

* STRENGTH OF MATERIALS (OR) SOLID MECHANICS

Solid mechanics is a science which deals with the behaviour of solids under loads [static, dynamic], It deals with the condition that stress is lesser than strength [Safe working Condition]

→ properties of materials

1. STRENGTH & Ability of a material to Resist load [Tension, Compression, Shear, gradual load, Sudden load, Impact load, point load, UDL, S.V.L] without undergoing deformation.

2. STIFFNESS = load resisting capacity of a material per unit deflection

$$K = \frac{W}{y}$$

→ stiffness is a measure of Young's modulus [E].

3. ELASTICITY & Ability to regain dimensions when applied load is removed

4. PLASTICITY & Ability to retain dimensions when applied load is removed

5. DUCTILITY = Ability of a material to Resist tensile load by undergoing permanent deformation without failure. material with large deformation at failure is ductile

Ductility factor = $\frac{\text{Strain at fracture or failure}}{\text{Strain at yield}}$

Best ductile materials are gold, silver, tungsten, platinum, & bronze.

$$D.F = \frac{\epsilon_{\text{failure or ultimate}}}{\epsilon_{\text{yield}}} \rightarrow \text{for mild steel}$$

$$D.F = 250$$

Ductile materials are deformed into wires.

6. MALLEABILITY = property of material by which it can be beaten & rolled into sheets & plates is called malleability.

Ability of material to resist compressive load by undergoing permanent [plastic] deformation without failure.

→ All ductile materials are malleable

Ex: plastic response of material to compressive force is malleability.

7. HARDNESS: Ability of a material to resist scratches, indentations, abrasion etc. These materials are difficult to machine [cutter] but can be utilised for cutting other materials.

Ex: MoS₂, HCS, HSS [High Speed Steel], Cubic boron nitride, diamond

8. TOUGHNESS: Strength + Ductility

Toughness modulus ← 2. Static test → Toughness is area under stress-strain curve upto ultimate point

3. Dynamic test → Toughness is Energy absorbed by material upto ultimate point

4. Resisting impact load without failure is known as Impact Strength (or) Toughness

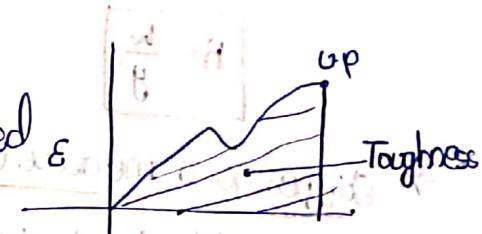
- Total energy absorbed by a body within elastic limit is called "Resilience".
- Ability of material to absorb energy when it is deformed elastically & release energy upon unloading is "Resilience".
- Max energy stored by a body at elastic limit is "proof Resilience".
- proof Resilience Ratio with unit volume is known as "modulus of resilience".

10. BRITTLENESS: Ability to fail under impact loads.

Ex: glass, cast iron etc.

11. CREEP: It depends on few parameters

1. Constant stress
2. strain vs time
3. temperature



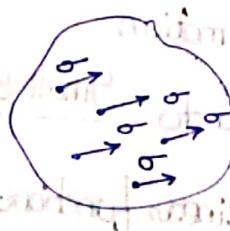
- Change in length of material with time under Constant load is "Creep".
- Continuous deformation with time under Sustained loading is "Creep".
- slow and progressive deformation under Constant stress at high temperatures is Called Creep.
- 12. FATIGUE After no. of Cycle of Reversal loads failure of material is "Fatigue failure".
- A member which is Subjected to Reversible tensile comp. σ may fail at a stress lower than σ_{Ultimate} of material. This property is fatigue of material.

• whenever a member is failed within ultimate stress at a point is due to Endurance σ or fatigue failure.

⇒ ASSUMPTIONS IN STRENGTH OF MATERIALS

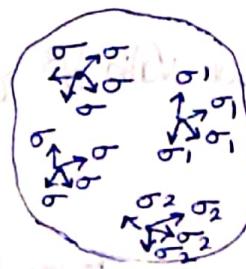
1. material is Continous [no voids, gaps, defects]
2. material is homogeneous & Isotropic

→ Homogeneous =
↓ = ↓
Some origin



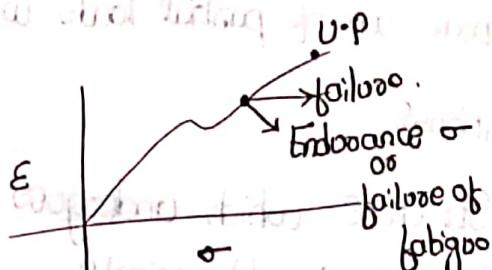
A material having Identical properties throughout its volume in any one direction is Said to be "homogeneous".
Ex- wood, Iron, Copper, Ag.
Ex- for non homogeneous - Alloys.

→ ISOTROPIC =
↓ = ↓
Same directionality



⇒ material which exhibit same elastic properties in all directions at one point is Called isotropic

→ Elastic Constants are identical in all directions at one point



Sys: Iron, Copper, Silver, glass, steel, brass, etc.

* Wood is "anisotropic" (or) "nonisotropic" bcz [layered, elongated grains (or) fibrous materials are anisotropic].
→ Orthotropic properties in 1st direction at a point are not same



here $\sigma_x / \sigma_y / \sigma_z$

3. Weight of material is neglected

① * * * * *
SUPER POSITION PRINCIPLE IS VALID

Algebraic sum of partial loads will be equal to net load

Conditions :-

1. Structure which undergoes S.P. is valid.

↳ structurally determinate

↳ structure obeys Hooke's law

2. S.P. is valid when materials are

↳ elastic

↳ linearly elastic

↳ metals

↳ homogeneous

3. Structures subjected to S.P. is valid

↳ Small deformation

↳ Large deformation

Small slopes & deflections [negligible]

4. S.P. is valid for material subjected to

deflections are linear functions of applied forces.

5. S.P. is valid when deflections are linear functions of applied forces.

6. S.P. is not valid for Impact loads but valid for gradual loads

7. St. Venant's principle is valid :-

Sudden change in any dimension causes stress concentration & can be prevented by using fillets or "chamfers". [PL to converted to UDL]

- * The elastic limit & young's modulus decreases with increase in temp.
- * yield point is not effected by temp.
- * Decreasing order of ductility
- wrought iron > mild steel > Cast iron > pig iron.
- * for a beam cooling wdl the strain energy will be maximum in case of Cantilever beam

* $\alpha_{boass} > \alpha_{steel}$ & $\alpha_{Cu} > \alpha_{steel}$

* $\alpha_{Al} > \alpha_{boass} > \alpha_{copper} > \alpha_{mildsteel}$

coefficient of linear expansion

also note that the coefficient of linear expansion is proportional to temperature and it increases with increase in temp.

Boatman effect occurs at

(Or when material contracts more than it expands)

→ heat of cooling in one part of the object is less than that of other parts

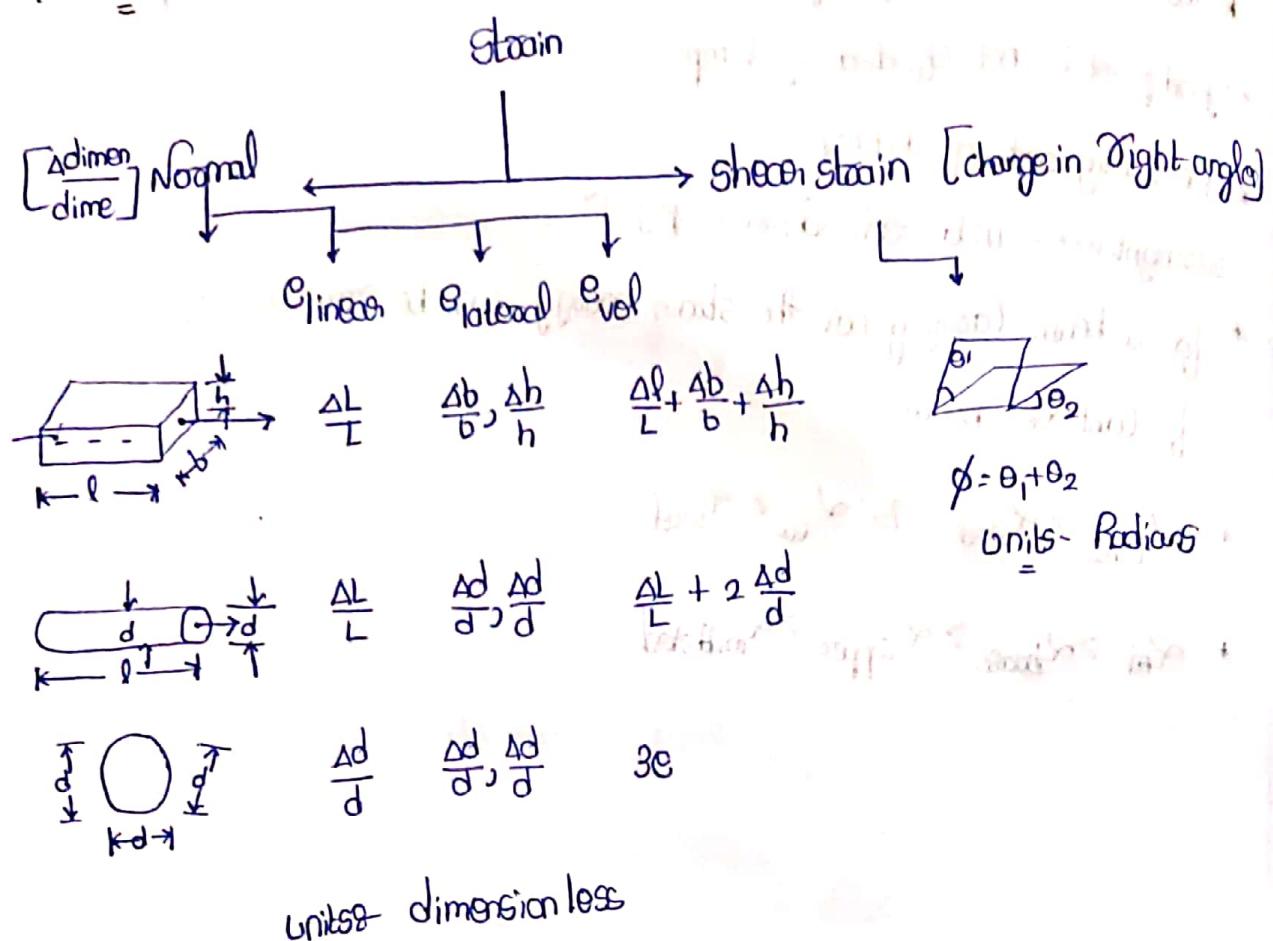
→ so it causes all part to move

→ longitudinal length change

→ transverse length change

→ shear stress

\Rightarrow STRAIN -



1. Unit of vol. strain = No units

2. parameter which relates Simple stress & strain is Young's modulus

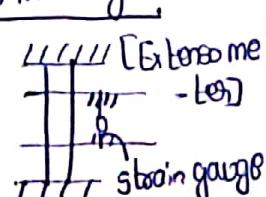
3. Intensity of unit stress causing unit strain is "E"

\Rightarrow STRESS - STRAIN CORRELATION

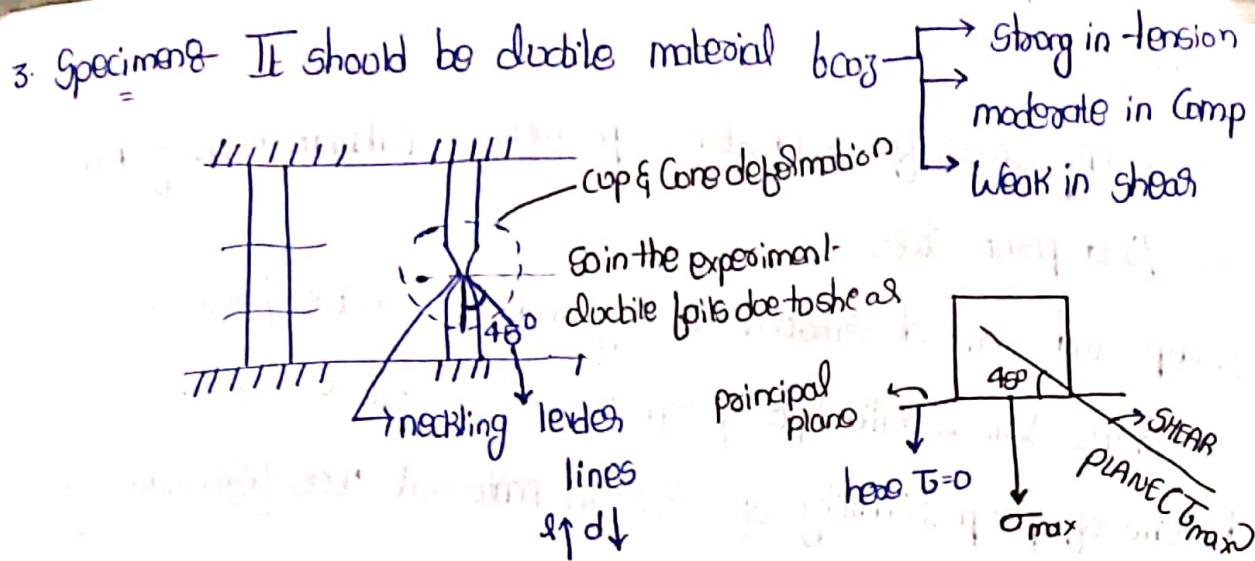
1. Tension test

2. Universal testing machine [strain oriented machine]

* Gauge length & It is the distance over in which change in length is measured. it is equals $5.64\sqrt{A_0}$ where A_0 is the original CS area

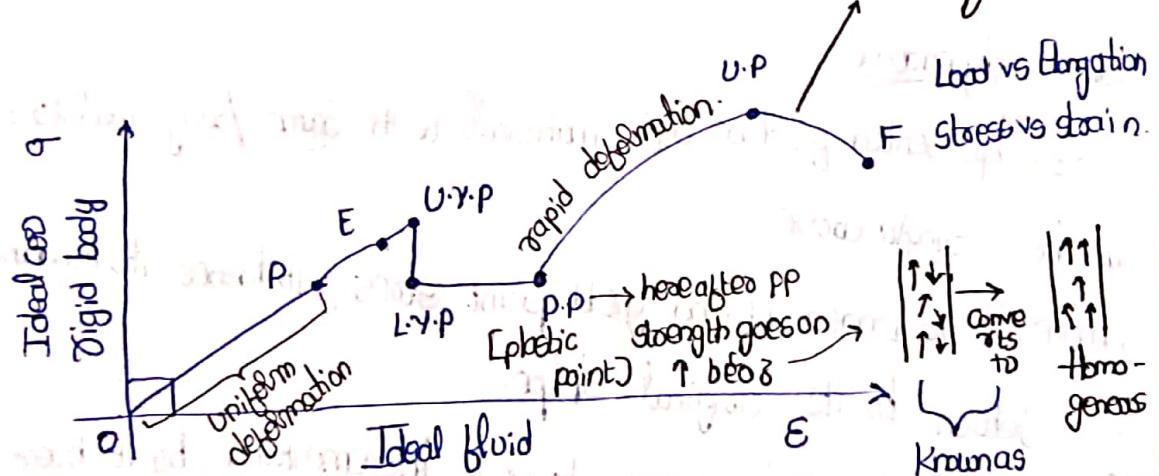


Note &	depend	Independent
	Lgauge	L _{total}
	A _{original}	Shape of CS



$$\Rightarrow \% \text{ of Elongation} = \left[\frac{L_f - L_0}{L_0} \right] \times 100$$

$$\Rightarrow \% \text{ of Reduction of Area} = \left[\frac{A_0 - A_f}{A_0} \right] \times 100$$



- permanent deformation starts from yield point
- Temporary " ends at Elastic limit" (blue, L.Y.P to P.P)

O-P → linear elastic

P-E → non-linear elastic

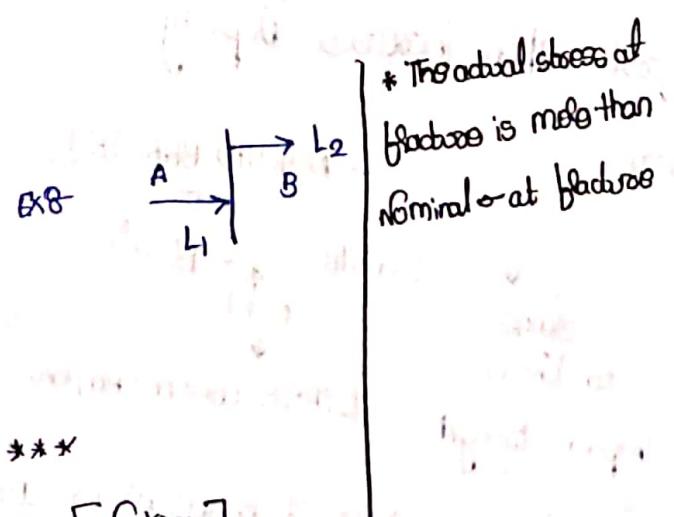
E-U.Y.P → coincident

U.Y.P - L.Y.P → yielding

L.Y.P - P.P → plastic zone

P.P - U.P → strain hardening ***

U.P - Failure → strain softening [Necking]



NOTE8

- 5) Working stress for mild steel Specimen is calculated using F.O.S on Yield point Stress.
- 3) Cup and Cone deformation usually indicates ductile failure.
- 3) Hooke's Law is valid upto proportionality limit [$\sigma \propto E$]
- 4) law of proportionality of $\sigma \propto E$ in material was formulated by Hook Robert

5) Ratio of strain at fracture to that of yield for mild steel is 250

6) For most metals $\sigma \propto E$ Relation Under elastic limit follows

Linear dependence

7) $\sigma \propto E$ Represented for two diff materials with same Young's modulus

will be Single Curve

5) material is loaded beyond yield point stress, it losses its tendency to return to its original shape

6) A thin mild steel wire is loaded incrementally till it breaks, Extension noted with increase in load will be increasing uniformly

first then increases rapidly.

10)

NOTCHED BAR TEST

done
to know

Ductile \rightarrow Brittle

Notch

STRESS CONCENTRATION



IZOD TEST

Impact strength

7) Toughness modulus of material is area of Stress-strain Curve upto failure.

$$(2) F_{\text{c}})_{\text{material}} = \frac{\sigma_{\text{Ultimate}}}{\sigma_{\text{Working}}}$$

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13) Sequence 8

1) $\sigma =$ Proportionality Limit $\rightarrow E \cdot L \rightarrow Y \cdot L \rightarrow$ Failure.
in local endo. tons.

proportionality

1) A mild steel specimen is tested under tension a Continuous graph load & Extension is obtained. A load at which there is Considerable extension without increase in load is lower yield point

Fos) ductile = lower yield stress

$$15) \text{ Tension Coefficient} = \frac{\text{Force}}{\text{Length}}$$

16) Tenacity = Tensile strength.

16) Tensile strength = $\frac{\text{maximum load}}{\text{original area}}$ of the test piece is ultimate

ratio of maximum load to original weight of the glass

Ques Is residual stress is deformation stress

Ques

Q) Residual stress is deformation stress.

(a) when the change in length takes place, is the strain is linear [load is

(g) when the change in length takes place in the direction of length] [Safe Condition]

$$\text{in direction of length} \\ \text{ii) } \sigma_{\text{Working}} < \sigma_{\text{yield (deformation)}} \quad [\text{safe condition}]$$

• Working = Yield point
• Set is irreversible deformation in body.

(ii) Permanent set is irrecoverable deformation in body.

(iii) A 100mm wide, 10 mm thick, is having hole 10mm, $F = 9 \text{ kN}$ then
 C.I.d. hole will be —

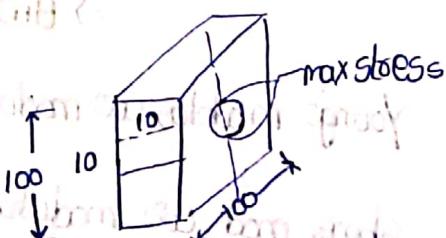
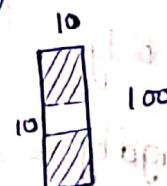
(a) A 100mm wide, 10mm thick
Max. one Section passing through. Centre of hole will be

$\text{sel} \quad A_1 - A_2$

$$\Rightarrow 100x \cdot 10 - 10x^2$$

$$A = 900$$

$$\therefore \sigma = \frac{900}{9} = 10 \text{ MPa}$$



23) If Carbon Content increases then ductility decreases.

