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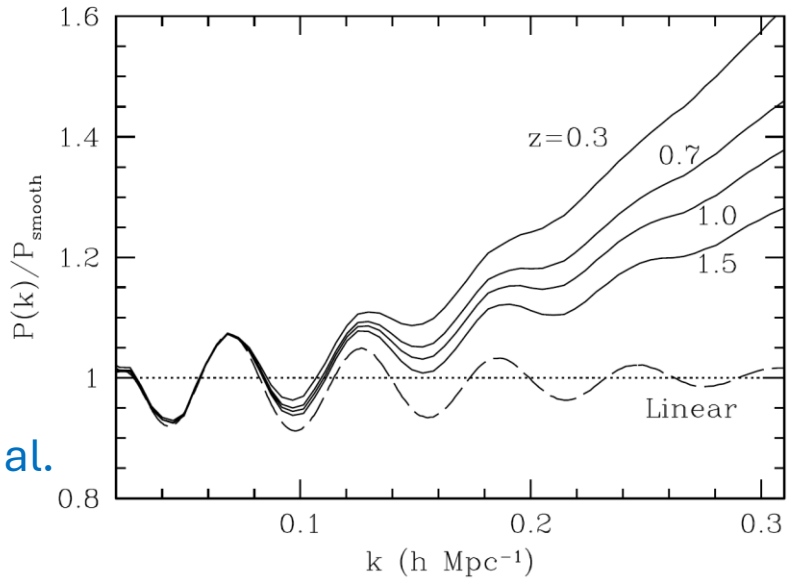
Non-linear structure formation

Non-linearity

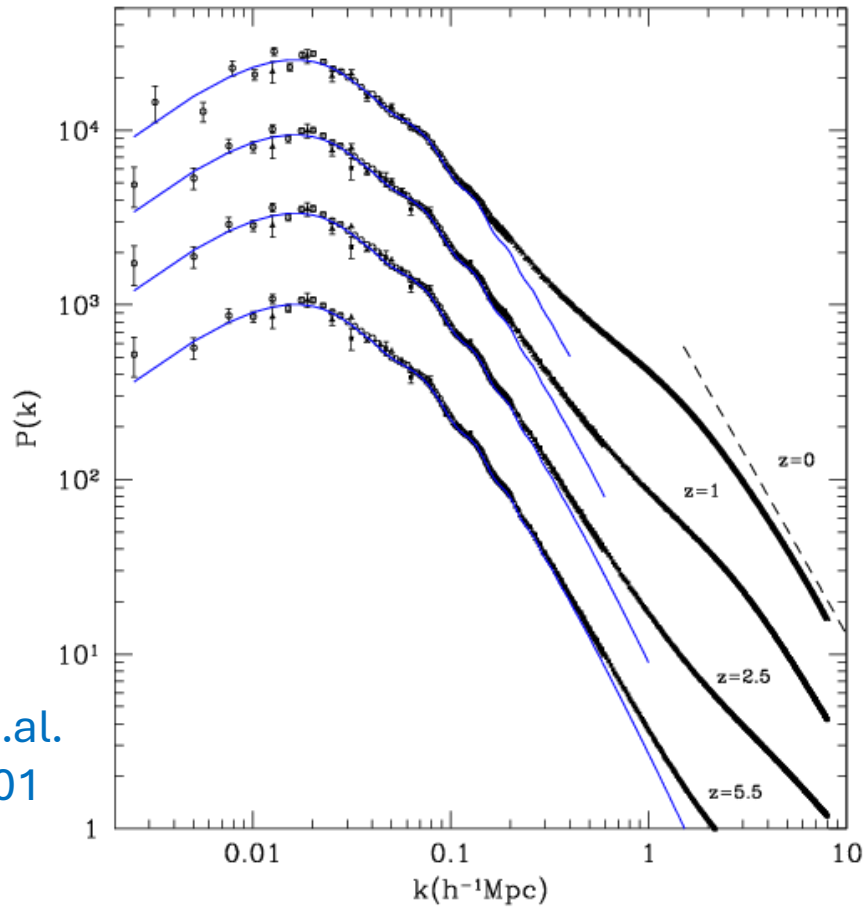
- Density perturbations become non-linear at late times on small scales

Power spectrum

$$\left\langle \Delta_m(\vec{k})\Delta_m(\vec{k}')\right\rangle = (2\pi)^3\,\delta^{(3)}(\vec{k}+\vec{k}')P(k)$$



Weinberg et.al. 1201.2434



Klypin et.al. 1411.4001

Importance of non-linearity

Taruya et.al. 1006.0699

- Redshift distortions

Power spectrum in redshift space

$$P(k,\mu),\quad \mu = \vec{n}\cdot\vec{k}/k$$

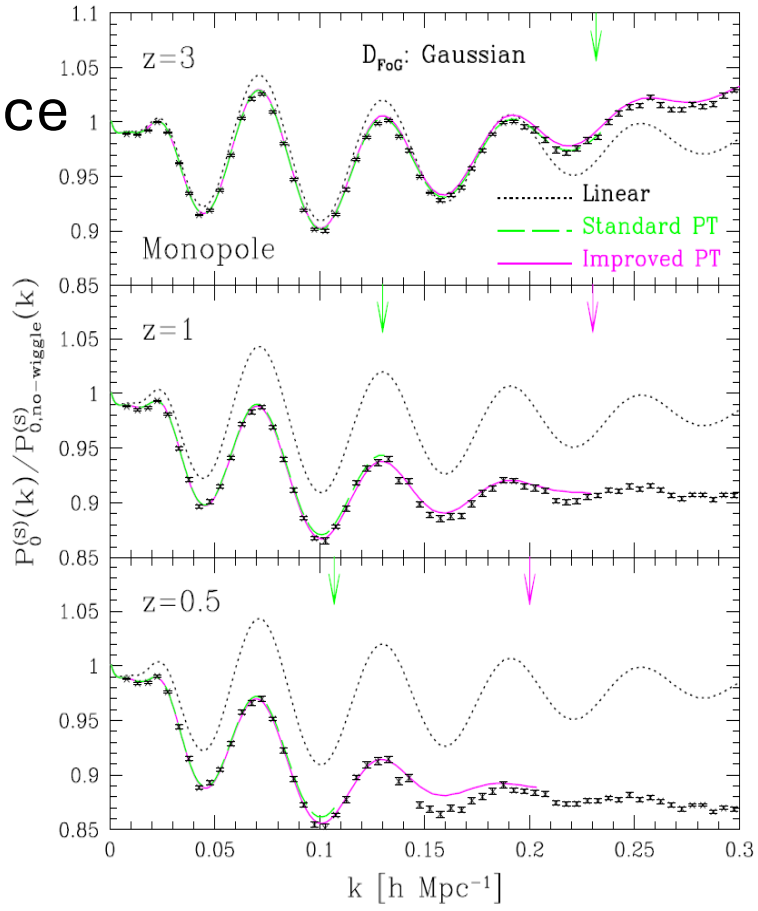
$$P(k,\mu) = \sum_l P_l(k) L_l(\mu)$$

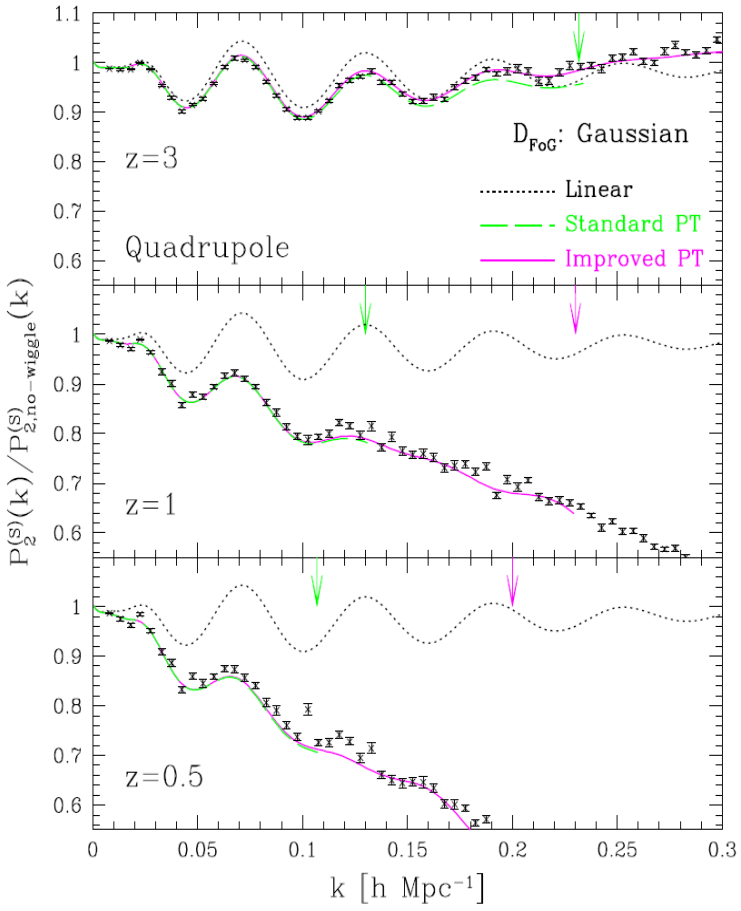
linear theory (Kaiser)

$$\delta^s(k) = \Delta_m(k)\left(1 + f\mu^2\right)$$

$$P_0(k) = \left(1 + \frac{2}{3}f + \frac{1}{5}f^2\right)P(k)$$

$$P_2(k) = \left(\frac{4}{3}f + \frac{4}{7}f^2\right)P(k)$$



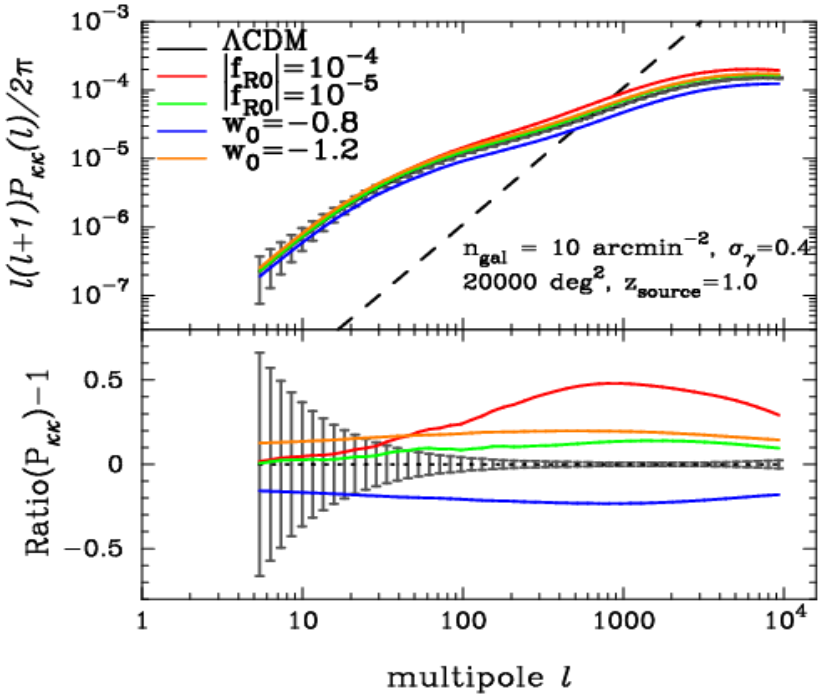


Importance of non-linearity

- Weak lensing

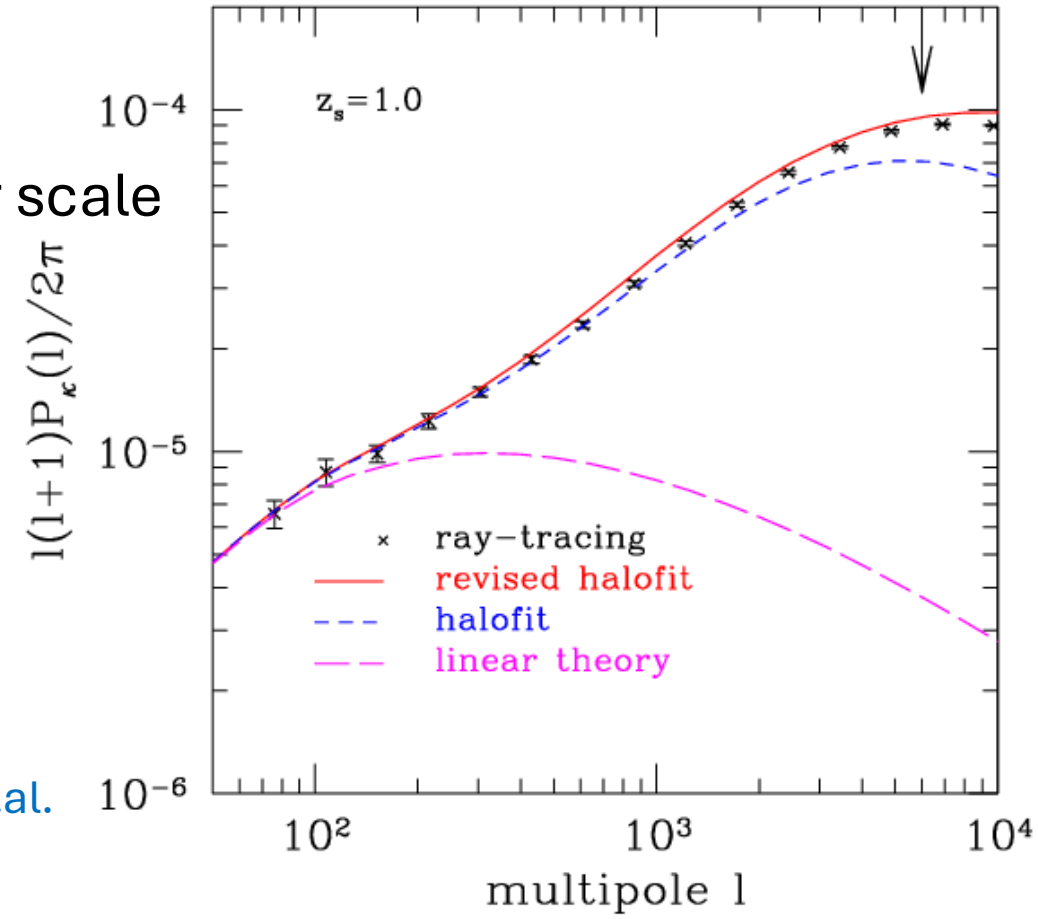
  Convergence power spectrum

  most information comes from non-linear scale



Shirasaki et.al. 1508.02104

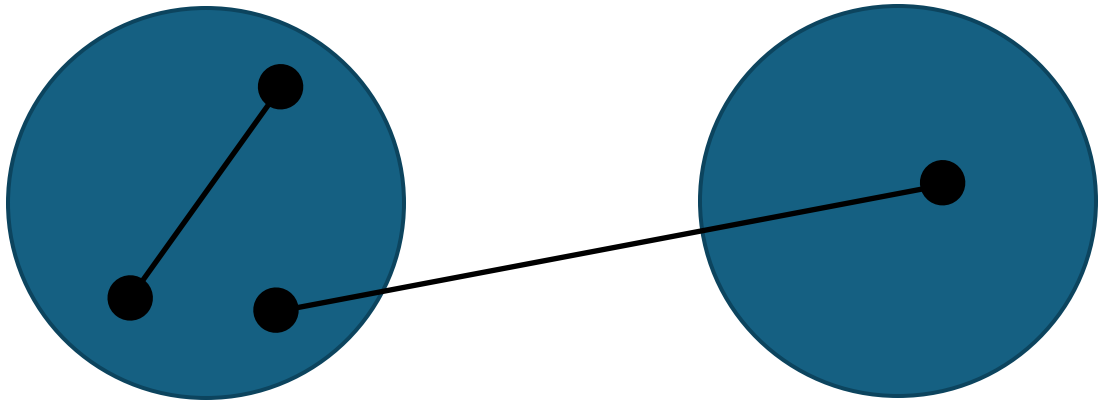
Takahashi et.al. 1208.2701



Halo model

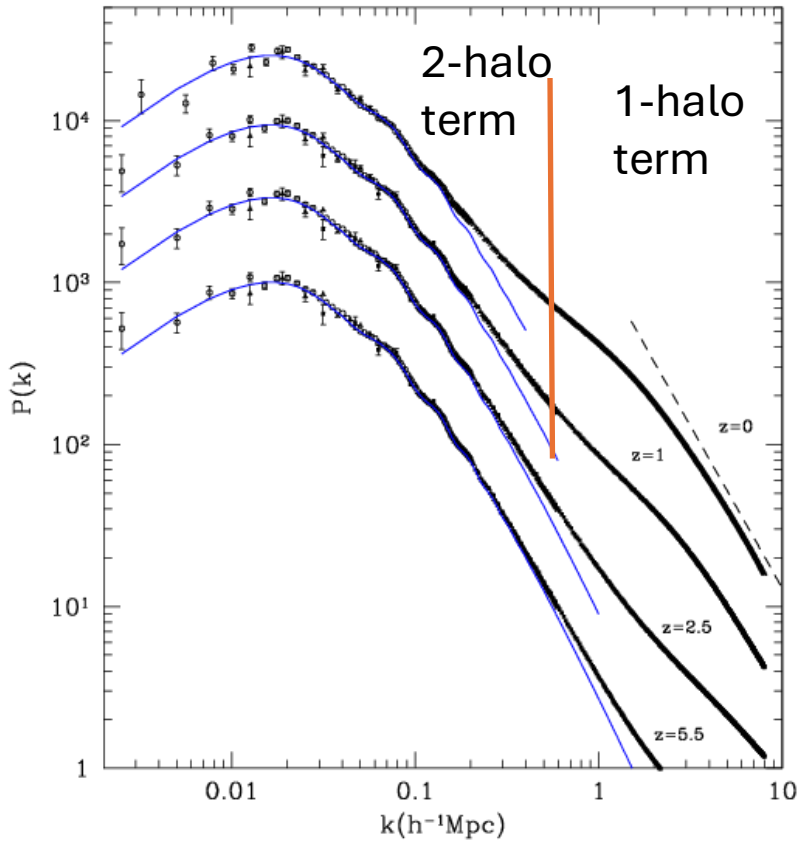
- The non-linear power spectrum from distributions and properties of dark matter halos

