SUPER STANDARDS

SS-1

If $A+B=90°$ are called complimentary angles.

(i) $\sin^2 A + \cos^2 B = 1$

(ii) $\cos^2 A + \cos^2 B = 1$

(iii) $\tan A \tan B = 1$

(iv) $\cot A \cot B = 1$

(v) $\sin A = \cos B$; $\cos A = \sin B$

(vi) $\tan A = \cot B$; $\cot A = \tan B$.

Problems (SS-1)

(1) $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 90°$

(a) $\frac{19}{2}$ (b) 9 (c) $17/2$ (d) 8

Sol: Shortcut (i) Angles → A.P → Sum $= 90°$ (F.A + L.A)

Step-1 No of terms

$n = \frac{L-F}{d} + 1$ (or) $n = \frac{L.A}{F.A}$ } used for consecutive terms.

Step ②: $\frac{\text{no of terms } (n)}{2}$ + extra terms.

Sol: $n = \frac{\cancel{85}^{17}}{\cancel{5}} = \frac{17}{2} + \sin^2 90°$

$= \frac{17}{2} + 1 = \frac{19}{2}$.

→ $\cos^2 5° + \cos^2 10° + \cos^2 15° + \ldots + \cos^2 90° =$ ________

Sol: $\frac{\cancel{85}}{\cancel{5}}$ $\frac{17}{2} + 0 = 17/2$.

→ $\sin^2 3° + \sin^2 6° + \sin^2 9° + \ldots + \sin^2 87°$

Sol: $\frac{\cancel{87}^{29}}{\cancel{3}} = 29$.

→ $\cos^2 9^\circ + \cos^2 18^\circ + \cos^2 27^\circ + \ldots\ldots + \cos^2 81^\circ + \cos^2 90^\circ = \_\_\_\_$

Sol $\frac{81+9}{9} = \frac{9}{2} + 0 = 9/2$

→ $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \sin \ldots\ldots + \sin^2 180^\circ = \_\_\_\_$

Sol $2\left(\frac{n}{2}\right) + \sin^2 90^\circ + \sin^2 180^\circ$

$= \frac{85}{5}$ (17) $= 17 + 1 + 0 = 18$

→ $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 360^\circ = \_\_\_\_$

Sol $2\left(\frac{n}{2}\right) + \cos^2 90^\circ + \cos^2 180^\circ + \cos^2 270^\circ + \cos^2 360^\circ$

$2\left(\frac{85}{5}\right)$ (17) $= \cancel{34 + 0 + (-1) + 0 + 1}$ $\quad 34 + 0 (-1)^2 + 0 + (1)^2$

$= \cancel{34 + 0 + (-1) + 0 + 1}$ $\quad = 34 + 1 + 1$

$= \cancel{34}$ $\quad = 36$

→ $\cot\left(\frac{\pi}{24}\right) \cot\left(\frac{3\pi}{24}\right) \cot\left(\frac{5\pi}{24}\right) \cot\left(\frac{7\pi}{24}\right) \cot\left(\frac{9\pi}{24}\right) \cot\left(\frac{11\pi}{24}\right) = \_\_\_\_$

Sol $\frac{11\pi}{24} + \frac{\pi}{24} = \frac{12\pi}{24} = \frac{\pi}{2}$

$\cot A \cdot \cot B = 1$

$\Rightarrow 1 \times 1 \times 1 = 1$

→ $\tan\left(\frac{\pi}{24}\right) \tan\left(\frac{3\pi}{24}\right) \ldots\ldots \tan\left(\frac{11\pi}{24}\right) \tan\left(\frac{16\pi}{24}\right) = \_\_\_\_$

Sol $1 \times \tan\frac{16\pi}{24} = \frac{\tan 2\pi}{3} = \tan\left(\frac{2\pi}{3}\right) = \frac{2 \times 180}{3}$ (60)

$= \tan 120^\circ$

$= \tan(90 + 30^\circ)$

$= -\cot 30^\circ = -\sqrt{3}$

→ $\ln(\tan 1^\circ) + \ln(\tan 2^\circ) + \ln(\tan 3^\circ) + \ln \ldots\ldots + \ln\tan(89^\circ)$.

Sol $\ln 1 \times 1 \times 1 \times 1 \ldots 1$ $\quad \log 1$

$\cdot \tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ$

$1 \times 1 \ldots$

$-\pi$

ii, iii, iiii

$\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \cdot --- \tan 89^\circ$

$1 \times 1 --- \times 1 \times 1 = 1$

3 – II   If $A+B = 180^\circ$ (A, B are supplementary angles) then

(i) $\cos A + \cos B = 0$

(ii) $\sin A - \sin B = 0$

(iii) $\tan A + \tan B = 0$

(iv) $\cot A + \cot B = 0$

$\cos 0^\circ + \cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{2\pi}{7}\right) + ---- \cos\left(\frac{7\pi}{7}\right) = \_\_\_\_$

Sol: $0 + \frac{7\pi}{7} = \pi$

$0 + 0 + ---- 0 + 0 = 0$

$\tan\left(\pi/7\right) + \tan\left(\frac{2\pi}{7}\right) + \tan\left(3\pi/7\right) + ------ + \tan\left(\frac{6\pi}{7}\right) = \_\_\_\_$

$\frac{6\pi}{7} + \frac{\pi}{7} = \frac{7\pi}{7} = \pi$

$= 3 \times 0 = 0$

S.S – TH   If $A+B = 45^\circ / 225^\circ / ---- (4n+1)\cdot\frac{\pi}{4}$: $n \in I$ then

(i) $(1+\tan A)(1+\tan B) = 2$

(ii) $(1-\cot A)(1-\cot B) = 2$

If $A+B = 135^\circ / 315^\circ / ---- (4n-1)\frac{\pi}{4}$: $n \in I$ then

(i) $(1+\cot A)(1+\cot B) = 2$

(ii) $(1-\tan A)(1-\tan B) = 2$

$\Rightarrow (1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ) ---- (1+\tan 45^\circ) = 2^n \Rightarrow n = \_\_\_\_$

Sol: $1+44 = 45 \Rightarrow 2^{22} \cdot (1+45^\circ)$

$= 2^{22} \cdot 2 = 2^{23}$

$\Rightarrow \frac{(1+\cot 22^\circ)(\cot 23^\circ)}{\cot 22^\circ \cot 23^\circ}$ $(1+\tan A)(1+\tan B) = 2$

$1+\frac{1}{\cot A}$

Sol 2 $\frac{\cot A+1}{\cot A}$ $\frac{\cot B+1}{\cot B}$ $\frac{(1+\cot 22^\circ)(\cot 23^\circ+1)}{\cot 22^\circ \cot 23^\circ}$

$\Rightarrow$ Eamcet

$\Rightarrow$ If $A < \frac{\pi}{16}$; $(1+\tan A)(1+\tan 4A) = 2$ then $A =$ ______

Sol

$A + 4A = \frac{\pi}{4}$

$5A = \frac{\pi}{4} \Rightarrow A = \frac{\pi}{20}$

S-S. IV If $A+B = 90^\circ$ then $\boxed{\tan A - \tan B = 2\tan(A-B)}$

(i) $\frac{\tan 70^\circ - \tan 20^\circ}{\tan 50^\circ} =$ ______

Sol $\frac{2\tan 50^\circ}{\tan 50^\circ} = 2$

(ii) $\tan 50^\circ - \tan 40^\circ = 2\cot(k)^\circ \Rightarrow k =$ ______

Sol $2\tan 10^\circ = 2\cot k$

$\cancel{2\cot 8}$

$\cancel{2}\cot 80^\circ = \cancel{2}\cot k$

$\boxed{k = 80^\circ}$

$\Rightarrow$ If $\alpha+\beta = \pi/2$; $\beta+\gamma = \alpha$ then $\tan\beta + 2\tan\gamma =$ ______

Sol $\tan\alpha - \tan\beta = 2\tan(\alpha-\beta)$

$\alpha - \gamma = \beta$

$\tan\alpha - \tan\beta = 2\tan\gamma$

$\alpha = \beta+\gamma$

$\tan\beta + 2\tan\gamma = \tan\alpha$

$\alpha - \beta = \gamma$

$\tan A + \tan B + k \tan A \tan B = k$; where $k = \tan(A+B)$.

$\tan A - \tan B - k \tan A \tan B = k$; where $k = \tan(A-B)$.

$\tan 56^\circ - \tan 11^\circ - \tan 56^\circ \tan 11^\circ = \underline{+1}$

$\tan 15^\circ + \tan 30^\circ + \tan 30^\circ + \tan 15^\circ = \underline{1}$

$\tan 110^\circ + \tan 10^\circ - \sqrt{3} \tan 10^\circ \tan 110^\circ = \underline{-\sqrt{3}}$

$\tan 55^\circ - \tan 25^\circ + \frac{1}{\sqrt{3}} \tan 55^\circ \tan 25^\circ = \underline{-\frac{1}{\sqrt{3}}}$,,

S - VI :- (I) If $A+B+C = 180^\circ / 360^\circ / 0^\circ / ---- n\pi$ $n \in I$ then

(1) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ $(\Sigma \tan A = \Pi \tan A)$.

(2) $\Sigma \cot A \cot B = 1$

(II) If $A+B+C = 90^\circ / 270^\circ / ---- (2n+1)\pi/2$: $n \in I$ then.

(1) $\cot A + \cot B + \cot C = \cot A \cot \cot C$.

(2) $\Sigma \tan A \tan B = 1$.

