LAPLACE TRANSFORMATION

→ conversion

Ecet → ① or ② questions

Complex equation ⟹ Algebraic equation

↳ Definition :- $f(t) \rightarrow 0-\infty \int xe^{-st}$

in book

$$\int e^{-st} f(t)\, dt$$

Good Definition

↳ $f(t)$ ⟹ Well defined function of 't'.

→ Time interval

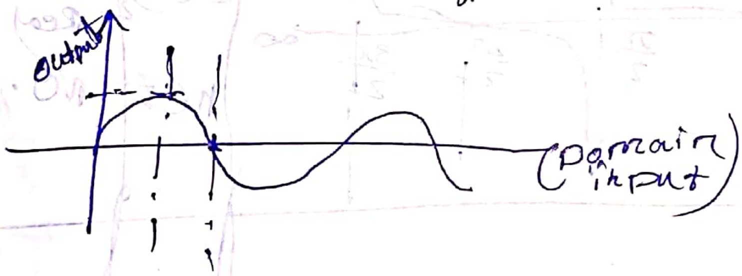
& $\int_0^\infty e^{-st} f(t)\, dt$ → Exist

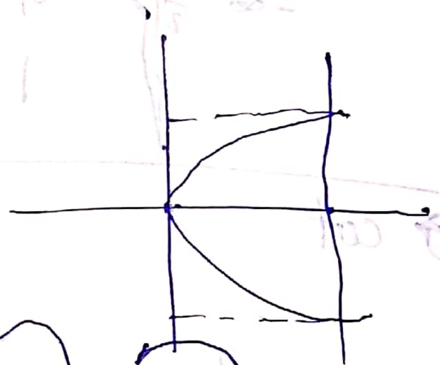
'S' ⟹ Complex number frequency parameter

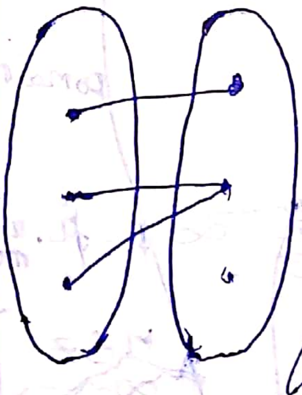
It is called as Laplace transformations.

Functions :- (mapping) ⟹ relationship between input & output

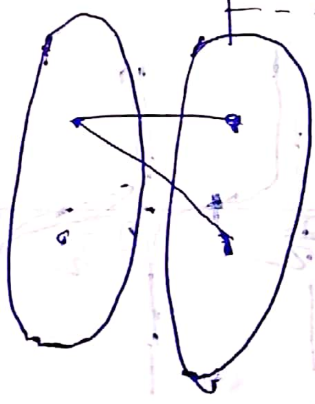
→ Each input must have one output only







Function



One ⟹ one

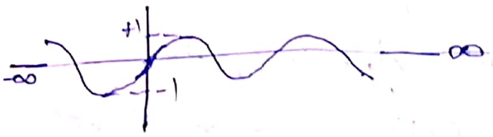
Many ⟹ one

These are functions

~~Many~~ one ⟹ Many

Not a function

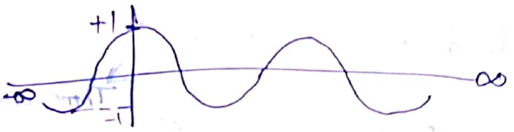
① Sin



Domain: $(-\infty, \infty)$ Real No

Range: $[-1, 1]$

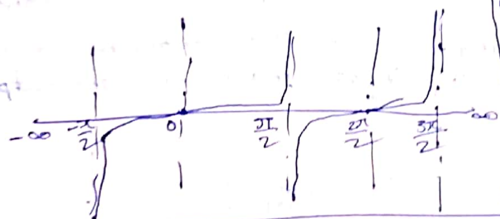
② Cos



Domain: $(-\infty, \infty)$ Real no.

Range: $[-1, 1]$

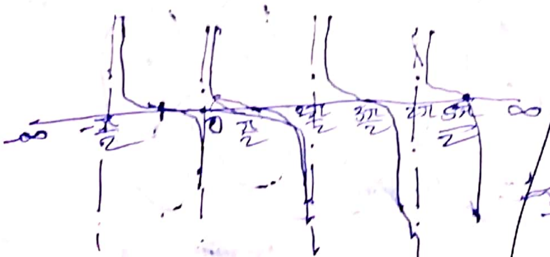
③ tan



Domain: $R - \{x$ where, $x = (2n+1)\frac{\pi}{2}$, $n \in Z\}$

Range: $(-\infty, \infty)$ Real No.

④ cot

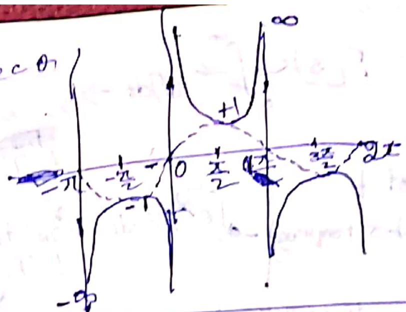


Domain: $R - \{x : x = n\pi\}$, $n \in Z$

Range: $(-\infty, \infty)$ Real No.

Real numb. +1 -1

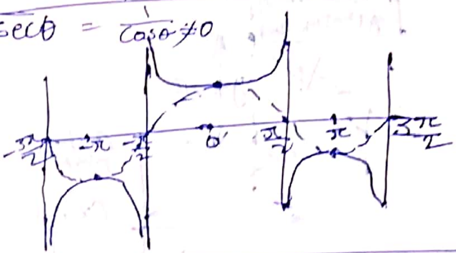
Cosec θ



Domain: $R - \{x : x = n\pi,\ n \in Z\}$

Range: $(-\infty, -1] \cup [1, \infty)$ $R - (-1, 1)$

$\sec\theta = \frac{1}{\cos\theta} \neq 0$



Domain: $R - \{x : x = (2n+1)\frac{\pi}{2}\}$ $n \in Z$

Range: $(-\infty, -1] \cup [1, \infty)$ $R - (-1, 1)$

$L\{f'(t)\} = f(0) - F(s)$; $\int_0 e^{-st} f(t)\,dt \rightarrow$ Universal L.T

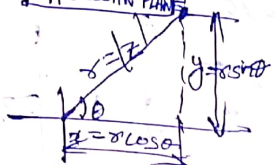
$f(t) \Rightarrow$ function of t

$s \Rightarrow$ Complex number frequency parameter

$\int_{-\infty}^{\infty} e^{-st} f(t)\,dt \rightarrow$ Bilateral L.T.

Complex number

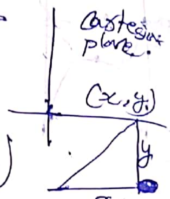
ARGAND PLANE
GAUSSIAN PLANE



$\boxed{z = x + iy}$

Rectangular form

(x, y) Real Numbers



i = iota

$\boxed{i^2 = -1}$

[Fourth root of unity $(i^3 = i^2 \times 1)$

$= -i$

$i \rightarrow i$

$i^2 \rightarrow -1$

$i^3 \rightarrow -i$

$i^4 \rightarrow +1$

$z = x + iy$

$\Downarrow$

$z = r\cos\theta \; i(r\sin\theta)$

$\boxed{z = r(\cos\theta + i\sin\theta)}$

$\boxed{z = re^{i\theta}}$ Polar form

$\Rightarrow e^{i\theta} = (\cos\theta + i\sin\theta)$

$e^{-i\theta} = (\cos\theta - i\sin\theta)$

Linearity principle:-

Proof:
$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$

$L\{c_1 f(t) + g(t)\}$

$c_1 \int_0^{\infty} f(t)\,dt \times e^{-st} + c_2 \int_0^{\infty} g(t)\,dt \times e^{-st}$

$c_1 L\{f(t)\} + c_2 L\{g(t)\}$

(1) $L\{k\}$

$L \int_0^{\infty} k \times 1 \times e^{-st}\,dt$

$k \int_0^{\infty} e^{-st}\,dt \rightarrow$ simple answers. i.e.; $\frac{1}{s}$

$= k\left[\frac{e^{-st}}{-st}\right]_0^{\infty}$

$= \frac{-k}{s}\left[e^{-s(\infty)} - e^{-s(0)}\right]$

$= \frac{-k}{s}\left[e^{-\infty} - e^{0}\right]$

$= \frac{-k}{s}\left[\frac{1}{e^{\infty}} - e^{0}\right]$

$= \frac{-k}{s}[0 - 1] = \frac{k}{s}$

(2)

$L\{e^{at}\}$

$\int_0^\infty e^{at}.e^{-st}\,dt$

$= \int_0^\infty e^{at-st}\,dt$

$= \int_0^\infty e^{t(-s+a)}\,dt$ → simple answer, ie, → $\frac{1}{s-a}$

$= \int_0^\infty e^{-\boxed{(s-a)}t}\,dt$

$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$

$= \frac{-1}{s-a}\left[e^{-\infty} - e^{0}\right]$

$= \frac{-1}{s-a}\left[0-1\right] = \frac{1}{s-a}$

(3)

$L\{e^{-at}\}$

$= \int_0^\infty e^{-at}.e^{-st}\,dt$ → simple answer, ie, → $\frac{1}{s+a}$

$= \int_0^\infty e^{-\boxed{(s+a)}t}\,dt$

$= \frac{-1}{(s+a)}\left[e^{-\infty} - e^{0}\right]$

$= \frac{-1}{s+a} \times -1 = \frac{1}{s+a}$

(4)

$L\{\sin at\} = \int_0^\infty e^{-st}.\sin at\,dt$

put => $a = -s$, $b = a$, $t=$

$I = \int_0^\infty e^{ax} \sin bx\,dx$

ILATE

$e^{ax} \Downarrow dV$, $\sin bx \Downarrow u$

$V = \frac{e^{ax}}{a}$, $du = b\cos bx$

$I = \sin bx . \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}. b\cos bx$

$I = \sin bx . \frac{e^{ax}}{a} - \frac{b}{a}\int e^{ax}.\cos bx$ $u = \cos bx$, $du = -\sin bx$, $dV = e^{ax}$, $V = \frac{e^{ax}}{a}$

$I = \sin bx . \frac{e^{ax}}{a} - \frac{b}{a}\left[\cos bx \frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} \times (-\sin bx)\right]$

$I = \sin bx . \frac{e^{ax}}{a} - \frac{b}{a}\left[\cos bx \frac{e^{ax}}{a} + \frac{b}{a}\int e^{ax} \sin bx\right]$

$I = \sin bx . \frac{e^{ax}}{a} - \frac{b}{a}\left[\cos bx \frac{e^{ax}}{a} + \frac{b}{a} I\right]$

$\sin bx . \frac{e^{ax}}{a} - \frac{b\cos bx\, e^{ax}}{a \cdot a} - \frac{b^2}{a^2} I = I$

$\sin bx . \frac{e^{ax}}{a} - \frac{b}{a}\cos bx \frac{e^{ax}}{a} = I\left(1 + \frac{b^2}{a^2}\right)$

$\frac{e^{ax}}{a}\left(a\sin bx - \frac{b}{\cancel{a}}\cos bx\right) = I\left(\frac{a^2+b^2}{a^2}\right)$

$\frac{e^{ax}(a\sin bx - b\cos bx)}{\cancel{a^2}} = \frac{I(a^2+b^2)}{\cancel{a^2}}$

$\therefore I = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$

$$\therefore I = \frac{e^{ax}(a\sin bx + b\cos bx)}{a^2+b^2}$$

Substitutes values

$$I = \left[\frac{e^{-st}}{(-s)^2+a^2}\left(-s\sin at + a\cos at\right)\right]_0^\infty$$

$$I = \left[\frac{e^{-\infty}}{''} \cdot ''\right] - \left[\frac{e^0}{s^2+a^2}(0+a)\right]$$

$$I = 0 - \left[\frac{a}{s^2+a^2}\right] = \frac{a}{s^2+a^2}$$

(5) Solve $L\{\cos at\} = \int_0^\infty e^{-st}\cos at\, dt = \frac{s}{s^2+a^2}$

(6) $L\{e^{iat}\}$

Sol:- $\int_0^\infty e^{iat-st}$

$$= \int_0^\infty e^{-(s-ia)t}$$

$$\text{Sol} = \frac{1}{s-ia} \cdot \frac{s+ia}{s+ia} = \frac{s+ia}{s^2-(i^2a^2)}$$

$$= \frac{s+ia}{s^2-(-a^2)}$$

$$= \frac{s+ia}{s^2+a^2}$$

$$\therefore = \boxed{\frac{s}{s^2+a^2}} + i\boxed{\frac{a}{s^2+a^2}}$$

Hence proved $e^{i\theta} = \cos\theta + i\sin\theta$

$L[\sinh at]$

Sol:-

$$\Rightarrow L\left\{\frac{e^{at}-e^{-at}}{2}\right\}$$

$$= \frac{1}{2}L\{e^{at}-e^{-at}\}$$

$$= \frac{1}{2}L\{e^{at}\} - L\{e^{-at}\}$$ we already proved

$$= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right)$$

$$= \frac{1}{2}\left(\frac{s+a-s+a}{s^2-a^2}\right)$$

$$= \frac{1}{2}\left(\frac{2a}{s^2-a^2}\right) = \frac{a}{s^2-a^2}$$

Formulas

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$\mathcal{L}\{t^n\} = \int_0^\infty e^{-st}\cdot t^n$

$\int u\,dv = uv - \int v\,du$

$= \frac{t^n e^{-st}}{-s} - \int \frac{e^{-st}}{-s} n t^{n-1}$

$\therefore \mathcal{L}\{t^n\} = \left[\frac{t^n e^{-st}}{-s}\right]_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} n t^{n-1}$

$= \left[\frac{t^n e^{-st}}{-s}\right]_0^\infty - \frac{n}{-s}\int_0^\infty e^{-st}\cdot t^{n-1}$

$= \frac{1}{-s}\left[t^n e^{-st}\right]_0^\infty + \frac{n}{s}\int_0^\infty e^{-st}\cdot t^{n-1}$

$= \frac{1}{-s}\left[\infty^n e^{-\infty} - 0^n e^{-s\cdot 0}\right] + \frac{n}{s}\int_0^\infty e^{-st}\cdot t^{n-1}$

$= -\frac{1}{s}[0-0] + \frac{n}{s}\int_0^\infty e^{-st}\cdot t^{n-1}$

$= \frac{n}{s}\int_0^\infty e^{-st}\cdot t^{n-1}$

According to $\mathcal{L}\{t^n\} = \int_0^\infty e^{-st}\cdot t^n$

$= \frac{n}{s}\int_0^\infty e^{-st}\cdot t^{n-1} = \underline{\mathcal{L}\{t^{n-1}\}}$

