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Fundamental factor models

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Part of the Quantopian Lecture Series:

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Fundamentals are data having to do with the asset issuer, like the sector, size, and expenses of the company. We can use this data to build a linear factor model, expressing returns on any asset as

$$R_t = a_t + b_{t1} F_1 + b_{t2} F_2 + \ldots + b_{tK} F_K + \epsilon_t$$

There are two different approaches to computing the factors F_j, which represent the returns associated with some fundamental characteristics, and the factor sensitivities b_{tj}.

Approach 1: Portfolio Construction

In the first, we start by representing each characteristic of interest by a portfolio: we sort all assets by that characteristic, then build the portfolio by going long the top quantile of assets and short the bottom quantile. The factor corresponding to this characteristic is the return on this portfolio. Then, the b_{ij} are estimated for each asset i by regressing over the historical values of R_i and of the factors.

We'll use the canonical Fama-French factors for this example, which are the returns of portfolios constructred based on fundamental factors.

We start by getting the fundamentals data for all assets and constructing the portfolios for each characteristic:

Import some libraries.

In [1]:
```
import numpy as np
import pandas as pd
from quantopian.pipeline.data import morningstar
import statsmodels.api as sm
from statsmodels import regression
import matplotlib.pyplot as plt
import scipy.stats
```

Set the date range for which we want data.

In [2]:
```
start_date = '2011-1-1'
end_date = '2012-1-1'
```

Using the Pipeline API to Fetch Data

The pipeline API is a very useful tool for factor analysis. We use it here to get data for our analysis. Specifically, we want the daily values of book to price ratio and market cap for every security. But we also do several other useful filtering steps which are detailed in code comments.

In [3]:
```
import numpy as np
from quantopian.pipeline import Pipeline
from quantopian.pipeline.data import morningstar
from quantopian.pipeline.data.builtin import USEquityPricing
from quantopian.pipeline.factors import CustomFactor, Returns

# Here's the raw data we need, everything else is derivative.

class MarketCap(CustomFactor):
    # Here's the data we need for this factor
    inputs = [morningstar.valuation.shares_outstanding, USEquityPricing.close]
    # Only need the most recent values for both series
    window_length = 1

    def compute(self, today, assets, out, shares, close_price):
        # Shares * price/share = total price = market cap
        out[:] = shares * close_price
```

```
class BookToPrice(CustomFactor):
    # pb = price to book, we'll need to take the reciprocal later
    inputs = [morningstar.valuation_ratios.pb_ratio]
    window_length = 1

    def compute(self, today, assets, out, pb):
        out[:] = 1 / pb

def make_pipeline():
    """
    Create and return our pipeline.

    We break this piece of logic out into its own function to make it easier to
    test and modify in isolation.

    In particular, this function can be copy/pasted into research and run by itself.
    """
    pipe = Pipeline()

    # Add our factors to the pipeline
    market_cap = MarketCap()
    # Raw market cap and book to price data gets fed in here
    pipe.add(market_cap, "market_cap")
    book_to_price = BookToPrice()
    pipe.add(book_to_price, "book_to_price")

    # We also get daily returns
    returns = Returns(inputs=[USEquityPricing.close], window_length=2)
    pipe.add(returns, "returns")

    # We compute a daily rank of both factors, this is used in the next step,
    # which is computing portfolio membership.
    market_cap_rank = market_cap.rank()
    pipe.add(market_cap_rank, 'market_cap_rank')

    book_to_price_rank = book_to_price.rank()
    pipe.add(book_to_price_rank, 'book_to_price_rank')

    # Build Filters representing the top and bottom 1000 stocks by our combined ranking system.
    biggest = market_cap_rank.top(1000)
    smallest = market_cap_rank.bottom(1000)

    highpb = book_to_price_rank.top(1000)
    lowpb = book_to_price_rank.bottom(1000)

    # Don't return anything not in this set, as we don't need it.
    pipe.set_screen(biggest | smallest | highpb | lowpb)

    # Add the boolean flags we computed to the output data
    pipe.add(biggest, 'biggest')
    pipe.add(smallest, 'smallest')

    pipe.add(highpb, 'highpb')
    pipe.add(lowpb, 'lowpb')

    return pipe
```

```
/usr/local/lib/python2.7/dist-packages/IPython/kernel/__main__.py:11: NotAllowedInLiveWarning: The fundame
ntals attribute valuation.shares_outstanding is not yet allowed in broker-backed live trading
```

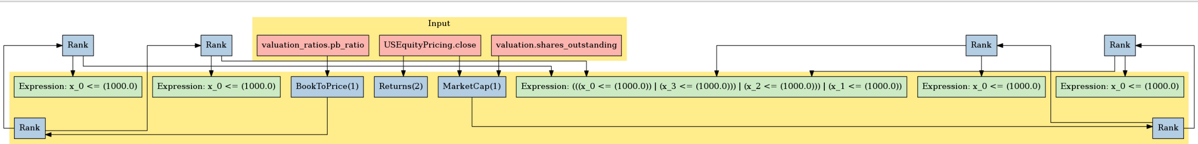
Now we initialize the pipeline.

