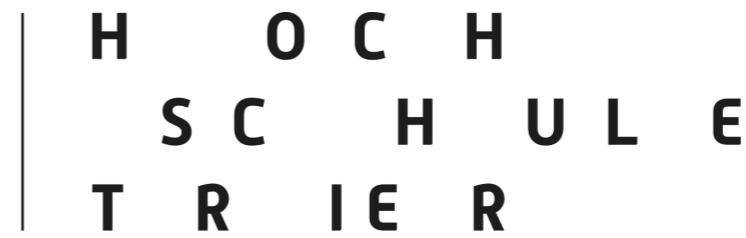


# **Context-Aware Clause Selection Using Symbol Name Meanings in Theorem Proving**

Claudia Schon  
Hochschule Trier  
[c.schon@hochschule-trier.de](mailto:c.schon@hochschule-trier.de)

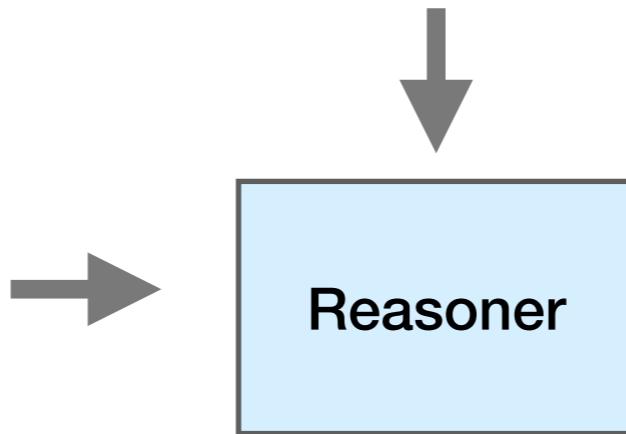
Trier University  
of Applied Sciences



# Automated Reasoning

```
subclass(bed, furniture)
subclass(doublebed, bed)
subclass(pillow, artifact)
subclass(blanket, fabric)
...
 $\forall x (instance(x, hammock) \rightarrow (material(x, fabric) \wedge ...))$ 
...
 $\forall x, y, z ((subclass(x, y) \wedge subclass(y, z)) \rightarrow subclass(x, z))$ 
subclass(weapon, device)
subclass(device, artifact)
```

Knowledge Base



Goal:

Are hammocks beds?

$\forall x (instance(x, hammock) \rightarrow instance(x, bed))$

Reasoner

# Automated Reasoning

```
subclass(bed, furniture)
subclass(doublebed, bed)
subclass(pillow, artifact)
subclass(blanket, fabric)
...
```

```
∀x(instance(x, hammock) →
  (material(x, fabric) ∧ ...
```

...

```
∀x, y, z
  ((subclass(x, y) ∧ subclass(y, z)) →
    subclass(x, z))
```

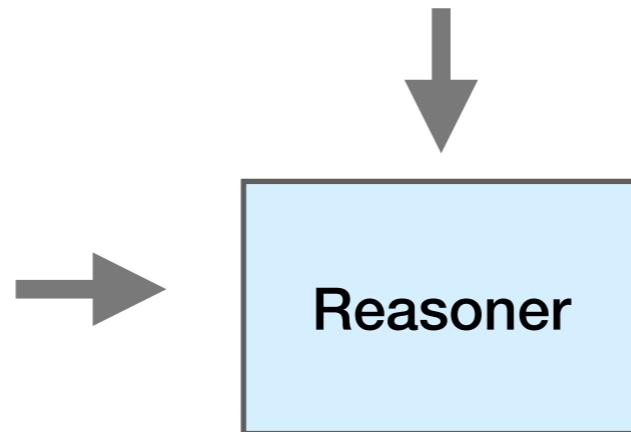
```
subclass(weapon, device)
subclass(device, artifact)
```

Knowledge Base

Goal:

Are hammocks beds?

$\forall x(instance(x, hammock) \rightarrow instance(x, bed))$



Computes inferences!

Reasoner

# Automated Reasoning

*subclass(bed, furniture)*

*subclass(doublebed, bed)*

*subclass(pillow, artifact)*

*subclass(blanket, fabric)*

...

$\forall x(\text{instance}(x, \text{hammock}) \rightarrow (\text{material}(x, \text{fabric}) \wedge \dots))$

...

$\forall x, y, z ((\text{subclass}(x, y) \wedge \text{subclass}(y, z)) \rightarrow \text{subclass}(x, z))$

*subclass(weapon, device)*

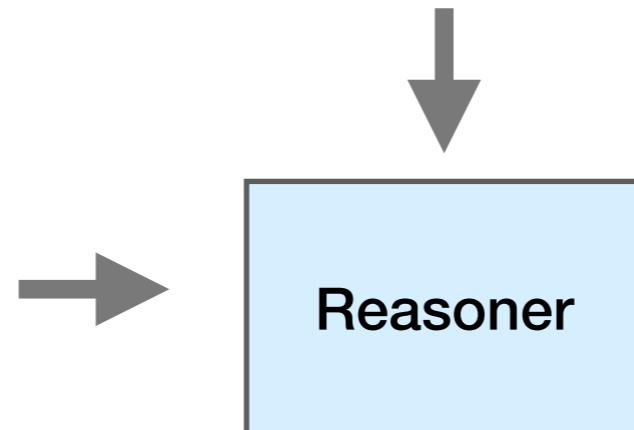
*subclass(device, artifact)*

Knowledge Base

Goal:

Are hammocks beds?

$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$



Computes inferences!

**Does not take the meaning of symbols into account!**

# Automated Reasoning

$s(b, f)$   
 $s(db, b)$   
 $s(p, a)$   
 $s(bl, fa)$   
...

$\forall x(in(x, h) \rightarrow$   
 $(m(x, fa) \wedge ...$

...

$\forall x, y, z$   
 $((s(x, y) \wedge s(y, z)) \rightarrow$   
 $s(x, z))$

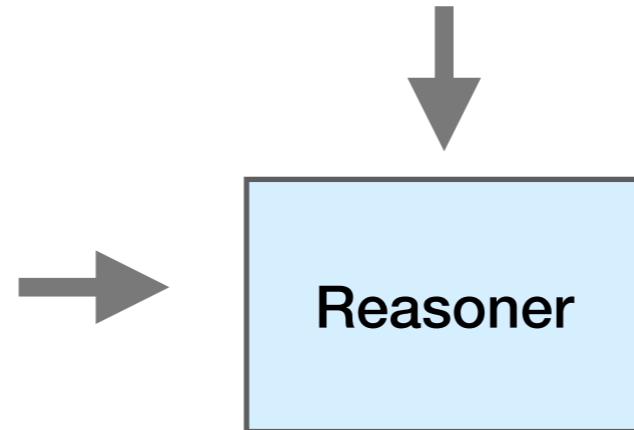
$s(w, de)$   
 $s(de, a)$

Knowledge Base

For the prover, the proof task looks like this:

Goal:

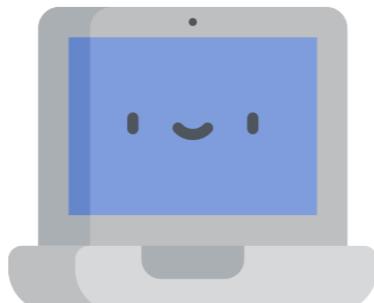
$$\forall x(in(x, h) \rightarrow in(x, b))$$



Computes inferences!

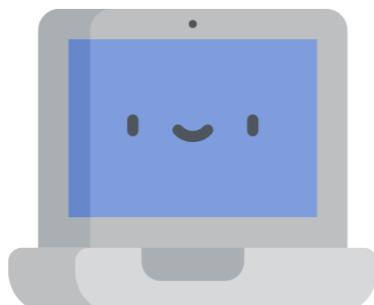
Does not take the meaning of symbols into account!

# Automated Reasoning: Are Hammocks Beds?



Reasoner: PyRes

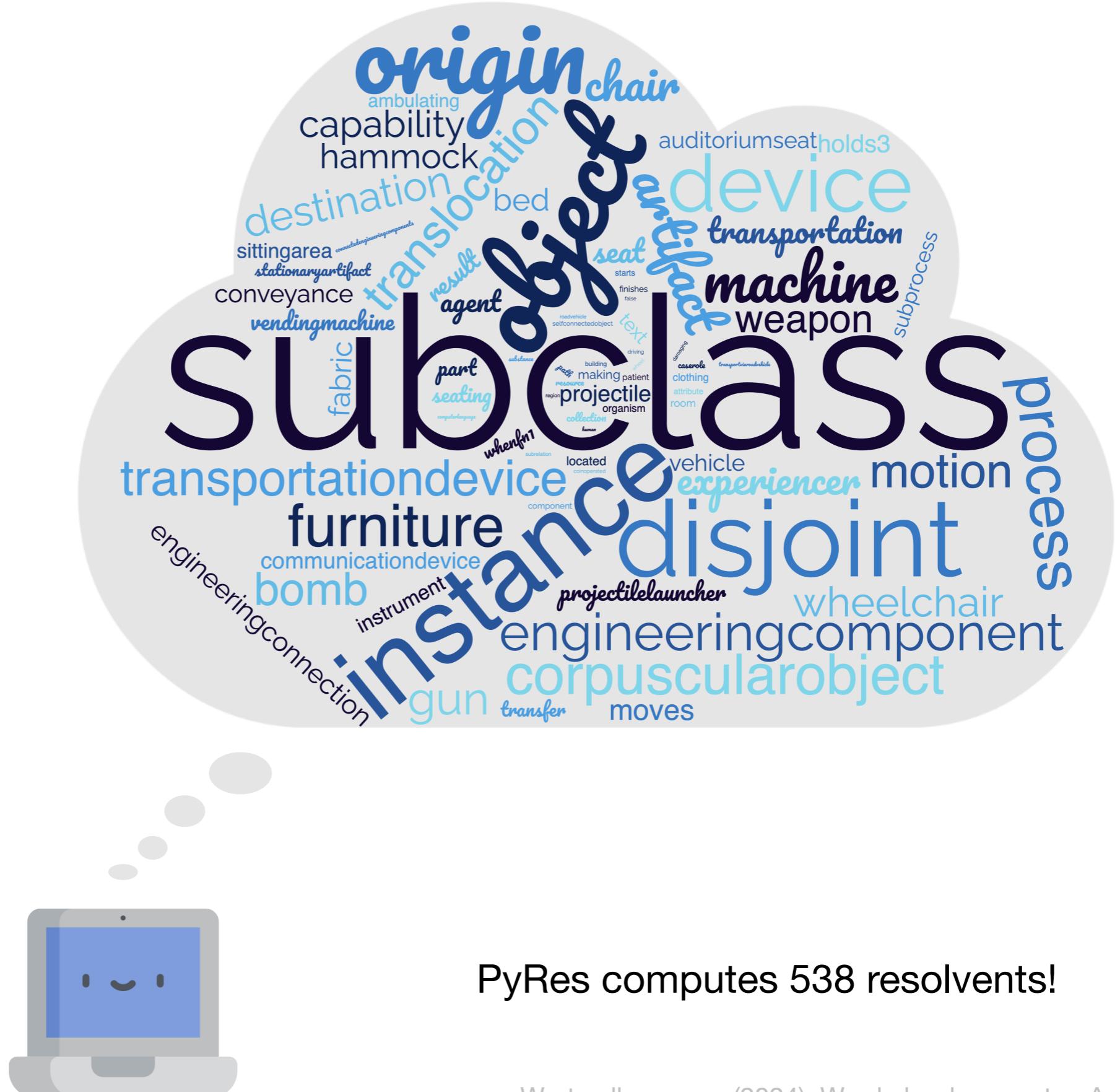
# Automated Reasoning: Are Hammocks Beds?



Reasoner: PyRes

PyRes computes 538 resolvents!

# Automated Reasoning: Are Hammocks Beds?



# PyRes computes 538 resolvents!



# Reasoner: PyRes

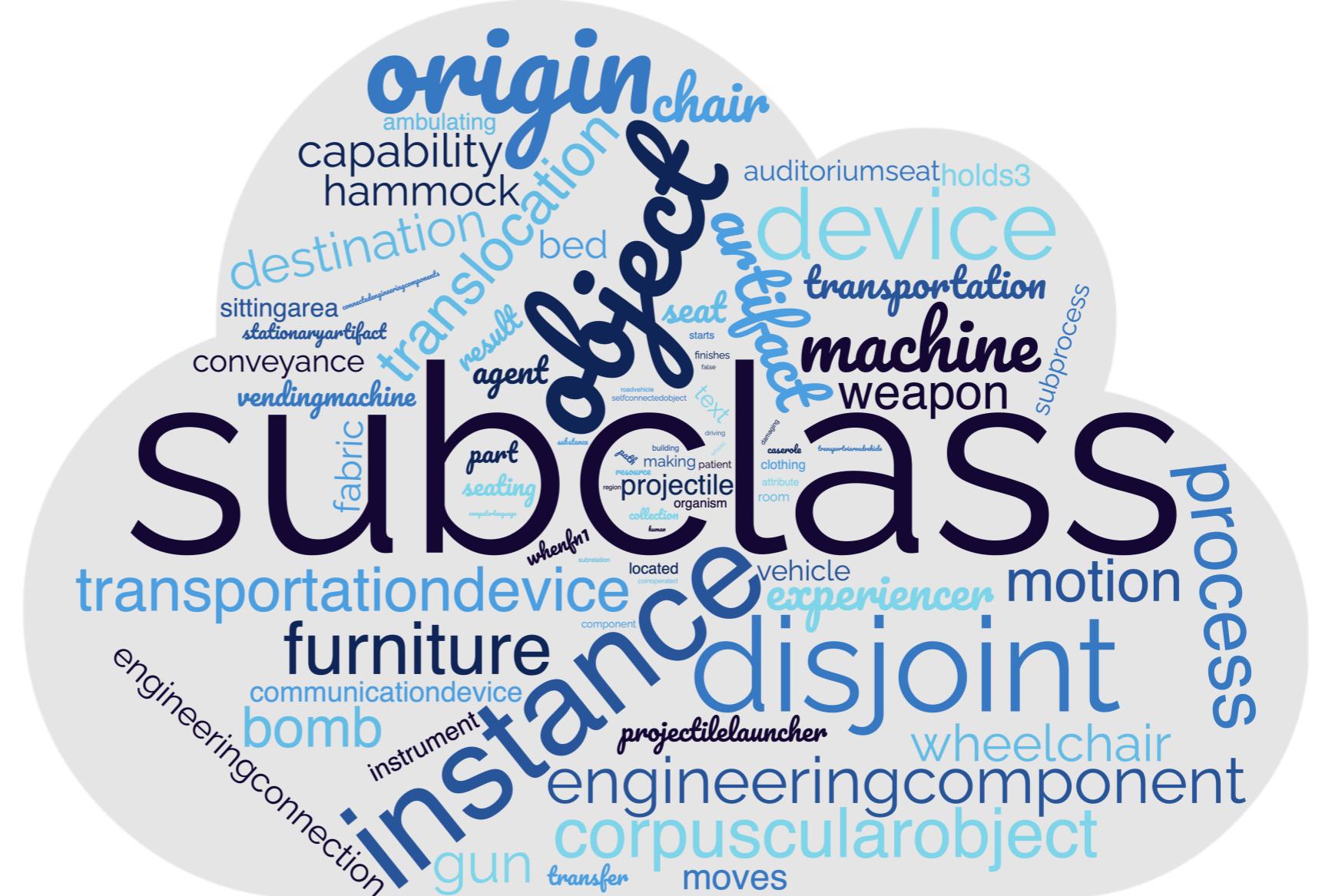
vendingmachine  
text  
driving  
wheel  
roadvehicle  
selfconnectedobject  
substance  
building  
making patient  
region projectile organism  
path resource collection  
region projectile organism  
collection human  
located coinoperated  
component  
whenfn1 subrelation  
engineeringconnection  
communicationdevice  
furniture  
bomb  
instrument  
instandance  
gun  
engineered  
corpus  
move  
transfer  
instancing  
part seating  
computerlanguage

# weapon

# Automated Reasoning: Are Hammocks Beds?



Reasoner: PyRes



PyRes computes 538 resolvents!

