A Gentle Introduction to Bilateral Filtering and its Applications

SIGGRAPH2007

"Fixing the Gaussian Blur": the Bilateral Filter

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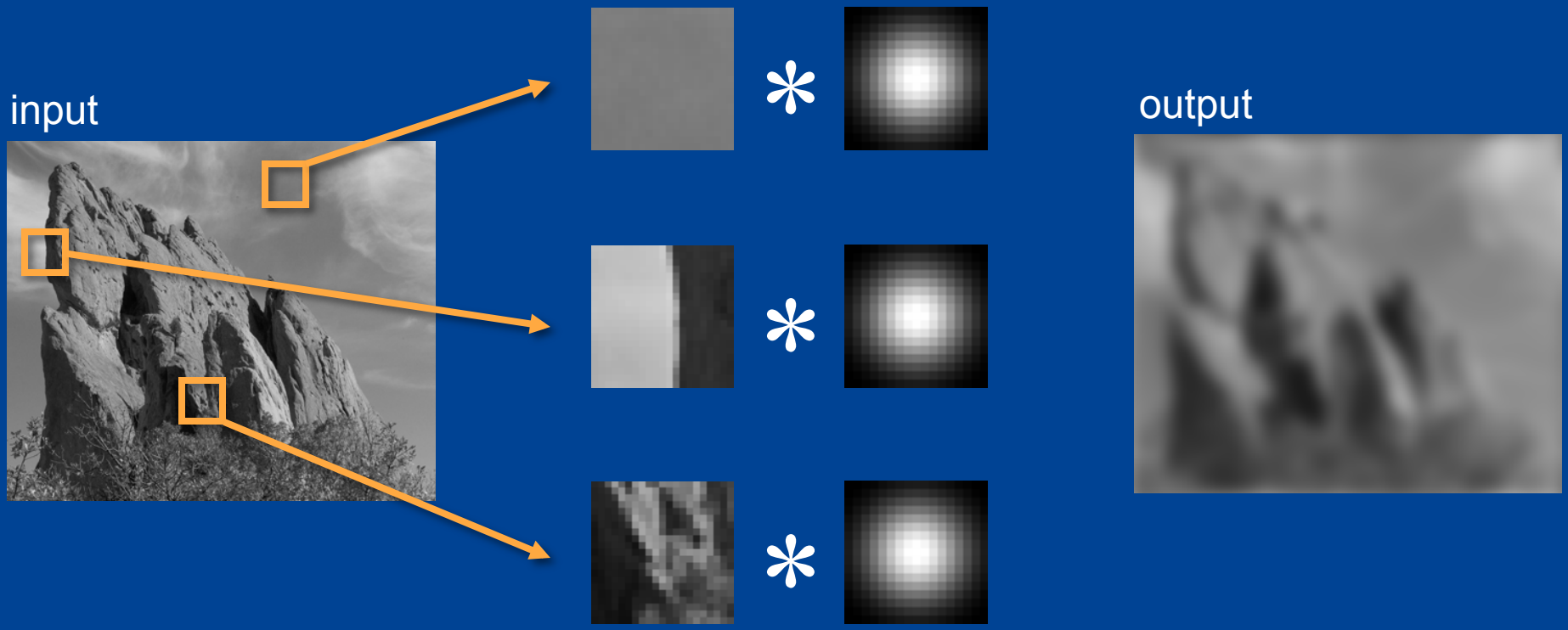
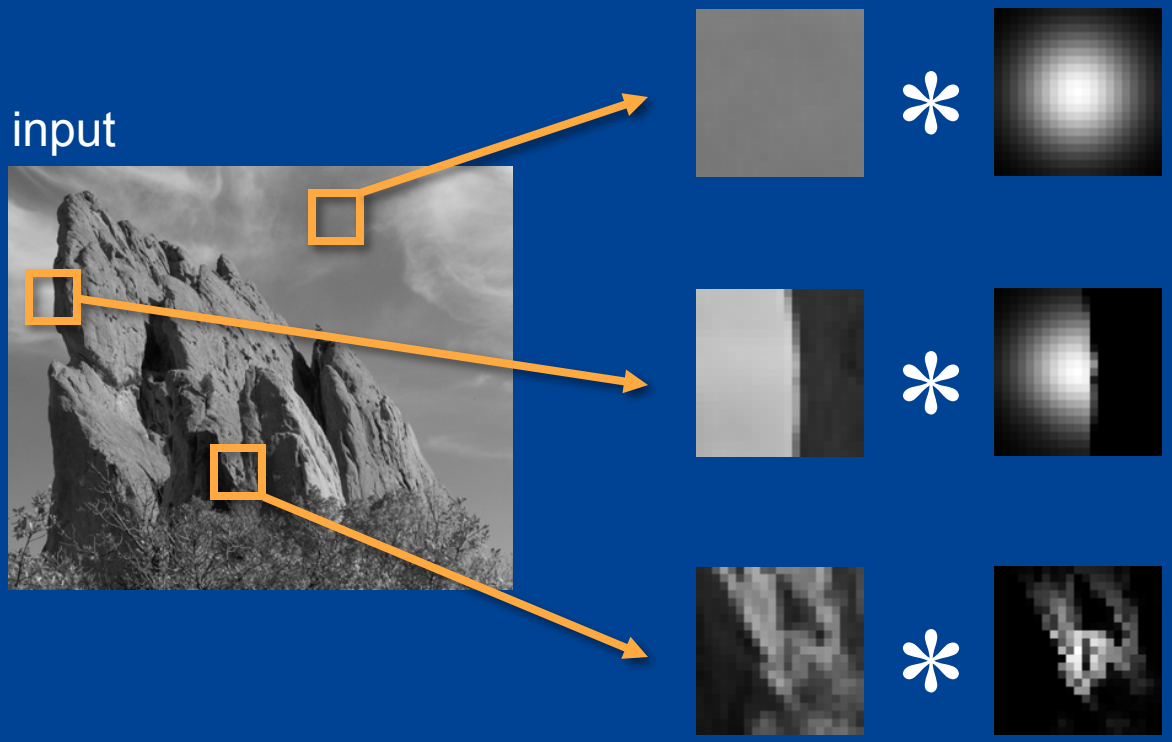
Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

