DIFFERENTIAL EQUATIONS

Mixed promblems

- Basic Definitions
- Order deg. of d.E
- Formation of D.E
- Sol. of the D.E
- n^{th} order 1^{st} degree D.Eqn

Definition:

1. Let $y = f(x)$ be a continous funct^n, then rate of change (or) derivative of y w.r.t x is $\frac{dy}{dx} = y_1 = y' = Dy = f'(x)$

slope of tangent line to the curve $y=f(x)$

First order derivative or Differential coefficient or differential

$\frac{d^2y}{dx^2} = y_2 = y'' = D^2y = f''(x)$ - concavity, 2nd order der.

$\frac{d^ny}{dx^n} = y_n = y^n = D^ny = f^n(x)$ - n^{th} order derivative.

order means max. no of derivatives of given funct^n. $f(x)^n$

2. Let $u = f(x, y)$ be an implicit funct^n.

D.V & I.VS

then $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \longrightarrow$ 1^{st} order P.D

$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}$ —— 2nd " "

$\frac{\partial^n u}{\partial x^n}, \frac{\partial^n u}{\partial y^n}$ —— n^{th} " "

3. Diff. Eq^n: An eqn involving dependent variable, independent variable & ~~coe~~ diff^l. coefficient

or dependent variable with one or more independent variable.

An eqn is

Types of D.E

1. Ordinary Diff. eqn [O.D.E]

A diff. eqn in which diff^{l} coefficient w.r.t single independent variable.

2. Partial diff. eqn [P.D.E]

A diff. eqn in which diff^{l} coefficient w.r.t two (or) more independent variable.

Ex: $x+y=4$ — Algebraic eqn

$2x+3y+4z=0$ — "

$x = \sin x$ — non - " [transcendental eqn]

x^2+x+c — $\ln$ eqn

$\frac{dy}{dx} = 4[x^2+y^2]$ — D.E

Q) Which of the following is not a D.E

a) $\frac{dy}{dx} = a(a)$ → Parameter — not a fixed value

✓ b) $\frac{d^2y}{dx^2} = K$ — D.E where K is fixed value. [O.D.E]

c) $x\frac{dy}{dx} + y\frac{d^2y}{dx^2} = x$ [It is eq to 0 zero]

d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 5u$ → P.DE → D.E

Linear D.E [L.D.E]

A D.E is set a be LDE if it satisfies the following condition

i) degree of dependent variable & its derivative should be 1

ii) Dependent variable and its derivative shouldnot be multiplied to eachother

iii) There is no transcendental functions of dependent variable and its derivative.

NOTE: If the D.E is non linear, then the D.E will not satisfy any one of the above conditions.

Standard form of L.D.E

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \frac{d^{n-2} y}{dx^{n-1}} + \dots + a_n y = x\ X \text{ [capital x)}$$

where a_0, a_1, a_2, a_n, X are const. (or) func^n of 'x'. — ①

i) if X = 0 . then ① is Homo. L.D.E

ii) if X ≠ 0 then ① is Non. " L.D.E

Which of the following is non L.D.E

a) $\frac{1 d^2 y}{dx^2} + (2x^2-1)\frac{dy}{dx} + 3y = 6x^2$ —— L.D.E (Non.H.D.E)

b) $\frac{dy}{dx} + \sqrt{y} = 7x^3$ ⟶ Non - NONH

c) $y\frac{dy}{dx} + 3 = 0$ ⟶ $y\frac{dy}{dx} + 0.y = -3$ [non.H]

d) $\left(\frac{dy}{dx}\right)^2 + 4y = 7$ ⟶ Non L.D.E - Non - H.

e) b,c,d

Which of the following Non. H.L.D.E

a) $\frac{dy}{dx} + e^{-y} = 3$ —— " " "

b) $\frac{dy}{dx} + e^{-x} = 0$ —— " " " + L.D.E

c) $\frac{dy}{dx} + xy^2 = 3$ —— " " "

d) $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = \sin y$ —— " " "

1. Order of the Eq^n 1.(Highest ~~degree~~) derivative present in D.E)
2. The no. of independent parameters in the eq^n = order of D.E
3. Max. no. of possible derivative is called order
4. Order of L.D.E is any +ve integer
5. Order is always +ve

Degree of D.E .

1. The highest power of highest order derivative present in D.E when the eq^n is a polynomial in all diff. coefficients
2. While finding degree diff! coefficients shouldn't contains radicals & fractions
3. Degrees always +ve integer
4. Without degree diff. eq^n can exist, but without order D.E can't exist
5. The degree of L.D.E is always 1
6. Every 1st degree D.E need not be L.D.E

case 1 : [Direct promblems]

	Order	Degree
1. $x\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^3 + 4y = 6$	2	1
2. $\left(\frac{d^3y}{dx^3}\right)^4 + x\left(\frac{d^3y}{dx^3}\right)^5 = 100x^6$	3	5
3. $\left(\frac{dy}{dx}\right) + ky = 3x$	1	1

case 2 : [Diff.^{le} coeffecient involving radicals and fractions]

$\left[\frac{dy}{dx}\right]^{\text{fractions}}$ —— $2/3$, $3/2$ ____ radicals

$\frac{1}{dy/dx}$, $\left(\frac{dy}{dx}\right)^{-3}$ ____ fractions

e) $\left(\frac{dy}{dx}\right)^2 = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^3}$ — order — 2, degree — 3

$$\left[\frac{dy}{dx}\right]^4 = 1 + \left(\frac{d^2y}{dx^2}\right)^3$$

$\boxed{y_1^4 = 1 + (y_2)^3} \longrightarrow$ Poly. in D.E

e) $y_2^{3/2} - y_1^{1/2} - 4 = 0$ — order — 2

$(y^{3/2} = 4 + y^{1/2})^2$

$[y^3 = 16 + y^1 + 8y^{1/2}]^2$ — degree - 6

Q3. $\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = 4$ Order = 1, degree = 2

$\left(\frac{dy}{dx}\right)^2 + 3 = 4\frac{dy}{dx}$

	O	D
Q4. $\left(\frac{dy}{dx}\right)^2 = 3x + 4y$	1	2
Q5. $\left(\frac{dy}{dx}\right)^{3/4} = x$	1	1
Q6. $\left(\frac{d^2y}{dx^2}\right)^{2/3} = y$	2	1
Q7. $\left(\frac{dy}{dx}\right)^{2/3} = 4\frac{d^2y}{dx^2}$	2	3

case 3: Not a Poly. in Diff^l eq^n

Poly $\boxed{f(x,y)\frac{d^ny}{dx^n} + f_2(x,y)\frac{d^{n-1}y}{dx^{n-1}} + \dots + f_n(x,y)y = x}$ → poly in Diff. Eq^n

	O	D
Q8. $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$	1	1
Q9. $\cos\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$	1	not . defined
Q10. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = e^{dy/dx}$	2	"

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Case 4:

Types of Quantities

- Variables $(x, y, z, u, v, \ldots)$
  - Dep. V
  - Inde V

  $y = x^2 + 4$

  y — D.V, x^2+4 — I.V

- constants:
  - Abs. const. (or) fixed const.

    Ex: All real no.

    $\boxed{1}$
  - Arbitrary const. [Parameters]
    - D.P

      $c_1 + c_2 = c$

      $c_1 c_2 = c$

      $\frac{c_1}{c_2} = c$
    - I.P

      $c_1 + c_2 x$

      Indepe

$y = ae^{x} + be^{-x}$ (a, b)

2. I.P

$\Rightarrow ax + by + c = 0$ (a, b, c)

a → P, x → V, b → P, y → V, c → P

I.P $= \boxed{2}$

$\frac{a}{c}x + \frac{b}{c}y + 1 = 0$

$Ax + By + 1 = 0$

(A, B)

$\Rightarrow y^2 = 4ax$ (a)

y → V, 4 → C, a → P, x → V

Q11. Find the order of the D.E

$y = (c_1 + c_2)e^x + c_3 e^{-x}$

Order = no. of I.P's

$c e^x + c_3 e^{-x}$

I.P's = 2

The order

12) $y = c_1 \cos^2 x + c_2 \cos 2x - c_3 \sin^2 x + c_4$ $[c_1, c_2, c_3, c_4]$

a) 1 b) 2 c) 3 d) 4

$y = c_1[1 - \sin^2 x] + c_2 \cos 2x - c_3 \sin^2 x + c_4$

$= c_1 - \sin^2 x[c_1 + c_3] + c_2 \cos 2x + c_4$

$C - \sin^2 x (A) + B \cos 2x$

$C - B \cos 2x$

2 I.P's

Q13) O. of $y = (c_1 + c_2)e^{x + c_3} + (c_4 - c_5)e^{x + c_6}$ $[c_1\ c_2\ c_3\ c_4\ c_5\ c_6]$

a) 6 b) 1 c) 2 d) 3

$y = (c_1 + c_2)e^{(x + c_3)} + (c_4 - c_5)e^{x + c_6}$

~~$c_1 e^x + c_2 e^x$~~

$C e^x . e^{c_3} + B e^x e^{c_6}$

$e^x[\underbrace{C.e^{c_3}}_{A} + \underbrace{Be^{c_6}}_{B}]$

$e^x(D)$

Order = 1

The order and degree of : Q.14)