Klypin et.al. arXiv:1411.4001



2 halo term
halo correlation function

1 halo term
halo profile



Bhattacharya et.al. 1005.2239

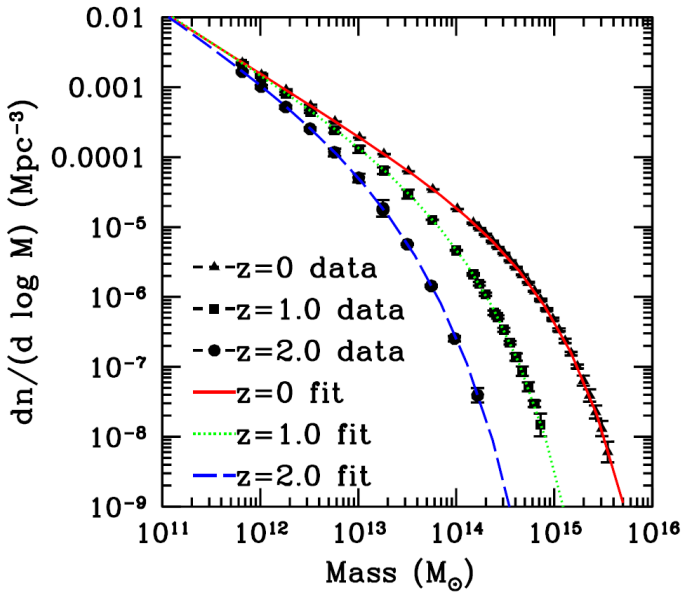
Halo model power spectrum

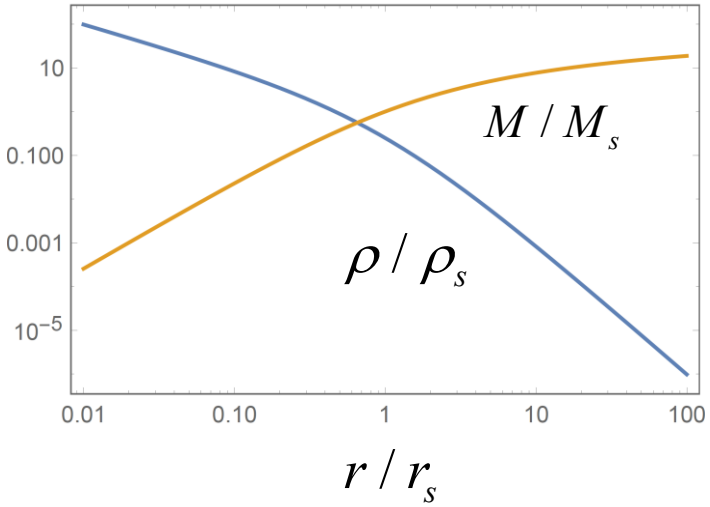
- Ingredients

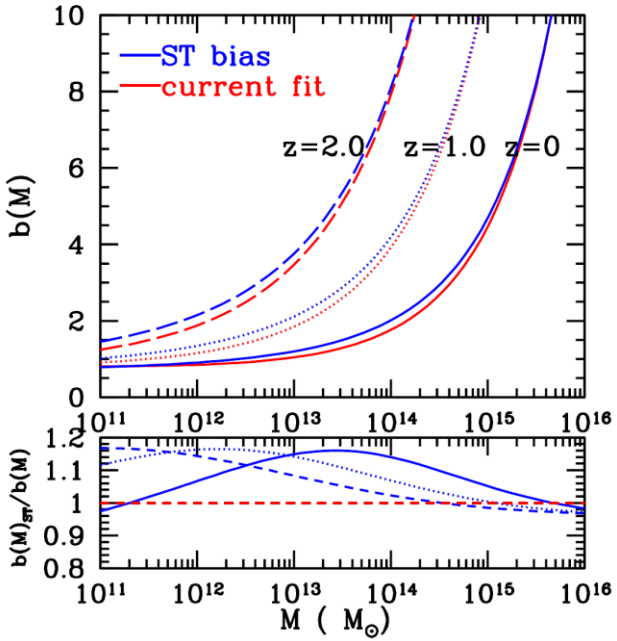
  halo mass function $n(m)$

  halo power spectrum $P_{hh}(k|m_1, m_2) = b(m_1)b(m_2)P_L(k)$

  density profile $u(k|m) = \int_0^{r_V} dr 4\pi r^2 \frac{\sin kr}{kr} \frac{\rho(r|m)}{m}$







Variance

©Wayne Hu

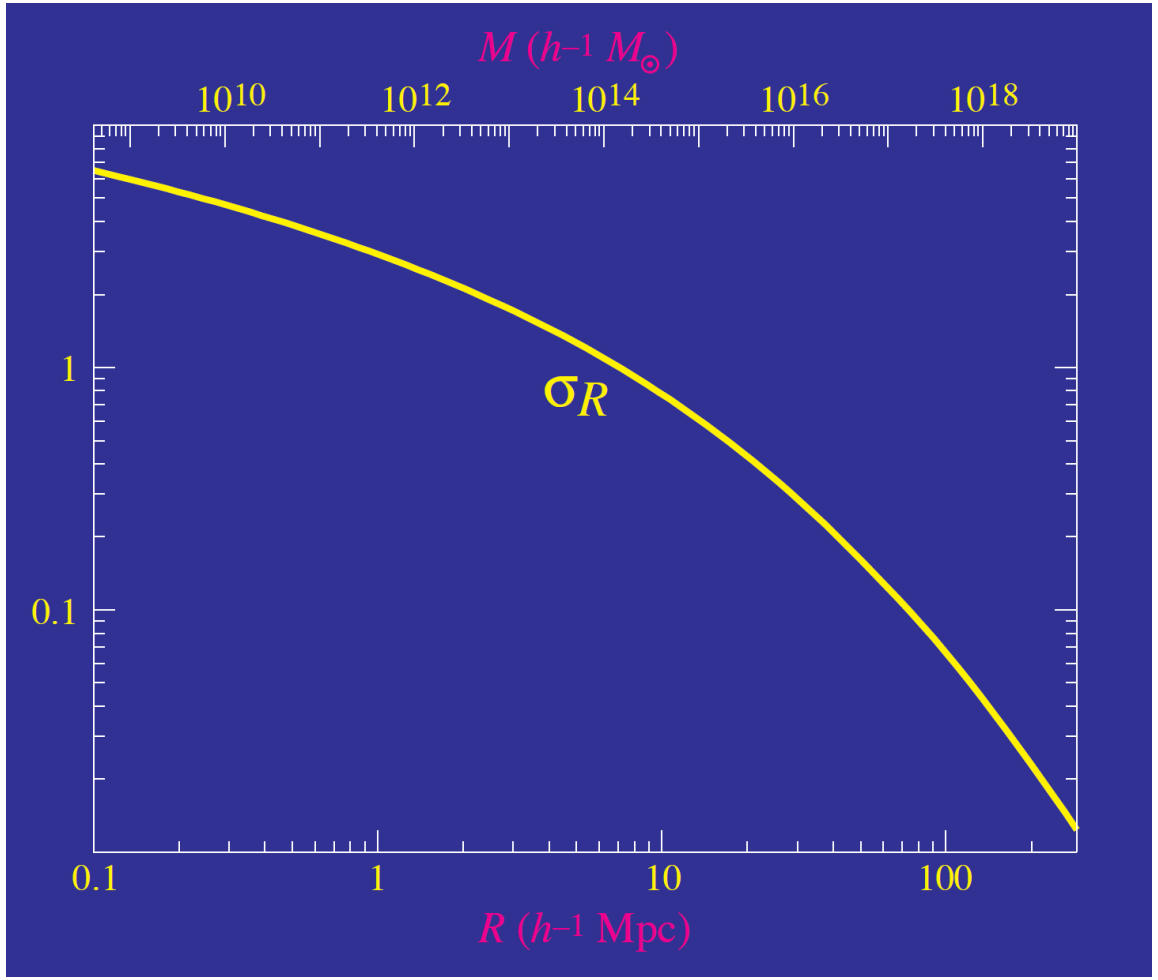
- Variance

All information is encoded in the variance for the Gaussian perturbations

$$\delta_R = \int d^3x' W_R(|\boldsymbol{x} - \boldsymbol{x}'|)\delta(\boldsymbol{x}')$$

$$W_R = 1, \quad r \leq R \qquad R = \left(\frac{3M}{4\pi}\right)^{1/3}$$
$$W_R = 0, \quad r > R$$

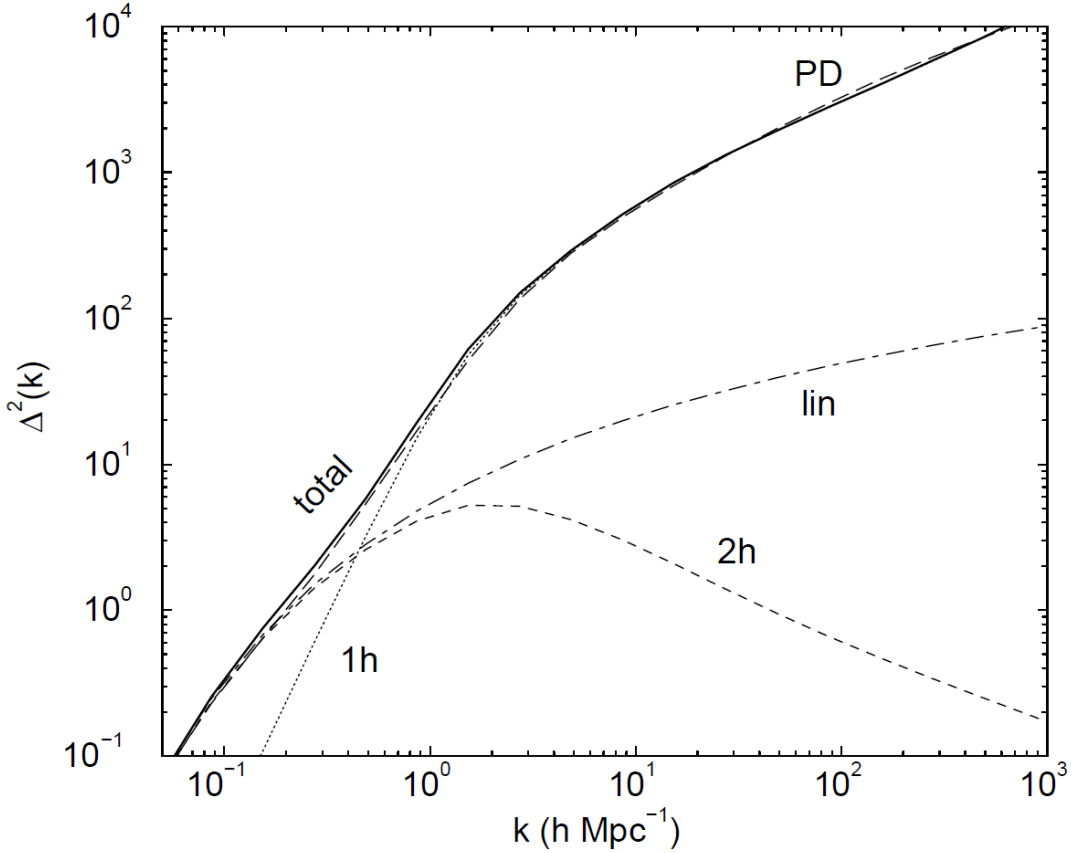
$$\sigma_R^2(M) = \langle \delta_R^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P(k) |\hat{W}_R(k)|^2$$

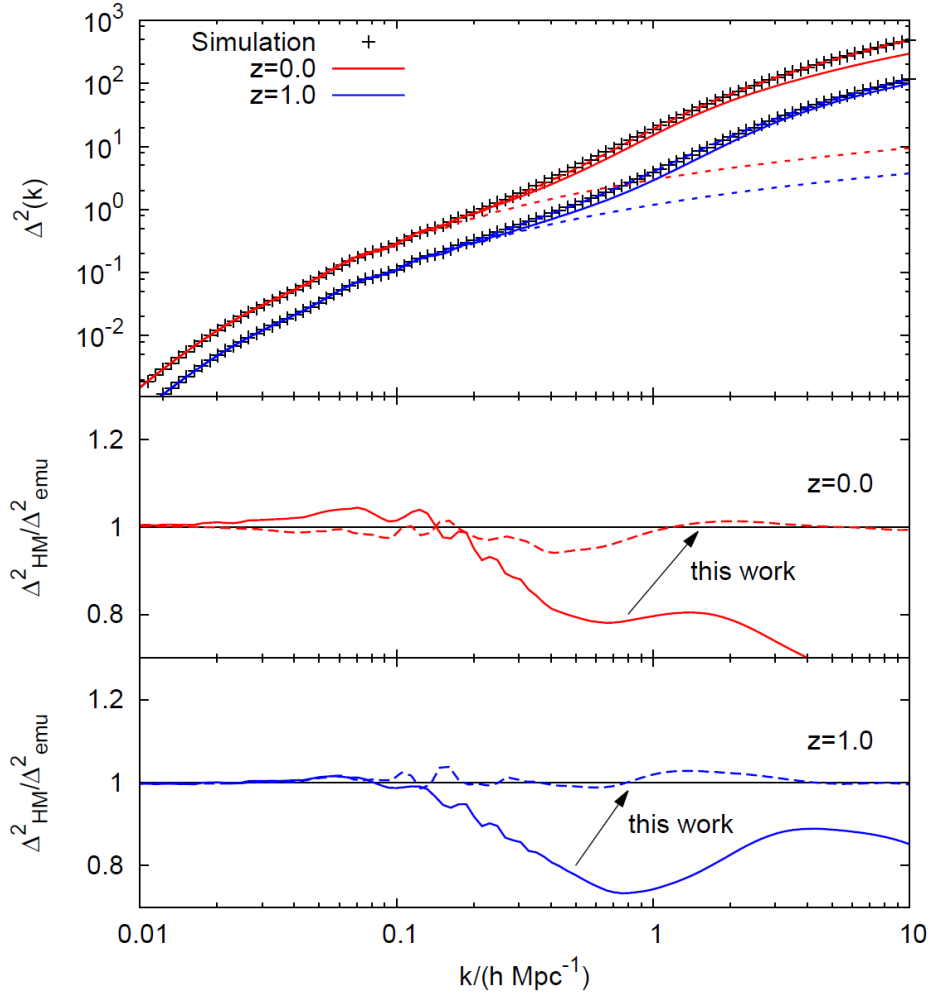


Halo model power spectrum

$$\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$$

- Halo model can explain the shape of non-linear power spectrum but not very accurate





HMcode (2020)

Mead et.al. 2009.01858

$$P_{\rm mm}^{\rm 2H}(k,z) = P_{\rm dwl}(k,z)\left[1 - f\frac{(k/k_{\rm d})^{n_{\rm d}}}{1+(k/k_{\rm d})^{n_{\rm d}}}\right] \qquad P_{\rm mm}^{\rm 1H}(k,z) \to P_{\rm mm}^{\rm 1H}(k,z) \times \frac{(k/k_*)^4}{1+(k/k_*)^4}$$

$$\rho_{\rm c}(M,r) \propto \frac{1}{r/(\nu^{\eta} r_{\rm s})[1+r/(\nu^{\eta} r_{\rm s})]^2} \qquad c(M,z) = B\left[\frac{1+z_{\rm f}(M,z)}{1+z}\right]$$

$$\Delta^2_{\rm HM}(k,z) = \left[[\Delta^2_{\rm 2H}(k,z)]^{\alpha} + [\Delta^2_{\rm 1H}(k,z)]^{\alpha}\right]^{1/\alpha}$$

12 fitted parameters

Parameter	Explanation	Equation	Default value	Fitted functional form or value
$k_{\rm d}$	Two-halo term damping wavenumber	16	-	$0.05699 \times \sigma_{8,\rm cc}^{-1.089}(z)\, h{\rm Mpc}^{-1}$
f	Two-halo term fractional damping	16	0	$0.2696 \times \sigma_{8,\rm cc}^{0.9403}(z)$
$n_{\rm d}$	Two-halo term damping power	16	-	2.853
k_*	One-halo term damping wavenumber	17	0	$0.05618 \times \sigma_{8,\rm cc}^{-1.013}(z)\, h{\rm Mpc}^{-1}$
η	Halo bloating	19	0	$0.1281 \times \sigma_{8,\rm cc}^{-0.3644}(z)$
B	Minimum halo concentration	20	4	5.196
α	Transition smoothing	23	1	$1.875 \times (1.603)^{n_{\rm cc}^{\rm eff}}$

N-body simulation

- Distributions of collisonless Dark Matter (DM) particles are described by the phase space distribution function $f(\boldsymbol{x}, \boldsymbol{p}, \tau)$
- The conservation of the distribution function

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{\partial \boldsymbol{x}}{\partial \tau} \nabla f + \frac{\partial \boldsymbol{p}}{\partial \tau} \frac{\partial f}{\partial \boldsymbol{p}} = 0$$

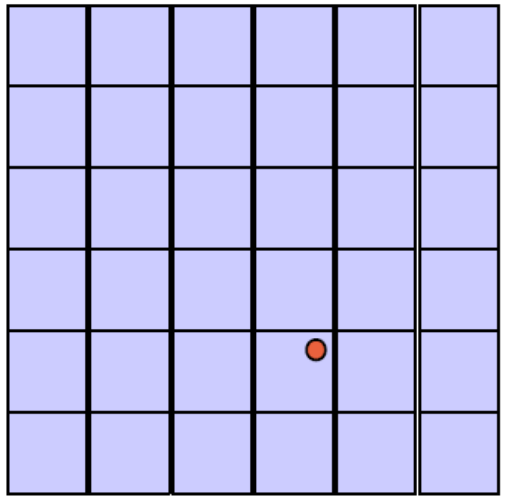
- Equations of motion of dark matter particles

$$\frac{d\boldsymbol{x}}{d\tau} = \frac{\boldsymbol{p}}{ma(\tau)}$$

$$\frac{d\boldsymbol{p}}{d\tau} = -a(\tau) m \nabla \Psi$$

where a is the scale factor, m is the mass of dark matter particles, Ψ is the Newtonian potential and τ is the conformal time