Bilateral Filter [Aurich 95, Smith 97, Tomasi 98]
No Averaging across Edges



The kernel shape depends on the image content.

Bilateral filter

- Tomasi and Manduci 1998]
  - http://www.cse.ucsc.edu/~manduchi/Papers/ICCV98.pdf
- Developed for denoising
- Related to
  - SUSAN filter [Smith and Brady 95] http://citeseer.ist.psu.edu/smith95susan.html
  - Digital-TV [Chan, Osher and Chen 2001] http://citeseer.ist.psu.edu/chan01digital.html
  - sigma filter http://www.geogr.ku.dk/CHIPS/Manual/f187.htm
- Full survey: http://people.csail.mit.edu/sparis/publi/2009/fntcgv/Paris_09_Bilateral_filtering.pdf

Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels.**

new

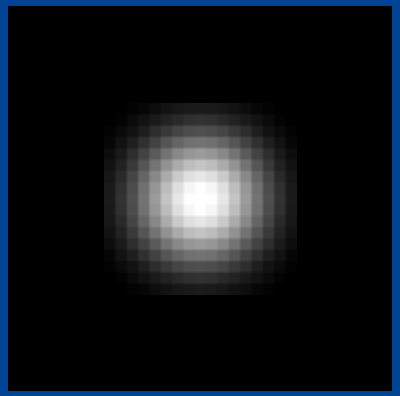
not new