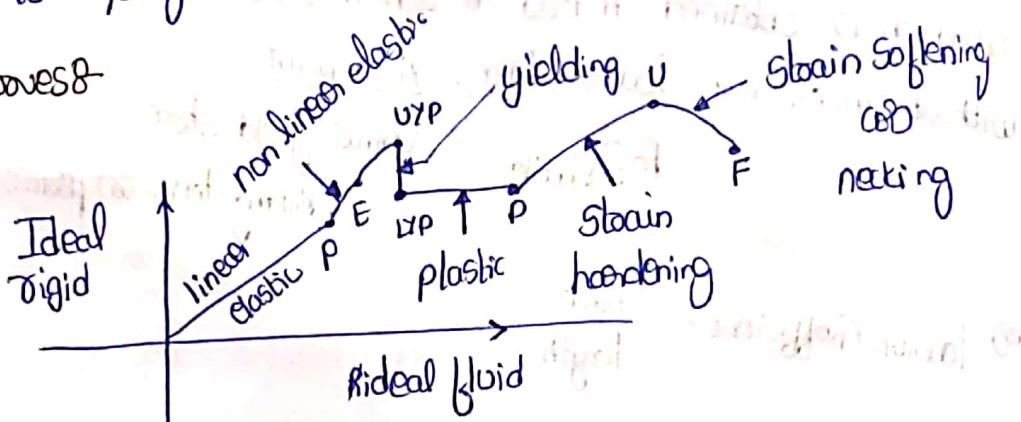
23) If carbon content increases material becomes less ductile, ie

23) If carbon content increases material becomes less ductile. ie
24) High yield strength increases material becomes less ductile.
yield point \uparrow permanent deformation \uparrow Ductility \uparrow

q6) strain hardening of material is increase in stress with strain beyond the yield stress

q7) when mild stress is stretched upto strain hardening state & load is released Recovery path of stress-strain curve will have slope equal to Young's modulus.

\Rightarrow Types of Curves



Rigid body

Ideal fluid

Linear elastic

Linear elastic necking

Rigid necking

Rigid plastic

Elastoplastic

Linear elastic st. hardening

Linear elastic yielding

\Rightarrow Elastic Constants

1. Young's mod \Rightarrow modulus of elasticity = $\frac{\text{Linear } \sigma}{\text{Linear } \epsilon} = E$

2. Shear mod \Rightarrow modulus of rigidity = $\frac{\text{Shear } \sigma}{\text{Shear } \epsilon} = G$

3. Bulk mod \Rightarrow dilation Constant = $\frac{\text{Dilat. } \sigma}{\text{Volumetric strain}} = K$

Direct stress is also known as Vol stress

Diamond	E
Steel	200 GPa
Cu	120
Brass [Gr-I]	100
Aluminium	70 [Seventy].
Tinber [Rubber]	10

Q) Max Elasticity is in E

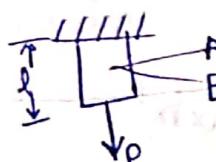
a) Rubber \rightarrow diamond \rightarrow steel \rightarrow Cu

\Rightarrow NOTE 8

For isotropic materials $\rightarrow [E > R > G]$ [i.e. only for brittle i.e. $\mu < 0.25$]

\Rightarrow RIGIDITY \propto E

② Axial Rigidity :- $[AE]$ i.e. $\Delta l \downarrow \text{as } AE \uparrow$ i.e. $\Delta l \propto \frac{1}{AE}$



$$E = \frac{\sigma}{\epsilon}$$

$$\frac{\Delta l}{l} = \frac{P}{AE}$$

$$\Delta l = \frac{Pl}{AE}$$

units - N

③ Flexural Rigidity \propto EI^2 .

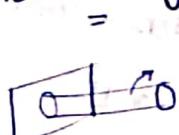


Here $EI^2 \uparrow$ $\theta \downarrow$

$$y = \frac{w l^3}{3EI}$$

units \propto $N \cdot mm^2$

④ Torsional Rigidity \propto GJ



Here $GJ \uparrow \theta \downarrow$

$$J = \frac{GJ}{l}$$

units \propto $N \cdot mm^2$

\Rightarrow Poisson's Ratio \propto $\mu \propto \frac{1}{m}$

Ratio of lateral strain to linear strain with sign changed

$$\mu = \frac{\text{E}_\text{lateral}}{\text{E}_\text{linear}}$$

1. $\mu = -ve$ for genetic materials.

2. $\mu = 0$ for Cork.

3. $\mu = +ve$ for all engg materials.

4. $\mu = 0.5$ for Elastic incompressible material [$\delta V = 0$ i.e. Rubber, fluids]
↳ \downarrow Indicates Constant volume.

5. μ for isotropic material = 0.25

6. μ for hard & brittle material < 0.25

7. μ for soft materials > 0.25

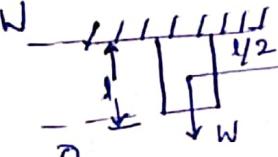
8. μ for steel = 0.27 to 0.3.

9. μ for Cast iron = but $\mu < 0.25$ here $0.21 \approx 0.25$.
(a) 0.27 (b) 0.31 (c) 0.33 (d) 0.36

10. μ of Aluminium = 0.33

11. μ of Concrete = 0.15

⇒ Self weight deformation


$$WKT \quad \Delta l = \frac{PL}{AE} = \frac{W \cdot L}{2AE}$$

$$= \frac{\gamma V l}{2AE} = \frac{\gamma A x l \times l}{2AE}$$

$$\Delta l = \frac{Pl}{2AE}$$

or

$$\Delta l = \frac{\gamma l^2}{2E}$$

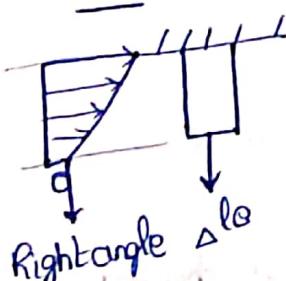
* Independent on area

* dependent on l^2

→ Note 8 $\Delta l = \frac{1}{3} \Delta l$ cylinders
cone

$$\Delta l_{cone} = \frac{1}{3} \times \frac{\gamma L^2}{2E} = \frac{\gamma L^2}{6E \cdot 11}$$

⇒ SELF WEIGHT σ [IN PRISMATIC BAR]



$$\sigma = \frac{W}{A} = \frac{\gamma A l}{A}$$

$$\sigma = \gamma l'$$

* Independent on area

* dependent on l'

NOTE 8

Independent

dependent

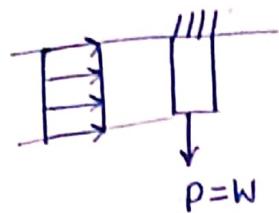
l^2

l'

→ Self weight Δl

→ Self weight σ

\Rightarrow WEIGHT LESS PRISMATIC BAR 8



$$\sigma = \frac{P}{A} = \frac{W}{H}$$

\Rightarrow Bending =

① $\Rightarrow \Delta l = \frac{4PL}{C\pi D^2 E}$

② $\Rightarrow \Delta l = \frac{Pl}{abE}$

③ $\Rightarrow \Delta l = \frac{Pl}{(b-b)tE} \log \left[\frac{b}{b-t} \right]$

\Rightarrow Important points 8

1) Independent no. of elastic Constants

① Isotropic $\rightarrow 2[*] [E, G] \text{ or } [K, \mu] \text{ or } [G, \mu]$

② Orthotropic $\rightarrow 9$

③ Anisotropic $\rightarrow 21$

2) For non-dilatent [INCOMPRESSIBLE] $H_{max} = 0.5$

3) $\mu = 0.25, \frac{E}{G} = ?$

$\therefore \frac{E}{G} = 2[1+\mu] \rightarrow \frac{E}{G} = 2[1.25] \Rightarrow \frac{E}{G} = 2.50$

$$\Rightarrow \frac{E}{G} = 2.5 \parallel.$$

\Rightarrow RELATIONSHIP σ

① $\Rightarrow E = 2G[1+\mu]$

④ $\mu = \frac{3K-2G}{6K+2G}$

② $E = 3K[1-2\mu]$

③ $E = \frac{9KG}{3K+G}$

4) If $K=G$ then $\mu = \dots$

$$\text{Sol} \quad \mu = \frac{3K-2G}{6K+2G} = \frac{1}{8} \text{ II}$$

5) Relation b/w 3 Elastic Constants $E = \frac{9KG}{3K+G}$

6) $E=2G$ then $K = \dots$

$$\text{Sol} \quad 2G = \frac{3KG}{3K+G} \Rightarrow 3K+G = \frac{3K}{2} \Rightarrow G = \frac{3K-6K}{2} \Rightarrow \text{Wrong}$$

WKT $E=2G[1+\frac{1}{\mu}]$ so $E=2G$ then $\mu=0$

$$\text{Hence WKT } E=3K[1-\frac{2}{\mu}] \Rightarrow K=\frac{E}{3}$$

7) $G=80$, $K=160$, $\mu=?$

$$\text{Sol} \quad \mu = \frac{3K-2G}{6K+2G} = \frac{420-160}{840+160} = \frac{260}{1000} = 0.26 \text{ II. } \frac{160}{260}$$

\Rightarrow THERMAL STRESSES

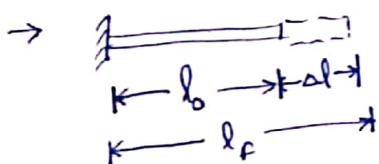
a. Thermal stress temp. stresses are secondary stresses

b. Coefficient of thermal expansion

$$\epsilon = \alpha (\Delta T)$$

$$\epsilon = \alpha (\Delta T)$$

$$\alpha = \frac{\epsilon}{\Delta T} = \frac{\Delta (\text{dimension})}{(\text{original dimension}) \times (\Delta T)}$$



$$\frac{l_f - l_0}{l_0 (\Delta T)} = \alpha$$

$$l_f - l_0 = l_0 \alpha (\Delta T)$$

$$l_f = l_0 + l_0 \alpha (\Delta T)$$

$$l_f = l_0 [1 + \alpha (\Delta T)]$$

$$l_f = l_0 [1 + \epsilon]$$

3. THERMAL STRESSES

$$\epsilon = \alpha (\Delta T)$$

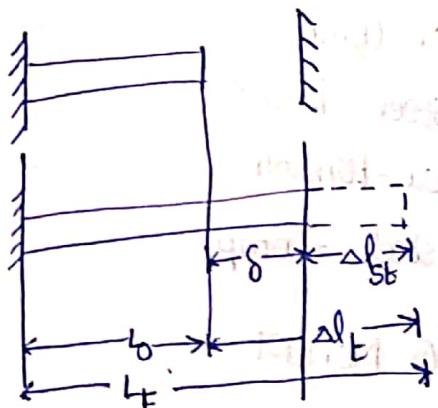
$$\frac{\sigma}{E} = \alpha (\Delta T)$$

$$\boxed{\sigma = E \alpha (\Delta T)}$$

* A beam is heated & expands freely [No stress]

5.  $\text{Temp} \uparrow \rightarrow \text{Comp} \sigma$ } $\text{Temp} \downarrow \rightarrow \text{Tens} \sigma$ } $\left. \begin{array}{l} h \\ \text{Thermal} \end{array} \right\} \sigma$'s

\Rightarrow Thermal σ 's with yield supports



$$\Delta l_{st} = l_t - l_0$$

$$\Delta l_{st} = L_0 \alpha (\Delta T) - \Delta$$

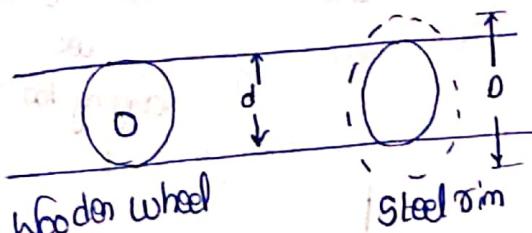
$$\frac{\Delta l_{st}}{L} = \frac{L \alpha (\Delta T)}{L} - \frac{\Delta}{l_0}$$

$$\boxed{\epsilon = \alpha (\Delta T) - \frac{\Delta}{l_0}}$$

$$\frac{\sigma}{E} = \alpha (\Delta T) - \frac{\Delta}{l_0}$$

$$\boxed{\sigma = E \left[\alpha (\Delta T) - \frac{\Delta}{l_0} \right]}$$

\Rightarrow HOOP STRESS (OR) CIRCUMFERENTIAL STRESS



Steel rim allowed to cool from t_o to t_d over wheel then

* Wooden wheel \rightarrow Comp

* Steel rim \rightarrow Tension.

$$\therefore \epsilon = \frac{\pi t_o - \pi t_d}{\pi d} \Rightarrow \alpha (\Delta T) = \frac{d \Delta}{d}$$

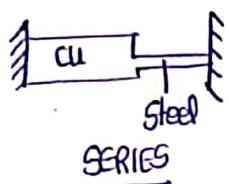
$$\Delta T = \frac{1}{\alpha} \left[\frac{D-d}{d} \right]$$

$$\Rightarrow \frac{\sigma}{E} = \frac{D-d}{d}$$

$$\sigma = E \left[\frac{D-d}{d} \right]$$

\Rightarrow COMPOSITE MEMBERS [$\alpha_{cu} > \alpha_{steel}$] & $\alpha_{buck} > \alpha_{steel}$

(1)

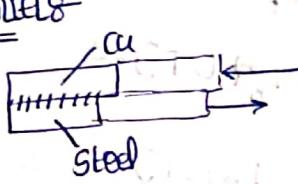


$t \uparrow \Rightarrow$ Both core in Comp

$t \downarrow \Rightarrow$ Both core in tension

(2)

PARALLEL



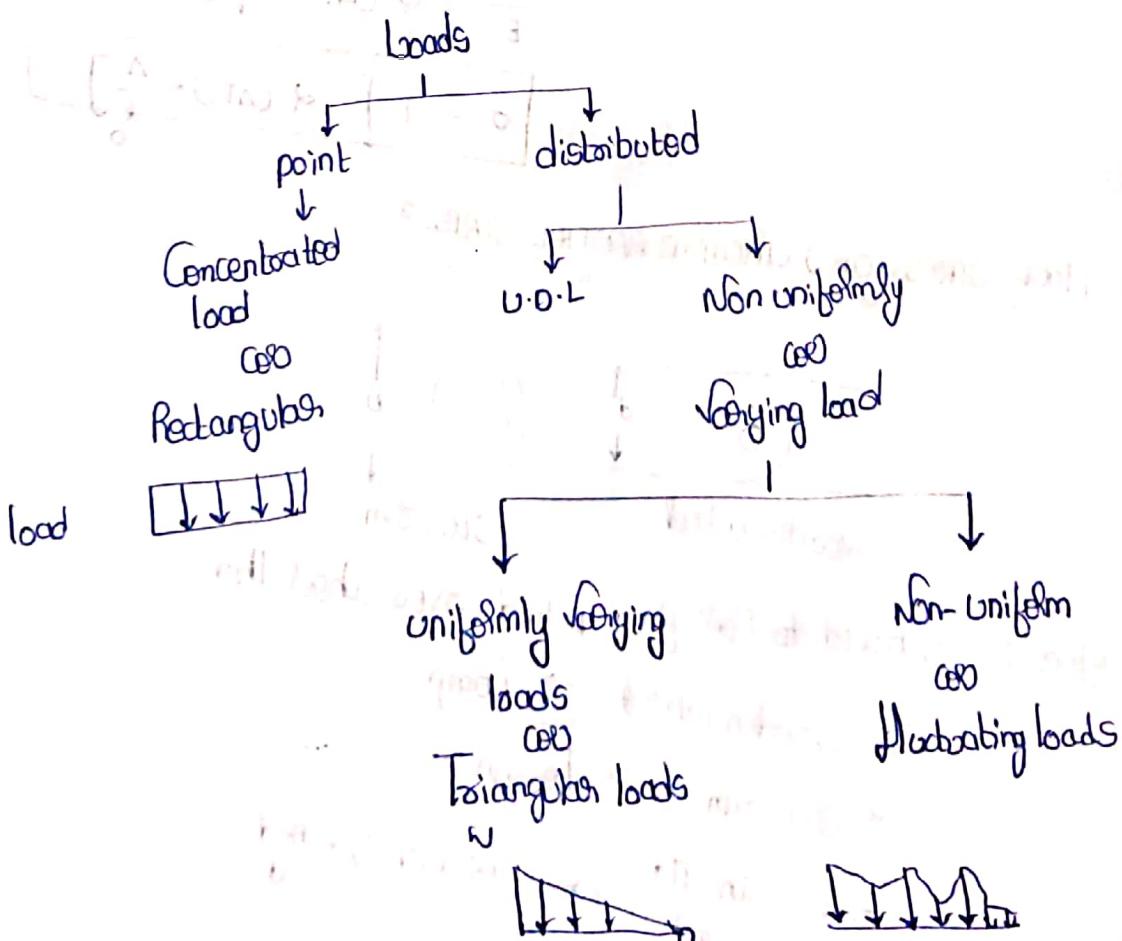
$t \uparrow \rightarrow$ Cu - Comp

Steel - Tension

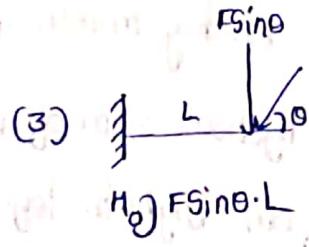
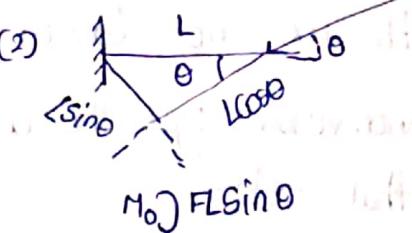
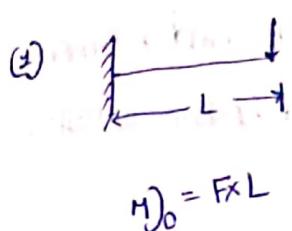
$t \downarrow \rightarrow$ Cu - Tension

Steel - Comp

* SHEAR FORCE & BENDING MOMENT



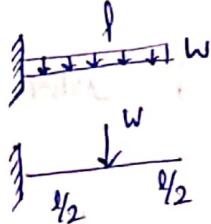
Moment = Force \times arm of moment



(4) moment of at a point is Concentrated point

$$M_0 = M_H.$$

$\rightarrow U \cdot D \cdot L \text{ To P.L.B}$

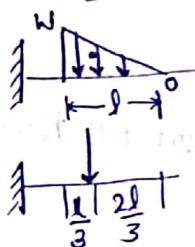


point of application = Centroid of area

$$w \times l = \text{Area} \text{ centroid} = l/2$$

so, Area = wl
Centroid = $\frac{l}{2}$

$\rightarrow U \cdot V \cdot L \text{ To P.L.B}$

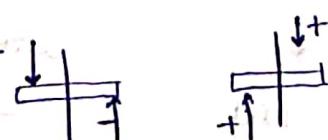


$$\text{Area} = \frac{1}{2} \times b \times h = \frac{1}{2} \times w \times l$$

Centroid from base = $\frac{l}{3}$. [ie from w].

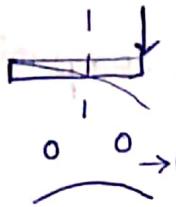
\rightarrow Shear force & Sign Convention

Algebraic Sum of all the forces from right to left [vice Versa] upto that section is called shear force of that section.

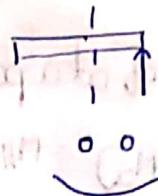


→ Bending moment & Sign Convention

Algebraic sum of all the moments caused by forces and moments from right to left [vice versa] upto the Considered Section is called Bending moment at that Section.



$\downarrow \leftarrow$ Hoggling [-ve]



$\uparrow \leftarrow$ Sagging [+ve]

⇒ Features of SFD & BMD

1) If there is a point load at a point acting on a beam, at a Section

SF changes Suddenly & BM has no change.

