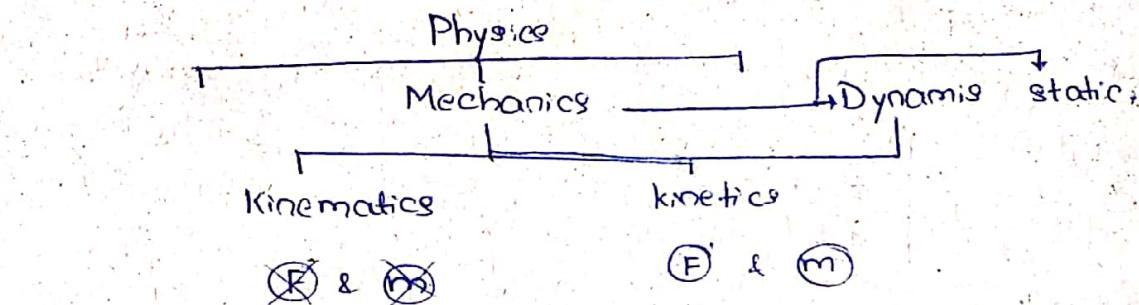


## → KINEMATICS

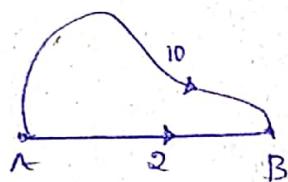
It is the sub- which deals with the motion of the bodies without regarding the force and mass.



→ Displacement : The change in position of a particle is called displacement & it is a vector with only 1 direction.

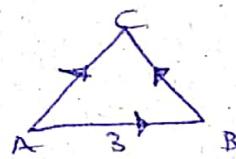
→ Distance : The length of the path covered by the body is called distance. It is scalar.

1. When a body reaches to its starting position, its displacement will be zero.
2. It is possible for a body to have distance without displacement.
3. It is not possible to have displacement without distance.



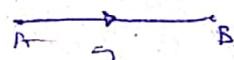
$$\text{Disp} = 0 \text{ m}$$

$$\text{Dist} = 10 \text{ m}$$



$$\text{Disp} = 0 \text{ m}$$

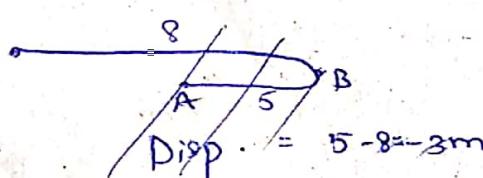
$$\text{Dist} = 9 \text{ m}$$



$$\text{Disp.} = 2 \text{ m}$$

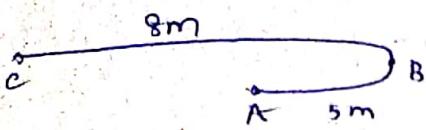
$$\text{Dist.} = 2 \text{ m}$$

$$\text{Disp.} \leq \text{Distance} ; \quad \frac{\text{Disp.}}{\text{Dist.}} = \leq 1$$



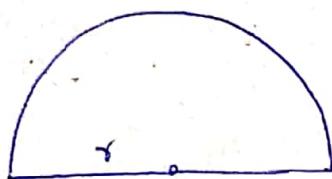
$$\text{Disp.} = 15 - 8 = 7 \text{ m}$$

→ Disp. may be -ve but  
Dist. always be +ve.