In [4]:
```
pipe = make_pipeline()
```

We can visualize the dependency graph of our data computations here.

In [5]:
```
pipe.show_graph('png')
```

Out[5]:



This function will allow us to run the pipeline.

In [6]:
```
from quantopian.research import run_pipeline
```

Now let's actually run it and check out our results.

In [7]:
```
# This takes a few minutes.
results = run_pipeline(pipe, start_date, end_date)
results
```

Out[7]:

		biggest	book_to_price	book_to_price_rank	highpb	lowpb	market_cap	market_cap
2011-01-03 00:00:00+00:00	Equity(2 [AA])	True	0.991867	3622	False	False	1.573937e+10	4373
	Equity(21 [AAME])	False	2.052967	4523	True	False	4.520648e+07	578
	Equity(24 [AAPL])	True	0.167400	493	False	True	2.957765e+11	4701
	Equity(31 [ABAX])	False	0.257898	856	False	True	6.002049e+08	2396
	Equity(37 [ABCW])	False	0.277500	953	False	True	2.601996e+07	370
	Equity(51 [ABL])	False	2.364066	4581	True	False	2.415793e+05	2
	Equity(53 [ABMD])	False	0.235200	759	False	True	3.632957e+08	1985
	Equity(58 [SERV])	False	1.666944	4402	True	False	8.785592e+06	109
	Equity(62 [ABT])	True	0.297699	1039	False	False	7.410661e+10	4626
	Equity(64 [ABX])	True	0.344994	1276	False	False	5.297974e+10	4599
	Equity(67 [ADSK])	True	0.193900	598	False	True	8.684483e+09	4138
	Equity(76 [TAP])	True	0.857633	3267	False	False	9.346545e+09	4175
	Equity(88 [ACI])	True	0.460893	1819	False	False	5.696619e+09	3993
	Equity(100 [IEP])	False	1.077354	3835	True	False	3.059538e+09	3653
	Equity(106 [ACU])	False	0.853315	3256	False	False	2.914872e+07	403
	Equity(107 [ACV])	True	0.360101	1336	False	False	3.663793e+09	3768
	Equity(112 [ACY])	False	1.571092	4350	True	False	2.793301e+07	390
	Equity(114 [ADBE])	True	0.367202	1382	False	False	1.565831e+10	4371
	Equity(117 [AEY])	False	0.949668	3503	False	False	3.185216e+07	433
	Equity(122							

Great, we have all the data. Now we need to compute the returns of our portfolios over time. We have the daily returns for each equity, plus whether or not that equity was included in any given portfolio on any given day. We can combine that information in the following way to yield daily portfolio returns.

Step 1: Subset our results into only data belonging to our 'biggest' portfolio.

In [8]:
```
results[results.biggest]
```

Out[8]:

		biggest	book_to_price	book_to_price_rank	highpb	lowpb	market_cap	market_cap
2011-01-03 00:00:00+00:00	**Equity(2 [AA])**	True	0.991867	3622	False	False	1.573937e+10	4373
	Equity(24 [AAPL])	True	0.167400	493	False	True	2.957765e+11	4701
	Equity(62 [ABT])	True	0.297699	1039	False	False	7.410661e+10	4626
	Equity(64 [ABX])	True	0.344994	1276	False	False	5.297974e+10	4599
	Equity(67 [ADSK])	True	0.193900	598	False	True	8.684483e+09	4138
	Equity(76 [TAP])	True	0.857633	3267	False	False	9.346545e+09	4175
	Equity(88 [ACI])	True	0.460893	1819	False	False	5.696619e+09	3993
	Equity(107 [ACV])	True	0.360101	1336	False	False	3.663793e+09	3768
	Equity(114 [ADBE])	True	0.367202	1382	False	False	1.565831e+10	4371
	Equity(122 [ADI])	True	0.301296	1057	False	False	1.125325e+10	4246
	Equity(128 [ADM])	True	0.829669	3193	False	False	1.922429e+10	4421
	Equity(154 [AEM])	True	0.267001	903	False	True	1.200374e+10	4266
	Equity(157 [AEG])	True	3.364738	4677	True	False	1.065935e+10	4229
	Equity(161 [AEP])	True	0.798722	3100	False	False	1.728033e+10	4399
	Equity(166 [AES])	True	0.808669	3131	False	False	9.602998e+09	4188
	Equity(168 [AET])	True	0.843526	3229	False	False	1.221105e+10	4277
	Equity(185 [AFL])	True	0.458695	1802	False	False	2.660740e+10	4486
	Equity(197 [AGCO])	True	0.602011	2401	False	False	4.714631e+09	3908
	Equity(205 [AGN])	True	0.221602	704	False	True	2.085580e+10	4439
	Equity(216							

Step 2: Get returns.