**Why should we draw unrelated inferences in automated reasoning?**

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$

$\neg w \vee d$   
 $\neg d \vee a$   
 $\neg h \vee f$   
 $\neg h \vee s$   
...

Clause set  $S$

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$

$\neg w \vee d$

$\neg d \vee a$

$\neg h \vee f$

$\neg h \vee s$

...

$h$

$\neg b$

Clause set  $S$

# Given-Clause Algorithmus

**Task:** proof that  $h \rightarrow b$  follows from clause set  $S$

$\neg w \vee d$   
 $\neg d \vee a$   
 $\neg h \vee f$   
 $\neg h \vee s$   
...

$h$

$\neg b$

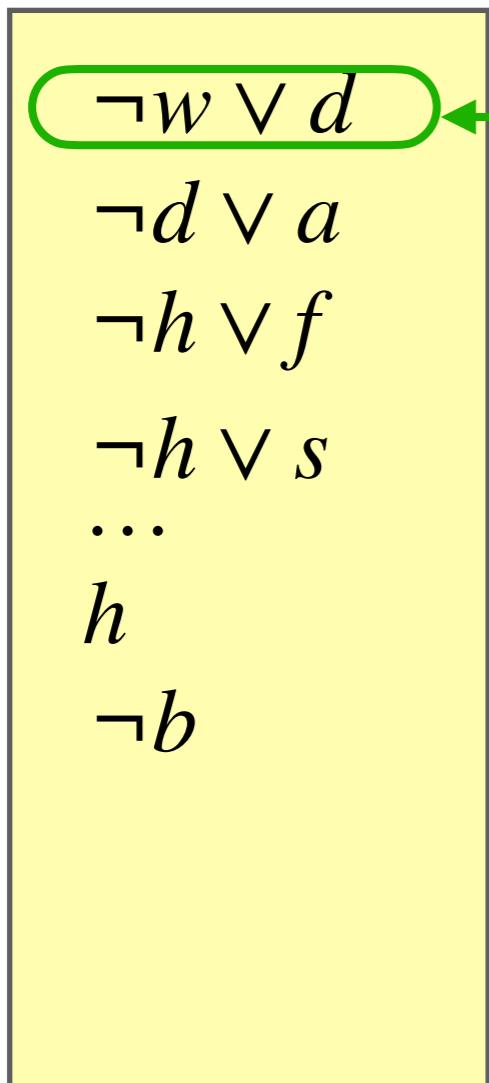


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



Select a given-clause

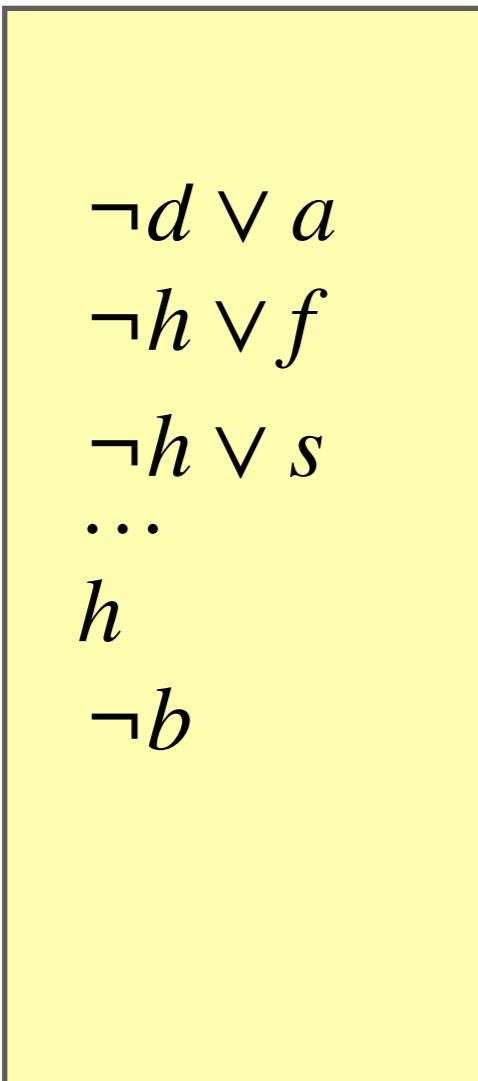


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg w \vee d$   
**Select a given-clause**



$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$

$$\neg d \vee a$$

$$\neg h \vee f$$

$$\neg h \vee s$$

...

$$h$$

$$\neg b$$

$\neg w \vee d$   
**Select a given-clause**

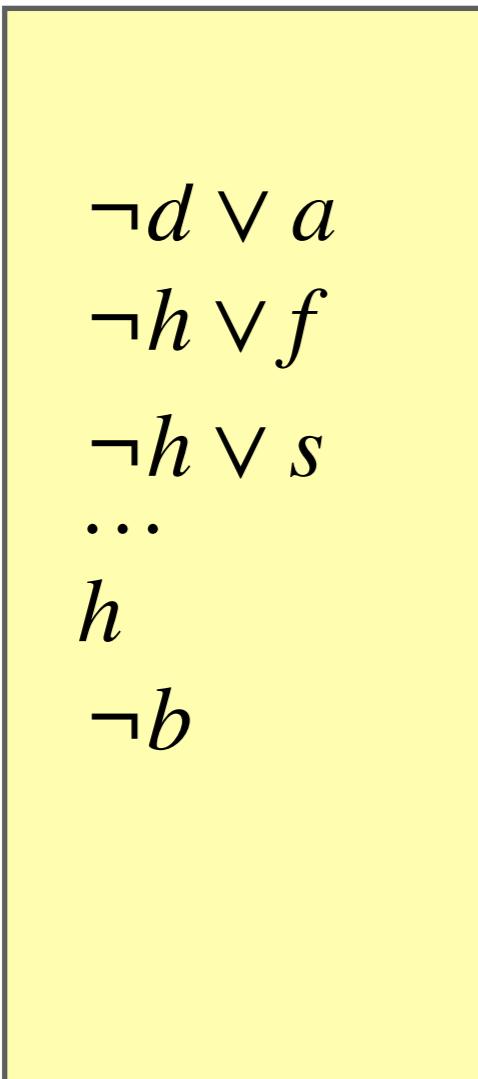
**Add all inferences of the given clause and clauses in  $P$  to  $U$ .**

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



**Select a given-clause**

**Add all inferences of the given clause and clauses in  $P$  to  $U$ .**

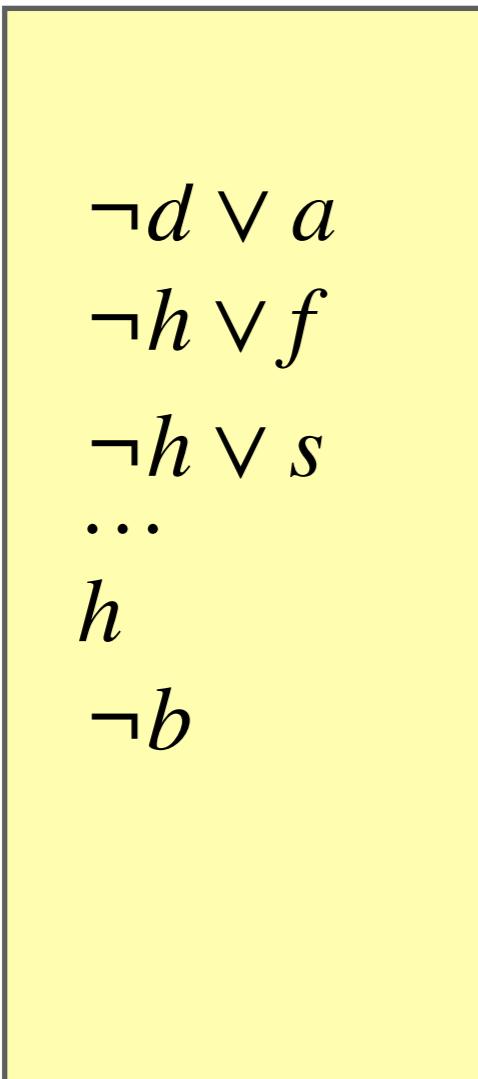


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



Select a given-clause

Add all inferences of the given clause and clauses in  $P$  to  $U$ .