→ If $A+B+C = 360^\circ$, then $\cot(A/4) + \cot(\frac{B}{4}) + \cot(\frac{C}{4}) = \cancel{\cot(\frac{A}{4}) \cot(\frac{B}{4})}$ ______

Sol) $A+B+C = 360^\circ$

$\Rightarrow \frac{A}{4} + \frac{B}{4} + \frac{C}{4} = 90^\circ$.

$\therefore \cot \frac{A}{4} \cdot \cot \frac{B}{4} \cdot \cot \frac{C}{4} = \Pi \cot \frac{A}{4}$. ,,

→ $\tan 11^\circ \tan 35^\circ + \tan 35^\circ \tan 44^\circ + \tan 44^\circ \tan 11^\circ = 1$

Sol) $\cancel{11+35+35+}$ $11 + 35 + 44 = 90$.

$\Rightarrow \Sigma \tan A \tan B = 1$ $= 1$,,

→ $\cancel{\tan 5x + \tan 5x + \tan}$ → $\tan 5x - \tan 3x - \tan 2x$.

Sol) $\tan 5x + \tan(-3x) + \tan(-2x)$ $\therefore 5x - 3x - 2x = 0$

$= \cancel{\Sigma \tan A}$ $\Sigma \tan 5x = \tan 5x \tan(-3x) \tan(-2x)$,,

$= \Pi \tan 5x$,

→ In a Δ^{le} ABC $\sum \cot A \cot B$ = ______

Sol 1 ,,

→ In a Δ^{le} ABC $\tan A + \tan B + \tan C$ = ______

Sol $\sum \tan A = \pi \tan A$,,

SS - VII ① If $0 < k\theta < \pi$ then $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}+\sqrt{2+2\cos(k\theta)}}}}$

$= 2\cos\left(\frac{k\theta}{2^n}\right)$ → last angle → no of sq roots

② $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+n \text{ roots}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$

(i) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$ = ______

Sol
$2\cos\left(\frac{\pi}{2^{4+1}}\right) = \cancel{2\cos\left(\frac{\pi}{2^5}\right) = \cos\frac{\pi}{32}}$,,

$= 2\cos\frac{\pi}{2^5} = 2\cos\left(\frac{\pi}{32}\right)$,,

(ii) $0^\circ < \theta < \frac{\pi}{8}$ then $\sqrt{2+\sqrt{2+\sqrt{2+2\cos(8\theta)}}}$ = ______

Sol
$2\cos\frac{8\theta}{2^3} = 2\cos\theta$,,

$\sqrt{2+\sqrt{2+2\cos\theta}}$

$2\cos\frac{\theta}{2^2} = 2\cos\frac{\theta}{4}$,,

→ In a Δ^{le} ABC $\Sigma \cot A \cot B$ = ______

Sol 1 ,,

→ In a Δ^{le} ABC $\tan A + \tan B + \tan C$ = ______

Sol $\Sigma \tan A = \pi \tan A$,

SS – VII ① If $0 < k\theta < \pi$ then $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2}+\sqrt{2+2\cos(k\theta)}}}}$

$= 2\cos\left(\frac{k\theta}{2^n}\right)$ → last angle → no of sq roots

② $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+n \text{ roots}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$

(i) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$ = ______

Sol $2\cos\left(\frac{\pi}{2^{4+1}}\right) = \cancel{2\cos\left(\frac{\pi}{2^5}\right)} = \cancel{\cos\frac{\pi}{32}}$,,

$= 2\cos\frac{\pi}{2^5} = 2\cos\left(\frac{\pi}{32}\right)$,,

(ii) $0° < \theta < \frac{\pi}{8}$ then $\sqrt{2+\sqrt{2+\sqrt{2+2\cos(8\theta)}}}$ = ______

Sol $2\cos\frac{8\theta}{2^3} = 2\cos\theta$,,

iii) $\sqrt{2+\sqrt{2+2\cos\theta}}$

Sol $2\cos\frac{\theta}{2^2} = 2\cos\frac{\theta}{4}$,

S.S – $\tan\theta$ + =

S-S-8 :- G.P

$\tan\theta + 2\tan\theta + 4\tan\theta + 8\tan\theta + \cdots + 2^{n-1}\tan(2^{n-1}\theta) + 2^n\cot(2^n\theta)$

$= \cot\theta$.

$(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)\cdots(1+\sec 2^n\theta) = \tan 2^n\theta \cdot \cot\theta$.

$\tan(\theta/2)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\cdots(1+\sec 2^n\theta) = \tan(2^n\theta)$

$(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)\cdots(2\cos 2^{n-1}\theta-1) = \frac{2\cos 2^n\theta+1}{2\cos\theta+1}$

$\tan\theta + \tan\left(\frac{2\pi}{3}+\theta\right) + \tan\left(\frac{4\pi}{3}+\theta\right) = 3\tan 3\theta$

$\cot\theta + \cot\left(\frac{2\pi}{3}+\theta\right) + \cot\left(\frac{4\pi}{3}+\theta\right) = 3\cot(3\theta)$.

If $A+B=60^\circ$ then (i) $\sin^2 A + \sin^2 B + \sin A \sin B = \frac{3}{4}$.

$A = 30^\circ$ $B = 30^\circ$ $\quad \frac{1}{4} + \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2}$

(ii) $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$.

(8) = If $A - B = 60^\circ$ then (i) $\sin^2 A + \sin^2 B - \sin A \sin B = \frac{3}{4}$.

(ii) $\cos^2 A + \cos^2 B - \cos A \cos B = \frac{3}{4}$

(9) $\sin^2\theta + \sin^2(60^\circ-\theta) + \sin^2(60^\circ+\theta) = 3/2$.

(10) $\cos^2\theta + \cos^2(60^\circ-\theta) + \cos^2(60^\circ+\theta) = 3/2$

Important bits based on SS 8 :-

→ $\tan\left(\frac{\pi}{5}\right) + 2\tan\left(\frac{2\pi}{5}\right) + 4\tan\left(\frac{4\pi}{5}\right) + 8\cot\left(\frac{8\pi}{5}\right) =$ ________

Sol: $\cot\frac{\pi}{5}$

→ $\tan x + 2\tan 2x + 4\tan 4x + 8\tan 8x =$ ________

Sol: $\cot x - 16\cot 16x$.

→ $(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta) = \tan 8\theta \cdot \cot\theta$.

→ $\tan\left(\frac{\pi}{16}\right) + 2\tan\left(\frac{\pi}{8}\right) + 4\tan\left(\frac{\pi}{4}\right) =$ ______

Sol $\cot\left(\frac{\pi}{16}\right) - 8\cancel{\cot\left(\frac{\pi}{2}\right)}$ 0

$= \cot\left(\frac{\pi}{16}\right)$ "

→ $\cos^2 16^\circ + \cos^2 76^\circ - \cos 16^\circ \cos 76^\circ =$ ______

Sol $76 - 16 = 60^\circ$ ∴ $\frac{3}{4}$ "

→ $\frac{\sin^3 10^\circ + \cos^3 20^\circ}{\sin 10^\circ + \cos 20^\circ}$

Sol $a^3+b^3 = (a+b)(a^2+b^2-ab)$

$\Rightarrow \frac{a^3+b^3}{a+b} = \boxed{a^2+b^2-ab}$

$= \sin^2 10^\circ + \cos^2 20^\circ - \sin 10^\circ \cos 20^\circ$

$= \sin^2 10^\circ + \sin^2 70^\circ - \sin 10^\circ \sin 70^\circ$

$= 70 - 10 = 60^\circ$

$= \frac{3}{4}$

→ If $\tan\theta + \tan(120^\circ+\theta) + \tan(240^\circ+\theta) = 3$ then $\tan 3\theta =$ ______

(a) 1 (b) 2 (c) 3 (d) -1

$\cancel{3}\tan 3\theta = \cancel{3}$

$\tan 3\theta = 1$

"

TYPE:- 5 Formula based & trig values

* SUPER PROBLEMS *

apter -1

$\tan(-945^\circ) = -\tan 945^\circ = -\tan(90\times10+45^\circ)$
$= -\cot 45^\circ = -1$

$\sin(-495^\circ) = -\sin 495^\circ = -\sin(90\times5+45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$

$\cot(840^\circ) = \cot(90\times9+30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$

$\sec(-885^\circ) = \sec(90\times9-15^\circ) = \operatorname{cosec} 15^\circ =$

$\operatorname{cosec}(-1200^\circ) = -\operatorname{cosec}(90\times13+30^\circ) = -\sec 30^\circ = -\frac{2}{\sqrt{3}}$

$\cos(1050^\circ) = \cos(90\times11+60^\circ) = \sec 60^\circ = 2$

$\sin(-1110^\circ) = -\sin 1110^\circ = -\sin(90\times12+30^\circ)$
$= -\sin 30^\circ = -\frac{1}{2}$

495 − 450 = 45; 990 − 885 = 105; 900 − 885 = 15; 13; 1200 − 1170 = 30; 1050 − 990 = 60; 1110 − 1080 = 30

ick -1

1) $T\text{-}R(k\pi \pm \theta)$ → k is odd (k = 1) = $T.r(\pi\pm\theta)$

→ k is even = $T.r(\pm\theta)$ (neglect $k\pi$)

→ ~~If both present then cancel k of both.~~

2) $\tan/\sin(\pm\pi, \pm 2\pi, \pm 3\pi \dots \pm n\pi) = 0$

3) $\cos(\pm\pi, \pm 3\pi, \pm 5\pi \dots) = -1$

4) $\cos(0^\circ, \pm 2\pi, \pm 4\pi \dots) = +1$

5) $\tan$ (odd multiple of "$\pi/2$") = ~~not~~ undefined.

