

Christian Alrabbaa<sup>1</sup> Franz Baader<sup>1</sup> Raimund Dachselt<sup>2</sup> Alisa Kovtunova<sup>1</sup> Julián Méndez<sup>2</sup>

<sup>1</sup> Chair of Automata Theory, TU Dresden

<sup>2</sup> Interactive Media Lab Dresden, TU Dresden

# The Concrete Evonne: Visualization Meets Concrete Domain Reasoning

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# Introduction

Description Logic  $EL_{\perp}$

- Atomic Concepts:  $DeliveryDrone, ChargingStation$
- Complex Concepts:  $Component \sqcap \exists locatedOn.LandingGear$
- Axioms:  $LargeBatteryDrone \sqsubseteq Drone \sqcap \exists hasPart.LargeBattery$

# Introduction

Concrete Domain  $D_{\mathbb{Q}_{diff}}$

- $batteryPercentage_1 = 0.75$
- $batteryPercentage_2 > 0.3$
- $batteryPercentage_1 - 0.2 = batteryPercentage_2$

# Introduction

Concrete Domain  $D_{\mathbb{Q}_{diff}}$

- $batteryPercentage_1 = 0.75$
- $batteryPercentage_2 > 0.3$
- $batteryPercentage_1 - 0.2 = batteryPercentage_2$

Concrete Domain  $D_{\mathbb{Q}_{lin}}$

- $4 * normalDischargeRate + highDischargeRate = 30$
- $8 * normalDischargeRate = 30$

# Introduction

Description Logics  $EL_{\perp}[D_{\mathbb{Q}, diff}]$  &  $EL_{\perp}[D_{\mathbb{Q}, lin}]$

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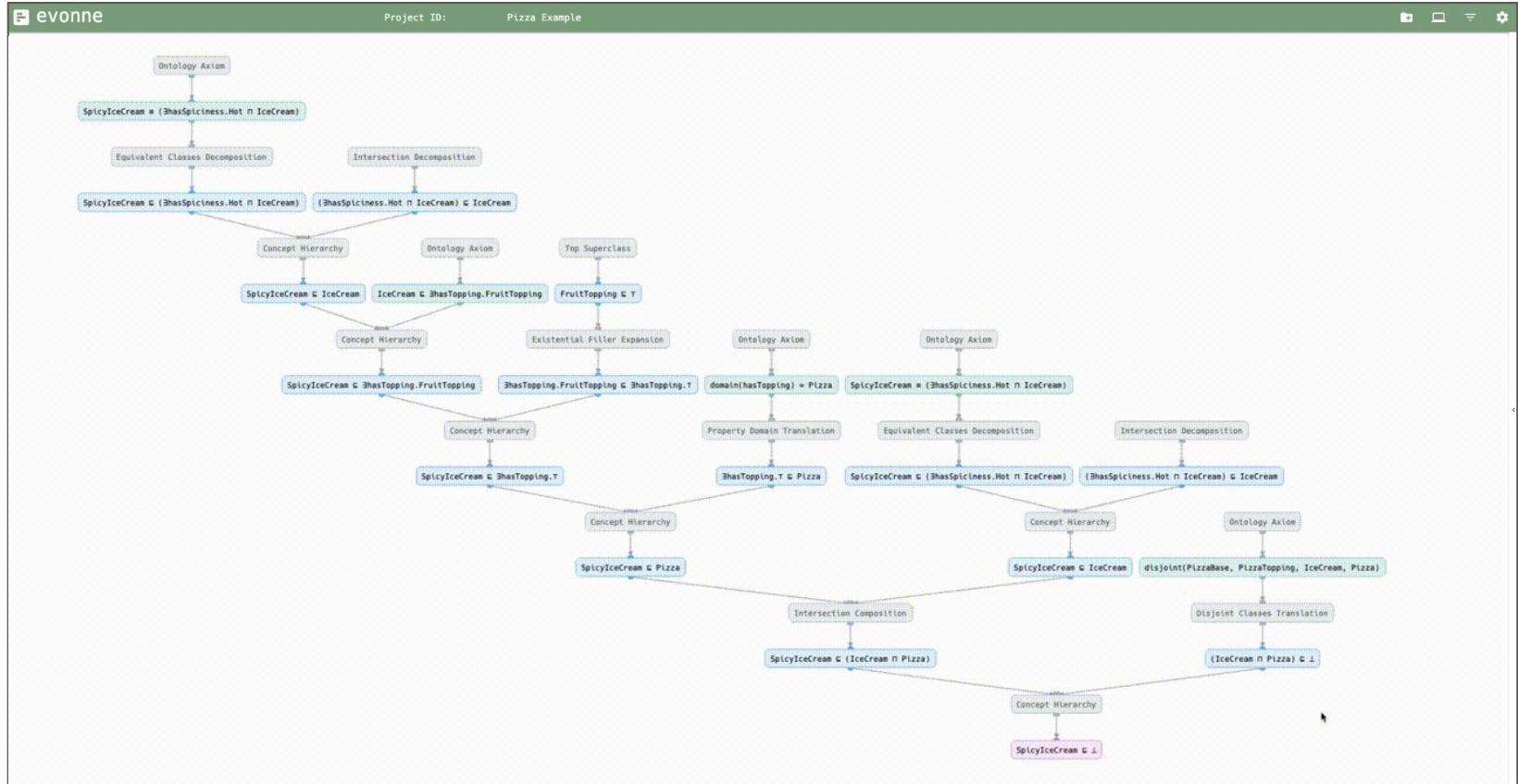
- $\exists \text{hasPart}(\text{Battery} \sqcap [\text{batteryPercentage} > 0.25])$