Particle mesh (PM)



- Particle Mesh method

  solution for the potential

$$\Psi(\boldsymbol{x}) = \int d\boldsymbol{x}' g(|\boldsymbol{x} - \boldsymbol{x}'|)\rho(\boldsymbol{x}'), \quad g(x) = -\frac{G}{|x|}$$

  Fourie transformation

$$\hat{\Psi}(k) = g(k)\hat{\rho}(k)$$

  Compute force by going back to real space $\boldsymbol{F} = -\nabla\Psi$

  the force resolution is limited by the mesh size

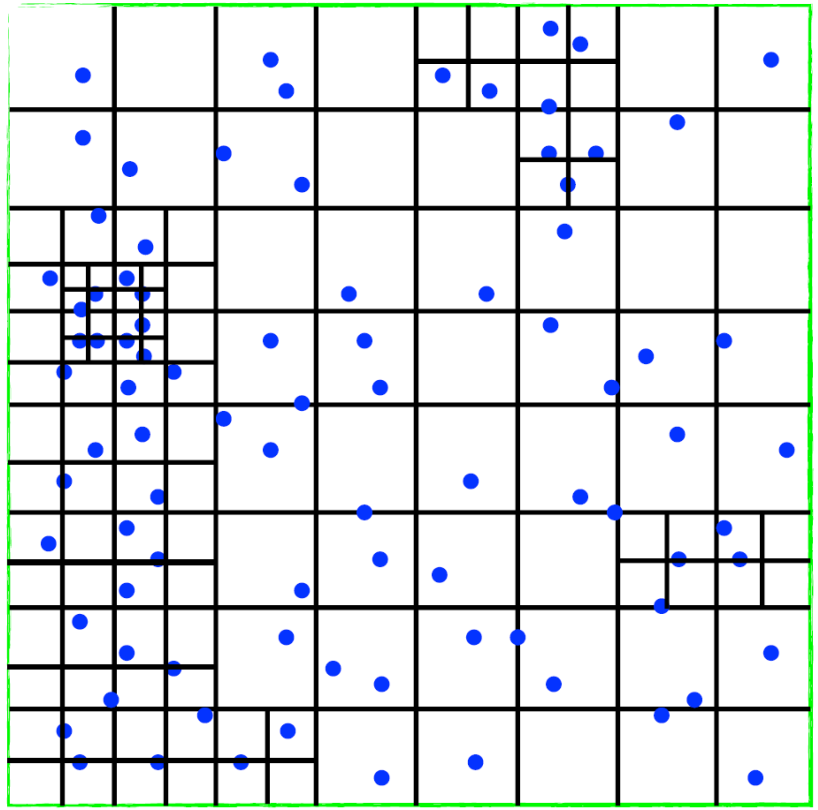
Adaptive mesh (AMR)

- In higher density regions, finer meshes are create
- Solve the Poisson equation by discretisation

$$\frac{\partial \Psi}{\partial x} = \frac{1}{h}(\Psi_{i+1,j,k} - \Psi_{i,j,k})$$

$$\nabla^2 \Psi = \frac{3}{2}\Omega_m(\rho - \bar{\rho})$$

$$\frac{1}{h^2}\left[\Psi_{i+1,j,k} + \Psi_{i-1,j,k} + \ldots - 6\Psi_{i,j,k}\right] = \frac{3}{2} a\ \Omega_m(\rho_{i,j,k} - 1)$$



Find a solution using relaxation method

example: RAMSES by Teyssier http://www.itp.uzh.ch/~teyssier/ramses/RAMSES.html

Comparisons

- Comparisons between three codes

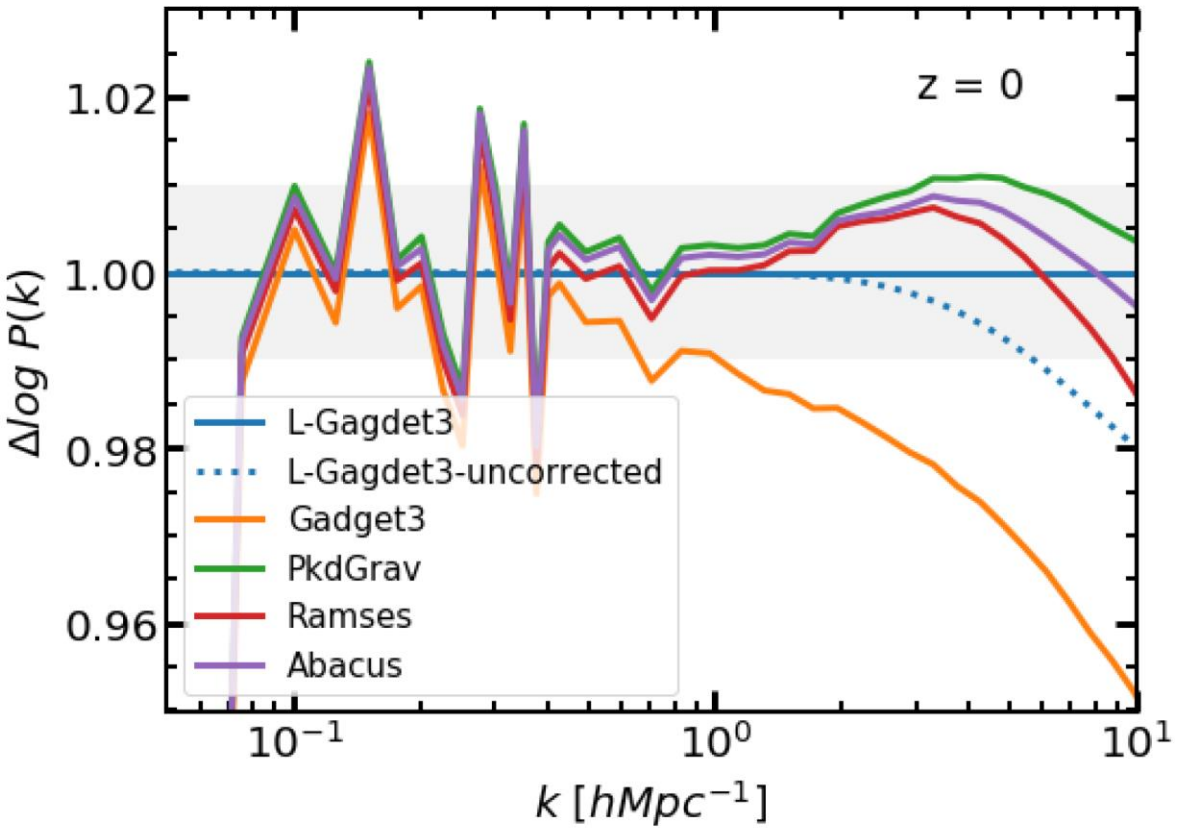
  RAMSES (AMR)
  PKD-Grav3 (tree)
  Gadget (tree-PM)
  Abacus

1% agreement up to k =5 h/Mpc

A careful numerical settings is required to achieve this level of agreements

Schneider et.al.
1503.05920

2004.06245



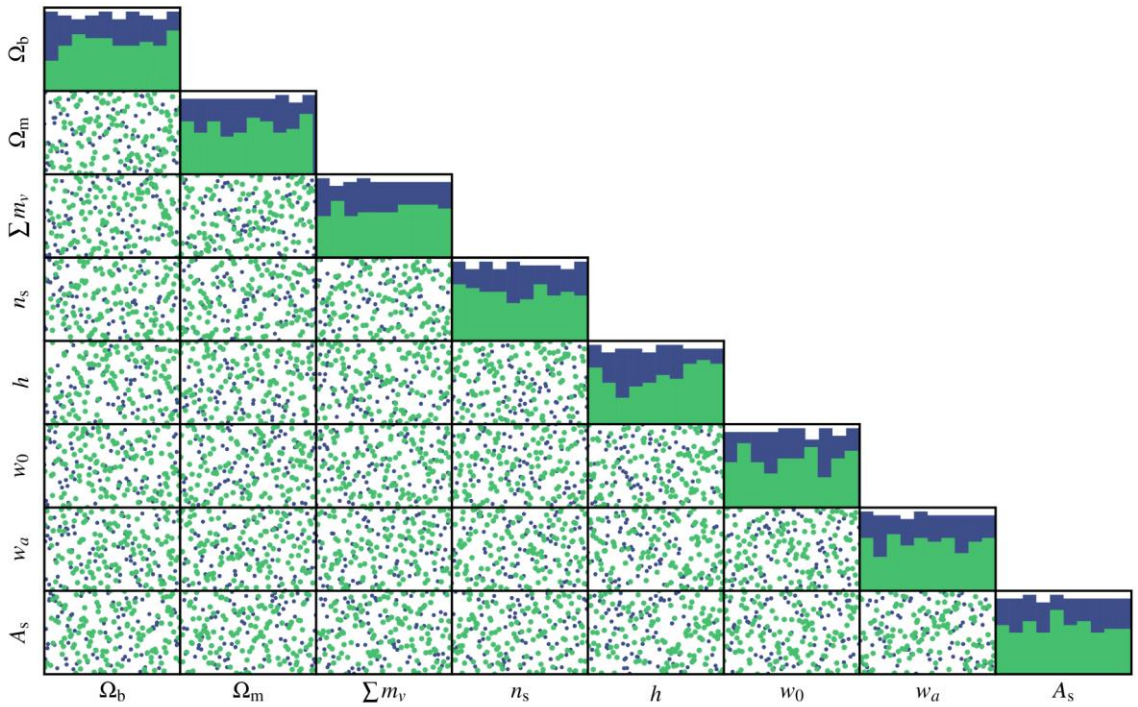
Emulators

Interpolation of P(k)

- Euclid Emulator 2 Euclid collaboration 2010.11288

108 simulations by PKDGRAV
PCA + Polynomial Chaos Expansion

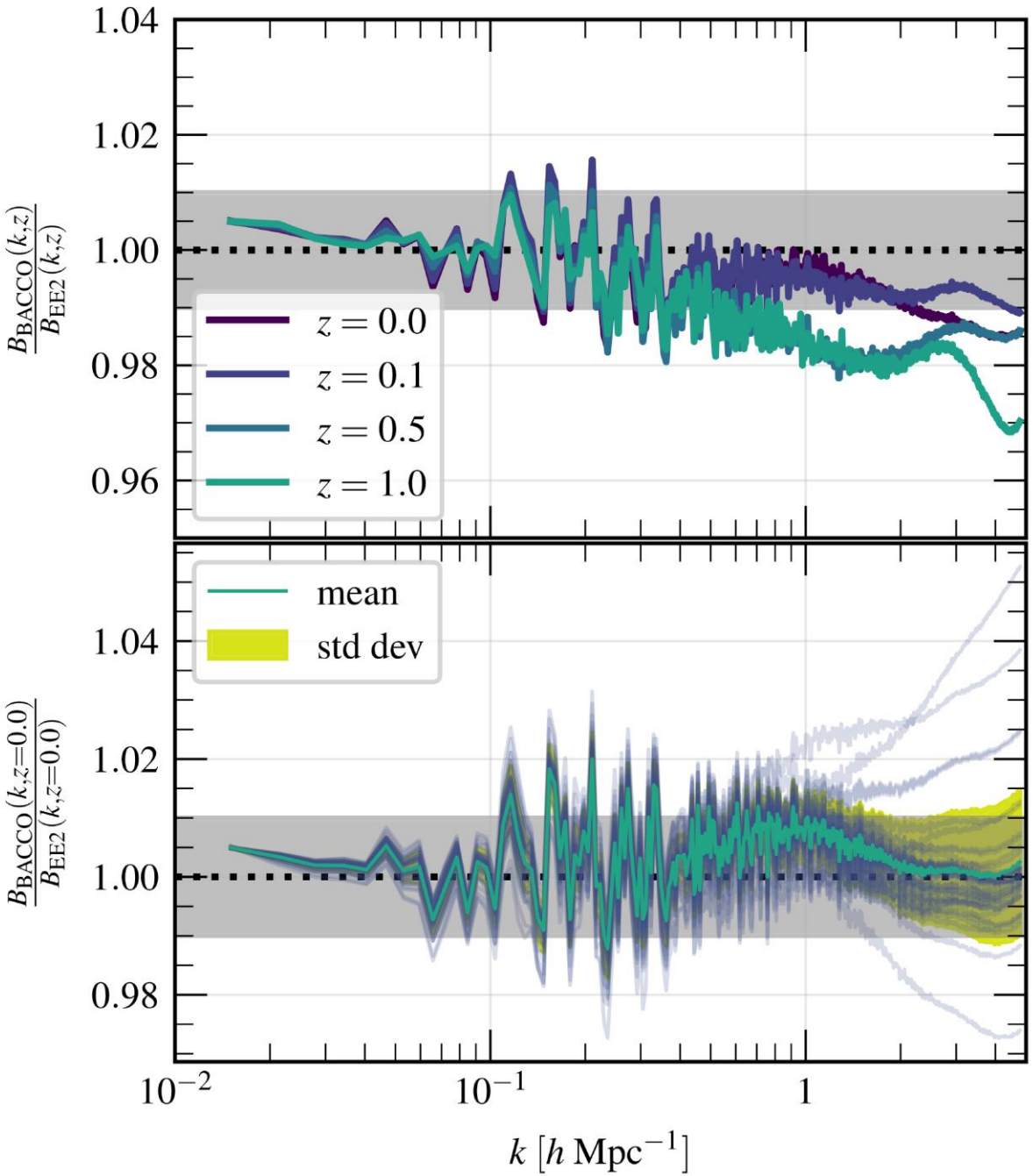
Parameter	Min.	Max.	Center
h	0.61	0.73	0.67
$\Omega_{\rm b}$	0.04	0.06	0.05
$\Omega_{\rm m}$	0.24	0.40	0.32
$n_{\rm s}$	0.92	1.0	0.96
$A_{\rm s}$	1.7×10^{-9}	2.5×10^{-9}	2.1×10^{-9}
$\sum_\nu m_\nu$	0.0 eV	0.15 eV	0.075 eV
w_0	-1.3	-0.7	-1.0
w_a	-0.7	0.7	0.0



Emulators

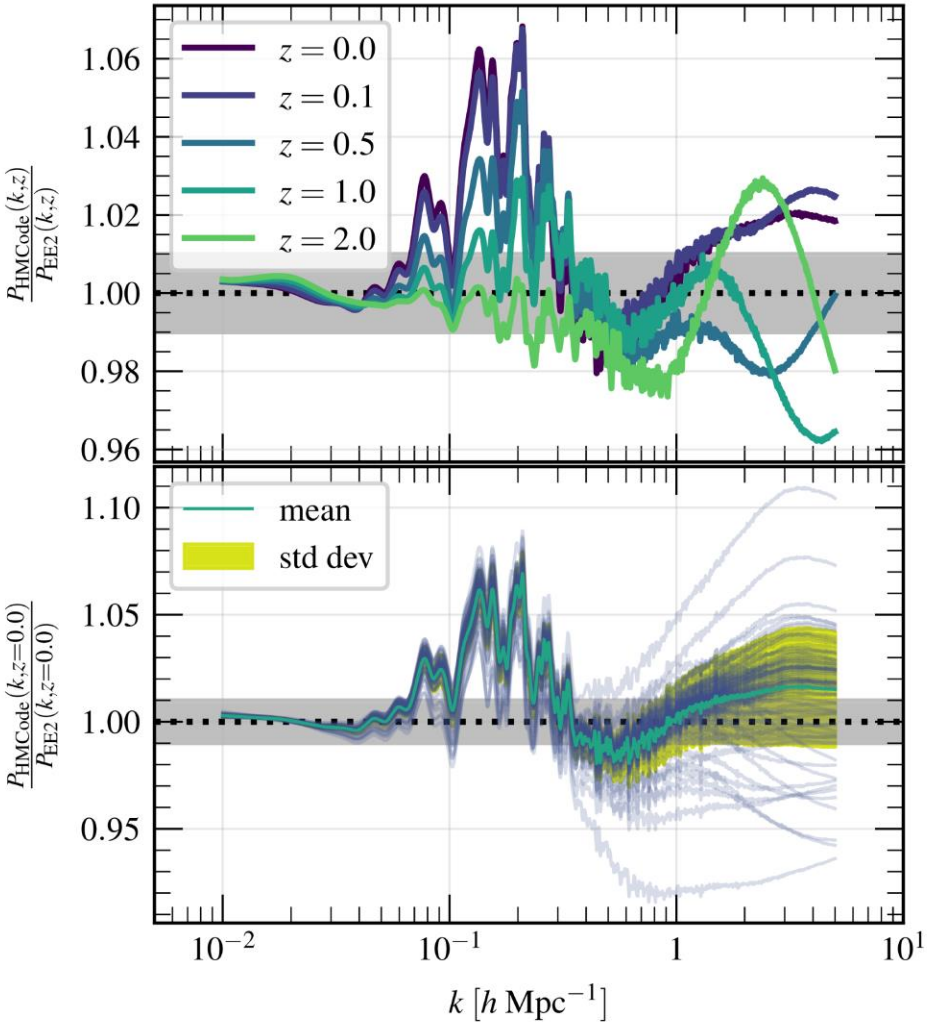
- Bacco Angulo et.al. 2004.06245
  6 simulations by L-Gadget
  + cosmology rescaling algorithm
  PCA + Gaussian process