6) $\cot$ (mult of "π") = undefined.

7) $\sin(4n+1)\pi/2 = 1$

8) $\sin(4n-1)\pi/2 = -1$

9) $\cos(2n+1)\pi/2 = 0$

→ $\sin(131\pi) = 0$

→ $\cos(111\pi) = -1$

→ $\tan(-33\pi) = 0$

→ $\sin(-1345\pi/2) = \frac{\overset{336+(1)}{1345}}{4} = -1$

→ $\tan(19\pi/2) = \cancel{0}$ U.D

→ $\cos(-2014\pi) = +1$

→ $\cot(2014\pi) = U.D$

→ $\sin(33\pi/2) = 1$ 4n8+1

→ $\sin\left(35\frac{\pi}{2}\right) = -1$ 4n8-1

→ $\cos(1999\pi/2) = 0$

→ $\cos(-2119\pi/2) = 0$

→ $\sec(2120\pi) + \sec(2121\pi - \theta)$

Sol $\sec(\cancel{2120}\pi) + \sec(\cancel{2121}\pi - \theta)$

$= \sec\theta + \sec(\pi - \theta)$

$= \sec\theta + -\sec\theta = 0$.

(even π) cancel both $\times$ $\times$

(odd π) cancel ~~only~~ only ~~k~~ k $\times$ $\times$

⇒ $\tan(-3141\pi - \theta) + \tan(2130\pi - \theta)$

Sol ~~$-\tan(3141\pi+\theta) + \tan(2130\pi - \theta)$~~

~~$= \pi \cdot = \tan\theta +$~~

Sol $-\tan(\cancel{3141}\pi + \theta) + \tan(\cancel{2130}\pi - \theta)$

$= -\tan\theta - \tan\theta = -2\tan\theta$.

Trick - 2 :- $\left(k \cdot \frac{\pi}{p} \pm \theta\right) = T\text{-}R\left(Q\cdot\pi + R\frac{\pi}{p} \pm \theta\right)$

1. $\tan\left(\frac{31\pi}{9}\right)$

Sol $9)31(3$ $\frac{27}{4}$ $= \tan\left(\cancel{3}\pi + 4\frac{\pi}{9}\right)$

$= \tan\frac{4\pi}{9} = \frac{4\times\cancel{180}^{20}}{\cancel{9}}$

$= \tan 80^\circ = \cot 10^\circ$.

2) $\cos(-11\pi/4) = \frac{-1}{\sqrt{2}}$.

Sol $+\cos\frac{11\pi}{4} = \frac{4)11(2}{\frac{8}{3}} = \frac{\cancel{2\pi}+3\pi}{4} = +\cos\left(\frac{3\pi}{4}\right) = \frac{3\times\cancel{180}^{90}}{\cancel{4}_2}$

$= +\cos(90+45) = +\frac{1}{\sqrt{2}}$, $= +\cos 135$

$= +\cos\frac{\cancel{270}\,135°}{\cancel{2}}$ $= $ ~~$\cos\frac{3\times180}{4}$~~

$\cot(111\pi/2 + \theta)$ $\qquad \frac{55}{\frac{111}{2}}$ $\qquad R = 1$, $Q = 55$ $\qquad$ ②

$\cot(\cancel{55}\pi + \frac{\pi}{2} + \theta)$

$\cot(\frac{3\pi}{2} + \theta)$

$= \cot(270° + \theta)$

$= \cot(90 \times 3 + \theta)$

$= -\tan\theta$,,

$\rightarrow \sin(71\,\pi/2 - \theta)$ $\qquad$ 2) $71(35$ Q, 61, 11, 10, 1 R, $\sin 270$, $90 \times 3 + \theta$, $-\cos\theta$

$= \sin(\cancel{35}\pi + \frac{\pi}{2} - \theta)$

$= \sin(\frac{3\pi}{2} - \theta)$

$= -\cos\theta$,,

T-P-(3)

$\left\{ \begin{array}{l} \sin\theta = \frac{c}{b} \\ \cos\theta = \frac{m}{n} \\ \tan\theta = p/q \end{array} \right.$ ... To find Remain's T-R

① $0° < \theta < \pi/2$ $\quad \sin\theta = \frac{60}{61}$, $\tan\theta =$ ______

Sol $\cos\theta = \frac{\sqrt{60+61 \times 61-60}}{61}$ (S. Trick)

$\cos\theta = \frac{\sqrt{121}}{61} = \frac{11}{61}$. $\qquad \tan\theta = \frac{60}{11}$,,

② If $\pi < \theta < 3\pi/2$; $\cos\theta = \frac{8}{17}$ then $\sin\theta =$ ______

Sol $\theta = 3^{rd}$ $\qquad \boxed{\sin\theta = \frac{-15}{17}}$

$\frac{\sqrt{25 \times 9}}{17} = \cancel{\frac{5 \cdot 3}{17} = \frac{2}{17}}$,, $= -\frac{5 \times 3}{17} = \frac{-15}{17}$,,

③ If $\tan\theta = \frac{3}{4}$ then $\sin\theta =$ ______

Sol $\tan\theta$ + in 1st & 3rd quadrant $= \pm\frac{3}{5}$,,

⇒ ④ - $\sec\theta = -5/3$ ($\theta \in Q_2$) then $\sin\theta$ = ______

Sol

$\cos\theta = -3/5$, $\sin\theta$ = $= \frac{4}{5}$

⇒ If $0 < \theta < 2\pi$, $\csc\theta + 2 = 0$ then θ = ______

Sol

$\csc\theta = -2$ → Q_3 $\theta = \pi + \pi/6 = \frac{7\pi}{6}$

→ Q_4 $\theta = 2\pi - \pi/6 = 11\pi/6$

$\sin\theta = -\frac{1}{2}$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

∴ $\theta = \frac{7\pi}{6}$ or $11\pi/6$

⇒ If $\sec\theta + \tan\theta = \frac{1}{5}$ then $\sin\theta$ = ______ & θ lies in ______ Q.

Sol

$\sec^2\theta - \tan^2\theta = 1$

$$\sec\theta + \tan\theta = \frac{1}{5}$$

$$\sec\theta - \tan\theta = 5$$

$$2\sec\theta = \frac{1}{5} + 5$$

$$2\sec\theta = \frac{26}{5} \Rightarrow \sec\theta = \frac{26}{5} \times \frac{1}{2} = \sec\theta = \frac{13}{5} \rightarrow Q_1, Q_4$$

$$2\tan\theta = \frac{1}{5} - 5 \Rightarrow \frac{1-25}{5} = \frac{-24}{5}$$

$$2\tan\theta = \frac{-24}{5} \Rightarrow \tan\theta = \frac{-24}{5} \times \frac{1}{2} = \frac{-12}{5} \rightarrow Q_2, Q_4$$

Common is Q_4.

∴ θ lies in IVth Quadrant

$\sin\theta = -\frac{12}{13}$ ⇒ $\sin\theta = -\frac{12}{13}$

If $\csc x - \cot x = 2013$ then Quadrant in which θ lies ______

Sol

$2\csc x = 2013 + \frac{1}{2013}$ = ⊕ → Q_1, → Q_2

$2\cot x = 2013 - \frac{1}{2013}$ = ⊖ → Q_2, → Q_4

∴ $\boxed{\theta \text{ lies in } Q_2}$

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$$ then θ lies in ______ Quadrant. (3)

$$\sqrt{\frac{1+\cos\theta}{(1-\cos\theta)}\cdot\frac{1+\cos\theta}{1+\cos\theta}} = \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{1-\cos^2\theta}} = \frac{|1+\cos\theta|}{|\sin\theta|} \qquad (\because \sqrt{x^2} = |x| \neq x)$$

$$\frac{|1+\cos\theta|}{|\sin\theta|} = \operatorname{cosec}\theta + \cot\theta \qquad \text{Here } \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$$

$\therefore$ $|\sin\theta|$ is positive then θ lies in I^{st} or II^{nd} Quadrant

$\therefore$ $\theta \in I$ or II Quadrant.

$\Rightarrow$ $$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$$ then θ lies in ______ Quadrant.

Sol $$\frac{\sqrt{(1-\sin\theta)^2}}{\sqrt{1-\sin^2\theta}} = \frac{|1-\sin\theta|}{|\cos\theta|} = \sec\theta - \tan\theta.$$

$\therefore$ $\theta \in I^{st}$ & IV^{th} Quadrant.

$\Rightarrow$ $K = (1+\sec A)(1+\sec B)(1+\sec C) = (\sec A - 1)(\sec B - 1)(\sec C - 1)$ then $k = ?$

(a) ± 1 (b) ± 2 (c) ± 3 (d) $\pm \tan A \tan B \tan C$.

Sol $K = A = B$ $\qquad \sec^2 A - \tan^2 A = 1$

$k^2 = AB \Rightarrow (\sec^2 A - 1)(\sec^2 B - 1)(\sec^2 C - 1)$

$k^2 = \tan^2 A \tan^2 B \tan^2 C$

$\therefore k = \pm \tan A \tan B \tan C$

$\Rightarrow$ If $k = (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$.

Sol $k^2 = (\sec^2 A - \tan^2 A) \ldots \ldots$

$= (1)(1)(1)$

→ If $8\sin^2 x + 5\cos^2 x = 3$ then $\cot x =$ ______

Sol $8\sin^2 x + 5\cos^2 x = 3$

divide by $\cos^2 x$ on both sides

$= 8\tan^2 x + 5 = \frac{3\sec^2 x}{\cancel{\sec^2 x}}$

$= 8\tan^2 x + 5 = 3(1+\tan^2 x)$

$= 8\tan^2 x + 5 = 3 + 3\tan^2 x \Rightarrow 5\tan^2 x = -2 \therefore \tan^2 x = -\frac{2}{5}$

$x \in \phi$

$\cot x =$ no sol^n.