Then $\mathcal{L}\{t^n\} = \frac{n}{s}\mathcal{L}\{t^{n-1}\}$

$\mathcal{L}\{t^{n-1}\} = \frac{n-1}{s}\mathcal{L}\{t^{n-2}\}$

and,
$\mathcal{L}\{t^n\} = \frac{n}{s}\times\frac{n-1}{s}\times\frac{n-2}{s} \cdots \times\frac{n-(n-2)}{s}\times\frac{n-(n-1)}{s}\cdot\mathcal{L}\{t^{n-n}\}$

$= \frac{n}{s}\times\frac{n-1}{s}\times\frac{n-2}{s} \cdots \frac{2}{s}\times\frac{1}{s}\times\mathcal{L}\{t^0\}$

$= \frac{n}{s}\times\frac{n-1}{s}\times\frac{n-2}{s} \cdots \frac{2}{s}\times\frac{1}{s}\times\mathcal{L}\{1\}$

$= \frac{n}{s}\times\frac{n-1}{s}\times\frac{n-2}{s} \cdots \frac{2}{s}\times\frac{1}{s}\times\left(\frac{1}{s}\right)$

$\therefore = \frac{n!}{s^n}\times\frac{1}{s} = \frac{n!}{s^{n+1}}$

$$\boxed{\therefore \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}}$$

Gamma Function :-

$\Gamma(n) = \int_0^\infty e^{-x}\cdot x^{n-1} \Rightarrow$ similar to $\int_0^\infty e^{-st}\cdot t^{n-1} = \mathcal{L}\{t^{n-1}\}$

$= \frac{n-1}{s}\times\frac{n-2}{s}\times\cdots\frac{2}{s}\times\frac{1}{s}\,\mathcal{L}\{t^{n-n}\}$

$= \frac{(n-1)!}{s}\times\mathcal{L}\{t^{n-n}\}$

$\Gamma(n) = \frac{(n-1)!}{s^{n+1}} = (n-1)!$

$\Gamma(n+1) = \frac{(n+1-1)!}{s^{n+1}} = n!$

$\Gamma(n) = (n-1)!$

$\Gamma(n+1) = n!$

Ex :- $\Gamma(5) = (5-1)! = 4! = 4\times3\times2\times1$

$\Gamma(3) = (3-1)! = 2! = 2\times1$

$5\Gamma(5) = 5[(5-1)!] = 5\times4! = 5\times4\times3\times2\times1 = 5!$

$\therefore 5\Gamma(5) = 5!$

Note:- $\boxed{n\Gamma(n) = n!}$

$$\boxed{\Gamma(n+1) = n\Gamma(n) = n!}$$

Q: 0!

Sol:- $0! = \Gamma(0+1) = \Gamma(1) = \int_0^\infty e^{-x} \cdot x^{1-1}$ $\quad [\because n! = \Gamma(n+1)]$

$= \int_0^\infty e^{-x} \cdot x^{0} = \int_0^\infty e^{-x}$

$= \left[\frac{e^{-x}}{-1}\right]_0^\infty$

$= \frac{1}{-1}\left[e^{-\infty} - e^{0}\right]$

$= \frac{1}{-1}\left[0 - 1\right]$

$\therefore 0! = \frac{-1}{-1} = 1$

$\Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1}$

$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-x} \cdot x^{\frac{1}{2}-1} = \int_0^\infty e^{-x} \cdot x^{-\frac{1}{2}} dx$ $\quad \boxed{\begin{array}{l}\text{Put}\\ x = u^2\\ dx = 2u\,du\end{array}}$

$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty e^{-u^2} \cdot (u^2)^{-\frac{1}{2}} 2u\,du = 2\int_0^\infty e^{-u^2} \cdot u^{-1} \cdot u\,du$

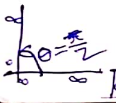
$\Gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-u^2} du$

$\Gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-u^2} du$, $\Gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-v^2} dv$

multiplying these both, we get

$\left(\Gamma\left(\frac{1}{2}\right)\right)^2 = 4\int_0^\infty \int_0^\infty e^{-u^2} \cdot e^{-v^2} du\,dv$

Convert $\boxed{\text{Cartesian form} - \text{Polar form}} \rightarrow \boxed{r, \theta}$



$\boxed{du\,dv = r\,dr\,d\theta}$ $\quad \boxed{\text{Put} \ldots = dt}$ $\quad$ and $\boxed{u^2 + v^2 = r^2}$

$\left[\Gamma\left(\frac{1}{2}\right)\right]^2 = 4\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_0^\infty e^{-(u^2+v^2)} \cdot du\,dv$

~~$= 4\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{\infty} e^{t} \cdot (-2r)\,dr$ $\quad (\because -r^2 = t)$~~

$= 4\int_0^{\frac{\pi}{2}} \int_0^\infty e^{-(u^2+v^2)} \cdot r \cdot dr\,dt$

$= 4\int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{r=0}^{r=\infty} e^{-r^2} \cdot dr \cdot dt$

Put $-r^2 = t$

$-2r\,dr = dt$

$\mathcal{L}\{t^{-\frac{1}{2}}\}$

$$\mathcal{L}\{t^{n}\} = \frac{n!}{s^{n+1}} = \frac{n\Gamma(n)}{s^{n+1}} = \frac{\Gamma(n+1)}{s^{n+1}}$$

$$= \cancel{\frac{-\frac{1}{2}\Gamma(\frac{1}{2})}{s^{-\frac{1}{2}+1}}} = \frac{\Gamma(-\frac{1}{2}+1)}{s^{-\frac{1}{2}+1}}$$

$$= \cancel{\frac{-\frac{1}{2}\Gamma(-\frac{1}{2})}{s^{\frac{1}{2}}}} = \frac{\Gamma(\frac{1}{2})}{s^{\frac{1}{2}}}$$

$$= \frac{\sqrt{\pi}}{\sqrt{s}} = \sqrt{\frac{\pi}{s}}$$

$\mathcal{L}\{t^{\frac{1}{2}}\}$

$$\mathcal{L}\{t^{n}\} = \frac{n!}{s^{n+1}} = \frac{n\Gamma(n)}{s^{n+1}}$$

$$= \frac{\frac{1}{2}\Gamma(\frac{1}{2})}{s^{\frac{1}{2}+1}}$$

$$= \frac{\frac{1}{2}\sqrt{\pi}}{s^{\frac{3}{2}}} = \frac{\sqrt{\pi}}{2\times s^{\frac{3}{2}}}$$

$$\Gamma\{\tfrac{3}{2}\} = \Gamma\{\tfrac{1}{2}+1\}$$

$$\boxed{\Gamma(n+1) = n\,\Gamma(n)}$$

$$\therefore \Gamma(\tfrac{1}{2}+1) = \tfrac{1}{2}\Gamma(\tfrac{1}{2})$$

$$= \tfrac{1}{2}\times\sqrt{\pi}$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

$$\Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\times\sqrt{\pi}$$

$$\Gamma(\tfrac{5}{2}) = \Gamma(\tfrac{3}{2}+1) = \tfrac{3}{2}\Gamma(\tfrac{3}{2}) = \tfrac{3}{2}\times\tfrac{1}{2}\times\sqrt{\pi}$$

$$\Gamma(\tfrac{7}{2}) = \Gamma(\tfrac{5}{2}+1) = \tfrac{5}{2}\Gamma(\tfrac{5}{2}) = \tfrac{5}{2}\times\tfrac{3}{2}\times\tfrac{1}{2}\times\sqrt{\pi}$$

$$\Gamma(\tfrac{15}{2}) = \Gamma(\tfrac{13}{2}+1) = \tfrac{13}{2}\Gamma(\tfrac{13}{2}) = \tfrac{13}{2}\times\tfrac{11}{2}\times\tfrac{9}{2}\times\tfrac{7}{2}\times\tfrac{5}{2}\times\tfrac{3}{2}\times\tfrac{1}{2}\times\sqrt{\pi}$$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$ and $\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$

$$n\Gamma(n) = \Gamma(n+1)$$

$$-\tfrac{1}{2}\Gamma(-\tfrac{1}{2}) = \Gamma(-\tfrac{1}{2}+1)$$

$$\Gamma(-\tfrac{1}{2}) = \frac{\Gamma(\frac{1}{2})}{-\frac{1}{2}}$$

$$\Gamma -\tfrac{1}{2} = \frac{\sqrt{\pi}}{-\frac{1}{2}} = -2\sqrt{\pi}$$

~~$\Gamma(\frac{1}{2}) = \sqrt{\pi}$~~

~~$\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$~~

Basic Formulas

① $\mathcal{L}\{1\} = \frac{1}{s}$

② $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

③ $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$

④ $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

⑤ $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

⑥ $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$

⑦ $\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$

⑧ $\mathcal{L}\{\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}}$

⑨ $\mathcal{L}\{\frac{1}{\sqrt{t}}\} = \sqrt{\frac{\pi}{s}}$

⑩ $\mathcal{L}\{e^{iat}\} = \mathcal{L}\{\cos at\} + \mathcal{L}\{i.\sin at\} = \frac{s}{s^2+a^2} + i.\frac{a}{s^2+a^2} = \frac{s+ia}{s^2+a^2}$

*⑪ $n! = \Gamma(n+1) = n\Gamma(n)$

*⑫ $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} = \frac{\Gamma(n+1)}{s^{n+1}} = \frac{n\Gamma(n)}{s^{n+1}}$

⑬ $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

⑭ $\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$

~~⑮ $\mathcal{L}\{t^{\frac{1}{2}}\} = \frac{\sqrt{\pi}}{2s^{3/2}}$~~

~~$\mathcal{L}\{t^{-\frac{1}{2}}\} = \sqrt{\frac{\pi}{s}}$~~

~~$s^4 - 5 \times 4 \times 3 \times 2 \times 1$~~

~~$s^5 - 1$~~

~~$5 \times 4 \times 3 \times 2 \times 1$~~

~~$4 \times 3 \times 2 \times 1$~~

Basics

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\cos 2\theta = 1-2\sin^2\theta$

$\cos\theta = 1-2\sin^2\frac{\theta}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\theta/2}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta}{1-3\tan^2\theta}$

$A+B=C \qquad A-B=D$

① & ②

Sum

$$\begin{aligned} A+B &= C \\ A-B &= D \\ \hline 2A &= C+D \end{aligned}$$

$$\boxed{A = \frac{C+D}{2}}$$

Diffrence

$$\begin{aligned} A+B &= C \\ -\ A-B &= D \\ (-)\ (+) &\ \ (-) \\ \hline 2B &= C-D \end{aligned}$$

$$\boxed{B = \frac{C-D}{2}}$$

then

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A.\sin B$

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) = \sin C + \sin D$

$2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = \sin C - \sin D$

$2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) = \cos C + \cos D$

$2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = \cos D - \cos C$

① $L\{t^3-3t^2+2\} = \frac{6}{s^4} - \frac{6}{s^3} + \frac{2}{s}$

② $L\{e^{2t}+4t^3+\cos 3t\} = \frac{1}{s-2} + \frac{24}{s^4} + \frac{s}{s^2+9}$

③ $L\{e^{2t}-4t^3+2\sin 3t\} = \frac{1}{s-2} - \frac{24}{s^4} + \frac{2s}{s^2+a^2}$

④ $L\{e^{t}-e^{-3t}\} = \frac{1}{s-1} - \frac{1}{s+3} = \frac{4}{(s-1)(s+3)}$

⑤ $L\left\{\frac{e^{-at}-1}{a}\right\} = \frac{1}{a} \times \left[\frac{1}{s+a} - \frac{1}{s}\right] = \frac{-1}{(s+a)s}$

⑥ $L\{(t+1)^2\} = L\{t^2+1+2t\} = \frac{2}{s^3} + \frac{1}{s} + \frac{2}{s^2}$

⑦ $L\{4e^{5t}+6t^3-3\sin 4t+2\cos 2t\} = \frac{4}{s-5} + \frac{36}{s^4} - \frac{12}{s^2+4^2} + \frac{2s}{s^2+2^2}$

⑧ $L\{3t^2+4\sin 2t+3\cos 2t-1\} = \frac{6}{s^3} + \frac{8}{s^2+2^2} + \frac{3s}{s^2+2^2} - \frac{1}{s}$

⑨ $L\{e^{2t}+4t^3-3\sin 2t+2\cos 2t\} = \frac{1}{s-2} + \frac{24}{s^4} - \frac{6}{s^2+2^2} + \frac{2s}{s^2+2^2}$

⑩ $L\{9e^{-2t}+5\cos 2t+5\sin 3t\} = \frac{9}{s+2} + \frac{5s}{s^2+2^2} + \frac{15}{s^2+3^2}$

⑪ $L\{t^3-2\sin 3t+\cos 3t\} = \frac{6}{s^4} - \frac{6}{s^2+3^2} + \frac{s}{s^2+3^2}$

⑫ $L\{\sin^2 3t\} = L\left(\frac{1-\cos 2(3t)}{2}\right) = \frac{1}{2}L(1-\cos 2(3t)) = \frac{1}{2}\left(\frac{1}{s}-\frac{s}{s^2+6^2}\right)$

⑬ $L\{\sin^3 t+e^{2t}\} = L\left\{\frac{3\sin\theta-\sin 3\theta}{4}\right\} = \frac{1}{4}\left\{\frac{3}{s^2+1^2} - \frac{3}{s^2+3^2}\right\} + \frac{1}{s-2}$

⑭ $L\{\cos^2 2t\} = L\left(\frac{1+\cos 2(2t)}{2}\right) = \frac{1}{2}\left\{\frac{1}{s}+\frac{s}{s^2+2^2}\right\}$

⑮ $L\{\cos^2 3t\} = L\left(\frac{1+\cos 2(3t)}{2}\right) = \frac{1}{2}\left[L(1)+L(\cos 6t)\right]$

$= \frac{1}{2}\left(\frac{1}{s}+\frac{s}{s^2+6^2}\right)$

⑯ $L\{\sin(\omega t+\theta)\} = L\{\sin\omega t\cos\theta + \cos\omega t\sin\theta\}$

$= \cos\theta\, L\{\sin\omega t\} + \sin\theta\, L(\cos\omega t)$

$= \cos\theta \times \frac{\omega}{s^2+\omega^2}$

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$L\{\sin 2t . \sin 3t\} = \frac{1}{2}\{2\sin 3t . \sin 2t\}$

$\Rightarrow \frac{1}{2} L\{\cos(3t-2t) - \cos(3t+2t)\} = \frac{1}{2}\left[\frac{s}{s^2+1^2} - \frac{s}{s^2+5^2}\right]$

$L\{\sin 2t . \cos t\} = \frac{1}{2}\{2\sin 2t . \cos t\} = \frac{1}{2} L\{\sin(3t) + \sin(t)\}$

$L\{\cos 6t . \cos 3t\} = \frac{1}{2}\{2\cos 6t \cos 3t\} = \frac{1}{2}\left[\frac{3}{s^2+a^2} + \frac{1}{s^2+a^2}\right]$