2) If there is no load b/w two points SF Remains Constant [Horizontal line] and bending moment changes linearly [straight inclined line]

3) If there is udl b/w two points SF changes linearly & BM changes parabolically.

4) If there is u.v.l b/w two points SF changes parabolically and BM changes Cubical parabola.

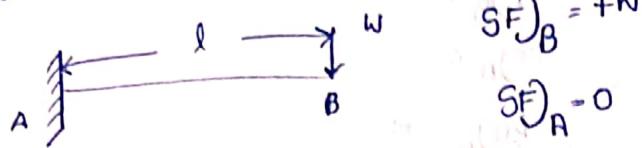
5) If there is Concentrated moment at a Section then SF has no change

but BM changes Suddenly.

	S.F.D	B.M.D
↓ no load	Suddenly —	no change / or \
U.D.L \leftarrow ∴	/ \ or \	/ P
 ∴	no change P	Sudden change Cubical parabola.

① CANTILEVER

② Cantilever beam is Subjected to point load.



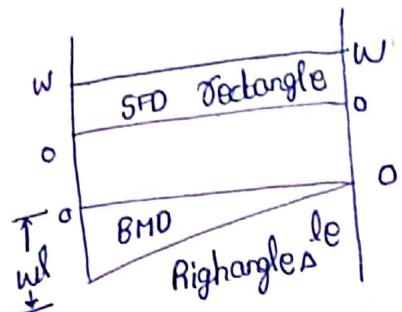
$$SF)_B = +w$$

$$SF)_A = 0$$

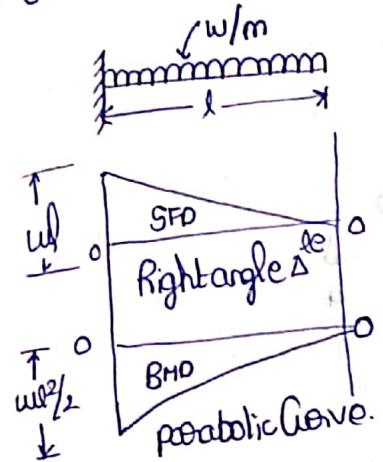
$$SF)_{\text{Just before A}} = w$$

$$BM)_B = 0$$

$$BM)_A = wl$$



③ Cantilever Subjected to U.O.L.



$$SF)_B = 0$$

$$SF)_x = wx^2$$

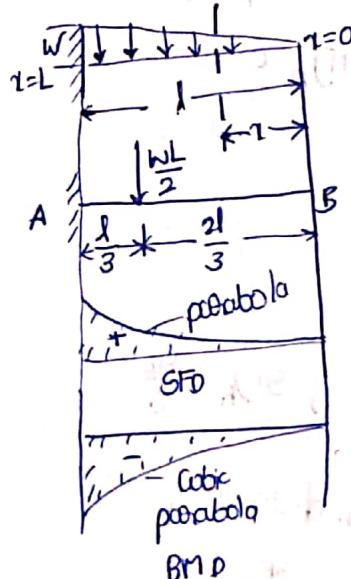
$$SF)_A = 0$$

$$SF)_{\text{Just before A}} = wl$$

$$BM)_B = 0$$

$$BM)_A = \frac{wl^2}{2}$$

④ Cantilever Subjected to U.V.L



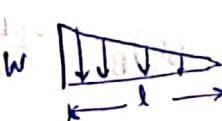
$$SF)_B = 0$$

$$SF)_A = \frac{wl}{2}$$

$$BM)_B = 0$$

$$BM)_A = -\frac{wl^2}{6}$$

Now Considering x distance from free end



$$\frac{w}{l} = \frac{w}{x} \Rightarrow w' = \frac{wx}{l}$$

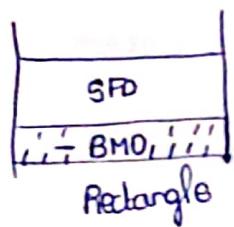
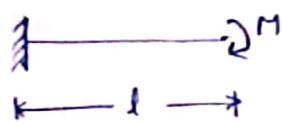
Area of loaded diagram

$$= \frac{1}{2} \times x \times \frac{wx}{l} = \frac{wx^2}{2l}$$

$$SF = \frac{wx^2}{2L}$$

$$BM = \frac{wx^3}{6L}$$

(b) Cantilever Subjected to moment &



$$SF)_B = 0$$

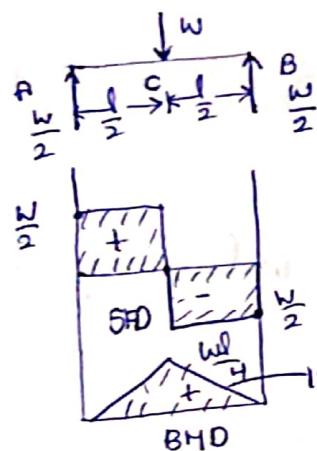
$$SF)_A = 0$$

$$BM)_B = -M$$

$$BM)_A = -M$$

② SS BEAM :-

1. SS beam Subjected to point load at Center



$$SF)_B = -\frac{w}{2}$$

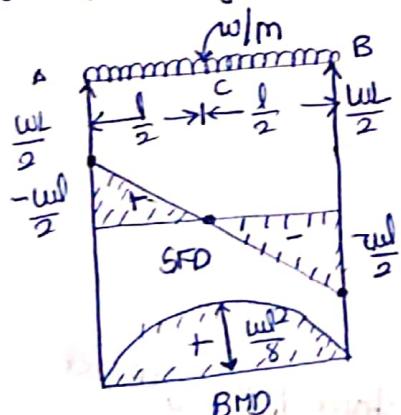
$$SF)_C = -\frac{w}{2} + w = \frac{w}{2}$$

$$SF)_A = \frac{w}{2}$$

$$BM)_B = BM)_A = 0$$

$$BM)_C = \frac{w}{2} \times \frac{l}{2} = \frac{wl}{4}$$

2. SS beam Subjected to udl L



$$SF)_B = -\frac{wl}{2}$$

$$SF)_C = 0$$

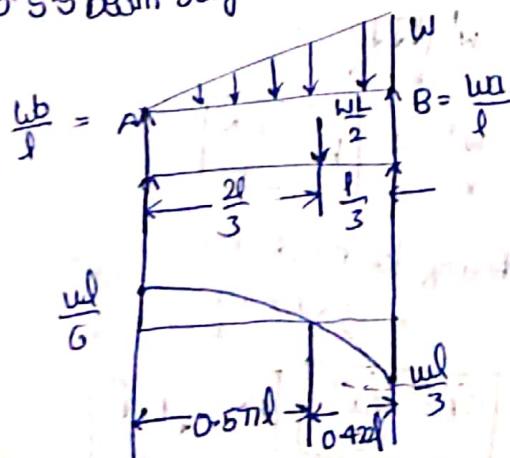
$$SF)_A = \frac{wl}{2}$$

$$BM)_B = 0$$

$$BM)_C = \frac{wL^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$

$$BM)_A = 0$$

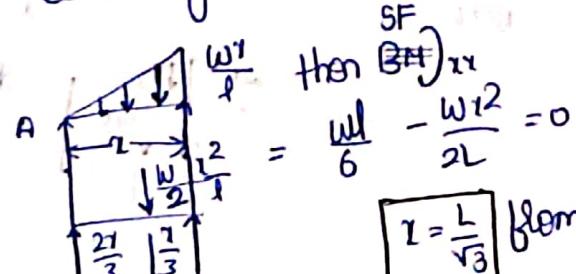
3. SS beam Subjected to uvl L



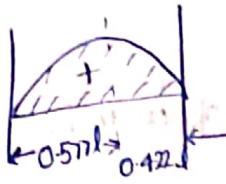
$$SF)_B = -\frac{wl}{3}$$

$$SF)_A = \frac{wl}{6}$$

Considering certain distance 'x', $SF = 0$



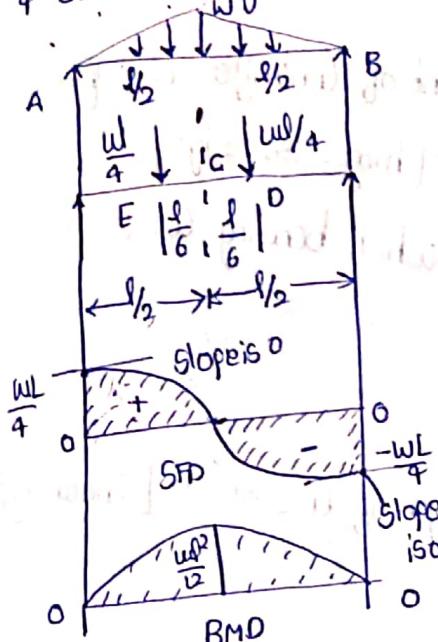
$$I = \frac{L}{\sqrt{3}} \text{ from A}$$



$$\begin{aligned}
 \text{Now } BM_{xx} &= \frac{wl}{6}(x) - \frac{wl^2}{2L} \left(\frac{x}{3}\right) \\
 &= \frac{wl}{6} \left[\frac{L}{3}\right] - \left[\frac{wl^3}{6L}\right] \\
 &= \frac{wl}{6} \left(\frac{L}{\sqrt{3}}\right) - \frac{w}{6L} \left(\frac{L^3}{\sqrt{3} \times \sqrt{3} \times \sqrt{3}}\right) \\
 &= \frac{wl^2}{6\sqrt{3}} \left[1 - \frac{L}{3}\right]
 \end{aligned}$$

$$BM_{max} = \frac{wl^2}{9\sqrt{3}} = 0.064wl^2 \text{ from A}$$

4. gg beam Subjected Symmetric U.V.L



$$\text{Hence Total load} = \frac{wl}{4} \times 2 = \frac{wl}{2}$$

$$R_A = R_B = \frac{wl}{4}$$

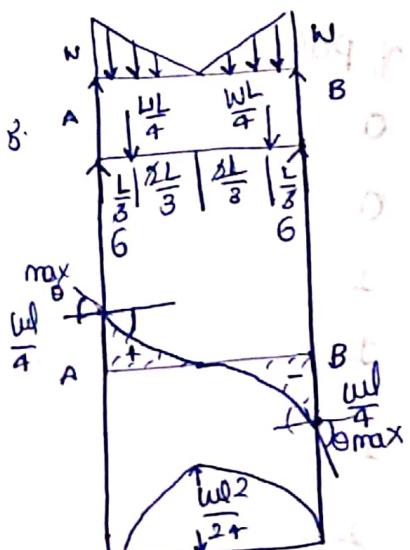
$$SF_E = -\frac{wl}{4}, SF_D = -\frac{wl}{4} + \frac{wl}{4} = 0$$

$$SF_E = \text{Not necessary}, SF_A = \frac{wl}{4}$$

$$BM_{B=0} = BM_A$$

$$\begin{aligned}
 BM_A &= \frac{wl}{4} \times \frac{l}{2} - \frac{wl}{4} \times \frac{l}{6} = \frac{wl^2}{8} - \frac{wl^2}{24} \\
 &= \frac{3wl^2 - wl^2}{24} = \frac{2wl^2}{24} = \frac{wl^2}{12}
 \end{aligned}$$

$$BM_{max} = \frac{wl^2}{12}$$



$$SF_B = -\frac{wl}{4}, SF_A = \frac{wl}{4}$$

$$\text{Max BM at Center} = \frac{wl}{4} \left[\frac{L}{2}\right] - \frac{wl}{4} \left(\frac{L}{3}\right)$$

$$BM_C = \frac{wl^2}{4} \left[\frac{1}{2} - \frac{1}{3}\right]$$

$$BM_C = \frac{wl^2}{24}$$

Hence, $BM_C = \frac{wl^2}{24}$

Notes

④ Rate of change of shear force with respect to length of beam is equals to intensity of load

$$\frac{dF}{dx} = w$$

⑤ Rate of change of bending moment with respect to length of beam is equals to shear force

$$\frac{dM}{dx} = F$$

⇒ point of Centraflexure

- 1. In a bending beam a point is known as "point of Centraflexure": If it is a location where bending moment is zero [changes its sign]
- 2. In a bending moment diagram it is a point which bending curve intersects "zero line"

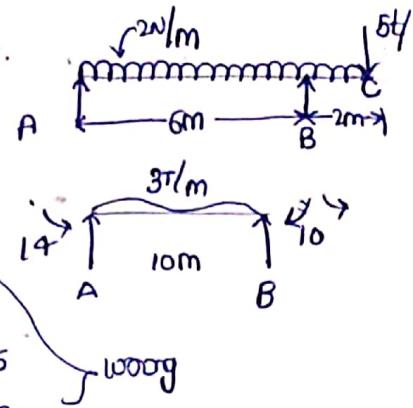
⇒ Point of inflection

IE is the point on the Curve at which sign of Curvature [Concavity] changes. [Deflection Curve changes sign]

⇒ Types of beam & No. of point of Centraflexure

	No. of POC
1. Cantilever beam	0
2. SS. beam	0
3. Single overhanging beam	1
4. propped Cantilever beam	1
5. double overhanging beam	2
6. Fixed beam	2
7. Continuous beam	Based on no. of Supports

$$Q) \text{pg no } 418 \quad AEE = 418 \quad BM_B = -5 \times 2 - 2 \times \frac{2}{2} = -14 \text{ t-m [Hanging].}$$



$$Q) \text{pg no } 418 \quad SF_A = 0.5 - 3 \times 10 =$$

$$R_B \times 10 - 10 + 3 \times \frac{10}{2} = 0$$

$$-10R_B - 10 + 15 = 0 \Rightarrow R_B = 0.5$$

$$-10R_B = -5 \quad R_B = 0.5$$

$$10R_B = 5 \quad 0.5 = 0.5$$

$$\Rightarrow \text{moments about } B=0 \Rightarrow R_A \times 10 - 14 - 3 \times \frac{10}{2} = 0 \Rightarrow R_A = \frac{44}{6} = 4.4 \text{ t-m [Wrong]}$$

$$R_A \times 10 - 14 - [3 \times 10] 5 + 10$$

$$R_A = 17.4 \text{ t-m}$$

$$Q) \text{pg no } 418 \quad AEE$$

$$\frac{\frac{wl^2}{2}}{\frac{wl^2}{8}} \Rightarrow \frac{\frac{1}{2}}{\frac{1}{8}} \Rightarrow \frac{8}{2} \Rightarrow 4:1 \parallel$$

$$Q) \text{pg no } 418 \quad ①$$

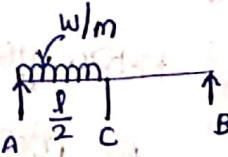
$$Q) \text{pg no } 418 \quad AEE$$

$$w \downarrow \Rightarrow -wl - \frac{wl^2}{2} \Rightarrow -wl \left[1 + \frac{l}{2} \right] \Rightarrow -wl \left[\frac{2+l}{2} \right] \text{ [Wrong]}$$

$$\Rightarrow -wl - \frac{wl}{2} \Rightarrow -wl \left[1 + \frac{1}{2} \right] \Rightarrow -1.5wl$$

$$Q) \text{pg no } 418 \quad -R_B \times l + \frac{wl^2}{2} = 0 \text{ [Wrong]} \quad R_B \times l = w \left(\frac{l}{4} \right)$$

$$R_B \cdot l = \frac{wl^2}{2} \Rightarrow R_B = \frac{wl}{2} \text{ [Wrong]}$$



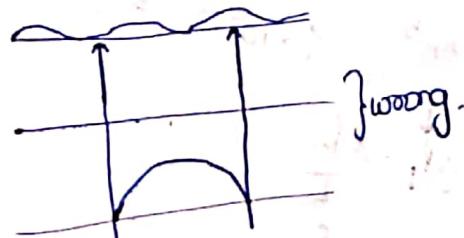
$$\Rightarrow R_B = \frac{wl}{8}, R_A = \frac{wl}{2} - \frac{wl}{8} = \frac{4wl - wl}{8} = \frac{3wl}{8}$$

$$SF_B = \frac{wl}{8} \quad SF_A = \frac{wl}{2}$$

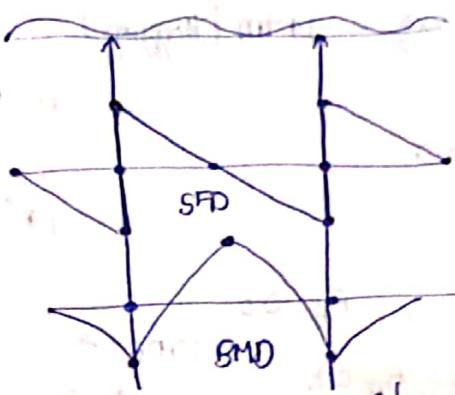
$$\text{Max BM} \rightarrow \text{Shear sign change} \Rightarrow \frac{3wl}{8} + \frac{wl}{2} = 0 \Rightarrow x = \frac{3l}{8} \quad \text{Wrong} \quad x = \frac{9}{4} l \quad 4) 30^{\circ}$$

$$\frac{wlx}{2} = \frac{3wl}{8} \quad \frac{3wl}{8}$$

$$Q) \text{pg no } 419$$



79



AEE

$$79) \quad 25x = 60, \text{ then } x = 2.4 \text{ m from Support.}$$

AEE

$$98) \quad \text{Diagram shows a beam with forces } P_C \text{ and } P \text{ at supports. Bending Moment } BM_C = \frac{P}{2} \cdot \frac{P}{4} = \frac{2P^2}{4} = \frac{P^2}{4} \text{ Nm.}$$

\Rightarrow IMPORTANT POINTS :-

$$\textcircled{1} \quad \varepsilon_2 = \frac{\sigma_z}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E}$$

$$\text{If It is 2 dimension } \Rightarrow \begin{cases} \varepsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \\ \varepsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} \end{cases} \quad \text{for Cubes}$$

$$\delta v = \Delta v = \frac{3P}{E} (1-2\mu) \times \text{volume}$$

$$\textcircled{2} \quad \frac{R}{E} = \frac{1}{3} \delta z$$

$$31) \text{ AEE 81 } E > KCG \text{ generally}$$

But here $R < E$ i.e. $\mu < 0.25$ for brittle materials.

So option (b) 0.15 is ✓

$$42) \text{ AEE 81 } E_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma}{E} - \mu \frac{\sigma}{E} - \mu \frac{\sigma}{E}$$

$$E_v = \frac{\sigma}{E} (1-2\mu) \times 3 \times \text{volume}$$

$$E_v = \varepsilon (1-2\mu) \times 3, \text{ option -D} \checkmark$$

$$\text{So } K = \left[\frac{dP}{\Delta v} \right] \Rightarrow \Delta v = \frac{dP}{K} = \frac{100}{0.667 \times 10^5} = \frac{100}{\frac{2}{3} \times 10^3} = \frac{3}{2} \times 10^{-3} = 1.5 \times 10^{-3}$$

③ For Composite members :-



$$\sigma = \frac{P}{A}$$

$$P = \sigma A$$

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

$$\frac{\Delta l}{L} = \frac{\Delta l_2}{L}$$

$$\varepsilon_1 = \varepsilon_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} \rightarrow \text{modulus ratio}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = \begin{cases} E_{\text{Strong}} \\ E_{\text{Weak}} \end{cases} \quad \text{for modulus ratio}$$

89) AEE Pg - 168 $E_x = \frac{\sigma_x}{E} - H \frac{\sigma_y}{E} - H \frac{\sigma_z}{E}$

$$= \frac{\sigma}{E} - 0.25 \frac{\sigma}{E} - \frac{0.25 \sigma}{E}$$

$$E_x = \frac{\sigma}{E} [0.5]$$

then $\sqrt{\epsilon} = \epsilon_x + \epsilon_y + \epsilon_z = 1.5 \frac{\sigma}{E}$ the option 2 is ✓

100) AEE Pg 168

Total &

E, C, K, H

Indirect " "

E, H

K, H

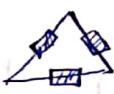
C, H

⇒ STRAIN ROSETTES

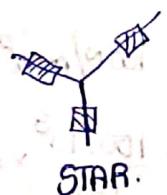
Arrangement of strain gauges to find linear strain



Rectangular



DELTA



STAR.