new

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

normalization factor

space weight

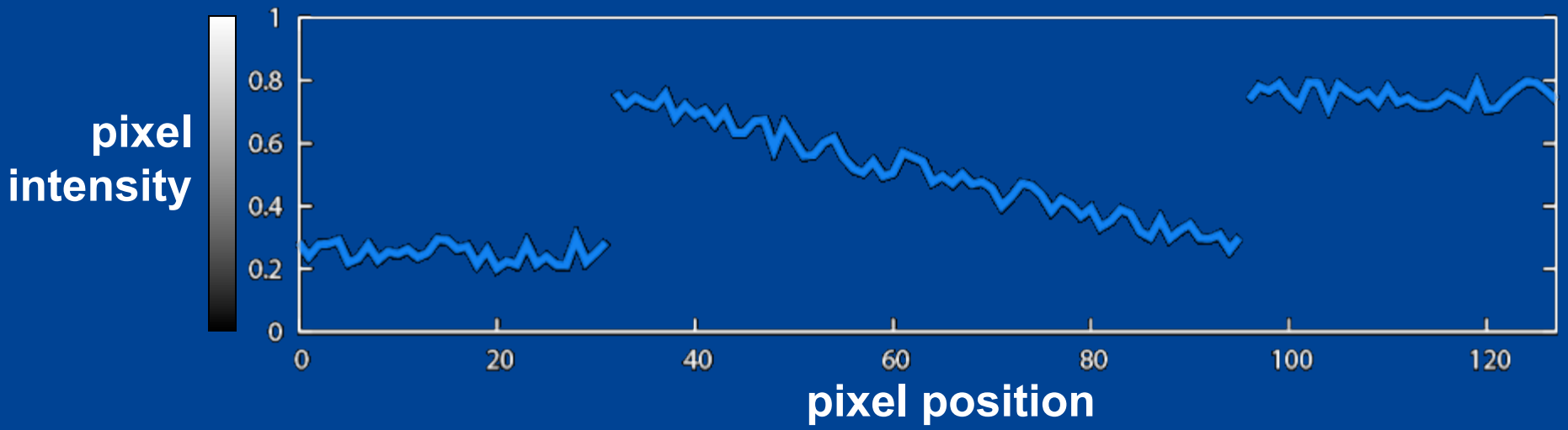


range weight

Illustration a 1D Image

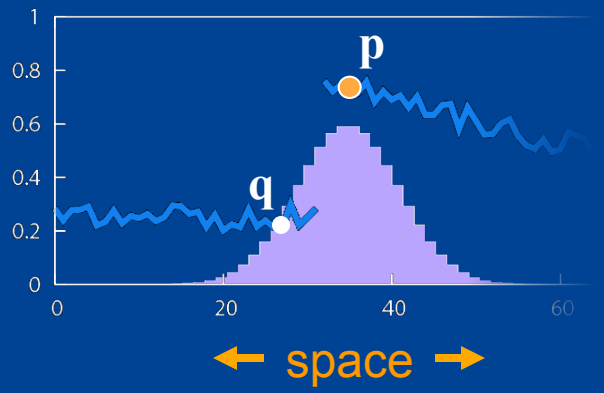
- 1D image = line of pixels

- Better visualized as a plot



Gaussian Blur and Bilateral Filter

Gaussian blur

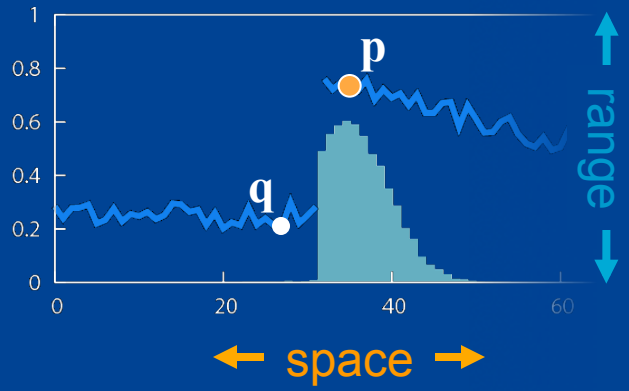


$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|) I_{\mathbf{q}}$$

space

Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



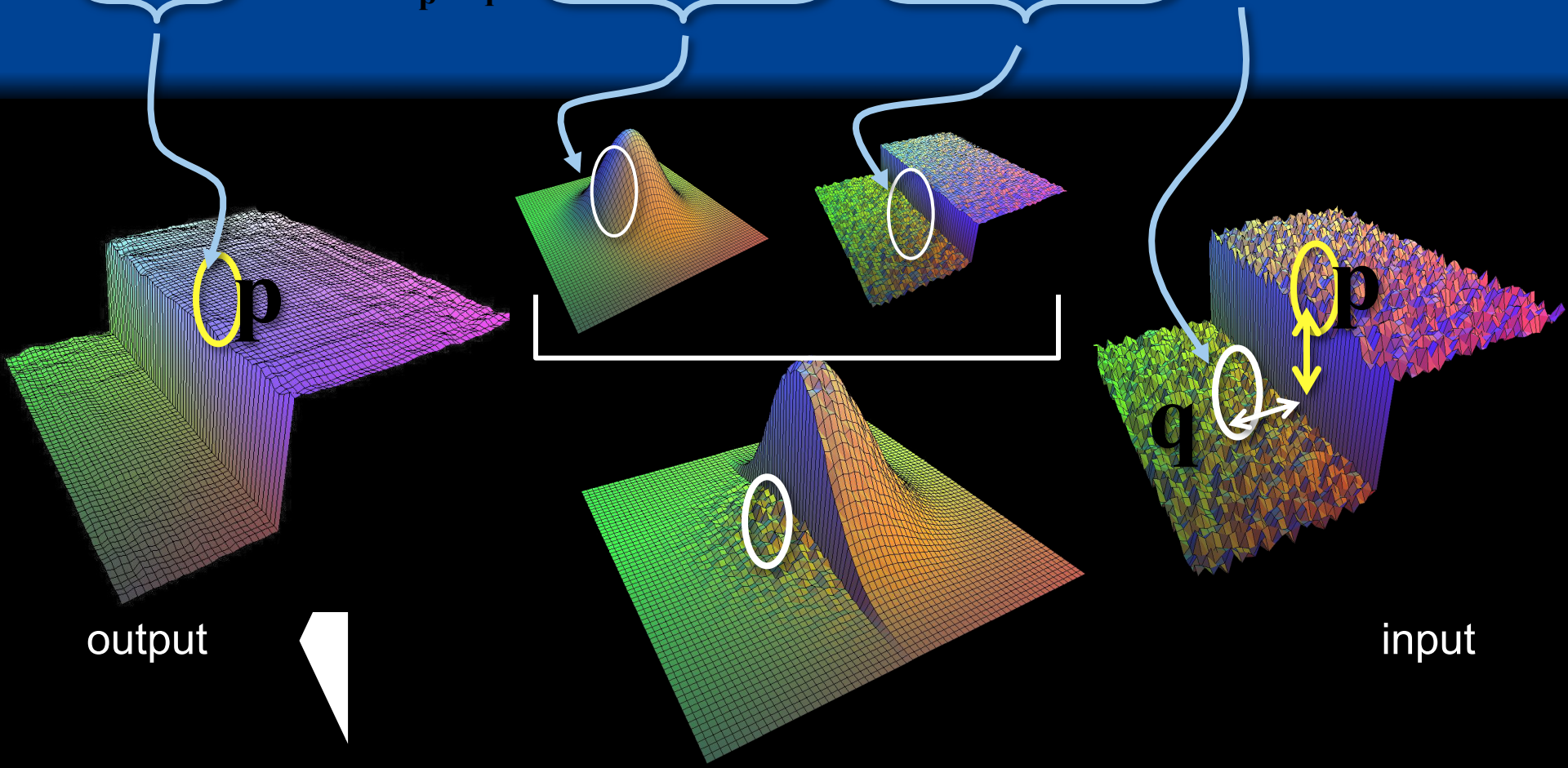
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$

space range

normalization

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$



reproduced from [Durand 02]