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 3\int y\,dx = \sin x$$

$$\frac{d}{dx}\left[\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 3\int y\,dx = \sin x\right]$$

$$\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + 3y = \cos x$$

order = 4

degree = 1

a) 3,1 b) 3,2 c) 4,1 d) 4,2 e) 3, not defined

Q.15) $\left[1 + y_2^{1/3}\right]^{1/2} = (y_1 \cdot y_3)^{1/3}$

O / D

3

$$\left[1 + y_2^{1/3} = (y_1 y_3)^{2/3}\right]^3$$

$$\cancel{1^3 + y_2^{(3/3)} = (y_1 y_3)^2}$$

$$(1 + y_2^{1/3})^3 = (y_1 y_3)^2$$

$$1 + y_2' + 3y_2^2 + 3y_2^{1/3} = y_1^2 y_3^2$$

$$1 + y_2 + 3y_2^2 - y_1^2 y_3^2 = 3y_2^{1/3}$$

C.O.B.S

$$\left[1 + y_2 + 3y_2^2 - y_1^2 y_3^2\right]^3 = \left[3y_2\right]^1$$

degree = 6

Q16) $\sqrt{1-x^2} + \sqrt{1-y^2} = 2020\,a(x-y)$

a) 1,2 b) 1,1 c) 1,3 d) 1,4

$\sqrt{1-x^2} + \sqrt{1-y^2} = 2020a(x-y)$

Form a D.E to get degree

$x = \sin\alpha \quad ; \quad \alpha = \sin^{-1}x$

$y = \sin\beta \quad ; \quad \beta = \sin^{-1}y$

$\cos\alpha + \cos\beta = 2020a(\sin\alpha - \sin\beta)$

$2\cancel{\cos\left(\frac{\alpha+\beta}{2}\right)}\cos\left(\frac{\alpha-\beta}{2}\right) = 2020a\left(2\cancel{\cos\left(\frac{\alpha+\beta}{2}\right)}\cos\left(\frac{\alpha-\beta}{2}\right)\right)$

$\cot\left[\frac{\alpha-\beta}{2}\right] = 2020a$

$\alpha - \beta = 2\cot^{-1}[2020a]$

$\sin^{-1}x - \sin^{-1}y = k$

D.O.BS

$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Order = 1 degree = 1

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Formation of D.E

Let $f(x, y, c_1, c_2, c_3 \dots c_n) = 0$

be represents n-parameters family of curves with $c_1, c_2, c_3 \dots c_n$ are 'n' independent parameters then D.E of (I) as follows

Step 1. Find no. of independent parameters [say (n)]

Step 2. Diff.te eqn (I) to n times will get n+1 eqns

Step 3. Eliminate n independent parameters with the help of n+1 eqns.

Step 4. Will get n^{th} order differential eqn to the family of curves

i.e $g\left[x\, y\, \frac{dy}{dx}\, \frac{d^2y}{dx^2}\, \frac{d^3y}{dx^3} \dots \frac{d^ny}{dx^n}\right] = 0 \underset{quest.}{\overset{sol.}{\rightleftarrows}} f\left(x, y, c_1, c_2, c_3 \dots c_n\right)$

Q. Find the diff' eqn of $y = ax^2 + bx + c$ [a, b, c]

NOTE: sometimes in the process of diff.tn indp. parameters may get eliminated.

$$\frac{y = ax^2 + bx + c}{c} \quad - 1$$

$$(y/c) = (a/c)x^2 + (b/c)x + 1$$

$$D = Ax^2 + Bx + 1$$

$n = 3$

$$\frac{dy}{dx} = 2ax + b \quad - 2$$

$$\frac{d^2y}{dx^2} = 2a \quad - 3$$

$$\frac{d^3y}{dx^3} = 0 \quad - 4$$

$(n+1)$ eqns $= 4$

$\boxed{y_3 = 0}$ 3rd order of D.E

O = 3, D = 1

Q2. $y = Ae^x + Be^{2x} + Ce^{3x}$ [A, B, C]

=

$$y_1 = Ae^x + 2B e^{2x} + 3Ce^{3x} + Be^{2x} + 2Ce^{3x}$$

$$y_1 = y + Be^{2x} + 2Ce^{3x}$$

$$y_2 = y_1 + 2Be^{2x} + 6Ce^{3x}$$

$$y_2 = y_1 + (Be^{2x} + 2Ce^{3x}) + [Be^{2x} + 4Ce^{3x}]$$

$$y_2 = y_1 + y_1 - y + Be^{2x} + 2Ce^{3x} + 2Ce^{3x}$$

$$y_2 = y_1 + y_1 - y_1 + y_1 - y + 2Ce^{3x}$$

$$y_2 = 3y_1 - 2y + 2Ce^{3x}$$

$$y_2 = 3y - 2y + 2Ce^{3x} + y_2 - 3y_1 + 2y$$

$$= 2Ce^{3x}$$

$$y_2 - 3y_1 + 2y = 2Ce^{3x}$$

$$y_3 - 3y_2 + 2y_1 = 6Ce^{3x}$$

$$= 3(y_2 - 3y_1 + 2y)$$

$$\boxed{y_3 - 6y_2 + 11y_1 - 6y = 0}$$

$O = 3$; $D = 1$

Q3 Super trick [Functional Exponential]

$$\rightarrow y = ae^{3x} + be^{5x} \quad (\alpha = 3,\ \beta = 5)$$

$$y_2 - (\ \ )_{\alpha+\beta}\, y_1 + (\ \ )_{\alpha\beta}\, y = 0$$

$$y = ae^{x} + be^{2x} + ce^{3x}$$

$$y_3 - (\ \ )_{\alpha+\beta}\, y_2 + (\ \ )_{\alpha\beta+\beta\gamma+\gamma\alpha}\, y_1 - (\ \ )_{\alpha\beta\gamma}\, y = 0$$

$$y = (c_1 + c_2x)e^{-x}$$

$$y_2(\ \ )_{\alpha+\beta+\gamma} - (\ \ )_{\alpha\beta+\beta\gamma+\gamma\alpha}\, y_1 + (\ \ )_{\alpha\beta\gamma}\, y = 0$$

$$y = (c_1 + c_2x + c_3x^2)e^{x}$$

$$\cancel{y_2(\ \ ) - y_2(\ \ ) + y_1(\ \ ) - (\ \ )y = 0}$$

$$y_3 - (\ \ )y_2 + (\ \ )y_1 - (\ \ )y = 0$$

Q_4. Form D.E $y = e^x[A\cos x + B\sin x]$

$y = e^x[A\cos x + B\sin x]$

$y_1 = e^x[-A\sin x + B\cos x] + \underbrace{e^x[A\cos x + B\sin x]}$

$y_1 = e^x[-A\sin x + B\cos x] + y$

$y_2 = e^x[-A\cos x - B\sin x] + y_1 + e^x[-A\sin x + B\cos x]$

$y_2 = -y + y_1 + y_1 - y$

$y_2 = -2y + 2y_1 = 0$

$y_2 + 2y - 2y_1 = 0$

$O = 2 \quad ; \quad D = 1$

Super Trick 2 (Function of EXP + Trign.] Root are complex conjucate pairs

$\alpha + i\beta \quad , \quad \alpha - i\beta$

1. $y = e^{x}(C_1\cos x + C_2\sin x)$ $[\alpha = 1 \quad ; \beta = i1]$

$y_2(\quad) - (\quad)y_1 + (\quad)y = 0$

2 $y = A\cos x + B\sin x$

$y_2(\quad) - (\quad)y_1 + (\quad)y = 0$

3. $y = A\sin(\omega t + \alpha)$ (A, α)

$y_2(\quad) - (\quad)y_1 + (\quad)y = 0$

4. $y = e^{-x}(A\cos 2x + B\sin 2x)$

$y_2(\quad) - (\quad)y_1 + (\quad)y = 0$

$$\begin{array}{r} -1 + i2 \\ -1 - i2 \\ \hline (-2) \\ (5) \end{array}$$

$(-1)^2 - (i2)^2$

$1 + 4$

Q5. $y = ax^2 + bx^3$ (a, b) — ①

$y_1 = 2ax + 3bx^2$ —— ②

$y_2 = 2a + 6bx$ —— ③

using cramers rule

$$\begin{vmatrix} \text{a coeff} & \text{b coeff} & \text{funct}^n \\ x^2 & bx^3 & y \\ 2x & 3x^2 & y_1 \\ 2 & 6x & y_2 \end{vmatrix}$$

$$= x \begin{vmatrix} x^2 & bx^2 & y \\ 2x & 3x & y_1 \\ 2 & 6 & y_2 \end{vmatrix}$$

$$x \left| x^2(3xy_2 - 6y_1) - bx^2(2xy_2 - 2y_1) + y(12x - 6x) \right| = 0$$

$$x \left| x^2y_2 - 4x^2y_1 + 6xy \right| = 0$$

$$x^2y_2 - 4x^2y_1 + 6xy = 0$$

$O = 2$; $D = 1$

Formation of D.E of geometrical promblems:

1. Circles

i) Find the D.E of Family of concentric circles with centre at (0, 0)

→ Write the geometrical eq^n including parameter r - parameter ← radius

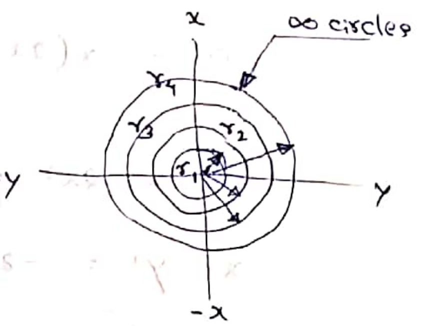
c (0,0) → fixed

$x^2 + y^2 = r^2$

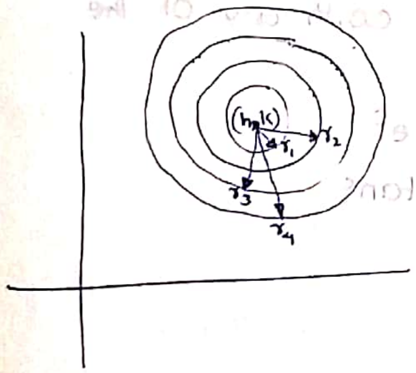
$n = 1$

$2x + 2yy' = 0$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$



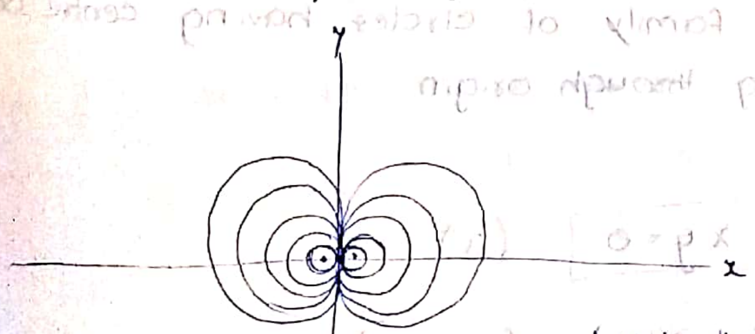
ii) Find the D.E of family of concentric circles with centre at (h, k)



$(x-h)^2 + (y-k)^2 = r^2$

$$\frac{dy}{dx} = \frac{-(x-h)}{y-k}$$

iii) Find the D.E of family of circles having centre on x-axis and passing through origin.



radius, $r = |\pm h| = h$

calca

$\Rightarrow (x \mp h)^2 + (y-0)^2 = h^2$

$x^2 + y^2 + \cancel{h^2} - 2hx = \cancel{h^2}$

$$\boxed{x^2 + y^2 + \lambda x = 0}$$

where $\lambda = \pm 2h$

$$-\lambda = \frac{x^2 + y^2}{x}$$

D.O.B.S

$$0 = \frac{x(2x + 2yy_1) - (x^2+y^2)}{x^2}$$

$$= 2x^2 - x^2 + 2xyy_1 - y^2$$

$$x^2 - y^2 = -2xyy_1$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}}$$

order = 1 ; degree = 1

NOTE: Arbitrary can be replaced with any of the parameter i.e.,

- $\log c$ $\quad e^c$
- $\sin c$ $\quad \tan c$
- $\cos c$

$\frac{1}{c}$, $\frac{1}{\lambda}$ $\pm 2h$

iv) Find the D.E of family of circles having centre on y-axis & passing through origin.

$\lambda = \pm 2k$

$$\boxed{x^2 + y^2 + \lambda y = 0} \quad (\lambda)$$

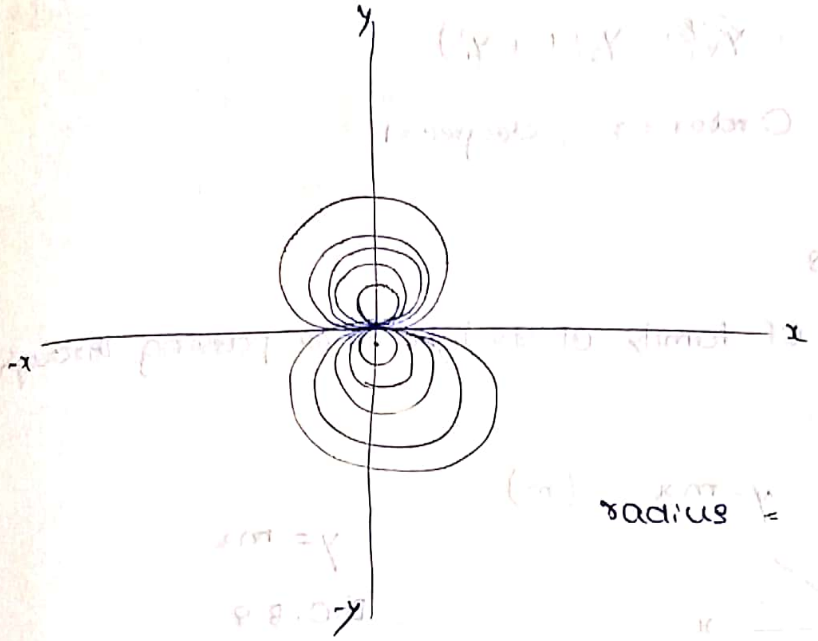
$$0 = y(2x + 2yy_1) - (x^2 + y^2)y_1$$

$$0 = 2xy + 2y^2y_1 - y_1x^2 - y_1y^2$$

$$0 = 2xy - x^2y_1 + y^2y_1$$

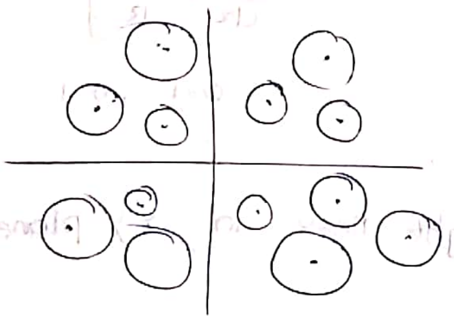
$2xy = y_1(y^2 - x^2)$

$$\boxed{\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}}$$ order • deg = 1



radius = $1 \pm k$

v) Find the D.E of family of ~~curve~~ circles in xy plane



3 parameters

$(x-h)^2 + (y-k)^2 = r^2$

(or)

$x^2 + y^2 + 2gx + 2fy + c = 0$

D.O.B.S w.r.t x

$2x + 2yy_1 + 2g + 2fy_1 + 0 = 0$

D.O.B.S w.r.t x

$1 + yy_2 + y_1^2 + 0 + fy_2 = 0$

$$-f = \frac{1 + y_1^2 + yy_2}{y_2}$$

$$\Rightarrow \quad 0 = \frac{y_1[0 + 2y_1y_2 + yy_3 + y_1y_2] - [1 + y_1^2 + yy_2]y}{y_1^2}$$

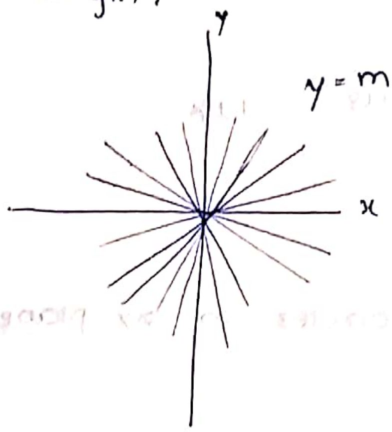
$$0 = 3y_1y_2^2 + yy_2y_3 - y_3 - y_3y_1^2 - y_3y_2^2y$$

$$3y_1y_1 = \cancel{y_1y_2} \; y_3(1 + y_1^2)$$

Order = 3 ; degree = 1

2 Straight lines

i) Find the D.E of family of straight lines passing through origin.