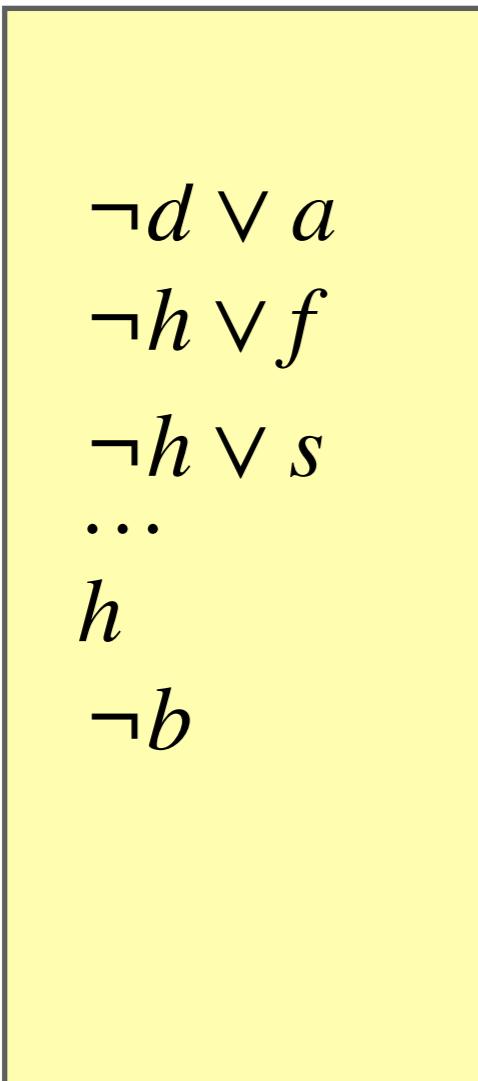


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



Select a given-clause

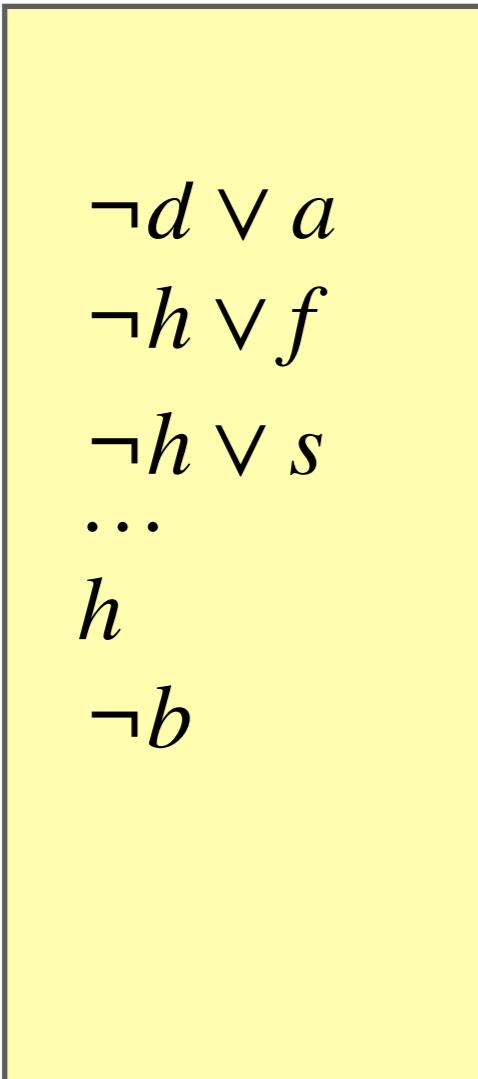


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



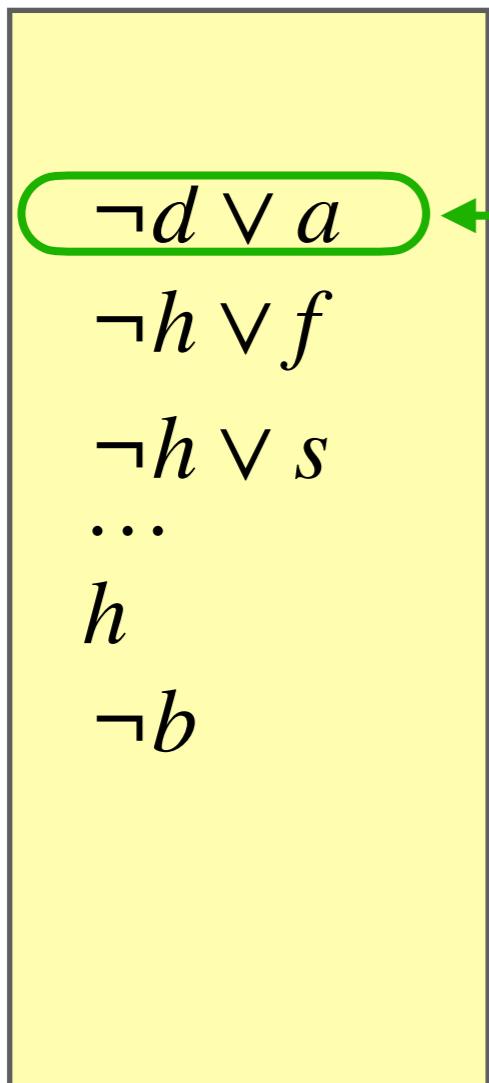
$U$ : set of unprocessed clauses



$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



Select a given-clause

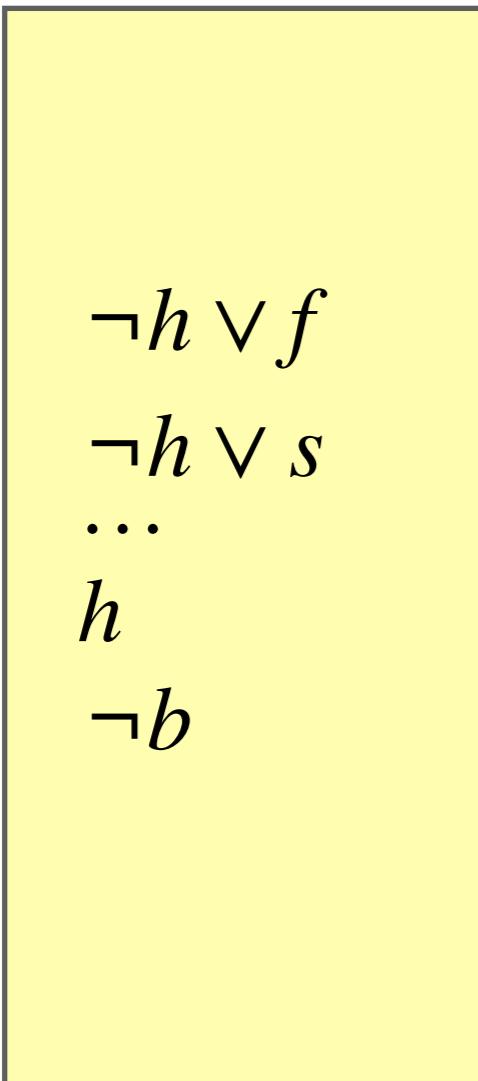


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg d \vee a$   
**Select a given-clause**

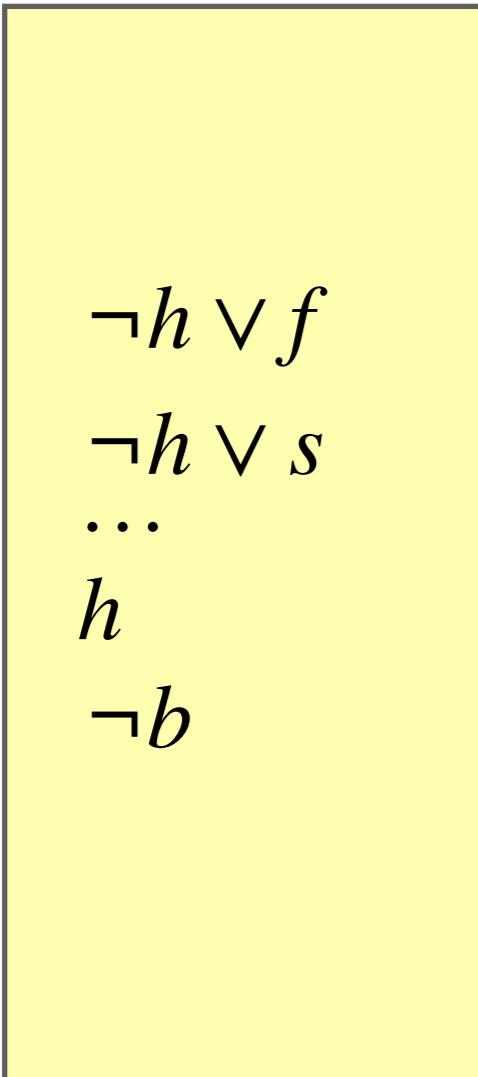


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg d \vee a$   
**Select a given-clause**



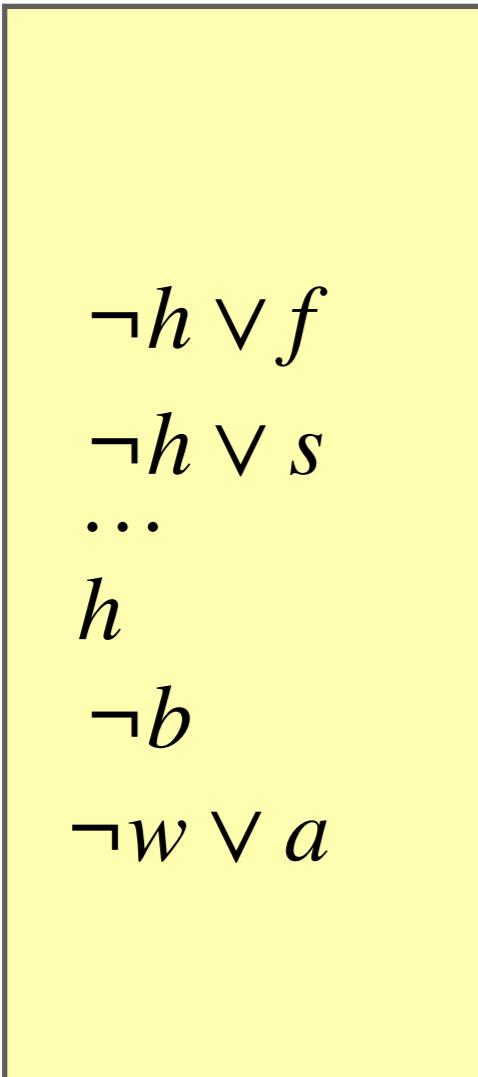
**Add all inferences of the given clause and clauses in  $P$  to  $U$ .**

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg d \vee a$   
Select a given-clause



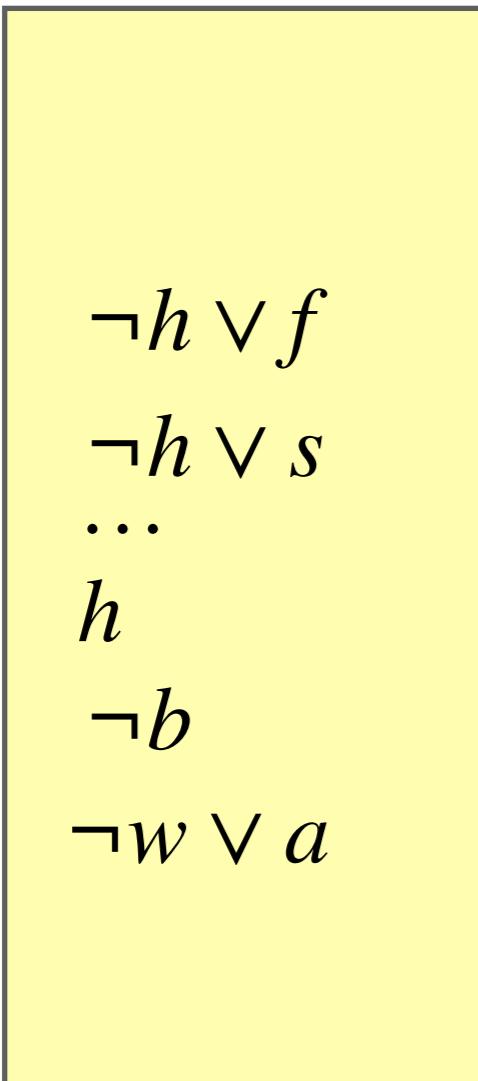
Add all inferences of the given clause and clauses in  $P$  to  $U$ .

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg d \vee a$   
**Select a given-clause**

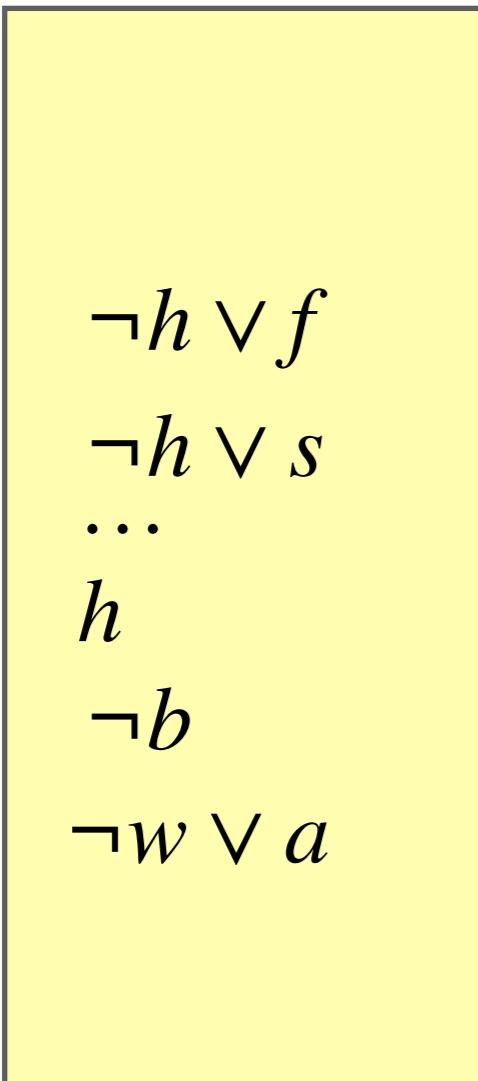


$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$



$\neg d \vee a$   
**Select a given-clause**



$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $h \rightarrow b$  follows from clause set  $S$

$\neg h \vee f$

$\neg h \vee s$

...

$h$

$\neg b$

$\neg w \vee a$

$\neg d \vee a$   
Select a given-clause

$\neg w \vee d$   
 $\neg d \vee a$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

Selection of given clause using heuristics.  
Does not take the meaning of symbol names into account.

# Given-Clause Algorithmus

**Task:** proof that  $\text{hammock} \rightarrow \text{bed}$  follows from the set of clauses

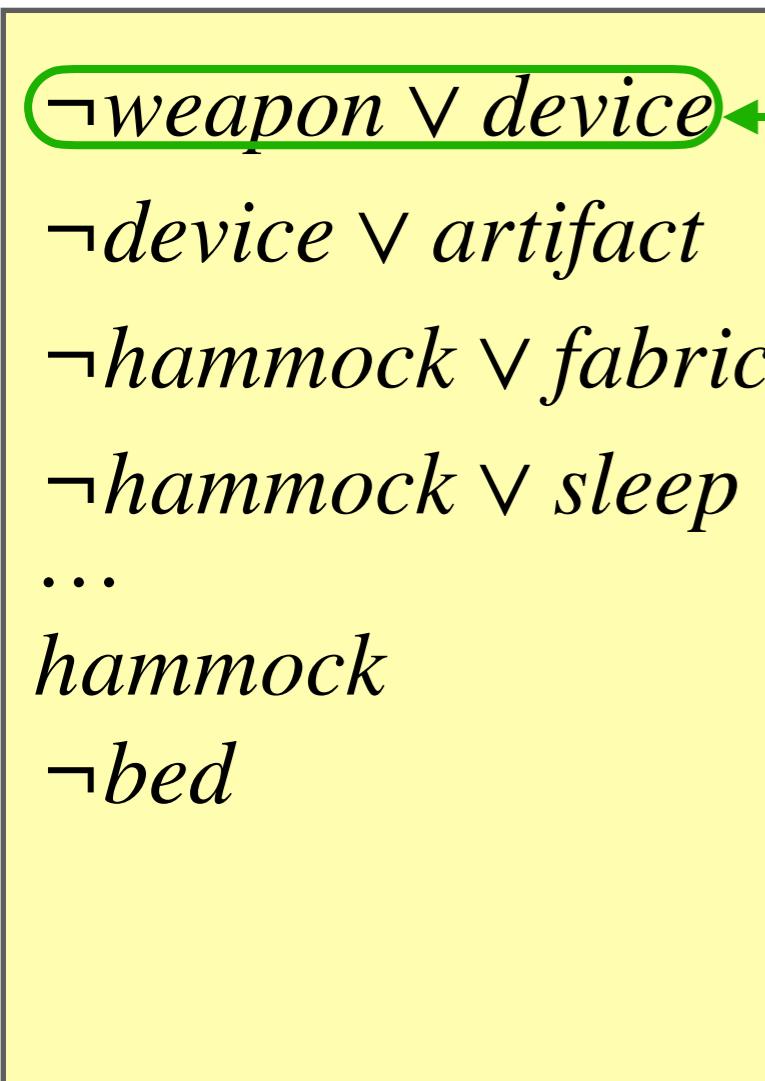
$\neg\text{weapon} \vee \text{device}$   
 $\neg\text{device} \vee \text{artifact}$   
 $\neg\text{hammock} \vee \text{fabric}$   
 $\neg\text{hammock} \vee \text{sleep}$   
...  
 $\text{hammock}$   
 $\neg\text{bed}$