Space and Range Parameters

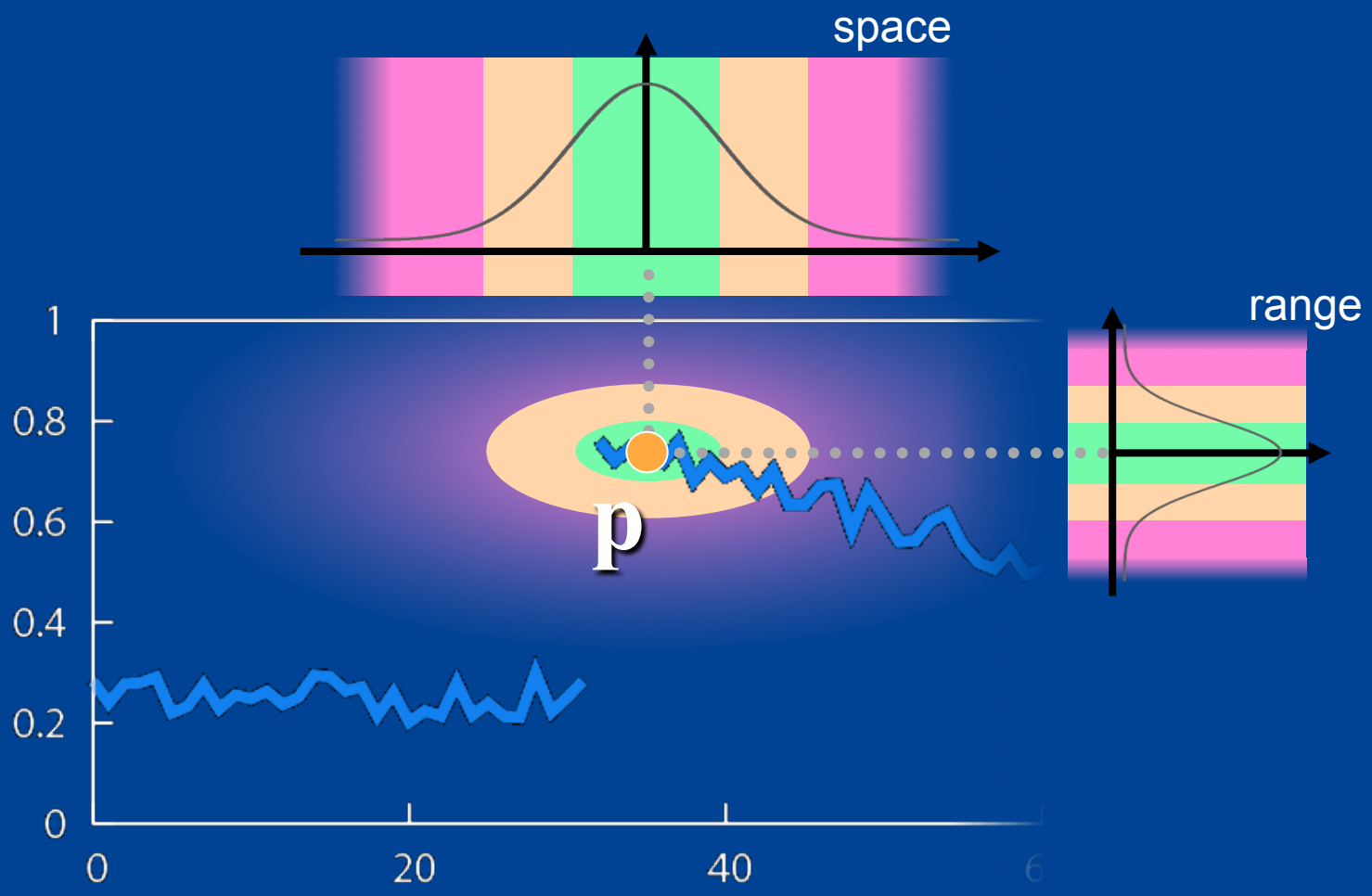
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.