In [9]:
```
results[results.biggest]['returns']
```

Out[9]:
```
2011-01-03 00:00:00+00:00  Equity(2 [AA])          0.013083
                           Equity(24 [AAPL])      -0.003769
                           Equity(62 [ABT])        0.007778
                           Equity(64 [ABX])        0.011217
                           Equity(67 [ADSK])      -0.014952
                           Equity(76 [TAP])       -0.004362
                           Equity(88 [ACI])       -0.005954
                           Equity(107 [ACV])      -0.000270
                           Equity(114 [ADBE])      0.006869
                           Equity(122 [ADI])      -0.007638
                           Equity(128 [ADM])       0.007028
                           Equity(154 [AEM])       0.002354
```

```
                            Equity(157 [AEG])       0.008210
                            Equity(161 [AEP])      -0.002495
                            Equity(166 [AES])      -0.002048
                            Equity(168 [AET])       0.002628
                            Equity(185 [AFL])       0.006238
                            Equity(197 [AGCO])     -0.002363
                            Equity(205 [AGN])       0.000000
                            Equity(216 [HES])       0.001308
                            Equity(239 [AIG])       0.001738
                            Equity(273 [ALU])       0.006826
                            Equity(300 [ALK])      -0.013041
                            Equity(328 [ALTR])     -0.009460
                            Equity(337 [AMAT])     -0.006369
                            Equity(338 [BEAM])     -0.007411
                            Equity(351 [AMD])       0.006143
                            Equity(353 [AME])      -0.002034
                            Equity(357 [TWX])       0.003117
                            Equity(368 [AMGN])     -0.011345
                                                      ...
2012-01-03 00:00:00+00:00  Equity(38965 [FTNT])    0.006925
                            Equity(39053 [CIT])    -0.007120
                            Equity(39073 [CIE])    -0.009591
                            Equity(39095 [CHTR])    0.001408
                            Equity(39347 [ST])     -0.006427
                            Equity(39495 [SDRL])   -0.001806
                            Equity(39499 [VIP])     0.002114
                            Equity(39546 [LYB])    -0.010360
                            Equity(39612 [SIX])    -0.003625
                            Equity(39778 [QEP])     0.002394
                            Equity(39994 [NXPI])   -0.002597
                            Equity(40338 [SMFG])    0.001821
                            Equity(40445 [LPLA])    0.002626
                            Equity(40573 [FRC])    -0.002603
                            Equity(40616 [MMI])    -0.000258
                            Equity(40755 [NLSN])    0.001350
                            Equity(40852 [KMI])     0.019658
                            Equity(41047 [HCA])     0.035228
                            Equity(41150 [APO])     0.009764
                            Equity(41242 [ARCO])   -0.005329
                            Equity(41416 [KOS])     0.000815
                            Equity(41451 [LNKD])   -0.011471
                            Equity(41462 [MOS])     0.003521
                            Equity(41484 [YNDX])    0.009641
                            Equity(41491 [FSL])    -0.023148
                            Equity(41636 [MPC])     0.001203
                            Equity(41886 [FNV])    -0.008592
                            Equity(42023 [XYL])    -0.011565
                            Equity(42118 [GRPN])   -0.037488
                            Equity(42173 [DLPH])    0.005602
Name: returns, dtype: float64
```

Step 3: Group by day and take the mean. This is pretty deep into pandas logic, so if you don't understand this on first pass it is recommended to check out pandas' documentation on all the functions used. Especially `groupby`, which is very useful. Keep in mind that the index in our results is a `MultiIndex` rather than a regular `Index`, that can complicate things.

In [10]:
```
results[results.biggest]['returns'].groupby(level=0).mean()
```

Out[10]:
```
2011-01-03 00:00:00+00:00   -0.000173
2011-01-04 00:00:00+00:00    0.010987
2011-01-05 00:00:00+00:00   -0.005017
2011-01-06 00:00:00+00:00    0.004201
2011-01-07 00:00:00+00:00   -0.003209
2011-01-10 00:00:00+00:00   -0.001847
2011-01-11 00:00:00+00:00    0.000640
2011-01-12 00:00:00+00:00    0.006782
2011-01-13 00:00:00+00:00    0.009905
2011-01-14 00:00:00+00:00   -0.000804
2011-01-18 00:00:00+00:00    0.005200
2011-01-19 00:00:00+00:00    0.004477
2011-01-20 00:00:00+00:00   -0.012755
2011-01-21 00:00:00+00:00   -0.006107
2011-01-24 00:00:00+00:00    0.000013
2011-01-25 00:00:00+00:00    0.007606
2011-01-26 00:00:00+00:00   -0.001106
2011-01-27 00:00:00+00:00    0.009396
2011-01-28 00:00:00+00:00    0.003167
2011-01-31 00:00:00+00:00   -0.017886
2011-02-01 00:00:00+00:00    0.009216
```