$= \frac{1}{2}\left[\frac{s}{s^2+6^2} + \frac{s}{s^2+3^2}\right]$

(20) $L\{\cosh^2 3t\} = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2-36}\right]$

(21) Find $L\{f(t)\}$. if $f(t) = \begin{cases} 1 & 0<t<2 \\ 2 & t>2 \end{cases}$

sol:- $\int_0^2 e^{-st}(1)\,dt + \int_2^\infty e^{-st}(2)\,dt$

$= \int_0^2 e^{-st}dt + 2\int_2^\infty e^{-st}dt$

$= \left[\frac{e^{-st}}{-s}\right]_0^2 + 2\left[\frac{e^{-st}}{-s}\right]_2^\infty$

$= \frac{1}{-s}\left[e^{-2s} - 1\right] + 2\frac{1}{-s}\left[0 - e^{-2s}\right]$

$= -\frac{1}{s}\left[e^{-2s} - 1\right] + \frac{2}{-s}\left[-e^{-2s}\right]$

$= -\frac{e^{-2s}}{s} + \frac{1}{s} + \frac{2e^{-2s}}{s}$

$= \frac{1}{s}\left[-e^{-2s} + 1 + 2e^{-2s}\right]$

$= \frac{1}{s}\left[e^{-2s} + 1\right]$

(22) $L\{f(t)\}$, when $f(t) = \begin{cases} e^t & 0<t<1 \\ 0 & t>1 \end{cases}$

$\int_0^1 e^{-st} . e^t\,dt + \int_1^\infty e^{-st}(0)\,dt$

$= \int_0^1 e^{-st+t} + \int_1^\infty e^{-st} \times 0\,dt$

$= \int_0^1 e^{-(s-1)t} + \int_0^\infty e^{-st} \times (0) = 0$

$= \frac{1}{-(s-1)}\left[e^{-s-1} - e^0\right] = \frac{e^{-(s-1)} - 1}{-(s-1)}$

$= \frac{1 - e^{-(s-1)}}{s-1}$

Formula

$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx + b\cos bx\right]$

$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$

(23) $\int_0^\pi e^{-st}\sin 2t = \left(\frac{e^{-st}}{s^2+2^2}\left[-s\sin 2t - 2\cos 2t\right]\right)_0^\pi$

$= \frac{e^{-s\pi}}{s^2+2^2}\left[0 - 2(1)\right] - \frac{e^0}{s^2+2^2}\left[0 - 2\right]$

$= \frac{e^{-s\pi}}{s^2+2^2}\left[-2\right] - \frac{1}{s^2+a^2}\left[-2\right]$

$= \frac{1}{s^2+a^2}\left[-2e^{-s\pi} + 2\right]$

$= \frac{2}{s^2+a^2}\left[1 - e^{-s\pi}\right]$

Exercise

(24) $t^2 + ab + b = \frac{2}{s^3} + \frac{ab}{s} + \frac{b}{s}$

(25) $t^3 - 3t + 5 = \frac{6}{s^4} - \frac{3}{s^2} + \frac{5}{s}$

(26) $e^{3t} - e^{-3t} = \frac{1}{s-3} - \frac{1}{s+3}$

(27) $2e^{3t} - e^{-3t} = \frac{2}{s-3} - \frac{1}{s+3}$

(28) $3e^{2t} + 4e^{-3t} = \frac{3}{s-2} + \frac{4}{s+3}$

(29) $t^3 + 5\cos t = \frac{6}{s^4} + \frac{5s}{s^2+1}$

(30) $5t + 2e^{t} + 8\sin 3t = \frac{5}{s^2} + \frac{2}{s-1} + \frac{24}{s^2+3^2}$

(31) $3\sin 4t - 2\cos 5t = \frac{12}{s^2+16} - \frac{2s}{s^2+25}$

(32) $3\sin 4t + 4\cos 3t = \frac{12}{s^2+16} + \frac{4s}{s^2+9}$

(33) $3\cosh(5t) - 4\sinh(5t) = \frac{3s}{s^2-25} - \frac{20}{s^2-25}$

(34) $e^{at+b} = \frac{e^b}{s-a}$ $\Rightarrow L(e^{at}\cdot e^{b}) = e^{b} L\{e^{at}\} = \frac{e^b}{s-a}$

(35) $3t^2 + 2\cos 2t + e^{-t} = \frac{3\times 2}{s^3} + \frac{2s}{s^2+2^2} + \frac{1}{s+a}$

(36) $t^3 + \cos 3t - e^{3t} = \frac{6}{s^4} + \frac{s}{s^2+9} - \frac{1}{s-3}$

(37) $e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t = \frac{1}{s-2} + \frac{24}{s^4} - \frac{6}{s^2+3^2} + \frac{3s}{s^2+9}$

(38) $4e^{2t} + 6t^3 - 2\cos 5t = \frac{4}{s-2} + \frac{36}{s^4} - \frac{2s}{s^2+25}$

(39) $t^2 + \sinh(2t) + \sin 2t = \frac{2}{s^3} + \frac{2}{s^2-4} + \frac{2}{s^2+4}$

(40) $\sin at + t^2 + 5e^{-3t} = \frac{a}{s^2+a^2} + \frac{2}{s^3} + \frac{5}{s+3}$

(41) $e^{-3t} + 4\cosh t + \sin 2t = \frac{1}{s+3} + \frac{4s}{s^2-1} + \frac{2}{s^2+4}$

(42) $t^4 + e^{2t} + 2\sin 2t = \frac{24}{s^5} + \frac{1}{s-2} + \frac{4}{s^2+4}$

(43) $(a+bt)^2 = L\{a^2 + b^2t^2 + 2abt\} = \frac{a^2}{s} + \frac{2b^2}{s^3} + \frac{2ab}{s^2}$

(44) $(t^2+1)^2 = L(t^4 + 2t^2 + 1) = \frac{24}{s^5} + \frac{4}{s^3} + \frac{1}{s}$

(45) $(e^{3t} - e^{-3t})^2 = L\{(e^{3t})^2 + (e^{-3t})^2 - 2e^{3t-3t}\} = L\{e^{6t}\} + L\{e^{-6t}\} - 2L\{1\}$
$= \frac{1}{s-6} + \frac{1}{s+6} - \frac{2}{s}$

(46) $\sin(at+b) = L(\sin at\cos b + \cos at \sin b)$
$= \cos b\, L\{\sin at\} + \sin b\, L\{\cos at\}$
$= \frac{\cos b \times a}{s^2+a^2} + \frac{\sin b \times s}{s^2+a^2} = \frac{a\cos b + s\sin b}{s^2+a^2}$

(47) $\cos(\omega t - \alpha) = L\{\cos\omega t\cos\alpha + \sin\omega t\cdot\sin\alpha\}$
$= \cos\alpha\, L\{\cos\omega t\} + \sin\alpha\, L\{\sin\omega t\}$
$= \frac{\cos\alpha \cdot s}{s^2+\omega^2} + \frac{\sin\alpha \times \omega}{s^2+\omega^2}$

(48) $\sin^2 t \Rightarrow \frac{1}{2} L[1-\cos 2\theta] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+2^2}\right]$

(49) $\cos^2 t = \frac{1}{2} L[1+\cos 2\theta] = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4a^2}\right]$

(50) $\sin^2 at = \frac{1}{2} L[1-\cos 2(a)t] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4a^2}\right]$

(51) $\cos^2 at = \frac{1}{2} L[1+\cos 2(a)t] = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4a^2}\right]$

(52) $\sin^3(2t) \Rightarrow L\left[\frac{3\sin(2t) - \sin 3(2t)}{4}\right] = \frac{1}{4} L\{3\sin(2t) + \sin 6t\}$
$= \frac{1}{4}\left[\frac{6}{s^2+4} + \frac{6}{s^2+9}\right]$

(53) $\sinh^2(2t) \Rightarrow L\left\{\frac{1-\cosh 2t}{2}\right\} = \frac{1}{2}L\left\{\frac{1}{s} - \cosh 2t\right\} = \frac{1}{2}\left(\frac{1}{s} - \frac{2}{s^2-4}\right)$

(54) $\cosh^2(2t) \Rightarrow L\left\{\frac{1+\cos 2t}{2}\right\} = \text{''} = \frac{1}{2}\left\{\frac{1}{s} + \frac{2}{s^2-4}\right\}$

(55) $\sin t.\cos t = \frac{1}{2} L(2\sin t.\cos t) = \frac{1}{2} L(\sin 2t) = \frac{1}{2} \times \frac{2}{s^2+2^2} = \frac{1}{s^2+4}$

(56) $\sin 2t.\cos 3t =$

(57) $(\sin t + \cos t)^2$

(58) $\sin 2t \cos 3t$

(59) $\sin 5t \cos 3t$

(60) $\sin 8t \cos 4t$

(61) $\cos t \cos 2t \cos 3t$

(62) $\sin t \sin 2t \sin 3t$

(63) $\cos 6t \cos 4t \sin t$

(64) $\sinh^2(2t)$

(65) $\cos t \cos 2t \cos 3t$

(66) $\sin t \sin 2t \sin 3t$

(67) $\cos 6t \cos 4t \sin t$

Any doubts
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(64) $\sinh^2 2t$

$= \frac{1}{2}[\cosh(4t) - 1]$

$= \frac{1}{2} L(\cosh 4t - 1)$

$= \frac{1}{2}\left[\frac{s}{s^2-16} - \frac{1}{s}\right]$

$= \frac{1}{2}\left[\frac{s^2 - s^2 + 16}{s(s^2-16)}\right]$

$= \frac{1}{2}\left[\frac{16}{s(s^2-16)}\right] = \frac{8}{s(s^2-16)}$

(or) $\sinh x = \frac{e^{x} - e^{-x}}{2}$

$\sinh^2(2t) = \left[\frac{e^{2t} - e^{-2t}}{2}\right]^2$

$= \frac{1}{4}\left[e^{4t} + e^{-4t} - 2e^{2t}.e^{-2t}\right]$

$= \frac{1}{4} L\left[e^{4t} + e^{-4t} - 2\right]$

$= \frac{1}{4}\left[\frac{1}{s-4} + \frac{1}{s+4} - \frac{2}{s}\right]$

$\cancel{= \frac{1}{4}\left[\frac{s^2+16+s^2+16+s^2+4^2}{s(s^2-4^2)}\right]}$

$\cancel{= \frac{1}{4}\left[\frac{3s^2-16}{s(s^2-16)}\right]}$

$= \frac{1}{4}\left[\frac{s+4+s-4}{s^2-4^2} - \frac{2}{s}\right]$

$= \frac{1}{4}\left[\frac{2s^2 - 2(s^2-16)}{s(s^2-4^2)}\right]$

$= \frac{1}{4} \times 2\left[\frac{s^2 - s^2 + 4^2}{s(s^2-16)}\right]$

$= \frac{1}{2}\left[\frac{16}{s(s^2-16)}\right] = \frac{8}{s(s^2-16)}$

(65) $\cos t \cos 2t \cos 3t$

$= \frac{1}{2}[2\cos t \cos 2t]\cos 3t$

$= \frac{1}{2}[\cos 3t + \cos t]\cos 3t$

$= \frac{1}{2}[\cos^2 3t + \cos t.\cos 3t]$

$= \frac{1}{2}\left[\frac{1}{2}(1+\cos 2(3t)) + \frac{1}{2}(2\cos t \cos 3t)\right]$

$= \frac{1}{2}\left[\frac{1}{2}(1+\cos 6t) + \frac{1}{2}(\cos 4t + \cos 2t)\right]$

$= \frac{1}{2} \times \frac{1}{2} L\left[(1+\cos 6t) + L(\cos 4t + \cos 2t)\right]$

$= \frac{1}{4}\left[\left(\frac{1}{s} + \frac{s}{s^2+36}\right) + \left(\frac{s}{s^2+16} + \frac{s}{s^2+4}\right)\right]$

(66) sint . sin2t . sin3t

Sol:- $\frac{1}{2}(2\sin t \cdot \sin 2t)\sin 3t$

$\frac{1}{2}(\cos t - \cos 3t)\sin 3t$

$= \frac{1}{2}\left[\cos t \sin 3t - \cos 3t \sin 3t\right]$

$= \frac{1}{2}\times\frac{1}{2}\left[2\cos t \sin 3t - 2\sin 3t\cos 3t\right]$

$= \frac{1}{4}\left[\sin 4t - \sin(-2t) - \sin 2(3t)\right]$

$= \frac{1}{4}\left[\sin 4t + \sin 2t - \sin 6t\right]$

$= \frac{1}{4}\left[\frac{4}{s^2+16} + \frac{2}{s^2+4} - \frac{6}{s^2+36}\right]$

(67) cos6t cos4t . sin7t

$= \frac{1}{2}\left(\cos(10t) + \cos(2t)\right)\sin t$

$= \frac{1}{2}\left[\cos 10t \sin t + \cos 2t \sin t\right]$

$= \frac{1}{4}\left[2\cos 10t \sin t + 2\cos 2t \sin t\right]$

$= \frac{1}{4}\left[\sin(11t) - \sin(9t) + \sin(3t) - \sin(t)\right]$

$= \frac{1}{4}\left[\frac{11}{s^2+121} - \frac{9}{s^2+81} + \frac{3}{s^2+9} - \frac{1}{s^2+1}\right]$

(68) $f(t) = \begin{cases} t & 0<t<1 \\ 0, & t>1 \end{cases}$

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \times f(t)\,dt$

$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st} \cdot t + \int_1^\infty e^{-st}(0)\,dt \rightarrow 0$

$\mathcal{L}\{f(t)\} = \left[t \cdot \frac{e^{-st}}{-s} - \int \frac{e^{-st}}{-s} \times 1\,dt\right]_0^1 \qquad du = 1,\ v = \frac{e^{-st}}{-s}$

$= \left[\frac{t \cdot e^{-st}}{-s} - \frac{1}{-s}\int e^{-st}dt\right]_0^1$

$= \left[\frac{te^{-st}}{-s} + \frac{1}{s}\frac{e^{-st}}{(-s)}\right]_0^1$

$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^1$

$= \left[\frac{e^{-s}}{-s} - \frac{e^{-s}}{s^2}\right] - \left(0 - \frac{e^0}{s^2}\right)$

$= \frac{e^{-s}}{-s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}$

(69) $\mathcal{L}\, f(t) = ? \Rightarrow$ If $f(t) = \begin{cases} t/k & 0<t<k \\ 1 & t>k \end{cases}$

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

$= \int_0^k e^{-st}\frac{t}{k}\,dt + \int_k^\infty e^{-st}(1)\,dt$

$= \frac{1}{k}\times\int_0^k e^{-st} t\,dt + \left[\frac{e^{-st}}{-s}\right]_k^\infty$