Answers of AEE8

$$168) T = \frac{F_g}{A} = \frac{F_g}{\pi d^2} \Rightarrow F_g = T \pi d^2 = 30 \times \pi \times 20 \times 20 = 31,699 \text{ kg.}$$

$$169) \sigma = \frac{F}{A} \Rightarrow 140 \times 10^6 = \frac{500 \times 10^3}{\frac{\pi}{4} \left(D - \frac{4D}{5} \right)^2} = \frac{500 \times 10^3}{\frac{\pi}{4} \left(\frac{D}{5} \right)^2} \quad \text{Wrong} \quad \text{Rif} \Rightarrow \frac{2D}{10} = \frac{1D}{5}$$

$$\Rightarrow 140 \times 10^6 = \frac{500 \times 10^3}{\frac{\pi}{4} \cdot \frac{D^2}{25}} \Rightarrow D^2 = \frac{500 \times 10^3 \times 25 \times 4}{140 \times 10^6} = \frac{500 \times 10^3 \times 100}{140 \times 10^6} \quad \text{Wrong} \quad \Rightarrow D = \frac{1}{5} D \quad \frac{D}{5} = \frac{1D}{5}$$

$$\Rightarrow D^2 = \frac{5}{14} \quad \text{Wrong}$$

$$\Rightarrow 140 \times 10^6 = \frac{500 \times 10^3}{\frac{\pi}{4} \left(D - 0.8D^2 \right)} \Rightarrow D = 17 \text{ mm.}$$

$$210) \frac{\sigma}{\sigma_b} = \frac{200}{100} = 2 \text{ / } .$$

$$199) E = 2 \times 10^6 \text{ kg/cm}^2 = 2 \times 10^6 \times 10^4 \text{ N/mm}^2$$

$$= 2 \times 10^5 \text{ N/mm}^2$$

$$E = \frac{\sigma}{\epsilon} \quad \Rightarrow 2 \times 10^5 = \frac{\frac{P}{I} \times 14^4}{\frac{45}{45}}$$

$$(\text{or}) \quad \frac{5}{P} \times 10^4 \times 4$$

$$P = \frac{45 \times 14 \times 22}{22} \quad \leq \quad \frac{45}{100000} = \frac{P \times 10^4 \times 4}{14 \times 2 \times 10^8 \times 2}$$

$$\Rightarrow \frac{1}{10} = \frac{P \times 4 \times 1}{\pi \times 14 \times 45} \quad \Rightarrow P = 99 \times \pi$$

$$pg-19 \& 17) WKT, \mu = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{linear}}} \Rightarrow \mu = \frac{\Delta D/A}{\frac{4P}{\pi D^2 E}} \Rightarrow \Delta D = \frac{4HPD}{\pi D^2 E}$$

$$\Delta D = \frac{4PH}{\pi D^2} \quad //$$

$$Q18 \rightarrow pg-20) \frac{\Delta l_{sw}}{\Delta l_{\text{load}}} = \frac{\frac{1}{2}wl/AE}{wl/AE} = \frac{1}{2} \Rightarrow 1:2$$

$$Q31 \rightarrow pg-20) \Delta l = \frac{\gamma L^2}{2E} \Rightarrow \frac{3.2}{1000} = \frac{50 \times 4^2}{2 \times E} \Rightarrow E = \frac{50 \times 4 \times 1000 \times 10}{32 \times 2}$$

$$E \Rightarrow \frac{25 \times 5000}{50 \times 10000 \times 16} \quad //$$

$$Q38-pg-20) \Delta l = \frac{PL}{AE}, \Delta l \propto \frac{1}{AE}$$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{(AE)_1}{(AE)_2} \Rightarrow \frac{(AE)_1}{\left[\frac{AE}{2}\right]} = 2 \Rightarrow \boxed{\Delta L_2 = 2y}$$

$$Q39-pg20) \gamma = 100 \text{ kN/m}^3, \sigma = 100 \text{ N/mm}^2$$

$$\sigma = \gamma \times L \Rightarrow 100 = \frac{100 \times 10^3}{10^9} \times L \Rightarrow L = \frac{10^{11}}{10^5} \text{ mm} = 10^6 \text{ mm} = 10^3 \text{ m.}$$

Q46), Compound bars can be analyzed based on equilibrium & Compatibility Conditions

3. Compatibility Conditions Cos Eqn generally deals with displacement, ϵ

Slope etc

q.(Comp) \rightarrow Generally used Condition is Same deflection or Same ϵ

Q55-pg22) % Reduction of $\frac{1}{E}$ % Elongation [ductility] //

A	B
% R - 60%	% R - 40%
$E = \text{less}$ ductile less	$E = \text{more}$ ductile more

$\therefore B$ is more ductile than A.

q6) strain energy theory [Not in syllabus]

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2H[\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1]$$

Hence for Cobe $\sigma_y^2 = 3\sigma^2[1-2H]$

$$\sigma = \frac{\sigma_y}{\sqrt{3[1-2H]}}$$

q7) Material with H is increase then material becomes soft then volumetric change also increases

q8) $50+P$ 30+40 then $50+P=70$ then $P=20$

$$\boxed{1}$$

should be 1 should be

q9) $A\Delta = \frac{PL}{AE} \rightarrow$ then

$$\Delta L = \frac{PL}{AE_1 + A_2 E_2}$$

$$\text{Hence } \Delta L = \frac{PL}{AE_1 + AE_2} = \frac{1}{3} \frac{PL}{AE}$$

q10) Generally based on the real values of the materials

$$T_{max} = \frac{\text{Normal stresses}}{2}$$

$$q) \quad \begin{array}{c} B \\ | \\ A \uparrow \frac{2L}{3} \quad \downarrow \frac{L}{3} \end{array} \quad C \Rightarrow BG = \frac{P2L/3}{L} = P \frac{2}{3} \quad \& \quad AB = \frac{P \frac{2}{3} L}{3L} = \frac{P}{3}$$

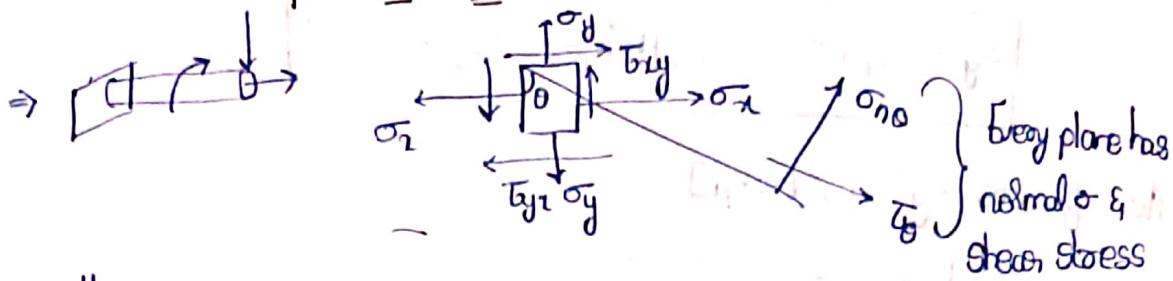
$$\text{then } \frac{AB}{BC} = \frac{P \frac{2}{3}}{\frac{P}{3}} = \frac{2}{1} \rightarrow 2:1:1.$$

* THERMAL'S 8

(Comp)

$$q) \quad E = \alpha t, \quad \frac{\sigma}{E} = \alpha t \Rightarrow \sigma = E \alpha t = 2 \times 10^6 \times 1.5 \times 10^{-6} \times 20 = 60 \text{ kg/cm}^2 (\text{Tens})$$

* COMPLEX STRESSES



then $\sigma_{n\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

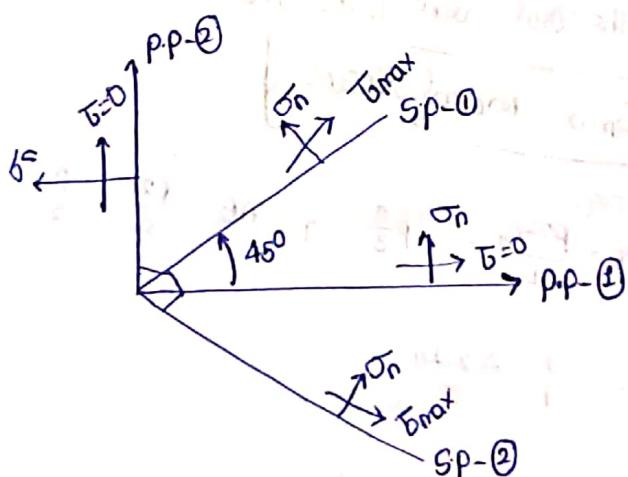
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin \theta - \tau_{xy} \cos \theta$$

⇒ Principal stresses [$\tau=0$] plane is principal plane &

$$0 = \frac{\sigma_x - \sigma_y}{2} \sin \theta - \tau_{xy} \cos \theta$$

$$\frac{\sigma_x - \sigma_y}{2} \sin \theta = \tau_{xy} \cos \theta$$

$$\theta_p = \frac{1}{2} \tan^{-1} \left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right]$$



⇒ Max principle principal stress theory [Rankine's theory] [Brittle material]

$$\sigma_{\max} \leq \sigma_{yp} \text{ [Safe]}$$

$$\begin{cases} \sigma_1 & \sigma_{\max} \\ \sigma_2 & \sigma_{\min} \end{cases} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$A \pm \sqrt{B}$$

\Rightarrow Max principal stress theory [Ductile material] [Gross / Tresca / Coulombs]

= shear = = = =

$$T_{\max} \leq T_{\text{yp}} \text{ [safe]}$$

$$T_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{A + \sqrt{B} - (A - \sqrt{B})}{2} = \sqrt{B}$$

$$T_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + T_{xy}^2}$$

Note: σ_n = nominal average stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2}$$

$$T_{\max} = \frac{\sigma_x - \sigma_y}{2}$$

$$\sigma_{\max} = \sigma_n \pm T_{\max}$$

↑ theory of failure

σ_{\min}

\Rightarrow Rankines [p. stress] $\sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + T_{xy}^2}$

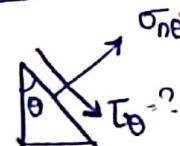
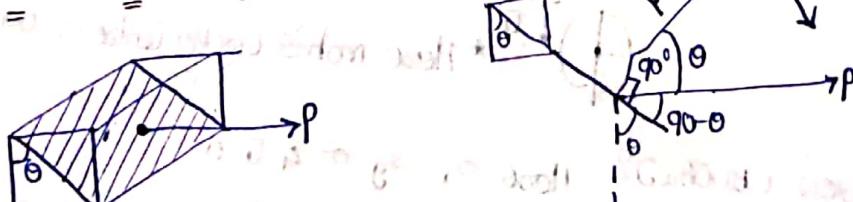
$$\Rightarrow \text{Guests } [T_{\max}] = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + T_{xy}^2}$$

$$\Rightarrow \text{St. Venants [principle & theory]} = \sigma_{\max} = \sigma_1 - H\sigma_2 \quad [\text{for 2-body}]$$

$$\Rightarrow \text{Strain energy [Haigh's theory]} = \sigma_{\max} = \sigma_1^2 + \sigma_2^2 - 2H\sigma_1\sigma_2 \quad [\text{for 2-body}]$$

$$\Rightarrow \text{Von Mises} \rightarrow \text{Distortion energy} = \sigma_{\max} = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \quad [\text{for 2-body}]$$

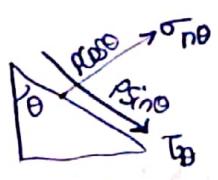
$$\Rightarrow \text{UNIDIRECTIONAL TENSILE LOAD } (P)$$



$$A = A_0 \cos \theta \rightarrow A_0 = \frac{A}{\cos \theta}$$

$$\sigma_{n\theta} = \sigma \cos^2 \theta$$

Now



$$\Rightarrow \sigma_{n\theta} = \frac{P_n}{A_n} = \frac{P \cos \theta}{A \cos \theta}$$

$$\Rightarrow \tau_\theta = \frac{P_t}{A_t} = \frac{P \sin \theta}{A \cos \theta}$$

$$T_\theta = \frac{\sigma}{2} \sin 2\theta$$

$$\text{for } \sigma_{n\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + T_{xy} \sin 2\theta$$

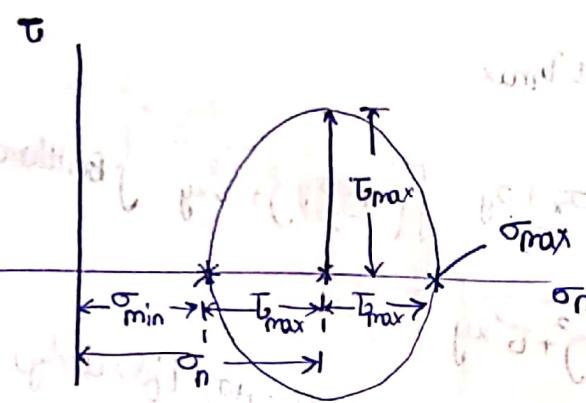
We know that $\sigma_y = T_{xy} = 0$

$$\text{then } \sigma_{n\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = \frac{\sigma}{2} [1 + \cos 2\theta] = \boxed{\frac{\sigma \cos^2 \theta}{2}}$$

$$T_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - T_{xy} \cos^2 \theta$$

$$\boxed{T_\theta = \frac{\sigma}{2} \sin 2\theta}$$

\Rightarrow MOHR'S CIRCLE



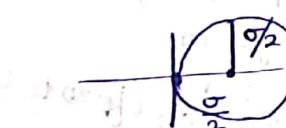
Here Radius = T_{max}

$$\sigma_{max} = \sigma_n + T_{max}$$

$$\sigma_{min} = \sigma_n - T_{max}$$

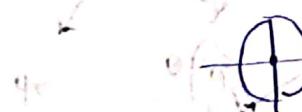
$$\text{Centre } [\sigma_n, 0]$$

① PURE TENSION \Rightarrow Here $\sigma_x = \sigma$, $\sigma_y = 0$ & $T_{xy} = 0$ i.e. $\rightarrow \sigma_x = \sigma$



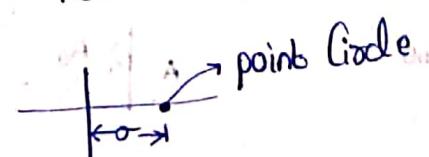
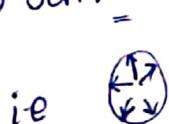
* These Mohr's Circle touches the origin

② PURE SHEAR \Rightarrow Here $\sigma_x = \sigma_y = 0$ & $T_{xy} = T$ i.e. $\downarrow \uparrow$ $\leftarrow \rightarrow$



* Here Mohr's Circle Centre is origin

③ SURFACE TENSION (BUBBLE) \Rightarrow Here $\sigma_x = \sigma_y = \sigma$ & $T = 0$

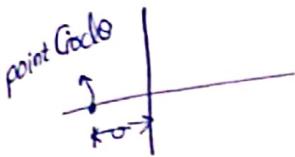


* positive (σ_n) axis, & point Circle

④ HYDROSTATIC PRESSURE SUBMERGED BODY \Rightarrow



i.e. $\sigma_x = \sigma_y = -\sigma$ & $T_{xy} = 0$



* negative ConCaxis & point Circle.

diagonal & off diagonal elements

$$\Rightarrow \text{Here } \sigma_x = \epsilon_x, \sigma_y = \epsilon_y \text{ & } \tau_{xy} = \frac{\phi_{xy}}{2}$$

Then, ① $\begin{cases} \epsilon_{\max} \\ \epsilon_{\min} \end{cases} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$

② $\frac{\phi_{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$

$$\phi_{\max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + (\phi_{xy})^2}$$

③ $\epsilon_n = \frac{\epsilon_x + \epsilon_y}{2}$

\Rightarrow 3D SPATIAL STRESS SYSTEMS

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{yz} \\ \tau_{xy} & \sigma_y & \tau_{zx} \\ \tau_{yz} & \tau_{zx} & \sigma_z \end{bmatrix}$$

stress & strain components

tensors

magnitude

& more than 1 direction

DETERMINANT OF MATRIX

$$\text{Hence, } \tau_{xy} = \tau_{yx}, \tau_{zy} = \tau_{yz}, \text{ & } \tau_{zx} = \tau_{xz}$$

$$\Rightarrow 2D \begin{bmatrix} \sigma_x & \tau_{xy} \\ \sigma_{yx} & \sigma_y \end{bmatrix} \Rightarrow \text{magnitude} = \sqrt{\sigma_x \sigma_y - \tau_{xy}^2}$$

* SYSTEM TOTAL σ & Components

Independent σ Components

10	1
20	4
30	9

1

3

6

No. of strain gauges

Required to know
charge in length.

* TORSION IN SHAFTS

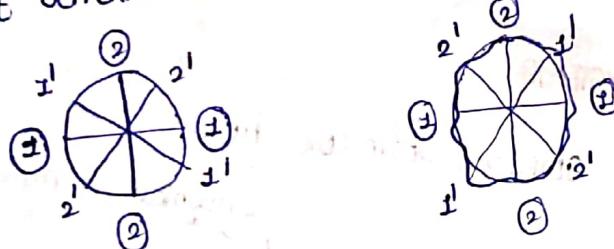
- Torsion is twisting about the axis
- Couple in the C.S. of beam causes torsion
- B.M takes place along the Centroidal axis
- Couple in the plane of beam causes bending
- pure torsion is in axial force = 0 i.e. $\Sigma x = 0$

∴ Bending i.e. $\Sigma y = 0$

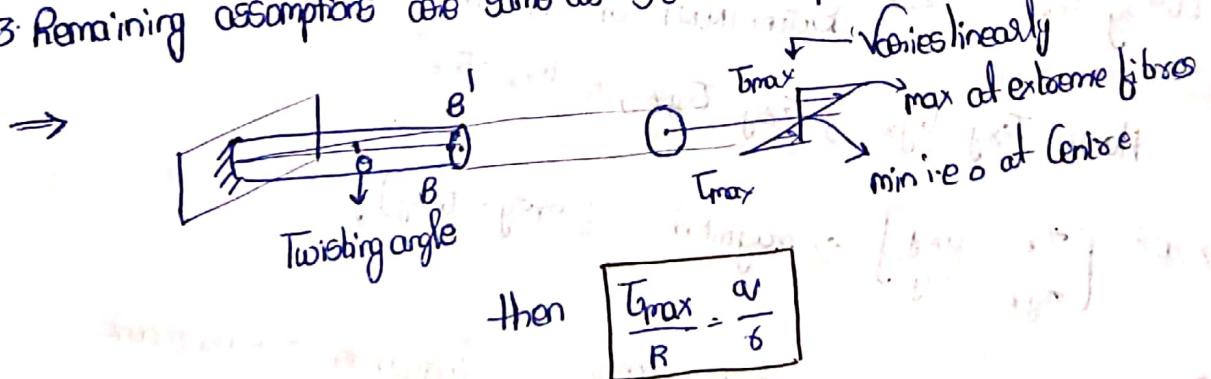
$$\text{and } T_{xy} = T$$

* ASSUMPTIONS

- 1. Torque is uniform along the shaft
- 2. C.S. of shaft remains same before and after application of load.



- 3. Remaining assumptions are same as G.O.M assumptions.