# Introduction

Description Logics  $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$  &  $EL_{\perp}[D_{\mathbb{Q}_{lin}}$  ]

- $\exists \text{hasPart}(\text{Battery} \sqcap [ \text{batteryPercentage} > 0.25 ])$
- $\text{Drone} \sqcap [ 2 * \text{highDischargeRate} = 30 ] \sqsubseteq \text{LargeBatteryDrone}$

# Evonne



# $EL_{\perp}[D_{\mathbb{Q},lin}]$ Explanations

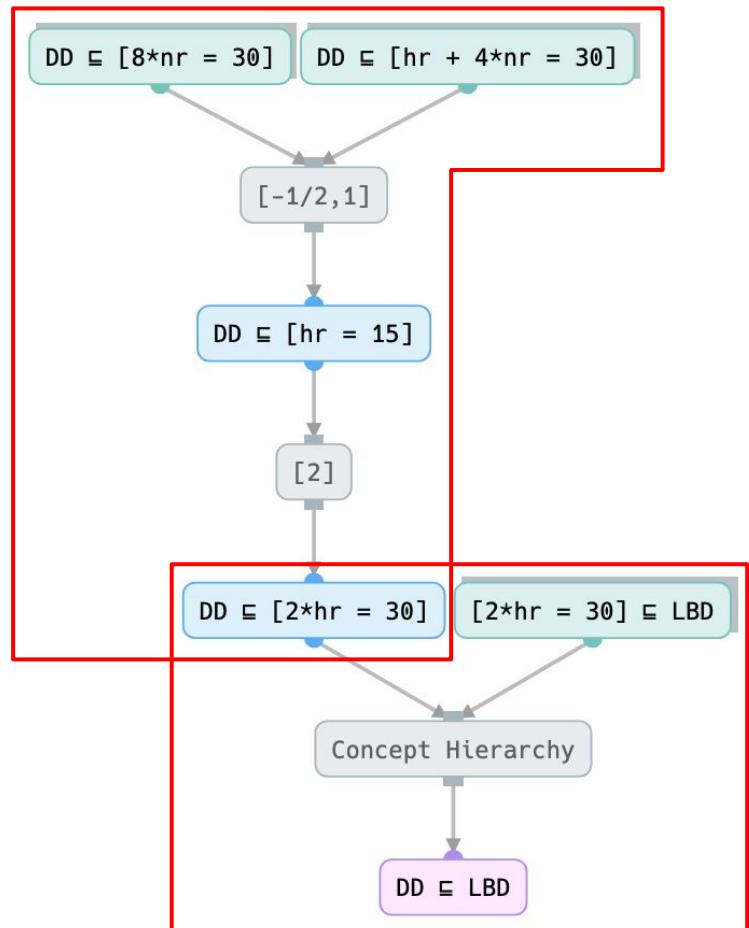
$$DD \sqsubseteq [8 * nr = 30]$$

$$DD \sqsubseteq [hr + 4 * nr = 30]$$

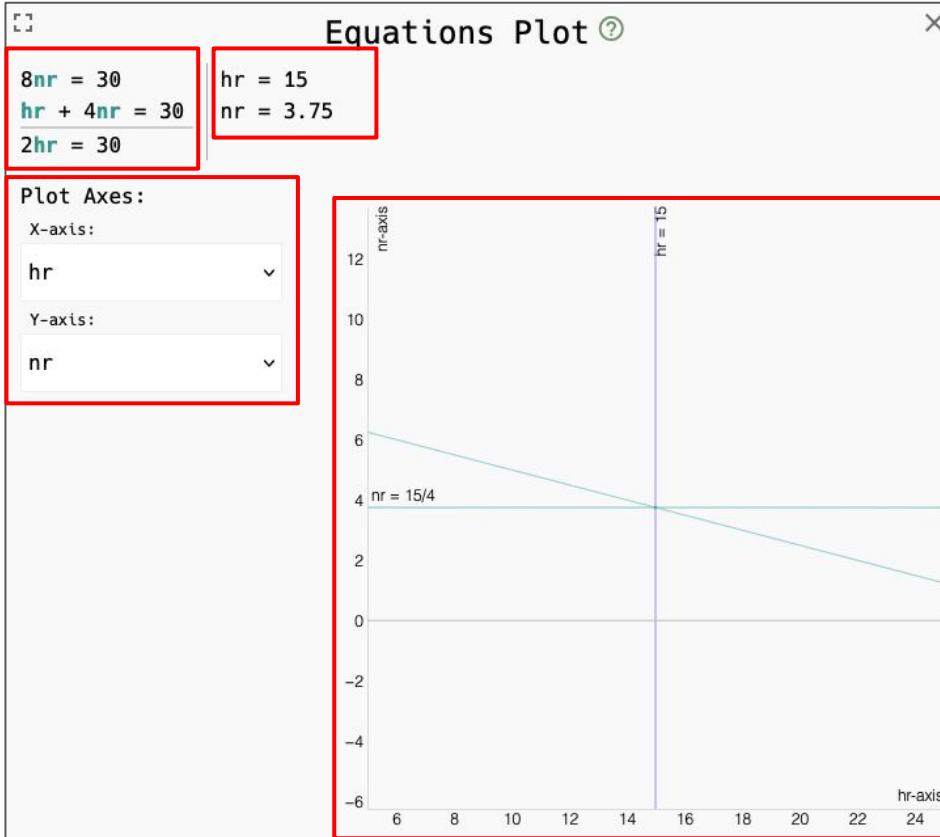
$$[2 * hr = 30] \sqsubseteq LBD$$

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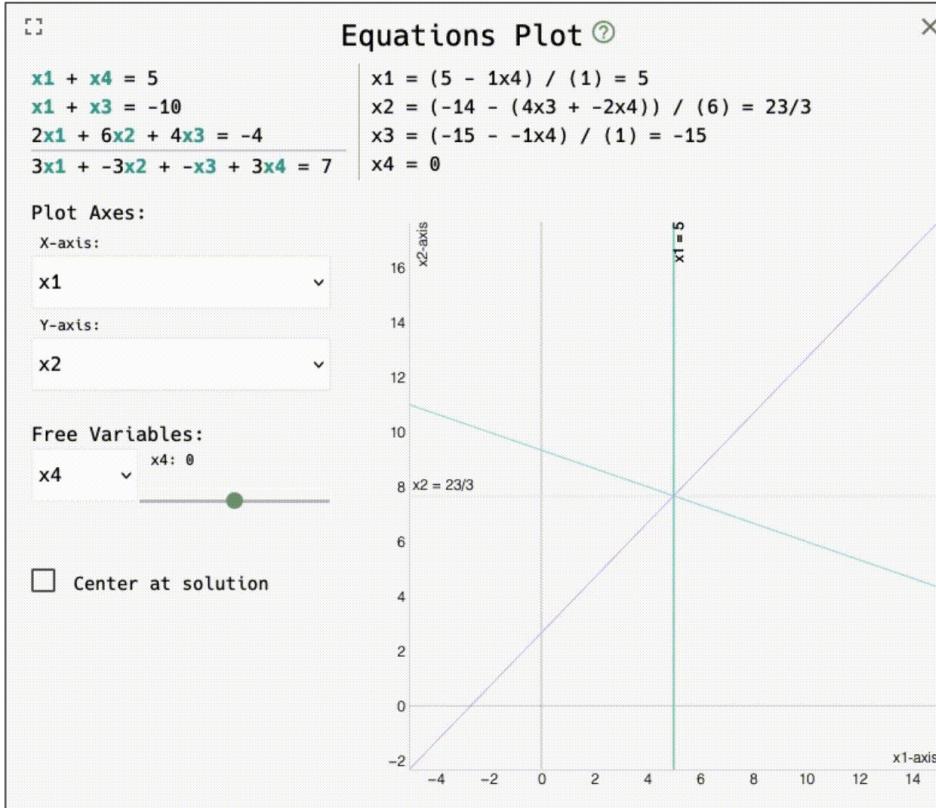
$$DD \sqsubseteq LBD$$



