Flipped classroom in theoretical computer science

A case study for equational logic

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Why am I here?

- One of the last PhD students of Anna :)
- Anna and I taught a lot together, and also did research.
- I learned a lot from her.

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Today's plan

Simulate what I learned form Anna about teaching to teach you about our reserach :).

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$$\frac{x = y}{y = x}$$

Can we make one more equation?

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Correct! $(1 + 1 = 2)$

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I give you $1 + 1 = 2$ and:

$$\frac{x_1 = y_1 \ , \ x_2 = y_2}{x_1 + x_2 = y_1 + y_2}$$

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Correct! $(1 + 1 + 2 = 2 + 1 + 1)$

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One step further

You start from the number 0, and you can only use the +2 operation.
You can use:

$$x = x \qquad x_1 + x_2 = x_2 + x_1 \qquad \frac{x = y}{y = x} \qquad \frac{x_1 = y_1 \ , \ x_2 = y_2}{x_1 + x_2 = y_1 + y_2}$$

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For 5 mins

Try to write your own numbers and equations. :) I will walk around.

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Main question

Can any two equal terms be proven equal by using our axiom set?

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Can any two equal terms be proven equal by using our axiom set?

Method:

- Find a property you believe all of our equations will have.
- Show all "basic" terms/equations have this property.
- Assume that all equations you can build in n steps have this property.
- Show that if you apply one more step the property is preserved.

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2. Show all "basic" terms/equations have this property: 0 is even.

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Any suggestion for a property?

1. Property: All the numbers will be even.
2. Show all "basic" terms/equations have this property: 0 is even.
3. Everything we can build in n steps has this property.
4. Show that if you apply one more build step the property is preserved.

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Any suggestion for a property?

1. Property: All the numbers will be even.
2. Show all "basic" terms/equations have this property: 0 is even.
3. Everything we can build in n steps has this property.
4. Show that if you apply one more build step the property is preserved.

Give it a shot?

After n steps we have managed to prove that $x = y$, and x and y are even. What is the next build step?

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- How do we make/prove new equations? - Axiomatic method

A process algebra + equivalences (bisimulations) + axioms = We can prove programs equal to each other!

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Why would anybody do this?

- Compiler optimization.
- Correctness proofs..

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The Anna works

- 10s of publications about all sorts of algebras, numbers, regular expressions, and many more!
- On the Two-Variable Fragment of the Equational Theory of the Max-Sum Algebra of the Natural Numbers.
- Some possitive some negative results.
- So much fun :).

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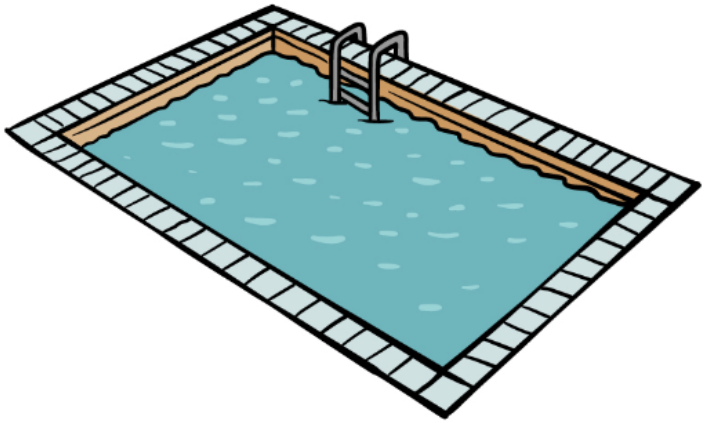

Thank you for your attention!

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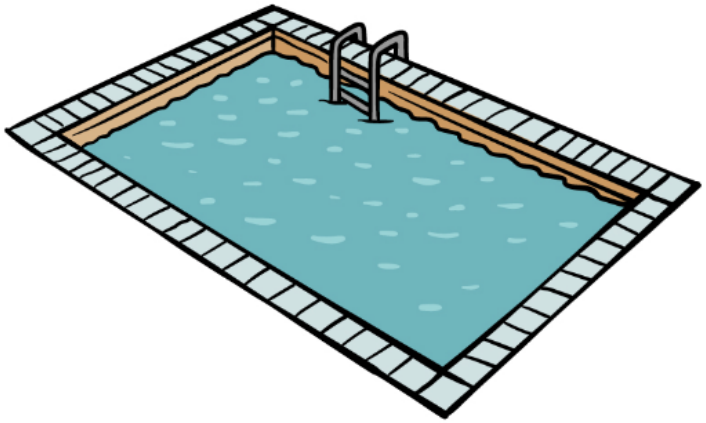

Questions?

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