⇒ Torsional Eqns

$$\frac{I}{J} = \frac{T}{R} = \frac{G\theta}{J}$$

Torsional Rigidity

$$I = \frac{GJ}{L}$$

$GJ \uparrow \rightarrow \theta \downarrow$
slope \downarrow

Flexural Rigidity

$$\frac{EI}{L} = M$$

$EI \uparrow \rightarrow$ stiffness
 \downarrow deflection
 \downarrow slopes

→ DESIGN

21/05/19

§ SHAFTS

$$\text{power} = \frac{\pi T I N T}{60} \quad \text{where } T \text{ is the Torque in N-m}$$

p is the power in watts

$$\text{then } P = \frac{\pi T I N T}{4500} \quad \text{where } T \rightarrow \text{kgm}$$

P → H.P

Here $\frac{P_h}{P_s}$ [Power of hollow]

P_s [" " solid]

$$\frac{\text{Torque}_h}{\text{Torque}_s} = \frac{\frac{\pi p h}{4}}{\frac{\pi p s}{4}} = \frac{\text{Strength}_h}{\text{Strength}_s}$$

$$\frac{W_h}{W_s} = \frac{A_h}{A_s}$$

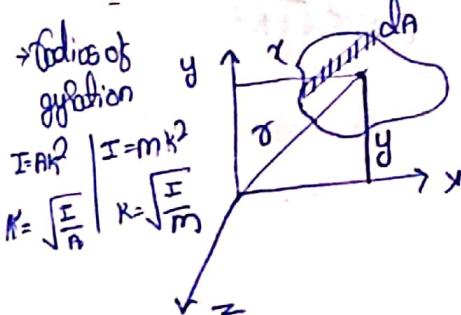
$\Rightarrow 1 \text{hp} = 746 \text{ watts}$

→ Inertia It is the resistance to the linear acceleration

$$F = m a \quad \text{Inertia}$$

→ Moment of inertia It is the resistance to angular acceleration is $T = I \alpha$

→ SECOND MOMENT OF AREA



$$I = \int A^2 \quad I = m x^2$$

$$K = \sqrt{\frac{I}{m}} \quad K = \sqrt{\frac{I}{m}}$$

$$\text{Then } I_{xx} = \int y^2 dA$$

$$I_{yy} = \int x^2 dA$$

$$I_{zz} = \int z^2 dA$$

$$\rightarrow \text{II axis theorem} \Rightarrow I_{AB} = I_G + Ah^2$$

$$\perp \text{axis theorem} \Rightarrow J = I_{zz} = I_{xx} + I_{yy}$$

Solid shaft

Here Design $\frac{I}{J} = \frac{T}{R} \Rightarrow T = J \times \frac{\sigma}{R} = T Z_p \text{ i.e. } T = T Z_p$

W.R.B $Z_p = \frac{\pi d^3}{16}$ then the $T = T \cdot \frac{\pi d^3}{16}$

Now $T = \frac{16T}{\pi d^3}$

Hollow shaft $\rightarrow T = T \times \frac{\pi}{160} [D^4 - d^4] = T \times \frac{\pi D^4}{160} \left[1 - \left(\frac{d}{D}\right)^4\right]$

$$\Rightarrow T = T \times \frac{\pi D^3}{16} [1 - k^4] \quad \left[\because k = \frac{d}{D}\right]$$

$$T = \frac{16T}{\pi D^3 [1-K^4]}$$

→ Composition of shafts

$$\textcircled{1} \quad N_h = N_s \quad | \quad \text{Strength}_h = \frac{Z_p h}{Z_p s}$$

$$A_h = A_s \quad | \quad \text{Strength}_s = \frac{Z_p s}{Z_p h}$$

$$d_h = d_s$$

Then $\frac{\pi}{4}(D^2 - d^2)^2 = \frac{\pi}{4}d_s^2$

$$d_s = (D^2 - d^2)^{1/2}$$

$$d_s^3 = [D^2 - d^2]^{3/2} = [D^2 - d^2] [D^2 - d^2]^{1/2}$$

Then $\frac{Z_p h}{Z_p s} = \frac{\frac{\pi}{160}(D^4 - d^4)}{\frac{\pi}{16}D^3 d_s^3}$

$$\frac{Z_p h}{Z_p s} = \frac{D^4 - d^4}{D^3 d_s^3} = \frac{D^4 - d^4}{D^3 d_s^3 [D^2 - d^2]^{1/2}}$$

$$= \frac{D^4 [1 - K^4]}{D^3 d_s^3 [D^2 - d^2] \sqrt{D^2 - d^2}}$$

$$= \frac{(D^2 - K^2)^2}{(1 - K^2)^2 \sqrt{1 - K^2}}$$

$$\frac{Z_p h}{Z_p s} = \frac{1 + K^2}{\sqrt{1 - K^2}}$$

② here $\text{Strength}_h = \text{Strength}_s$

then $\frac{N_h}{N_s} = \frac{A_h}{A_s} = ?$

then $Z_p h = Z_p s$

$$\frac{\pi}{160}(D^4 - d^4) = \frac{\pi}{16}d_s^3$$

$$d_s = \left[\frac{D^4 - d^4}{D} \right]^{1/3}$$

then $\frac{A_h}{A_s} = \frac{\frac{\pi}{4}(D^2 - d^2)}{\frac{\pi}{4}d_s^2} = \frac{D^2 [1 - K^2]}{D^4 [1 - K^4]^{2/3}} = \frac{1 - K^2}{(1 - K^4)^{2/3}}$

$$\text{then } \frac{A_h}{A_s} = \frac{W_h}{W_s} = \frac{1-K^2}{(1-K^4)^{2/3}}$$

→ Condition

② When $W_h = W_s$ & $A_h = A_s$ then

$$\frac{\text{strength}_h}{\text{strength}_s} = \frac{1+K^2}{\sqrt{1-K^2}}$$

③ $d_s = D$

$$\frac{\text{strength}_h}{\text{strength}_s} = K^4$$

④ $\text{strength}_h = \text{strength}_s$ then $\frac{W_h}{W_s} = \frac{A_h}{A_s} = \frac{1-K^2}{(1-K^4)^{2/3}}$

If $W_h = W_s$ then Hollow shaft \geq Solid shaft

becos  then for hollow dia increases w.r.t. solid diameter.

If $d_s = D$ then $\text{strength}_h \leq \text{strength}_s$

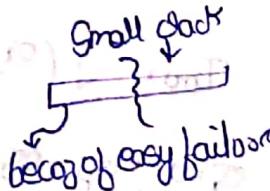
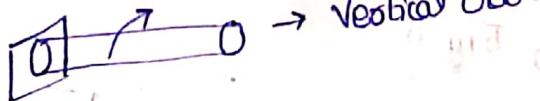
becos 

If $d_s = D$, head $d = 0.5D$ then

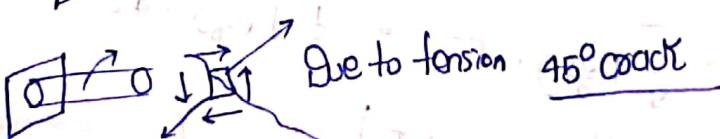
$$\frac{\text{strength}_h}{\text{strength}_s} = 1-K^4 = 1-\frac{1}{16} = \frac{15}{16}$$

⇒ FAILURE CRITERIA

1. For ductile



2. For brittle



DESIGN

For bending

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$M = \sigma_b \cdot \frac{I}{y} = \sigma_b \times \frac{\frac{\pi}{4} \times D^4}{\frac{D^3}{3}} = \frac{\pi D^3}{32} \sigma_b$$

$$\sigma_b = \frac{32M}{\pi D^3}$$

for hollow

$$\sigma_b = \frac{32M}{\pi D^3}$$

for solid

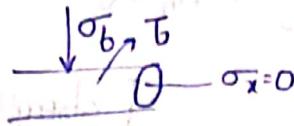
① Torsion

$$T = \frac{16T}{\pi d^3} \text{ and for solid}$$

$$T = \frac{16T}{\pi d^3 (1 - K^4)} \text{ for hollow}$$

→ Combined stress &

② brittle cracking



$$\text{N.R.T } \sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{\sigma_b}{2} + \sqrt{\left[\frac{\sigma_b}{2}\right]^2 + \tau^2}$$

$$= \frac{1}{2} [\sigma_b + \sqrt{\sigma_b^2 + 4\tau^2}]$$

$$= \frac{1}{2} \left[\frac{32M}{\pi d^3} + \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4 \left(\frac{16T}{\pi d^3}\right)^2} \right]^2 \text{ torsional equivalent}$$

$$\sigma_{b\max} = \frac{32}{\pi d^3} \left[\frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right) \right] \text{ mechanical bending equivalent}$$

$$\sigma_{b\max} = \frac{32M_e}{\pi d^3} \quad \text{where } M_e \rightarrow \text{mechanical equivalent}$$

$$M_e = \frac{1}{2} (M + \sqrt{M^2 + T^2})$$

② Guest's Rule

$$T_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

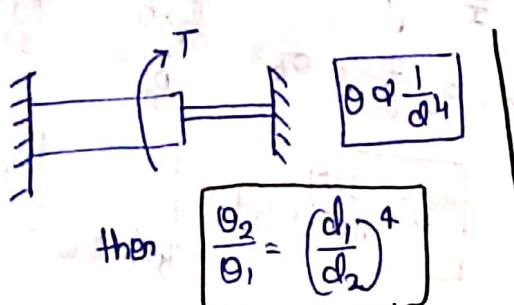
$$\text{Here } \sigma_x = 0, \sigma_y = \sigma_b, \tau_{xy} = \tau$$

$$\text{then } T_{\max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \frac{1}{2} \sqrt{\sigma_b^2 + 4\tau^2}$$

$$T_{\max} = \frac{16T_e}{D^3 \pi} \quad \text{where } T_e \text{ is Torsional equivalent}$$

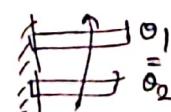
$$T_e = \sqrt{M^2 + T^2}$$

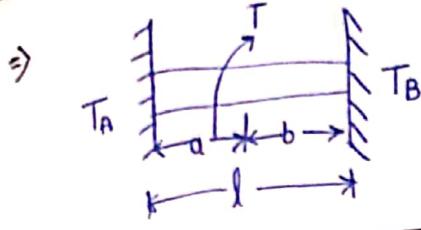
⇒ SHAFTS IN SERIES



⇒ shafts in \propto 1/d^4

$$\theta_1 = \theta_2$$

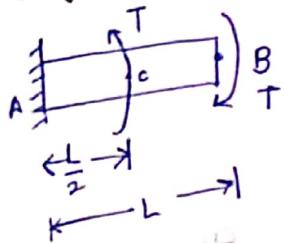




Here $T = T_A + T_B$

$$T_B = \frac{T_a}{\frac{l}{2}} \text{ & } T_a = \frac{T_b}{\frac{b}{2}}$$

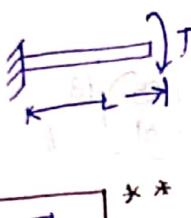
\Rightarrow Angular twist



WKT Angular twist

$$\theta = \frac{TL}{GJ}$$

Here Considering



then $\theta_B - \theta_A = \frac{TL}{2GJ}$

$$\theta_B - \theta_A = \frac{TL}{GJ} \quad \left(\frac{L}{2} - \frac{L}{2} \right) \frac{1}{GJ}$$

$$\theta_B - \theta_A = \frac{TL}{2GJ}$$

WKT Stiffness $= k = w/\theta$

Toosional stiffness $= T/\theta$

Toosional rigidity $= GJ$

then $\frac{T}{\theta} = \frac{GJ}{L}$

$$\frac{T}{\theta} = \frac{GJ}{L}$$

Toosional stiffness

Toosional rigidity

Q) A shaft transmits 50HP at 60 rpm. If max torque is 30% greater than mean torque then, max torque in kg/m^2 is —

$$\text{Sol: } S_0 = \frac{2 \times \pi \times 60 \times T}{4500} \Rightarrow T = \frac{20 \times 60 \times 150}{2 \times \pi \times 60} = \frac{20 \times 150}{\pi} = \frac{3000}{\pi}$$

$$T_{max} = \frac{30}{100} \times \frac{3000}{\pi} = \frac{9000}{\pi} \quad \text{wrong } T_{max} = 1.3 \times \frac{3000}{\pi} = \frac{3900}{\pi} \text{ Nm.}$$

$$28 \text{ Q AEEG} \quad \sigma_{\max} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\text{Now } \sigma_b = \frac{32M}{\pi D^3} = \frac{32 \times 2500 \pi}{\pi \times (10^2)^3} = 80 \text{ MPa}$$

Given $\tau = 30 \text{ MPa}$

$$\begin{aligned} \sigma_{\max} &= \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} \\ \sigma_{\min} &= 40 \pm 50 \Rightarrow 90 \text{ MPa} \text{ and } -10 \text{ MPa} \end{aligned}$$

AEE Q-29) Given $T_h = T_s$

$$f_z z_p h = f_z z_p s$$

$$D_0^3 \frac{\pi}{16} (C_1 - K^4) = \frac{\pi}{16} D^3$$

$$D_0^3 \left(1 - \frac{16}{81}\right) = D^3$$

$$\left(\frac{D}{D_0}\right)^3 = \frac{81-16}{81} = \frac{65}{81} \Rightarrow \frac{D}{D_0} = \left(\frac{65}{81}\right)^{1/3}$$

Q-30 - $\left(\frac{T_s - T_h}{100 T_s} \right) \times 100$

- $U = \frac{P^2 l}{2 A E}$ for this chapter

$$U = \frac{P^2 l}{2 G J}$$

generally $U = \frac{1}{2} \rho a l$

* THEORY OF SIMPLE BENDING

→ ASSUMPTIONS & [Euler-Bernoulli]

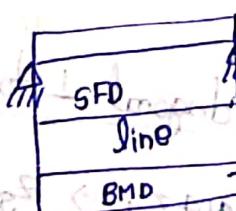
- 1. It was developed by Euler.
- 2. Transverse section of beam which is plane before bending will remain plane after bending.
- 3. It is valid for static load with no residual stress.
- 4. Plane section hypothesis in simple bending states, assumes bending strain is proportional to distance from neutral axis.
- 5. Strain varies linearly along depth.
- 6. Radius of curvature is four times greater than deflection of beam.
- 7. It is valid for steel members.
- 8. It is valid when cross section do not warp [bent / twist].

⇒ BM Eqns

$$\frac{M}{I} = \frac{\sigma_b}{Y} = \frac{E}{R}$$

Bending σ is also known as skin σ

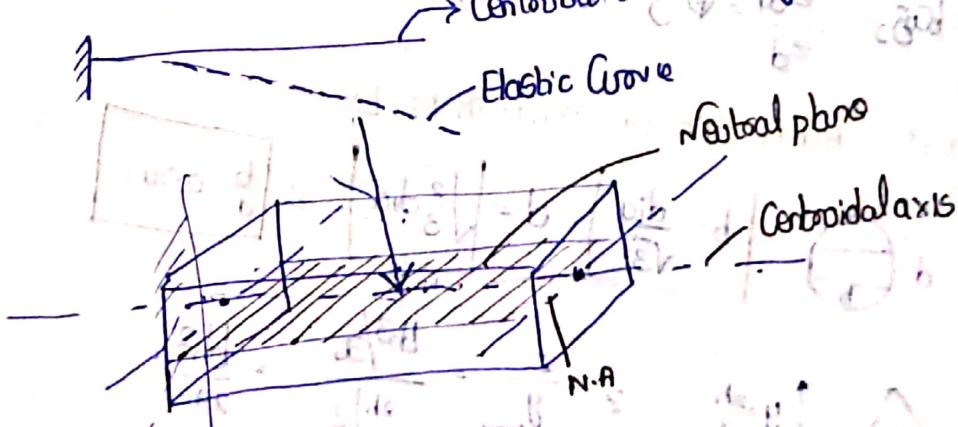
pure bending



pure bending $S.F = 0$

$B.M$ is non-zero constant

Centroidal axis coincides with longitudinal axis.



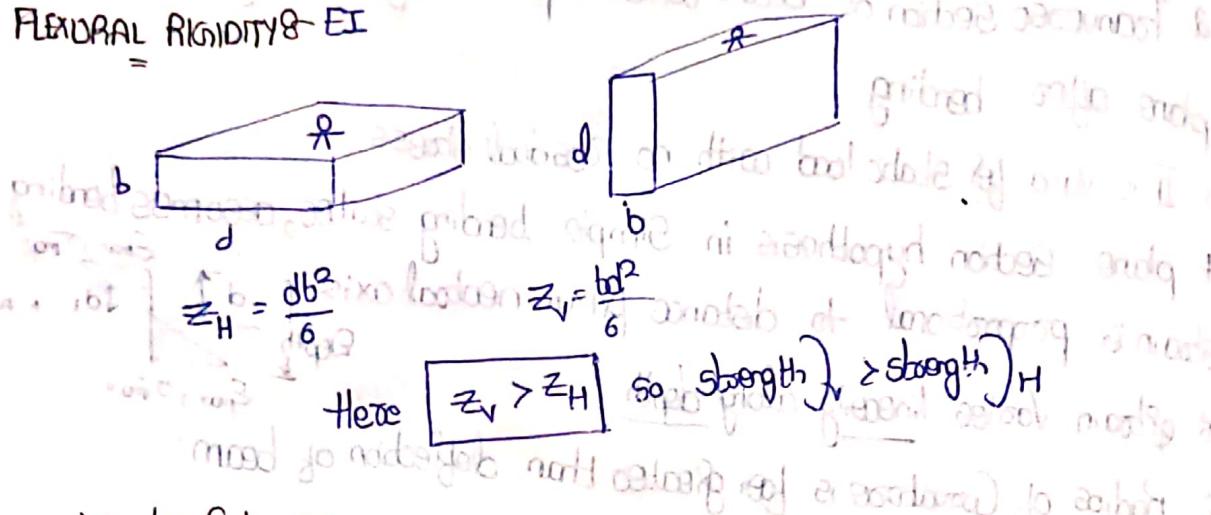
Beam of uniform CS subjected to uniform $B.M$ throughout its length
then deflection shape of beam is Circular arc

\Rightarrow Section modulus [First moment of area]

$$z = \frac{I}{y} \text{ where } y \text{ is the distance from N.A. to outermost layer}$$

Significance $\propto z$ then \uparrow strength in bending.

\Rightarrow FLEXURAL RIGIDITY $\propto EI$



\Rightarrow Z for diff Sections

$$(1) \quad \text{Square} \quad z = \frac{I}{y} = \frac{\frac{a^3}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

$$(2) \quad \text{Diamond} \quad z = \frac{I}{y} = \frac{\frac{a^3}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}}$$

* Square is more strength than Diamond \perp to the base figure

$$\text{but, } \frac{Z_{sq}}{Z_d} = \sqrt{2} \text{ i.e. } Z_{sq} = 1.414 Z_d \Rightarrow Z_{sq} = 41.4 \% \text{ stronger than } Z_d$$

\Rightarrow STRONGEST BEAM CAN BE CUTTED FROM LOG

$$d = \text{dia} \quad b = \frac{\text{dia}}{\sqrt{3}} \quad d = \sqrt{\frac{2}{3}} \text{ dia} \quad \text{or, } \frac{b}{d} = 0.707$$

$$(3) \quad \text{Triangle} \quad z = \frac{I}{y_{max}} = \frac{\frac{bh^3}{36}}{\frac{2h}{3}} = \frac{bh^2}{24}$$

$$(4) \quad \text{Circle} \quad z = \frac{I}{y_{max}} = \frac{\frac{\pi d^3}{32}}{d} = \frac{\pi d^2}{32}$$

(5)

\Rightarrow BEAM OF UNIFORM STRENGTH & Maximum bending stress is same at every section along its longitudinal axis.

$$M = \frac{\text{const}}{Z}$$

for question
for rectangle $M = bd^2$ then $M = b \epsilon I$ Mod^2 then $d \propto \sqrt{m}$ *

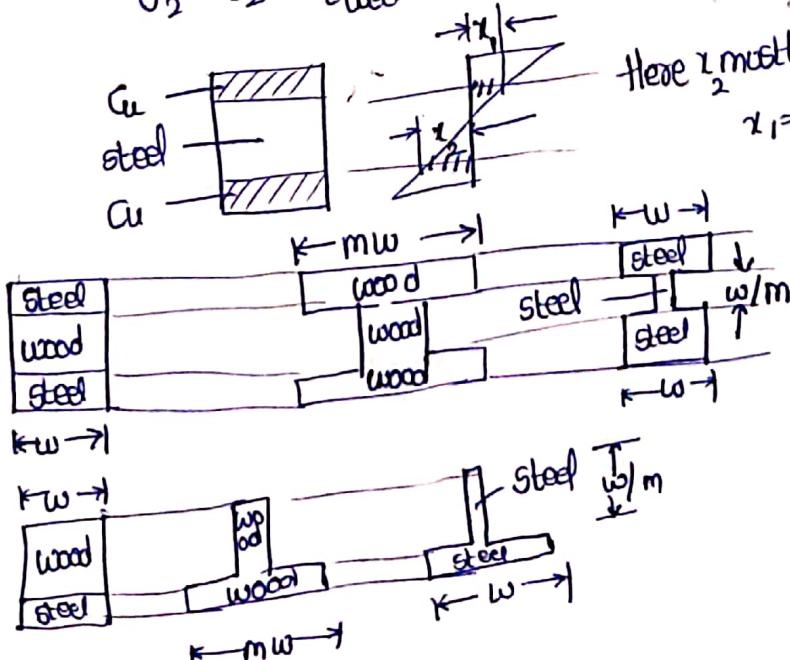
NOTE: The real, prismatic beams are not uniform strength
a. Uniform strength is achieved by pure bending only.