$= \frac{1}{k}\times\left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^k - \frac{1}{s}\left[e^{-st}\right]_k^\infty$

$= \frac{1}{k}\left[\frac{ke^{-sk}}{-s} - \frac{e^{-sk}}{s^2} \ldots\right] \ldots \frac{1}{s}\left[e^{-s\infty} - e^{-sk}\right]$

$$= \frac{1}{k}\left[\frac{te^{-sk}}{-s} - \frac{e^{-sk}}{s^2} - \frac{1}{s^2}\right] - \frac{1}{s}\left[0 - e^{-sk}\right]$$

$\boxed{e^{-\infty}=0}$

(70) $f(t) = \begin{cases} 0, & 0<t<1 \\ t, & 1<t<2 \\ 0, & t>2 \end{cases}$

$$\mathcal{L}f(t) = \cancel{\int_0^1 e^{-st}(0)\,dt} + \int_1^2 e^{-st}(t) + \cancel{\int_2^\infty e^{-st}(0)\,dt}$$

$$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_1^2$$

$$= \left[\frac{2e^{-2s}}{-s} - \frac{e^{-2s}}{s^2}\right] - \left[\frac{e^{-s}}{-s} - \frac{e^{-s}}{s^2}\right]$$

(71) $f(t) = \begin{cases} (t-1)^2, & t>1 \\ 0, & t<1 \end{cases}$

$$\mathcal{L}f(t) = \int_0^\infty e^{-st}f(t)\,dt$$

$$= \int_1^\infty e^{-st}(t-1)^2\,dt + \cancel{\int_0^1 e^{-st}(0)\,dt}$$

$$= \int_1^\infty e^{-st}(t^2+1-2t)\,dt$$

$u = t^2+1-2t$, $dv = e^{-st}$

$du = 2t-2$, $v = \frac{e^{-st}}{-s}$

$$= \int_1^\infty e^{-st}(t)^2 + e^{-st}(1) - e^{-st}\,2t$$

$$= \int_1^\infty e^{-st}t^2\,dt + \int_1^\infty e^{-st}\,dt - 2\int_1^\infty e^{-st}t$$

$$= \left[\frac{t^2e^{-st}}{-s} - \frac{2te^{-st}}{s^2} + \frac{2e^{-st}}{s^3}\right]_1^\infty + \left[\frac{e^{-st}}{-s}\right]_1^\infty - 2\left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_1^\infty$$

u		v
$+t^2$		e^{-st}
$-2t$		$\frac{e^{-st}}{-s}$
$+2$		$\frac{e^{-st}}{s^2}$
0		$\frac{e^{-st}}{-s^3}$

$$= \left[0 - \left[\frac{e^{-s}}{-s} + \frac{-2e^{-s}}{s^2} + \frac{2e^{-s}}{-s^3}\right]\right] - \frac{1}{s}\left[\cancel{e^{-\infty}} - e^{-s}\right]$$

$$= \left[\frac{e^{-s}}{s} + \frac{2e^{-s}}{s^2} + \cancel{\frac{2e^{-s}}{s^3}}\right] + \frac{e^{-s}}{s} - \frac{2e^{-s}}{s^2}$$

$$= \left[\frac{e^{-s}}{s} + \frac{e^{-s}}{s} - \frac{2e^{-s}}{s} + \frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s^2}\right] + \frac{2e^{-s}}{s^3}$$

$$= \left[\cancel{\frac{2e^{-s}}{s} - \frac{2e^{-s}}{s}} + \cancel{\frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s^2}}\right] + \frac{2e^{-s}}{s^3}$$

$$= \frac{2e^{-s}}{s^3}$$

(72) $f(t) = \begin{cases} 0, & 0<t<1 \\ 1, & 1<t<2 \\ 2, & t>2 \end{cases}$

$$\mathcal{L}f(t) = \cancel{\int_0^1 e^{-st}(0)\,dt} + \int_1^2 e^{-st}(1)\,dt + \int_2^\infty e^{-st}(2)\,dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_1^2 + 2\left[\frac{e^{-st}}{-s}\right]_2^\infty$$

$$= \frac{1}{-s}\left[\left(e^{-2s} - e^{-s}\right) + 2\left(e^{-\infty} - e^{-2s}\right)\right]$$

$$= \frac{1}{-s}\left[e^{-2s} - e^{-s} - 2e^{-2s}\right]$$

$$= \frac{1}{s}\left[-\frac{e^{-2s}}{s} + \frac{e^{-s}}{s}\right] \Rightarrow \frac{1}{s}\left[e^{-2s} + e^{-s}\right]$$

$$\frac{-s^2 e^{-s}}{s^3} + \frac{e^{-s}}{s} = \frac{-s^2e^{-s} + s^2e^{-s}}{s^3}$$

$$= \frac{-s^2 e^{-s} + s^2 e^{-s}}{s^3}$$

(13) $F(t) = \begin{cases} \cos t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases}$

First shifting theorem :-

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$

$L\{e^{at} f(t)\} = \int_0^\infty e^{-st} e^{at} . f(t) = \int_0^\infty e^{-(s-a)t} f(t) = \bar{f}(s-a)$

$L\{t^n\} = \frac{n!}{s^{n+1}} \Rightarrow L\{e^{at} t^n\} = \left.\frac{n!}{s^{n+1}}\right]_{s \to s-a}$

$= \frac{n!}{(s-a)^{n+1}}$

$L\{e^{at} . \sin bt\} = \left.\frac{b}{s^2+b^2}\right]_{s \to s-a} = \frac{b}{(s-a)^2+b^2}$

$L\{e^{at} . \cos bt\} = \left.\frac{s}{s^2+b^2}\right]_{s \to s-a} = \frac{s-a}{(s-a)^2+b^2}$

$L\{e^{at} . \sinh bt\} = \left.\frac{b}{s^2-b^2}\right]_{s \to s-a} = \frac{b}{(s-a)^2-b^2}$

$L\{e^{at} . \cosh bt\} = \left.\frac{s}{s^2-b^2}\right]_{s \to s-a} = \frac{s-a}{(s-a)^2-b^2}$

$L\{e^{-at} \sin bt\} = \left.\frac{b}{s^2+b^2}\right]_{s \to s+a} = \frac{b}{(s+a)^2+b^2}$

$L\{e^{-at} \cos bt\} = \left.\frac{s}{s^2+b^2}\right]_{s \to s+a} = \frac{s+a}{(s+a)^2+b^2}$

$L\{e^{-at} \sinh bt\} = \left.\frac{b}{s^2-b^2}\right]_{s \to s+a} = \frac{b}{(s+a)^2-b^2}$

$L\{e^{-at} \cosh bt\} = \left.\frac{s}{s^2-b^2}\right]_{s \to s+a} = \frac{s+a}{(s+a)^2-b^2}$

Exercise

(1) $L\{t^3 . e^{-3t}\} = \left.\frac{3!}{s^3}\right]_{s \to s+3} = \left.\frac{6}{s^4}\right]_{s \to s+3} = \frac{6}{(s+3)^4}$

(2) $L\{\sin 4t . e^{-2t}\} = \left.\frac{4}{s^2+4^2}\right]_{s \to s+2} = \frac{4}{(s+2)^2+16}$

(3) $L\{e^{3t} \sinh 4t\} = \left.\frac{4}{s^2-4^2}\right]_{s \to s-3} = \frac{4}{(s-3)^2-16}$

(4) $L\{e^{-3t} \cos 4t\} = \left.\frac{s}{s^2+4^2}\right]_{s \to s+3} = \frac{s+3}{(s+3)^2+16}$

(5) $L\{e^{-t} \cos 2t\} = \left.\frac{s}{s^2+2^2}\right]_{s \to s+1} = \frac{s+1}{(s+1)^2+4}$

(6) $L\{e^{-t}(3\sinh 2t - 5\cosh 2t)\} = \frac{6}{(s+1)^2-2^2} - \frac{5s}{(s+1)^2-2^2}$

$= \frac{6-5s}{(s+1)^2-4}$

(7) $L\{e^{2t}(3\sin 4t - 4\cos 4t)\} = \frac{12}{(s-2)^2+4^2} - \frac{4s}{(s-2)^2+4^2}$

$= \frac{12-4s}{(s-2)^2+16}$

⑨ $\mathcal{L}\{e^{2t}(3\sin 4t - 4\cos 4t)\} = \frac{12}{(s-2)^2+16} - \frac{4s}{(s-2)^2+16} = \frac{12-4s}{(s-2)^2+16}$

⑩ $\mathcal{L}\{7e^{5t}\} = \frac{7}{s-5} = \frac{5s+0}{(s-15)^6}$

⑩ $\mathcal{L}\{(t+2)^2 e^t\} = \mathcal{L}(t)^2 + \mathcal{L}(2t) + \mathcal{L}(4)$

$= \frac{2}{(s-1)^3} + \frac{2}{(s-1)^2} + \frac{4}{s-1}$

⑪ $\mathcal{L}\{e^{-t}\cos^2 t\} = \mathcal{L}\left\{\frac{1}{2}\left[1+\cos 2t\right]\right\} = \frac{1}{2}\left[\mathcal{L}(1)+\mathcal{L}(\cos 2t)\right]$

$= \frac{1}{2}\left[\frac{1}{s+1} + \frac{s}{(s+1)^2+4}\right]$

⑬ $\mathcal{L}\{e^{4t}\cdot \sin 2t\cos t\} = \frac{1}{2}\,|2\sin 2t\cdot\cos t| = \frac{1}{2}\sin 3t + \sin t$

$= \frac{1}{2}\mathcal{L}(\sin 3t) + \frac{1}{2}\mathcal{L}(\sin t)$

$= \frac{1}{2}\cdot\frac{3}{(s-4)^2+9} + \frac{1}{2}\times\frac{1}{(s-4)^2+9}$

$= \frac{1}{2}\left(\frac{3}{(s-4)^2+9} + \frac{1}{(s-4)^2+1}\right)$

⑬ $\mathcal{L}\{\cosh at\,\sin bt\} = \frac{1}{2}(e^{at}+e^{-at})\sin bt$

$= \frac{1}{2}\mathcal{L}(e^{at}\sin bt + e^{-at}\sin bt)$

$= \frac{1}{2}\frac{b}{(s-a)^2+b^2} + \frac{b}{(s+a)^2+b^2}$

⑭ $\sinh 3t\cos^2 t$:

$= \frac{1}{2}\left[e^{3t}-e^{-3t}\right]\cos^2 t$

$= \frac{1}{2}\left[e^{3t}\cos^2 t - e^{-3t}\cos^2 t\right]$

$= \frac{1}{2}\left[\frac{1}{2}\left[(1+\cos 2t)\right]_{s\to s-a} - \frac{(1+\cos 2t)}{2}\right]_{s\to s+a}$

$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{s-a}+\frac{s}{(s-a)^2+4}\right) - \frac{1}{2}\left(\frac{1}{s+a}+\frac{s}{(s+a)^2+4}\right)\right]$

$\frac{1}{2}\times\frac{1}{2}\left[\frac{1}{s-3} + \frac{s}{(s-3)^2+4} - \frac{1}{s+3} - \frac{s+3}{(s+3)^2+4}\right]$

⑮ $te^{t} = \left.\frac{1}{s^2}\right]_{s\to s+1} = \frac{1}{s^2+1+25}$

⑯ $t^2e^{2t} = \frac{2}{(s-2)^3}$

⑰ $t^3e^{-3t} = \frac{6}{(s+3)^4}$

⑱ $t^3e^{2t} = \frac{6}{(s-2)^4}$

⑲ $e^{-at}[1-at] = \frac{1}{s+a} - \frac{a}{(s+a)^2}$

⑳ $e^{t}[t^2-6t+7] = \frac{2}{(s-1)^3} - \frac{6}{(s-1)^2} + \frac{7}{s-1}$

㉑ $e^{-2t}[t^2-6t+7] = \frac{2}{(s+2)^3} - \frac{6}{(s+2)^2} + \frac{7}{s+2}$

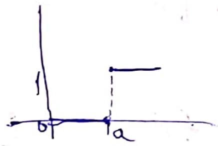
㉒ $e^{t}\sin 2t = \frac{2}{(s+1)^2+4} = \frac{2}{s^2+1+2s+4} = \frac{2}{s^2+2s+5}$

㉓ $e^{3t}\sin t = \frac{3}{(s+2)^2-25)+9}$

㉔ $e^{-2t}\cos 3t = \frac{(s+2)}{(s+2)^2+2s+1}$

Unit step function [Heaviside's unit function]

$$u(t-a) = \begin{cases} 0, & t<a \\ 1 & t>a \end{cases}$$



$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^\infty e^{-st}(u(t-a))\,dt$$

$$= \int_0^a e^{-st}(0) + \int_a^\infty e^{-st}(1)\,dt$$

$$= \int_a^\infty e^{-st}\,dt = \frac{[e^{-st}]_a^\infty}{-s} = \frac{e^{-\infty} - e^{-as}}{-s}$$

$$= \frac{0 - e^{-as}}{-s}$$

$$= \frac{-e^{-as}}{-s} = \frac{e^{-as}}{s} \#$$

① $a=0$

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} = \frac{1}{s} \#$$

Second shifting theorem:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$$

$$\mathcal{L}\{g(t)\} = ? \quad g(t) = \begin{cases} 0 & t<a \\ f(t-a) & t>a \end{cases}$$

$$\cancel{\int_0^a e^{-st}(0)}^{\,0} + \int_a^\infty e^{-st} f(t-a)\,dt$$

Put
$t - a = u$
$t = u+a$
$dt = du + 0$
$\therefore dt = du$

$$= \int_a^\infty e^{-st} f(t-a)\,dt$$

$$= \int_a^\infty e^{-s(u+a)} f(u)\,du$$

$$= \int_a^\infty e^{-su} \cdot e^{-sa} \cdot f(u)\,du$$

$$= e^{-as} \boxed{\int_a^\infty e^{-su} f(u)\,du} \to \bar{f}(s)$$

$$= e^{-as} \int_0^\infty e^{-su} f(u)\,du$$

$$= e^{-as} \int_0^\infty e^{-st} f(t)\,dt$$

$$\boxed{\mathcal{L}\{g(t)\} = e^{-as}\bar{f}(s)}$$

$$\mathcal{L}\{f(t)\} = \bar{f}(s)$$

$$\boxed{\mathcal{L}\{u(t-a)\, f(t-a)\} = e^{-as}\bar{f}(s)}$$

Find $\mathcal{L}\{g(t)\}$ where $g(t) = \begin{cases} 0, & t<4 \\ (t-4)^3 & t>4 \end{cases}$

$$= \cancel{\int_0^4 e^{-st}(0) + \int_4^\infty e^{-st}}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} = \bar{f}s$$

$$\therefore \mathcal{L}\{t^3\} = \frac{6}{s^4}$$

$$\mathcal{L}\{g(t)\} = \frac{e^{-as} \times 6}{s^4}$$

Find the $L\{g(t)\} = \begin{cases} 0, & 0<t<\frac{\pi}{2} \\ \sin t, & t>\frac{\pi}{2} \end{cases}$