$y = mx \quad (m)$

$y = mx$

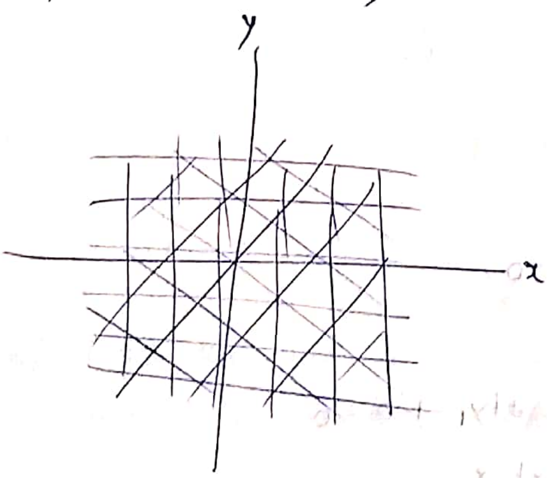
D.O.B.S

$$\frac{dy}{dx} = m = \frac{y}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}$$

O = 1 ; D = 1

ii) Find the family of straight lines in x-y planes



$y = mx + c \quad [m, c]$

(or)

$\frac{x}{a} + \frac{y}{b} = 1 \quad [a, b]$

(or)

$ax + by + 1 = 0 \quad [a, b]$

$y = mx + c$

D.O.B.S. w.r.t x

$$\frac{dy}{dx} = m(1) + 0$$

$$\boxed{\frac{d^2y}{dx^2} = 0}$$

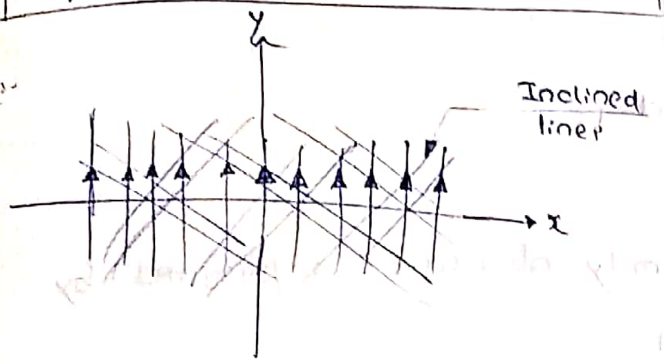
order = 2

degree = 1

iii) Find the D.E of Non-horizontal lines in x-y plane

$\boxed{\text{Eq}^n \text{ of horizontal line, } y = k}$

Sol:-



General eqn of lines

$ax + by = 1 \quad [a \neq 0]$

$ax = 1 - by$

$x = \frac{1}{a} - \frac{b}{a}y$

$\boxed{x = A + By}$ (A, B)

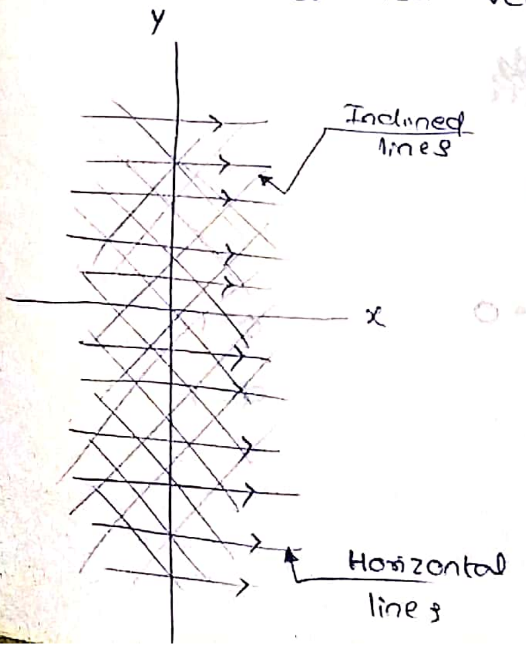
D.O.B.S w.r.t x

$\frac{dy}{dx} = 0 + B$

$\boxed{\frac{d^2y}{dx^2} = 0}$ order = 2

degree = 1

iv) Find the D.E of Non-vertical line x-y plane



$\boxed{\text{For vertical line } x = k}$

For non-vertical lines

$ax + by = 1 \quad [b \neq 0]$

$by = 1 - ax$

$y = \frac{1}{a} - \frac{a}{b}x$

$\boxed{y = A + Bx}$ (A, B)

$$\frac{dy}{dx} = 0 + B$$

$$\boxed{\frac{d^2y}{dx^2} = 0}$$

$O = 2 \quad ; \; d = 1$

v) The D.E of family of curves represented by

i) $Ax^2 + By^2 = 1$

D.O.B.S

$$2Ax + 2Byy_1 = 0$$

D.O.B.S

$$0 = \frac{x(yy_2 + y_1^2) - yy_1}{x^2}$$

$$xyy_2 + xy_1^2 = yy_1$$

Order = 2

Degree = 1

★★★) JEE mains previous questio

$$y^2 = 2c(x + \sqrt{c})$$

$$2y\frac{dy}{dx} = 2c\left[\cancel{\frac{dx}{dx}} + 0\right]$$

1

$$\cancel{2}y\frac{dy}{dx} = \cancel{2}c\,\cancel{\frac{dy}{dx}}$$

$$yy' = c\cancel{y}$$

$$c\cancel{y} - yy' = 0$$

$$\boxed{c = yy'}$$

O = 1

d = 1

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$$y^2 = 2yy'\left(x + \sqrt{yy'}\right)$$

$$y = 2y_1\left[x + \sqrt{yy_1}\right]$$

$$(y - 2xy_1)^2 = 4y_1^2 \times yy_1$$

$$(y - 2xy_1)^2 = 4y_1^3 y$$

$O = 1 \quad ; \; D = 3$

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Method - 4

Homogeneous D.Eq^n

H.F =

Ex: 1) $f(x,y) = \dfrac{x^2+y_2}{x3-y_2}$ — HF / — NHF $\Rightarrow \dfrac{HF}{NHF} = NHF$

2) $f(x,y) = \dfrac{x-y}{x+y}$ — HF / — HF $= \dfrac{HF}{HF} = HF$ T.D = 0

3) $f(x,y) = \dfrac{x^2+y_2}{x^3+y_3}$ — HF / — HF $= HF$ $\dfrac{D=2}{D=3}$

T.D = N.D − D.D =

= 2 − 3

= −1

Standard form of H.D.E

Condition I : $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is H.D.E

then T.D = 0 [f(x.y) & g(x,y) be two HF of same degree]

$$\frac{dy}{dx} = \frac{x^n h(y/x)}{x^n H(y/x)}$$

$$= G(y/x)$$

~~→ Tip for case 1:~~

→ Tip for case 1 : In RHS of H.D.E $\underbrace{\text{always} \quad \ln[x,y]}_{\text{final answer}}$

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

$$= F(y/x)$$

1. then put y = v ; x = 1
   & y = vx

2. $\frac{dy}{dx} = v + x\frac{dv}{dx}$

3. apply V-S-M to get sol. of H.D.E

→ Tip for case 2 :

$$\frac{dx}{dy} = \frac{f(x,y)}{g(x,y)} = F(x/y)$$

then 1 put $\frac{x}{y} = v$; In R.H.S x = v ; y = 1
x = vy

2 $\frac{dy}{dx} = v + y\frac{dv}{dy}$

3. apply V-S-M will get sol.

1Q) Solve the D.E $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$

H.D.E

$$v + x\frac{dv}{dx} = \frac{(1+v)^2}{2}$$

$$x\frac{dv}{dx} = \frac{1+v^2+\cancel{2v}-\cancel{2v}}{2}$$

$$\int \frac{2}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

$$2\tan^{-1}(v) = \ln(x) + \ln(c)$$

$$2\tan^{-1}\left(\frac{y}{x}\right) = \ln(x+c)$$

2Q) Solve D.E $x\frac{dy}{dx} = y[\log y - \log x + 1]$

$$\cancel{v + x\frac{dy}{dx}}$$

$$\frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$$

$\Rightarrow$ T.D = 0

$$v + x\frac{dv}{dx} = v[\log v + 1]$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

$$\log[\log v] = \ln x + \ln c$$

$$\log\left(\log\frac{y}{x}\right) = \ln x + \ln c$$

$$\log(y/x) = cx$$

$$\frac{y}{x} = e^{cx}$$

$$\boxed{y = x.e^{cx}}$$

→ 3Q) $(1+e^{x/y})\,dx + e^{y/x}\left[1 - \frac{x}{y}\right]dy = 0$

$$\frac{dx}{dy} = \frac{e^{x/y}[x/y - 1]}{1 + e^{x/y}}$$

$$v + y\frac{dv}{dy} = \frac{e^v[v-1]}{1+e^v}$$

$$\frac{dv}{dy} = \frac{e^v(v-1) - v \cdot ve^v}{1 + e^v}$$

$$= \frac{\cancel{ve^v} - e^v - v - \cancel{ve^v}}{1 + e^v}$$

$$\frac{dv}{dy} = \frac{-[e^v + v]}{1 + e^v}$$

$$\int \frac{1 + e^v}{-(e^v + v)} dv = \int \frac{dy}{y}$$

$$-\log|e^v + v| = \log y + \log c$$

$$\frac{1}{e^v + v} = cy$$

$$e^{x/y} + x/y = \frac{c}{y}$$

$$\boxed{ye^{x/y} + x = c}$$

⇒ Method 5

Non. H. D.Eq^n

[Repeated obj model]

Standard form

N.H.DE

$$\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2} \Longrightarrow \text{N.H.DE}$$

NHDE

Case I:

condition $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$ (say)