\Rightarrow Equivalent cross methods

(i) Comp members

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} = \frac{E_{\text{strong}}}{E_{\text{weak}}} = \text{modulus ratio.}$$

Here γ_2 must be equal to γ_1 at junctions.
 $\gamma_1 = \gamma_2$



* Non homogeneous \rightarrow Homogeneous

Dimensions alter cons/beg.

$$5AEQ) \quad \begin{array}{c} 1600 \\ \text{---} \\ 15\text{cm} \end{array} \quad \begin{array}{c} 1600 \\ \text{---} \\ 16-2 \end{array} \quad \frac{1600}{15} = \frac{600}{15-1} \\ \text{then } h = 10\text{cm}$$

$$7AEQ) \quad \begin{array}{c} R = \frac{d}{2} \\ \sigma = \frac{E}{R} \end{array} \quad \text{then} \quad \frac{\sigma}{\frac{d}{2}} = \frac{E}{\frac{d}{2}} \quad \sigma = \frac{Ed}{d}$$

8AEQ) bxd of n times \Rightarrow bxd all dimensions double or triple

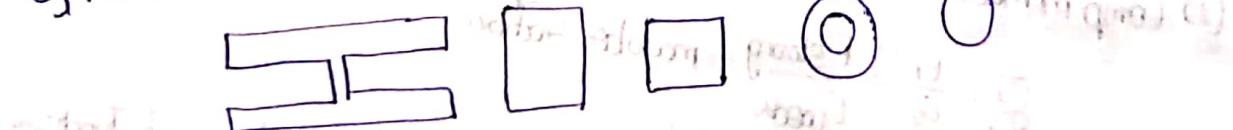
$$\text{then ratio is } \frac{z_1}{z_2} = \frac{nbd^2}{\frac{b^2d^2}{6}} = \frac{1}{n} \text{ II.}$$

$$12AEQ) \quad \frac{M}{I} = \frac{F}{Y} = \frac{E}{R} \quad \text{then } \frac{1}{R} \propto \frac{1}{I}$$

$$\frac{c_1}{c_2} = \frac{I_2}{I_1} = n^2 = 9 \text{ II.}$$

$$28AEQ) \quad \begin{array}{c} b \\ h \end{array} \quad \Rightarrow \frac{h}{2} \quad \begin{array}{c} b \\ b \end{array} \quad \frac{s_2}{s_1} = \frac{z_2}{z_1} = \frac{\frac{2b}{4} \times \frac{1}{6}}{\frac{b}{6}} = \frac{1}{2}$$

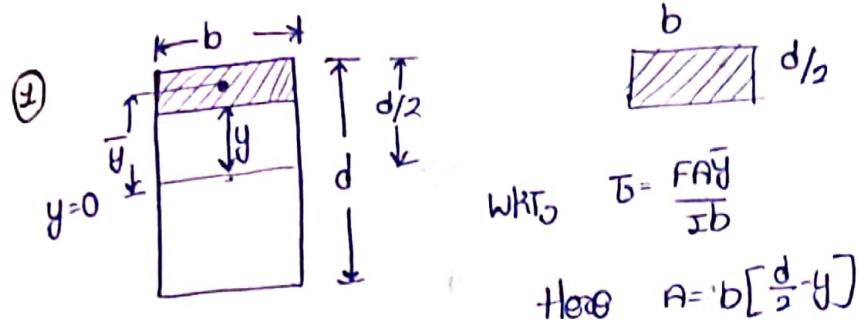
from
44) Neutral it must have maximum area



and that is the answer



⇒ FLEXURAL SHEAR STRESS VARIATIONS



If there is a small vertical cut of thickness \bar{y} at height y from the bottom, then $\bar{y} = \frac{1}{2} \left(\frac{d}{2} + y \right)$

Then eqn becomes $T = F \times b \left[\frac{d}{2} - y \right] \left[\frac{d}{2} + \bar{y} \right] \times \frac{1}{2}$

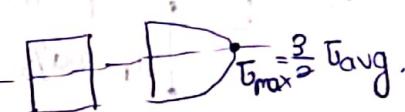
$$T = \frac{6F}{bd^3} \left[\frac{d^2}{4} - y^2 \right]$$

i.e. $T \propto y^2$ → parabola

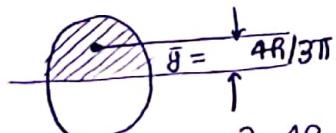
If $y=0$ then $T = \frac{6F}{bd^3} \left[\frac{d^2}{4} \right] \Rightarrow T_{\max} = \frac{3F}{2} \left(\frac{1}{bd} \right) \rightarrow T_{\text{avg}}$

$$T_{\max} \text{ at } y=0 \text{ is } \frac{3}{2} T_{\text{avg}}$$

If $y=d/2$ then $T=0$ then T diagram would be,



② CIRCLES



$$T = \frac{F \times \frac{\pi R^2}{2} \times \frac{4R}{3\pi}}{\frac{\pi R^4}{4} \times 2R} = \frac{4}{3} \frac{F}{\pi R^2}$$

$$T_{\max} = \frac{4}{3} T_{\text{avg}}$$

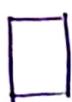
$T \propto \frac{1}{R^2}$ → parabolic



=> SECTION

T_{max}/T_{avg}

T_{NA}/T_{avg}



$$\frac{9}{2}$$

$$\frac{3}{2}$$



$$\frac{9}{3}$$

$$\frac{4}{3}$$



$$\frac{3}{2}$$

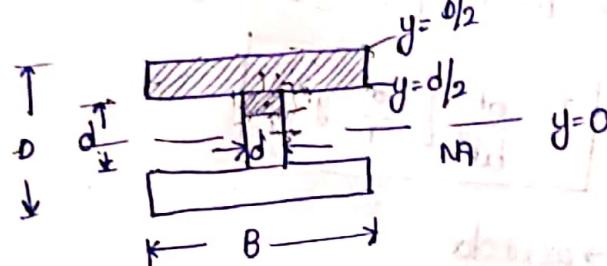
$$\frac{4}{3}$$



$$\frac{9}{8}$$

$$1$$

⑧ I-S E C T I O N S



$$T = \frac{F}{J_b} [A\bar{y}]$$

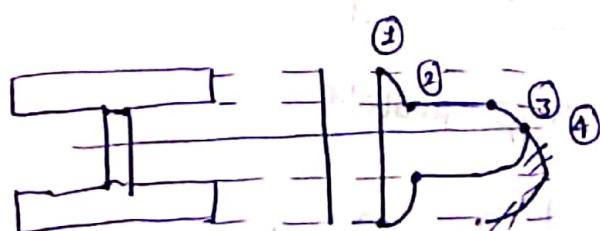
$$\rightarrow \text{Diagram of a rectangular cross-section of width } B \text{ and height } d. \text{ The neutral axis (NA) is at } y_1 = d+d/2. \text{ The area } A = \frac{B}{2} [d^2 - d^2] = \frac{1}{4} B d^2.$$

$$\rightarrow \text{Diagram of a rectangular cross-section of width } B \text{ and height } d. \text{ The neutral axis (NA) is at } y_2 = \frac{1}{2} [\frac{d}{2} + d]. \text{ The area } A_2 = \left(\frac{d}{2} - y\right) b, y_2 = \frac{1}{2} [\frac{d}{2} + d].$$

$$\Rightarrow T = \frac{F}{J_b} [A\bar{y}] = \frac{F}{J_b} [A_1\bar{y}_1 + A_2\bar{y}_2]$$

$$\boxed{T = \frac{F}{J_b} \left[\frac{B}{8} (d^2 - d^2) + \frac{b}{2} \left(\frac{d^2}{4} - y^2 \right) \right]}$$

$$T_{max} = \frac{F}{8J_b} [B(d^2 - d^2) + bd^2]$$



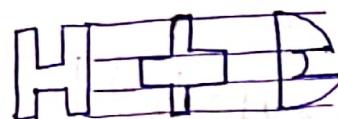
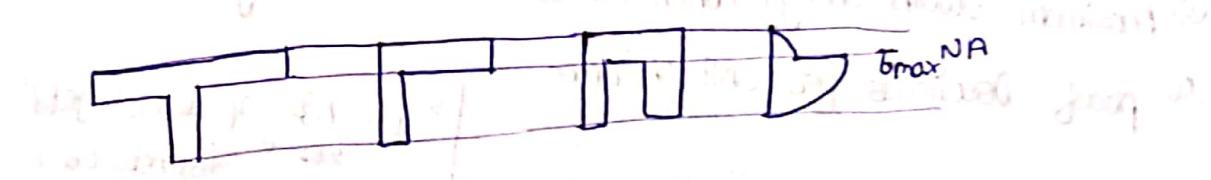
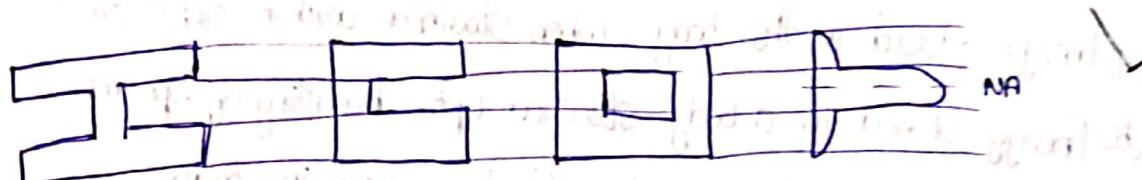
① At top of flange $y = \frac{D}{2}$ then $\bar{\sigma} = 0$

② At End of flange i.e. $B=0$ then $\bar{\sigma} = \frac{F(C^2-d^2)}{8I}$

③ At start of web i.e. $T=0$ then $\bar{\sigma} = \frac{F}{8Jb}[B(C^2+d^2)]$

④ $T_{max} = \frac{F}{8Jb}[B(C^2-d^2)+bd^2]$

$\Rightarrow Z = \frac{FAY}{Ib}$ then $Z \propto \frac{1}{b}$



* In flanged beam web is to resist shear, flange is to resist bending

① In flanged beam web is to resist shear, both I & Box Section are Equally

② For Vertical load without torsion both I & Box Section are Equally

Strengths [with same depth & Cross-sectional area]

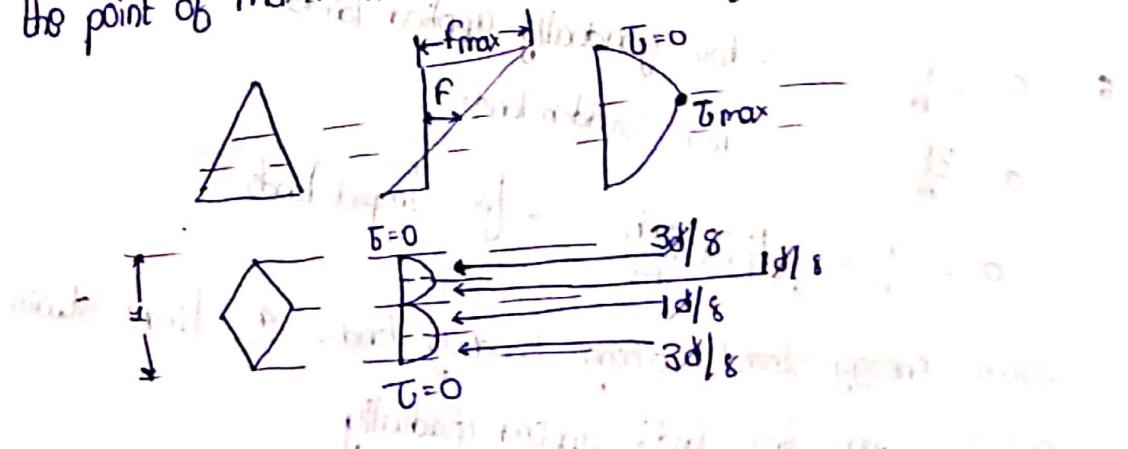
③ If slight torsion is acting then Box Section is better

+ Bending is prime Criteria for design and shear is Secondary.

④ At the point of maximum bending stress, shear stress must be zero

⑤ At the point of maximum shear stress bending need not be zero

⑥ At the point of maximum shear stress bending need not be zero



\Rightarrow STRAIN ENERGY

1. Energy stored in a body within elastic limit is known as "strain energy".

2. Total strain energy stored in a body is known as "Resilience".

3. Maximum strain energy which can be stored in a body is "proof Resilience".

4. Strain energy is the

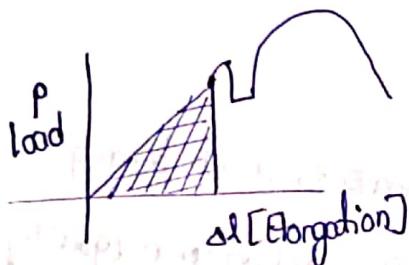
(a) Energy stored in the body when strained within elastic limit.

(b) Energy stored in a body strained upto breaking of Specimen.

(c) Maximum strain energy which can be stored in a body.

(d) proof Resilience per unit Volume.

5.



$$\text{then} \quad \text{strain energy} [U] = \frac{1}{2} P \Delta l$$

$$\& \text{strain energy} = \int_0^{\frac{M^2}{2EI}} d\epsilon$$

$$U = \frac{P^2 A l}{2 A^2 E} = \frac{P^2 A l}{A^2 2 E}$$

$$\& U = \frac{1}{2} \times \sigma \times \epsilon \times V$$

$$U = \frac{P^2}{2 E} \times V$$

$$U = \frac{P^2 l}{2 A E}$$

$$\& \text{for shafts} \quad U = \frac{1}{2} \frac{I L^2}{2 G J}$$

$$U = \frac{1}{2} \times \sigma \times \epsilon \times V$$

$$6. \quad \sigma = \frac{P}{A}$$

\rightarrow for gradually applied loads

$$\sigma = \frac{2P}{A}$$

\rightarrow for sudden loads

$$\sigma = \frac{P}{A} + \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2EP_h}{AL}}$$

\rightarrow for Impact loads

7. Strain energy stored in a body suddenly loaded is $\frac{4}{3}$ times strain energy when same load is applied gradually.

8. Strain energy stored in body when load is applied gradually is $\frac{1}{2} \times E$
stress in sudden load is $\frac{2P}{A}$

⇒ SPRINGS

Springs are elastic members whose function is to store energy when load is applied by temporary deformation and comes back to original position when it is removed.

→ Applications

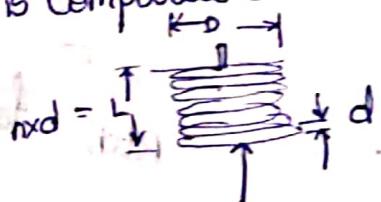
1. To Control vibration & forces
2. To Control motion in Link mechanism.

3. To apply forces
4. To measure forces

Ex: Cam followers, Springs in weight balances, dynamometers, Springs in clocks, watches, Suspension Systems etc.

- Types of springs
 ↳ Helical spring ↳ Conical Spring
 ↳ Torsional spiral spring ↳ Leaf Spring

① Solid length: When the compression spring is compressed until all the coils come in contact with each other



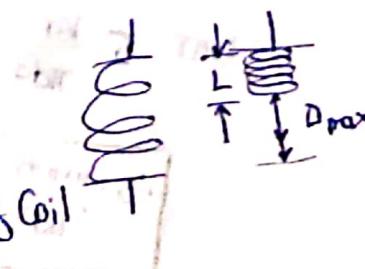
$$L_{solid} = n \cdot d$$

n = no. of Coils, d = dia of wire

D = Dia of Coil

② Free length: It is the length of spring in uncomressed state

$$\text{Free length} = L + \Delta_{max} \\ = n \cdot d + \Delta_{max}$$



* ③ Spring index: Ratio of mean diameter of coil

to diameter of wire

$$c = \frac{D}{d}$$

④ Spring Rate (R) Spring Constant (Co), Stiffness & load acting per unit deflection

$$R = \frac{W}{\Delta}$$

(1) IN SERIES

Two parallel

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

(OR)

$$K_{eqn} = \frac{K_1 K_2}{K_1 + K_2}$$

(2) IN PARALLEL

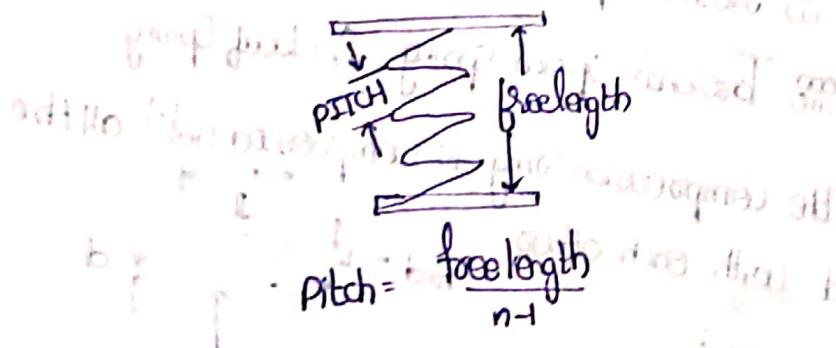
$$K_1 \parallel K_2$$

$$K_{eq} = K_1 + K_2$$

NOTE :-

Closed coil helical spring is cut into two equal parts stiffness of each spring compared to original spring is doubled.
If n equal parts stiffness increases by n times.

⑤ Pitch It is the axial distance b/w coils in uncomressed state



⑥ DESIGN OF SPRINGS

Closed coil helical spring is designed similar to shaft subjected to torque

$$\text{WKT } T = \frac{16T}{\pi d^3}$$

$$T = \frac{16WR}{\pi d^3}$$



⑦ Angle of twist

$$\text{WKT, } \theta = \frac{GJ}{I}$$

$$\Rightarrow \theta = \frac{TL}{GJ}$$

$$\Rightarrow \theta = \frac{WR(2\pi R)}{G \times \frac{\pi}{3} d^4}$$

$$\theta = \frac{64WR^2}{Gd^4}$$

$$\boxed{\theta = \frac{64WR^2n}{Gd^4}}$$



$$\boxed{L = 2\pi R}$$

where n = no. of Coils.

We know that displacement of lines = θ

$$\text{linear velocity} = \theta \omega$$

$$\text{linear Acceleration} = \theta \alpha$$

Then ⑧ DEFLECTIONS

$$S = R\theta$$

$$\Delta = R \left[\frac{64WR^2n}{Gd^4} \right]$$

then

$$\boxed{\Delta = \frac{64WR^3n}{Gd^4}}$$

$$\textcircled{1} \text{ Stiffness } K = \frac{W}{\Delta} = \frac{W}{\frac{64WR^3n}{Gd^4}}$$

$$\boxed{K = \frac{Gd^4}{64R^3n}}$$

\Rightarrow LEAF SPRINGS These springs are used in railway wagons, coaches and road vehicles

- 2. Centre line of all the leafs are initially circular with same radius
- 3. All leafs becomes flat when spring is subjected to maximum load

Called proof load

$$\textcircled{1} \quad \sigma_b = \frac{3WL}{2nbt^2}$$

where b, t are breadth & thickness of plate

$$\textcircled{2} \quad A = \frac{3WL^3}{8nEbt^3}$$

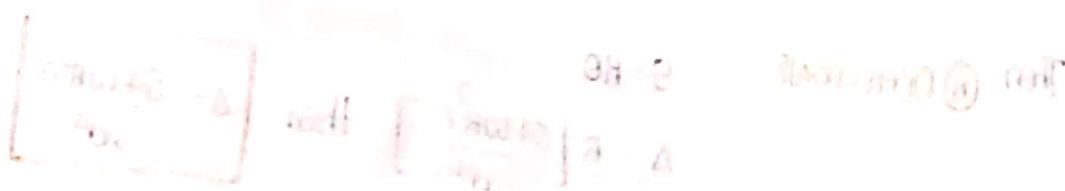
where

POINTS

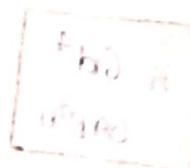
1. Flat spiral spring is used in watches

2. Closed coil helical Spring is subjected to torque about its axis

- 3. Spring wires should experience bending stress.
- 3. Stress induced in a wire of closed coil helical spring subjected to axial loads
 - (a) torsional shear
 - (b) bending stress
 - (c) Transverse direct stress
 - (d) all of the above
- 4. Maximum shear stress occurs on the outermost fibres in closed coil helical spring [Similar to shaft]



maximum shear stress = $\frac{F_{max} D}{2 \pi r^3}$



Maximum shear stress in torsion $\tau = \frac{T r}{I_p}$

If axial load is applied then shear stress will be increased due to increase of bending stress due to axial load.



