The math question is to take an spheroid as defined by WGS84, take a heading and a distance and find a time zone.

Step one is to find the new longitude and lattiude based off the old one. For a spherical coordinate system:

$$d\vec{r} = dr\hat{r} + rd\phi\hat{\phi} + r\sin\phi d\theta\hat{\theta}$$

In WGS84, the earth is defined as a reference ellipsoid with parameters a and $1/f$, for the semi-major axis and the inverse flattness. The 'radius' r is defined as:

$$\rho(\phi) = \frac{a}{\sqrt{1-(2f-f^2)\sin^2\phi}}$$

Since r is only function of ϕ, we want to integrate only across that when possible to simplify it. $\hat{l}$ is the distance travelled under the defined heading.

$$\int_{\vec{r}_0}^{\vec{r}_1} \hat{l}\cdot d\vec{r} = \int_{\phi_0}^{\phi_1} \frac{d\rho}{d\phi}d\phi + \rho d\phi + \rho\sin\phi\frac{d\theta}{d\phi}d\phi$$

The two unknown functions $\frac{d\rho}{d\phi}$ and $\frac{d\theta}{d\phi}$ can be defined from the definition of the ellipsoid surface $\rho(\phi)$ and the heading (a constant h) respectively.

$$l = \int_{\vec{r}_0}^{\vec{r}_1} \hat{l}\cdot d\vec{r} = \rho(\phi_1) - \rho(\phi_0) + \int_{\phi_0}^{\phi_1} \rho + h\rho\sin\phi d\phi$$

Ideally, this would have an analytic solution, but the alone ρ term makes it (by definition) an elliptic integral (specifically the incomplete elliptic integral of the first kind). The secondary term can be integrated using a change of variables leading to an inverse tangent function. This unfortunately makes solving for ϕ an optimization problem, as an analytic solution cannot be calculated.

$$l = \int_{\vec{r}_0}^{\vec{r}_1} \hat{l}\cdot d\vec{r} = \rho(\phi_1) - \rho(\phi_0) + a(F(\phi_1, 2f-f^2) - F(\phi_0, 2f-f^2)) + \int_{\phi_0}^{\phi_1} h\rho\sin\phi d\phi$$

The last term can be dealt with separately. Writing ρ as a function of cos to properly change variables.

$$h\int_{\phi_0}^{\phi_1} \frac{a}{\sqrt{1-(2f-f^2)\sin^2\phi}}\sin\phi d\phi = ha\int_{\phi_0}^{\phi_1} \frac{\sin\phi}{\sqrt{(1-f)^2+(2f-f^2)\cos^2\phi}}d\phi$$

With $u = \cos\phi$

$$h\int_{\phi_0}^{\phi_1} \frac{a}{\sqrt{1-(2f-f^2)\sin^2\phi}}\sin\phi d\phi = ha\int_{u_1}^{u_0} \frac{1}{\sqrt{(1-f)^2+(2f-f^2)u^2}}du$$

$$= \frac{ha}{1-f}\int_{u_1}^{u_0} \frac{1}{\sqrt{1+\frac{(2f-f^2)}{(1-f)^2}u^2}}du$$

$g = \sqrt{\frac{(2f-f^2)}{(1-f)^2}}u$

$$= \frac{ha}{\sqrt{2f-f^2}}\int_{g_1}^{g_0} \frac{1}{\sqrt{1+g^2}}dg = \frac{ha}{\sqrt{2f-f^2}}(\sinh^{-1}(g_0) - \sinh^{-1}(g_1))$$

Using the definitions of g and u the original definition in ϕ can be written out. This function of $\tan^{-1}$, F and $\rho(\phi)$ will be minimized for a specified distance l to yield the proper ϕ_1. With the heading, this can be used to calculated θ_1. These can be used for time zone.

One thing to note is that great care must be taken going over a pole. I am not sure if it will break so I will likely have to code in a special try catch for it.