$$\text{Disp.} = -3\text{m}$$

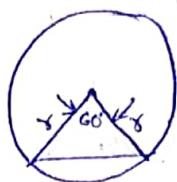
$$\text{Dist.} = 13\text{m}$$



$$\text{Disp.} = 2\pi$$

$$\text{Dist.} = \pi r$$

$$\theta = 180^\circ = \pi \text{ rad}$$



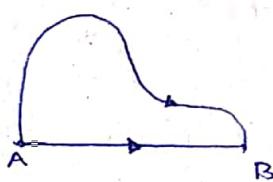
$$\text{Disp.} = r$$

$$\text{Dist.} = \pi r / 3$$

$$\text{Disp. : Dist.} = 3 : \pi$$

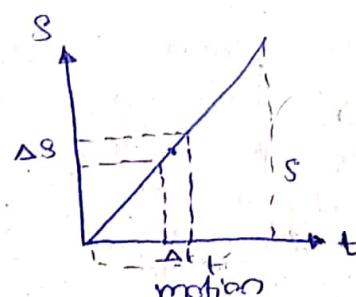
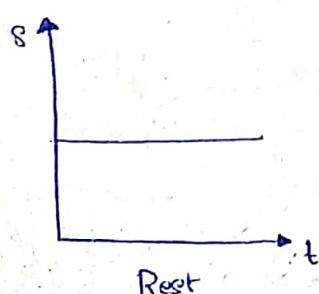
$$\theta = 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$\left. \begin{array}{l} \text{Disp. in a circle} = \omega \times r \times \sin \frac{\theta}{2} \\ \text{Dist. " " " } = r \times \theta \end{array} \right\}$$



$$\text{Velocity} = \text{Disp.} / \text{time}$$

$$\text{Speed} = \text{Dist.} / \text{time}$$



Graphical Representation

- Velocity is defined as rate of change of disp. [vector qty]
- Speed " " " " " " " " " " " " dist. [scalar qty]
- angle of slope,  $\tan \theta = \frac{s}{t} = V$

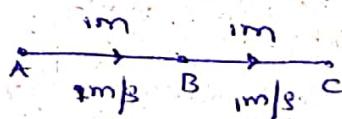
Instantaneous velocity

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

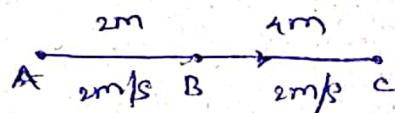
$$V = \frac{ds}{dt}$$

- Uniform velocity:

When magnitude & direction of velocity does not change it is uniform velocity

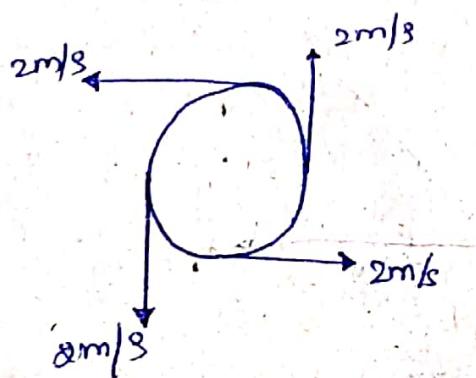


- Non uniform velocity: When magnitude of velocity changes or direction changes or both changes, then it is non-uniform velocity.