All other aspects of torsion apply here.

1988
1989

1990 1991 1992

1993 1994 1995

1996 1997

1998 1999

2000 2001 2002

2003 2004 2005

2006 2007 2008

2009 2010 2011

2012 2013 2014

2015 2016 2017

2018 2019 2020

2021 2022 2023

2024 2025 2026

2027 2028 2029

2030 2031 2032

2033 2034 2035

2036 2037 2038

2039 2040 2041

2042 2043 2044

2045 2046 2047

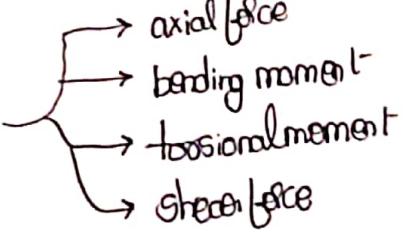
2048 2049 2050

2051 2052 2053

2054 2055 2056

2057 2058 2059

$\Rightarrow \text{SIMPLE} \sigma \& E <$

1. Strain energy for sudden load = $\frac{WL^2}{AE}$ & $\frac{PL^2}{AE} = \underline{P \times \delta L}$
2. For beam carrying u.d.l the strain energy will be maximum in case of Cantilever beam.
3. A linear force deformation relation is obtained in materials having & following Hooke's law
4. The ratio of stresses produced by suddenly applied & gradual load on a bar is $1+1=2$
5. The strain energy produced by suddenly applied & gradual load on a bar is $2+2=4$
6. Tangent modulus of elasticity is the slope of tangent drawn at a point in σ & E curve
7. Strain energy of beam depends upon

8. When a member is subjected to axial tensile loads, the greatest normal σ is equal to $\frac{2 \times G}{\sqrt{3}}$

9. If $P = \text{Total load on Composite member}$
then $P_1 = \frac{PA_1E_1}{A_1E_1 + A_2E_2 + \dots}$

$$P_2 = \frac{PA_2E_2}{A_1E_1 + A_2E_2 + \dots}$$

10. The strain energy stored in a spring when subjected to greatest load without going permanently distorted is called proof load
Dissilience

11. The greatest load which a spring can carry without getting permanently distorted is proof load
12. If the percentage of elongation is $> 5\%$, material is ductile & if it is $< 5\%$, material is brittle
13. In a Composite beam, a couple is produced.
14. Strain energy when subjected to B.M is $\int_0^L \frac{M^2}{2EI} dx = \text{Change in thickness}$
15. In Izod test Specimen is Supported as Cantilever beam & In Charpy test Specimen is Supported as Simply Supported beam
16. Total strain energy upto fracture is "Toughness"
17. Strain energy is inversely proportional to Bulk modulus
18. Spring Constant is the Ratio of load per unit deflection
19. Beams Subjected to transverse force
20. The yield moment of a Gross Section is defined as the moment that will just produce the yield stress at outermost fibers of section
21. If the stress in each Gross Section of pillar is equal to its working stress it is called body of equal strength
22. The stress along the Contact Surface of Net a member is bearing
23. In Brinell hardness test the type of indenter is hard steel ball
24. In Compression test the fracture in Cast iron would occur along oblique 45° plane
25. $T = \frac{\sigma_{\text{Normal}}}{2} \times \left[\frac{\delta}{2} = \epsilon \right] \rightarrow \text{Ratio of shear strain} = \text{Normal } E$
26. Strain energy = Just storing work done energy
Resilience = storing work done ^{re} & releasing upon unloading
27. A cylindrical bar of volume V and is subjected to a tensile force in longitudinal direction. If μ is μ and longitudinal E then final volume of bar is $\underline{(1+\mu)(1-\mu) V}$

28) If Young's modulus of elasticity is determined for mild steel in tension & compression, the two values will have $E_t/E_c = 1$

29) A rubber band is elongated to double its initial length, its strain is 0.693

30) If modulus of elasticity is zero then material is plastic

31) Compressibility is Reciprocal of K

32) For rectangular C section from Crookes Section then $\frac{b}{d} = 0.707$

33) In Creep test, Specimen is subjected to Uniaxial tension

34) A tapering bar [dia of end sections being d_1 & d_2] & bar of uniform Gross Section & have the same length and are subjected to same axial pull. Both the bars will have the same extension if $d = \sqrt{d_1 d_2}$

35) For elastic material $E = k$

36) Young's modulus do not change with increase in Carbon Content

37) If all the dimensions of a bar are increased in the proportion n:1 then the proportion with which the Max stress produced in prismatic bar by own weight will increase in Tension n:1

38) True stress σ is related with Engg stress (σ_0) as $\frac{\sigma}{\sigma_0} = \frac{1}{1+E}$

39) Strain energy is due to changes in strain

40) Strain energy is always a two scalar quantity

41) Strain energy is the same whether it is Compression or Tension.

42) Strain energy is the same whether it is Compression or Tension.

43) As per elastic theory, $F_{cs} = \frac{Yield\sigma}{Working\sigma}$

44) The stress along the Contact Surface of a Rivet and the members is bearing stress

45) The actual breaking stress of a material is greater than the ultimate strength

⑯ For the materials which do not have a yield point from a tension test, yield point is defined as the stress at which permanent set reaches the value 0.2%.

⑰ The lower yield point is more significant than upper yield point, bcz it is less influenced by shape of Specimen

⑱ The brittleness test is done on brittle material & Crack is 45° w.r.t. in oblique plane
↓
Compression

⑲ The Assumption that the Gross Section plane before bending remain plane after bending means the strain in the fibres are proportional to their distances from neutral axis.

⑳ The Assumption that material is homogeneous and isotropic and has same values of E in ten & Comp implies that stress & strain in all fibres

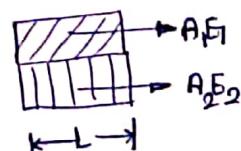
㉑ Stress necessary to cause a non proportional G.O. Perimeter Extension equal to a fixed percentage of gauge length is proof stress.

㉒ $\frac{\Delta L}{L} = \text{Axial stiffness of bar}$

㉓ $\frac{\Delta L}{L} = \text{flexural stiffness}$

㉔ Equivalent young's modulus of parallel Composite member is

$$\text{Equivalent } = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}$$



and the app will be able to read it by using the `get()` method.

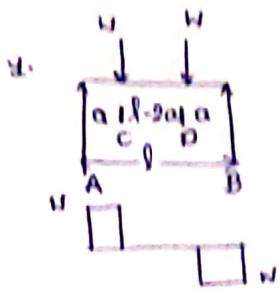
and the author of the original work.

soil microflora and soil fauna) and *biochemical factors* (e.g. organic matter).

It is a good idea to have your doctor prescribe a daily multivitamin supplement.

the first time I have ever seen a bird

\Rightarrow SFD & BMD



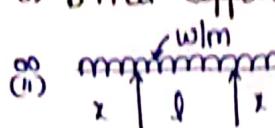
$$SF_{max} \text{ at } A \& B = W \cdot a - W$$

$$B.M_{max} \text{ at } C \& D = W \cdot a^2 / 4$$



Q. In overhanging beams :-

i) B.M at Support is always -ve.

(ii)  $B.M_{max} \text{ at Centre} = +\frac{w(z^2 - 4z^2)}{2}$ at midspan

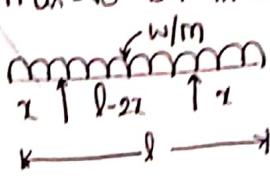
$B.M_{max} \text{ at Support} = -\frac{wz^2}{2}$

Support moment is always -ve P.O.C is $z = \frac{l}{2} - \frac{\sqrt{l^2 - 4a^2}}{2}$ from Supports

(iii) Max tve B.M equal to M_{b1} -ve B.M in below G.S.

when $z = 0.207l \rightarrow$

$$z = \frac{l}{2\sqrt{2}}$$



3) B.M is max where SF is zero & changes its sign.

4. $\frac{dF}{dz} = -N$ & $\left(\frac{dM}{dz}\right) = F$, where F = shear force
↓
slope of BM Graph

5. If SF is Constant b/w two sections, B.M varies linearly

6. If SF is zero b/w two sections, B.M Constant b/w Sections

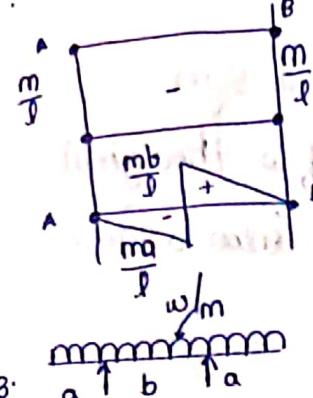
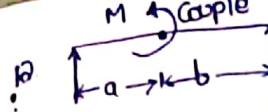
7. If BM is zero, then SF is zero

8. In a Cantilever the BM at any section of beam is equals to the area of SFD b/w the free end and the section.

9. The shear force on a beam is proportional to Coordinate of the axis

10. change in B.M is the area of SFD

11. change in SF is the area of loading



* Rectangular S.F.D

* Both the +ve BM are max at point of application of the Couple.

13. Thus BM at mid span is $\frac{M}{b} \cdot \frac{a}{2}$ & $a = \frac{b}{2}$

14. The expression for the elastic curve at free end of Cantilever beam when subjected to udl is $WL^4/8EI$

15. → P.O.C is at $\frac{L}{2\sqrt{3}}$ from Centre.

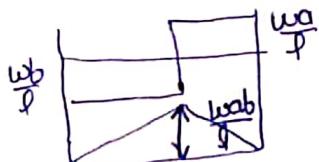
16. then SF at P.O.C is $\frac{M}{L}$

17. In a rigid jointed frame the joints considered as to rotate only as a whole

$$18. \left. \begin{array}{c} M \\ \hline A \quad B \end{array} \right) \Rightarrow S_B = \frac{M L^2}{2EI}$$

19. $M = \left(\frac{wl}{2} \cdot r - \frac{w^2}{2} \right)$ } general expression for udl on beam

20. → In ss beam under load SF is zero



21. When SF changes its sign, then BM is max

22. In BM at any section = Area of S.F.D below free end Section x y

23. SF & Co-ordinate of axis is $\frac{1}{R}$

24. → BM occurs at Support

- & → BM occurs under load

8. BMD \rightarrow  } ss beam & moment applied at
the right end.

Q6. The slope of BM changes its sign when SF changes its sign

Q7. A Simply Supported beam AB carries udl of w throughout the span. what Concentrated load should be applied at Center to cause same b.m. as udl is $\frac{wL^2}{9}$

Same b.m. as udl is $\frac{wL^2}{9}$ ~~using the same condition~~

Now we have to find the value of w which gives all signs of slopes in

order to get the same b.m. as udl. ~~using the same condition~~

Let's consider two cases between the

center and the left end of the beam.

Case 1: Slope is zero at center and

increasing with bending moment at

left end and decreasing with bending moment at right end.

$$\frac{M}{E} \propto \frac{\theta}{\delta}$$

Then we can say that if the slopes are zero at center and increasing with bending moment at left end and decreasing with bending moment at right end.

Case 2: Slope is zero at left end and increasing with bending moment at

right end and decreasing with bending moment at center.

Then we can say that if the slopes are zero at left end and increasing with bending moment at right end and decreasing with bending moment at center.

Case 3: Slope is zero at right end and increasing with bending moment at

left end and decreasing with bending moment at center.

Then we can say that if the slopes are zero at right end and increasing with bending moment at left end and decreasing with bending moment at center.

Case 4: Slope is zero at center and increasing with bending moment at

both ends and decreasing with bending moment at both ends.

⇒ BENDING STRESSES <

1. Due to bending moments, bending stresses co-developed
2. When a beam is subjected to constant bending moment, then it is uniform strength of beam then $\boxed{d\sigma/dm}$
3. When a beam is subjected to pure bending assumes a shape of arc of Circle
4. Two beams one of Circular Section and the other a Square Section of same length same strength the ratio of length same strength the ratio of weights of Circular Section to Square is 1.118
5. If two beams of Circular Section and the others Square Section have equal area of Gross Section If Subjected to bending then Square Section is more economical than Circular Section.
6. The Ratio of flexural strength of Square Section with its two sides horizontal to its diagonal horizontal is $\sqrt{2}$
7. The intensity of direct longitudinal stress in the Gross Section at any point distant r from neutral axis, it is proportional to r
8. The Curvature at any point ($\frac{1}{R}$) along the Curve representing the deflected shape of a beam is given by $\pm \frac{dy}{dx^2} = \frac{M}{EI}$, hence denominator has negligible value, hence $\frac{1}{R} = \frac{M}{EI} = \frac{dy}{dx^2} \pm \frac{\pm \frac{dy}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$
9. Moment of Resistance = Strength αz
10. From $\frac{M}{I} = \frac{F}{y} = \frac{E}{R}$ has $\boxed{F = \frac{My}{R}}$, hence outermost fibres are having max bending σ .
11. If Two beam of equal Gross Section area & Subjected to equal bending moment then Square Section is stronger than Circle

a most efficient & economical section of beam is I-Section to Resist bending

13. The Ratio of length and diameter of a Simply Supported uniform Circular beam which experiences max bending or equal to tensile or due to same load at mid span is $\frac{1}{2}$

14. The bending stress at the outer fibres is known as "principle of"

15. Keeping the breadth as Constant, depth of a Cantilever length 'l' of uniform strength loaded w.d.l is $\sqrt{\frac{3w}{\sigma_b}} \times l$ at fixed end

16. Keeping the depth 'D' Constant the width of Cantilever length 'l' of uniform strength loaded with w.d.l is $\frac{3w}{\sigma_b^2} \times D^2$ at fixed end

17. The Ratio of length and depth of a ss beam which experiences max bending stress equal to tensile or due to same load at mid span is

$\frac{2}{3}$

18. The Ratio of moment of inertia about the neutral axis to the distance of the most distant point of section from neutral axis is Section modulus

19. $F = \frac{dM}{dx}$ rather than $F = \frac{M}{Z}$

20. The permissible stress in fillet welds used during erection is deduced by soil.

21. The slenderness ratio of facing bars should not be exceed 145

22. The load carrying capacity is primarily governed by Slenderness Ratio

23. The loss in prestress in the tendons due to Creep in Concrete is given as 5 to 8%

24. Twisting moment is applied in plane of c.s about $\frac{z^2}{8}$ axis

25. In the Case of thin Cylinders, the stress distribution across the thickness of wall is uniform

- (18) The spring used in Spring balances is close coiled helical spring
(19) The stiffness factor for a beam fixed at one end & free at other is $\frac{3EI}{L}$

(20) Thick cylinders are analysed based on Poisson's theory

(21) for ductile materials, the most appropriate failure theory is
Maximum shear stress theory

(22) The maximum shear stress in a thin tube is half of Avg σ [shear stress]

Reason: variation of stress will remain uniform throughout the length of the tube. The outer boundary will experience maximum shear stress.

Reason: deformation is proportional to shear stress and the maximum shear stress is constant at the outermost point of the tube.

Reason: At equilibrium state, shear stress is linear function of shear strain. If shear stress is zero, shear strain is also zero.

Reason: given two other failure theories, Maximum shear stress theory is more conservative than the others.

Reason: maximum shear stress theory is more conservative than the others because it provides minimum safety factor among all the theories.

Reason: maximum shear stress theory is more conservative than the others because it provides minimum safety factor among all the theories.

an infant brought about by egg and hatching of the
embryo developed which was

absorbed by the mother and passed on to the next
egg plus other remains of the embryo.

return of part of food will have been carried out in
either addition of eggs different to those eaten and
thus & more material left over after

egg eat body leaving

body with no egg left over

digestion still leaving with no egg mass or body left

body with no egg left over

body with no egg left over

had all digested & dead protein used for new
material and body is reduced to nothing

with remains left all period of hatching & metabolism

now to prove that this is so with protein made
from a mixture of protein and a mixture of protein

body body body
body body body

\Rightarrow SHEAR STRESSES

↓ If the vertical shear force given rise to horizontal shear force. Then it is Called Complementary shear

a. $T = \frac{FAY}{Ib}$ & Horizontal shear = $T_h = \frac{FAY}{I}$ per unit width

b. Shear is Carried by web & bending is Carried by flange.

c. The max stress is 50% more than T_{avg} for Rectangular Section

d. Then max T is 33.33% more than T_{avg} for Circular Section

e. For Hollow Circular Section $a_{max} = T_{max} = 2 T_{avg}$

f) T is Max at Centroid

g. for \square^{100} T at y^2 distance from N.A is $\frac{6F}{bd^3} \left[\frac{d^2}{4} - y^2 \right]$

h. The variation of shear stress on a plane parallel to the neutral plane in a beam is Carried by u.d.l is Linear

i. For O^{100} T at y^2 from N.A is $\frac{16}{3} \frac{F}{bd^4} \times \frac{1}{\pi} \left[d^2 - 4y^2 \right] = \frac{16}{3} \frac{F}{Td^4} (d^2 - 4y^2)$

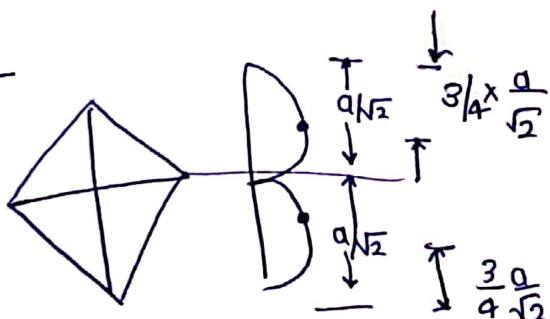
j. A beam of Δ^{100} Gross Section of base b & height h . The T will be Max at distance of $\frac{h}{2}$ from bottom

k. When a beam is Subjected to bending the most economical section is

Square

l. A beam of Square Section of area a^2 is held Such that one of its diameters is Vertical. The Maximum T will develop at a distance h where

h is $\frac{3}{4} \times \frac{a}{\sqrt{2}}$



most of section is Goolar, Sque, Yedanguba, & the rest is at middepth

With regard to the question of the nature of the "mysteries" of the Greeks, we have

and the next day we were able to get the first 1000000 units.

Impact of the operation

It was a huge success.

We had to add another 1000000 units of capacity by the end of the year.

It was a great success.

We had to add another 1000000 units of capacity by the end of the year.

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⇒ DEFLECTION OF BEAMS <=

1) The curved shape of the axis is called elastic curve (or) elastic axis.
 Curved shape of deflection points.

2) Beams of uniform strength $\Rightarrow b = \frac{6M}{Fd^2}$

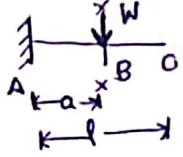
3) $\Delta l = \frac{l^2}{8R}$ at centre I_B , beam of length l bending into circular arc.

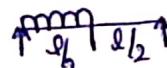
$$2 \Theta_A = \frac{Ml}{2EI} \quad 2 \Theta_B = -\frac{Ml}{2EI} \quad \& \quad \Delta_{max} = y_{max} = \frac{Ml^2}{8EI} = \frac{M^2}{M_A + M_B}$$

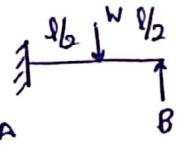
4) $-CM = EI \frac{d^2y}{dx^2}$

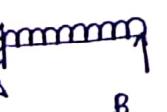
then $\int M = EI \int \left(\frac{d^2y}{dx^2} \right) \Rightarrow \int M = EI \frac{dy}{dx} \} \text{slope}$

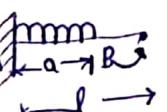
$\int \int M = EI \int \int \frac{dy}{dx^2} \Rightarrow \int M = EI \cdot y \} \text{deflection.}$

5)  $\Delta_B = \frac{w a^3}{2EI} + \frac{w a^2}{2EI} [l-a]$

6)  then Symmetric Centre = $\frac{1}{2} \left(\frac{5}{384} \frac{wl^4}{EI} \right) = \frac{wl^4}{EI} \times \frac{5}{768}$

7)  $R_B = \frac{35}{16} \cdot w$
P.O.C = $\frac{3}{11} l$ from A [fixed end]

8)  P.O.C = $\frac{3}{4} l$ from free end. (or) $\frac{l}{4}$ from fixed end.