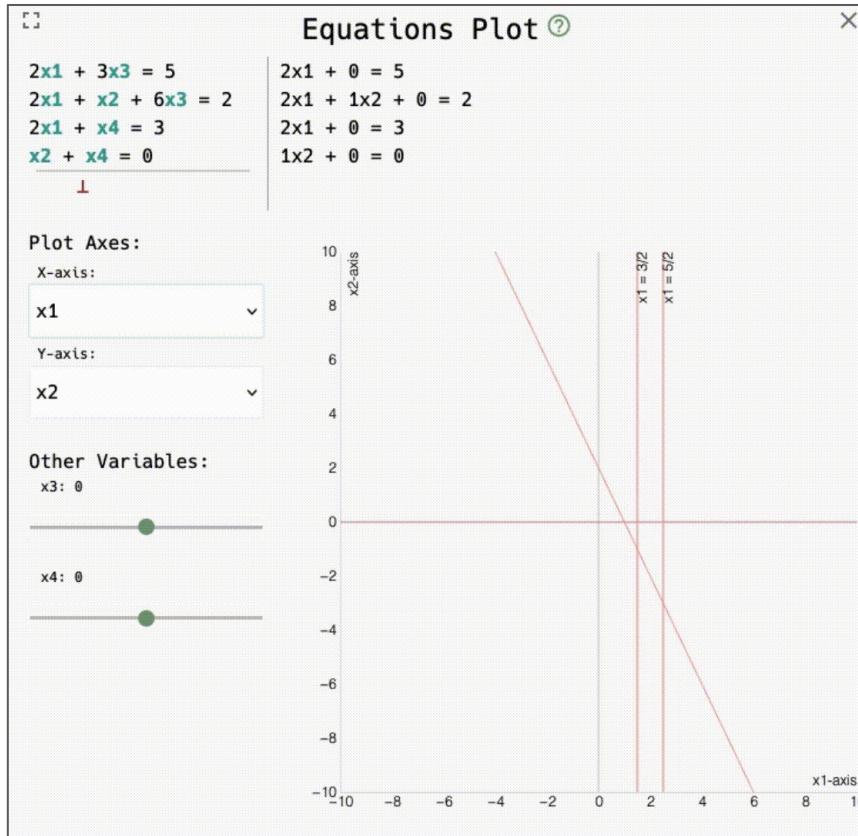
# $EL_{\perp}[D_{Q,lin}]$ Explanations



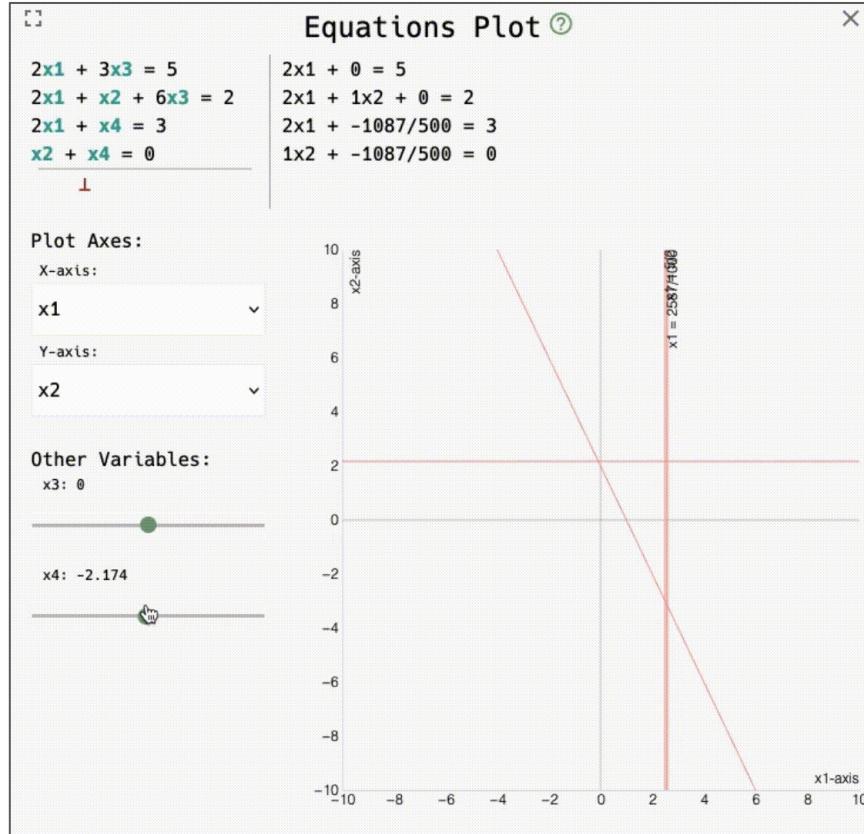