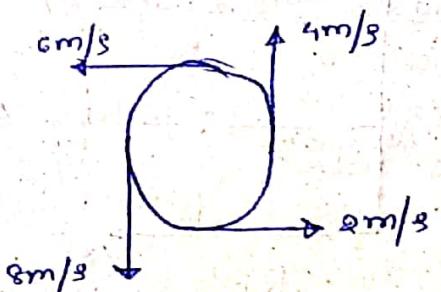


- When a body has uniform velocity it must have uniform speed

- When a body has uniform speed it is not necessarily to have uniform velocity



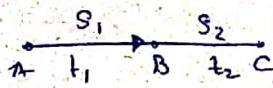
Uniform speed  
Non - u velocity



Non-uniform velocity  
" " " " speed

## AVERAGE VELOCITY

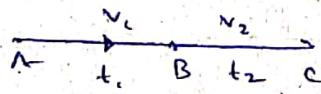
Case 1:  $s, t$  given



$$A.V = \frac{\text{total dist}}{\text{total time taken}}$$

$$= \frac{s_1 + s_2}{t_1 + t_2}$$

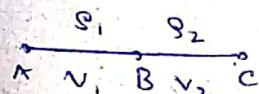
Case 2:  $v_1, t_1$  given



$$A.V = \frac{s}{t} \Rightarrow s = vt$$

$$= \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

Case 3:  $s, v$  given

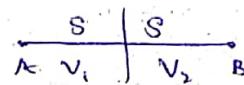


$$V = \frac{s}{t} \Rightarrow \frac{1}{t} = \frac{s}{v}$$

$$AV = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$$

$$= \frac{v_1 v_2 (s_1 + s_2)}{s_1 v_2 + s_2 v_1}$$

Case 4: When the first half with  $v_1$  & next half with  $v_2$  velocities

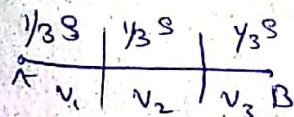


$$s_1 = s_2 = s$$

$$AV = \frac{v_1 v_2 (s_1 + s_2)}{s(v_1 + v_2)}$$

$$= \frac{v_1 v_2}{v_1 + v_2}$$

Case 5: When three equal parts with  $v_1, v_2, v_3$  velocities



$$s_1 = s_2 = s_3 = s$$

$$AV = \frac{s + s + s}{\frac{s}{v_1} + \frac{s}{v_2} + \frac{s}{v_3}}$$

$$= \frac{3v_1 v_2 v_3}{v_2 v_3 + v_1 v_3 + v_1 v_2}$$

Case 6: When  $u, v$  given

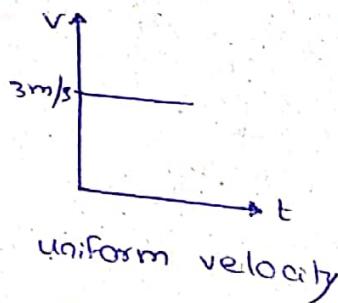


$$AV = \frac{u + v}{2}$$

→ Acceleration: The rate of change of velocity

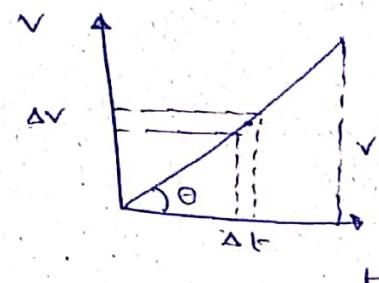
$$a = \frac{\Delta v}{t} \text{ m/s}^2$$