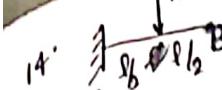
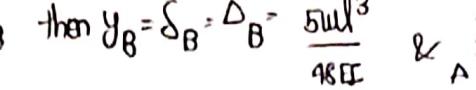
9)  then $\Delta_{B max} = \frac{w a^4}{8EI} + \frac{w a^3}{6EI} [l-a]$

10. The differentiation of elastic lines is B.M

11. The Second differentiation of elastic line is load

12. $\frac{d\theta}{dx} = SF$

13)  then $\theta_B = \frac{wl^3}{24EI}$ & $\Delta_B = \frac{wl^4}{30EI}$

14.  then $y_B = \delta_B = \Delta_B = \frac{5wl^3}{48EI}$ &  then $y_C = \delta_C = \frac{7wl^4}{384EI}$

15. The ratio of the deflection of the free end of a Cantilever is due to an isolated load at $\frac{1}{3}$ rd & $\frac{2}{3}$ rd of Span is $\frac{2}{7}$

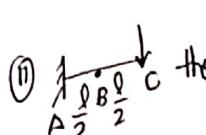
⑥ In ss beam \rightarrow Support-slope max & under load- $\Delta y_{max} \& \theta = 0$

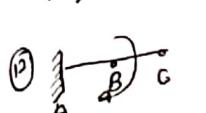
In Cantilever \rightarrow free end- $\Delta y_{max} \& \theta_{max}$

fixed beam \rightarrow At fixed end -slope & $\Delta y = 0$

\rightarrow At pos - slope is max

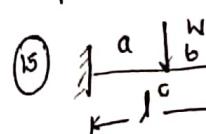
\rightarrow Under load- Δy_{max}

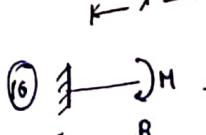
⑦  then $y_B = \delta_B = \frac{5wl^3}{48EI}$

⑧  then $y_C = \delta_C = \frac{3ML^2}{2EI}$

⑨ A ss beam, Then $\frac{\Delta y_{max}}{F_{bending}} = \frac{l^2}{6Ed}$

⑩ Slope and deflection of beam varying flexural Rigidity may be computed by Conjugate mtd & Plane stress method also.

⑪  then $y_0 = \frac{Wa^2b^2}{3EIl^3}$

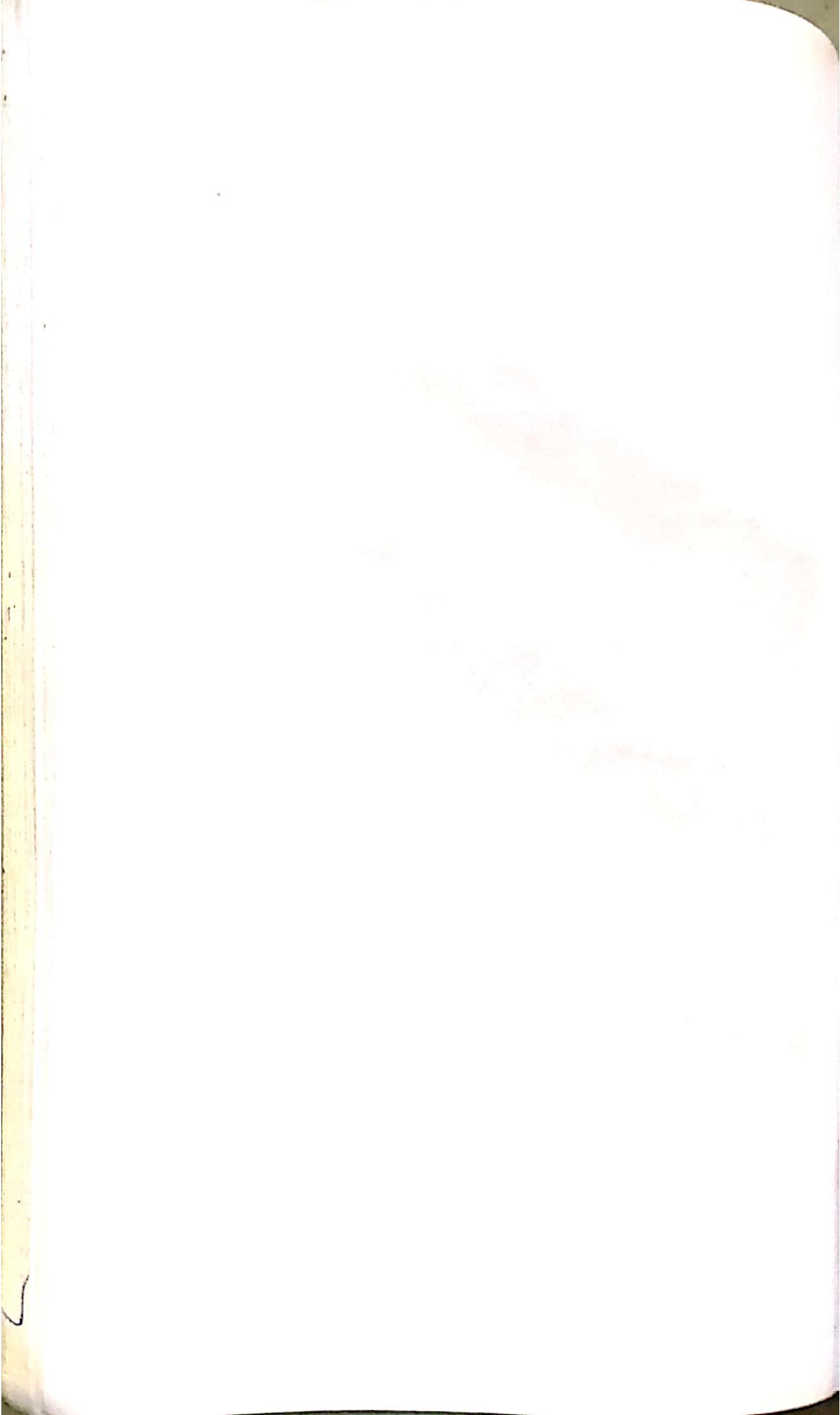
⑫  then $\theta_B = \frac{Ml}{EI}$ & $\Delta y_{Bmax} = \frac{Ml^2}{2EI}$

⑬ The deflection in any beam as per Castiglione's theorem is $\int \frac{\eta^2}{2EI} d\gamma$

⑭ When $\theta=0$ i.e y is max

Monotonia formosa ist ja relativ oft zu sehen
in Gräsern die weit voneinander entfernt
oder einzeln stehen. Einige Formen sind sehr
einfach und ohne besondere Farbe oder Ausdehnung
aber andere haben eine auffällige Farbe oder Ausdehnung
oder eine auffällige Form oder ein
sonstiges Merkmal.

Die Formen sind nach Größe
in drei Gruppen unterteilt:
1. Die kleinen Formen, die nur
etwa 10 cm hoch werden.
2. Die mittleren Formen, die
etwa 20 cm hoch werden.
3. Die großen Formen, die
etwa 30 cm hoch werden.
Die kleinen Formen sind meistens
zweigig verzweigt und haben
eine einfache Form, während die
großen Formen meistens
eine komplizierte Form haben.
Die mittleren Formen sind
meistens einfach verzweigt und
haben eine einfache Form, während die
großen Formen meistens
eine komplizierte Form haben.



$\Rightarrow \underline{\text{TORSION}}$

1. The ratio of strain energy stored by a hollow shaft of external dia d and internal dia d' to that strain energy stored by a solid shaft of dia d per unit vol is $\frac{d^3 + d'^3}{d^3}$

2. Torque Required to produce unit angle of twist per unit length is

Torsional Rigidity

3. Torsional strength of a shaft is proportional to $\frac{J}{R}$ = polar section modulus

4. ~~A~~ A Solid Circular shaft has been subjected to pure torsion moment

The ratio of max shear to max normal σ is $1:1$

5. When a rectangular bar is subjected to torsion, max τ will occur at middle of larger side

6. ~~A~~ A Circular shaft is subjected to torque T which is half of the bending moment applied, then the ratio of Max bending stress & Max shear stress is 2 } happens uniformly

7. The statement that plane Section before twisting will be plane even after twisting is applicable for only hollow & Circular Sections

8. A shaft is subjected to a bending moment M & torque T simultaneously.

The ratio of Max bending stress to Max shear stress is $\frac{2M}{T}$

9. The strength of the shaft = Maximum torque = power transmitted by shaft

10. The ratio of max shear stresses developed in a solid shaft of dia d' & a hollow shaft of External dia b' & internal dia

of same torque is $\frac{d'^4 - d^4}{d'^4}$

⑪ The Ratio of moment of resistance of solid to hollow shaft is $\frac{D^4}{D^4 - d^4}$

⑫ The Ratio of $\frac{J}{R} =$ polar section modulus \propto Torsion Section modulus

⑬ $P = \frac{2\pi NT}{60}$ Watt & $P = \frac{2\pi NT}{60,000}$ KW

Here N = Revolutions per minute

⑭ $P = TXW$ $\rightarrow \omega = \text{Angular Speed in rad/sec} = \frac{2\pi N}{60}$

After returning, I took the following notes
about what was observed during my stay.

On the 1st day of May, I saw the first pair of
nesting birds.

On the 2nd day of May,

I saw the first pair of nestlings with
feathers about ready to fly.

On the 3rd day of May, I saw the first pair of nestlings
with feathers about ready to fly.

On the 4th day of May, I saw the first pair of nestlings
with feathers about ready to fly.

On the 5th day of May, I saw the first pair of nestlings
with feathers about ready to fly.

On the 6th day of May, I saw the first pair of nestlings
with feathers about ready to fly.

On the 7th day of May, I saw the first pair of nestlings
with feathers about ready to fly.

On the 8th day of May, I saw the first pair of nestlings
with feathers about ready to fly.

⇒ COLUMN & STABILITY

∴ Rankine's formula: $P_0 = \frac{F_c A}{1 + \alpha \lambda^2}$
where $\alpha = \frac{f_c}{\pi^2 E}$

2. Rankine's formula for eccentric columns

$$P = \frac{F}{\left(1 + \frac{ey_0}{R^2}\right)}$$

3. Secant formulae for eccentric columns

$$F_{max} = P_0 \left[1 + \frac{ey_0}{R^2} \sec \frac{1}{2} \sqrt{\frac{P}{EI}} \right]$$

① The basis of ISI Code formula is Secant formulae

⑤ Euler's formula is not valid when $\lambda \leq 80$

⑥ The Critical length of Column corresponds to neutral equilibrium.

⑦ Rankine's Constant for timber = $\frac{1}{160}$

⑧ For max strength of a Column $I_{xx} = I_{yy}$

⑨ If $\lambda \leq 80 \rightarrow$ Short columns

$80 \leq \lambda \leq 120 \rightarrow$ Medium Columns

$120 \geq \lambda \rightarrow$ Long Columns

→ DAMS & RETAINING WALLS ←

① For walls, the angle of repose is "0".

② To avoid tension at base 8-

$$\text{i) for } \Delta^{\text{less}} \quad b = \frac{H}{\sqrt{3}}$$

$$\text{ii) for } \square^{\text{less}} \quad b = \frac{H}{\sqrt{5}}$$

$$\text{iii) for } \triangle^{\text{less}} \quad ab + b^2 = a^2 + \frac{H^2}{9}$$

③ For No sliding Condition 8-

$$\text{i) for } \Delta^{\text{less}} \quad b = \frac{H}{\mu s}$$

$$\text{ii) for } \square^{\text{less}} \quad b = \frac{H}{2\mu s} + H\alpha$$

$$\text{iii) for } \triangle^{\text{less}} \quad b = \frac{H}{\mu s} - a$$

μs is sp. gravity of dam material

④ The plane of repose is inclined at $(\frac{\pi}{4} + \frac{\phi}{2})$ with horizontal

⑤ As per Rankine's theory, the inclination of the resultant stress with the normal to the plane $> \phi$

$$\text{⑥ As per Rankine's theory} \quad \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} < \sin \phi$$

$$\text{⑦ min. depth of foundation} \Rightarrow h = \frac{P}{A\gamma_s} \left[\frac{1 - \sin \phi}{1 + \sin \phi} \right]^2$$

P = Total load on wall

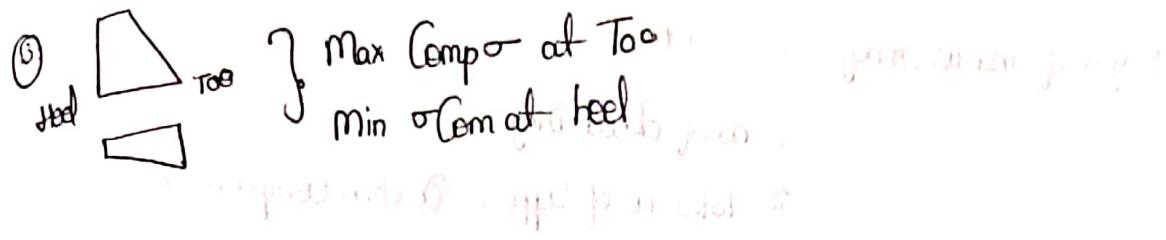
⑧ In Coulomb's graphical method the total active earth pressure is equal to the active pressure Δ^{less} multiplied by Specific wt of Earth

⑨ For Cohesive soils, $\phi = 0^\circ$

⑩ For Sand $\phi = 30^\circ$

⑪ The total active earth pressure for a retaining wall subjected to surcharge is $\frac{1}{2} k_a w e^{H^2} + k_b w H$

① Resultant force = $\sqrt{\text{pressure}^2 + \text{wt of dam}^2}$.



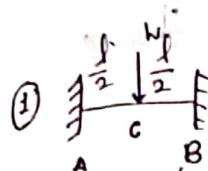
=> FRAMES <=

① Degree of indecomposability = $D_S = 3c\sigma - 3$

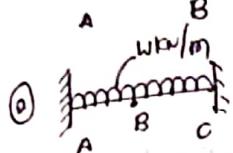
c = no. of closed rings

σ = Total no. of Support Decision Components

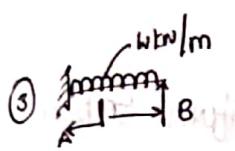
\Rightarrow AXED, CONTINUOUS & PROPPED CANTILEVER



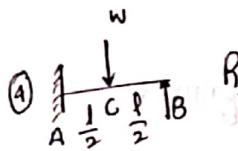
then Max BM at A, = $\frac{wL^2}{8}$ | p.o.c at $\frac{L}{4}$ from supports (fixed support)



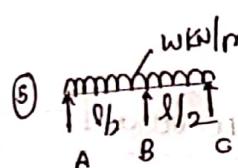
Max BM_{ABC} = $\frac{wL^2}{12}$ & BM_C = $\frac{wL^2}{24}$ | p.o.c at $0.25L$ from supports & $\frac{L}{2\sqrt{3}}$ from Centre



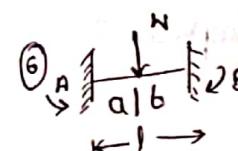
Max BM occurs @ Centre | p.o.c is $\frac{l}{4}$ from fixed end & $\frac{3l}{4}$ from free end



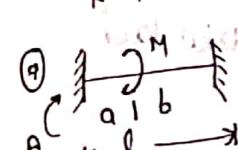
Max BM Occurs at $\frac{3}{8}l$ from | p.o.c is $\frac{3L}{11}$ from fixed end & $\frac{2}{11}L$ from free end



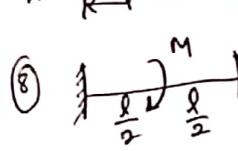
$R_B = \frac{5wL}{8}$ & $M_B = \frac{5wL^2}{32}$ [Hogging]



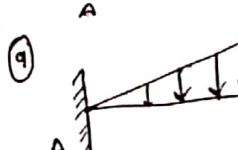
$M_{AB} = \frac{wab^2}{l^2}$ & $M_{BA} = \frac{w8b^3 + 4ab^3}{l^2}$ (due to lateral deflection about axis B)



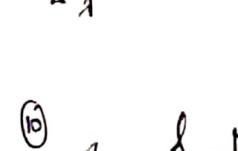
$M_{AB} = \frac{Mb}{l^2} (2a+b)$ & $M_{BA} = \frac{Ma}{l^2} (2b-a)$



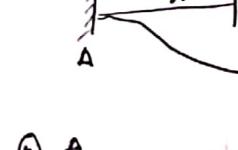
then $M_{AB} = M_{BA} = \frac{M}{4}$



$M_{AB} = -\frac{wl^2}{30}$



$M_{BA} = \frac{wl^2}{30}$



$M_{AB} = \frac{w(l-b)^2}{l^2}$



$M_{BA} = \frac{3w(l-b)^2}{l^2}$

Right component must vanish to zero

→ OTHERS ←

④ The effect of arching a beam is to reduce bending moment throughout

⑤ Cylinders thin - $\frac{Pd}{t} < 20$

thick - $\frac{Pd}{t} > 30$

⑥ Circumferential (C) hoop stress = $\frac{Pd}{2t}$ } P is internal fluid pressure

⑦ Longitudinal stress = $\frac{Pd}{4t}$ } Causing tensile σ

⑧ Beams of uniform strength are preferred to those of uniform section because these are economical for large spans

⑨ Longitudinal cracks observed in timber beams are due to shear failure below layers

Cracks = shear failure.

⑩ The flexure formula $\frac{M}{I} = \frac{F}{A} = \frac{E}{R}$ is valid for static loads with no residual σ

⑪ When a close coiled helical spring is subjected to an axial compressive load the material will be subjected to shear σ

⑫ When an open coiled helical spring when subjected to an axial load, the material is subjected to B.M & Twisting moment

⑬ The $\frac{\delta_v}{v} = \epsilon_v = \frac{\sigma d}{2tE} \left(\frac{5}{2} - \frac{2}{m} \right)$ in thin cylinders

$$\epsilon_v = \frac{Pd}{4tE} \left(5 - \frac{4}{m} \right)$$

⑭ Due to hoop stress longitudinal cracks are formed.

⑮ Max shear stress in cylindrical shell = $\tau_{max} = \sigma_{max} = \frac{Pd}{8t}$

⑯ North light roof truss is used when light is Galleon

⑰ When the stiffness of column increases, then its effective length decreases.

⑱ Young's modulus of Concrete is obtained from Compressive Strength

test on Concrete Cube

- 6) The maximum B.M. for a prismatic beam of length L and carrying a total vol of $\frac{W}{N}$ may be taken as $\frac{WL}{10}$
- 7) The ratio of GJ of a solid shaft to that of hollow shaft of same material and weight per unit length. If the internal diameter for hollow shaft is half the external diameter is $\frac{3}{5}$
- 8) The Conjugate beam for fixed beam is ss beam
- 9) Thick cylinders are analyzed by poisson's theory.
- 10) The Spring used in Spring balances is close coiled helical spring
- 11) For ductile material, the most appropriate failure theory is Maximum shear stress theory
- 12) The stiffness factor for a beam fixed at one end & other end free is $\frac{3EI}{L}$
- 13) A free body diagram is ~~of~~ the diagram of the body ~~as~~ a part of the body in isolated equilibrium
- 14) Compared to bending deformation, shear deformation is small
- 15) In an overhanging beam with equal overhangs on either side
- 16) Conjugate beam of fixed beam is ssbeam
 " of roller support is hinged support
- | | | |
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| 26) Tension test

Compression test

Torsion test | Ductile
45° Crack
bulging
90° Crack | Brittle
90° Crack
45° Crack
45° Crack |
|--|--|--|