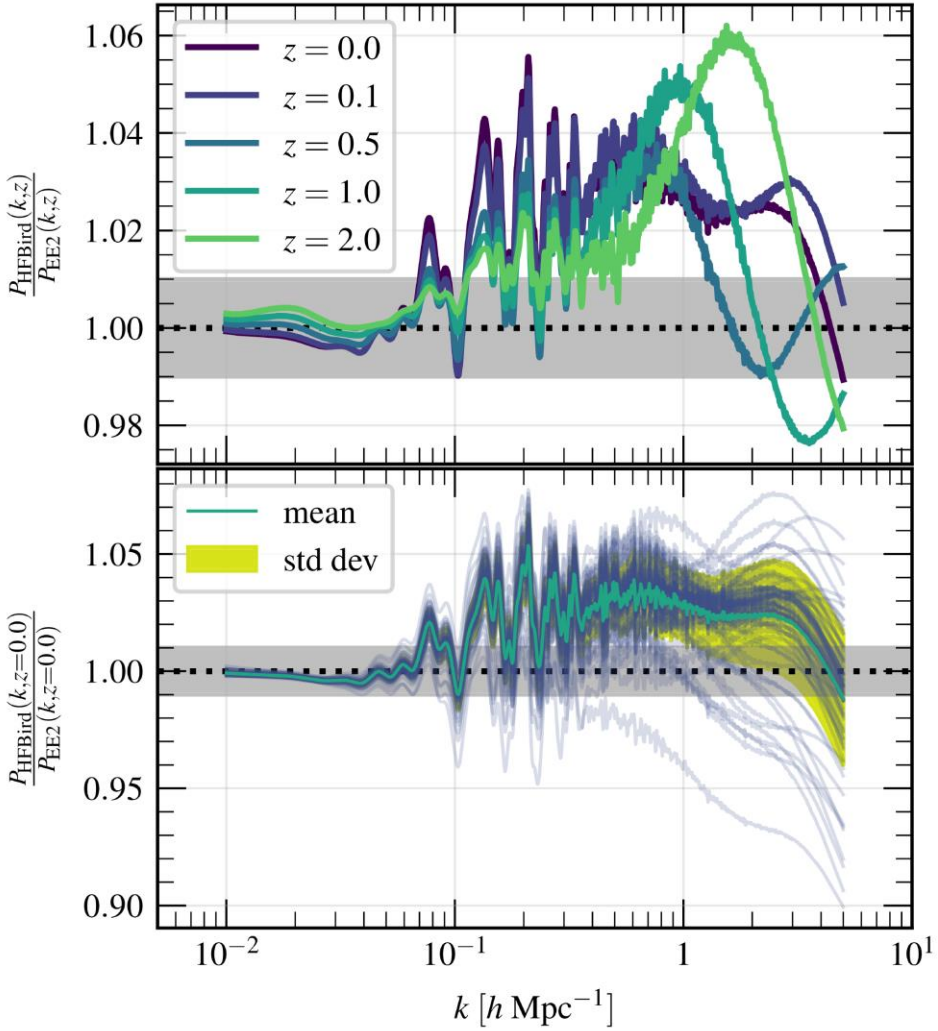
Parameter	Min.	Max.
h	0.6	0.8
$\Omega_{\rm b}$	0.04	0.06
$\Omega_{\rm m}$	0.23	0.40
$n_{\rm s}$	0.92	1.01
σ_8	0.73	0.9
$\sum_\nu m_\nu$	0.0 eV	0.4 eV
w_0	−1.15	−0.85
w_a	−0.3	0.3



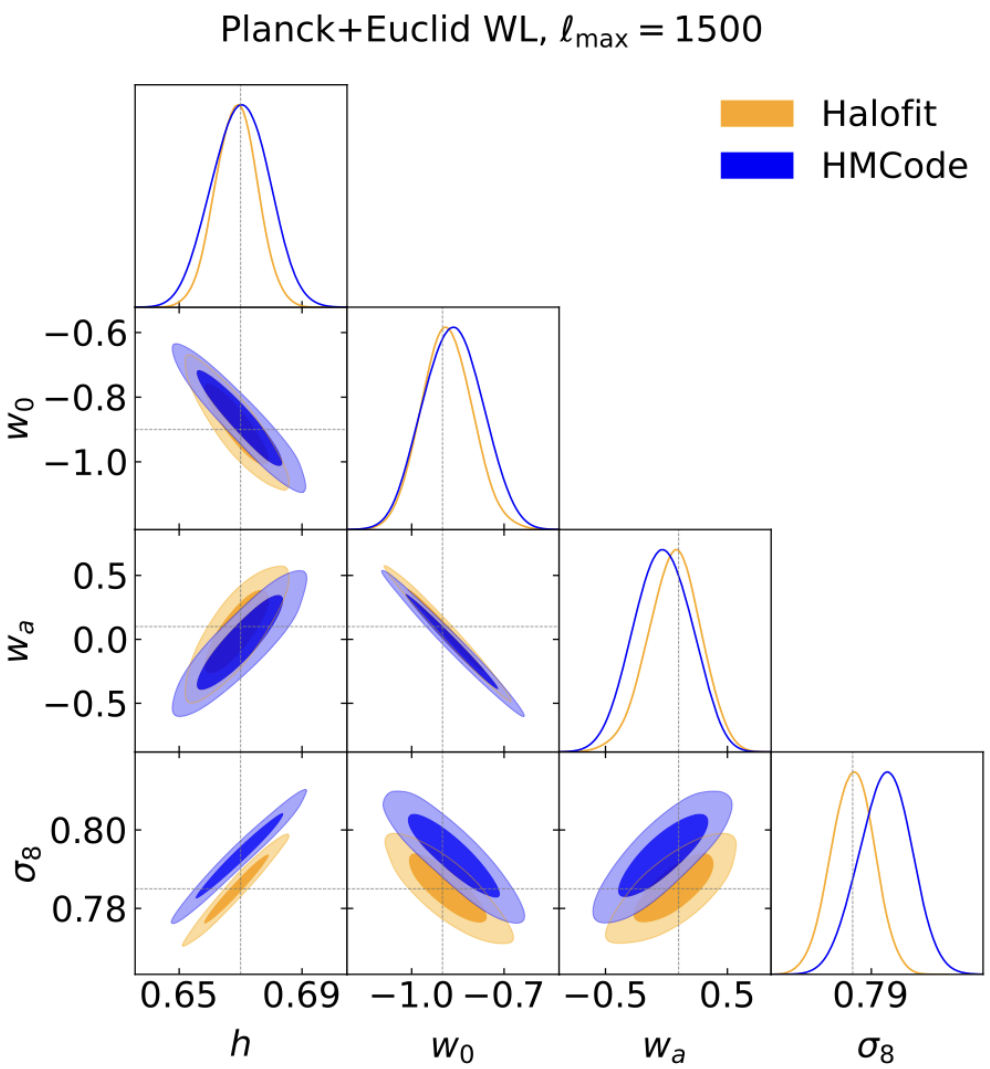
Comparison with Hmcode and halofit

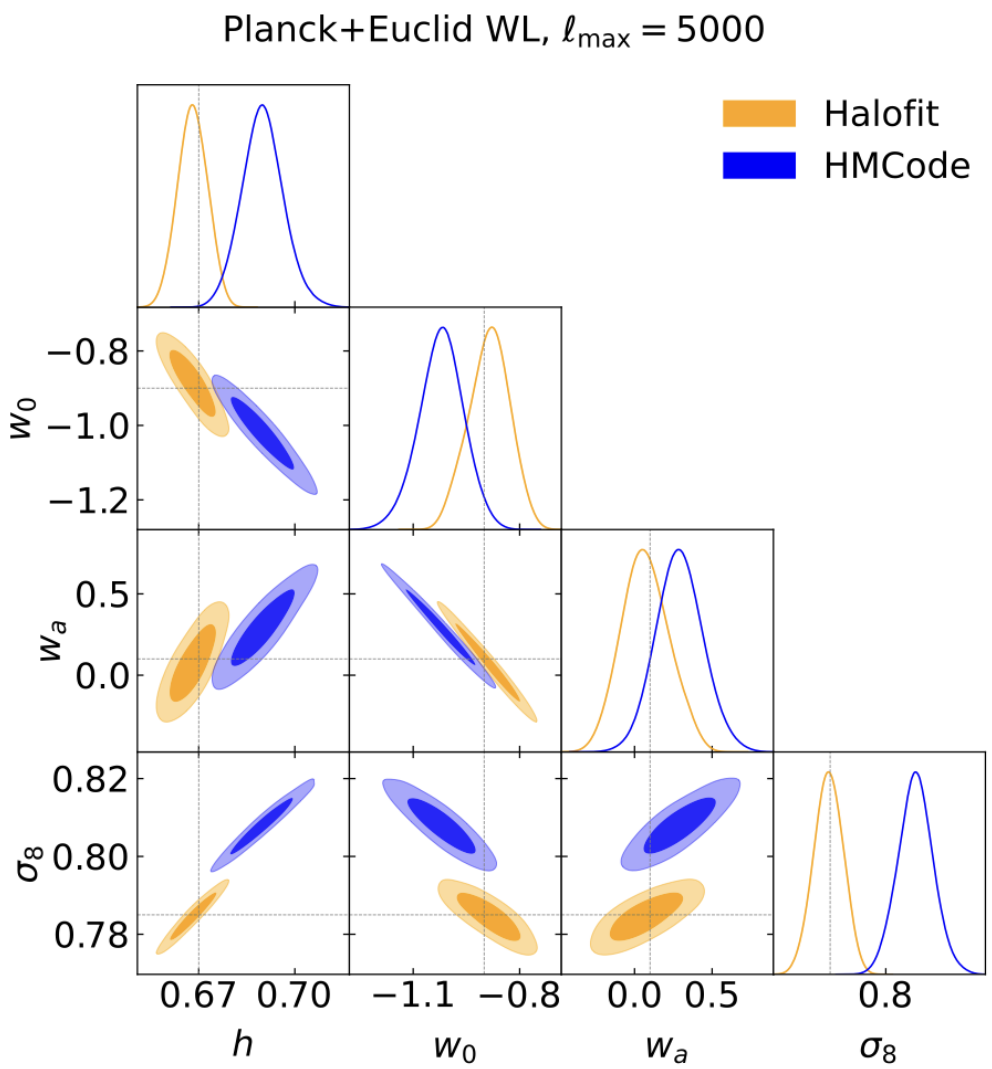
Euclid collaboration 2010.11288





Parameter bias

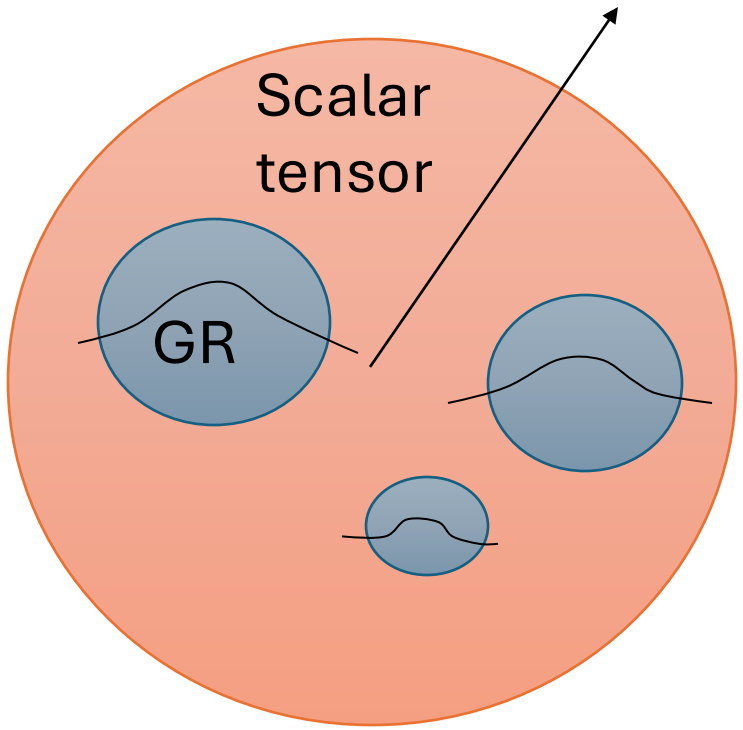




Nonlinear structure in modified gravity

Behaviour of gravity in modified gravity

There regimes of gravity



In most models, the scalar mode obeys non-linear equations describing the transition from the scalar tensor theory on large scales to GR on small scales

$$\rho_{crit} \approx 10^{-29} g / cm^3,$$

$$\rho_{galaxy} \approx 10^{-24} g / cm^3,$$

$$\rho_{solar} \approx 10 g / cm^3$$

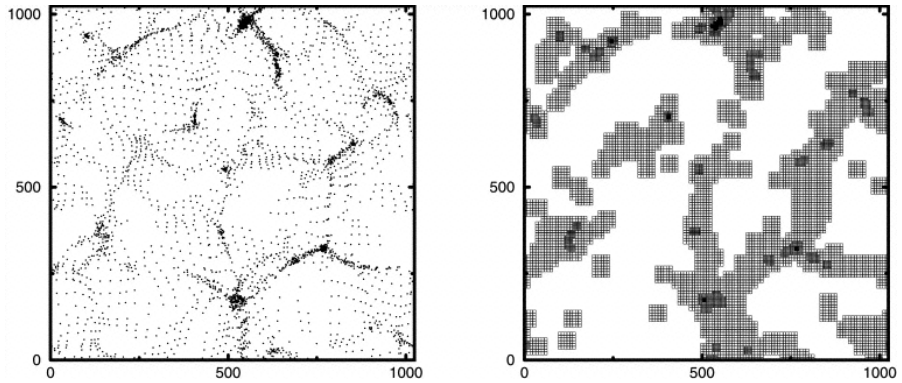
Understandings of non-linear clustering require N-body simulations where the non-linear scalar equation needs to be solved

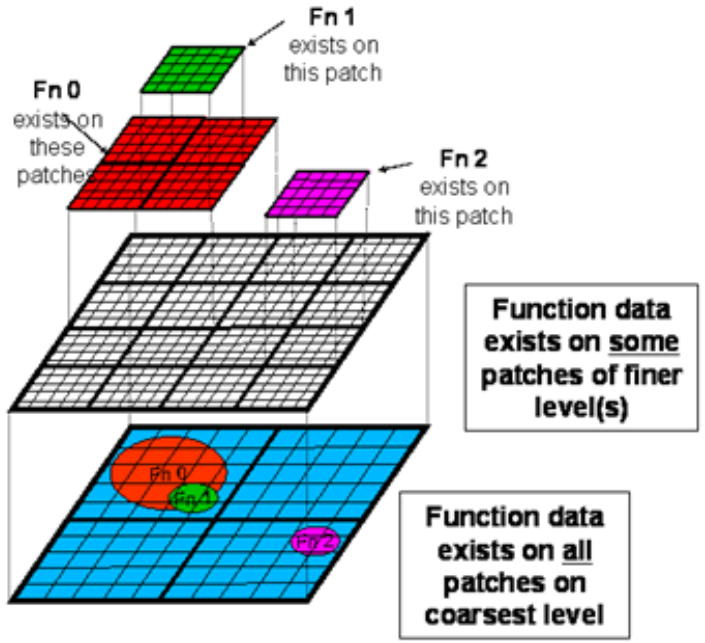
N-body Simulations for MG

- Multi-level adaptive mesh refinement
- solve Poisson equation using a linear Gauss-Seidel relaxation
- add a scalar field solver using a non-linear Gauss Seidel relaxation

ECOSMOG Li, Zhao, Teyssier, KK JCAP1201 (2012) 051
MG-GADGET Puchwein, Baldi, Springel MNRAS (2013) 436 348
ISIS Llinares, Mota, Winther A&A (2014) 562 A78
DGPM, Schmidt PRD80, 043001
Oyaizu et.al. PRD78 12354, PRD79 083518

Modified Gravity Simulations comparison project
Winther, Shcmidt, Barreira et.al. arXiv: 1506.06384





Example – f(R) gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{R + f(R)}{16\pi G} + L_m\right]$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\delta f_R \qquad f_R = \frac{df(R)}{dR}$$

$$\nabla^2\delta f_R = -\frac{1}{3}\delta R(f_R) - \frac{8\pi G}{3}\rho$$

Two limits

- GR $\delta f_R = 0$ $\delta R(f_R) = -8\pi G\rho$ ➡ $\nabla^2\Psi = 4\pi G\rho$
- Scalar-Tensor(ST) $\delta R = 0$ ➡ $\nabla^2\Psi = \frac{4}{3}\times 4\pi G\rho$

  the fifth force has a similar strength as gravity

Linear regime

- Linearise the equation

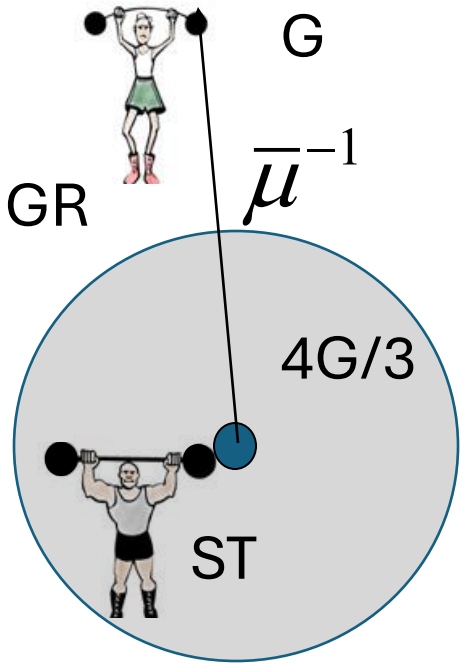
$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta \rho_{\mathrm{M}}$$

The fifth force does not propagate beyond the Compton wavelength $\bar{\mu}^{-1}$ (GR limit)

$$\nabla^2 \Psi = 4\pi G \rho$$

Below the Compton wavelength, gravity is enhanced (ST limit)