→ Graphical representation



$$a = \frac{\Delta v}{t}$$

$$a = \frac{0}{t} = 0$$



acceleration = ~~area~~

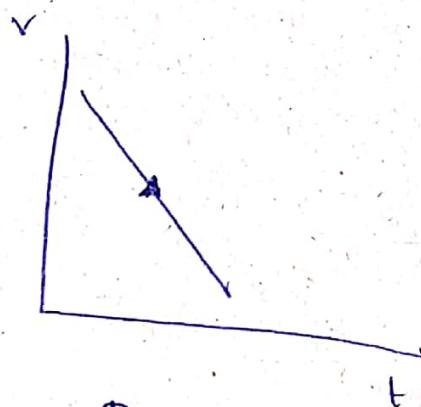
angle of slope

$$\tan \theta = \frac{v}{t} = a$$

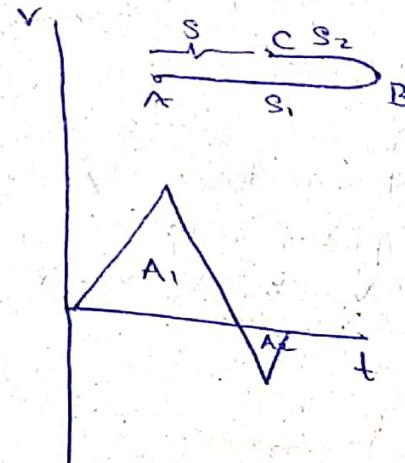
Instantaneous accn

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$a = \frac{dv}{dt}$$



Retardation



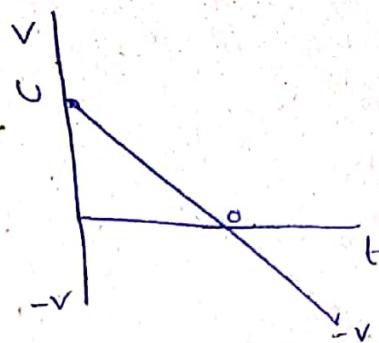
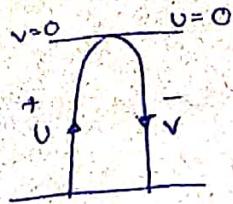
area of the curve  
gives displacement

$$S = A_1 - A_2$$

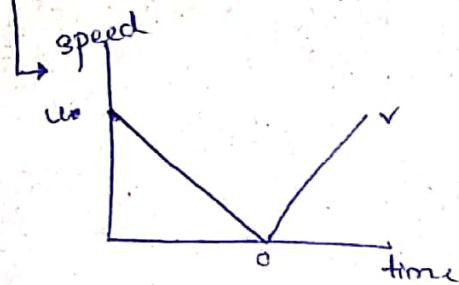
=

$$S = \frac{1}{2} [A_1^2 - A_2^2]$$

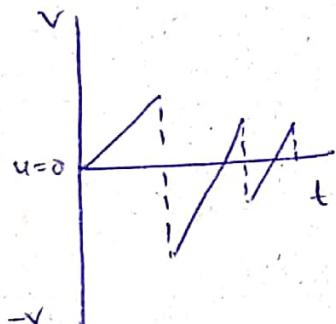
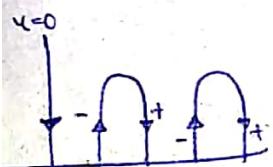
### Projected body



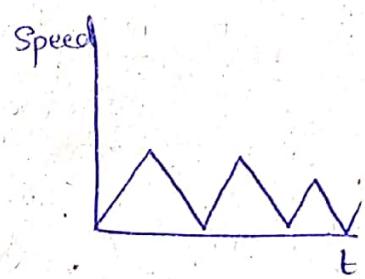
in terms of speed



### For a freely falling body



in terms speed/time



Eqn of motion:

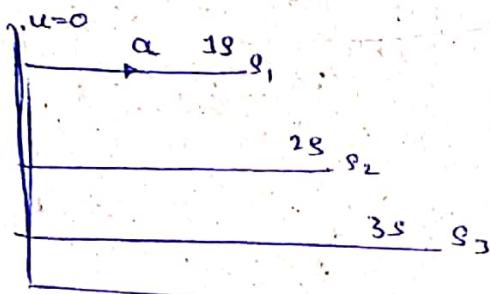
$$v = u + at$$

$$1. \quad v = u + at \\ \text{if } u=0, \quad a = \text{const.}$$

$$v = at$$

2.  $s = ut + \frac{1}{2}at^2$   
 If  $u=0$ ;  $a=\text{const.}$

$$s \propto t^2$$



$$s = (v-at)t + \frac{1}{2}at^2$$

$$s = vt - at^2 + \frac{1}{2}at^2$$

$$s = vt + \frac{1}{2}at^2$$

$$s_1 : s_2 : s_3 = 1 : 4 : 9$$

$$v = u+at$$

$$u = v-at$$

$$s = ut + \frac{1}{2}at^2$$

3.  $v^2 - u^2 = 2as$

If  $v=0$ ;  $a=\text{const.}$

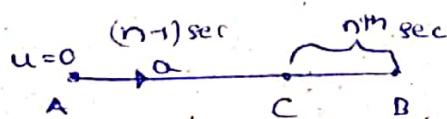
$$0 - u^2 = - 2as$$

$$s \propto u^2$$

→ Disp. = Avg. velocity  $\times$  time

$$s = \left( \frac{u+v}{2} \right) \times t$$

4. Disp. in the  $n^{\text{th}}$  second



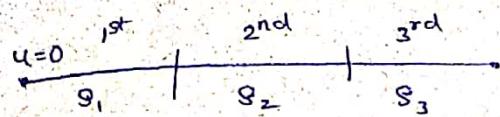
$$s_n = u + a(n - \frac{1}{2})$$

$$= u + \frac{1}{2}a(2n - 1)$$

If  $u=0$ ;  $a=\text{const.}$

$$s_n \propto 2n - 1$$

Disp. in successive seconds



$$s_1 : s_2 : s_3 = 1 : 3 : 5$$

$$s_2 = \frac{s_1 + s_3}{2}$$

$$s_n = s - s_{(n-1)}$$

From graph

$$\tan \theta = \frac{v-u}{t} = a$$

$$v-u = at$$

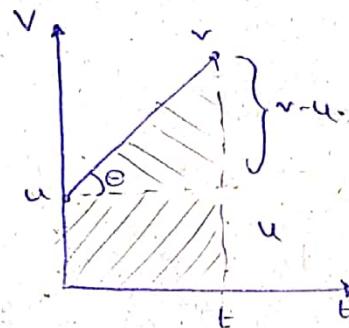
$$v = u + at$$

Area of curve [disp.]

$$s = ut + \frac{1}{2}(v-u)t$$

$$= ut + \frac{1}{2}(at)t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$



By differentiation

$$a = \frac{dv}{dt}$$

$$\int_u^v dv = a \int_0^t dt$$

$$[v]_u^v = a [t]_0^t$$

$$v-u = at$$

$$v = u + at$$

$$v = \frac{ds}{dt}$$

$$ds = v \times dt$$

$$\int_u^v ds = a \int_0^t dt$$

$$ds = (u+at)dt$$

$$ds = u dt + at dt$$

$$\int_0^t ds = u \int_0^t dt + a \int_0^t t dt$$

$$s = ut + \frac{at^2}{2}$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$a = \frac{dv}{ds} \cdot v$$

$$ads = v dv$$

$$a \int_s^S ds = \int_u^v v dv$$

$$as = \left[ \frac{v^2}{2} \right]_u^v$$

$$as = \frac{v^2 - u^2}{2}$$

$$\boxed{v^2 - u^2 = 2as}$$

→ When a body s, disp. in  $n_1$  sec & S<sub>2</sub>, disp. in  $n_2$  sec  
then its accn is \_\_\_\_\_

$$S_n = u + a(n - \gamma_2)$$

$$S_1 = u_0 + a(n_1 - \gamma_2)$$

$$S_2 = u + a(n_2 - \gamma_2)$$

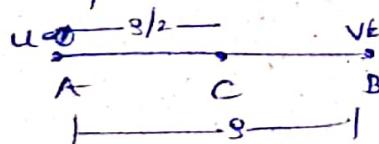
$$S_2 - S_1 = a(n_2 - n_1)$$

$$a \rightarrow | S_1 | S_2 | \frac{s_2}{n_2} |$$

$$\boxed{a = \frac{S_2 - S_1}{n_2 - n_1}}$$

When a body crosses two points A & B with velocities of  $v_1$  &  $v_2$   
then, the velocity a) midpoint of A & B b) half the time