→ If $\sin x + \sin^2 x = 1$ then $\cos^8 x + 2\cos^6 x + \cos^4 x =$ ______

Sol ~~$\sin x = 1 - \sin^2 x$~~ ~~(S.O.B.S)~~ ~~$= \cos^2 x$~~

$\sin x = 1 - \sin^2 x = \cos^2 x$

(S.O.B.S)

$\sin^2 x = \cos^4 x$

$1 - \cos^2 x = \cos^4 x$

$1 = \cos^4 x + \cos^2 x$

(S.O.B.S)

$1 = \cos^8 x + \cos^4 x + 2\cos^6 x$ //

$= 1$

→ If $\cos x + \cos^2 x = 1$ then $\sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x$.

Sol $\cos x = 1 - \cos^2 x = \sin^2 x$

S.O.B.S

$1 - \sin^2 x = \sin^4 x$

$1 = \sin^4 x + \sin^2 x$

C.O.B.S

$1^3 = \sin^{12} x + \sin^6 x + 3(\sin^6 x)(\sin^4 x + \sin^2 x)$

$1 = \sin^{12} x + \sin^6 x + 3\sin^{10} x + 3\sin^8 x$ //

$\sin x + \sin^2 x = 1$ then $a\cos^{12}x + b\cos^8 x + c\cos^{10}x + d\cos^6 x - 1 = 0$ then

$\frac{b+c}{a+d} = ?$

$\sin x = 1 - \sin^2 x$

S.O.B.S

$1 - \cos^2 x = \cos^4 x$

$1 = \cos^4 x + \cos^2 x$

C.O.B.S

$1 = \cos^{12}x + \cos^6 x + 3\cos^6 x\,(\cos^4 x + \cos^2 x)$

$= \cos^{12}x + \cos^6 x + 3\cos^{10}x + 3\cos^8 x - 1 = 0$

$a = 1,\ b = 3,\ c = 3,\ d = 1$

$\frac{b+c}{a+d} = \frac{3+3}{1+1} = 3$

⇒ If $\sin x + \sin^2 x + \sin^3 x = 1$ then $\cos^6 x - 4\cos^4 x + 8\cos^2 x =$ ______

$\sin x + \sin^3 x = 1 - \sin^2 x$

$\sin x + \sin^3 x = \cos^2 x$

S.O.B.S

$1 - \cos^2 x + \sin^6 x = \cos^2 x$

$1 - \cos^2 x + (1 - \cos^2 x)^3 = \cos^2 x$

$1 - \cos^6 x - 3(1)\cos^2 x\,(1 - \cos^2 x) = \cos^2 x - 1 + \cos^2 x$

$1 - \cos^6 x - 3\cos^2 x + 3\cos^4 x - \cos^2 x + 1 - \cos^2 x = 0$

$1 - \cos^6 x - 5\cos^2 x + 3\cos^4 x + 1 = 0$

$-\cos^6 x - 5\cos^2 x + 3\cos^4 x + 2 = 0$

$\cos^6 x + 5\cos^2 x - 3\cos^4 x - 2 = 0$

Ans = 4

then eliminate θ

→ If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$

Sol: Note :- 1 parameter = 2 eqns
2 parameters = 3 eqns
elimination

two eqns are ⊥r to each others then

$x^2+y^2 = a^2\cos^2\theta + b^2\sin^2\theta + \cancel{2ab\sin\theta\cos\theta}$

$= a^2\sin^2\theta + b^2\cos^2\theta - \cancel{2ab\sin\theta\cos\theta}$

⇒ $x^2+y^2 = a^2+b^2$ .. (locus of circle)

Q2) (Exam)
⇒ If $m = \tan\theta + \sin\theta$; $n = \tan\theta - \sin\theta$ then $(m^2-n^2)^2$ = ______

(a) $4mn$ (b) $8mn$ ✓(c) $16mn$ (d) $16m^2n^2$.

Sol) $\theta = 45°$

$m = 1+\frac{1}{\sqrt{2}}$, $n = 1-\frac{1}{\sqrt{2}}$

LHS $(m^2-n^2)^2$

$= \left(\left(1+\frac{1}{\sqrt{2}}\right)^2 - \left(1-\frac{1}{\sqrt{2}}\right)^2\right)^2$
$a^2 \quad - \quad b^2$

$=$

$= \left(\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2 - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2\right)^2$

$= \left[\frac{2+1+4\sqrt{2}}{2} - \left(\frac{2+1-4\sqrt{2}}{2}\right)\right]^2$

$= \left(\frac{2+1+4\sqrt{2}-2-1+4\sqrt{2}}{2}\right)^2$

$= \frac{(4\sqrt{2})^2}{2^2} = \frac{16\times2}{4} = \frac{\cancel{32}\,8}{\cancel{4}} = 8.$

RHS $= 4\left(1+\frac{1}{\sqrt{2}}\right)\left(1-\frac{1}{\sqrt{2}}\right)$

$= 4\left(1-\frac{1}{2}\right)$

$= \cancel{4}^{2}\left(\frac{1}{\cancel{2}}\right)$

(a) $= 2$ ✗ (b) $4mn\times2$ $= 2\times2 = 4$ ✗ (c) $4mn\times4$ $= \cancel{4\times4=16}$ $2\times4 = 8$ ✓ (d) 16 ✗

If $x = r\cos\theta \cos\phi$; $y = r\sin\phi \cos\theta$; $z = r\sin\theta$ then $x^2+y^2+z^2 =$ ______

3 eq'ns → 2 parameters

$x^2+y^2+z^2 = r^2\cos^2\theta\cos^2\phi + r^2\sin^2\phi\cos^2\theta + r^2\sin^2\theta$

$= r^2[\cos^2\theta(1) + \sin^2\theta]$

$= r^2$.

If $x = c\tan\theta$; $y = c\cot\theta$ then eliminate para ______

$xy = c^2\tan\theta\cot\theta$ (multiply each other)

$\therefore xy = c^2$ → Reccongular curve

locus →

Compou C & D rule $\left(\text{If } \frac{a}{b} = \frac{c}{d} \implies \frac{a+b}{a-b} = \frac{c+d}{c-d} \right)$

(1) If $\frac{\tan\alpha}{\tan\beta} = \frac{m}{n}$ then $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{m+n}{m-n}$

(2) If $\frac{\sin\alpha}{\sin\beta} = \frac{m}{n}$ then $\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = \frac{m+n}{m-n}$

(3) If $\cot\alpha \cdot \cot\beta = \frac{m}{n}$ then $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{m+n}{m-n}$

(4) If $\frac{\cos\alpha}{\cos\beta} = \frac{m}{n}$ then $-\cot\left(\frac{\alpha+\beta}{2}\right)\cdot\cot\left(\frac{\alpha-\beta}{2}\right) = \frac{m+n}{m-n}$

ex: If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a}{b}$ then $\frac{\tan\alpha}{\tan\beta} =$ ______

So $\frac{\tan\left(\frac{\alpha+\beta+\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta-\alpha+\beta}{2}\right)} = \frac{a+b}{a-b}$

$= \frac{\tan\alpha}{\tan\beta}$

→ If $\cos(x-y) = 3\cos(x+y)$ then $\tan x \tan y =$ ______

Sol: $\frac{\cos(x-y)}{\cos(x+y)} = \frac{3}{1}$

$= -\cot\left(\frac{x-y+x+y}{2}\right)\cot\left(\frac{x-y-x-y}{2}\right)$

$= (+\cot x)(+\cot y) = \frac{3+1}{3-1} = \frac{4}{2} = 2.$

$= \tan x \tan y = \frac{1}{2}$

→ If $m\tan(\theta-30^\circ) = n\tan(\theta+120^\circ)$ then $\cos 2\theta =$ ______

Sol $\frac{m}{n} = \frac{\tan(\theta+120^\circ)}{\tan(\theta-30^\circ)} = \frac{\tan\alpha}{\tan\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{m+n}{m-n}$

$= \frac{\sin(\theta+120^\circ+\theta-30^\circ)}{\sin(\theta+120^\circ-\theta+30^\circ)} = \frac{\sin(2\theta+90^\circ)}{\sin(150^\circ)} = \frac{\cos 2\theta}{\frac{1}{2}}$

$= \frac{m+n}{m-n} = \frac{\cos 2\theta}{\frac{1}{2}} \Rightarrow 2\cos 2\theta = \frac{m+n}{m-n}$

$\therefore \cos 2\theta = \frac{m+n}{2(m-n)}$

If $\sin B = \frac{1}{5}\sin(2A+B)$ then $\frac{\tan(A+B)}{\tan A} =$ ______

$\cancel{\frac{\sin(A+B)}{\sin B}} = \frac{\sin(2A+B)}{\sin B} = \frac{5}{1}$

$\frac{\tan\left(\frac{2A+B+B}{2}\right)}{\tan\left(\frac{2A+B-B}{2}\right)} = \frac{\tan(A+B)}{\tan(A)} = \frac{5+1}{5-1} = \frac{6}{4} = \frac{3}{2}$

(6.)A (8)

If $\frac{\cos x}{\cos(x-2y)} = \lambda$ then $\tan(x-y)\ \tan y$ = ______

$-\cot\left(\frac{x+x-2y}{2}\right) \cdot \cot\left(\frac{x-x+2y}{2}\right) = \frac{\lambda}{1}$