$U$ : set of unprocessed clauses

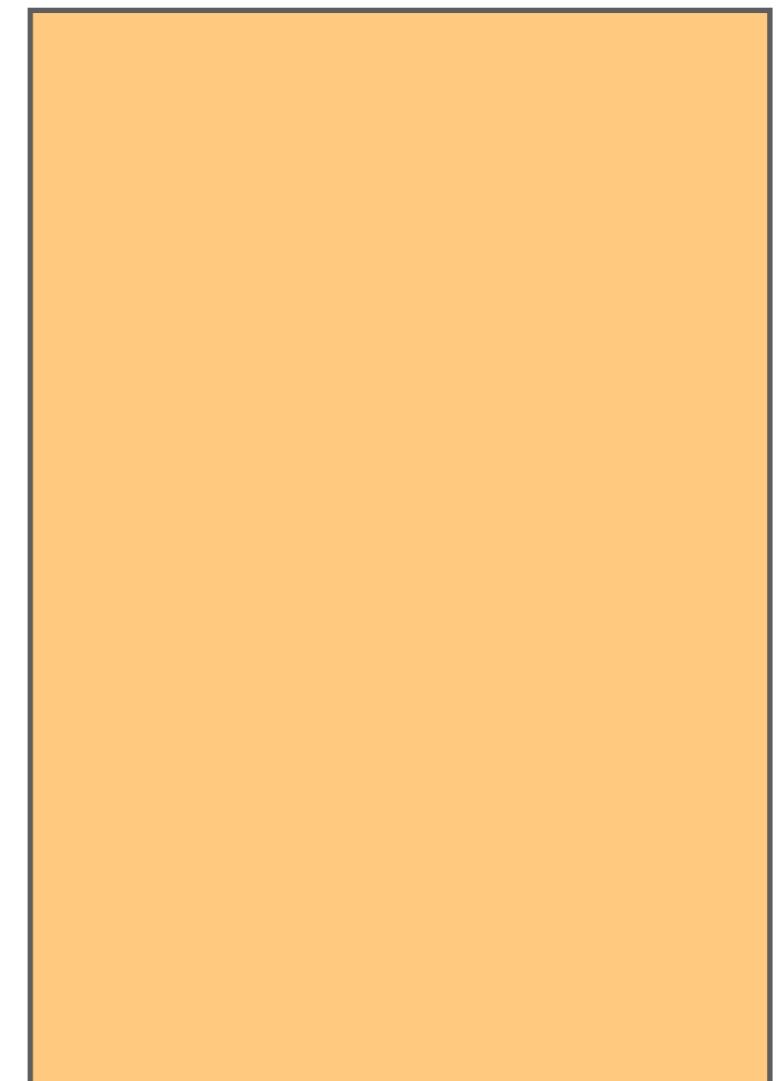
$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $\text{hammock} \rightarrow \text{bed}$  follows from the set of clauses



Select a given-clause



$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

$\neg weapon \vee device$   
**Select a given-clause**

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

$\neg weapon \vee device$   
**Select a given-clause**

**Add all inferences of the given clause and clauses in P to U.**

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

**Select a given-clause**

**Add all inferences of the given clause and clauses in P to U.**

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clauses Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

**Select a given-clause**

**Add all inferences of the given clause and clauses in P to U.**

$\neg weapon \vee device$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

**Select a given-clause**

$\neg weapon \vee device$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
 $\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

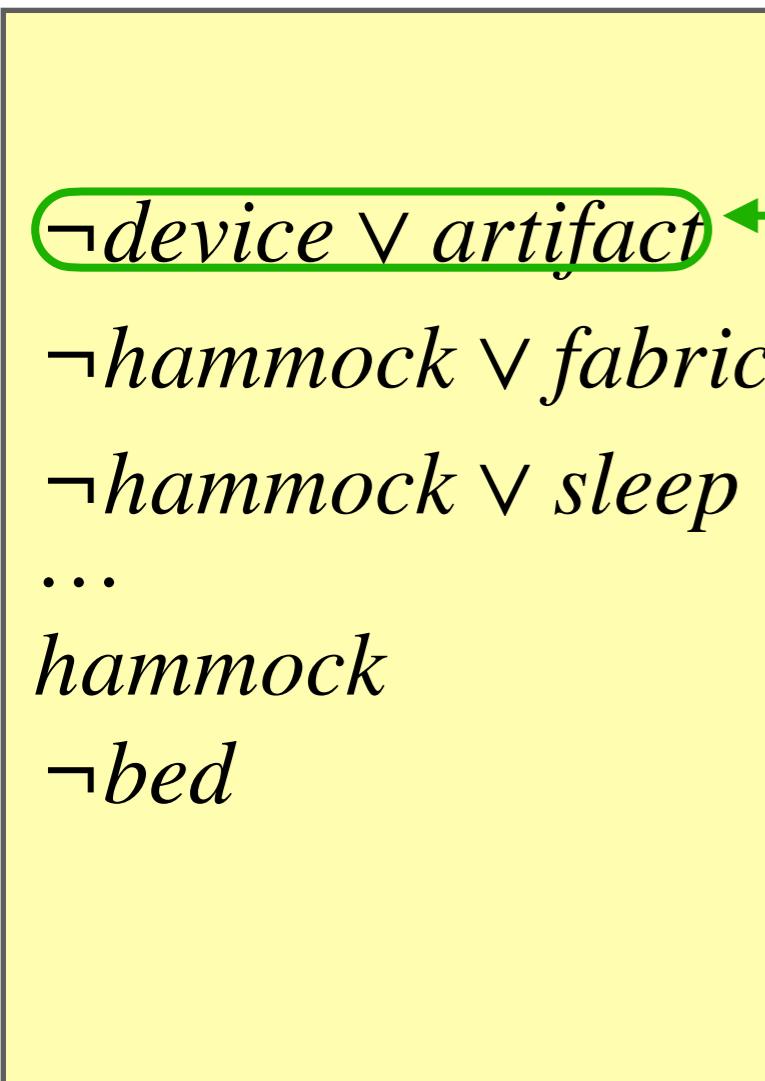
$\neg weapon \vee device$

$U$ : set of unprocessed clauses

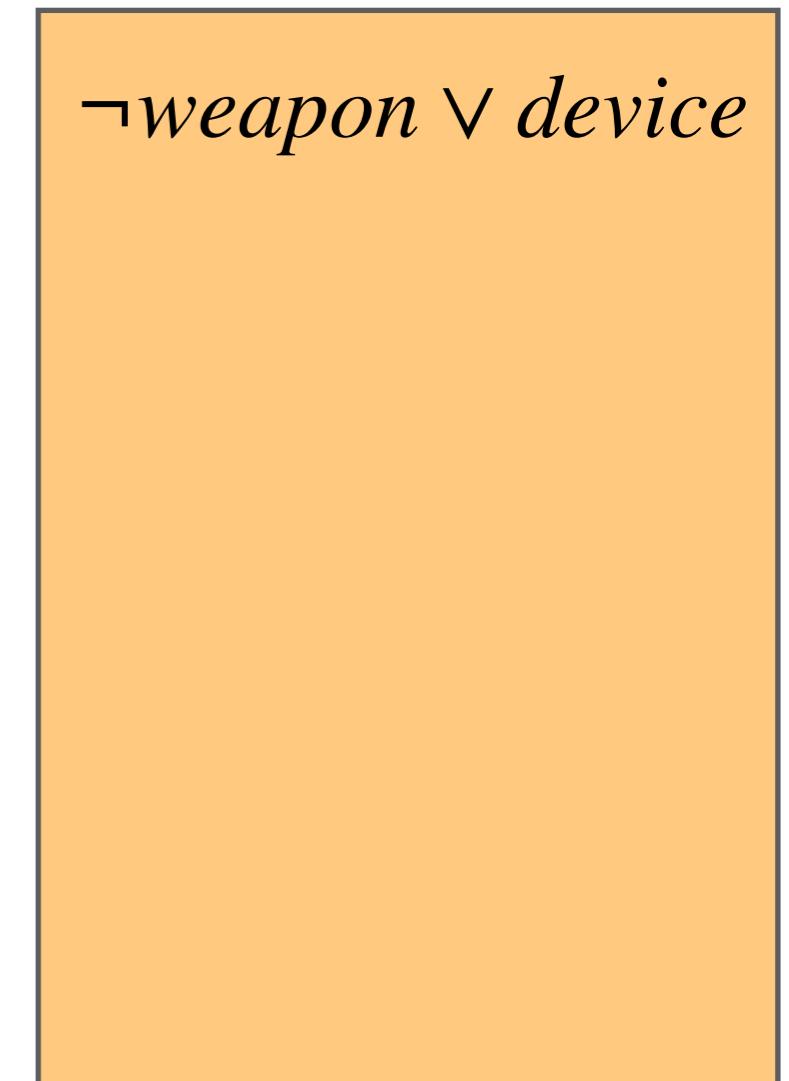
$P$ : set of processed clauses

# Given-Clauses Algorithmus

Task: proof that  $hammock \rightarrow bed$  follows from the set of clauses



Select a given-clause



$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

$\neg device \vee artifact$   
**Select a given-clause**

$\neg weapon \vee device$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

$\neg device \vee artifact$   
**Select a given-clause**

$\neg weapon \vee device$   
 $\neg device \vee artifact$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$

$\neg device \vee artifact$   
**Select a given-clause**

**Add all inferences of the given clause and clauses in P to U.**

$\neg weapon \vee device$   
 $\neg device \vee artifact$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$   
 $\neg weapon \vee artifact$

$\neg device \vee artifact$   
**Select a given-clause**

**Add all inferences of the given clause and clauses in P to U.**

$\neg weapon \vee device$   
 $\neg device \vee artifact$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg hammock \vee fabric$   
 $\neg hammock \vee sleep$   
...  
 $hammock$   
 $\neg bed$   
 $\neg weapon \vee artifact$

$\neg device \vee artifact$   
**Select a given-clause**

$\neg weapon \vee device$   
 $\neg device \vee artifact$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

# Given-Clause Algorithmus

**Task:** proof that  $hammock \rightarrow bed$  follows from the set of clauses

$\neg device \vee artifact$   
**Select a given-clause**

$\neg hammock \vee fabric$

$\neg hammock \vee sleep$

...

$hammock$

$\neg bed$

$\neg weapon \vee artifact$

$\neg weapon \vee device$

$\neg device \vee artifact$

$U$ : set of unprocessed clauses

$P$ : set of processed clauses

**Idea: Use clauses with symbols similar to the proof task as given clause!**

# Symbol Name Heuristics

**Idea: Use clauses with symbols similar to the proof goal as given clause!**

# Symbol Name Heuristics

Idea: Use clauses with symbols similar to the proof goal as given clause!

How to determine similarity?

# Symbol Name Heuristics

Idea: Use clauses with symbols similar to the proof goal as given clause!

How to determine similarity?

We use Word Embeddings!

# Meaning of Symbol Names using Word Embeddings

“a word is characterized by the company it keeps” (Firth, 1957)

Word embedding: a function  $f: Voc \rightarrow \mathbb{R}^n$

# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|}$$

assuming that both  $u$  and  $v$  are non-zero.

# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\sum_{i=1}^n u_i \cdot v_i}{\sqrt{\sum_{i=1}^n (u_i)^2} \cdot \sqrt{\sum_{i=1}^n (v_i)^2}} = \cos(\phi)$$

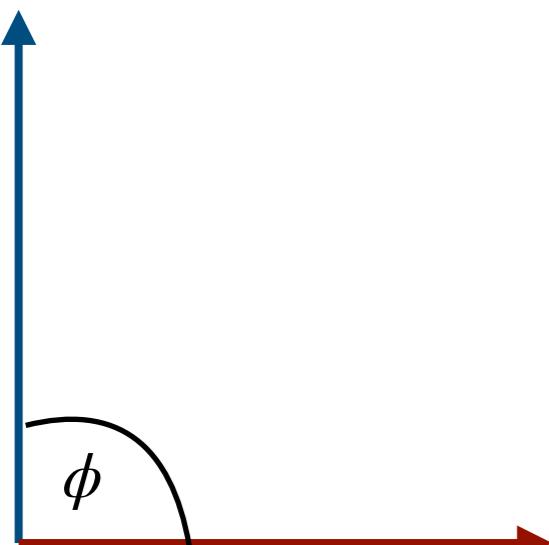
assuming that both  $u$  and  $v$  are non-zero.

# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\sum_{i=1}^n u_i \cdot v_i}{\sqrt{\sum_{i=1}^n (u_i)^2} \cdot \sqrt{\sum_{i=1}^n (v_i)^2}} = \cos(\phi)$$

assuming that both  $u$  and  $v$  are non-zero.



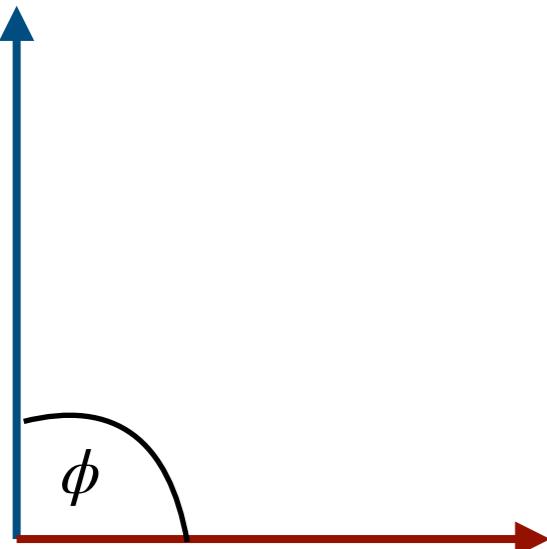
$$\cos(\phi) = 0$$

# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\sum_{i=1}^n u_i \cdot v_i}{\sqrt{\sum_{i=1}^n (u_i)^2} \cdot \sqrt{\sum_{i=1}^n (v_i)^2}} = \cos(\phi)$$

assuming that both  $u$  and  $v$  are non-zero.



$$\cos(\phi) = 0 = \text{cosine\_similarity}(u, v)$$

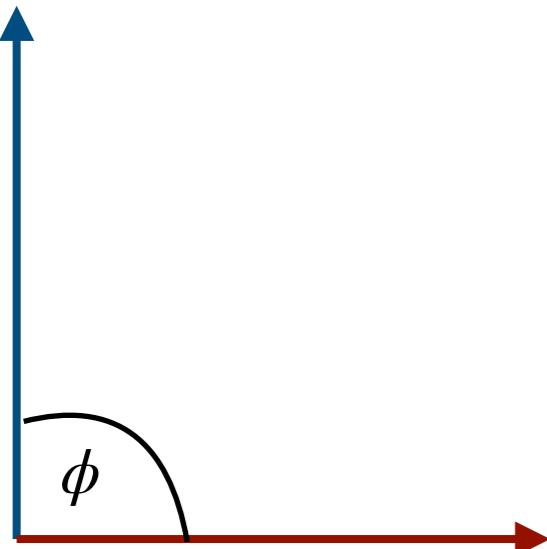
$u$  and  $v$  are very different

# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

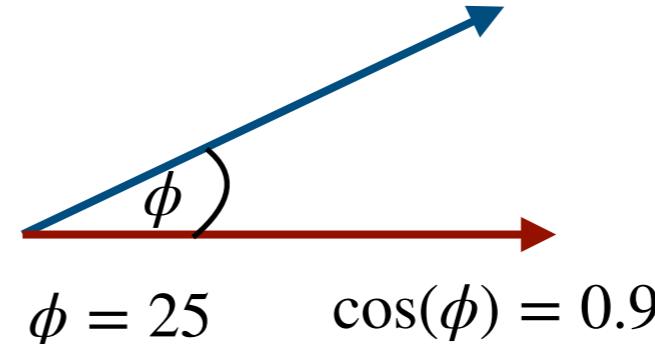
$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\sum_{i=1}^n u_i \cdot v_i}{\sqrt{\sum_{i=1}^n (u_i)^2} \cdot \sqrt{\sum_{i=1}^n (v_i)^2}} = \cos(\phi)$$

assuming that both  $u$  and  $v$  are non-zero.