$\Downarrow$

$\cos(t-\frac{\pi}{2})$

$\sin t = \cos(\frac{\pi}{2}-t)$

$= \cos(t-\frac{\pi}{2})$

$= \cos(t-\frac{\pi}{2})$

$\boxed{\cos(-\theta)=\cos\theta}$

$\therefore L\{\cos(t-\frac{\pi}{2})\}$

Assume $a=0$

$= L\{\cos t\} = \frac{s}{s^2+1}$

$\therefore L\{g(t)\} = \frac{e^{-\frac{\pi}{2}s}}{s^2+1}$

---

(3) $g(t) = \begin{cases} 0, & \text{if } t<\frac{2\pi}{3} \\ \cos(t-\frac{2\pi}{3}), & \text{if } t>\frac{2\pi}{3} \end{cases}$

$L\{\cos(t)\} = \frac{s}{s^2+1}$

$L\{g(t)\} = \frac{e^{-\frac{2\pi}{3}s}s}{s^2+1}$

(4) $L\{4u(t-\pi)\cos t\}$

$L\{u(t-a)f(t-a)\} = e^{-as}F(s)$

$-\cos t = \cos(\pi-t)$

$-\cos t = \cos[-(\pi-t)]$

$-\cos t = \cos(t-\pi)$

$\cos t = -\cos[t-\pi]$

$\therefore L\{-4u(t-\pi)\cos(t-\pi)\}$

$-4\,L\{u(t-\pi)\cos(t-\pi)\}$

$L\{\cos t\} = \frac{s}{s^2+1}$

$\therefore L\{4u(t-\pi)\cos t\} = \frac{-4e^{-\pi s}\times s}{s^2+1}$

---

(5) $g(t) = \begin{cases} 0 & 0<t<b \\ t-b & t>b \end{cases}$

$L\{f(t)\} = L\{t\} = \frac{1}{s^2}$

$L\{g(t)\} = e^{-as}F(s) = \frac{e^{-bs}}{s^2}$

---

(6) $g(t) = \begin{cases} 0 & t<a \\ e^{t-a} & t>a \end{cases}$

$F(s) = L\{e^t\} = \frac{1}{s-1}$

$g(t) = \frac{e^{-as}\times 1}{s-1} = \frac{e^{-as}}{s-1}$

⑥ $g(t) = \begin{cases} 0 & t < \frac{\pi}{3} \\ \cos\left(t - \frac{\pi}{3}\right) & t > \frac{\pi}{3} \end{cases}$

$= \frac{s e^{-\frac{\pi}{3}s}}{s^2+1}$

⑦ $g(t) = \begin{cases} 0 & t < \frac{2\pi}{3} \\ \sin\left[t - \frac{2\pi}{3}\right] & t > \frac{2\pi}{3} \end{cases}$

$\mathcal{L}(\sin t) = \frac{1}{s^2+1}$

$g(t) = \frac{e^{-\frac{2\pi}{3}s}}{s^2+1}$

$\cancel{g(t) = \begin{cases} 0 & t < \frac{3\pi}{2} \\ \left(t - \frac{3\pi}{2}\right) & t > \frac{3\pi}{2} \end{cases}}$

$g(t) = \begin{cases} 0 & t < 3 \\ \sinh(t-3) & t > 3 \end{cases}$

$= \frac{e^{-3s}}{s^2-1}$

$g(t) = \begin{cases} 0 & t < 4 \\ \cancel{4-t} & t > 4 \end{cases}$

$= e^{-(t-4)}$

$\mathcal{L}\{e^{-t}\} = \frac{1}{s+1}$

$\therefore g(t) = \frac{e^{-4s}}{s+1}$

$\mathcal{L}\{5u(t-2)\, u(t-2)^2\}$ $\quad \mathcal{L}\{t^2\} = \frac{2}{s^3}$

$5 \times 2 \times \frac{2}{s^3} \times e^{-2s} = \frac{20 e^{-2s}}{s^3}$

$g(t) = \begin{cases} 0 & t < \frac{\pi}{2} \\ \cos^2\left(t - \frac{\pi}{2}\right) & t > \frac{\pi}{2} \end{cases}$

$\mathcal{L}\{\cos^2 t\} = \mathcal{L}\{\frac{1}{2}(1 + \cos 2t)\}$

$= \frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+4}\right)$

$\therefore g(t) = \frac{e^{-\frac{\pi}{2}s}}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$

$g(t) = \begin{cases} 0 & t < \frac{\pi}{2} \\ 1 - \cos^2\left(t - \frac{\pi}{2}\right) & t > \frac{\pi}{2} \end{cases}$

Change of Scale property :-

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$$

$$\mathcal{L}\{f(at)\} = \int_0^\infty e^{-st} f(at)\,dt$$

Put,

$at = u$ | $t = 0,\ u = 0$

$t = \frac{u}{a}$ | $t = \infty,\ u = \infty$

$f(at) = f(u)$

$at = u$

$a\,dt = du$

$\therefore dt = \frac{1}{a} \times du$

$$= \frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)u} f(u)\,du$$

$$= \frac{1}{a}\int_0^\infty e^{-\left(\frac{s}{a}\right)u} f(u)\,du = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$$

$$= \frac{1}{a} \times \bar{f}(s)_{s \to \frac{s}{a}}$$

~~out of syllabus~~

$$\mathcal{L}\left\{f\left(\frac{t}{a}\right)\right\} = a \cdot \bar{f}(s)_{s \to as}$$

① $\mathcal{L}\{f(t)\} = \frac{20-4s}{s^2-4s+20}$ Find $\mathcal{L}\{f(3t)\}$

Here, $a = 3$

$$\mathcal{L}\{f(at)\} = \frac{1}{a} \cdot \bar{f}(s)_{s \to \frac{s}{a}} = \frac{s}{3}$$

$$= \frac{1}{3} \cdot \left[\frac{20 - \frac{4s}{3}}{\left(\frac{s}{3}\right)^2 - \frac{4s}{3} + 20}\right]$$

$$= \frac{1}{3}\left[\frac{\frac{60-4s}{3}}{\frac{s^2}{9} - \frac{4s}{3} + \frac{20}{1}}\right]$$

$$= \frac{1}{3}\left[\frac{\frac{60-4s}{3}}{\frac{s^2-12s+180}{9}}\right]$$

$$= \frac{\frac{60-4s}{9}}{\frac{s^2-12s+180}{9}} = \frac{60-4s}{s^2-12s+180}$$

② $\mathcal{L}\{\cos^2 t\} = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$ Find $\mathcal{L}\{\cos^2 kt\}$

Given, $\bar{f}(s) = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4}\right]$

$$\mathcal{L}\{f(at)\} = \frac{1}{a} \cdot \bar{f}(s)_{s \to \frac{s}{a}} = \frac{s}{k}$$

$$= \frac{1}{k} \times \frac{1}{2}\left[\frac{k}{s} + \frac{\frac{s}{k}}{\frac{s^2+4k^2}{k^2}}\right]$$

$$= \frac{1}{k}\times\frac{1}{2}\left[\frac{k}{s} + \frac{s}{k} \times \frac{k^2}{s^2+4k^2}\right]$$

$$= \frac{1}{2k}\left[\frac{k}{s} + \frac{ks}{s^2+4k^2}\right]$$

$$= \frac{1}{2}\left[\frac{k}{ks} + \frac{ks}{k(s^2+4k^2)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+4k^2}\right]$$

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③ If $\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$ ⇒ find $\mathcal{L}\left\{\frac{\sin at}{t}\right\}$

$\cancel{\mathcal{L}\{f(s)\} = \tan^{-1}\left(\frac{1}{s}\right)}$

$\cancel{\mathcal{L}\{f(at)\} = \frac{1}{a}\tan^{-1}\left(\frac{1}{as}\right)}$

Sol;– $a\,\mathcal{L}\left\{\frac{\sin at}{at}\right\}$

$= a \times \mathcal{L}(f(at))$

$= a \times \left[\frac{1}{a} \times \bar{f}(s)_{s \to \frac{s}{a}}\right]$

$= a \times \left[\frac{1}{a} \times \tan^{-1}\left(\frac{1}{\frac{s}{a}}\right)\right]$

$= \tan^{-1}\left(\frac{a}{s}\right)$

---

④ $\mathcal{L}\{f(t)\} = \frac{1}{s}e^{-\frac{1}{s}}$ find $\mathcal{L}\{e^{-t}f(3t)\}$

$\mathcal{L}\{f(3t)\} = \frac{1}{3}\cdot\left[\frac{1}{\frac{s}{3}}\cdot e^{-\frac{1}{\frac{s}{3}}}\right]$

$= \frac{1}{\cancel{3}}\cdot\left[\frac{\cancel{3}}{s}\cdot e^{-\frac{3}{s}}\right]$

$\mathcal{L}\{f(3t)\} = \frac{e^{-\frac{3}{s}}}{s}$

$\mathcal{L}\{e^{-t}f(3t)\} = \frac{e^{-\frac{3}{s+1}}}{s+1}$

⑤ $\mathcal{L}\{f(t)\} = \frac{s^2-s+1}{(2s+1)^2(s-1)}$ Find $\mathcal{L}\{f(2t)\}$

$\mathcal{L}\{f(at)\} = \frac{1}{a}\bar{f}(s)_{s \to \frac{s}{a}} = \frac{s}{2}$

$a=2$

$\mathcal{L}\{f(2t)\} = \frac{1}{2}\cdot\left[\frac{\frac{s^2}{4}-\frac{s}{2}+1}{\left[\frac{2s}{2}+1\right]^2\left(\frac{s}{2}-1\right)}\right]$

$= \frac{1}{2}\left[\frac{\frac{s^2-2s+4}{4}}{s^2+1^2+2s\left(\frac{s-2}{2}\right)}\right]$

$= \frac{1}{2}\left[\frac{s^2-2s+4}{\cancel{4}\cdot 2} \times \frac{\cancel{2}}{(s^2+1+2s)(s-2)}\right]$

$= \frac{s^2-2s+4}{4(s^2+1+2s)(s-2)}$

$= \frac{s^2-2s+4}{4(s^3+s+2s^2-2s^2-2-4s)}$

$= \frac{s^2-2s+4}{4(s^3-3s-2)}$

$\mathcal{L}\{f(2t)\} = \frac{1}{4}\left[\frac{s^2-2s+4}{s^3-3s-2}\right]$

---

⑥ $\mathcal{L}\{f(t)\} = \frac{1}{s}e^{-\frac{1}{s}}$ find $\mathcal{L}\{f(5t)\}$

$\mathcal{L}\{f(5t)\} = \frac{1}{5}\cdot\left(\frac{5}{s}e^{-\frac{5}{s}}\right)$

$\mathcal{L}\{f(5t)\} = \frac{1}{\cancel{5}}\times\frac{\cancel{5}}{s}e^{-\frac{5}{s}} = \frac{e^{-\frac{5}{s}}}{s}$

⑦ $\mathcal{L}\{f(t)\} = \frac{20-4s}{s^2-4s+20}$ Find $\mathcal{L}\{e^{-t}f(2t)\}$.

$$\mathcal{L}\{f(2t)\} = \frac{1}{2}\left[\frac{20-\frac{4s}{2}}{\frac{s^2}{4}-\frac{4s}{2}+20}\right]$$

$$= \frac{1}{2}\left[\frac{20-2s}{\frac{s^2-8s+80}{4}}\right]$$

$$= \frac{1}{2}\left[\frac{(20-2s)\times 4\,2}{s^2-8s+80}\right]$$

$$\mathcal{L}\{f(2t)\} = \frac{40-4s}{s^2-8s+80}$$

The n,

$$\mathcal{L}\{e^{-t}f(2t)\} = \left.\frac{40-4s}{s^2-8s+80}\right]$$ where, $s \to s+a = s+1$

$$= \frac{40-4s-4}{(s+1)^2-8s-8+80}$$

$$= \frac{40-4s-4}{s^2+1+2s-8s+72}$$

$$\mathcal{L}\{e^{-t}f(2t)\} = \frac{40-4s-4}{s^2-6s+73}$$

$$\mathcal{L}\{e^{-t}f(2t)\} = \frac{36-4s}{s^2-6s+73}$$

⑧ ~~Given~~, $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$

$$\mathcal{L}\{f(at)\} = \frac{1}{a}\left[\frac{1}{s^2+1}\right]_{s \to \frac{s}{a}}$$

$$= \frac{1}{a}\left[\frac{1}{\frac{s^2}{a^2}+1}\right]$$

$$= \frac{1}{a}\left[\frac{1}{\frac{s^2+a^2}{a^2}}\right]$$

$$= \frac{1}{a}\left[\frac{a^2}{s^2+a^2}\right]$$

$$\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$$

$$\mathcal{L}\{\cos at\} = \frac{1}{a}\left[\frac{s}{s^2+1}\right]_{s \to \frac{s}{a}} = \frac{1}{a}\left[\frac{\frac{s}{a}}{\frac{s^2+a^2}{a^2}}\right]$$

$$= \frac{1}{a}\left[\frac{s \times a}{s^2+a^2}\right]$$

$$= \frac{s}{s^2+a^2}$$

$$\mathcal{L}\{\cosh at\} = \frac{1}{a}\left[\frac{s}{s^2-1}\right]_{s \to \frac{s}{a}} = \frac{1}{a}\left[\frac{\frac{s}{a}}{\frac{s^2-a^2}{a^2}}\right] = \frac{1}{a}\left[\frac{s \times a}{s^2-a^2}\right] = \frac{s}{s^2-a^2}$$

$\mathcal{L}\{\sinh at\}$

$$\mathcal{L}\{e^{at}\} = \frac{1}{a}\left[\frac{1}{s-1}\right]_{s \to \frac{s}{a}} = \frac{1}{a}\left[\frac{a}{s-a}\right] = \frac{1}{s-a}$$

$\mathcal{L}\{e^{-at}\}$

Laplace transform of derivatives

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} \frac{d}{dt}(f(t))$$

$$\int u\,dv = uv - \int v\,du$$

$$= e^{-st} f(t) - \int f(t) e^{-st} \times (-s)$$

$$\left[e^{-st} f(t)\right]_0^\infty + s\int_0^\infty e^{-st} f(t)\,dt$$

$$[0 - 1f(0)] + s\,\mathcal{L}\{f(t)\}$$

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$L\{f'(t)\} = sL\{f(t)\} - f(0)$