a) mid point of A & B



$$v^2 - u^2 = 2as$$

$$a = \frac{v^2 - u^2}{2s} \quad \text{--- (1)}$$

$$a = \frac{v^2 - u^2}{s} \quad \text{--- (2)}$$

$$a = \frac{v^2 - u^2}{s} = \frac{v^2 - u^2}{2s}$$

$$= \frac{v^2 + u^2 - 2v^2 - 2u^2}{2s} = \frac{v^2 - u^2}{2s}$$

$$v^2 = \frac{v^2 + u^2}{2}$$

$$v = \sqrt{\frac{v^2 + u^2}{2}}$$

b) at mid-time

$$v = u + at$$

$$a = \frac{v-u}{t}$$

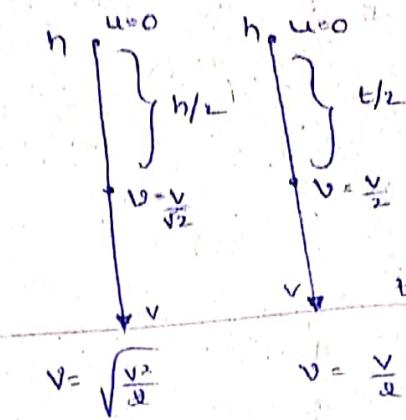
$$a = \frac{v-u}{t/2}$$

$$a = \frac{v-u}{t} = \frac{v-u}{t/2}$$

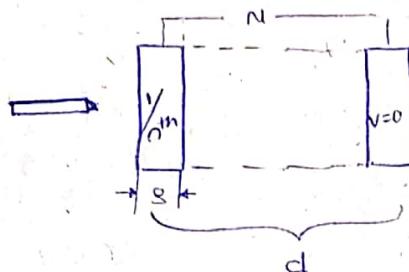
$$\Rightarrow av - ate = 2(v-u)$$

$$\Rightarrow v = \frac{v+u}{2}$$

For a freely falling body



When a bullet loses  $\frac{1}{n^{th}}$  velocity when it passes through a plank of thickness  $s$



i. The min. no. of planks required to stop the bullet,  $N = \frac{D^2}{2n-1}$

ii. The further dist. travelled by before it comes to rest

$$d = \left[ \left( \frac{n^2}{2n-1} \right) \times s \right] - s$$

$$[N-1] \times s$$

$$\text{Ex:- } \frac{1}{n^{th}} \sim \frac{1}{5}$$

$$N = \frac{n^2}{2n-1} = \frac{5^2}{2 \times 5 - 1} \Rightarrow \frac{25}{9}$$

$$= 2.7$$

= 3 planks

$$d = (2.7 - 1) \times 10 [N-1] \times s$$

$$= 17 \text{ cm}$$

Disp.  $s = (t^2 - st + 9)m$ , the time taken to come to rest

$$V = \frac{ds}{dt}$$

$$V = st - s \quad [V=0]$$

$$st - s = 0$$

$$t = s/2$$

$$= 4$$

Disp. of a particle changes with time as  $t = \sqrt{x+3}$   
then if disp. when it comes rest

$$t = \sqrt{x} + 3$$

$$x = (t-3)^2$$

$$= t^2 - 6t + 9$$

$$V = \frac{dx}{dt}$$

$$= st - s \quad [at \ V=0]$$

$$t = 3s$$

$$x = 0$$

$$S = t^2 + st + 2m \quad \text{disp. during 3rd sec}$$

$$S = t^2 + st + 2m$$

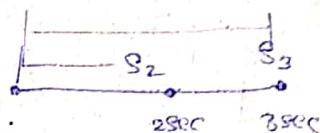
$$S_3 = 3^2 + 6 + 2$$

$$= 17m$$

$$S_2 = 2^2 + 2^2 + 2$$

$$= 10$$

$$S_3 - S_2 = 7m$$



$$S = t^2 + st + 2$$

$$V = \frac{ds}{dt} = st + 2$$

$$V = st + 2$$

at initial velocity 'u',  $t=0$

$$V = 2m/s$$

$$\alpha = \frac{dv}{dt} = 2m/s^2$$

$$S_n = u + \alpha \left( n - \frac{1}{2} \right)$$

$$= 2 + 2 \left[ 3 - \frac{1}{2} \right]$$

$$= 2 + (2 \times \frac{5}{2})$$

$$= 7m$$

Velocity of a particle changes with disp. as  $v = \sqrt{95 - 6x}$ ,  $a = ?$

$$a = \frac{dv}{dt} = \frac{d}{dx} \sqrt{95 - 6x}$$

$$\begin{aligned} &= \frac{-6}{2\sqrt{95 - 6x}} \\ &= \frac{-6}{2\sqrt{95 - 6x}} \times \frac{\sqrt{95 - 6x}}{\sqrt{95 - 6x}} \\ &= \frac{-6}{2\sqrt{95 - 6x}} \end{aligned}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$\Rightarrow \frac{-6}{2\sqrt{95 - 6x}} \times \frac{dx}{dt}$$