- range σ_r : “minimum” amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



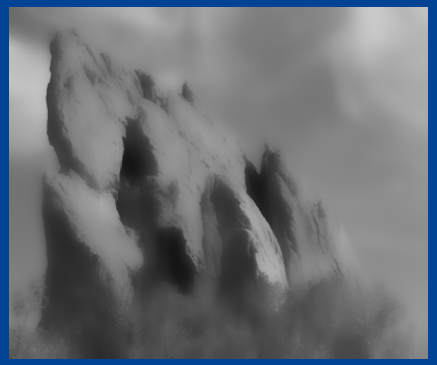
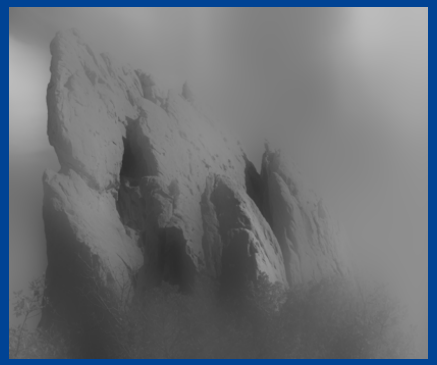
Bilateral filter

Noisy input

After bilateral filter

Exploring the Parameter Space

input

	$\sigma_r = 0.1$	$\sigma_r = 0.25$	$\sigma_r = \infty$ (Gaussian blur)
$\sigma_s = 2$			
$\sigma_s = 6$			
$\sigma_s = 18$			

Varying the Range Parameter

input

$\sigma_r = 0.1$ $\sigma_r = 0.25$ $\sigma_r = \infty$ (Gaussian blur)

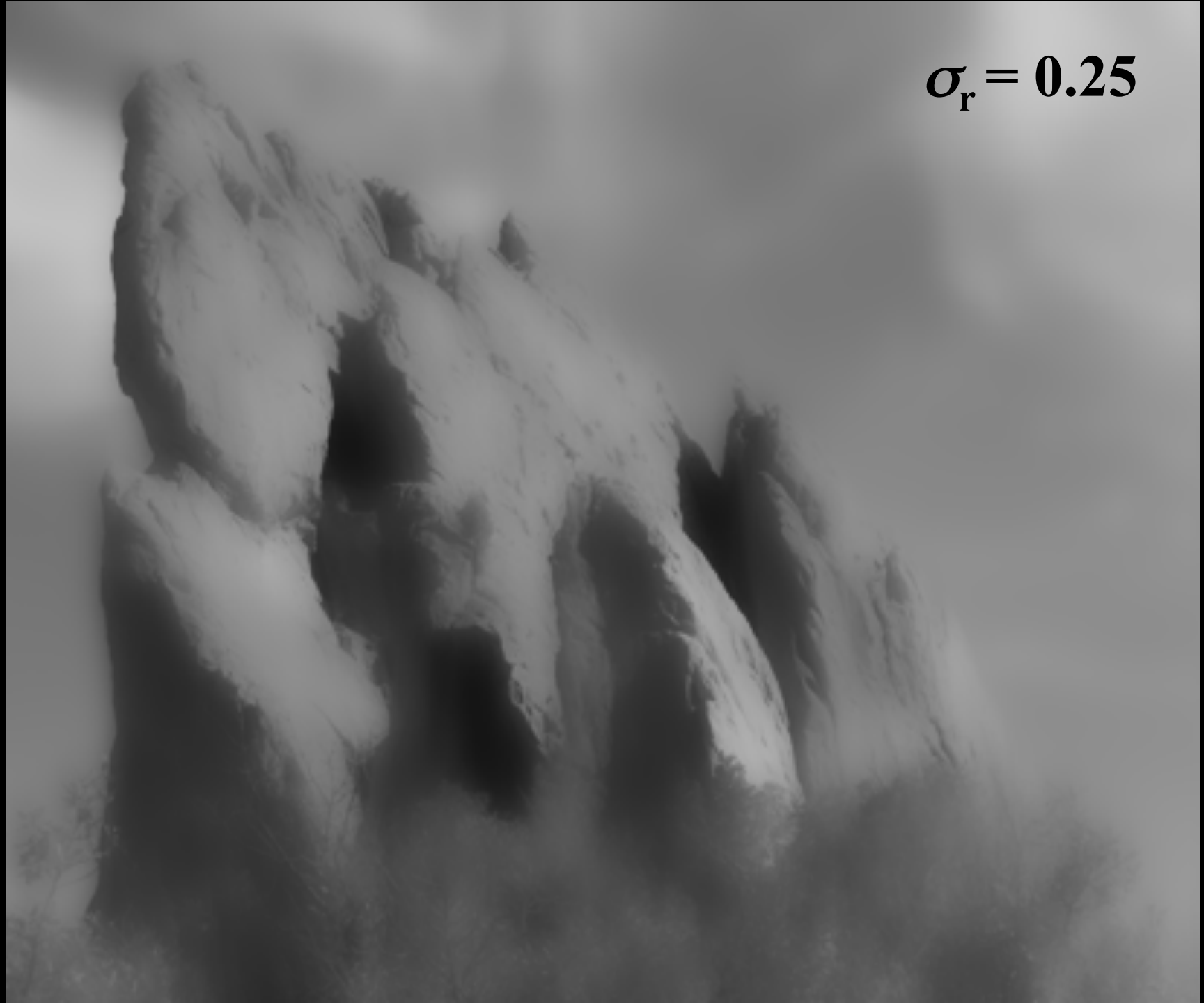
$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_r = 0.1$