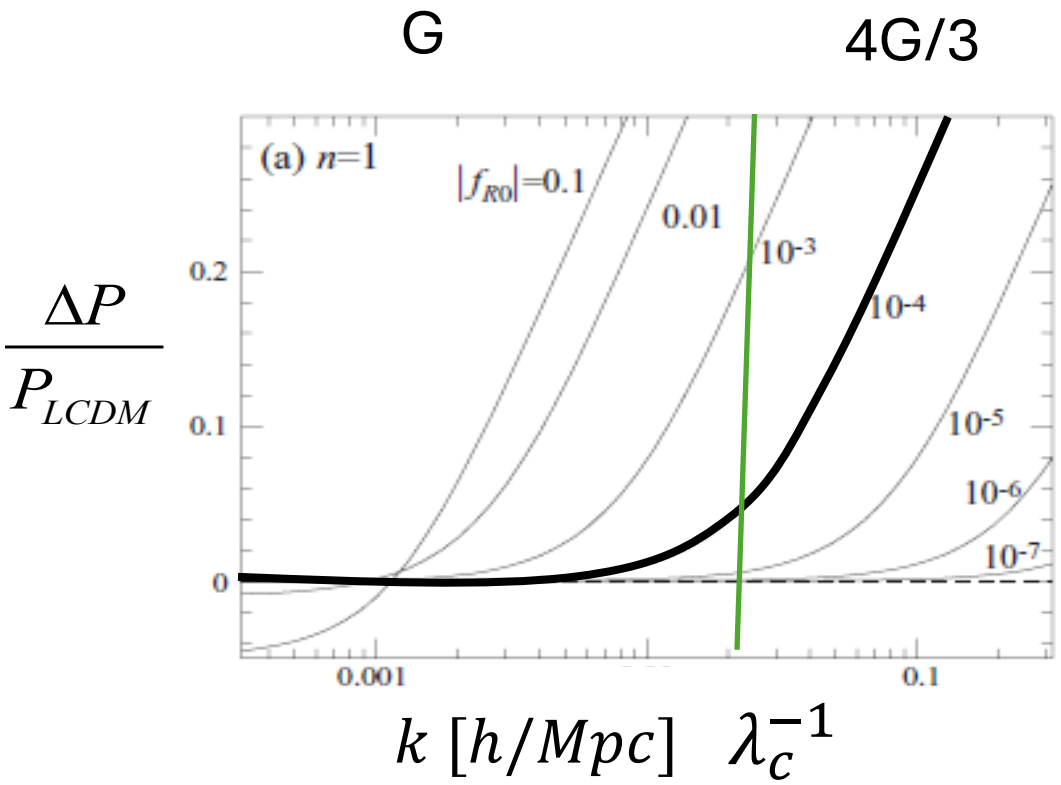
$$\nabla^2 \Psi = \frac{4}{3} \times 4\pi G \rho$$

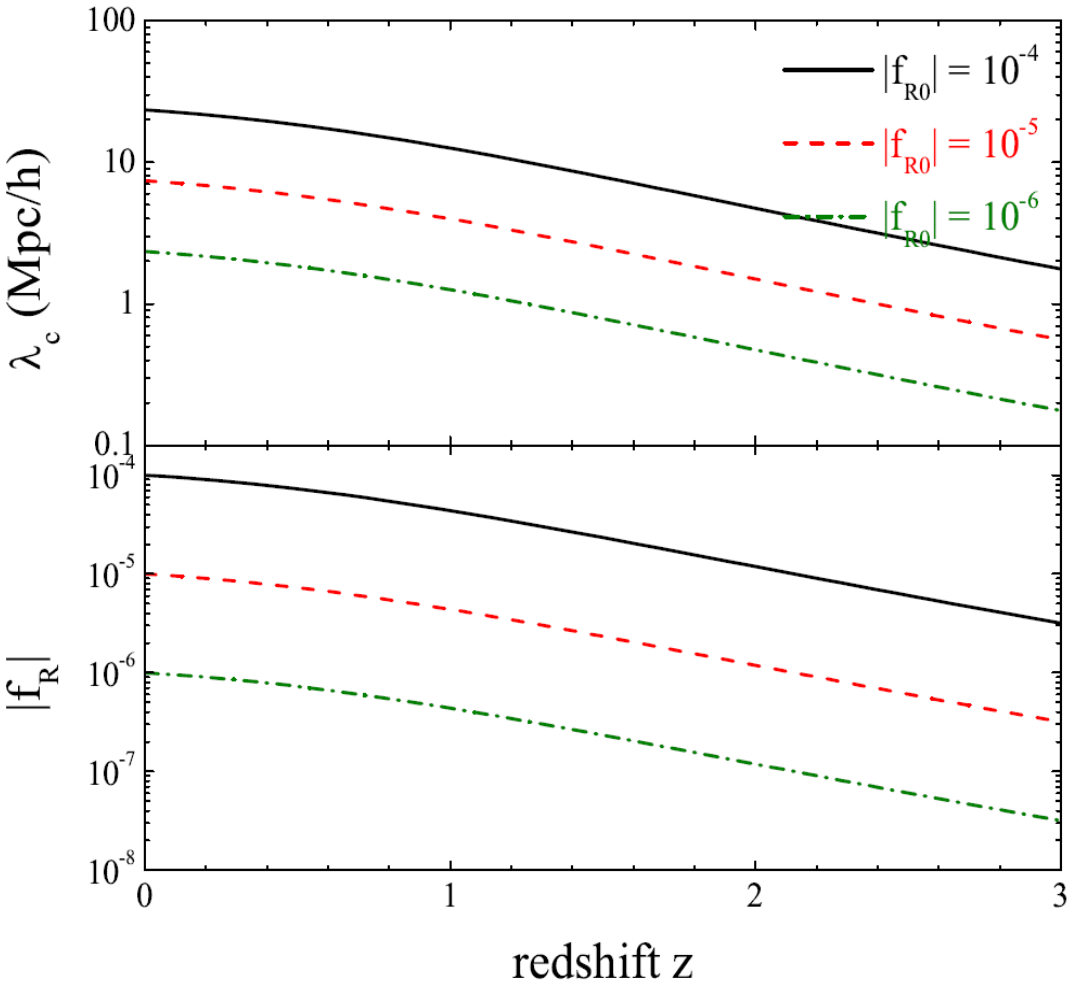


Linear predictions

$$f(R) = R - 2\Lambda - |f_{R0}| \frac{R_0^2}{R} \qquad |f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$$

- Linear predictions at z=0





Chameleon mechanism

$$f(R) = R - 2\Lambda - |f_{R0}| \frac{R_0^2}{R}$$

$$|f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$$

- Full f(R) simulations

  solve the non-linear scalar equation

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta\rho_{\rm M}]$$

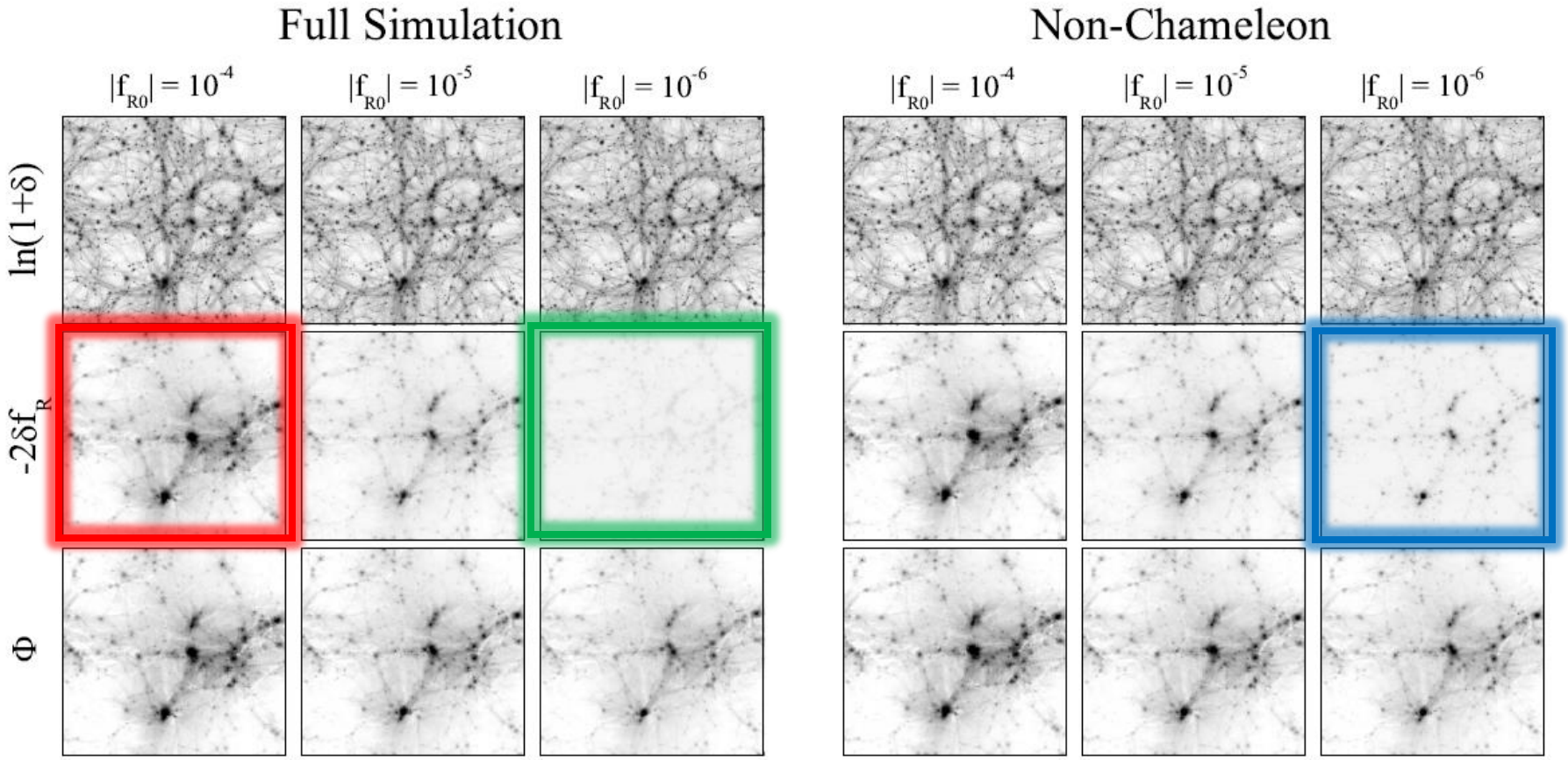
- Non-Chameleon simulations

  artificially suppress the Chameleon by linearising the scalar equation to remove the Chameleon effect