$= -\cot\left(\frac{2x-2y}{2}\right)\ \cot\left(\frac{2y}{2}\right)$

$= -\cot(x-y)\ \cot y = \frac{\lambda+1}{\lambda-1}$

$\frac{-(-1+\lambda)}{-(\lambda+1)}$

$\frac{\lambda-1}{\lambda+1}$

$= \cancel{\tan(x-y)\ \tan y = -\left(\frac{\lambda-1}{\lambda+1}\right)}$

$= \cancel{\tan(x-y)\ \tan y = 1-\lambda}$

$= +\cot(x-y)\ \cot y = \frac{\lambda+1}{+(-\lambda+1)}$

$= \cancel{\cot r}$ $\boxed{\tan(x-y)\ \tan y = \frac{1-\lambda}{1+\lambda}}$

→ If $\tan\theta_1 = k\ \cot\theta_2$ then $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}$

$\cancel{\frac{\tan\theta_1}{\tan\theta_2}} = \cancel{\frac{\cot\theta_2}{\cot\theta_1}} = \frac{1}{k}$ $\qquad \frac{1}{k} = \frac{\cot\theta_2}{\tan\theta_1}$

$\cancel{\cot\theta_1 = k\cot\theta_2}$ $\qquad = \frac{1}{k} = \cot\theta_1 \cdot \cot\theta_2$

$\Rightarrow \cancel{\cot\theta_1}$

cot

$= \frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} = \frac{1+k}{1-k}$

→ If $3\tan A\ \tan B = 1$ then $2\cos(A+B)$ = ______

Sol $\tan A\ \tan B = \frac{1}{3}$

$\cot A\ \cot B = \frac{3+1}{3-1} \Rightarrow \frac{\cos(A-B)}{\cos(A+B)} = 2$

$= \frac{4}{2} = 2.$

$\Rightarrow \boxed{2\cos(A+B) = \cos(A-B)}$

Important problems on C & D rule (Eamcet Probl...

→ If $\cos\alpha = \frac{2\cos\beta - 1}{2-\cos\beta}$ $(0<\alpha, \beta<\pi)$; $\alpha+\beta = \pi$ then $\tan(\frac{\alpha}{2})$?

$\boxed{\beta = \pi - \alpha}$

Sol $\frac{\cos\alpha}{1} = \frac{2\cos\beta - 1}{2-\cos\beta}$

$\frac{\cos\alpha+1}{\cos\alpha-1} = \frac{2\cos\beta - 1 + 2 - \cos\beta}{2\cos\beta - 1 - 2 + \cos\beta} \Rightarrow \frac{\cos\beta+1}{3\cos\beta-3} = \frac{1+\cos\beta}{3\cos\beta-3}$

$= \frac{\cancel{2}\cos^2(\frac{\alpha}{2})}{-\cancel{2}\sin^2(\alpha/2)} \Rightarrow \frac{\cancel{2}\cos^2(\beta/2)}{+3(-\cancel{2}\sin^2\beta/2)} = \cot^2\alpha/2 = \frac{1}{3}\cot^2(\beta/2)$

$\cos\alpha+1$

$$\boxed{\begin{aligned}\cos\alpha+1 &= 2\cos^2\alpha/2\\ \cos\alpha-1 &= -2\sin^2\alpha/2\end{aligned}}$$

$\tan^2\alpha/2 = 3\tan^2\beta/2$

$\tan^2(\alpha/2) = 3\tan^2(\frac{\pi}{2}-\frac{\alpha}{2})$

$\tan^2(\frac{\alpha}{2})\tan^2(\frac{\alpha}{2}) = 3$

$\tan^4(\frac{\alpha}{2}) = 3$

~~$\tan\frac{\alpha}{2} = \pm 3^{\frac{1}{4}}$~~

$\boxed{\tan\frac{\alpha}{2} = 3^{\frac{1}{4}}}$ //

→ If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta}$ ~~tan~~ then $\tan(\theta/2) = ?$

Sol $\frac{\cos\theta}{1} = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta}$

$\frac{\cos\theta+1}{\cos\theta-1} = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta}$

$= \frac{\cancel{2}\cos^2\theta/2}{-\cancel{2}\sin^2\theta/2} \Rightarrow -\cot^2\theta/2 = \frac{\cancel{\cos\alpha} - \cancel{\cos\beta} + 1 - \cancel{\cos\alpha\cos\beta}}{\cos\alpha - \cos\beta - 1 + \cos\alpha\cos\beta}$

$= \frac{\cos\alpha(1-\cos\beta) + (1-\cos\beta)}{\cos\alpha(1+\cos\beta) - (1+\cos\beta)}$

Most Important formulas :-

(1) $\sum \tan A \tan B = \dfrac{1-\cos(A+B+C)}{\cos A \cos B \cos C}$

(2) $\sum \cot A \cot B = 1 + \dfrac{\sin(A+B+C)}{\sin A \sin B \sin C}$

:- Advanced level problems :-

(1) In a $\triangle^{le}$ ABC $\sin A \sin B \sin C = \frac{1}{3}$ then $\sum \cot A \cot B =$ ______

Sol: $\sum \cot A \cot B = \dfrac{1+\sin(A+B+C)}{\sin A \sin B \sin C} = 1 + \dfrac{\sin(180)}{\frac{1}{3}}$ $1 + \dfrac{0}{\frac{1}{3}} = 1$

(2) In a $\triangle^{le}$ ABC $\cos A \cos B \cos C = \frac{1}{3}$ then $\sum \tan A \tan B =$ ______

Sol: $\sum \tan A \tan B = \cancel{1-\cos(A+B+C)}$ $1 - \dfrac{\cos(A+B+C)}{\cos A \cos B \cos C}$

$= \cancel{1-\cos(180)}$ $1 - \dfrac{\cos 180^\circ\ (-1)}{\frac{1}{3}}$

$= 1+3 = 4$

-) ~~In a $\triangle^{le}$ ABC~~

-) $\tan 10^\circ \tan 50^\circ + \tan 50^\circ \tan 70^\circ - \tan 10^\circ \tan 70^\circ =$ ______

Sol: take $A = -10^\circ$ $B = 50^\circ$ $C = -70^\circ$ (logic at last term) of "–" side).

$= \tan(-10^\circ)\tan 50^\circ + \tan 50^\circ \tan(-70^\circ) - (\tan(-10^\circ)\tan(-70^\circ)) = 1 - \dfrac{\cos(-10^\circ+50^\circ-70^\circ)}{\cos(-10^\circ)\cos(50^\circ)\cos(-70^\circ)}$

$= 1 - \dfrac{\cos(-30^\circ)}{\cos 10^\circ \cos(60-10^\circ)\cos(60+10^\circ)}$

$= 1 - \dfrac{\cos 30^\circ}{\frac{1}{4}\cos 3(10^\circ)}$

$=$ ~~$\tan 10^\circ \tan 50^\circ - \tan$~~

$= -[\tan 10^\circ \tan 50^\circ + \tan 50^\circ \tan 70^\circ - \tan 10^\circ \tan 70^\circ]$ $= 1 - \dfrac{\cos 30^\circ}{\frac{1}{4}\cos 30^\circ} = 1-4 = -3$

$= +3$

$= 3$

$\rightarrow$ $\tan\left(\frac{2\pi}{7}\right)\tan\left(\frac{4\pi}{7}\right)+\tan\left(\frac{4\pi}{7}\right)\tan\left(\frac{8\pi}{7}\right)+\tan\left(\frac{2\pi}{7}\right)\tan\frac{8\pi}{7}$

Sol

$\Sigma \tan A \tan B = \dfrac{1-\cos(2\pi)}{\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\cos\left(\frac{8\pi}{7}\right)}$

$\frac{2\pi}{7}+\frac{4\pi}{7}+\frac{8\pi}{7}$

$\pi \cancel{\frac{14}{7}} 2 = 2\pi$

$= 1 \dfrac{\cancel{1}-\cos 2\pi}{\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\cos\left(\pi+\frac{\pi}{7}\right)}$

$= -\cos\frac{\pi}{7}$

$= 1 \dfrac{\cancel{1}+\cos 2\pi}{+\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos 4\frac{\pi}{7}}$

$= \dfrac{1+1}{\frac{-1}{2^3}} = 1+\dfrac{1}{\frac{-1}{2^3}} = 1-8 = -7$,,

$\Rightarrow$ $\cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 16^\circ \cot 76^\circ =$ ______

So

~~A=16°~~ $A=-16^\circ$ $b=44^\circ$ $c=-76^\circ$.