$a_1 = a_2 k$

$b_1 = b_2 k$

$c_1 = c_2 k$

$$\Rightarrow \frac{dy}{dx} = \frac{a_2 x k + b_2 k y + c_2 k}{a_2 x + b_2 y + c_1}$$

$$\frac{dy}{dx} = k$$

$$\int dy = k \int dx$$

$$** \quad \boxed{y = kx + d} \quad **$$

a) $\frac{dy}{dx} = \frac{2x + 3y + 4}{8x + 12y + 16}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{4} = k$$

$$\Rightarrow \quad y = \frac{1}{4}x + c$$

$$\boxed{x - 4y = c}$$

$$\Rightarrow \frac{dy}{dx} = \frac{m(ax + by) + c_1}{n(ax + by) + c_2}$$

$$t = ax + by$$

$$\frac{dt}{dx} = a + b\frac{dy}{dx}$$

$$\frac{dy}{dx} = \left[\frac{dt}{dx} - a\right] / b$$

$$\frac{\frac{dt}{dx} - a}{b} = \frac{mt + c_1}{nt + c_2}$$

$$\frac{dt}{dx} - a = \frac{mbt + bc_1}{nt + c_2}$$

$$\frac{dt}{dx} = \frac{mbt + bc_1 + nat + ac_2}{nt + c_2}$$

$$\frac{dt}{dx} = \frac{t[mb + na] + ac_2 + bc_1}{nt + c_2}$$

$$\int \frac{nt + c_2}{(mb + na)t + ac_2 + bc_1} dt = \int dx$$

$$\int \frac{ax + b}{cx + d} dx = \frac{a}{c}x - \frac{(ad - bc)}{c^2} \log|D.R|$$

$$\Rightarrow \frac{n}{mb + na} t - \left[\frac{nac_2 + nbc_1 - mbc_2\,nac_2}{(mb + na)^2}\right] \log|\text{denominator}| = x + c$$

$$\frac{n(ax+by)}{mb+na} - x - \frac{b(nc_1 - mc_2)}{(mb+na)^2}\log|D.R| = C'$$

$$\frac{nax + nby - mbx - nax}{mb+na} - \frac{b(nc_1 - mc_2)}{(mb+na)^2}\log|DR| = C'$$

$$\frac{b(ny - mx)}{mb+na} - \frac{b(nC_1 - mC_2)}{(mb+na)^2}\log|DR| = C'$$

$$(ny - mx)\,(mb+na) + (mc_2 - na)\log|DR| = C'$$

$$\Rightarrow \frac{(mx - ny)(mb+na)}{nc_1 - mc_2} + \log|(ax+by)(mb+na) + bc_1 + ac_2| =$$

$\Rightarrow$ Standard form

$$\frac{dy}{dx} = \frac{a_1x + b_2y + c_1}{a_2x + b_2y + c_2}$$

condition II $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Super Trick.

(bottom to top along horizontal)

$$\begin{array}{ccc} x & & y_1 \\ n & & m \\ C_2 & & C_1 \end{array} \qquad \begin{array}{l} ax + by \\ an + bm \\ ac_2 + bc_1 \end{array}$$

$$\left(\frac{mx - ny}{nc_1 - mc_2}\right)(an + bm) + \log\left|(ax+by)(an+bm) + ac_2 + bC_2\right|$$

a) $\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$

$= \frac{1(x-y)+3}{2(x-y)+5}$

x	y	x-y
2	1	1
5	3	2

$\left[\frac{x-2y}{1}\right] \times 1 + \log\left|(x-y)1+2\right| = C$

$x-2y + \log|x-y+2| = C$

Q₂) $(2x+2y+3)\frac{dy}{dx} = x+y+1$

$\frac{dy}{dx} = \frac{1(x+y)+1}{2(x+y)+3}$

x	y	x+y
2	1	3
3	1	4

$\frac{(x-2y)}{-1} \times 3 + \log\left|(x+y)3+4\right| = C$

$-(3x-6y) + \log|3x+3y+4| = C$

$6y-3x + \log|3x+3y+4| = C$

→ Case 3: $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$

condition $\frac{a_1}{a_2} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$

then $b_1 + a_2 = 0$

$$\Rightarrow \frac{dy}{dx} = -\left[\frac{ax + hy + g}{hx + by + f}\right]$$

Sol :- $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$

(conic section $\Delta \neq 0$)

a) $\frac{dy}{dx} = \frac{x - 2y + 1}{2x + y - 1}$

$$= -\left[\frac{-x + 2y - 1}{2x + y - 1}\right]$$

$a = -1$ $h = 2$ $g = -1$

$h = 2$ $b = 1$ $f = -1$

$\Rightarrow -1x^2 + 4xy + y^2 - 2x - 2y + C = 0$

$-x^2 + 4xy + y^2 - 2[x + y] + c = 0$

$x^2 - 4xy - y^2 + 2(x + y) - C = 0$

$\Rightarrow$ Condition 4:

IF $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\Rightarrow \frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

Procedure:

put $x = X + h \Rightarrow dx = dX$

$y = Y + k \Rightarrow dy = dY$

$$\boxed{\frac{dy}{dx} = \frac{dY}{dX}}$$

$$\frac{dY}{dX} = \frac{a_1X + b_1Y + \boxed{a_1h + b_1k + c_1}}{a_2X + b_2Y + \boxed{a_2h + b_2k + c_2}} \Rightarrow \text{N.H.D.E}$$

Make NHDE into HDE

$$\Rightarrow \left.\begin{aligned} a_1 h + b_1 k + c_1 &= 0 \\ a_2 h + b_2 k + c_2 &= 0 \end{aligned}\right\} \longrightarrow \text{find } [h, k]$$

$$\frac{h}{b_1 c_2 - b_2 c_1} = \frac{k}{a_2 c_1 - a_1 c_2} = \frac{1}{a_1 b_2 - a_2 b_1}$$

$$\frac{dY}{dX} = \frac{a_1 X + b_1 Y}{a_2 X + b_2 Y}$$

$\Rightarrow$ H.D.E by putting $Y = vX$ [Method 4]

On final answers

put $X = x - h$

$Y = y - k$

Q) The Non - H.D.Eqn $\frac{dy}{dx} = \frac{x + 2y + 3}{3x + 4y + 5}$ is converted into HDEqn then the transformation.

a) $x = X + 2$; $y = Y + 3$

b) $x = X - 2$; $y = Y - 1$

c) $x = X - 1$; $y = Y + 2$

d) $x = X + 1$; $y = Y - 2$

$$\frac{x + 2y + 3}{3x + 4y + 5} \qquad \frac{h}{10 - 12} = \frac{k}{9 - 5} = \frac{1}{4 - 6}$$

$$\frac{h}{-2} = \frac{k}{4} = \frac{1}{-2}$$

$h = 1$; $k = -2$

✓d) Ans) $x = X + 1$

$y = Y - 2$

case 4 can be used for all cases. { case I, II, III }

ECET - 2019

Q) If the substitution $x = X + h$, $y = Y + k$ transforms the

D.E $(y - x + 1)\,dy - (y + x + 2)\,dx = 0$ into HDE

then $(h, k) =$

a) $(1/2, 1/2)$ ✓ b) $(-1/2, -3/2)$ c) d)

$$\frac{dy}{dx} = \frac{y + x + 2}{y - x + 1}$$

$$= \frac{x + y + 2}{-x + y + 1}$$

$$\frac{h}{-1} = \frac{k}{-3} = \frac{1}{2}$$

$h = -1/2$; $k = -3/2$

Case 5: Subcase of C-4:

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ & diagnally same

i.e, $\frac{dy}{dx} = \frac{ax + by + c_1}{bx + ay + c_2}$

ShortCut sol:- $(X + Y)^{a-b} \times (X - Y)^{a+b} = C$

$X = x - h$; $Y = y - k$

$= x - 1$; $= y$

Q) Solve D.Eqn of $\frac{dy}{dx} = \frac{3x + 2y - 3}{2x + 3y - 2}$

$$\frac{h}{5} = \frac{k}{0} = \frac{1}{5}$$

$h = 1$; $k = 0$

$$(X+Y)^{a-b} \times (X-Y)^{a+b} = C_1$$

$$(x-1+y)^{1} \times (x-1-y)^{5} = C_1$$

$$(x+y-1)(x-y-1)^{5} = \text{const.}$$

9Q) Pg 28)

$$\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$$

$$= \frac{-7x+3y+7}{3x-7y-3}$$

case 5 :-

-9 - (49)

$$\frac{h}{40} = \frac{k}{0} = \frac{1}{40}$$

$$h = 1 \quad ; \quad k = 0$$

$$X = x-1 \quad ; \quad Y = y - \frac{21}{20}$$

$$[x-1+y]^{-10} \times [x-1-y]^{-4} = C_1$$

$$(x+y-1)^{-10}(x-y-1)^{-4} = \text{const.}$$

$$(x+y-1)^{10}(x-y-1)^{4} = c$$

Sq. Roots O.B.S

$$\Rightarrow (x+y-1)^{5}(x-y-1)^{2} = \text{constant.}$$

Method 6 : Linear Differential Eqns. [L.D.E]

Case 1 : Differentiation of y w.r.t x

$$\frac{dy}{dx} + Py = Q$$

- L.D.E in y
- P, Q are functions of 'x' or const.

