, ITMAX, EPS, GAMA

1.0 DP(I)

1.0 1.0 1.4142 GP1(I)

(2) 

Box 2. An example of input file for Problem 4.5

Example 4.5: Nonlinear analysis of a problem

1.0 0.0 0.0 0.0 0.0 0.0 AX0,AX1,AU1,AU2,AUX1,AUX2

0.0 0.0 -2.0 0.0 0.0 0.0 BX0,BX1,BU1,BU2,BUX1,BUX2

0.0 0.0 0.0 0.0 0.0 0.0 CX0,CX1,CU1,CU2,CUX1,CUX2

0.0 0.0 0.0 0.0 0.0 0.0 FX0,FX1,FX2

0.0 1.0 X0,AL

1 2 IEL, NEM

2 0 0 NSPV, NSSV, NSMB

1 1 1.0 ISPV(I,J),VSPV(I)

3 1 0.5

1 0 NONLIN, NPRNT

1 10 1.0E-03 0.0 NLS, ITMAX, EPS, GAMA

1.0 DP(I)

1.0 1.0 0.5 GP1(I)

Table 1. Numerical results of the validation case (Ex4.4.1)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X(m) | Direct Iteration | | | | | | Newton Iteration | | | | | |
| 2L | 4L | 8L | 1Q | 2Q | 4Q | 2L | 4L | 8L | 1Q | 2Q | 4Q |
| 0.000 | 0.99992 | 0.99987 | 0.99982 | 0.99993 | 0.99987 | 0.99980 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 | 0.99998 |
| 0.125 | - | - | 1.00760 | - | - | 1.00760 | - | - | 1.00780 | - | - | 1.00780 |
| 0.250 | - | 1.03070 | 1.03060 | - | 1.03080 | 1.03060 | - | 1.03080 | 1.03080 | - | 1.03080 | 1.03080 |
| 0.375 | - | - | 1.06790 | - | - | 1.06790 | - | - | 1.06800 | - | - | 1.06800 |
| 0.500 | 1.11800 | 1.11800 | 1.11800 | 1.11870 | 1.11800 | 1.11800 | 1.11800 | 1.11800 | 1.11800 | 1.11800 | 1.11800 | 1.11800 |
| 0.625 | - | - | 1.17920 | - | - | 1.17920 | - | - | 1.17920 | - | - | 1.17920 |
| 0.750 | - | 1.25000 | 1.25000 | - | 1.25000 | 1.25000 | - | 1.25000 | 1.25000 | - | 1.25000 | 1.25000 |
| 0.875 | - | - | 1.32880 | - | - | 1.32880 | - | - | 1.32880 | - | - | 1.32880 |
| 1.000 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 | 1.41420 |

Table 2. Numerical results of Problem 4.5

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X(m) | Direct Iteration | | | | | | Newton Iteration | | | | | |
| 2L | 4L | 8L | 1Q | 2Q | 4Q | 2L | 4L | 8L | 1Q | 2Q | 4Q |
| 0.000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 0.125 | - | - | 0.88857 | - | - | 0.88883 | - | - | 0.88855 | - | - | 0.88881 |
| 0.250 | - | 0.79821 | 0.79958 | - | 0.79924 | 0.80003 | - | 0.79819 | 0.79956 | - | 0.79921 | 0.80001 |
| 0.375 | - | - | 0.72686 | - | - | 0.72728 | - | - | 0.72683 | - | - | 0.72725 |
| 0.500 | 0.66002 | 0.66514 | 0.66631 | 0.66076 | 0.66675 | 0.66670 | 0.66000 | 0.66512 | 0.66629 | 0.66071 | 0.66673 | 0.66667 |
| 0.625 | - | - | 0.61511 | - | - | 0.61540 | - | - | 0.61509 | - | - | 0.61537 |
| 0.750 | - | 0.57063 | 0.57124 | - | 0.57132 | 0.57145 | - | 0.57062 | 0.57123 | - | 0.57131 | 0.57143 |
| 0.875 | - | - | 0.53324 | - | - | 0.53334 | - | - | 0.53323 | - | - | 0.53333 |
| 1.000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0.5000 | 0.50000 |