$L\{f''t\} = s^2L\{f(t)\} - sf(0) - f'(0)$

$L\{f'''t\} = s^3L\{f(t)\} - s^2f(0) - sf'(0) - f''(0)$

Find $L\{f(t)\}$ by theorem of Transform of Derivatives

① t^2

$0 = f(0) \leftarrow f(t) = t^2$

$0 = f'(0) \leftarrow f'(t) = 2t$

$f''(0) = 2 \leftarrow \boxed{f''(t) = 2}$

$L\{f''(t)\} = s^2L\{f(t)\} - s\cancel{f(0)}^0 - \cancel{f'(0)}^0$

$L(2) = s^2L\{t^2\}$

$\frac{2}{s} = s^2L(t^2)$

$\therefore L(t^2) = \frac{2}{s^3}$

② $L\{\sin^2 t\}$

Put, $\sin 2t$ $\cos\frac{dt}{dx}$

$\sin^2(0) = 0$

$f(0) \leftarrow f(t) = \sin^2 t$

$f'(t) = 2\sin t\cos t = \sin 2t$

$L\{f'(t)\} = sL\{f(t)\} - \cancel{f(0)}$

$L\{\sin 2t\} = sL\{\sin^2 t\}$

$\frac{2}{s(s^2+4)} = L\{\sin^2 t\}$

③ $t\sin at$

$L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0)$

$f(t) = t\sin at$, $f(0) = 0$

$f'(t) = \sin at + at\cos at$, $f'(0) = 0$

$f''(t) = a\cos at + a(\cos at + t(-a\sin at))$

$f''(t) = a\cos at + a\cos at - a^2t\sin at$

$f''(t) = 2a\cos at - a^2t\sin at$ $f''(0) \neq 0$

$\therefore L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0)$

$2aL\{\cos at\} - a^2L\{t\sin at\} = s^2L(t\sin at) - s\cancel{f(0)}^0 - \cancel{f'(0)}^0$

$2a\left(\frac{s}{s^2+a^2}\right) = s^2L\{t\sin at\} + a^2L\{t\sin at\}$

$\frac{2as}{s^2+a^2} = L\{t\sin at\}(s^2+a^2)$

$\therefore L\{t\sin at\} = \frac{2as}{(s^2+a^2)^2}$

④ $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}}e^{-\frac{1}{4s}}$, find $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$

Given, $f(t) = \sin\sqrt{t} \Rightarrow f(0) = 0$

$f'(t) = \frac{1}{2}\frac{\cos\sqrt{t}}{\sqrt{t}} \Rightarrow f(0) \neq 0$

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$\therefore \mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)^{0}$

$\frac{1}{2}\mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = s\times\frac{\sqrt{\pi}}{2s^{3/2}}e^{-\frac{1}{4s}}$

$\mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \frac{\sqrt{\pi}\cdot s^{1-\frac{3}{2}}\cdot \cancel{2}\times e^{-\frac{1}{4s}}}{\cancel{2}}$

$\mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\pi}\, s^{-\frac{1}{2}}\times e^{-\frac{1}{4s}}$

$\therefore \mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \frac{\sqrt{\pi}}{\sqrt{s}}e^{-\frac{1}{4s}} = \sqrt{\frac{\pi}{s}}e^{-\frac{1}{4s}}$

If $\mathcal{L}\left\{2\sqrt{\frac{t}{\pi}}\right\} = \frac{1}{s^{3/2}}$, prove that $\mathcal{L}\left\{\frac{1}{\sqrt{t\pi}}\right\} = \frac{1}{\sqrt{s}}$

$f(t) = 2\frac{\sqrt{t}}{\sqrt{\pi}}$ $\quad f(0) = 0$

$f'(t) = \frac{\cancel{2}}{\sqrt{\pi}}\times\frac{1}{\cancel{2}\sqrt{t}} = \frac{1}{\sqrt{t\pi}}$

$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)^{0}$

$\mathcal{L}\left\{\frac{1}{\sqrt{t\pi}}\right\} = s\times\frac{1}{s^{3/2}}$

$\mathcal{L}\left\{\frac{1}{\sqrt{t\pi}}\right\} = s^{1}\cdot s^{-3/2} = s^{1-\frac{3}{2}} = s^{-\frac{1}{2}} = \frac{1}{\sqrt{s}}$

$\therefore \mathcal{L}\left\{\frac{1}{\sqrt{t\pi}}\right\} = \frac{1}{\sqrt{s}}$

$\mathcal{L}\{t\sin at\} = \frac{2as}{(s^2+a^2)^2}$. P.T. That $\mathcal{L}\{\sin at + at\cos at\} = \frac{2as^2}{(s^2+a^2)^2}$

$f(t) = t\sin at \rightarrow f(0) = 0$

$f'(t) = \sin at + at\cos at \rightarrow f'(0) \neq 0$

$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)^{0}$

$\mathcal{L}\{\sin at + at\cos at\} = s\times\frac{2as}{(s^2+a^2)^2} = \frac{2as^2}{(s^2+a^2)^2}$

$\therefore$ Here proved.

Multiplication of powers of t:-

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-ts}f(t)\,dt$

$\mathcal{L}\{t.f(t)\} = ?$

D.O.B.S.

$\frac{d}{ds}\mathcal{L}\{f(t)\} = \frac{d}{ds}\int_0^\infty e^{-ts}f(t)\,dt$

Leibnitz rule :- According to this rule,

$$\frac{d}{d\alpha}\int_a^b f(x,y) = \int_a^b \frac{\partial}{\partial s} f(x,y)$$

$\therefore \frac{d}{ds}\mathcal{L}\{f(t)\} = \int_0^\infty \frac{\partial}{\partial s}\cdot e^{-ts}f(t)\,dt$

$\frac{d}{ds}\mathcal{L}\{f(t)\} = \int -t\times e^{-ts}f(t)\,dt$

$\therefore \frac{d}{ds}\mathcal{L}\{f(t)\} = -\int te^{-st}f(t)\,dt \rightarrow -\mathcal{L}\{t\,f(t)\}$

$\therefore \frac{d}{ds} L\{f(t)\} = -L\{t(f(t))\}$

$\therefore \boxed{L\{t(f(t))\} = (-1)\frac{d}{ds} L\{f(t)\}}$

$\therefore \boxed{L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} L\{f(t)\}}$

① $L\{t \sin at\}$

$L\{t\, f(t)\} = (-1)\frac{d}{ds} L\{f(t)\}$

$= (-1)\frac{d}{ds} L\{\sin at\}$

$= -1 \frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$

$= -a \frac{d}{ds}\left(s^2+a^2\right)^{-1}$

$L\{t f(t)\} = -a\left(\frac{-1}{(s^2+a^2)^2}\right)\times 2s = \frac{2as}{(s^2+a^2)^2}$

② $L\{t e^{t} \cosh t\}$

$L\{t.e^{t}\cosh t\} = (-1)\frac{d}{ds} L\{\cosh t\}$

$= (-1)\frac{d}{ds}\left(\frac{s}{s^2-1^2}\right)$

$= (-1)\left[\frac{(s^2-1^2)-2s^2}{(s^2-1^2)^2}\right]$

$= \left[\frac{2s^2-(s^2-1^2)}{(s^2-1^2)^2}\right]$

$= \left[\frac{s^2+1}{(s^2-1)^2}\right]$

$= \frac{s^2+1}{(s^2-1)^2}$ where, $s \to s+1$

$= \frac{(s+1)^2+1}{((s+1)^2-1)^2} = \frac{s^2+2s+2}{(s^2+2s)^2}$

③ $L\{t\cos 3t\}$

$L\{t\, f(t)\} = (-1)\frac{d}{ds} L\{f(t)\}$

$= (-1)\frac{d}{ds} L\{\cos 3t\}$

$= (-1)\frac{d}{ds}\left\{\frac{s}{s^2+3^2}\right\}$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu'-uv'}{v^2}$

$= (-1)\left[\frac{s^2+9-2s^2}{(s^2+9)^2}\right]$

$= \left[\frac{2s^2-s^2-9}{(s^2+9)^2}\right]$

$L\{t\cos 3t\} = \left[\frac{s^2-9}{(s^2+9)^2}\right]$

④ $L\{t^2 e^{2t}\}$

$L\{t^2 e^{2t}\} = (-1)^2 \frac{d^2}{ds^2} L\{e^{2t}\}$

$$L\{t^2e^{2t}\} = \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right)$$

$$= \frac{1}{(s-2)^2} \times \frac{2}{(s-2)^3}$$

④ $t^2\cos 3t$

$$L\{t^2\cos 3t\} = (-1)^2\frac{d^2}{ds^2}L\{\cos 3t\}$$

$$= 1\times\frac{d^2}{ds^2}\left(\frac{s}{s^2+9}\right)$$

$$= \frac{d}{ds}\left(\frac{s^2+9 - s\times 2s}{(s^2+9)^2}\right)$$

$$= \frac{d}{ds}\left(\frac{9-s^2}{(s^2+9)^2}\right)$$

$$= \frac{-2s(s^2+9)^2 - 2(9-s^2)(s^2+9)2s}{(s^2+9)^4}$$

$$= \frac{-2s(s^2+9)^2 - 2(9-s^2)(s^2+9)2s}{(s^2+9)^4}$$

$$= (s^2+9)\times\frac{-2s(s^2+9)-4s(9-s^2)}{(s^2+9)^4}$$

$$= \frac{-2s^3-18s-36s+4s^3}{(s^2+9)^3}$$

$$= \frac{2s^3-54s}{(s^2+9)^3}$$

⑤ $L\{t^3e^{-3t}\}$

$$L\{t^3e^{-3t}\} = (-1)^3\frac{d^3}{ds^3}L\{e^{-3t}\}$$

$$= -1\frac{d^3}{ds^3}\left(\frac{1}{s+3}\right)$$

$$= -1\frac{d^2}{ds^2}\left(\frac{-1}{(s+3)^2}\right)$$

$$= -1\times-1\frac{d^2}{ds^2}\left(\frac{1}{(s+3)^2}\right)$$

$$= \frac{d}{ds}\left(\frac{-2}{(s+3)^3}\right)$$

$$= -2\frac{d}{ds}\left(\frac{1}{(s+3)^3}\right)$$

$$= -2\left(\frac{-3}{(s+3)^4}\right) = \frac{6}{(s+3)^4}$$

⑥ $L\{t\cos 3t\cos 2t\} = \frac{1}{2}L\{t[2\cos 3t\cos 2t]\}$

$$= \frac{1}{2}L\{t[\cos(3t-2t)-\cos(3t+2t)]\}$$

$$= \frac{1}{2}L\{t[\cos t + \cos 5t]\}$$

$$= \frac{1}{2}L\{t\cos t\} + \frac{1}{2}L\{t\cos 5t\}$$

$$= (-1)\frac{1}{2}\frac{d}{ds}L\{\cos t + \cos 5t\}$$

$$= -\frac{1}{2}\left[\frac{s}{s^2+1} + \frac{s}{s^2+25}\right]$$

$=$

$L\{te^{-t}\sin 4t\}$ $= (-1)\frac{d}{ds}\left(\frac{4}{s^2+16}\right)$

$= -4\frac{d}{ds}\left(\frac{1}{s^2+16}\right)$

$= -4 \times \left[\frac{-1\times 2s}{(s^2+16)^2}\right]$

$= \frac{4\times 2s}{(s^2+16)^2}$ $s \to s+1$

$= \frac{4\times 2(s+1)}{[(s+1)^2+16]^2}$

$= \frac{8s+8}{(s^2+2s+17)^2} = \frac{8s+8}{(s^2+2s+17)^2}$

(8) $L\{te^{at}\sin bt\}$ $= (-1)\frac{d}{ds}\left(\frac{b}{s^2+b^2}\right)$

$= (-1)\times\left[\frac{b\cdot -1\times 2s}{(s^2+b^2)^2}\right]$

$= \frac{2s\times b}{(s^2+b^2)^2}$ where $s \to s-a$

$= \frac{2(s-a)b}{[(s-a)^2+b^2]^2}$

$= \frac{2sb-2ab}{[s^2+a^2-2as+b^2]^2}$

$= \frac{2b(s-a)}{(s^2+a^2-2as+b^2)^2}$

(9) $L\{te^{2t}\cos 5t\}$ $= (-1)\frac{d}{ds}\left(\frac{s}{s^2+25}\right)$

$= (-1)\left[\frac{s^2+25 - s\times 2s}{(s^2+25)^2}\right]$

$= -1\left[\frac{s^2+25-2s^2}{(s^2+25^2)^2}\right]$

$= (-1)\left[\frac{25-s^2}{(s^2+25)^2}\right]$

$= \frac{s^2-25}{(s^2+25)^2}$ $s \to s-2$

$= \frac{(s^2+4-2s)-25}{(s^2+4+25)^2}$

$= \frac{s^2-2s-21}{(s^2+4+25)^2}$

$L\{1\} = L\{t^0\} = \frac{0!}{s^{0+1}} = \frac{1}{s}$

(10) $L\{t^2\cosh at\}$ $= (-1)^2\frac{d^2}{ds^2}\left[\frac{s}{s^2-a^2}\right]$

$= \frac{d}{ds}\left[\frac{(s^2-a^2) - s\times 2s}{(s^2-a^2)^2}\right]$

$= \frac{d}{ds}\left[\frac{-s^2-a^2}{(s^2-a^2)^2}\right]$

$= (-1)\frac{d}{ds}\left[\frac{s^2+a^2}{(s^2-a^2)^2}\right]$

$= (-1)\left[\frac{(s^2-a^2)^2\times 2s - (s^2+a^2)2(s^2-a^2)\times 2s}{(s^2-a^2)^4}\right]$

$$= (-1) \times \left[\cancel{(s^2-a^2)} \times \frac{2s(s^2-a^2) - 4s(s^2+a^2)}{(s^2-a^2)^{\cancel{4}\,3}} \right]$$

$$= -1 \left[\frac{2s^3 - 2a^2 s - 4s^3 - 4a^2 s}{(s^2-a^2)^3} \right]$$

$$= \frac{4s^3 + 4a^2 s - 2s^3 + 2a^2 s}{(s^2-a^2)^3}$$

$$\cancel{= \frac{2s(2s^2 - 2a^2)}{s^2-a^2}}$$

$$= \frac{2s^3 + 6a^2 s}{(s^2-a^2)^3}$$

Laplace transformation of division by t

① $L\left\{\frac{\sin at}{t}\right\}$

$$= L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty L\{f(t)\}$$

$$= \int_s^\infty L\{\sin at\}$$

$$= a\int_s^\infty \frac{1}{s^2+a^2} = \left[a \cdot \frac{1}{a} \tan^{-1}\left(\frac{s}{a}\right) \right]_s^\infty$$

$$= \left[\tan^{-1}\left(\frac{s}{a}\right)\right]_s^\infty$$

$$= \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{a}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)$$

$$= \cot^{-1}\left(\frac{s}{a}\right)$$

② $L\left\{\frac{1-e^t}{t}\right\}$

$$\therefore L\{f(t)\} = L\{1\} - L\{e^{t}\}$$

$$= \frac{1}{s} - \frac{1}{s-1}$$

$$= \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right)$$

$$= \left(\log s - \log (s-1)\right)_s^\infty$$

$$= \left[\log \frac{s}{s-1}\right]_s^\infty$$

$$= \left[\log \frac{1}{\frac{s-1}{s}}\right]_s^\infty$$

$$= \left[\log \frac{1}{1-\frac{1}{s}}\right]_s^\infty$$

$$= \left[\log \frac{1}{1-\frac{1}{\infty}}\right] - \left[\log \frac{1}{1-\frac{1}{s}}\right]$$

$$= \log 1^{\nearrow 0} - \log \frac{s}{s-1}$$

$$= -\log \frac{s}{s-1} = \log\left(\frac{s}{s-1}\right)^{-1} = \log\left(\frac{s-1}{s}\right)$$