$= \cot(-16^\circ)\cot 44^\circ + \cot 44^\circ \cot(-76^\circ) - \cot(-16^\circ)\cot(-76^\circ) = 1 \dfrac{\cancel{1}+\sin(-48^\circ)}{\sin(-16^\circ)\sin(44^\circ)\sin(-76^\circ)}$

$-16^\circ+44^\circ-76^\circ$

$\frac{a}{2}$ $\frac{-\pi}{48}$

$= 44-92 = -48^\circ$

$= 1-\dfrac{\cancel{\sin 48^\circ}}{\frac{1}{4}\cancel{\sin 48^\circ}}$

$\Rightarrow 1-4 \Rightarrow -3$

$= -\left(\cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 16^\circ \cot 76^\circ\right) = +3$

$= 3$,,

MUL - SUB MUL - ANGLES *

rad	$\pi/24$	$\pi/12$	$\pi/10$	$\pi/8$	$\pi/5$	$3\pi/10$	$3\pi/8$	$2\pi/5$	$5\pi/12$	$11\pi/24$
degree	$7\frac{1}{2}^\circ$	15°	18°	$22\frac{1}{2}^\circ$	36°	54°	$67\frac{1}{2}^\circ$	72°	75°	82.5°
$\sin\theta$	x	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\cancel{\frac{\sqrt{10-2\sqrt{5}}}{4}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	x
$\cos\theta$	x	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\cancel{\frac{\sqrt{5}+1}{4}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	x
$\tan\theta$	$(\sqrt{3}-\sqrt{2})\times(\sqrt{2}-1)$	$2-\sqrt{3}$	x	$\sqrt{2}-1$	x	x	$\sqrt{2}+1$	x	$2+\sqrt{3}$	$(\sqrt{3}+\sqrt{2})\times(\sqrt{2}+1)$

$$\sin 22\tfrac{1}{2}^\circ = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$\cos 22\tfrac{1}{2}^\circ = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

→ degree to radians

$$\cancel{75}^{5}{}^\circ \times \frac{\pi}{\cancel{180}_{12}} = \frac{5\pi}{12}$$

radians to degree

$$= \frac{\pi}{8} \times \frac{180}{\pi} = 22.5^\circ$$

→ Find the Quadratic eqn whose roots $\tan 75^\circ$ & $\cot 75^\circ$.

Sol: To get eqn when roots given

$$x^2 - (\alpha+\beta)x + (\alpha.\beta) = 0$$

$$x^2 - (\tan 75^\circ + \cot 75^\circ)x + (\tan 75^\circ \cot 75^\circ)$$

1

$$= x^2 - (2+\cancel{\sqrt{3}} + 2-\cancel{\sqrt{3}})x$$

$$= x^2 - 4x + 1 = 0$$

→ Find the Q.Eqn whose roots are $\sin^2 18^\circ$, $\cos^2 36^\circ$.

Sol: $\sin^2 18^\circ + \cos^2 36^\circ = \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2}{4}$

$$= \frac{5+1-\cancel{2\sqrt{5}}+5+1+\cancel{2\sqrt{5}}}{16}$$

$$\frac{5-1}{16} + \frac{5+1}{16} \quad \frac{10}{16}$$

$\sin^2 18^\circ \cos^2 36^\circ$

$$= \frac{5+\sqrt{5}-\sqrt{5}-1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$x^2 - \left(\frac{5}{6}\right)x + \frac{1}{4} = 0$$

$$= x^2 - \frac{5x}{6} + \frac{1}{4} = 0$$

$\frac{6x^2-5x}{6} + \frac{1}{4} = 0$

$= 24x^2 - 20x + 6 = 0$

$= 6x^2 - 5x +$

Super
dele - 1
Frick - 2

$= \alpha+\beta = \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \frac{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2}{16} = \frac{5+1-\cancel{2\sqrt{5}} + 5+1+\cancel{2\sqrt{5}}}{16}$

$= \frac{\cancel{12}\,3}{\cancel{16}\,4} = \frac{3}{4}$

$\alpha.\beta = \left(\frac{\sqrt{5}-1}{4}\right)^2 \times \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \frac{(6-2\sqrt{5})(6+2\sqrt{5})}{4^2 \times 4^2} = \frac{6^2-(2\sqrt{5})^2}{(4)^4} = \frac{36-20}{4^4}$

$= \frac{\cancel{16}}{4^2 \times \cancel{4^2}} = \frac{1}{16}$

$= x^2 - \left(\frac{3}{4}\right)x + \frac{1}{16} = x^2 - \frac{3x}{4} + \frac{1}{16}$

$= \frac{4x^2-3x}{4} + \frac{1}{16} = \frac{64x^2-48x+4}{64}$

$= \frac{\cancel{4}(16x^2-12x+1)}{\cancel{64}\,16} = 0 = \boxed{16x^2-12x+1 = 0}$

Find the Value of $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$.

$\cancel{\sin^4(22}\ \sin^4(22\tfrac{1}{2}^\circ) + \sin^4(67\tfrac{1}{2}^\circ) + \sin^4(67\tfrac{1}{2}^\circ) + \sin^4(22\tfrac{1}{2}^\circ)$

$\Downarrow$ $\pi - \frac{3\pi}{8}$ $\Downarrow$ $\pi - \frac{\pi}{8}$

$= 2\left(\sin^4 22\tfrac{1}{2}^\circ + \sin^4 67\tfrac{1}{2}^\circ\right)$

$= 2\left(\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}\right)^4 + \left(\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}\right)^4$

$= \cancel{2}\left(\frac{(\sqrt{2}-1)^2 + (\sqrt{2}+1)^2}{\cancel{8}\,4}\right) = \frac{\cancel{2}(2+1)}{\cancel{4}\,2} = 3/2$

Super important problems

Trick - 1 power of sin/cos : is '4' then Ans $\frac{3n}{8}$ $n \in I^+$

Trick - 2 " " " is 2 then Ans $\frac{n}{2}$ $n \in I^+$

n is depends on last term

either $2n-1$ (or, $2(n-1)$
odd even

(1) $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right) =$ $2n-1=7$, $n=4 \Rightarrow \frac{3(4)}{8} = 3/2$

(2) $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$

(3) $\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{5\pi}{8}\right) + \sin^2\left(\frac{7\pi}{8}\right) =$ $2n-1=7$, $n=4 \Rightarrow \frac{n}{2} =$

(4) $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 2$

(5) $\sin^4\left(\frac{\pi}{16}\right) + \sin^4\left(\frac{3\pi}{16}\right) + \sin^4\left(\frac{5\pi}{16}\right) + \sin^4\left(\frac{7\pi}{16}\right) = 3/2$

(6) $\cos^4\left(\frac{\pi}{16}\right) + \cos^4\left(\frac{3\pi}{16}\right) + \cos^4\left(\frac{5\pi}{16}\right) + \cos^4\left(\frac{7\pi}{16}\right) = 3/2$

(7) $\sum_{n=1}^{4} \sin^2\left((2n-1)\frac{\pi}{16}\right) =$ ~~28+24 8~~ $2n-1=7$ $n=4 \Rightarrow 2$

(8) $\sum_{n=1}^{4} \cos^2\left((2n-1)\pi/16\right) = 2$

(9) $\cos^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{3\pi}{10}\right) + \cos^2\left(\frac{5\pi}{10}\right) + \dots \cos^2\left(\frac{19\pi}{10}\right) =$ $2n-1=19$, $n=10 \Rightarrow \frac{10}{2} = 5$

(10) $\sin^2\left(\frac{\pi}{10}\right) + \sin^2\left(\frac{3\pi}{10}\right) + \dots\dots + \sin^2\left(\frac{19\pi}{10}\right) = 5$

(11) $\sin^4\left(\frac{\pi}{10}\right) + \dots\dots + \sin^4\left(\frac{19\pi}{16}\right)$ is $\frac{3(10)}{8} = \frac{15}{4}$

(12) $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\frac{5\pi}{8}\right)\right)\cdot\left(1+\cos\frac{7\pi}{8}\right)$

Sol: S.C $\frac{1}{4}\sin^2\left(\frac{\pi}{2n}\right) \Rightarrow$ $4n-1=7$, $4n=8$, $n=2$ $= \frac{1}{4}\sin^2\left(\frac{\pi}{4}\right)$ $\frac{1}{2}$

(14) $\left(1+\cos\frac{\pi}{12}\right)\left(1+\cos\frac{5\pi}{12}\right)\left(1+\cos\frac{7\pi}{12}\right)\left(1+\cos\frac{11\pi}{12}\right)$

Sol

$4n-1=11 \quad n=3$

$\frac{1}{4}\sin^2\frac{\pi}{2n} \Rightarrow$

$= \frac{1}{4}\sin^2\frac{\pi}{6} = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16} \quad \frac{1}{16}$

(15) $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$

Sol

$\frac{1}{4}\sin^2\frac{\pi}{\cancel{2\left(\frac{5}{2}\right)}}$ $\quad \cancel{4n-1=9 \quad n=\frac{10 5}{4 2}} = \frac{5}{2}$.

$\cancel{\frac{1}{4}\sin^2\frac{\pi}{5}}$

$\cancel{= \frac{1}{4}\cdot\frac{10-2\sqrt5}{4^2}} \quad \cancel{\frac{1}{4}\cdot\frac{10-2\sqrt5}{16} \Rightarrow}$

$= 4n-1=9 \quad 4n=10 \Rightarrow n=\frac{5}{2}$

$= \frac{1}{4}\sin^2(36) = \frac{1}{4}\times\frac{10-2\sqrt5}{16} = \frac{\cancel{10}\,5}{\cancel{64}\,32} - \frac{\cancel{2}\sqrt5}{\cancel{64}\,32}$

$= \frac{5-\sqrt5}{32}\times\frac{5+\sqrt5}{5+\sqrt5} = \frac{5^2-5}{}$

Sol

$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\left(\pi-\frac{3\pi}{10}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{10}\right)\right)$

$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$

$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$= \sin^2\frac{\pi}{10}\sin^2\frac{3\pi}{10}$

$= \left(\frac{\sqrt5-1}{4}\right)^2\times\left(\frac{\sqrt5+1}{4}\right)^2$

$= 5$

$= \cancel{(a+b)^2\times(a-b)^2}$

$= (a^2-b^2)^2$

$= \left[\cancel{\left(\frac{\sqrt5-1}{4}\right)^2-\left(\frac{\sqrt5+1}{4}\right)^2}\right]^2$

$= \frac{(\sqrt5-1)^2(\sqrt5+1)^2}{4^4} = \frac{(5+1-2\sqrt5)(5+1+2\sqrt5)}{4^4}$