- IF = $e^{\int P dx}$

- Sol :-

$$y[I.F] = \int Q (I.F) dx + c$$

Case 2:

$$\frac{dx}{dy} + Px = Q$$

- L.D.E in x
- P, Q are funct. of y or c
- IF = $e^{\int P dy}$
- Sol :-

$$x[I.F] = \int Q (IF) dy + c$$

Case 3:

$$\frac{dt}{dx} + Pt = Q$$

- L.DE in t
- P, Q are funct of x or c
- IF = $e^{\int P dx}$
- Sol :-

$$t[I.F] = \int Q (IF) dx + c$$

NOTE : If the I.F of case-1 is $f(x)$

then $p = \frac{f'(x)}{f(x)}$

Q) $(ax + by + c)\frac{dy}{dx} = 1$

a) L.D.E in x
b) " " y
c) Non " " "
d) " " " x

$$\frac{dy}{dx} = \frac{1}{ax + by + c}$$

→ Py is not there in N [case 1]

$$\frac{dx}{dy} = ax + by + c$$

→ Px is there [case 2]

$$\frac{dx}{dy} - ax = by + c$$

Q) IF $\frac{dy}{dx} + Py = Q$ is $\sin x$ then $P = ?$

$$P = \frac{f'(x)}{f(x)}$$

$= \frac{[illegible]}{[illegible]}$

P = [illegible]

3(a) Solve the D.E. $(x + 2y^3)\frac{dy}{dx} = y$

$\frac{dy}{dx} = \frac{y}{x + 2y^3}$

Non linear in y

Linear in x

$\frac{dx}{dy} = \frac{x}{y} + 2y^2$

$\frac{dx}{dy} - \frac{x}{y} = 2y^2$

$P = -\frac{1}{y} \quad Q = 2y^2$

$\therefore \quad IF = e^{\int -1/y}$

$= e^{-\log y}$

$IF = 1/y$

$\therefore \quad x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y}\, dx$

$\frac{x}{y} = y^2 + c$

$x = y^3 + cy$

$x = y[y^2 + c]$

4) $(1+y^2)dx = (e^{\tan^{-1}y} - x)\,dy$

$\frac{dy}{dx} = \frac{1+y^2}{-x + e^{\tan^{-1}y}}$

$\frac{dx}{dy} = \frac{e^{\tan^{-1}y} - x}{1+y^2}$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$

$P = \frac{1}{1+y^2} \quad Q = \frac{e^{\tan^{-1}y}}{1+y^2}$

$I.F. = e^{\tan^{-1}y}$

Sol:- $x.e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \, x e^{\tan^{-1}y} \, dy$

$$t = e^{\tan^{-1}y}$$

$$\cancel{\frac{dt}{dy} = \frac{1}{1+y^2}} \qquad \frac{e^{\tan^{-1}y}}{1+y^2} dy = dt$$

$$\cancel{dy = (1+y^2)dt}$$

$$x.e^{\tan^{-1}x} = \int t \, dt$$

$$x e^{\tan^{-1}x} = \frac{(e^{\tan^{-1}y})^2}{2} + C$$

Method 7 :

Reduce to L.D.E [Non L.D.E or Bernoullis D.E]

$$\frac{dy}{dx} + Py = Qy^n$$

$$y^n$$

$$\Rightarrow \frac{1}{y^n}\frac{dy}{dx} + P.y^{(1-n)} = Q$$

Put $t = y^{(1-n)}$

$$\frac{dt}{dx} = (1-n) y^{-n} . \frac{dy}{dx}$$

$$\frac{1}{y^n} . \frac{dy}{dx} = \frac{1}{1-n} . \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{1-n} . \frac{dt}{dx} + Pt = Q .$$

$$\frac{dt}{dx} + (1-n)t . P = Q(1-n)$$

$P = (1-n)$

L.D.E in 't'

Q) Solve $x\frac{dy}{dx} + y = y^2 x^3 \log x$

~~$x\frac{dy}{dx}$~~
Divide with x

$$\frac{dy}{dx} + \frac{y}{x} = y^2 x^2 \log x$$

divide with y^2

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = x^2 \log x$$

~~t~~
multiply with -1

$$\frac{-1}{y^2}\frac{dy}{dx} + \left(\frac{-1}{x}\right)\left(\frac{1}{y}\right) = -x^2 \log x$$

$$t = \frac{1}{y}$$

$$\frac{dt}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dt}{dx} - \frac{t}{x} = -x^2 \log x$$

$$IF = e^{\int -1/x}$$

$$= 1/x$$

$$\Rightarrow \frac{1}{y}\cdot\frac{1}{x} = \int -x^{\cancel{2}} \log x \cdot \frac{1}{\cancel{x}} dx$$

$$\frac{1}{xy} = -\int x \log x \, dx$$

$$-\left[\cancel{\frac{x^2}{2}\log x + \log x}\right]$$

$$= -\frac{x^2}{2}\left[\ln x - \frac{1}{2}\right] + C$$

$$\left\{ \int x^n \ln x \, dx = \frac{x^{n+1}}{n+1}\cdot\left[\ln x - \frac{1}{n+1}\right] \right.$$

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Method 7:

Reduce to L.D.E

$\Rightarrow$ Non ~~H~~.L.D.E (or) Bernoulli's D.E

Q) $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$

$\sec^2 y \cdot \frac{dy}{dx} + 2x\tan y = x^3$

$\frac{d(\tan y)}{dx} + 2x = x^3$

$IF = e^{\int 2x}$

$= e^{x^2}$

$\Rightarrow \tan y \cdot e^{x^2} = \int x^3 \cdot e^{x^2}\, dx$

$t = x^2$

$\frac{dt}{dx} = 2x$

$dx = \frac{dt}{2x}$

$= \cancel{x^3} \cdot e^t \cdot \frac{dt}{2\cancel{x}}$

$= \frac{1}{2}\int t \cdot e^t\, dt$

$= \frac{1}{2}\left[e^t(t-1)\right] + c$

Q) Solve $\frac{dy}{dx}(x^2y^3 + xy) = 1$

$\frac{dy}{dx} = \frac{1}{x^2y^3 + xy}$

$\frac{dx}{dy} = x^2y^3 + xy$

$\frac{dx}{dy} - xy = x^2y^3$

div. with x^2

$\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$

$$\frac{-1}{x^2}\frac{dx}{dy} + \frac{1}{x}\cdot y = -y^3$$

$$\frac{d}{dy}\left(\frac{-1}{x}\right) + Py = Q$$

$$IF = e^{\int y}$$

$$= e^{y^2/2}$$

$$\Rightarrow \frac{-1}{x}\cdot e^{y^2/2} = \int y^3 . e^{y^2/2} . dy$$

$$'' = \frac{1}{2}\left[e^{y^2}\left[y^2-1\right]\right] + c$$

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N^{th} order L.D.E

Standard form:

$$a_0\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + a_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_ny = X \quad \text{--- (1)}$$

where $a_0, a_1, a_2 \dots a_n, X$ are constants or fn's of x.

(1) is called L.D.E with const. coefficients

(1) is called L.D.E with variable coefficients

⇒ if $X = 0$ (1) → H.L.D.E → (1) has only one sol. i.e Complementary sol. $y = y_c$

⇒ if $X \neq 0$ (1) N.H.L.D.E → (1) has two sol. i.e complementary sol. & particular sol.

Total sol. or G.S = $\boxed{y_T = y_c + y_p}$

Notations

let $\frac{dy}{dx} = Dy$, $\frac{d^2y}{dx^2} = D^2y, \dots, \frac{d^ny}{dx^n} = D^ny$

(1) ⇒ $\underbrace{\left[a_0D^n + a_1D^{n-1} + a_2D^{n-1} + \dots + a_n\right]}_{\text{Simplest form of L.D.E}} y = x$ — (2)

$$\boxed{f(D)y = x}$$

- Put $D = m$; $y = 1$ in (2)

⇒ $a_0m^n + a_1m^{n-1} + a_2m^{n-2} + \dots + a_n = 0$

Auxiliary ~~eqn~~ eqn [A.E]

It has altmost n roots.

Complementary fn of H.L.D.E with const coefficients

Root of A.E	C.F (or) (Y_c)
1. Roots are real and distinct $m_1, m_2, m_3, \ldots m_n$	$Y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \ldots c_n e^{m_n x}$
2. Two roots are real and eq. and rest are distinct $m\ m\ m_3\ m_4 \ldots$	$Y_c = (c_1 + c_2 x) e^{mx} + c_3 e^{m_3 x} + \ldots$
3. 3 equal real roots and rest are distinct $m\ m\ m\ m_4$	$Y_c = (c_1 + c_2 x + c_3 x^2) e^{mx} + c_4 e^{m_4 x} + \ldots$
4. Roots are complex, conjugate pair $\alpha + i\beta,\ \alpha - i\beta,\ m_3, m_4 \ldots$	$Y_c = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right] + c_3 e^{mx} + \ldots$
5. Repeated complex conj. roots and rest are real $\alpha \pm i\beta,\ \alpha \pm i\beta,\ m_5 \ldots$	$Y_c = e^{\alpha x}\left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right] + c_5 e^{m_5 x}$
6. Roots are conjugate surds rest are real $\alpha + \sqrt{\beta},\ \alpha - \sqrt{\beta},\ m_3 \ldots$	$Y_c = e^{\alpha x}\left[c_1 \cosh\sqrt{\beta}x + c_2 \sinh\sqrt{\beta}x\right] + c_3 e^{m_3 x} + \ldots$
7. Repeated conj. roots $\alpha \pm \sqrt{\beta},\ \alpha \pm \sqrt{\beta},\ m_5 \ldots$	$Y_c = e^{\alpha x}\left[(c_1 + c_2 x)\cosh\sqrt{\beta}x + (c_3 + c_4 x)\sinh\sqrt{\beta}x\right] + c_5 e^{m_5 x} \ldots$

2) if $\sin x$ is the sol of

$$\frac{d^4y}{dx^4} + \frac{2d^3y}{dx^3} + \frac{6d^2y}{dx^2} + \frac{2dy}{dx} + 5y = 0$$

then G.S

a) $y = C_1\cos x + C_2\sin x + C_3e^{3x} + C_4e^{2x}$

b) $y = C_1\cos x + C_2\sin x + C_3e^{-3x} + C_4e^{-2x}$

✓c) $y = C_1\sin x + C_2\cos x + e^{-x}[C_3\sin 2x + C_4\cos 2x]$

d) $y = C_1\sin x + C_2\cos x + C_3\sin 2x + C_4\cos 2x$

→ Verification method.