③ $L\left\{\frac{e^{2t} - e^{3t}}{t}\right\}$

$$\therefore = L\{e^{2t}\} - L\{e^{3t}\}$$

$$= \int_s^\infty \left(\frac{1}{s-2}\right) - \int_s^\infty \left(\frac{1}{s-3}\right)$$

$$= \left[\log(s-2) - \log(s-3)\right]_s^\infty \; ; = \log\left(\frac{s-2}{s-3}\right)_s^\infty$$

$$= \left[\log\left(\frac{s-2}{s-3}\right)\right]_s^{\infty}$$

$$= \cancel{\left[\log\left(\frac{s}{s-3} - \frac{2}{s-3}\right)\right]_s^{\infty}}$$

$$= \left[\log\left(\frac{\infty-2}{\infty-3}\right)^{\to 0} - \log\left(\frac{s-2}{s-3}\right)\right]$$

$$= -\log\left(\frac{s-2}{s-3}\right) = \log\left(\frac{s-3}{s-2}\right)$$

(4) $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$

$$= L\{\cos 2t\} - L\{\cos 3t\}$$

$$= \int_s^{\infty}\left[\frac{s}{s^2+4} - \frac{s}{s^2+9}\right]$$

$$= \frac{1}{2}\int_s^{\infty}\left[\frac{2s}{s^2+4} - \frac{2s}{s^2+9}\right]$$

we know $\int \frac{f'(x)}{f(x)} = \log f(x)$

$$= \frac{1}{2}\left[\log(s^2+4) - \log(s^2+9)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+9}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log\left(\frac{1+\frac{4}{s^2}}{1+\frac{9}{s^2}}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log\left(\frac{1+\frac{4}{\infty^2}^{\to 0}}{1+\frac{9}{\infty^2}^{\to 0}}\right) - \log\left(\frac{1+\frac{4}{s^2}}{1+\frac{9}{s^2}}\right)\right]$$

$$= \frac{1}{2}\left[\log(1)^{\to 0} - \log\left(\frac{s^2+4}{s^2+9}\right)\right]$$

$$= \frac{1}{2}\log\left(\frac{s^2+9}{s^2+4}\right) = \log\sqrt{\frac{s^2+9}{s^2+4}}$$

(5) $L\left\{\frac{\sin 3t \cdot \cos t}{t}\right\} = \frac{1}{2}\{2\sin 3t \cdot \cos t\} = \sin(a+b) + \sin(a-b)$

sol:—

$$= \frac{1}{2}\int_s^{\infty}\left[L\{\sin 4t\} + L\{\sin 2t\}\right]$$

$$= \frac{1}{2}\left[\int_s^{\infty}\left(4 \cdot \frac{1}{s^2+16} + \frac{2}{s^2+4}\right)\right]$$

$$= \frac{1}{2}\left[4\int_s^{\infty}\frac{1}{s^2+16} + 2\int_s^{\infty}\frac{1}{s^2+4}\right]$$

$$\cancel{= 3\int_s^{\infty}\frac{1}{1+\frac{9}{s^2}} + \int_s^{\infty}\frac{1}{1+\frac{1}{s^2}}}$$

$$\cancel{= 3\left[\log\frac{1}{1+\frac{9}{s^2}}\right]_s^{\infty} + \left[\log\frac{1}{1+\frac{1}{s^2}}\right]_s^{\infty}}$$

wrong

$$\cancel{= -3\log\left(\frac{s^2}{s^2+9}\right)}$$

$$= \frac{1}{2}\left[4 \times \frac{1}{4}\tan^{-1}\left(\frac{s}{4}\right) + 2 \times \frac{1}{2}\tan^{-1}\left(\frac{s}{2}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\frac{s}{4}\right) + \tan^{-1}\left(\frac{s}{2}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\left(\tan^{-1}(\infty) + \tan^{-1}(\infty)\right) - \left(\tan^{-1}\left(\frac{s}{4}\right) + \tan^{-1}\left(\frac{s}{2}\right)\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{\pi}{2}\right) - \tan^{-1}\left(\frac{s}{4}\right) + \tan^{-1}\left(\frac{s}{2}\right)\right]$$

$$= \frac{1}{2}\left[\pi - \tan^{-1}\left(\frac{s}{4}\right) - \tan^{-1}\left(\frac{s}{2}\right)\right]$$

(6) $L\left\{\frac{1-\cos t}{t}\right\} = L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty L\{f(t)\}$

$= \int_s^\infty L(1-\cos t)\}$

$= \int_s^\infty \left(\frac{1}{s} - \frac{1}{2}\frac{2s}{s^2+1}\right)$

$= \left[\log s - \frac{1}{2}\log(s^2+1)\right]_s^\infty$

$= \left[\log s - \log\sqrt{s^2+1}\right]_s^\infty$

$= \left[\log \frac{s}{\sqrt{s^2+1}}\right]_s^\infty$

$= \left[\log \frac{\infty}{\sqrt{\infty^2+1}} \to 0 - \log\frac{s}{\sqrt{s^2+1}}\right]$

$L\left\{\frac{1-\cos t}{t}\right\} = -\log\frac{s}{\sqrt{s^2+1}} = \log\frac{\sqrt{s^2+1}}{s}$

Imp ***
(7) $L\left\{\frac{1-\cos t}{t^2}\right\}$

Sol: $L\left\{\frac{1-\cos t}{t^2}\right\} = \frac{1}{t} L\left\{\frac{1-\cos t}{t}\right\} = \frac{1}{t} \times \left[\log\sqrt{\frac{s^2+1}{s^2}}\right]$

$= \int_s^\infty \left(\log\sqrt{\frac{s^2+1}{s^2}}\right)$

$= \int_s^\infty \log\sqrt{\frac{s^2+1}{s^2}} \times 1$ $\qquad = \left[\log\sqrt{\frac{s^2+1}{s^2}} \cdot 1\right]_s^\infty$

u $\qquad$ dv $\qquad \int dv = v = s$

$du = \frac{d}{ds}\left(\log\sqrt{\frac{s^2+1}{s^2}}\right)$

$du = \frac{1}{\sqrt{\frac{s^2+1}{s^2}}} \times \frac{1}{2\sqrt{\frac{s^2+1}{s^2}}} \times \frac{d}{ds}\cancel{(1)}^0 + \frac{d}{ds}\left(\frac{1}{s^2}\right)$

into

$du = \sqrt{\frac{s^2}{s^2+1}} \times \frac{1}{2} \times \sqrt{\frac{s^2}{s^2+1}} \times \frac{-2}{s^3}$

$du = -\frac{s^2}{s^2+1} \times \frac{1}{s^3}$

$\therefore \int \log\sqrt{\frac{s^2+1}{s^2}} \times 1 = \left[\log\sqrt{\frac{s^2+1}{s^2}} \times s\right]_s^\infty - \int_s^\infty s \times \frac{-1}{s(s^2+1)}$

$= \left[\log\left(\frac{s^2+1}{s^2}\right)^{\frac{1}{2}} \times s\right]_s^\infty + \int_s^\infty \frac{1}{s^2+1}$

$= \frac{s}{2}\left[\log\left(\frac{s^2+1}{s^2}\right)\right]_s^\infty + (1 \times \tan^{-1} s)_s^\infty$

$= \frac{s}{2}\left[\log\left(1+\frac{1}{\infty^2}\right) - \log\left(\frac{s^2+1}{s^2}\right)\right] + \left[\tan^{-1}\infty - \tan^{-1}s\right]$

$= \frac{s}{2}\left[\log(1)^{\to 0} - \log\left(\frac{s^2+1}{s^2}\right)\right] + \left[\frac{\pi}{2} - \tan^{-1}s\right]$

$= \frac{s}{2}\log\left(\frac{s^2}{s^2+1}\right) + \left[\frac{\pi}{2} - \tan^{-1}s\right]$

$= \frac{s}{2}\log\left(\frac{s^2}{s^2+1}\right) + \cot^{-1}(s)$

Laplace transform of integral of function

$L\left\{\int_0^t f(t)\,dt\right\}$

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

$g(t) = \int_0^t f(t)$

$g'(t) = f(t) \qquad [\because g(0) = 0]$

$L\{g'(t)\} = sL\{g(t)\} - \cancel{g(0)}^0$

$L\{g(t)\} = \frac{1}{s} L\{g'(t)\}$

$$\boxed{L\left\{\int_0^t f(t)\right\} = \frac{1}{s} L\{f(t)\}}$$

Ex: (1) $L\left\{\int_0^t \sin at\right\} = \frac{1}{s} \times L\{\sin at\} = \frac{1}{s} \times \frac{a}{s^2+a^2}$

② $L\left\{\int_0^t e^{-t}\cos t\, dt\right\}$

$= \frac{1}{s} \times L\{\cos t\}_{s \to s+1}$

$= \frac{1}{s} \times \left[\frac{s}{s^2+1}\right]_{s \to s+1}$

$= \frac{1}{s} \times \left[\frac{s+1}{(s+1)^2+1}\right] = \frac{1}{s}\times\frac{s+1}{(s^2+2s+2)}$

③ $L\left\{\int_0^t\int_0^t \cosh at\, dt\, dt\right\}$

$= \frac{1}{s^2} L\{\cosh at\}$

$= \frac{1}{s^2} \times \frac{s}{s^2-a^2} = \frac{1}{s(s^2-a^2)}$

④ $L\left\{\int_0^t te^{-t}\sin 4t\, dt\right\}$

$= \frac{1}{s} \times (-1) \times \frac{d}{ds}\left\{L\{\sin 4t\}_{s \to s+1}\right\}$

$= \frac{-1}{s} \times \frac{d}{ds}\left[\frac{4}{s^2+16}\right]_{s \to s+1}$

$= \frac{-1}{s} \times \frac{d}{ds}\left[\frac{4}{s^2+2s+17}\right]$

$= \frac{-1}{s} \times 4\frac{d}{ds}\left[\frac{1}{s^2+2s+17}\right]$

$= -\frac{1}{s} \times 4 \frac{(-1)}{(s^2+2s+17)^2} \times (2s+2)$

$= \frac{1}{s} \times \frac{4(2s+2)}{(s^2+2s+17)} = \frac{8(s+1)}{s(s^2+2s+17)}$

⑤ $L\left\{\int_0^t \frac{1-e^{-t}}{t}\, dt\right\}$

$\therefore ① \Rightarrow L\{1-e^{-t}\} = \int_s^\infty \frac{1}{s} - \frac{1}{s+1}$

$= \left[\log s - \log(s+1)\right]_s^\infty$

$= \left[\log \frac{s}{s+1}\right]_s^\infty$

$= \left[\log \frac{1}{1+\frac{1}{s}}\right]_s^\infty$

$= \log \frac{1}{1+\frac{1}{\infty}} - \log\frac{s}{s+1}$

$= \log(1) \to 0 - \log\frac{s}{s+1}$

$= \left(\log\frac{s+1}{s}\right) \times \frac{1}{s}$

$= \frac{1}{s} \times \log\left(\frac{s+1}{s}\right)$

⑥ $L\int_0^t te^{-t}\sin t\, dt$

$\therefore$ ① $\Rightarrow L\{\sin t\} = \frac{1}{s^2+1}$

② $\Rightarrow \left(\frac{1}{s^2+1}\right)_{s \to s+1} = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$

③ $\Rightarrow \frac{-1}{1} \times \frac{d}{ds}\left(\frac{1}{s^2+2s+2}\right) = \frac{-1 \times (-1)}{(s^2+2s+2)^2} \times (2s+2)$

$= \frac{2(s+1)}{(s^2+2s+2)^2}$

④ $\Rightarrow \frac{1}{s} \times \frac{2(s+1)}{(s^2+2s+2)^2} = \frac{2(s+1)}{s(s^2+2s+2)^2}$

$L\left\{\int_0^t \int_0^t \sin at \, dt\, dt\right\}$

$\therefore$ ① $\Rightarrow L\{\sin at\} = \frac{a}{s^2+a^2}$

② $\Rightarrow \frac{1}{s} \times \frac{a}{s^2+a^2}$

③ $\Rightarrow \frac{1}{s} \times \frac{1}{s} \times \frac{a}{(s^2+a^2)} = \frac{a}{s^2(s^2+a^2)}$

Evaluation of integrals by the laplace transform

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$

① $\Rightarrow \int_0^\infty t e^{-3t}\, dt$

$= \int_0^\infty e^{-3t} t \, dt$

$= \int_0^\infty e^{-st} \cdot f(t)\, dt = L\{t\} = \frac{1}{s^2} = \frac{1}{3^2} = \frac{1}{9}$

$\therefore s = 3, \; f(t) = t$

② $\int_0^\infty e^{-4t} \sin 3t \, dt$

$= L\{\sin 3t\} = \frac{3}{s^2+3^2}$ and $s = 4$

$= \frac{3}{4^2+9} = \frac{3}{16+9} = \frac{3}{25}$

③ $\int_0^\infty e^{-2t} \cos t \, dt$

$= L\{\cos t\}_{s=2} = \frac{s}{s^2+1} \quad [\because s=2] = \frac{2}{5}$

④ $\int_0^\infty t e^{-2t} \sin 3t \, dt$

$= L\{t \sin 3t\}$

$= 3 \times (-1) \times \frac{d}{ds}\left(\frac{1}{s^2+9}\right)$

$= -3 \times \frac{-1}{(s^2+9)^2} \times 2s$

$= \frac{6s}{(s^2+9)^2} = \frac{6 \times 2}{(2^2+9)^2} = \frac{12}{169}$ #

⑤ $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\, dt = \int_0^\infty \frac{e^{-at}}{t}\, dt - \int_0^\infty \frac{e^{-bt}}{t}\, dt$