$$\cos(\phi) = 0 = \text{cosine\_similarity}(u, v)$$

$u$  and  $v$  are very different

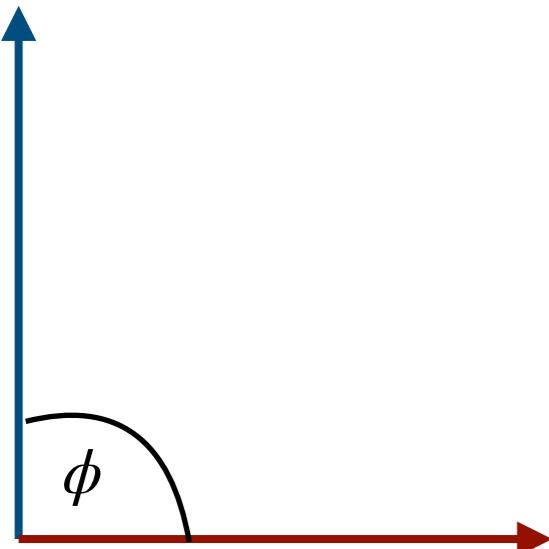


# Measuring Similarities

Let  $u, v \in \mathbb{R}^n$ . The cosine similarity of  $u$  and  $v$  is defined as

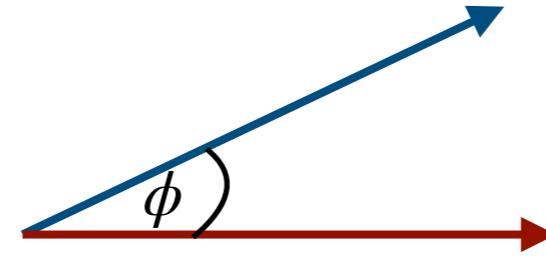
$$\text{cosine\_similarity}(u, v) = \frac{u \cdot v}{\|u\| \|v\|} = \frac{\sum_{i=1}^n u_i \cdot v_i}{\sqrt{\sum_{i=1}^n (u_i)^2} \cdot \sqrt{\sum_{i=1}^n (v_i)^2}} = \cos(\phi)$$

assuming that both  $u$  and  $v$  are non-zero.



$$\cos(\phi) = 0 = \text{cosine\_similarity}(u, v)$$

$u$  and  $v$  are very different



$$\phi = 25 \quad \cos(\phi) = 0.9 = \text{cosine\_similarity}(u, v)$$

$u$  and  $v$  are similar

# Word Embeddings: Similar Words

Comparing words to the word *bed* in the Numberbatch word embedding:

Word	Cosine Similarity to <i>bed</i>
bed	1.0000
pillow	0.6285
sleep	0.6225
blanket	0.4787
vehicle	0.1025
weapon	0.0201

# Word Embeddings: Similar Words

Comparing words to the word *bed* in the Numberbatch word embedding:

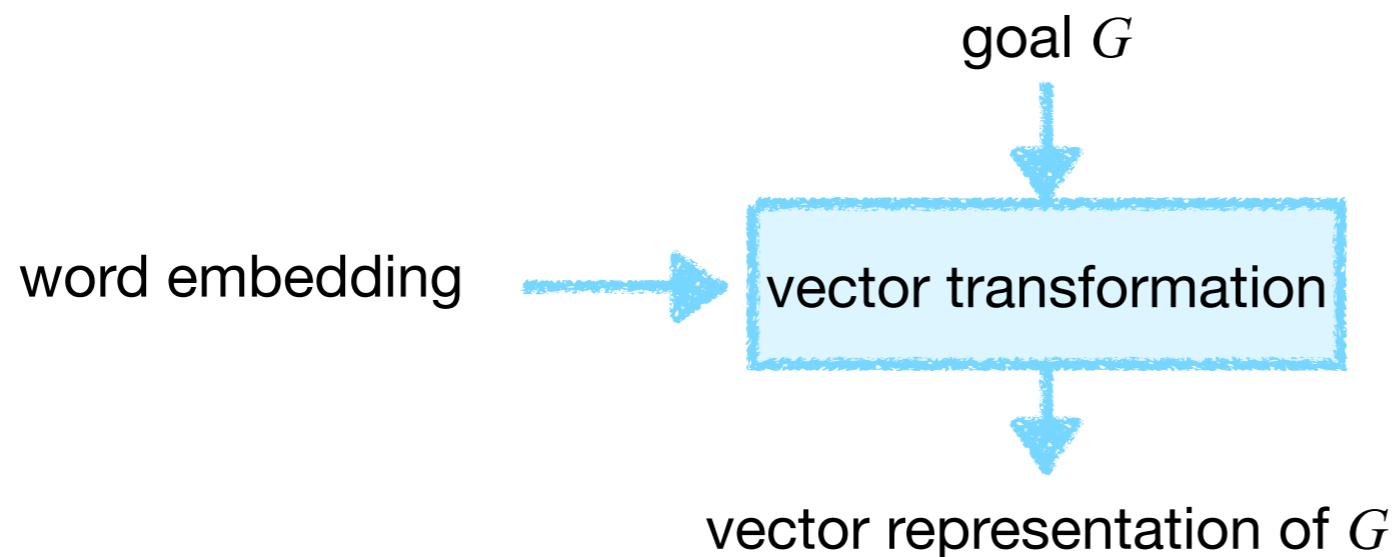
Word	Cosine Similarity to <i>bed</i>
bed	1.0000
pillow	0.6285
sleep	0.6225
blanket	0.4787
vehicle	0.1025
weapon	0.0201

We use these similarities to select the given clause!

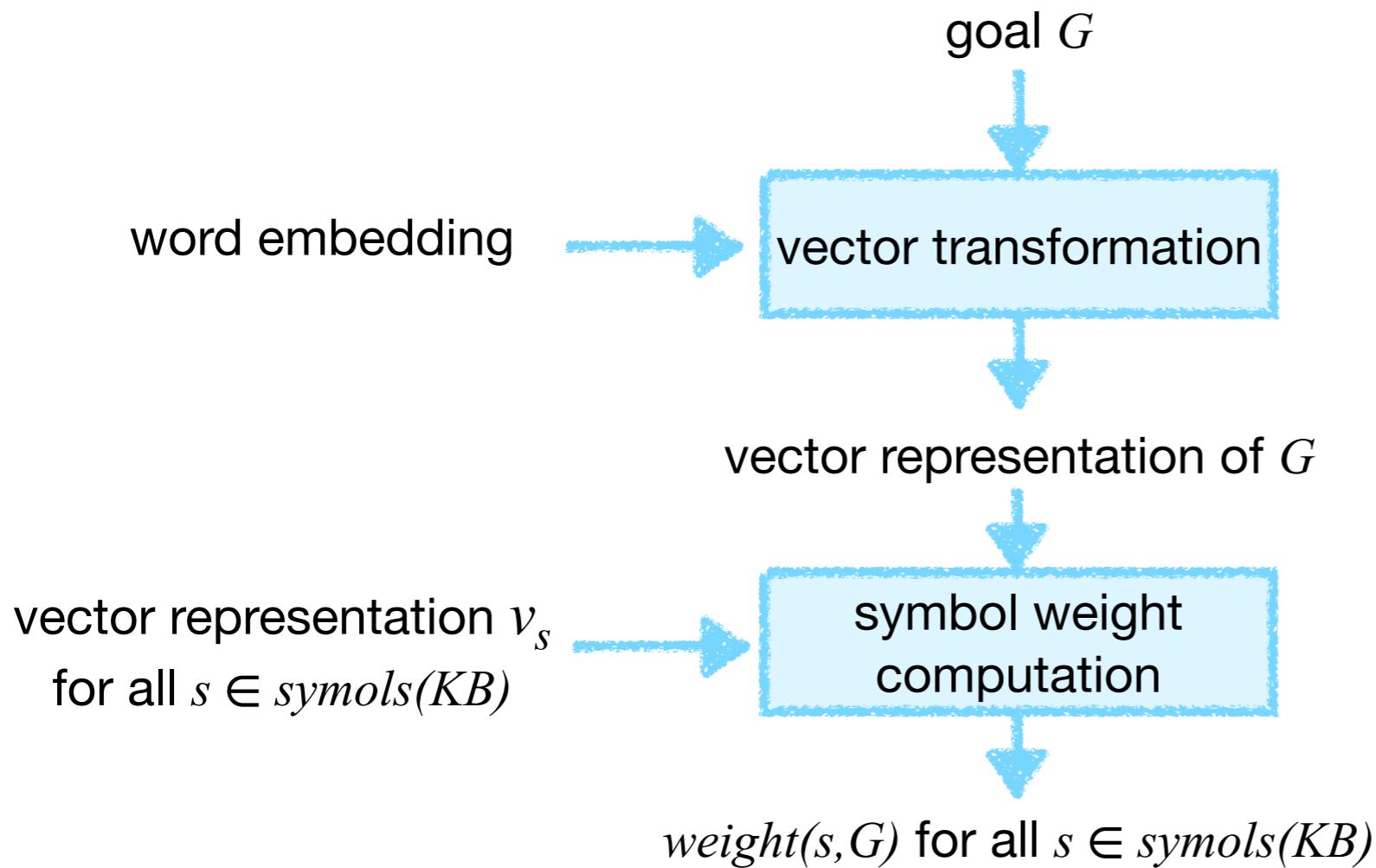
# Symbol Name Heuristic

goal  $G$

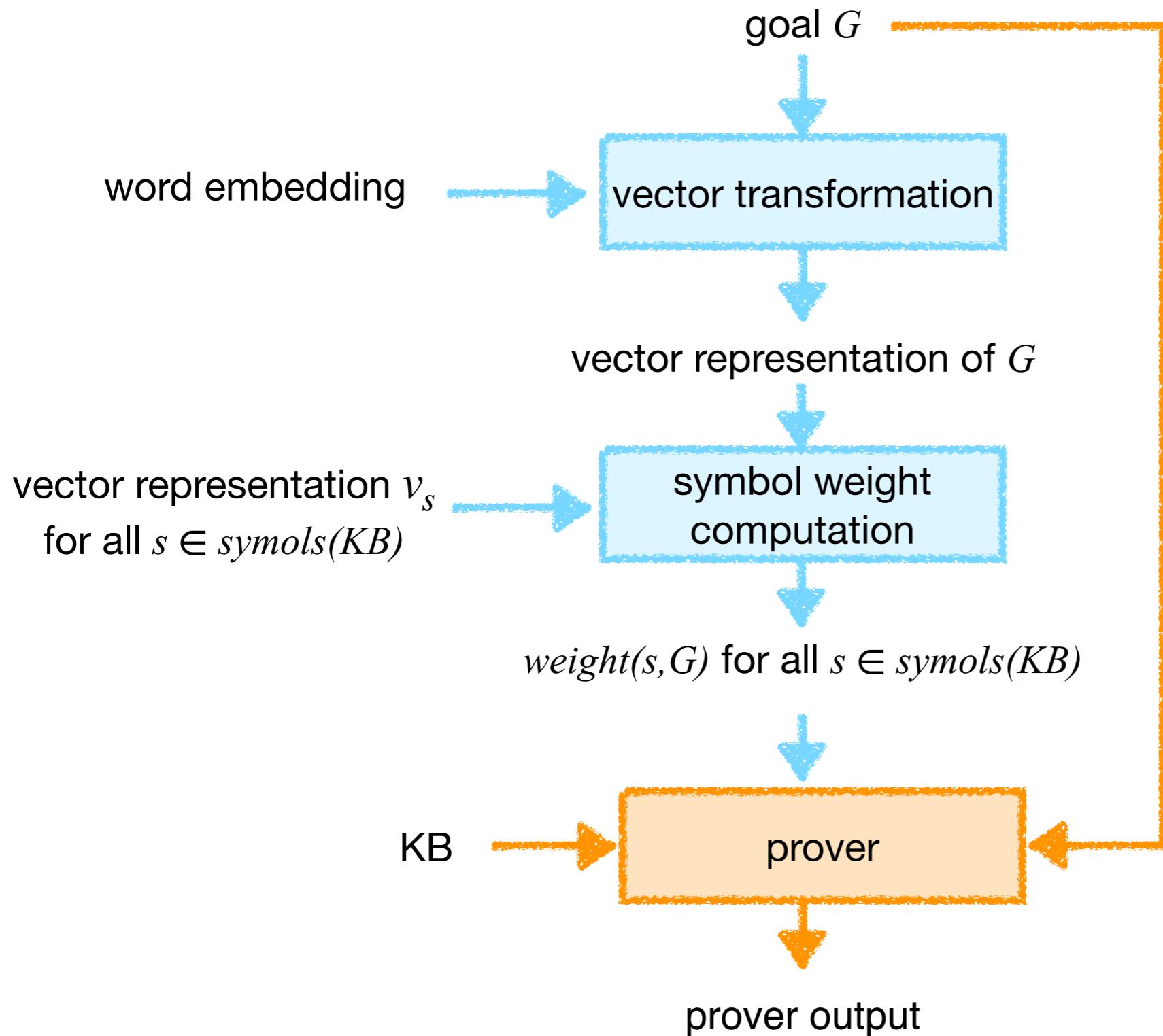
# Symbol Name Heuristic



# Symbol Name Heuristic



# Symbol Name Heuristic



# Representing Symbols and Formulae by Vectors

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

# Representing Symbols and Formulae by Vectors

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

$f : \text{Voc} \rightarrow \mathbb{R}^n$  a word embedding with vocabulary  $\text{Voc}$