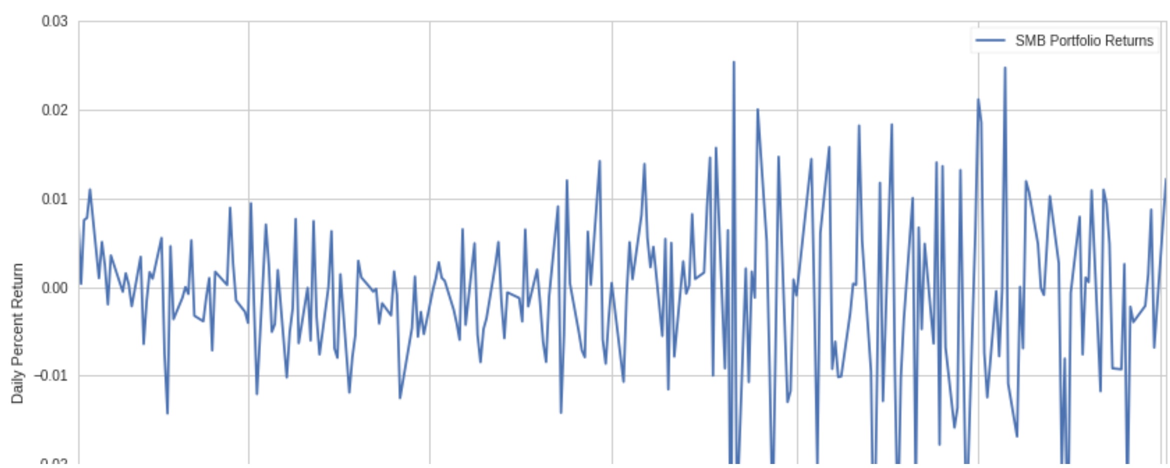
```
2011-02-02 00:00:00+00:00    0.016059
2011-02-03 00:00:00+00:00   -0.001468
2011-02-04 00:00:00+00:00    0.003532
2011-02-07 00:00:00+00:00    0.002991
2011-02-08 00:00:00+00:00    0.005747
2011-02-09 00:00:00+00:00    0.004054
2011-02-10 00:00:00+00:00   -0.004357
2011-02-11 00:00:00+00:00    0.001720
2011-02-14 00:00:00+00:00    0.006792
                               ...
2011-11-18 00:00:00+00:00   -0.019297
2011-11-21 00:00:00+00:00   -0.000848
2011-11-22 00:00:00+00:00   -0.018459
2011-11-23 00:00:00+00:00   -0.004176
2011-11-25 00:00:00+00:00   -0.025292
2011-11-28 00:00:00+00:00   -0.002621
2011-11-29 00:00:00+00:00    0.033148
2011-11-30 00:00:00+00:00    0.001859
2011-12-01 00:00:00+00:00    0.045931
2011-12-02 00:00:00+00:00   -0.001984
2011-12-05 00:00:00+00:00   -0.000097
2011-12-06 00:00:00+00:00    0.012671
2011-12-07 00:00:00+00:00   -0.001904
2011-12-08 00:00:00+00:00    0.000533
2011-12-09 00:00:00+00:00   -0.025552
2011-12-12 00:00:00+00:00    0.018515
2011-12-13 00:00:00+00:00   -0.017489
2011-12-14 00:00:00+00:00   -0.014710
2011-12-15 00:00:00+00:00   -0.013625
2011-12-16 00:00:00+00:00    0.004156
2011-12-19 00:00:00+00:00    0.005895
2011-12-20 00:00:00+00:00   -0.015466
2011-12-21 00:00:00+00:00    0.032779
2011-12-22 00:00:00+00:00    0.001803
2011-12-23 00:00:00+00:00    0.010930
2011-12-27 00:00:00+00:00    0.007011
2011-12-28 00:00:00+00:00   -0.000239
2011-12-29 00:00:00+00:00   -0.015062
2011-12-30 00:00:00+00:00    0.011664
2012-01-03 00:00:00+00:00   -0.001335
Name: returns, dtype: float64
```

Now run through this computation for each portfolio and get our final results.