$= \frac{5+1-2\sqrt5}{16}\times\frac{5+1+2\sqrt5}{16}$

$= \frac{6-2\sqrt5}{16}\times\frac{6+2\sqrt5}{16}$

$= \frac{2(3-2\sqrt5)}{16\,8}\times\frac{2(3+2\sqrt5)}{16\,8}$

$= \frac{9+6\sqrt5-6\sqrt5-20}{64}$

$= \frac{-11}{64}$

$= \frac{36+12\sqrt5-12\sqrt5-20}{16}$

$\tan 81° - \tan 63° - \tan 27° + \tan 9° =$ ______

$\tan 81° - \tan 63°$

$\tan 81° + \tan 9° - (\tan 63° + \tan 27°)$

$= \cot 9° + \tan 9° - (\cot 27° + \tan 27°)$

$= 2\operatorname{cosec} 18° - 2\operatorname{cosec} 54°$

$\boxed{\begin{array}{l} \cot A + \tan A = 2\operatorname{cosec} 2A \\ \cot A - \tan A = \dfrac{(1-\tan^2 A)2}{2\tan A} = 2\cot 2A \end{array}}$

$= 2\left(\frac{4}{\sqrt{5}-1}\right) - 2\left(\cancel{\sqrt{5}-1}\ \frac{4}{\sqrt{5}+1}\right)$

$= \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1} \Rightarrow \cancel{\frac{8\sqrt{5}-1-8-8\sqrt{5}+8}{(\sqrt{5})^2-1}} = \cancel{\frac{16\ 4\ 2}{24\ 6\ 3}} = \frac{2}{3}$

$= \cancel{8\left[\frac{\sqrt{5}+1 - \sqrt{5}+1}{24\ 3}\right]} \quad \cancel{\frac{2}{3}}$

$= 8\left[\frac{\sqrt{5}+1 - \sqrt{5}+1}{5-1}\right] = 2[2] = 4$,,

→ If $\cos A + \cos B + \cos C = 0$ then $\Sigma \cos 3A =$ ______

sol $\Sigma \cos 3A = \Sigma\, 4\cos^3 A - 3\cos A$

$= 4(\cos^3 A + \cos^3 B + \cos^3 C) - 3(\underbrace{\cos A + \cos B + \cos C}_{0})$

$= 4\ 3\cos A \cos B \cos C$

$= 12 \cos A \cos B \cos C$.

If $a+b+c=0 \Rightarrow \boxed{a^3+b^3+c^3 = 3abc}$,,

→ If $x + \frac{1}{x} = 2\cos\theta$ then $\frac{1}{2}\left(x^3 + \frac{1}{x^3}\right) =$ ______

sol C.O.B.S

$x^3 + \frac{1}{x^3} + 3\left(x\cdot\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$

$= 2(4\cos^3\theta - 3\cos\theta)$

$x^3 + \frac{1}{x^3} = 2(\cos 3\theta) \Rightarrow \frac{1}{2}\left(x^3 + \frac{1}{x^3}\right) = \cos 3\theta$,,

(2 diff angles α θ)

$\cos(\theta+\alpha)$

→ If $\cos^2\theta + \cos^2(\alpha+\theta) - 2\cos\alpha\cos\theta$ $\cos(\theta+\alpha)$

(a) $\sin^2\alpha$ (b) $\cos^2\alpha$ (c) $\tan^2\alpha$ (d) $\sec^2\alpha$.

Sol: put $\theta = 0°$

$= \cancel{1 + \cos^2\alpha - 2\cos\alpha \cos(0) + \cos\alpha}$ $1+\cos^2\alpha - 2\cos^2\alpha$

$= \cancel{1 + \cos^2\alpha - 2\cos\alpha + \cos\alpha}$ $1-\cos^2\alpha = \sin^2\alpha$

$= \cancel{1 + \cos^2\alpha - \cos\alpha}$

→ $2\sin^2 A + 4\cos(A+B)\sin A\sin B + \cos 2(A+B)$ is depends on ———

(a) A (b) B (c) A & B (d) A (or) B

Sol: $A = 0$

$= + 0 + + \cancel{\cos 2A} + \cos 2B$

$= 1 + \cos 2B$ ∴ depends on B

→ $\cos^2(\alpha-\beta) + \cos^2\beta - 2\cos(\alpha-\beta)\cos\alpha\cos\beta$ is independent on

(a) α (b) β (c) α & β (d) none.

Sol: ~~put $\alpha = 0°$~~ put $\beta = 0°$

$= \cos^2\alpha + 1 - 2\cos\alpha\cos\alpha$

$\cos^2\alpha + 1 - 2\cos^2\alpha$

$1 - \cos^2\alpha = \sin^2\alpha$

∴ depends on α independents on β.

(or)

put $\alpha = 0$

$\cancel{\cos^2\beta + \cos^2\beta - 2\cos\beta \cos\alpha}$

$= \cancel{2\cos^2\beta - 2\cos}$

$\cos^2\beta + \cos^2\beta - 2\cos\beta\cos\beta$

$\cancel{\cos^2\beta} + \cos^2\beta - \cancel{2\cos^2\beta}$

∴ $\beta = 0$

α is depends ; independs on β

"Root Concept" :- $s+c$ & $s-c$ concept.

$s = \sin A/2$ & $c = \cos(A)/2$

$|s+c| = \sqrt{1+\sin A}$ — ①

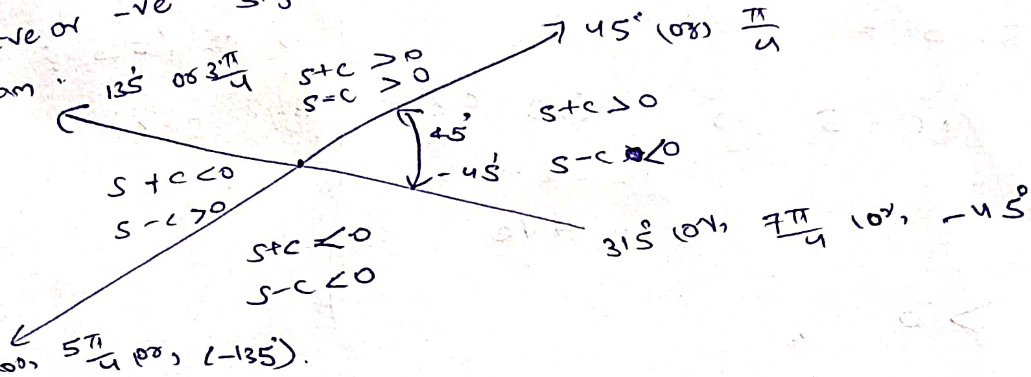
$s+c = \pm\sqrt{1+\sin A}$ — ②

$\sqrt{1-\sin A} = |s-c|$ — ③

$s-c = \pm\sqrt{1-\sin A}$ — ④

In pt ② & ④

The +ve or -ve sign can be determined with the help of below diagram :



→ $\sqrt{1+\sin A} + \sqrt{1-\sin A} = 2\cos(A/2)$ then $\frac{A}{2} \in$ ________ 2n terms.

Sol: $|s+c| + |s-c| = 2\cos(A/2)$

$\pm s+c + \pm s-c = 2\cos A/2$

$s+c-(s-c)$

$= s+\cancel{c}-s+\cancel{c} = 2s$

$\therefore = s+c > 0 \quad s-c < 0$

$\therefore x > 0 \quad y < 0 \rightarrow$ it is belongs to IV Quadrant

$= \left[\frac{\pi}{4}, -\frac{\pi}{4}\right]$ ..

(or)

$\left[2n+\frac{\pi}{4}, 2n-\frac{\pi}{4}\right]$ ..

→ $\sqrt{1+\sin A} - \sqrt{1-\sin A} = 2\sin(A/2)$ then $A/2 \in$ ________ In terms.

Sol: $|s+c| - |s-c|$ $\in \left[\frac{\pi}{4}, -\frac{\pi}{4}\right]$

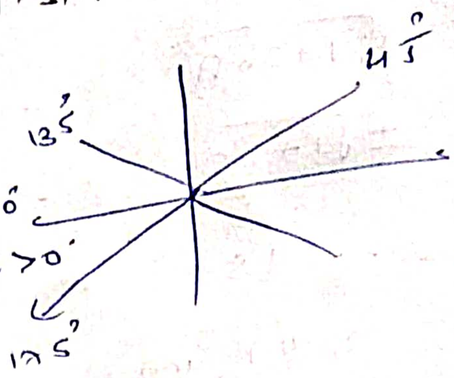
$= \cancel{+s+c - (s+-c)}$ $\quad s+c-(-(s-c))$

$= \cancel{s+c+s}$ $\quad = \cancel{s}\cancel{c}+s+\cancel{c}$ $\therefore$ $s+c>0 \quad s-c<0$

$= 2s$ $\quad x>0 \quad y<0$

$\rightarrow$ If $A = 348°$ then $\sqrt{1+\sin A} + \sqrt{1-\sin A} = \underline{\quad +ve \quad}$

Sol $\frac{A}{2} = 174°$.