# Representing Symbols and Formulae by Vectors

$symbols(F)$ : the set of predicate and function symbols in formula  $F$

$f : Voc \rightarrow \mathbb{R}^n$  a word embedding with vocabulary  $Voc$

assume  $symbols(F) \subseteq Voc$

# Representing Symbols and Formulae by Vectors

$symbols(F)$ : the set of predicate and function symbols in formula  $F$

$f : Voc \rightarrow \mathbb{R}^n$  a word embedding with vocabulary  $Voc$

assume  $symbols(F) \subseteq Voc$

The vector representation of a symbol  $s$  is

$$v_s = f(s)$$

# Representing Symbols and Formulae by Vectors

$symbols(F)$ : the set of predicate and function symbols in formula  $F$

$f: Voc \rightarrow \mathbb{R}^n$  a word embedding with vocabulary  $Voc$

assume  $symbols(F) \subseteq Voc$

The vector representation of a symbol  $s$  is

$$v_s = f(s)$$

The vector representation of a formula  $F$  is

$$v_F = \frac{\sum_{s \in symbols(F)} v_s}{|symbols(F)|}$$

# Symbol Weight Computation

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

$\text{symbols}(KB)$ : the set of predicate and function symbols occurring in  $KB$

# Symbol Weight Computation

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

$\text{symbols}(KB)$ : the set of predicate and function symbols occurring in  $KB$

$v_G$  the vector representation of goal  $G$

$v_s$  the vector representation of symbol  $s$

# Symbol Weight Computation

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

$\text{symbols}(KB)$ : the set of predicate and function symbols occurring in  $KB$

$v_G$  the vector representation of goal  $G$

$v_s$  the vector representation of symbol  $s$

The weight of symbol  $s$  w.r.t goal  $G$  is defined as:

$$\text{weight}(s, G) = 1000 - 1000 \cdot \text{cosine\_similarity}(v_s, v_G)$$

# Symbol Weight Computation

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

$\text{symbols}(KB)$ : the set of predicate and function symbols occurring in  $KB$

$v_G$  the vector representation of goal  $G$

$v_s$  the vector representation of symbol  $s$

The weight of symbol  $s$  w.r.t goal  $G$  is defined as:

$$\text{weight}(s, G) = 1000 - 1000 \cdot \text{cosine\_similarity}(v_s, v_G)$$

**Provers use clauses with  
symbols with a low weight first.**

# Symbol Weight Computation

$\text{symbols}(F)$ : the set of predicate and function symbols in formula  $F$

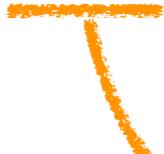
$\text{symbols}(KB)$ : the set of predicate and function symbols occurring in  $KB$

$v_G$  the vector representation of goal  $G$

$v_s$  the vector representation of symbol  $s$

The weight of symbol  $s$  w.r.t goal  $G$  is defined as:

$$\text{weight}(s, G) = \underline{1000} - 1000 \cdot \text{cosine\_similarity}(v_s, v_G)$$



**Provers use clauses with symbols with a low weight first.**

**We use default weights of 1000 for all symbols.**

# **Selection of Given-Clause**

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*

↑  
0.1232

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*

↑  
0.1232 0.0422

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*

↑      ↑      ↑  
0.1232 0.0422 0.1933

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

0.1232 0.0422 0.1933

*subClass(bed, furniture)*

0.1232

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

↑  
0.1232 0.0422 0.1933

*subClass(bed, furniture)*

↑  
0.1232 0.7261

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

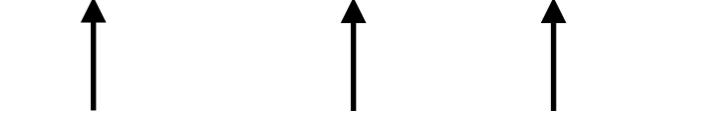
Clauses:

*subClass(weapon, device)*



0.1232 0.0422 0.1933

*subClass(bed, furniture)*



0.1232 0.7261 0.3323

Cosine similarity to the goal:

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

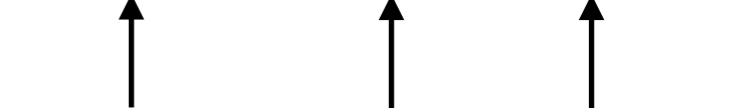
Clauses:

*subClass(weapon, device)*



0.1232 0.0422 0.1933

*subClass(bed, furniture)*



0.1232 0.7261 0.3323

Symbol weight:

# Selection of Given-Clause

Goal:

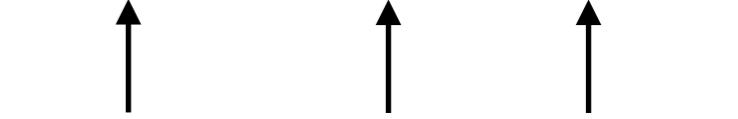
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*

*subClass(bed, furniture)*



Cosine similarity to the goal:

0.1232 0.0422 0.1933

0.1232 0.7261 0.3323

Symbol weight:

$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

*subClass(weapon, device)*  
0.1232 0.0422 0.1933  
876.8

*subClass(bed, furniture)*  
0.1232 0.7261 0.3323

Cosine similarity to the goal:

Symbol weight:

$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

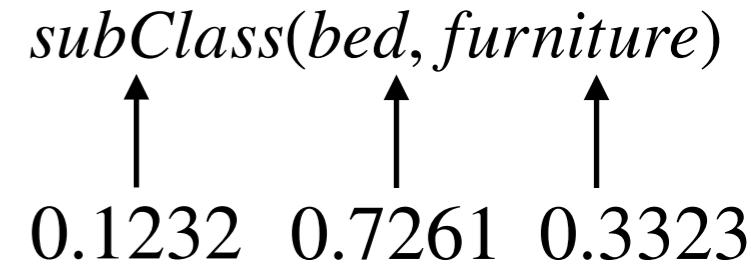
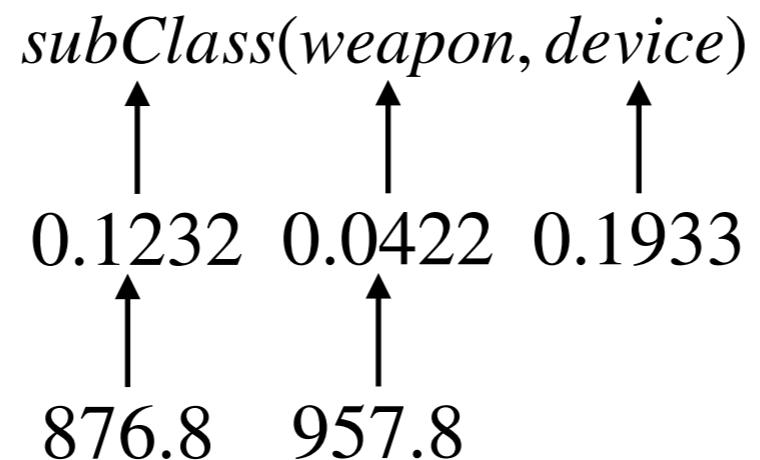
# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:



Cosine similarity to the goal:

Symbol weight:

$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

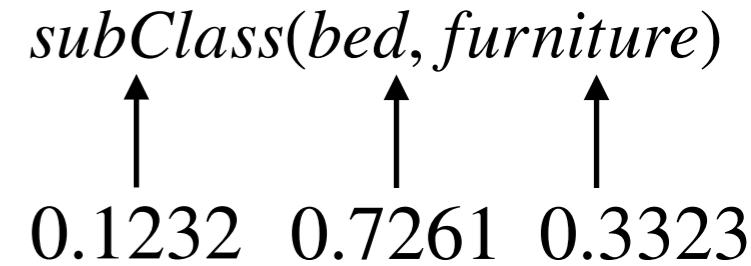
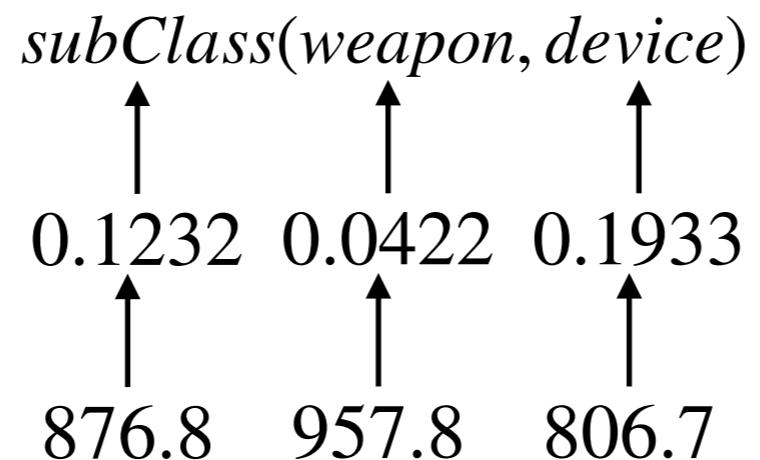
# Selection of Given-Clause

Goal:

Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:



Cosine similarity to the goal:

Symbol weight:

$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

# Selection of Given-Clause

Goal:

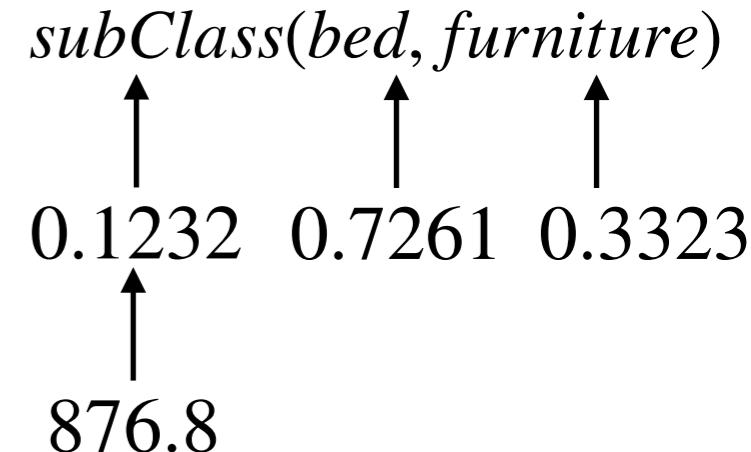
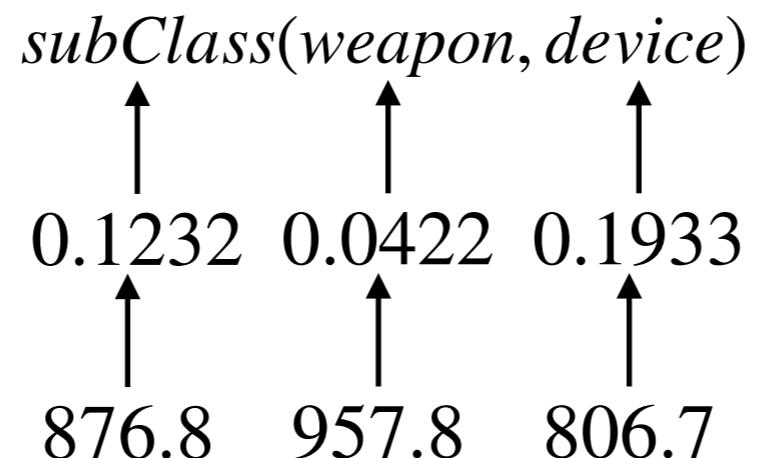
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

Cosine similarity to the goal:

Symbol weight:



$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

# Selection of Given-Clause

Goal:

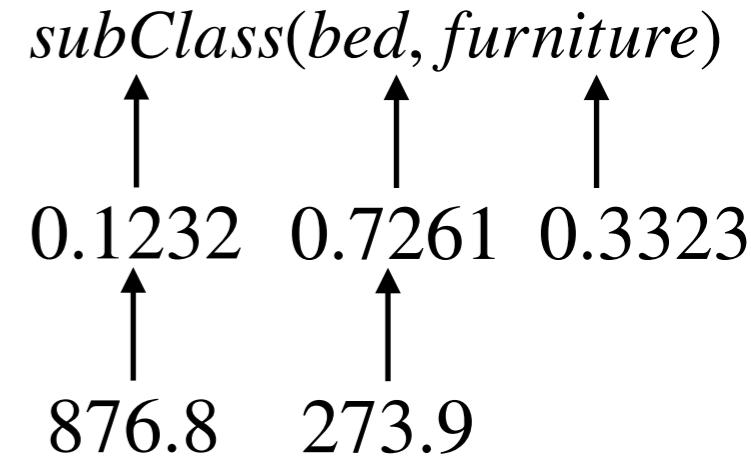
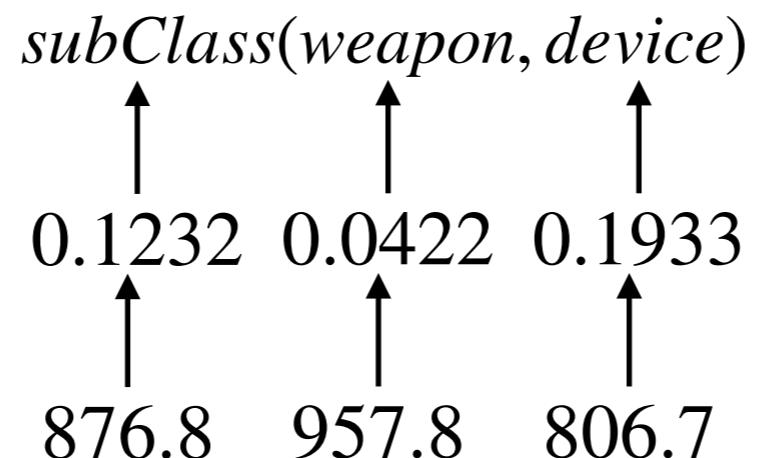
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

Cosine similarity to the goal:

Symbol weight:



$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

# Selection of Given-Clause

Goal:

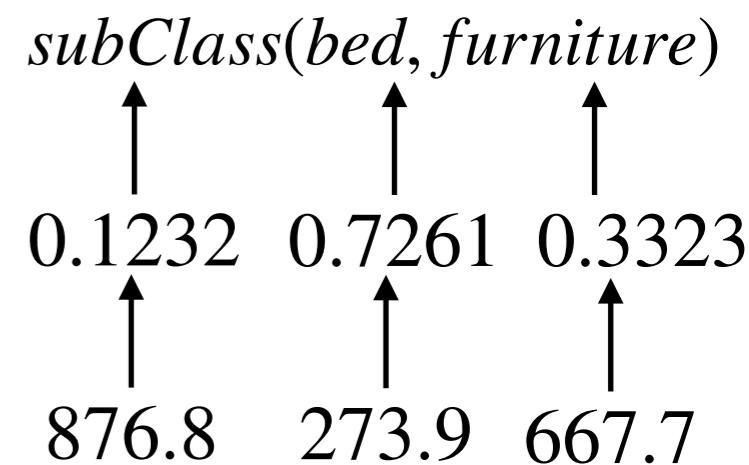
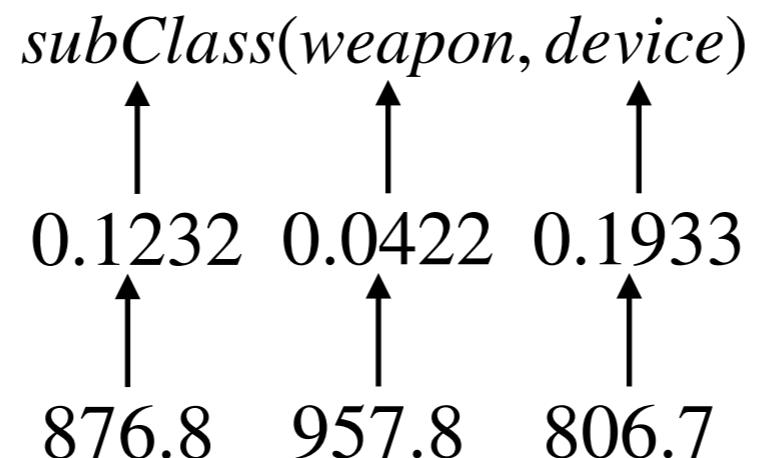
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

Cosine similarity to the goal:

Symbol weight:



$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{symbol}, v_{goal})$$

# Selection of Given-Clause

Goal:

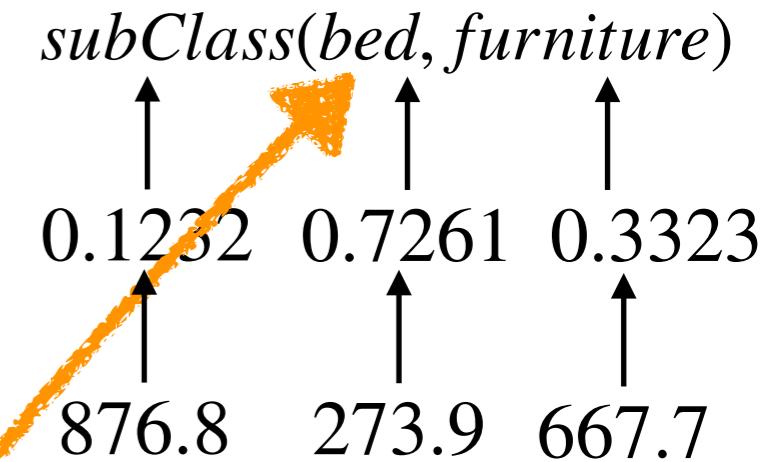
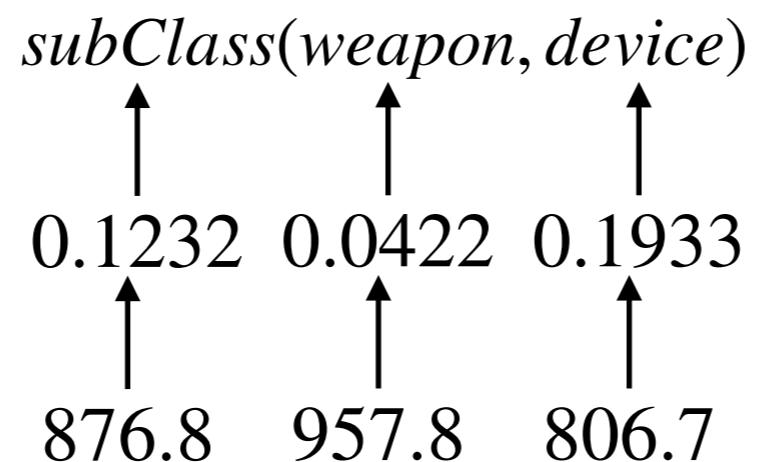
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

Cosine similarity to the goal:

Symbol weight:



$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{\text{symbol}}, v_{\text{goal}})$$

Given clause = clause with smallest similarity-clause-weight

# Selection of Given-Clause

Goal:

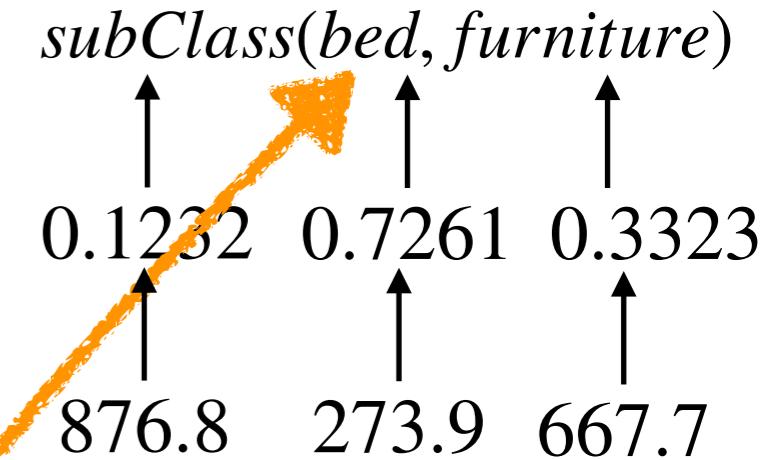
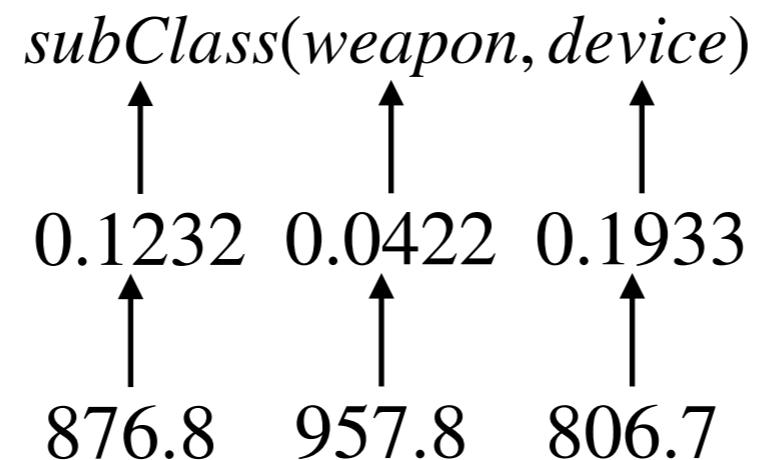
Are hammocks beds?

$$\forall x(\text{instance}(x, \text{hammock}) \rightarrow \text{instance}(x, \text{bed}))$$

Clauses:

Cosine similarity to the goal:

Symbol weight:

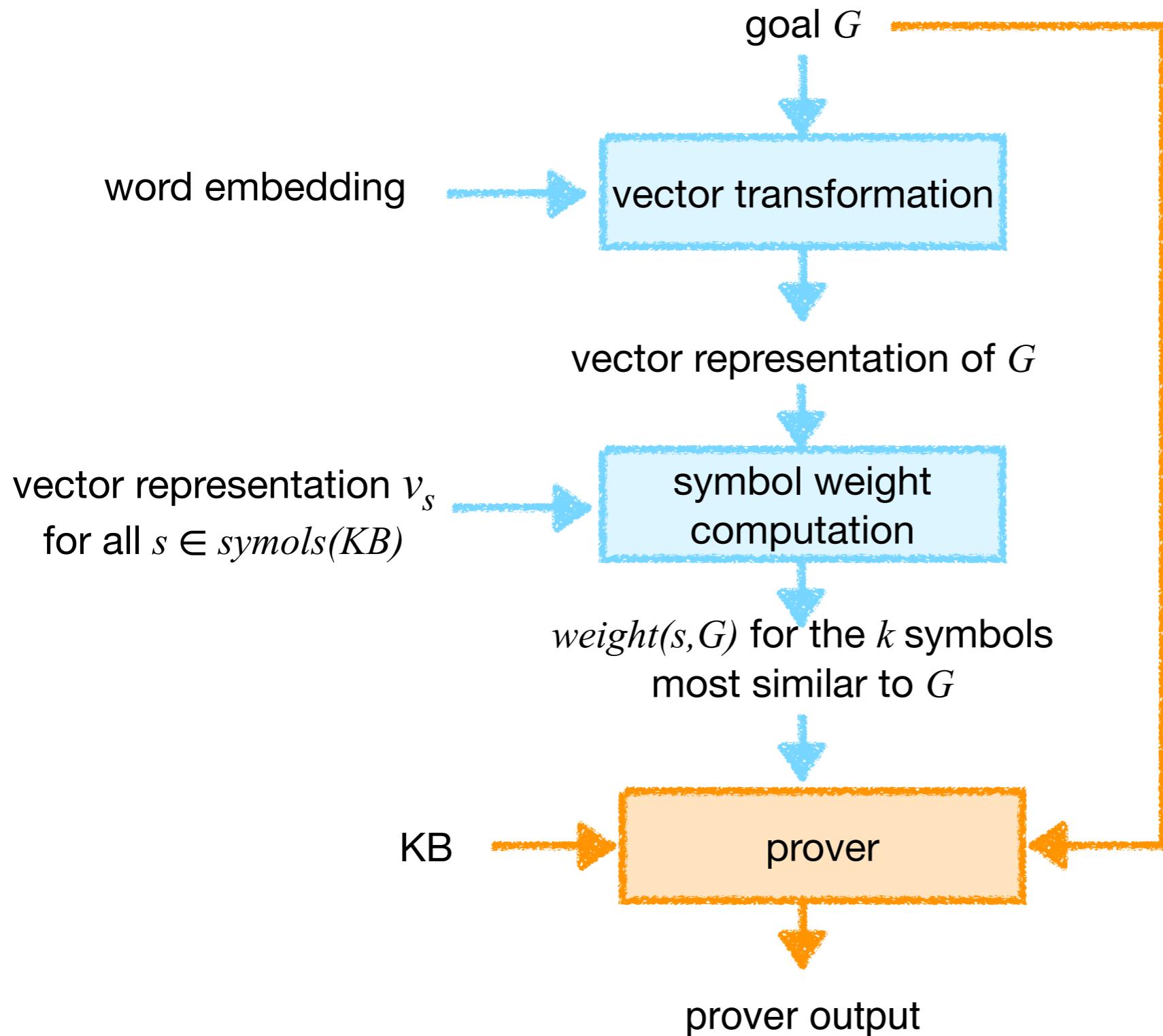


$$1000 - 1000 \cdot \text{cosine\_similarity}(v_{\text{symbol}}, v_{\text{goal}})$$

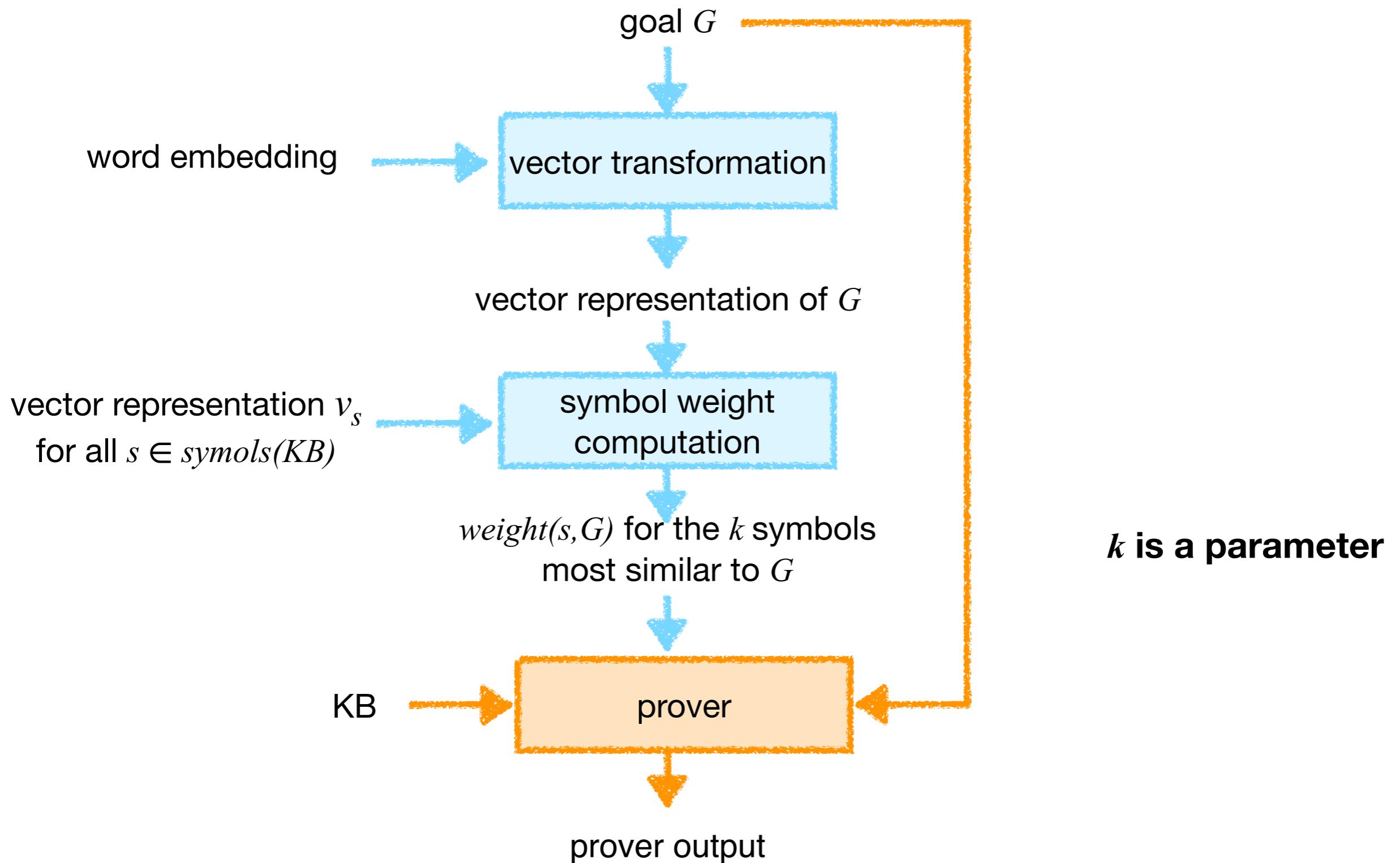
Given clause = clause with smallest similarity-clause-weight

We interleave the selection of the given clause by similarity-clause-weight with the selection of the oldest clause.