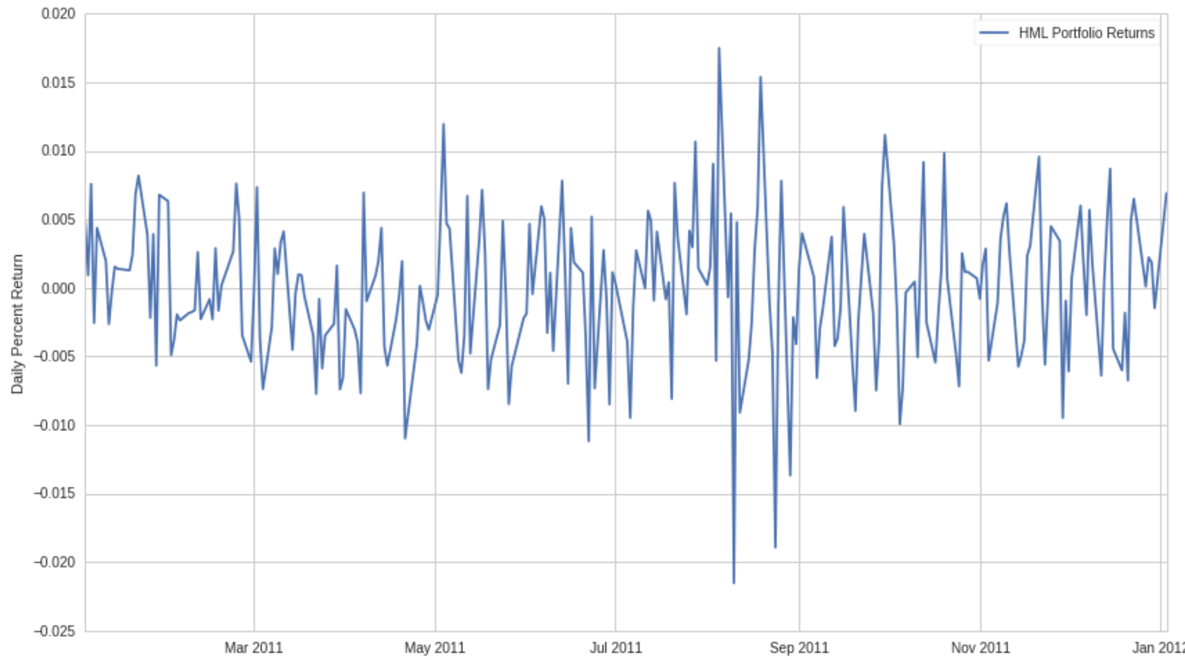
In [11]:
```
R_biggest = results[results.biggest]['returns'].groupby(level=0).mean()
R_smallest = results[results.smallest]['returns'].groupby(level=0).mean()

R_highpb = results[results.highpb]['returns'].groupby(level=0).mean()
R_lowpb = results[results.lowpb]['returns'].groupby(level=0).mean()

SMB = R_smallest - R_biggest
HML = R_highpb - R_lowpb
```

What were the daily returns?

In [12]:
```
plt.plot(SMB.index, SMB.values)
plt.ylabel('Daily Percent Return')
plt.legend(['SMB Portfolio Returns']);
```



In [13]:
```
plt.plot(HML.index, HML.values)
plt.ylabel('Daily Percent Return')
plt.legend(['HML Portfolio Returns']);
```



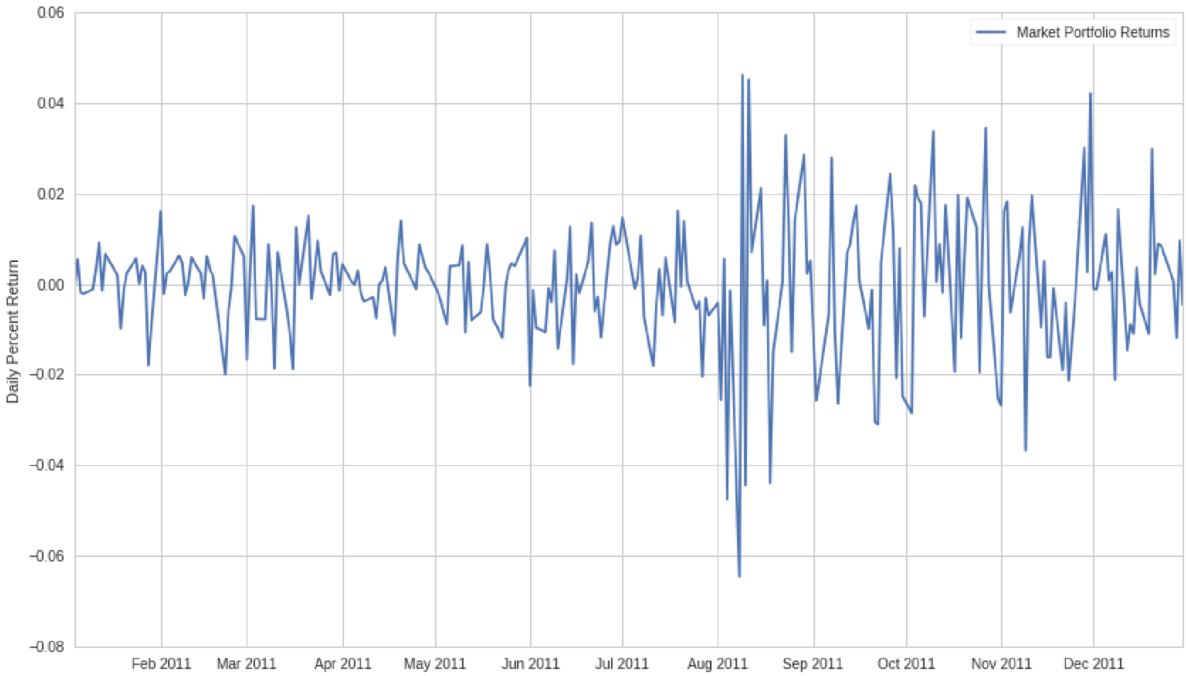
And what would it look like to hold these portfolios over time?

In [14]:
```
plt.plot(SMB.index, np.cumprod(SMB.values+1))
plt.ylabel('Cumulative Return')
plt.legend(['SMB Portfolio Returns']);
```

The last data we need are the daily returns on the broad market.