1	2	6	2	5
				-3
				0

X not used bcoz Ans: has complex roots.

1. Roots are a, b H.D.Eqn

$(D-a)$ $(D-b)$

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$$

2. Roots are a, b, c

$$\frac{d^3y}{dx^3} - (a+b+c)\frac{d^2y}{dx^2} + (ab+bc+ca)\frac{dy}{dx} - (abc)y = 0$$

3. Roots are a, b, c, d

$$\frac{d^4y}{dx^4} - \underbrace{(a+b+c+d)}\frac{d^3y}{dx^3} + (ab+bc+cd+ad)\frac{d^2y}{dx^2} - (abc+bcd+dab)\frac{dy}{dx} + (abcd)y = 0$$

$$\frac{d^4x}{dx^4} + \frac{2d^3y}{dx^3} + \frac{6d^2y}{dx^2} + \frac{2dy}{dx} + 5y = 0$$

Ans :- C :-

$a = 0 + i$ —— $C_1\sin x$

$b = 0 - i$ —— $C_2\sin x$

$c = -1 + 2i$ —— $e^{-x}(C_3\sin 2x)$

$d = -1 - 2i$ —— $e^{-x}(C_4\cos 2x)$

$a+b+c+d = -2$

$abcd = 5$

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→ Actual Method.

sinx, sol. of D.E

↳ $e^{0x}[c_1 \sin x + c_2 \sin x] + a, b$

Roots are $0+i$, $0-i$, a, b

Sum $= a+b = -2$

Product $ab = 5$

Solving above

$a = -1+2i$

$b = -1-2i$

$c = i$

$d = -i$

1. $y_3''' - 3y' + 2y = 0$

$(m^3 - 3m + 2)y = 0$

1	1	0	-3	2	
	0	1	1	-2	
1	1	1	-2		0
	0	1	2		
-2	1	2	0		
	0	-2			
	1	0			

$1, 1, -2$

$y = (c_1 + c_2 x)e^{x} + c_2 e^{-2x}$

2. $(D^4 + 8D^2 + 16)y = 0$

$(D^2)^2 + \cancel{2(2D)^2} + 4^2$

$2 \cdot 4 \cdot D^2$

$a^2 + 2ab + b_2$

$(D^2 + 4)^2 = 0$

$(m^2 + 4)^2 = 0$

$m^2 = \pm 2i$

$\cancel{(c_1 + c_2 x)e^{2x} + (c_3 + c_4 x)e^{-2x}}$

$y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$

8. 3) $(D^4 - 2D^3 - 3D^2 + 4D + 4)y = 0$

$m^4 - 2m^3 - 3m^2 + 4m + 4 = 0$

-1	1	-2	-3	4	4
	0	-1	3	0	-4
-1	1	-3	0	4	0
	0	-1	4	-4	
	1	-4	4	0	

$m^2 - 4m + 4 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m = 2, 2$

$\Rightarrow (C_1 + C_2 x)e^{2x} + (C_3 + C_4 x)e^{-x}$

~~$m^2 - 4m + 4 = 0$~~

~~$m = \frac{4 \pm \sqrt{16-4}}{2}$~~

$= \frac{4 \pm \sqrt{12}}{2}$

$= 2 \pm \sqrt{2}$

-1	1	-2	-3	4	4
	0	1	-1	-4	
	1	-1	-4	0	

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm\sqrt{-4}$

$= \pm\sqrt{-1} \times 2$

$\pm i2$

$\pm 2i$

5. $(D^4 + 4)y = 0$

$m^4 + 4 = 0$

~~$m^2 + 2$~~

4. $y'' - 2y' + 10y = 0$ $\quad y(0) = 4$ $\quad y'(0) = 1$

$m^2 - 2m + 10 = 0$

$m = \frac{2 \pm \sqrt{4 - 40}}{2}$

$= \frac{2 \pm i\sqrt{36}}{2}$

$= 1 \pm i3$

$y = e^x[C_1 \cos 3x + C_2 \sin 3x]$

$e^0[C_1 + 0] = 4$

$C_1 = 4$

$y' = e^x\left[C_1 \cos 3x + C_2 \sin 3x - 3C_1 \sin 3x + 3C_2 \cos 3x\right]$

$1 = [C_1 + 0 - 0 + 3C_2]$

$c_1 = 4$

$c_1 + 2c_2 = 1$

$\cancel{c_2} = -3/2$ $\quad c_2 = -1$

$y = e^x\left[4\cos 3x + \cancel{1}\sin 3x\right]$

$= e^x\left[4\cos 3x - \sin 3x\right]$

5) $\frac{d^4y}{dx^4} + 4y = 0$

$m^4 + 4 = 0$

$(m^2)^2 + 2^2 = 0$

$(m^2)^2 + 2^2 = 0$

$(m^2)^2 + 4m^2 - 4m^2 + 2^2 = 0$

$(m^2+2)^2 - (2m)^2 = 0$

$\Rightarrow m^2 + 2 - 2m = 0$

$m^2 + 2 + 2m = 0$

-2	1	0	0	0	4
	0	-2	4	8	-4
	1	-2	4	8	0

$-2 \pm \sqrt{4+8}$

6.) $(D^3 + D^2 + 100D + 100)y = 0$

$m^3 + m^2 + 100m + 100 = 0$

$m = -1$

-1	1	1	100	100
	0	-1	0	-100
-10	1	0	100	0
	0	-10	-100	
	1	10		

$m^2 + 100 = 0$

$m^2 + 10^2 = 0$

$(m^2 + 2m10 + 10^2) - 2m10$ $\quad b^2 =$

$(m+10)^2 - 20m = 0$ $\quad b = 10$

$m =$ $\quad 2ab = 10$

100

$\cancel{20} \times 10$

$\cancel{40}$ 5×20

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Linear Combinations of fn's →

$c_1 y_1 + c_2 y_2 + \dots c_n y_n$

→ if y_1, y_2 are two independent sol. of 2nd order HLDE

$$\frac{d^2y}{dx^2} + \alpha \frac{dy}{dx} + \beta y = 0 \quad \text{——} ①$$

then $y = c_1 y_1 + c_2 y_2$ is also sol. of ①

$\boxed{k y_1, k y_2 \text{ are dep. sol. of } y_1 \,\&\, y_2}$

[using superposition principle (or) linearity principle]

→ if y_1, y_2 are two independent sol. then

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$= 0$ dep. sol.

→ if y_1, y_2, y_3 ... 3 ~~indep. sol.~~ 3 indep. sol. then

$$\begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} \neq 0$$

$=$ dep. sol.

Ex: $\frac{d^2y}{dx^2} + y = 0$

Sol. $m^2 + 1 = 0$

$m = \pm i$

$= +i, -i$

$y = c_1 \cos x + c_2 \sin x$ —— dep. sol.

$y = c_1 f_1 + c_2 f_2$

$f_1 = \cos x$

$f_1' = -\sin x$

$f_1'' = -\cos x$

} two indep. sol.

7

Verify sol. same inde./dep.

$y_1 = \cos x$
$y_2 = \sin x$ > two indep. sol.

$$\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1 \neq 0$$

$y_1 = e^x$, $y_2 = 3e^x$

verify dep. sol.

$$\begin{vmatrix} e^x & 3e^x \\ e^x & 3e^x \end{vmatrix} = 0$$ dep. sol.

$\cos x$, $\sin x$, ——— sol.

$\cancel{+}2\cos x$, $2\sin x$ ——— dep. sol.

→Q) $\frac{d^2y}{dx^2} + y = 0$ then sol. of D.E can be

a) $\sin x, \cos x$ b) $2\sin x - 3\cos x$ c) $2\sin x, \cos x$ d) All of above.