$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta\rho_{\rm M}$$

Snapshots at z=0

- If the fifth force is not suppressed, we have $-2\delta f_R = \Phi.$



Fifth force is not suppressed

Chameleon is working

Compton wavelength is short

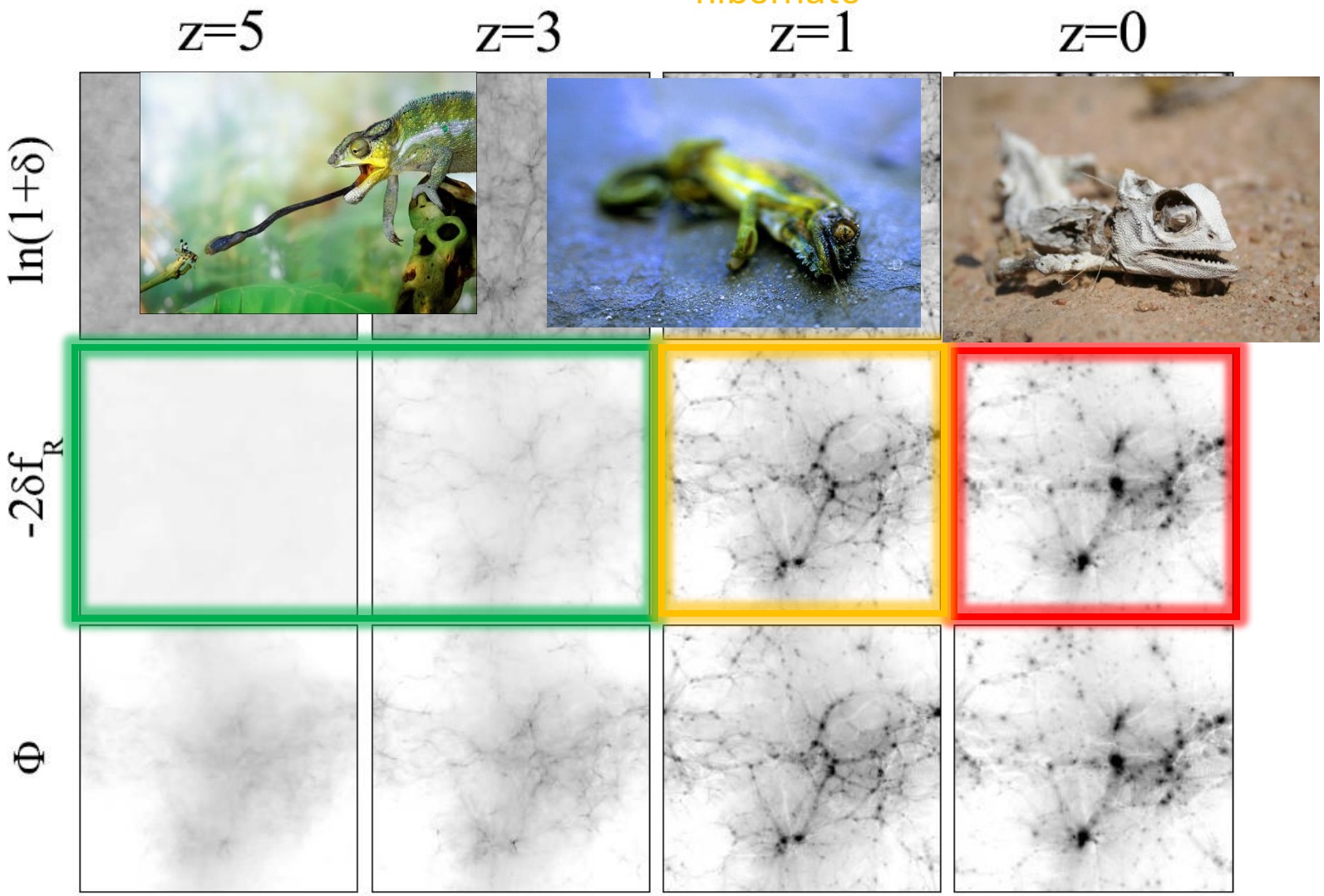
Snapshots

Chameleon is working

Chameleon starts to hibernate

Chameleon stops working

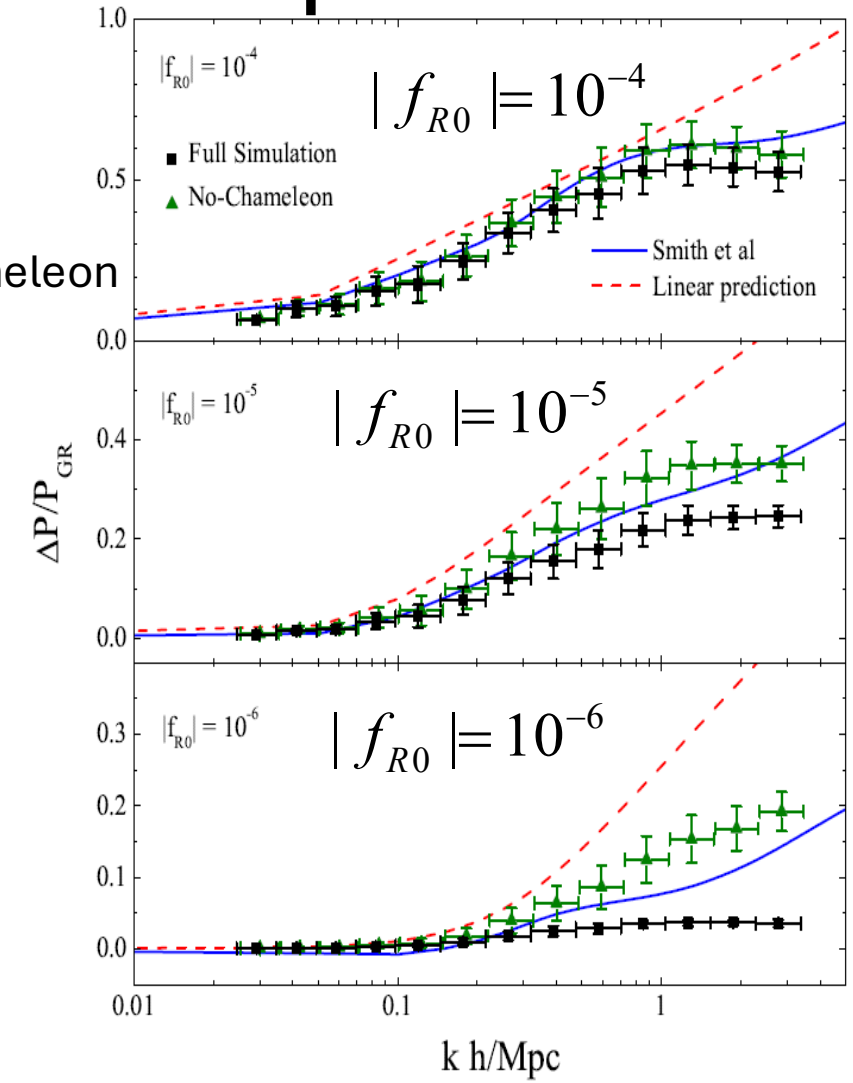
$|f_{R0}| = 10^{-4}$



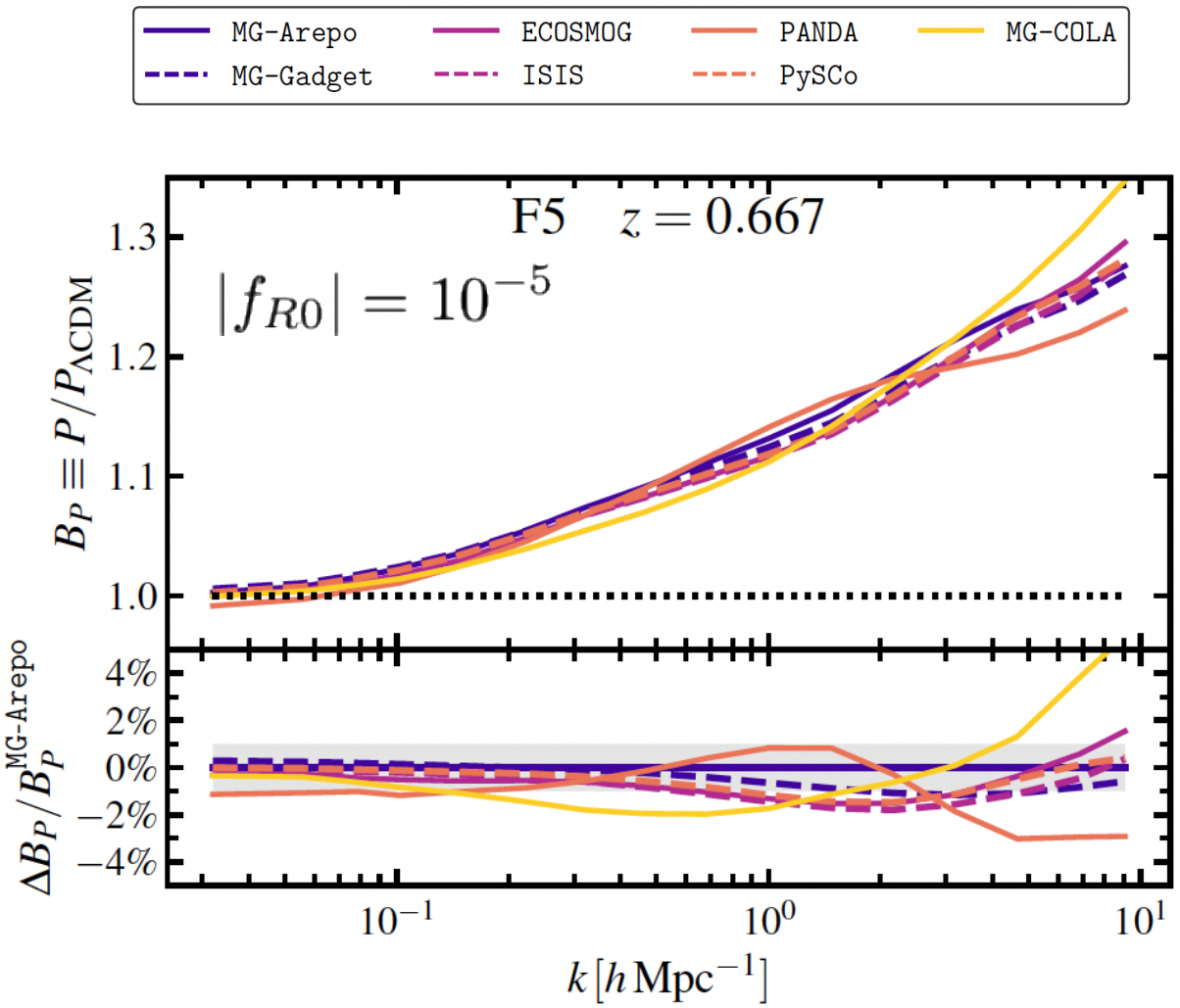
Power spectrum

full

Non-Chameleon



Zhao, Li, KK 1011.1257



Euclid: Adamek et.al. 2409.03522

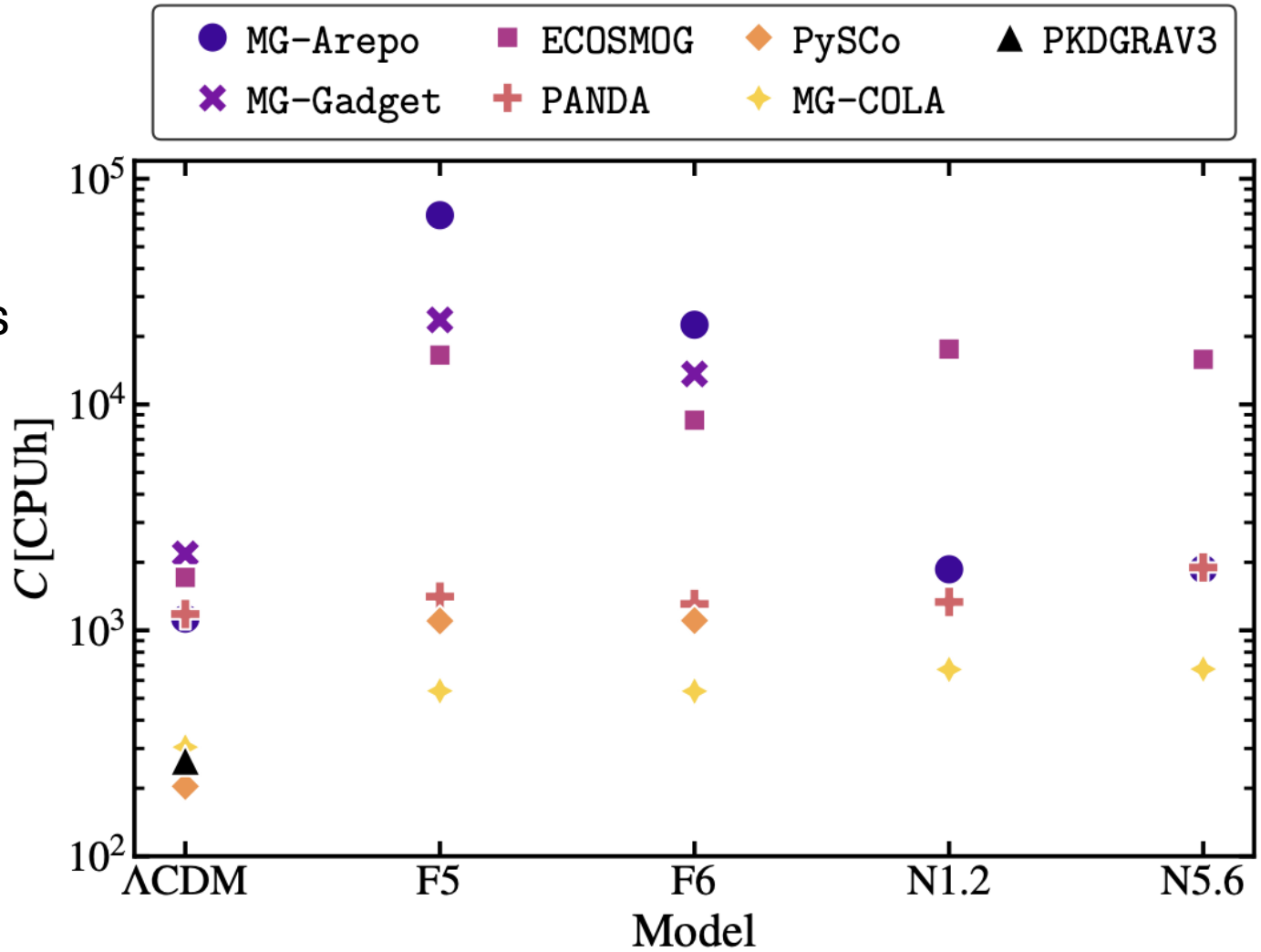
Comparison of CPU hours

Euclid: Adamek et.al. 2409.03522

$$B = 250\,h^{-1}\,\mathrm{Mpc} \quad N_{\mathrm{p}} = 512^3$$

- Codes that solve the scalar field exactly are up to O(100) more expensive to run f(R) simulations compared with LCDM

- Approximate methods help reduce the overhead of adding the scalar field significantly



Halo model approach

- ReACT Bose et.al. 2005.12184

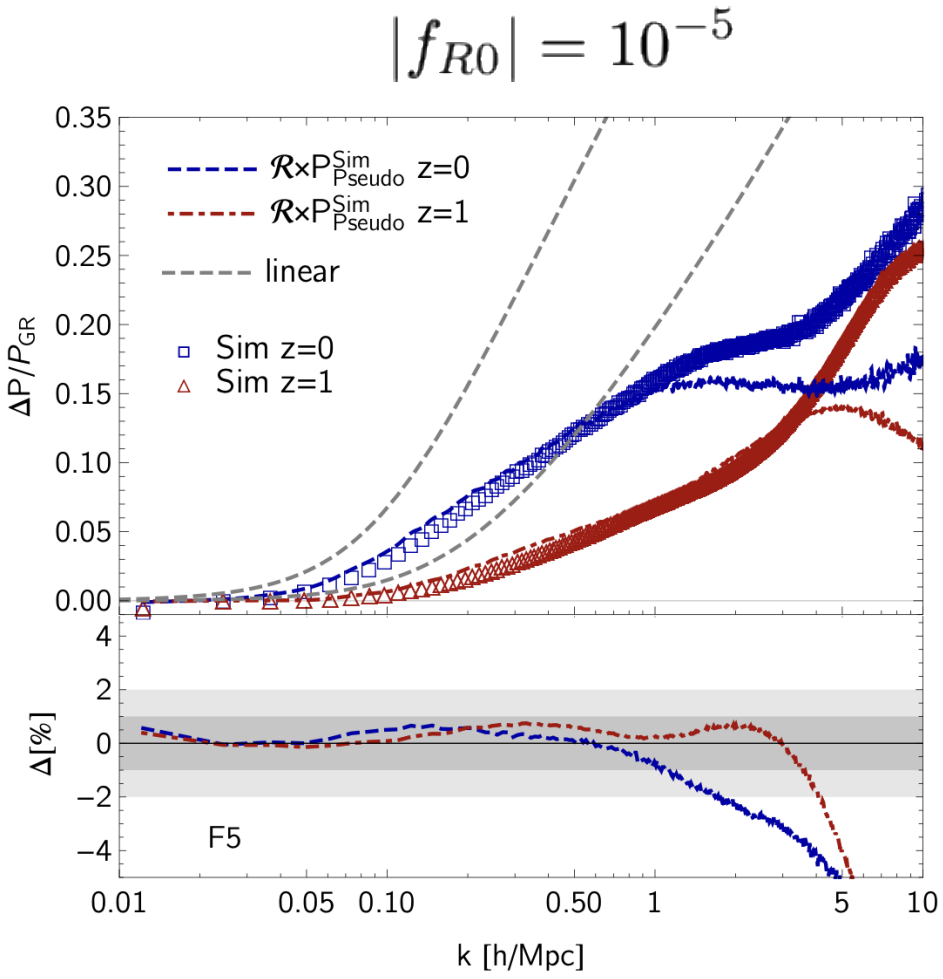
$$P_{\rm NL}(k,z) = \mathcal{R}(k,z) P_{\rm NL}^{\rm pseudo}(k,z)$$

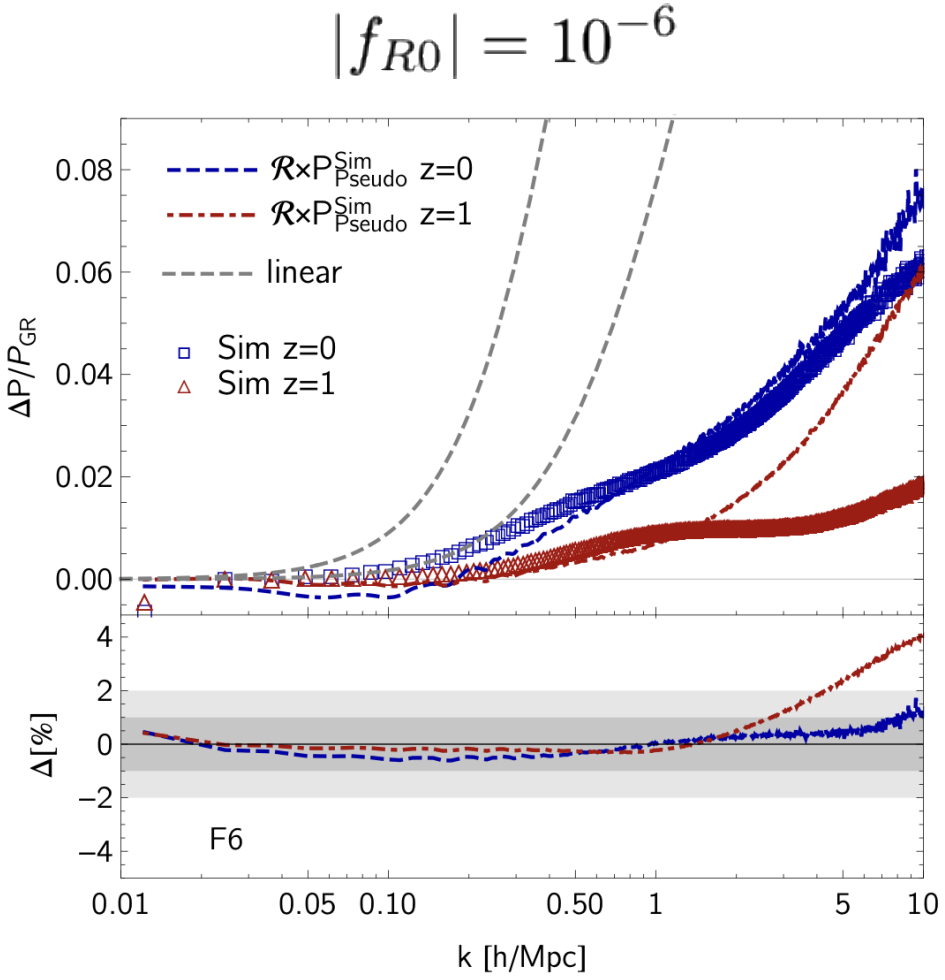
$P_{\rm NL}^{\rm pseudo}(k,z)$: LCDM power spectrum with the same linear power spectrum

$$\mathcal{R}(k,z) = \frac{[(1-\mathcal{E}(z))e^{-k/k_\star(z)} + \mathcal{E}(z)]P_{\rm L}(k,z) + P_{\rm 1h}(k,z)}{P_{\rm hm}^{\rm pseudo}(k,z)} \qquad \mathcal{E}(z) = \lim_{k\to 0} \frac{P_{\rm 1h}(k,z)}{P_{\rm 1h}^{\rm pseudo}(k,z)},$$

$P_{\rm 1h}(k,z)$: 1-halo term including non-linear modifications (i.e. screening) modelled by modified spherical collapse

ReACT


$|f_{R0}| = 10^{-5}$
$\mathcal{R}\times P^{\rm Sim}_{\rm Pseudo}$ z=0
$\mathcal{R}\times P^{\rm Sim}_{\rm Pseudo}$ z=1
linear
Sim z=0
Sim z=1
$\Delta P/P_{\rm GR}$
Δ[%]
F5
k [h/Mpc]


$|f_{R0}| = 10^{-6}$
$\mathcal{R}\times P^{\rm Sim}_{\rm Pseudo}$ z=0
$\mathcal{R}\times P^{\rm Sim}_{\rm Pseudo}$ z=1
linear
Sim z=0
Sim z=1
$\Delta P/P_{\rm GR}$
Δ[%]
F6
k [h/Mpc]

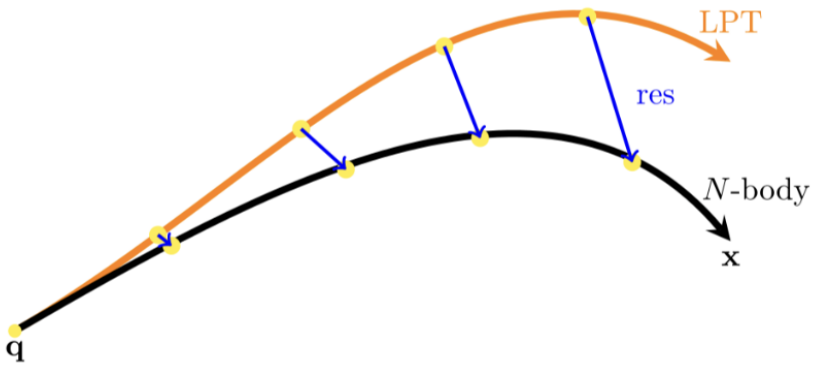
Approximate simulations

Tassev, Zaldarriaga, Eisenstein 1301.0322

- COLA (Comoving Lagrangian Acceleration)
  - 2nd order Lagrangian Perturbation Theory (LPT)

$$\vec{x}_{LPT} = \nabla\phi$$

$$\frac{d^2\delta\vec{x}}{d\tau^2} = -\vec{\nabla}\Phi_N - \frac{d^2\vec{x}_{\rm LPT}}{d\tau^2}$$



  - Simulations (Particle-Mesh)

Approximation for screening based on spherically symmetric solutions

$$F_\varphi = 2\,\beta^2\,F_{Newton}\,\varepsilon(\Psi, \nabla\Psi, \nabla^2\Psi)$$ Winther and Ferreira 1403.6492

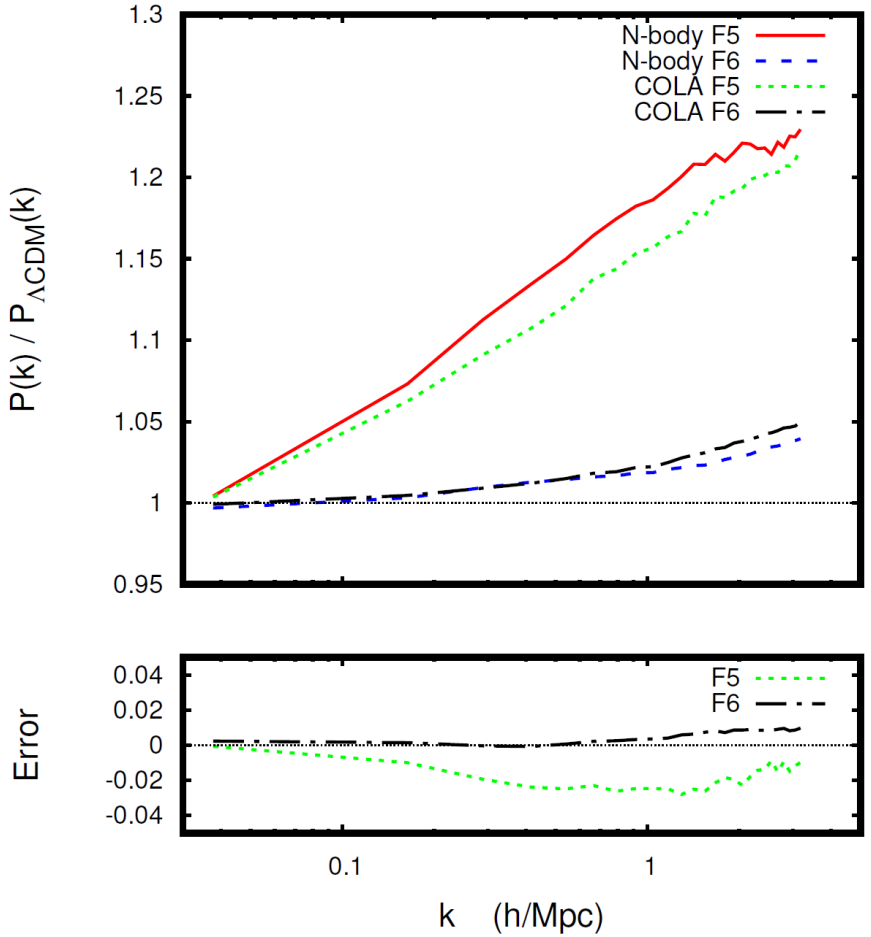
Valogiannis and Bean 1612.06469, Winther, Koyama et.al. 1703.00879