In [15]:
```
M = get_pricing('SPY', start_date='2011-1-1', end_date='2012-1-1', fields='price').pct_change()[1:]
```

In [16]:
```
plt.plot(M.index, M.values)
plt.ylabel('Daily Percent Return')
plt.legend(['Market Portfolio Returns']);
```



Actually Running the Regression

Now that we have returns series representing our factors, we can compute the factor model for any return stream using a linear regression. Below, we compute the factor sensitivities for returns on a tech portfolio.

In [17]:
```
# Get returns data for our portfolio
portfolio = get_pricing(['MSFT', 'AAPL', 'YHOO', 'FB', 'TSLA'],
                        fields='price', start_date=start_date, end_date=end_date).pct_change()[1:]
R = np.mean(portfolio, axis=1)
```

Put all the data into one dataframe for convenience.

In [18]:
```
# Define a constant to compute intercept
constant = pd.TimeSeries(np.ones(len(R.index)), index=R.index)

df = pd.DataFrame({'R': R,
              'M': M,
              'SMB': SMB,
              'HML': HML,
              'Constant': constant})
df = df.dropna()
```

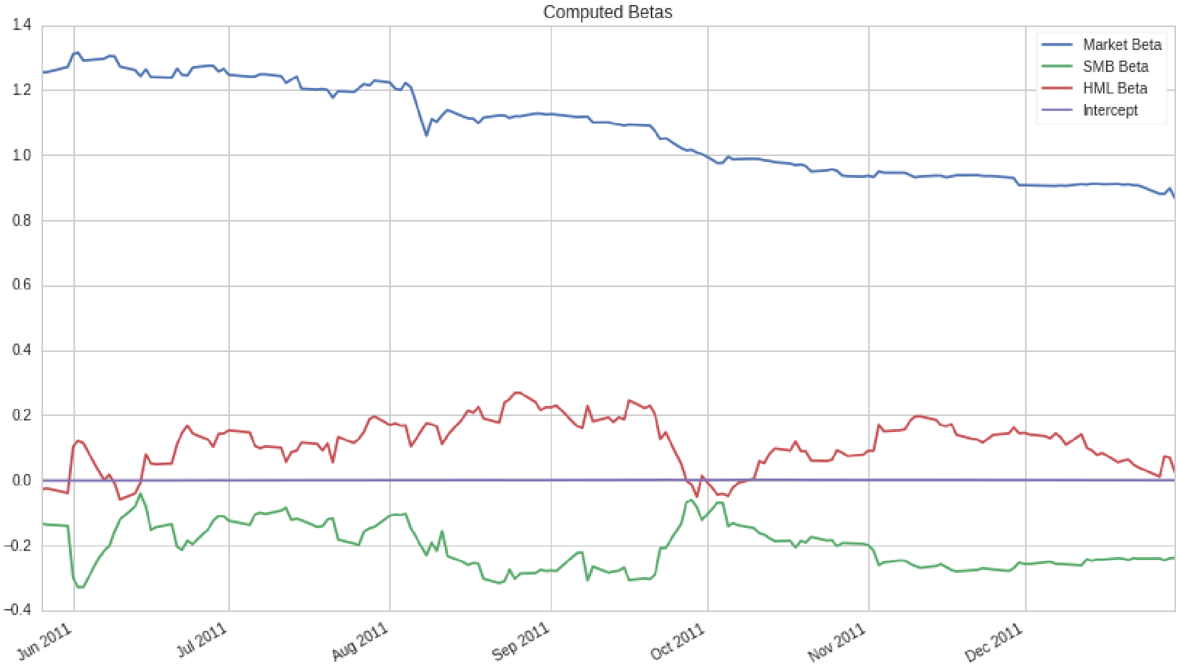
Perform the regression. You'll notice that these are the sensitivities over an entire year. It can be valuable to look at the rolling sensitivities as well to determine how stable they are.

In [19]:
```
# Perform linear regression to get the coefficients in the model
b1, b2, b3 = regression.linear_model.OLS(df['R'], df[['M', 'SMB', 'HML']]).fit().params

# Print the coefficients from the linear regression
print 'Historical Sensitivities of portfolio returns to factors:\nMarket: %f\nMarket cap: %f\nB/P: %f' %  (b1
, b2, b3)
```

```
Historical Sensitivities of portfolio returns to factors:
Market: 0.961804
Market cap: -0.203828
B/P: 0.050790
```

Let's perform a rolling regression to look at how the estimated sensitivities change over time.

In [20]:
```
model = pd.stats.ols.MovingOLS(y = df['R'], x=df[['M', 'SMB', 'HML']],
                               window_type='rolling',
                               window=100)
rolling_parameter_estimates = model.beta
rolling_parameter_estimates.plot();
plt.title('Computed Betas');
plt.legend(['Market Beta', 'SMB Beta', 'HML Beta', 'Intercept']);
```



Approach 2: Factor Value Normalization

This is also known as cross-sectional factor analysis.

Another approach is to normalize factor values each bar and see how predictive of that bar's returns they were. We do this by computing a normalized factor value b_{aj} for each asset a in the following way.

$$b_{aj} = \frac{F_{aj} - \mu_{F_j}}{\sigma_{F_j}}$$

F_{aj} is the value of factor j for asset a during this bar, μ_{F_j} is the mean factor value across all assets, and σ_{F_j} is the standard deviation of factor values over all assets. Notice that we are just computing a z-score to make asset specific factor values comparable across different factors.