# $EL_{\perp}[D_{\mathbb{Q},lin}]$ Explanations



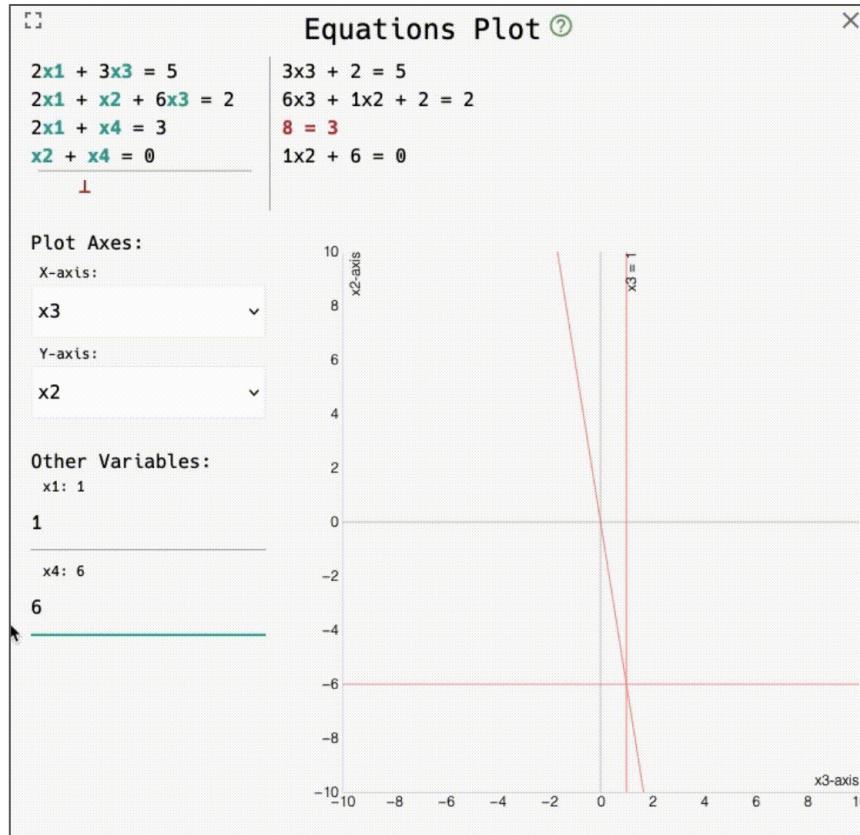
# $EL_{\perp}[D_{\mathbb{Q},lin}]$ Explanations