$L\{e^{-at}\} - L\{e^{-bt}\}$

$= \int_s^\infty \frac{1}{s+a} - \int_s^\infty \frac{1}{s+b}$

$= \left[\log(s+a)\right]_s^\infty - \left[\log(s+b)\right]_s^\infty$

$= \log\left(\frac{s+a}{s+b}\right)_s^\infty$

$= \log\left(\frac{1+\frac{a}{\infty}}{1+\frac{b}{\infty}}\right) - \log\left(\frac{s+a}{s+b}\right)$

$= -\log\left(\frac{s+a}{s+b}\right) = \log\left(\frac{s+b}{s+a}\right)$

⑥ $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} = \int_0^\infty e^{-0t}\left(\frac{\cos 6t - \cos 4t}{t}\right) dt$

$= \int_0^\infty e^{-0t} \cos 6t \, dt - \int_0^\infty e^{-0t} \cdot \cos 4t \, dt$

$= L\{\cos 6t\} - L\{\cos 4t\}$

$= \frac{s}{s^2+36} - \frac{s}{s^2+16}$

$= 6 \int_s^\infty \frac{1}{s^2+36} - 4 \int_s^\infty \frac{1}{s^2+16}$

$= 6 \times \frac{1}{6} \times \tan^{-1}\left(\frac{s}{6}\right) - 4 \times \frac{1}{4} \tan^{-1}\left(\frac{s}{4}\right)$

$= \left[\tan^{-1}\left(\frac{s}{6}\right)\right]_s^\infty - \left[\tan^{-1}\left(\frac{s}{4}\right)\right]_s^\infty$

$= \left[\tan^{-1}\left(\frac{\infty}{6}\right) - \tan^{-1}\frac{s}{6}\right] - \left[\tan^{-1}\left(\frac{\infty}{4}\right) - \tan^{-1}\frac{s}{4}\right]$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{6}\right) \ldots$

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$= \tan^{-1}(1) - \tan^{-1}(1) = 0$

(i) $\int_0^\infty e^{-2t} \cdot t \, dt$

~~(ii) $\int_0^\infty t^3 e^{-2t} dt$~~

Sol: $\int_0^\infty e^{-2t} \cdot t \cdot dt$

$= L\{t\} = \frac{1}{s^2}$ $\boxed{\text{and } s = 2}$ $\therefore \frac{1}{4}$

(ii) $\int_0^\infty t^3 \cdot e^{-2t} dt$

Here, $s = 2$, $f(t) = t^3$

$L\{t^3\} = \frac{3!}{s^4} = \frac{6}{2^4} = \frac{6^3}{16_8} = \frac{3}{8}$

(iii) $\int_0^\infty e^{-4t} \cos 3t \, dt$

Here, $s = 4$, $f(t) = \cos 3t$

$L\{\cos 3t\} = \frac{s}{s^2+9} = \frac{4}{16+9} = \frac{4}{25}$

(iv) $\int_0^\infty e^{-t} \sin 2t \, dt$ $s = 1$

Sol: $L\{\sin 2t\} = \frac{2}{s^2+4} = \frac{2}{(1)^2+9} = \frac{2}{5}$

(V) $\int_0^\infty e^{-2t} \cdot \sin 3t \, dt$

Sol:- $L\{\sin 3t\} = \frac{3}{s^2+9} = \frac{3}{2^2+9} = \frac{3}{13}$

(Vi) $\int_0^\infty e^{-3t} \sin 2t \, dt$

Sol:- $L\{\sin 2t\} = \frac{2}{s^2+4}\Big)_{s \Rightarrow 3} = \frac{2}{9+4} = \frac{2}{13}$

(vii) $\int_0^\infty e^{-2t} t \sin t \, dt$

$L\{\sin t\} = ① \frac{1}{s^2+a^2}$

$② (-1) \frac{d}{ds}\left(\frac{1}{s^2+a^2}\right)$

$= -\frac{(-1)}{(s^2+a^2)^2} \times 2s$

$= \frac{1}{(s^2+a^2)^2} \times 2s = \frac{1}{2^2+1} \times 2(2) = \frac{4}{5}$

(viii) $\int_0^\infty t \cdot e^{-3t} \sin t \, dt$

$\Rightarrow L\{\sin t\} = \frac{1}{s^2+1} = -\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

$= -\frac{-1}{(s^2+1)^2} \times 2s$

$= -\frac{-1}{(3^2+1)^2} \times 2 \times 3$

$= \frac{6}{100} = \frac{3}{50}$

(ix) $\int_0^\infty t \cdot e^{-2t} \cos t \, dt$

$\boxed{\frac{d}{dv} = \frac{vu' - uv'}{v^2}}$

$L\{\cos t\} = \frac{s}{s^2+a^2}$

$= (-1)\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$

$= -\left[\frac{s \times 2s + (s^2+a^2) \times 1}{(s^2+a^2)^2}\right]$

$= -\left(\frac{-2s^2 + s^2 + 1^2}{(s^2+a^2)^2}\right)$

$= -\left(\frac{1-s^2}{(s^2+a^2)^2}\right)$

$= \frac{s^2-1}{(s^2+1)^2}$

$= \frac{2^2-1}{(2^2+1)^2} = \frac{3}{25}$

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(x) $\int_0^\infty t\, e^{-3t}\cos t\, dt$

$= -\frac{d}{ds}\left(\frac{s}{s^2+1}\right) = -\left(\frac{(s^2+1) - s(2s)}{(s^2+1)^2}\right)$

$= -\left(\frac{-s^2+1}{(s^2+1)^2}\right)$

$= \frac{s^2-1}{(s^2+1)^2}\Big|_{s=3} = \frac{3^2-1}{(3^2+1)^2} = \frac{8}{100} = \frac{2}{25}$

(xi) $\int_0^\infty t^2 e^{-2t}\sin 2t\, dt$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

$= 2\,\frac{-1}{(s^2+4)^2}\times 2s$

$= \frac{d}{ds}\frac{-4s}{(s^2+4)^2} = -4\left(\frac{(s^2+4)^2 - 2s\times 2s(s^2+4)}{(s^2+4)^4}\right)$

$= -4\left(\frac{s^2+4-4s^2}{(s^2+4)^3}\right) = -4\left(\frac{400-32}{(20)^4}\right)$

$= -4\,\frac{(s^2+4)^2-4s^4-16s^2}{(s^2+4)^4} = -4\left(\frac{368}{160000}\right)$

$= -4\,\frac{s^4+16+8s^2-4s^4-16s^2}{(s^2+4)^4} = \frac{368}{40000}$

$= -4\left(\frac{-8s^2+16-3s^4}{(s^2+4)^4}\right)\Big|_{s=4} = \frac{92}{1000} = \frac{46}{500}$

$= \frac{23}{250}$

$= -4\times\left(\frac{16-8s^2-3s^4}{(s^2+4)^4}\right)$

$= -4\times\left(\frac{160-256}{160000}\right) = -4\times\left(\frac{16-144-768}{(20)^4}\right)$

$= \frac{-(-96)}{40000} = \frac{368}{40000}$

$= 4\times\left(\frac{4896}{160000}\right)$

$= \frac{96}{40000}$

$\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}\,dt$

$\Rightarrow \int_0^\infty e^{-0t}\left[\frac{e^{-t}-e^{-2t}}{t}\right]dt$

$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \int_s^\infty \frac{1}{s+1}\,ds - \int_s^\infty \frac{1}{s+2}$

$= [\log(s+1) - \log(s+2)]_s^\infty$

$= \left[\log\left(\frac{s+1}{s+2}\right)\right]_s^\infty$

$= \log\frac{1+\frac{1}{\infty}}{1+\frac{2}{\infty}} \to 0 - \log\left(\frac{s+1}{s+2}\right)$

$= -\log\left(\frac{s+1}{s+2}\right) = \log\left(\frac{s+2}{s+1}\right)$

$\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt$

$= L\{\cos 6t - \cos 4t\} = \frac{s}{s^2+36} - \frac{s}{s^2+16}$

$= \int_s^\infty \frac{s}{s^2+36}\,ds - \int_s^\infty \frac{s}{s^2+16}\,ds$

$= \frac{1}{2}\left[\int_s^\infty \frac{2s}{s^2+36} - \int_s^\infty \frac{2s}{s^2+16}\,ds\right]$

$= \frac{1}{2}\left[\log(s^2+36) - \log(s^2+16)\right]$

$= \frac{1}{2}\log\left(\frac{s^2+36}{s^2+16}\right)_0^\infty$

$= \frac{1}{2}\left(\log\frac{1+\frac{36}{\infty}}{1+\frac{16}{\infty}} \to 0 - \log\left(\frac{s^2+36}{s^2+16}\right)\right)$

$= \frac{1}{2}\left(\log\left(\frac{s^2+16}{s^2+36}\right)\right)$

$= \frac{1}{2}\log\left(\frac{4}{9}\right)$

$= \log\sqrt{\frac{4}{9}} = \log\left(\frac{2}{3}\right)$

$\int_0^\infty e^{-t}\frac{\sin t}{t}$

$L\{\sin t\} = \int_s^\infty \frac{1}{s^2+1}$

$\cancel{= \left[\log(s^2+1)\right]_s^\infty}$

$\cancel{= 0 - \log(s^2+1) = -\log(1+1)}$

$\cancel{= -\log(2) = \log\left(\frac{1}{2}\right)}$

$\cancel{\log\left(\frac{1}{2}\right)}$

$= \frac{1}{1}\times\left[\tan^{-1}(s)\right]_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}(s)$

$= \frac{\pi}{2} - \tan^{-1}(1) \quad = \cot^{-1}(s)$

$= \cot^{-1}(1) = \frac{\pi}{4}$

$\int_0^\infty e^{-t}\frac{\sin^2 t}{t} = L\left\{\frac{1-\cos 2t}{2}\right\}$

$= \frac{1}{2}\left\{\frac{1}{s} - \frac{1}{2}\frac{2s}{s^2+4}\right\}$

$\cancel{\frac{1}{2}\left[\frac{-1}{s^2} - \frac{s^2+4-s(2s)}{(s^2+4)^2}\right]}$

$\cancel{= \frac{1}{2}\left(\frac{-1}{s^2} - \frac{4-s^2}{s^2+4^2}\right)}$

$= \frac{1}{2}\left(\log s - \log\sqrt{s^2+4}\right)_s^\infty$

$= \frac{1}{2}\left(\log\frac{s^2}{\sqrt{s^2+4}}\right)_s^\infty$

$= \frac{1}{4}\left(\log\frac{s^2}{s^2+4}\right)_s^\infty$

$= \frac{1}{4}\left(0 - \log\frac{s^2}{s^2+4}\right)$

$= \frac{1}{4}\log\frac{s^2+4}{s^2}$

$= \frac{1}{4}\log\left(\frac{1+4}{1}\right) = \frac{1}{4}\log 5$

విదేశి యూనివర్శిటీలలో అడ్మిషన్ పొందుటకు అర్హత పరీక్షలు ఫీజు చెల్లింపుల పథకము

ప్రభుత్వము వారు విదేశాలలో చదువుకొనగోరే విద్యార్థులకు అంబేద్కర్ ఓవర్‌సీస్ విద్యానిధి పథకం ప్రవేశపెట్టారు. విదేశి యూనివర్శిటీలో చదువుకొనగోరే విద్యార్థులు ఇంగ్లీషులో సామర్థ్యం కొరకు TOFEL మరియు IELTS జనరల్ ఆటిట్యూడ్ టెస్ట్‌లు అయిన GRE/GMAT పరీక్షలలో మంచి మార్కులతో ఉత్తీర్ణులై ఉండాలి.

పై పరీక్షల కొరకు ఎంపిక చేయబడిన పేరొందిన కోచింగ్ ఇనిస్టిట్యూట్‌లో చేరగోరు విద్యార్థులకు ప్రభుత్వము వారు కోచింగ్ ఫీజు మంజూరు చేసెదరు డిగ్రీ ఉత్తీర్ణులైన లేదా డిగ్రీ ఫైనల్ ఇయర్ విద్యార్థులు ఈ పథకమునకు అర్హులు. కుటుంబ వార్షికాదాయము సంవత్సరమునకు రూ.2.00 లక్షల లోపు ఉండవలెను.

5 నుంచి 8 వ తరగతి చదువు ఎస్‌సి విద్యార్థులకు మెట్రిక్ పూర్వ ఉపకార వేతనములు

ప్రైమరీ స్థాయి నుండి సెకండరీ స్థాయి వరకు ఎస్‌సి విద్యార్థులు ప్రవేశించు దశలో డ్రాపవుట్‌లు నివారించటానికి డేస్కాలర్స్‌కు మాత్రమే వర్తించే విధంగా ఈ పథకం రూపొందించబడింది. ఈ పథకం క్రింద బాలురకు నెలకు రూ.100/- చొ॥న (10) నెలలు మరియు బాలికలకు నెలరు రూ.150/- చొ॥న (10) నెలలు ఉపకారవేతనం చెల్లించుటకు ప్రతిపాదించటమైనది. ఈ పథకము ద్వారా 6 లక్షలు మంది ఎస్‌సి విద్యార్థులకు లబ్ది చేకూర్చబడునట్లు ప్రతిపాదించడమైనది. 2013-14 సంవత్సరంలో (97,900) మంది విద్యార్థులకు ఉపకార వేతనము అందచేయడం జరిగింది.

ప్రతిభావంతులైన విద్యార్థుల పథకం

సాధారణంగా ఇతర విద్యార్థులతో సమానంగా పోటీని తట్టుకొని నిలబడటానికై ఎస్‌సి విద్యార్థులకు నాణ్యమైన విద్యన అందించడమే ఈ పథకం ప్రధాన లక్ష్యం. క్రమ శిక్షణలో మంచి పేరు సంపాదించి అత్యధిక విద్యా ప్రమాణాలు కలిగివున్న మిషనరీలు, ఇతర ప్రైవేటు పాఠశాలలు ఉత్తమమైనవిగా ఎంపిక చేసిన పాఠశాలలను ఈ పథకం క్రింద ఎంపిక చేస్తారు. ఈ పథకం క్రింద చేర్చుకున్న ప్రతి విద్యార్థికి ఏడాదికి రూ.20,000/- వరకు ఆర్థిక సహాయాన్ని మంజూరు చేస్తారు. ఆదాయ పరిమితి సం॥నకు గ్రామీణ ప్రాంతాల వారికి రూ.65,000/- పట్టణ ప్రాంతాల పారికి రూ. 75,000/- గా నిర్ధేశించారు. ఈ పథకంలో విద్యార్థులు 5వ తరగతి నుండి చేరవచ్చు. ఇది లాటరీ పద్ధతిలో ఎంపిక చేస్తారు. ప్రతి జిల్లాలో ఎంపిక చేసిన పాఠశాలల్లో (50) మంది ఎస్‌సి పిల్లలను చేర్చుకుంటారు. ఈ సంఖ్య 2014-15 నుండి (100) కి పెంచుతూ ప్రభుత్వం ఉత్తర్వులు జారీచేయడం జరిగింది.