The exceptions to this formula are indicator variables, which are set to 1 for true and 0 for false. One example is industry membership: the coefficient tells us whether the asset belongs to the industry or not.

After we calculate all of the normalized scores during bar t, we can estimate factor j's returns F_{jt}, using a cross-sectional regression (i.e. at each time step, we perform a regression using the equations for all of the assets). Specifically, once we have returns for each asset R_{at}, and normalized factor coefficients b_{aj}, we construct the following model and estimate the F_js and a_t

$$R_{at} = a_t + b_{a1}F_1 + b_{a2}F_2 + \dots + b_{aK}F_K$$

You can think of this as slicing through the other direction from the first analysis, as now the factor returns are unknowns to be solved for, whereas originally the coefficients were the unknowns. Another way to think about it is that you're determining how predictive of returns the factor was on that day, and therefore how much return you could have squeezed out of that factor.

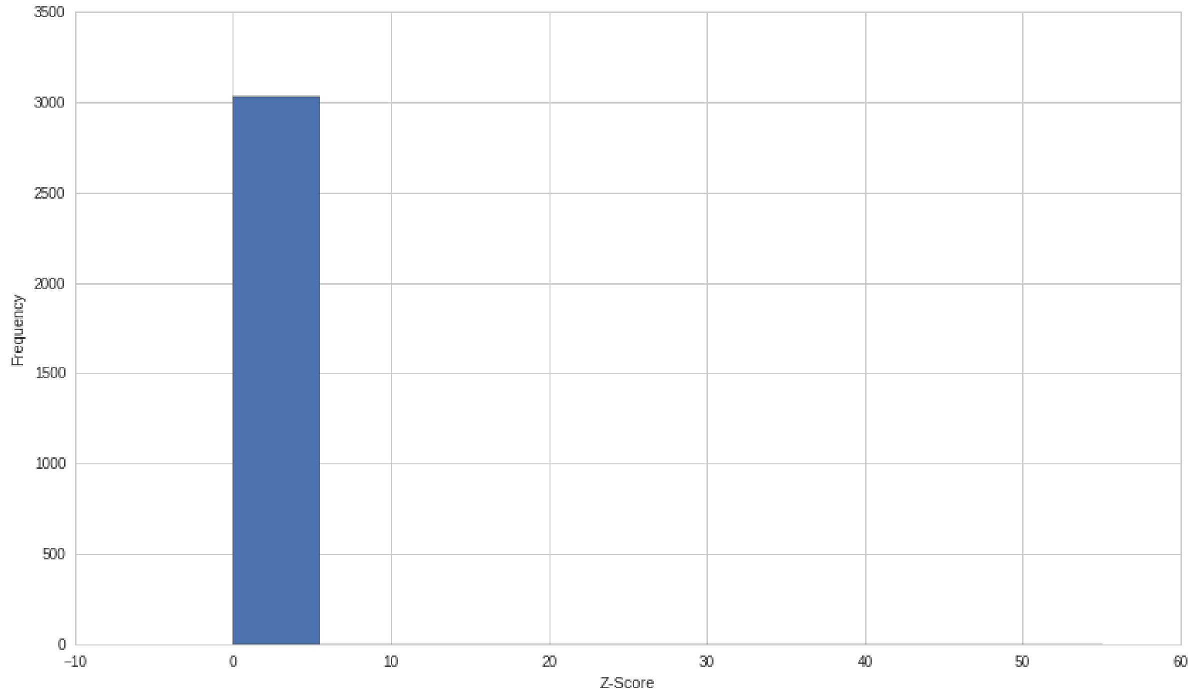
Following this procedure, we'll get the cross-sectional returns on 2011-01-03, and compute the coefficients for all assets:

Getting the Data

We already have the results of the previous pipeline call, so we can grab book to price information for 2011-1-3 pretty easily.

In [21]:
```
BTP = results['book_to_price'].loc['2011-1-3']
zscore = (BTP - np.mean(BTP)) / np.std(BTP)
zscore.dropna(inplace=True)

plt.hist(zscore)
plt.xlabel('Z-Score')
```

```
plt.ylabel('Frequency');
```



Problem: The Data is Weirdly Distributed

Notice how there are big outliers in the dataset that cause the z-scores to lose a lot of information. Basically the presence of some huge book to price datapoints causes the rest of the data to seem to occupy a relatively small area. We need to get around this issue using some data cleaning technique, here we're use winsorization.