Approximate method

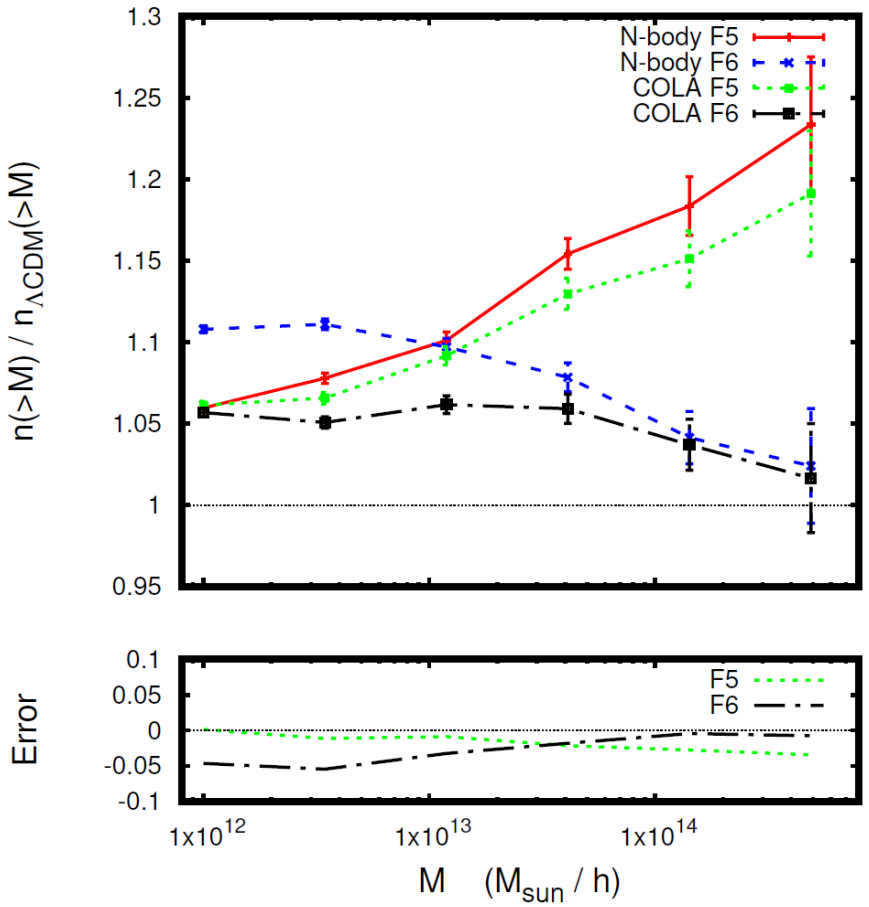
- COLA (2LPT + PM + screening approximation)

Winther et.al. 1703.00879

Power spectrum



mass function



Field level emulator

Saadeh, Koyama, Morice-Atkinson 2406.03374

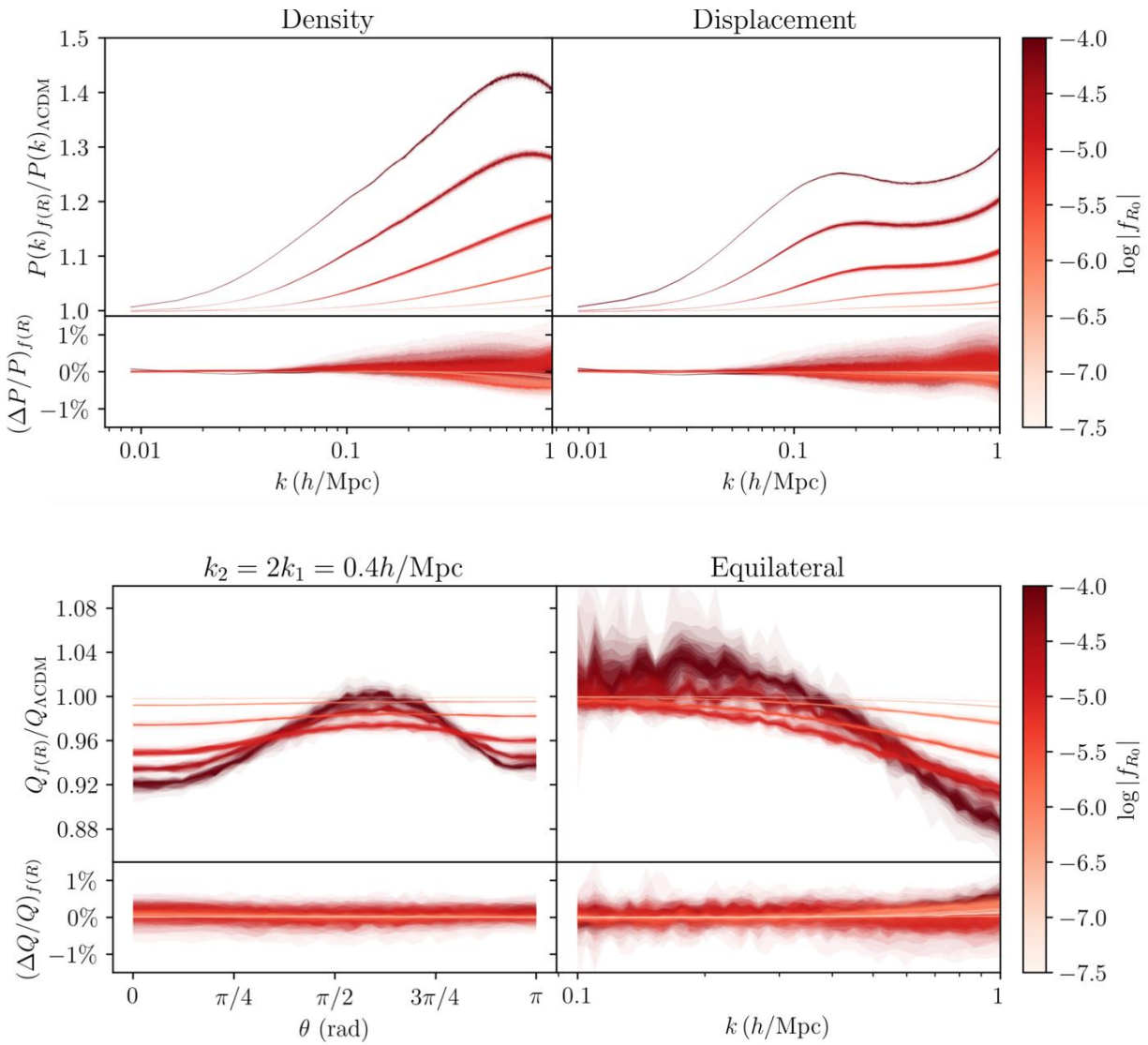
- map2map architecture

$$\Psi^{(\Lambda\mathrm{CDM})}(\mathbf{q};z=0) \mapsto \Psi^{(\mathrm{MG})}(\mathbf{q};z=0).$$

$$\mathbf{v}^{(\Lambda\mathrm{CDM})}(\mathbf{q};z=0) \mapsto \mathbf{v}^{(\mathrm{MG})}(\mathbf{q};z=0)$$

$\log_{10}\lvert f_{R_0}\rvert$	num sims training	num sims validation	num sims test	total
−4	371	25	25	421
−4.5	322	25	25	372
−5	273	25	25	323
−5.5	224	25	25	274
−6	176	25	25	226
−6.5	127	25	25	177
−7	78	25	25	128
−7.5	29	25	25	79
total	1600	200	200	2000

Same resolution as QUIJOTE N-body suite (Np=512^3, L=1Gpc)



Rescaling simulations

Angulo and White 0912.4277

1412.5195

Halo model ingredient

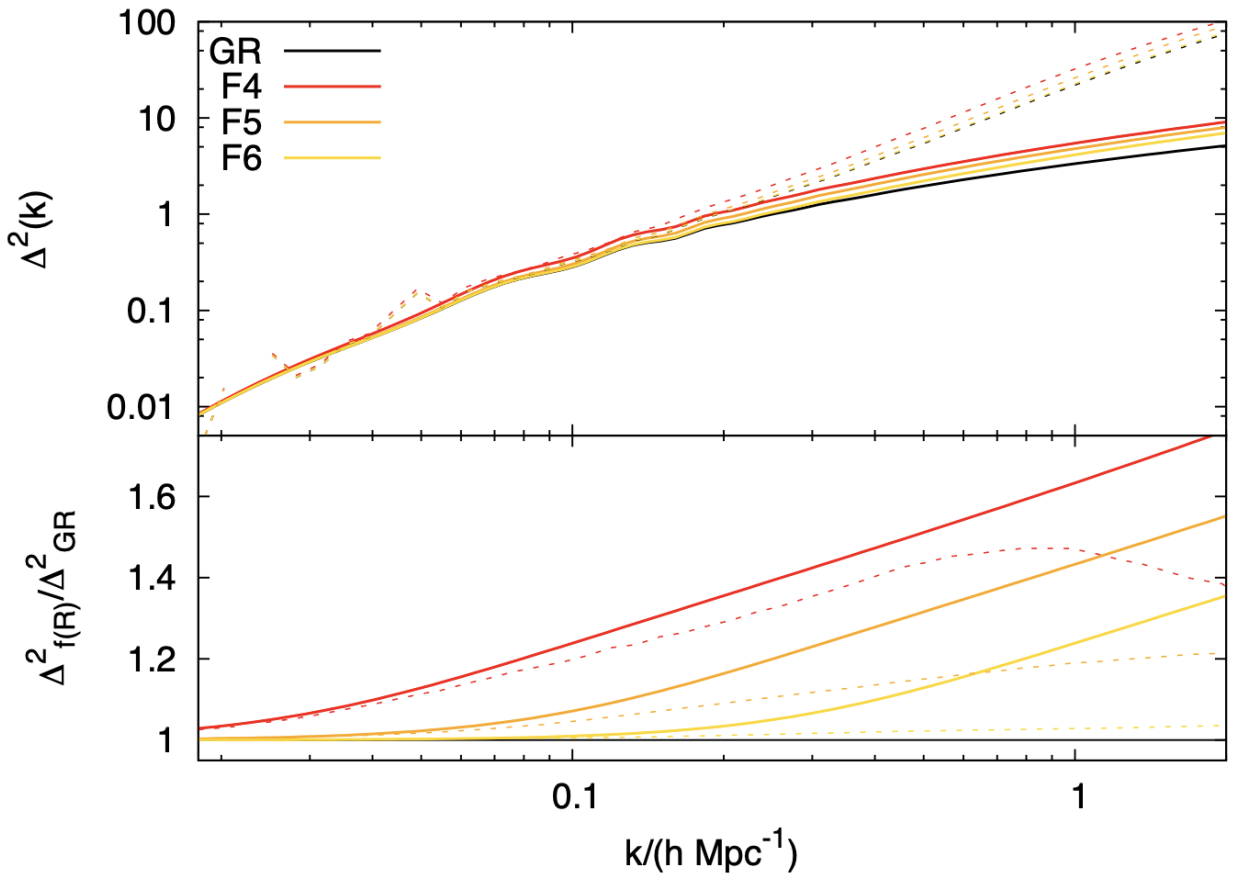
halo mass function, halo bias
density profile

halo mass function and bias are determined by

$$\sigma^2(R,z) = \int g^2(k,z)\Delta^2_{\rm lin}(k,0)W^2(kR)\,{\rm d}\ln k$$

$$\Delta^2(k) = \frac{4\pi V k^3}{(2\pi)^3}P(k)$$

$$W(x) = \frac{3}{x^3}(\sin x - x\cos x)$$



$g(k,z)$: growth function normalised to be 1 today

Rescaling

1412.5195

- Mapping
  an original simulation (z, L)
  a 'target' cosmology (z', L')

We find s and z so that

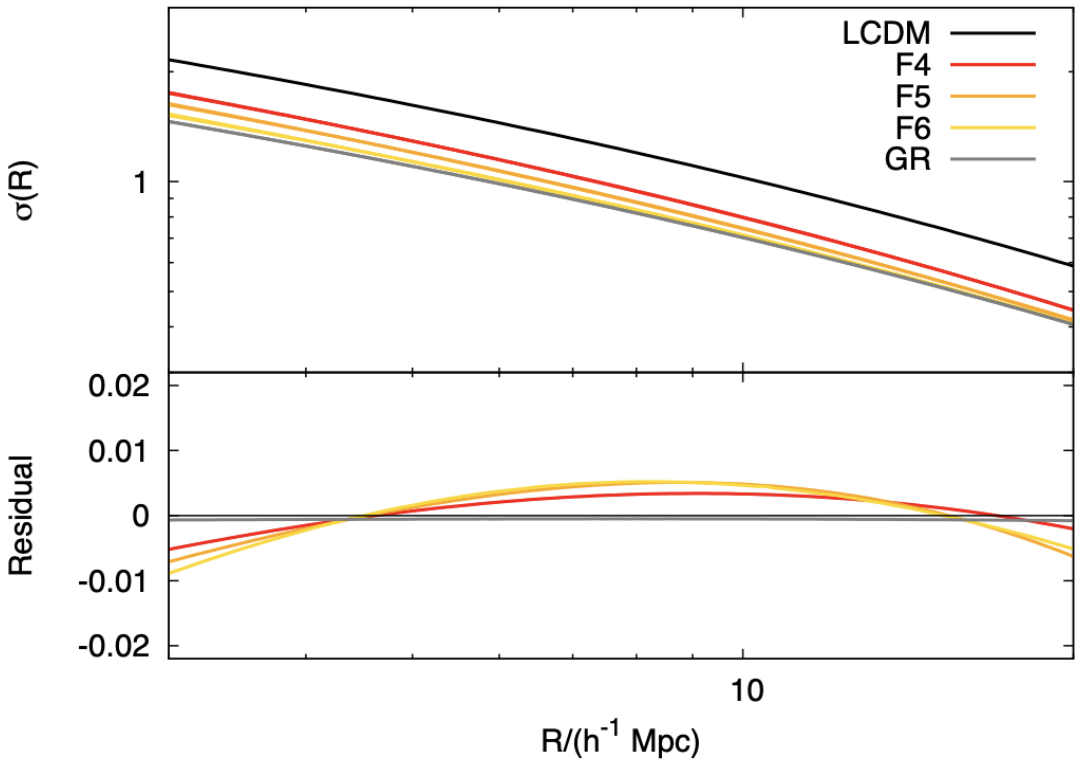
$$\sigma(R'/s, z) = \sigma'(R', z')$$

Original: LCDM $\sigma_8 = 1.2$

Target

Simulation	f_{R0}	True σ_8
GR	–	0.820
F6	-10^{-6}	0.834
F5	-10^{-5}	0.875
F4	-10^{-4}	0.940

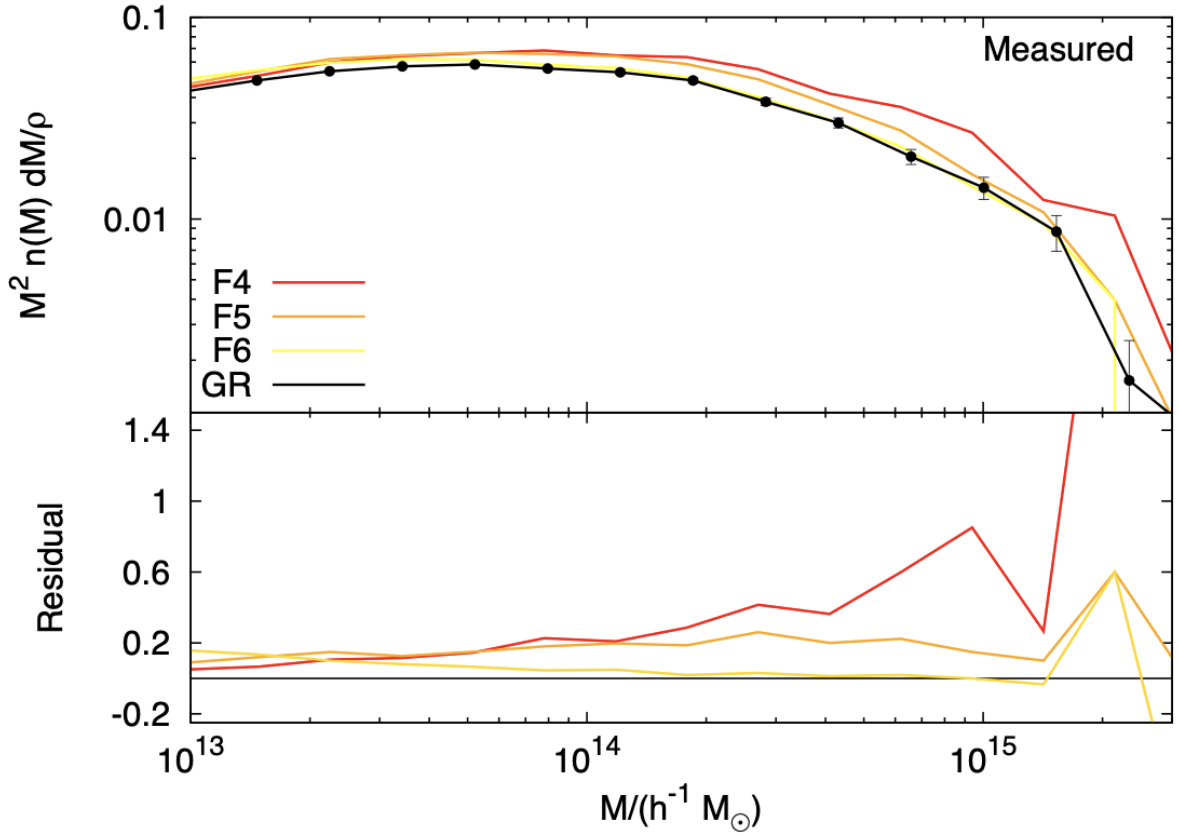
Target	s	L	z	$k_{\rm nl}$
GR	1.063	$482h^{-1}$ Mpc	0.844	$0.170h$ Mpc^{-1}
F6	0.956	$536h^{-1}$ Mpc	0.644	$0.164h$ Mpc^{-1}
F5	0.850	$602h^{-1}$ Mpc	0.381	$0.151h$ Mpc^{-1}
F4	0.838	$611h^{-1}$ Mpc	0.225	$0.136h$ Mpc^{-1}