# $EL_{\perp}[D_{Q,lin}]$ Explanations



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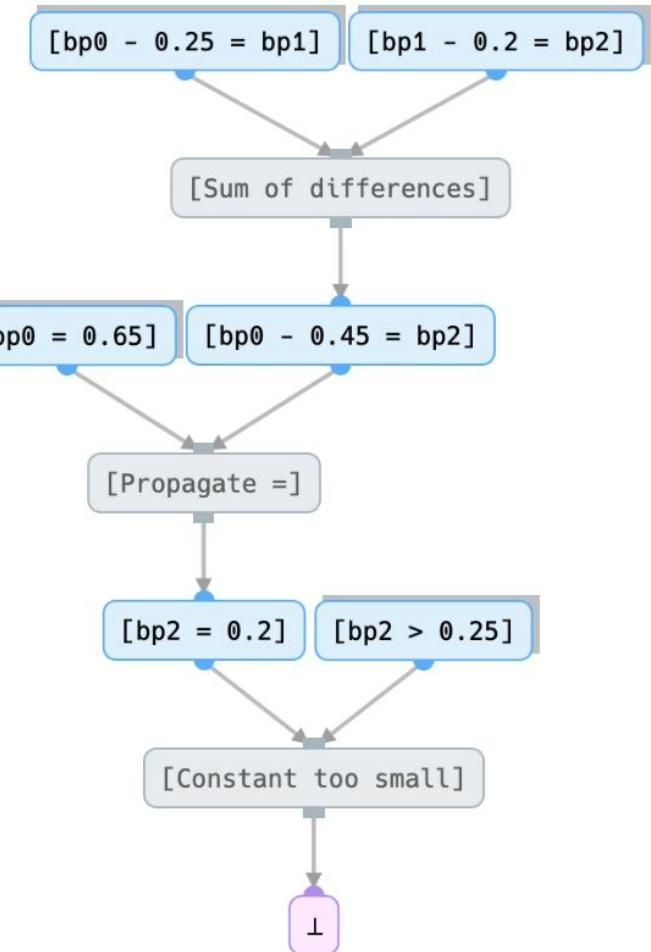
# $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$ Explanations

$$[bp_0 - 0.25 = bp_1] \quad [bp_1 - 0.2 = bp_2]$$

$$[bp_2 > 0.25] \quad [bp_0 = 0.65]$$

## Saturation Rules

$$\begin{array}{c} \frac{x = q \quad x = p}{\perp} R_{\neq} : q \neq p \quad \frac{x + q = y \quad y + p = z}{x + (q + p) = z} R_{+} \quad \frac{x + 0 = x}{x + 0 = x} R_0 \\ \frac{x + q = y \quad x + p = y}{\perp} R_{\neq}^+ : q \neq p \quad \frac{x = q \quad y = p}{x + (p - q) = y} R_{-} \quad \frac{x + q = y}{y + (-q) = x} R_{\leftrightarrow} \\ \frac{x = q \quad x > p}{\perp} R_{<} : q < p \quad \frac{x = q \quad x + p = y}{y = q + p} R_{=} \quad \frac{x > q \quad x + p = y}{y > q + p} R_{>} \end{array}$$



# $EL_{\perp}[D_{\mathbb{Q}^{diff}}]$ Explanations

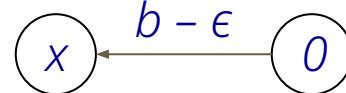
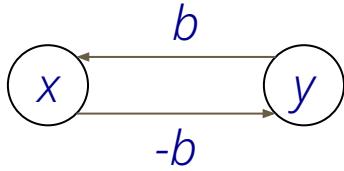
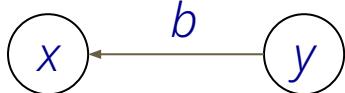
$$x - y \leq b$$

$$x - y = b$$

$$x < b$$

$$x - (0) < b$$

$$x - (0) \leq b - \epsilon$$



# $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$ Explanations

### Negative Cycle - CDP1

Premises:

$$\begin{aligned} \text{bp0} - 0.25 &= \text{bp1} \\ \text{bp1} - 0.2 &= \text{bp2} \\ \text{bp0} &= 0.65 \\ -\text{bp2} &\leq -0.25 - \epsilon \end{aligned}$$

Conclusion:

$$\perp$$

Cycle Value:  $-1/20 - \epsilon$

The diagram shows a directed graph with four nodes:  $\text{bp0}$ ,  $\text{bp1}$ ,  $\text{bp2}$ , and  $(0)$ . There are directed edges between these nodes with the following weights:

- $\text{bp0} \rightarrow \text{bp1}$ : weight  $0.25$
- $\text{bp1} \rightarrow \text{bp2}$ : weight  $-0.25$
- $\text{bp2} \rightarrow (0)$ : weight  $-0.25 - \epsilon$
- $(0) \rightarrow \text{bp0}$ : weight  $0.65$
- $\text{bp0} \rightarrow (0)$ : weight  $-0.65$
- $\text{bp2} \rightarrow \text{bp1}$ : weight  $0.2$
- $\text{bp1} \rightarrow (0)$ : weight  $-0.2$

# $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$ Explanations

Negative Cycle - CDP1 ⓘ

Premises:

- $bp0 - 0.25 = bp1$
- $bp1 - 0.2 = bp2$
- $bp0 = 0.65$
- $- bp2 \leq -0.25 - \epsilon$

Conclusion:

$\perp$

Cycle Value:  $-1/20 - \epsilon$

```
graph TD; bp0 -- "0.25" --> bp1; bp1 -- "-0.25" --> bp2; bp2 -- "0.2" --> (0); (0) -- "-0.65" --> bp0;
```

# $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$ Explanations

Negative Cycle - CDP1 ⓘ

Premises:

- $bp0 - 0.25 = bp1$
- $bp1 - 0.2 = bp2$
- $bp0 = 0.65$
- $- bp2 \leq -0.25 - \epsilon$

Conclusion:

$\perp$

Cycle Value:  $-1/20 - \epsilon$

Propagate a value for "bp2"!

◀ ▶ ⌂

# $EL_{\perp}[D_{\mathbb{Q}_{diff}}]$ Explanations

Negative Cycle - CDP1 ②

Premises:

$$x_4 + 0 = x_5$$
$$x_3 - 2 = x_2$$
$$x_2 - 1 = x_1$$
$$x_4 + 5 = x_3$$

Conclusion:

$$x_5 + 2 = x_1$$

Negated:

$$x_5 - x_1 \leq -2.0 - \epsilon$$
$$x_1 - x_5 \leq 2.0 - \epsilon$$

Cycle Value:  $-\epsilon$

```
graph TD; x5((x5)) -- "0" --> x4((x4)); x4 -- "5" --> x3((x3)); x3 -- "-5" --> x2((x2)); x2 -- "2" --> x1((x1)); x1 -- "-2" --> x5; x2 -- "-2 - \epsilon" --> x4; x1 -- "2 - \epsilon" --> x3;
```

# Evaluation

- Performance benchmark
  - ~100 proofs
- Two qualitative user studies
- Proofs vs alternative CD explanations
- 11 participants
- Results:
  - Visual explanations enhance understanding of numerical reasoning
  - Plots were preferred in the simple cases
  - Cycles were preferred over proofs

# Summary & Future Work

- Extend Evonne with  $EL_{\perp}[D_{\mathbb{Q}^{diff}}]$  &  $EL_{\perp}[D_{\mathbb{Q}^{lin}}]$  Proofs
- Alternative visual Explanations
  - Negative cycles
  - Equations plots
- Evaluation
  - Qualitative user studies
  - Performance benchmark
- Address issues from participants
- Explanation of CD non-entailments

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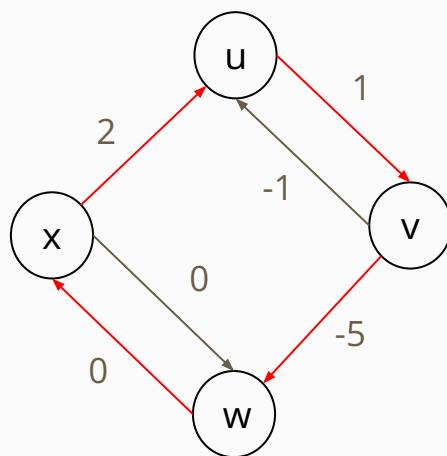
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$$\left\{ \begin{array}{l} w - v \leq -5 \\ x - w \leq 0 \\ u - x \leq 2 \\ v - u \leq 1 \end{array} \right.$$

→ ⊥

$$0 \leq -2$$



Weight = -2