$\sigma_r = 0.25$


$\sigma_r = \infty$
(Gaussian blur)

Varying the Space Parameter

input

$\sigma_r = 0.1$

$\sigma_r = 0.25$

$\sigma_r = \infty$ (Gaussian blur)

$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$

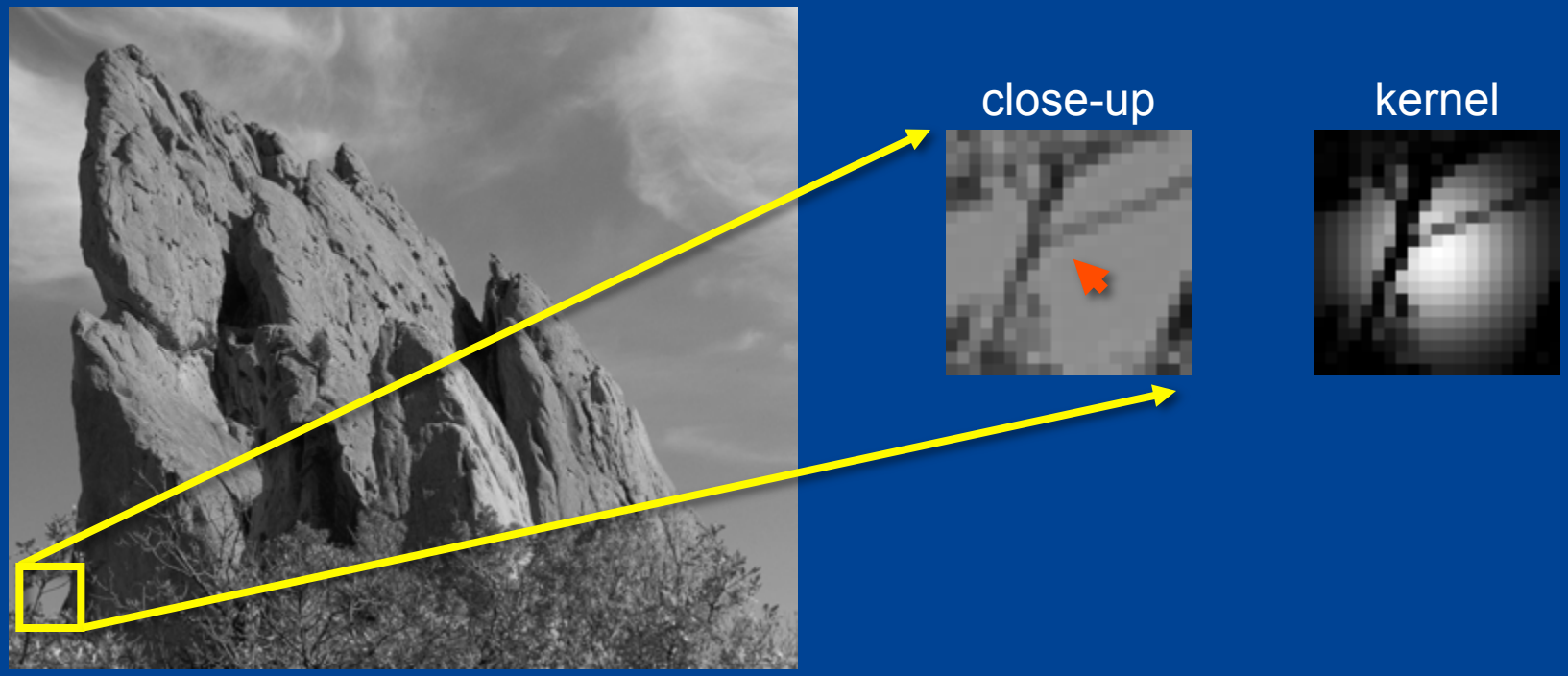
How to Set the Parameters

Depends on the application. For instance:

- space parameter: proportional to image size
  - e.g., 2% of image diagonal
- range parameter: proportional to edge amplitude
  - e.g., mean or median of image gradients
- independent of resolution and exposure

Bilateral Filter Crosses Thin Lines

- Bilateral filter averages across features thinner than ~$2\sigma_s$
- Desirable for smoothing: more pixels = more robust
- Different from diffusion that stops at thin lines



Bilateral Filtering Color Images

For gray-level images

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

intensity difference

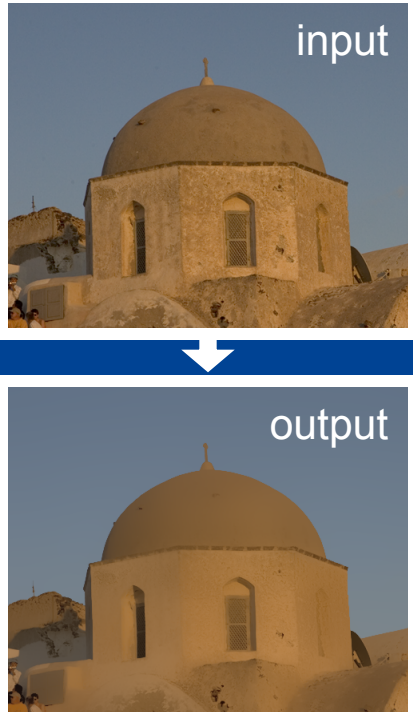
scalar

For color images

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(\|\mathbf{C}_{\mathbf{p}} - \mathbf{C}_{\mathbf{q}}\|) \mathbf{C}_{\mathbf{q}}$$

color difference

3D vector
(RGB, Lab)



Hard to Compute

- Nonlinear $$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$
- Complex, spatially varying kernels
  - Cannot be precomputed, no FFT…

- Brute-force implementation is slow > 10min

Basic denoising

Noisy input

Bilateral filter 7x7 window

Basic denoising

Bilateral filter

Median 3x3

Basic denoising

Bilateral filter

Median 5x5

Basic denoising

Bilateral filter

Bilateral filter – lower sigma

Basic denoising

Bilateral filter

Bilateral filter – higher sigma

Denoising

- Small spatial sigma (e.g. 7x7 window)
- Adapt range sigma to noise level
- Maybe not best denoising method, but best simplicity/quality tradeoff
  - No need for acceleration (small kernel)
  - But the denoising feature in e.g. Photoshop is better