$y = 2\sin x - 3\cos x$
$y' = 2\cos x + 3\sin x$
$y'' = -2\sin x + 3\cos x$
$y'' + y = 0$
$-\cancel{2\sin x} + \cancel{3\cos x} + \cancel{2\sin x} - \cancel{3\cos x} = 0$

$y = 2\sin x$
$y' = 2\cos x$
$y'' = -2\sin x$
$y'' + y = 2\sin x - 2\sin x = 0$

→Q) If $y = x$ is the sol. of $x^2y'' + xy' - y = 0$ 2nd ~~L.D.E~~

2nd L. indep. sol. of D.E is ___

$\cancel{x^2m^2 + xm - 1 = 0}$

$\cancel{m^2 + \frac{m}{x} - \frac{1}{x^2} = 0}$

$x \lfloor x^2$

$\frac{-x \pm \sqrt{x^2 - 4x^2}}{2}$

a) x^2 b) $\frac{1}{x}$ c) $\frac{1}{x^2}$ d) x^n

V-m

$y = x^2$

$y' = 2x$

$y'' = 2$

$x^2y'' + xy' - y = 0$

$x^2 \cdot 2 + x \cdot 2x - 2 \neq 0$

$2x^2 + 2x^2 - 2 \neq 0$

$y = 1/x$

$y' = -1/x^2$

$y'' = 2/x^3$

$x^2y'' + xy' - y = 0$

$\cancel{x^2}\frac{2}{x^{\cancel{3}}} + \cancel{x}\frac{-1}{x^{\cancel{2}}} - y = 0$

$\frac{2}{x} - \frac{1}{x} - \frac{1}{x} = 0$

$\frac{2}{x} - \frac{2}{x} = 0$

(or)

$$\begin{vmatrix} x & 1/x \\ 1 & -1/x^2 \end{vmatrix} = \frac{-1}{x} - \frac{1}{x} \neq 0$$

$x, 1/x$ are two L.I.D sol.

Going to flash back

S.T - 3

$y = Ax^m + Bx^n$ $[m \neq n]$

$\rightarrow x^2y'' - (m+n-1)xy' + (mn)y = 0$

$\rightarrow x^2y'' + xy' - y = 0$

$m+n-1 = -1$

$mn = 1$

$m+n = 0$

$m = 1, n = -1$

$y = Ax^1 + Bx^{-1}$

$x, 1/x$

ECET 2014

5) If $y_1 = e^x$, $y_2 = e^{-x}$ are two sol. of H.D.E

a) $y_3 = e^{2x}$, $y_4 = e^{-2x}$ are also " " "

b) $y_3 = xe^x$; $y_4 = xe^{-x}$ " " " " "

c) $y_3 = \cosh x$, $y_4 = \sinh x$ " " " " "

d) $y_3 = \cos x$, $y_4 = \sin x$ " " " " "

$y = ce^x + c'e^{-x}$

$y = (c_1+c_2)e^x + (c_2-c_1)e^{-x}$

$= \frac{c_1[e^x - e^{-x}] + c_2[e^x + e^{-x}]}{2}$

$y = A\sinh x + B\cosh x$

d) Roots

$+i, -i$

$\frac{d^2y}{dx^2} - (0)\frac{dy}{dx} - (-1)y = 0$

$\boxed{\frac{d^2y}{dx^2} - y = 0}$

$\cosh = \frac{e^x + e^{-x}}{2}$

$\sinh = \frac{e^x - e^{-x}}{2}$

c) Roots

$+\sqrt{1}, -\sqrt{1}$

$\frac{d^2y}{dx^2} - (0)\frac{dy}{dx} + (+1)y = 0$

$\boxed{\frac{d^2y}{dx^2} + y = 0}$

5) Find the no. of values of 'm' for which e^{mx} with the sol. of $\frac{D^2y}{dx^2} + 9\frac{dy}{dx} + 18y = 0$

a) 0 b) ∞ c) 2 d) none.

$\Rightarrow m^2 - 9m + 18 = 0$

$m^2 - 6m - 3m + 18 = 0$

$m = 6, 3$ —————— 2 solutions

$y = c_1e^{6x} + c_2e^{3x}$

$y = c_1e^{m_1x} + c_2e^{m_2x}$

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Q) If $y = C_1e^{-x} + C_2\cos x + C_3\sin x$

with sol. of $K_0y_3 + K_1y_2 + K_2y_1 + K_3y = 0$

then $K_0, K_1, K_2, K_3 =$ ______

roots $-1, +i, -i$

$D^3 - (-1)D^2 + (1)D - (-1) = 0$

$K_0 = 1 \quad K_1 = 1 \quad K_2 = 1 \quad K_3 = 1$

$\cancel{-i} + 1 + \cancel{i}$

$(-1\times i) + (i\times -i) + (-i\times$

$\cancel{-i} + 1 + \cancel{i}$

Q) If the sol. of $\frac{d^2y}{dx^2} - 40\frac{dy}{dx} + 111y = 0$

is $y = c_1e^{ax} + c_2e^{bx}$ then $|a-b| =$ ___

$\cancel{\frac{40 \pm \sqrt{1600 - 444}}{2}}$

$|a-b| = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{40^2 - 4(111)}$

$= \sqrt{1156}$

$= 34$

$|a-b| = 34$

$\cancel{1600}$
444
1156

$30 \times 30 = 900$

34×34: 900, 240, 16, $\boxed{1156}$

34×34: 900, 120, 120, 16, 1156

Q) The no. of sol. of $\left|\frac{dy}{dx}\right| + |y| = 0$

a) 0 ✓ b) 1 c) ∞ d) 2

$m + 1 = 0$

$m = -1$

$y = c_1e^{-x}$

$\frac{dy}{dx} + y = 0$

$P = 1 \quad Q = 0$

$IF = e^x$

G.S $\Rightarrow$

$$ye^x = \int 0\,dx + c$$

$$ye^x = C$$

$$\boxed{y = Ce^{-x}}$$

If ~~y = 0~~ $C = 0$

$$\left|\frac{dy}{dx}\right| + |y| = 0$$

$$\frac{dy}{dx} + 0 = 0 \checkmark$$

If $C = 1$

$$y = e^{-x}$$

$$|-e^{-x}| + |e^{-x}| \neq 0$$

ECET 2017

Q) $y = cx - c^2$ is GS of D.E

a) $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

b) $\frac{dy}{dx} = C$ ✓

c) $\frac{d^2y}{dx^2} = 0$

d) $\left(\frac{dy}{dx}\right)^2 + \frac{x\,dy}{dx} + y = 0$

$\frac{dy}{dx} = c$ put in (a) $c^2 - xc + y = 0$

$$\boxed{y = cx - c^2}$$

Rules for Finding Particular Integrals : $[Y_P]$

- $D = \frac{d}{dx}$ = Diff. operator
- $D^{-1} = \frac{1}{D}$ = integral (or) inverse operator
- $\frac{x}{D} = \int D$; $\frac{x}{D^2} = \iint x$; $\frac{x}{D^3} = \iiint x$
- $D(x) = \frac{d}{dx}(x)$; $D^2(x) = \frac{d^2}{dx^2}(x)$
- $\frac{x}{D-a} = e^{ax}\int xe^{-ax}\,dx$; $\frac{x}{D+a} = e^{-ax}\int xe^{ax}\,dx$

Rule 1

If $X = e^{ax+b} \mid k \mid a^{px+b} (a>0) \mid k\sinh(ax+b) \mid k\cosh(ax+b)$

If $X = e^{ax+b}$

if $F(D)y = e^{ax+b}$

$$y_p = \frac{e^{ax+b}}{f(a)} \quad \text{put } D = a \quad \& \quad f(a) \neq 0$$

$$y_p = \frac{x \cdot e^{ax+b}}{f'(a)} \quad \text{''} \quad f(a) = 0 \quad \& \quad f'(a) \neq 0$$

$$y_p = \frac{x^2 e^{ax+b}}{f''(a)} \quad \text{''} \quad f(a) = 0 \quad \& \quad f'(a) = 0 \quad \& \quad f''(a) \neq 0$$

till continue $DR \neq 0$

1. Find y_p of $(D^2 - 3D + 4)y = e^{2-3x}$

$$y_p = \frac{e^{2-3x}}{D^2 - 3D + 4}$$

$D = a$

$= -3$

$$= \frac{e^{2-3x}}{9 + 9 + 4}$$

$$\boxed{y_p = \frac{e^{2-3x}}{22}}$$

2. Find y_p of $(D-3)^4 y = 3e^{3x}$

$$\Rightarrow y_p = \frac{x^4 3e^{3x}}{24}$$

y_p of $(D-a)^n = ke^{ax}$

then $y_p = \dfrac{x^n \cdot ke^{ax}}{n!}$

3. y_p of $(D+1)^5 y = e^{-x}$

$$y_p = \frac{x^5 e^{-x}}{120}$$

6) $(D^2+3D+2)y = e^{e^x}$

$(D^2+2D+D+2)^2$

$$y_p = \frac{e^{e^x}}{(D-2)(D-1)}$$

D-

7) $(D^2+D)y = \frac{1}{1+e^x}$

8) $(D^2-4D+1)y = 2x$

+ Pg 13, 14, 16, 15