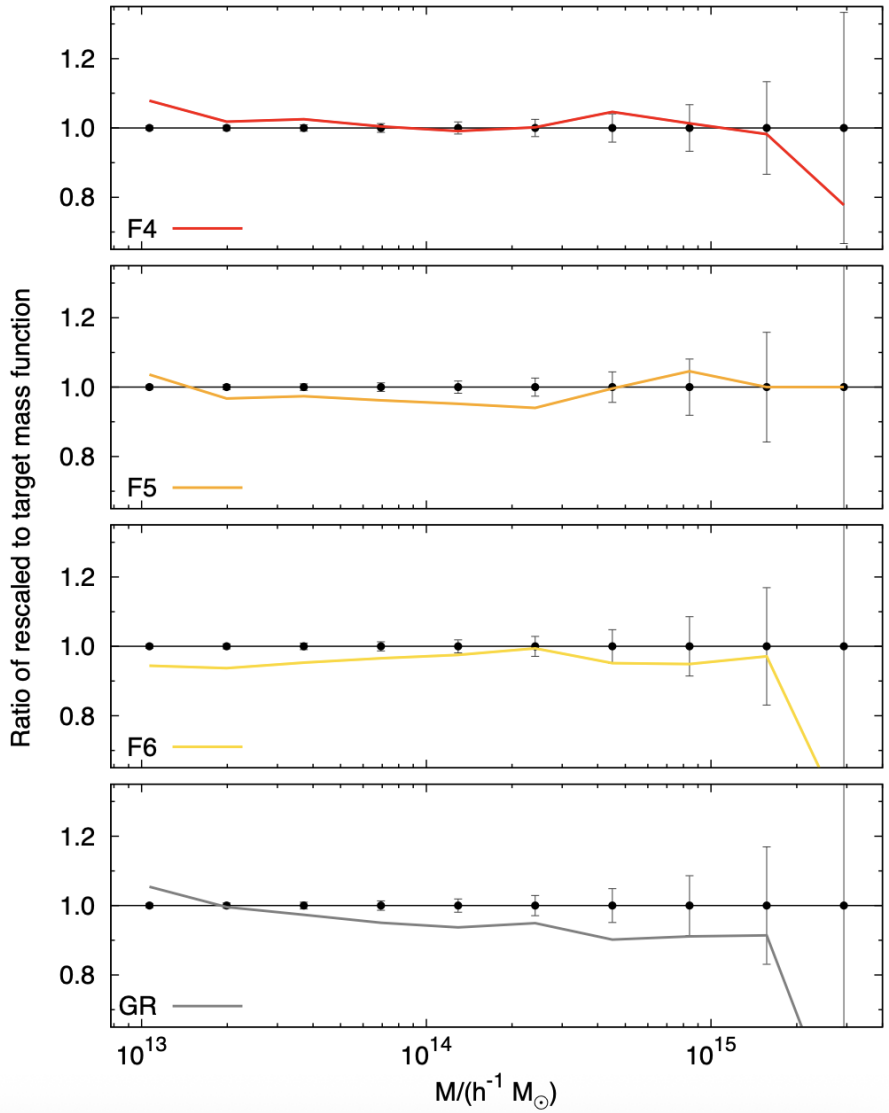
Mass function

1412.5195

- Mass function in target cosmologies



After rescaling



Power spectrum

1412.5195

- Linear power spectrum

To match the linear power spectrum, we use the Zel'dovich approximation

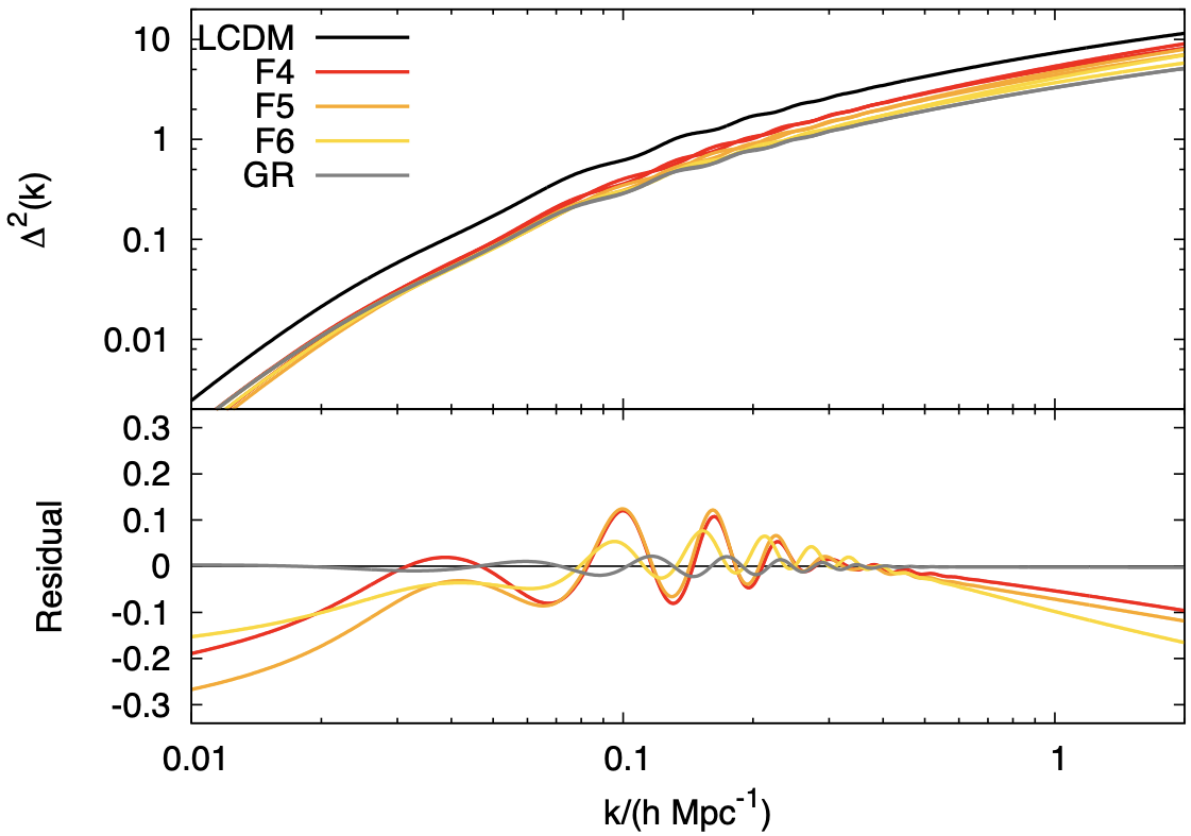
Eulerian position $\mathbf{x} = \mathbf{q} + \mathbf{f}$ $\delta = -\nabla \cdot \mathbf{f}$

f : displacement

$$\mathbf{x}'' = \mathbf{x}' + \delta\mathbf{f}'. \quad \delta\mathbf{f}'_{\mathbf{k}'} = \left[\sqrt{\frac{\Delta'^2_{\rm lin}(k', z')}{\Delta^2_{\rm lin}(sk', z)}} - 1\right] \mathbf{f}'_{\mathbf{k}'}$$

We apply Gussian smoothing

$$\exp(-k^2 R_{\rm nl}^2/2) \qquad \sigma'(R'_{\rm nl}, z') = 1$$



Matter power spectrum

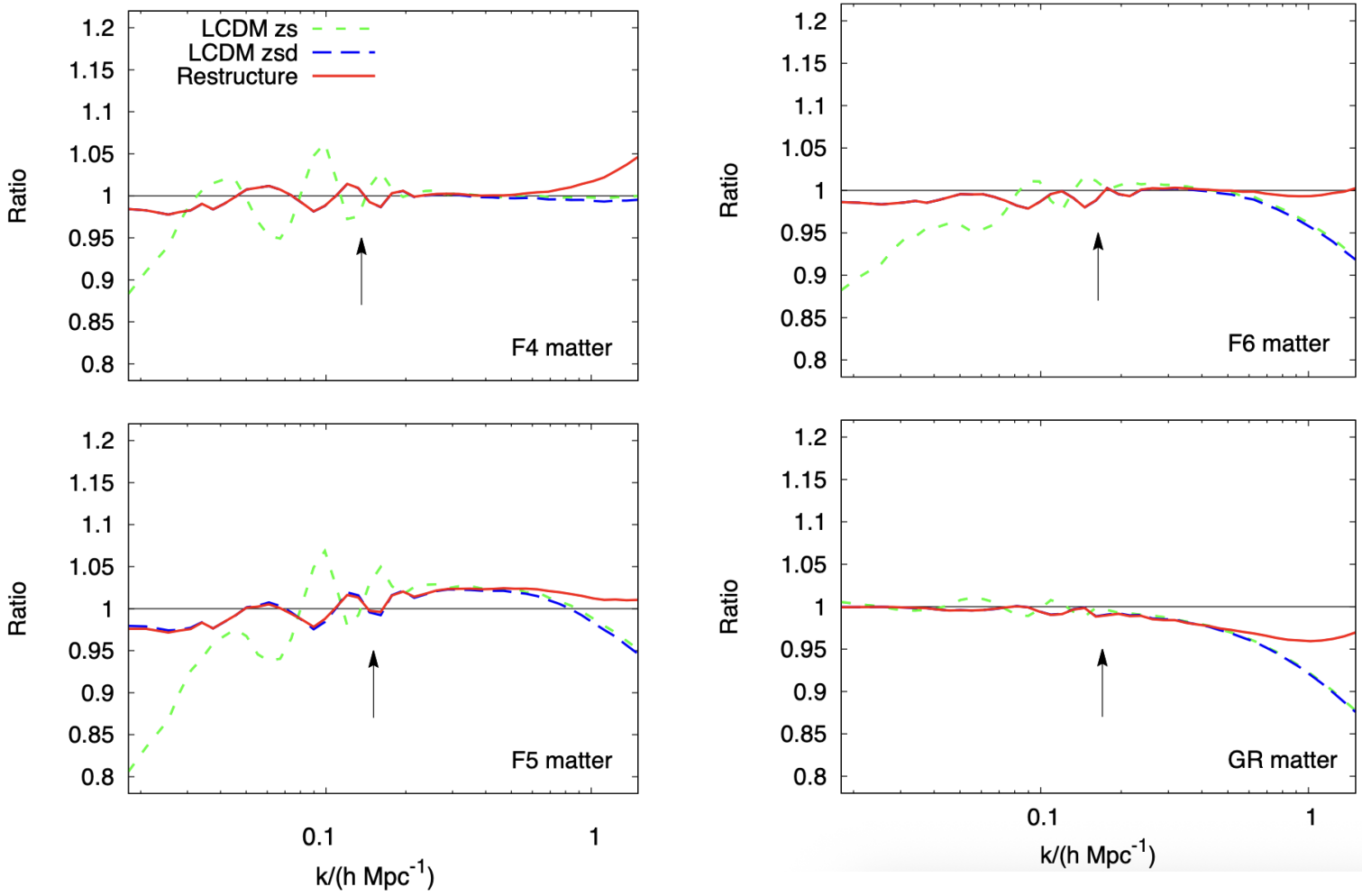
1412.5195

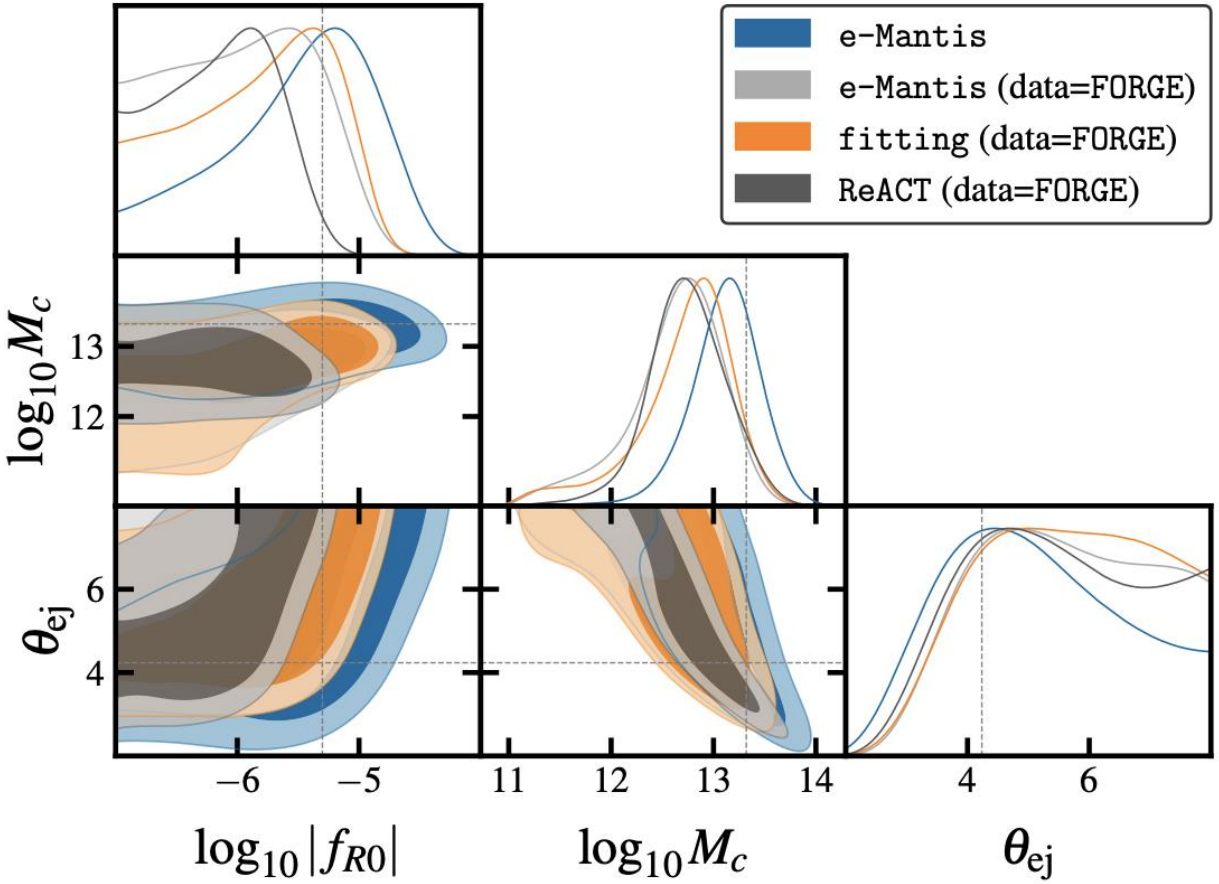
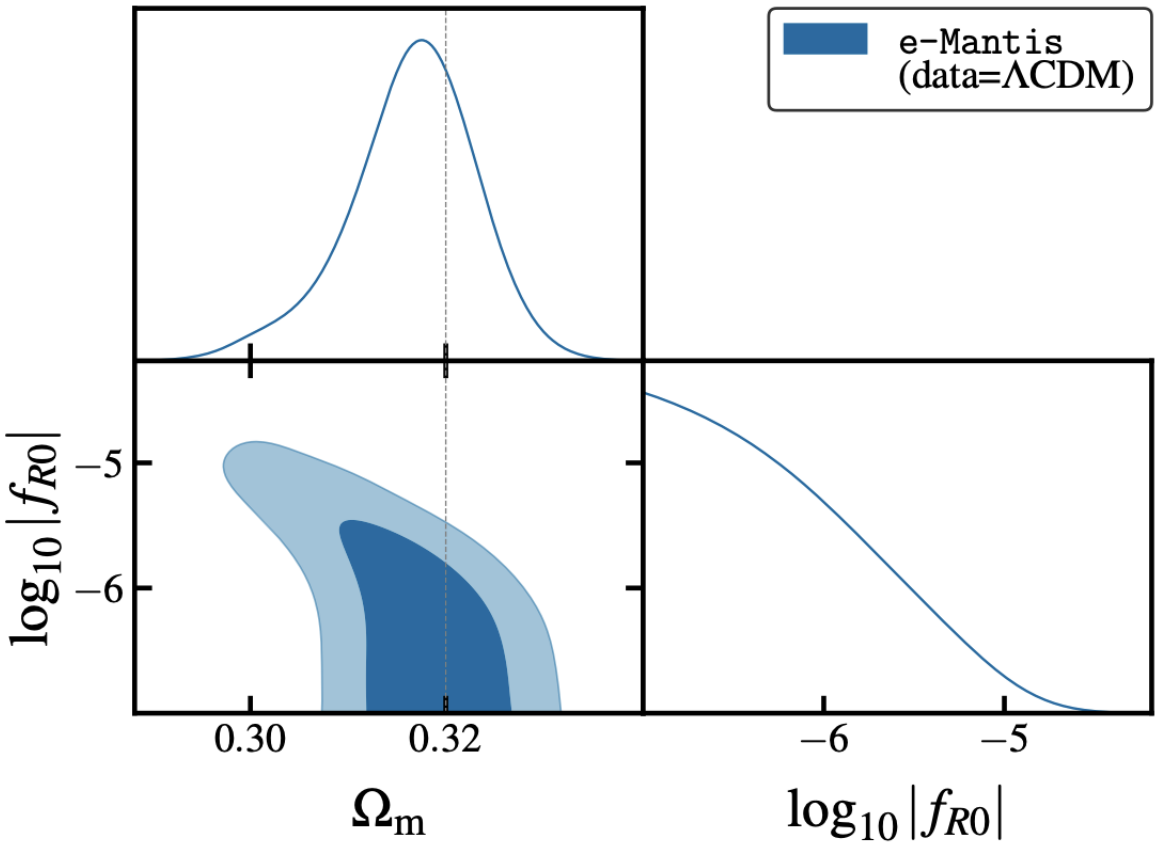
- Halo profile

Finally, we can restructure halo particles to reproduce the halo profile in the target simulations

- Velocities

We can apply a similar procedure to velocities



Euclid forecast

2409.03524