$= |s+c| + |s-c|$

$= -(s+c) + (s-c)$

$= \cancel{-s}-c \cancel{+s}-c = -2\cos\left(\frac{A}{2}\right) = 2\cos 6°.$ +ve

$\rightarrow \cos 10° - \sin 10° = \underline{\qquad}$

Sol $\cancel{|s+c|}$ $c - s \Rightarrow -(s-c)$

$= -(-ve)$

$= +ve$

$\therefore > 0$

$\rightarrow \sin 18° - \cos 18°$

Sol $s - c$

< 0

$\rightarrow 4\sin 27°$

Sol Consider $27°$ is lies in $-\frac{\pi}{4}$ to $\frac{\pi}{4}$

$\frac{A}{2} = 27°$

$A = 54°$

$= \sqrt{1+\sin A} - \sqrt{1-\sin A}$

$|s+c| - |s-c|$

$s+c > 0 \;:\; s-c < 0$

$= \sqrt{1+\sin 54°} - \sqrt{1-\sin 54°}$

$= \sqrt{1+\frac{\sqrt{5}-1}{4}} - \sqrt{1-\frac{\sqrt{5}+1}{4}}$

$= \frac{\sqrt{1+\sqrt{5}-1} - \sqrt{1-\sqrt{5}+1}}{2}$

$\cancel{\frac{\sqrt{6-1} - (\sqrt{-4}+1)}{2}} = \sqrt{10+}$

$= \sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}$

$\therefore 4\sin 27° = \sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}}$

Chapter - 4

RELATION BETWEEN COEEF'S & ROOTS :-

Model-1

If x_1 & x_2 are distinct roots of $a\cos x + b\sin x = c$ then $\tan\left(\frac{x_1+x_2}{2}\right) =$ ______

Logic :- Given eqn coeff argument is x.
then Question argument must be "$x/2$" form
then answer is $\frac{\text{Sin coeff}}{\text{Cos coeff}}$

$= b/a$ "

If α, β are the diff sol^n of ~~a a cos~~ $a\cos x + b\sin x = c$ then

(i) $\sin(\alpha+\beta)$

(ii) $\cos(\alpha+\beta)$

(iii) $\tan(\alpha+\beta)$

Sol using logic $\tan\frac{(\alpha+\beta)}{2} = b/a = t$

$\sin\theta = \frac{2\tan\theta}{1+\tan^2\theta}$; $\theta = \frac{(\alpha+\beta)}{2} = t$

$= \frac{2t}{1+t^2} = \frac{2(b/a)}{1+(b/a)^2} = \frac{\frac{2b}{a}}{1+\frac{b^2}{a^2}} = \frac{2b/a}{\frac{a^2+b^2}{a^2}}$

$= \frac{2b}{\cancel{a}} \times \frac{a^{\cancel{2}}}{a^2+b^2}$

$\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$.

$\cos(\alpha+\beta) = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-t^2}{1+t^2} = \frac{1-(b/a)^2}{1+(b/a)^2} = \frac{1-b^2/a^2}{1+b^2/a^2}$

$\cos(\alpha+\beta) = \frac{a^2-b^2}{\cancel{a^2}} \times \frac{\cancel{a^2}}{a^2+b^2} = \frac{a^2-b^2}{a^2+b^2}$

$\tan(\alpha+\beta) \quad \frac{2ab}{\cancel{a^2+b^2}} \times \frac{\cancel{a^2+b^2}}{a^2-b^2} = \frac{2ab}{a^2-b^2}$ "

→ If θ_1, θ_2 are two diff sol^n of $3\cos 2\theta + 4\sin 2\theta$ then $\tan(\theta_1+\theta_2) =$ ______

Sol $\tan\left(\frac{\theta_1+\theta_2}{2}\right) = 4/3$ "

→ If α, β are sol^n of $6\cos x + 8\sin x = 1$ then (i) $\sin\left(\frac{\alpha+\beta}{2}\right)$ (ii) $\cos\left(\frac{\alpha-\beta}{2}\right)$

Sol $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{\cancel{8}\,4}{\cancel{6}\,3} = \frac{4}{3}$

$\sin\left(\frac{\alpha+\beta}{2}\right) = 4/5$

$\cos\left(\frac{\alpha-\beta}{2}\right) = 3/5$. "

→ If x_1, x_2 are the sol^n of $a\cos 2x + b\sin 2x = c$ then $\tan(x_1+x_2)$

Sol b/a "

→ In a Δ^{le} PQR $\because \angle R = \pi/2$ if $\tan(P/2)$ $\tan(Q/2)$ are the roots of $ax^2+bx+c=0$ then

(a) $a+b=c$ (b) $b+c=a$ (c) $a+c=b$ (d) $a+b=c$

Sol $P+Q+R = 180^\circ$
a b c

$P+Q = 180-R$ $R=90^\circ$

$P+Q = 90^\circ$

$a+b = c$ "

→ If $\tan A$, $\tan B$ are the roots of x^2-bx+c then $\sin^2(A+B) =$ ______

Sol $\tan A + \tan B = -b/a = +b/1 = b$

$\tan A \tan B = c/a = \frac{c}{1} = c$.

$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B} = \frac{b}{1-c}$

$\sin^2(A+B) = \frac{1}{\csc^2(A+B)} = \frac{1}{1+\cot^2(A+B)} = \frac{1}{1+\frac{1}{\tan^2(A+B)}}$

$= \frac{1}{\frac{\tan^2(A+B)+1}{\tan^2(A+B)}} = \frac{\tan^2(A+B)}{\tan^2(A+B)+1}$

$$\frac{\left(\frac{b}{1-c}\right)^2}{\left(\frac{b}{1-c}\right)^2+1} = \frac{\left(\frac{b}{1-c}\right)^2}{\frac{b^2+(1-c)^2}{(1-c)^2}} = \frac{b^2}{b^2+(1-c)^2}$$

Model -2 :-

A.P . (or) G.P (or) H.P

$2b = a+c$ $\quad$ $b^2 = ac$ $\quad$ $\frac{2}{b} = \frac{1}{a}+\frac{1}{c}$

If $\sec(\theta+\alpha)$, $\sec\theta$, $\sec(\theta-\alpha)$ are in A.p then $\cos^2\theta =$ ____

Sol

$$2\sec\theta = \sec(\theta+\alpha) + \sec(\theta-\alpha)$$

$$\frac{2}{\cos\theta} = \frac{1}{\cos(\theta+\alpha)} + \frac{1}{\cos(\theta-\alpha)}$$

$$\frac{2}{\cos\theta} = \frac{\cos(\theta-\alpha)+\cos(\theta+\alpha)}{\cos(\theta+\alpha)\cos(\theta-\alpha)}$$

$$\frac{\cancel{2}}{\cos\theta} = \frac{\cancel{2}\cos\theta\cos\alpha}{\cos^2\theta-\sin^2\alpha}$$

$\Rightarrow \cos^2\theta - \sin^2\alpha = \cos^2\theta\cos\alpha$

$\Rightarrow -\sin^2\alpha = \cos^2\theta\cos\alpha - \cos^2\theta$

$\Rightarrow -\sin^2\alpha = \cos^2\theta(\cos\alpha - 1)$

$\Rightarrow +\sin^2\alpha = +\cos^2\theta(1-\cos\alpha)$

$\Rightarrow \cancel{\cos^2\alpha = \frac{1-\cos^2\theta}{1}}$ $\Rightarrow \cos^2\theta = \frac{1-\cos^2\alpha}{1-\cos\alpha}$

$$= \frac{\cancel{(1-\cos\alpha)}(1+\cos\alpha)}{\cancel{(1-\cos\alpha)}}$$

$$\cos^2\theta = 2\cos^2\frac{\alpha}{2}$$

$$\cos\theta = \pm\sqrt{2}\cos\left(\frac{\alpha}{2}\right)$$

$\Rightarrow$ If $\tan\beta = 2\sin\alpha\sin\gamma\,\mathrm{cosec}(\alpha+\gamma)$ then $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are in ______ . (a) A.p (b) G.p (c) H.p (d) A-G.p.

Sol

$$\tan\beta = \frac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$$

$$\tan\beta = \frac{2\sin\alpha\sin\gamma}{\sin\alpha\cos\gamma+\cos\alpha\sin\gamma} \Rightarrow \cancel{\frac{2}{\cot\gamma}} + \quad \frac{2}{\cot\gamma+\cot\alpha}$$

$$\frac{1}{\cot\beta} = \frac{2}{\cot\gamma + \cot\alpha} \Rightarrow \cot\gamma + \cot\alpha = 2\cot\beta$$

$$A + C = 2B$$

$$\Rightarrow \frac{1}{\tan\gamma} + \frac{1}{\tan\alpha} = \frac{2}{\tan\beta} \quad \text{are in A.P}$$

then $\tan\gamma$, $\tan\alpha$ $\tan\beta$ are in H.P

→ If $\cos 2B = \dfrac{\cos(A+C)}{\cos(A-C)}$ then $\tan A$ $\tan B$ $\tan C$ = ______

Sol $\cos 2B = \dfrac{\cos(A+C)}{\cos(A-C)}$ use C. & D rule

$$\frac{\cos 2B + 1}{\cos 2B - 1} = \frac{\cos(A+C) + \cos(A-C)}{\cos(A+C) - \cos(A-C)}$$

$$= \frac{\cancel{2}\cos^2 A/2}{-\cancel{2}\sin^2 A/2} = \frac{2\cos\left[\frac{A+C+A-C}{2}\right]\cos\left[\frac{A+C-A+C}{2}\right]}{-2\sin\left[\frac{A+C+A-C}{2}\right]\sin\left[\frac{A+C-A+C}{2}\right]}$$

$$-\cot^2 2B = \frac{\cancel{2}\cos A \cos C}{-\cancel{2}\sin A \sin C}$$

$$= +\cot^2 2B = 4\cot A \cot C$$

$$= B^2 = ac \quad \text{H.P}$$

$\therefore$ $\tan^2 2B = \tan A \tan C$ also in H.P