# Symbol Name Heuristic



# Symbol Name Heuristic



# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:

# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:
  - Collection of automatically generated tests

# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:
  - Collection of automatically generated tests
  - Each task: Goal for which it must be proved that it follows from the Adimen SUMO ontology

# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:
  - Collection of automatically generated tests
  - Each task: Goal for which it must be proved that it follows from the Adimen SUMO ontology
  - We randomly chose two disjoint sets of 1,000 of these proof tasks

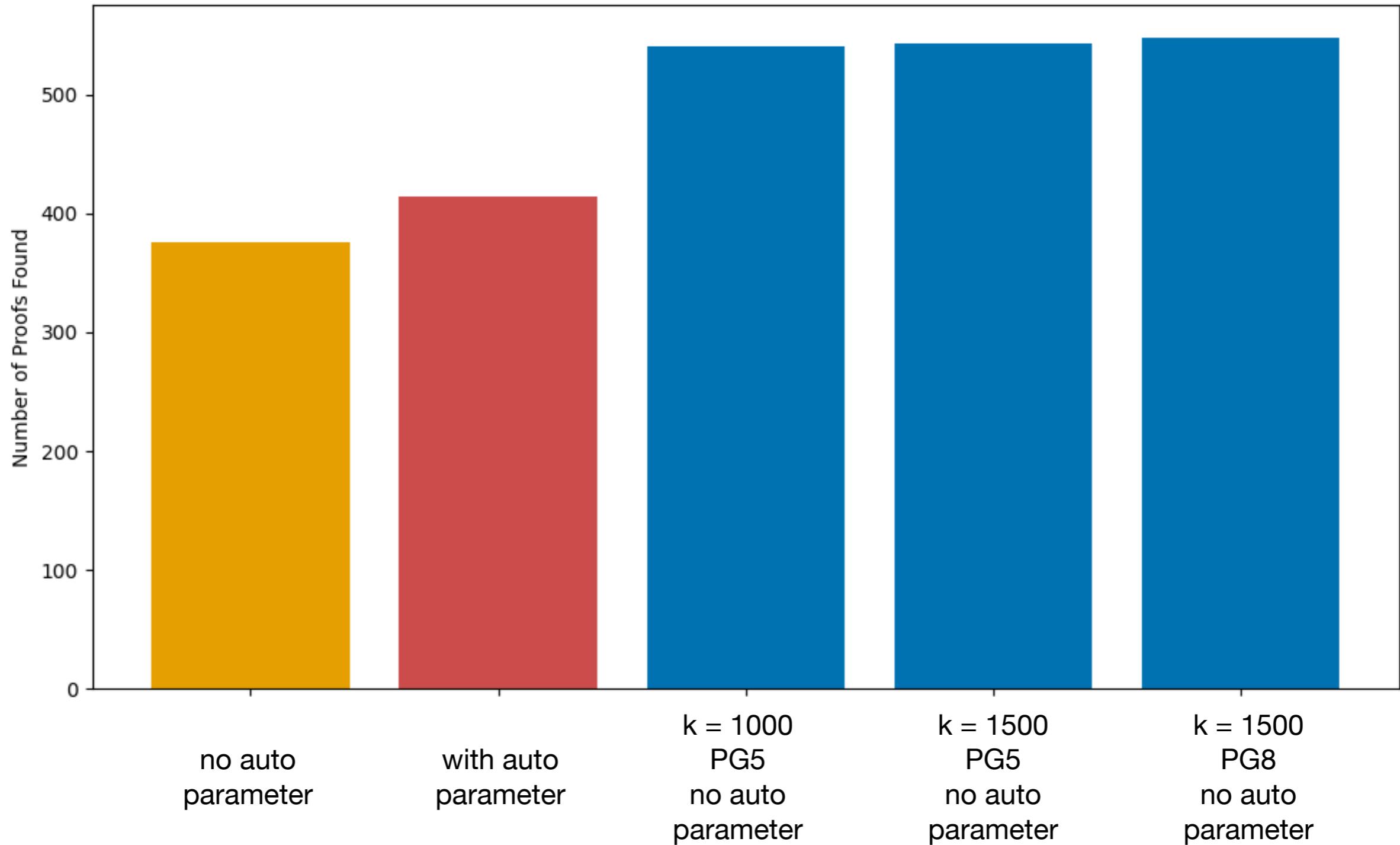
# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:
  - Collection of automatically generated tests
  - Each task: Goal for which it must be proved that it follows from the Adimen SUMO ontology
  - We randomly chose two disjoint sets of 1,000 of these proof tasks
    - First set: parameter-tuning set to determine interesting values for  $k$  and the pick given ratio

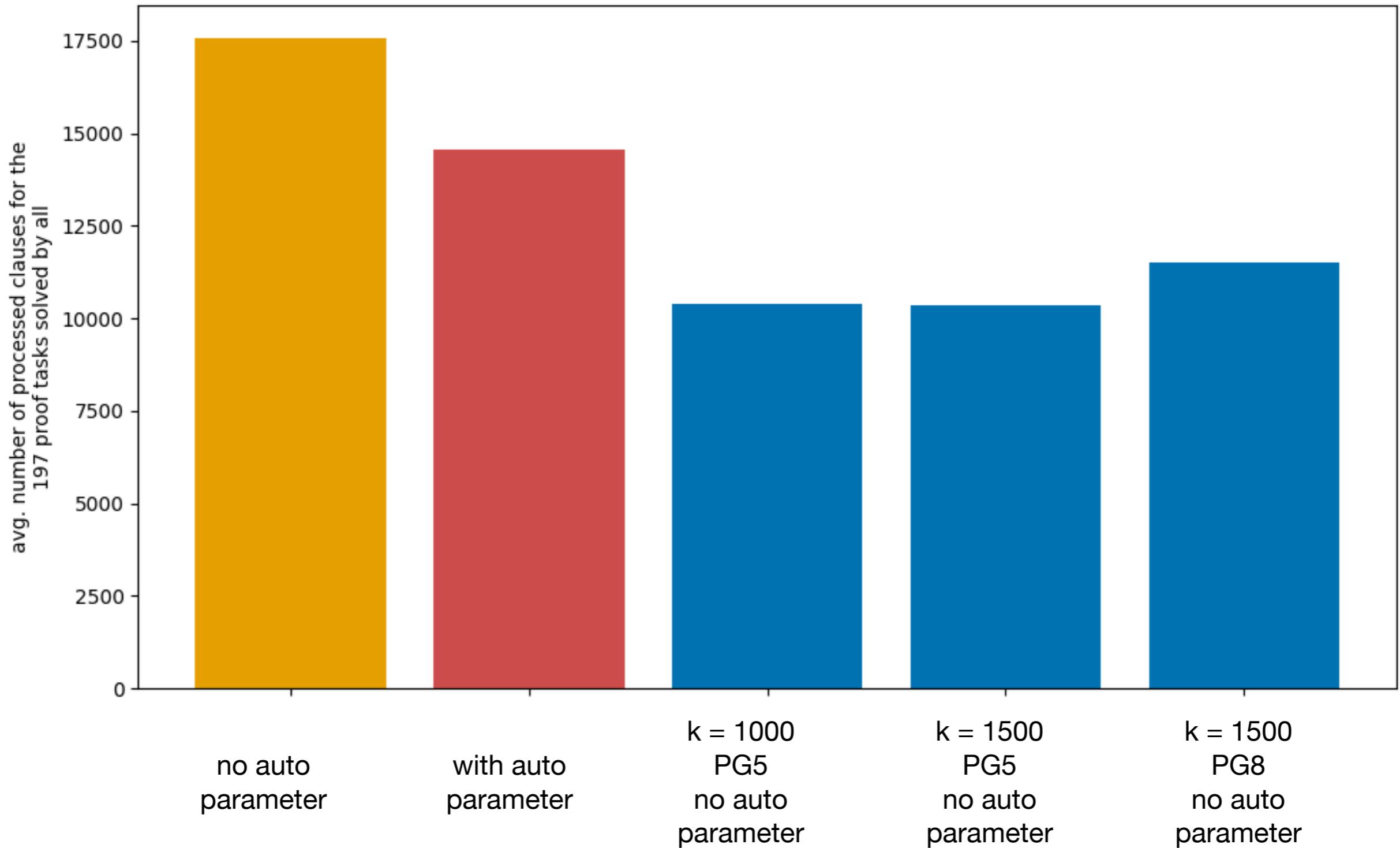
# Experimental Results

- Adimen SUMO WhiteBoxTruthTests:
  - Collection of automatically generated tests
  - Each task: Goal for which it must be proved that it follows from the Adimen SUMO ontology
  - We randomly chose two disjoint sets of 1,000 of these proof tasks
    - First set: parameter-tuning set to determine interesting values for  $k$  and the pick given ratio
    - Second set: evaluation set
- We use the theorem prover E (timeout 10 sec.)

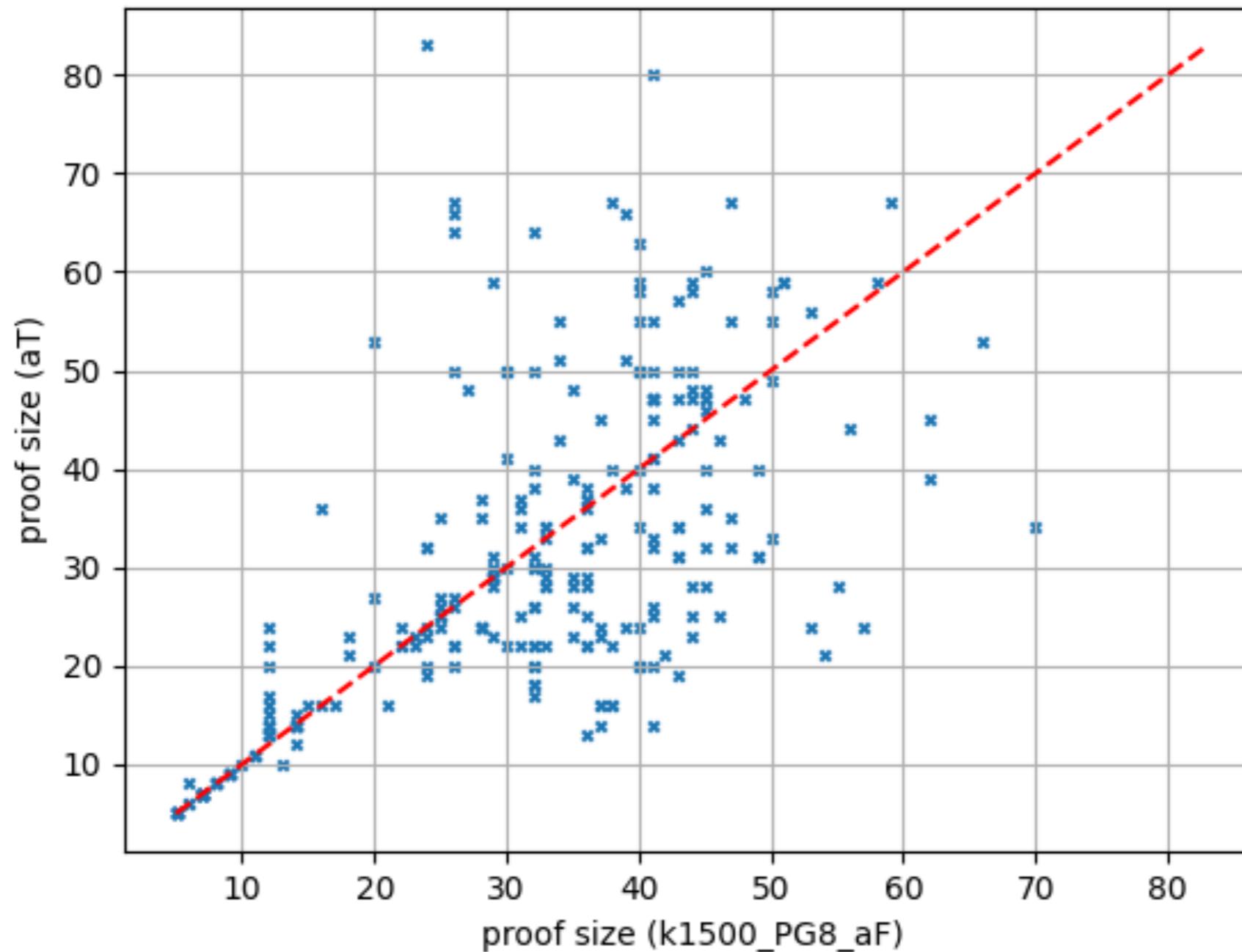
# Evaluation Set: Number of Proofs Found



# Evaluation Set: Average Number of Clauses Processed



# Evaluation Set: Comparison of Proof Sizes



# Conclusion and Future Work

**Symbol names carry important information which:**

- can be used to make the proof process more goal directed and

# Conclusion and Future Work

**Symbol names carry important information which:**

- can be used to make the proof process more goal directed and
- helps to find more proofs.

# Conclusion and Future Work

## **Symbol names carry important information which:**

- can be used to make the proof process more goal directed and
- helps to find more proofs.

## **Ongoing and Future work:**

- Investigate the difference of proofs found with or without the guidance by symbol names.
- Use the approach in other domains.

# Conclusion and Future Work

## Symbol names carry important information which:

- can be used to make the proof process more goal directed and
- helps to find more proofs.

## Ongoing and Future work:

- Investigate the difference of proofs found with or without the guidance by symbol names.
- Use the approach in other domains.