Winsorization

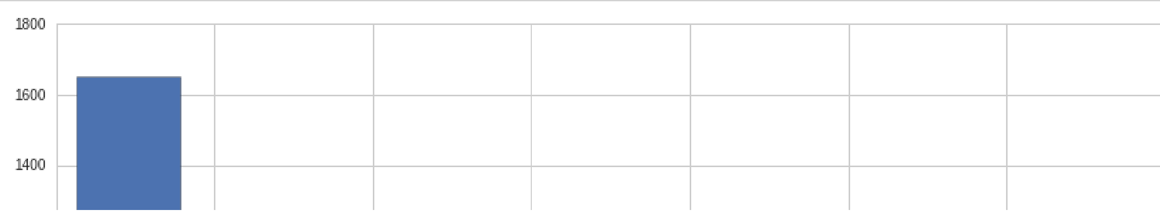
Winzorization takes the top $n\%$ of a dataset and sets it all equal to the least extreme value in the top $n\%$. For example, if your dataset ranged from 0-10, plus a few crazy outliers, those outliers would be set to 0 or 10 depending on their direction. Here is an example.

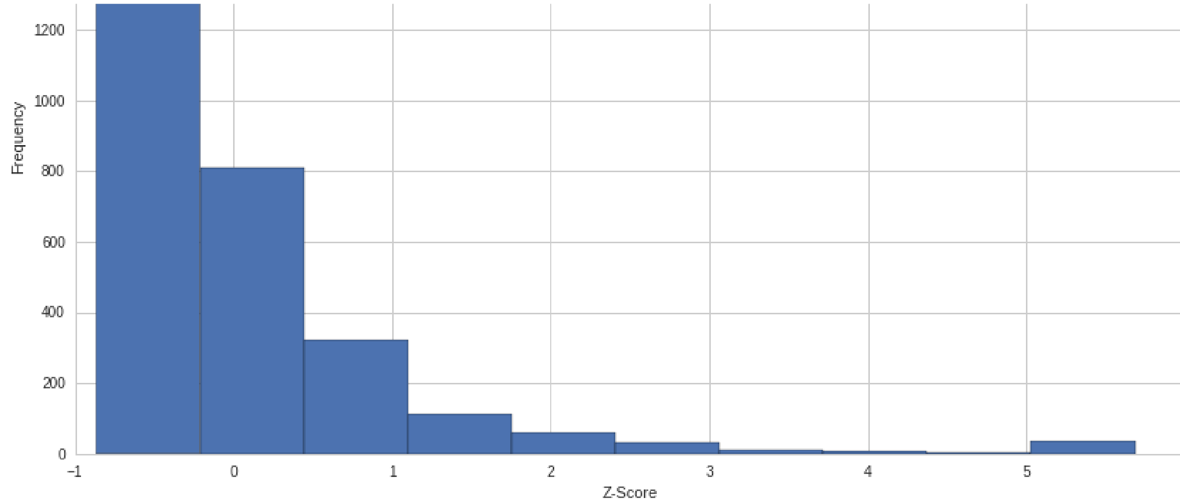
In [22]:
```
# Get some random data
X = np.random.normal(0, 1, 100)

# Put in some outliers
X[0] = 1000
X[1] = -1000

# Perform winsorization
print 'Before winsorization', np.min(X), np.max(X)
scipy.stats.mstats.winsorize(X, inplace=True, limits=0.01)
print 'After winsorization', np.min(X), np.max(X)
```

```
Before winsorization -1000.0 1000.0
After winsorization -1.83229157134 3.06505673118
```

This looks good, let's see how our book to price data looks when winsorized.

In [23]:
```
BTP = results['book_to_price'].loc['2011-1-3']
scipy.stats.mstats.winsorize(BTP, inplace=True, limits=0.01)
BTP_z = (BTP - np.mean(BTP)) / np.std(BTP)
BTP_z.dropna(inplace=True)

plt.hist(BTP_z)
plt.xlabel('Z-Score')
plt.ylabel('Frequency');
```





We need the returns for that day as well.

In [24]:
```
R_day = results['returns'].loc['2011-1-3']
```

Now set up our data and estimate F_j using linear regression.

In [25]:
```
constant = pd.TimeSeries(np.ones(len(R_day.index)), index=R_day.index)

df_day = pd.DataFrame({'R': R_day,
              'BTP_z': BTP_z,
              'Constant': constant})
df_day = df_day.dropna()

# Perform linear regression to get the coefficients in the model
F1 = regression.linear_model.OLS(df_day['R'], df_day['BTP_z']).fit().params
print F1
```

```
BTP_z    0.002035
dtype: float64
```

Finally, let's add another factor so you can see how the code changes.

In [26]:
```
MKT = results['market_cap'].loc['2011-1-3']
scipy.stats.mstats.winsorize(MKT, inplace=True, limits=0.01)
MKT_z = (MKT - np.mean(MKT)) / np.std(MKT)

constant = pd.TimeSeries(np.ones(len(R_day.index)), index=R_day.index)

df_day = pd.DataFrame({'R': R_day,
              'BTP_z': BTP_z,
              'MKT_z': MKT_z,
              'Constant': constant})
df_day = df_day.dropna()

# Perform linear regression to get the coefficients in the model
F1, F2 = regression.linear_model.OLS(df_day['R'], df_day[['BTP_z', 'MKT_z']]).fit().params
print F1, F2
```

```
0.00201998732392 0.000131209827362
```

To expand this analysis, you would simply loop through days, running this every day and getting an estimated factor return.

Using Fundamental Factor Modeling

Returns Prediction