$$a = \frac{-6}{2 \times (\frac{dx}{dt})} \times \frac{dx}{dt}$$

$$a = -3$$

$$v^2 - v_0^2 = 2ax$$

$$v^2 - 5^2 = 2(-3)x$$

$$a = -3 \text{ m/s}^2$$

Displacement of particle is  $s = t^3 - 12t^2 + 5t + 2 \text{ m}$ , the time taken to attain the max. velocity is

$$s = t^3 - 12t^2 + 5t + 2$$

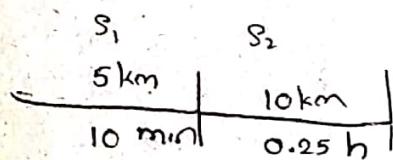
$$\frac{ds}{dt} = 3t^2 - 24t + 5$$

$$\frac{d^2s}{dt^2} = 6t - 24$$

$$a = 6t - 24 \quad [a=0]$$

$$6t = 24$$

$$t = 4 \text{ sec}$$



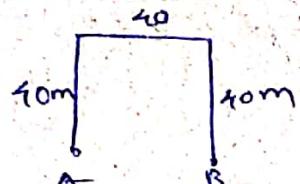
$$\text{Avg. velocity} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{5 + 10}{1.66 + 0.25}$$

$$= \frac{15}{1.91} \Rightarrow \frac{1500}{191}$$

$$= \frac{15}{\frac{1}{6} + \frac{1}{4}}$$

$$= \frac{\frac{12}{24} \times 18}{105} = 30 \text{ km/h}$$

$$= 10 \text{ m/s}$$



$$v = 12 \text{ m/s}$$

$$\bar{v} = \frac{40}{50/12 \times 3}$$

$$t = \frac{d}{s}$$

$$\bar{v} = \frac{s}{t}$$

$$= \frac{s}{(\text{d/s}) \times \text{total dist.}}$$

$$= \frac{40 \text{ m}}{(40/12) \times 3}$$

$$t = 4 \text{ seconds}$$

A car travels first  $\frac{1}{2}$  of the distance with  $40 \text{ km/hr}$  & next  $\frac{1}{2}$  with  $60 \text{ km/hr}$

$$\bar{v} = \frac{2 \cdot v_1 \cdot v_2}{v_1 + v_2}$$

$$= \frac{s_1 + s_2}{v_1 + v_2}$$

$$\left( \frac{s_1}{v_1} \right) + \left( \frac{s_2}{v_2} \right)$$

$$= \frac{2 \times 40 \times 60}{40 + 60}$$

$$= \frac{4800}{100} = 48 \text{ km/hr}$$

→ A body from rest its velocity is  $3 \text{ m/s}$  after  $6 \text{ sec}$ . Its velocity after  $8 \text{ sec}$

$$v = u + at$$

$$v = u + t$$

$$\frac{v_1}{t_1} = \frac{v_2}{t_2} \Rightarrow \frac{3}{6} = \frac{v_2}{8}$$

$$v_2 = 4 \text{ m/s}$$

→ The velocity of a body at an instant is  $10 \text{ m/s}$ , after  $5 \text{ sec}$  it becomes  $20 \text{ m/s}$ , its velocity  $3 \text{ sec}$  before the instant

$$u = ? \quad 10 \text{ m/s} \quad v = 20 \text{ m/s}$$

--- 3s --- I --- 5s ---

$$v = u + at$$

$$20 = 10 + a \times 5$$

$$a = 2 \text{ m/s}^2$$

$$v = u + at$$

$$10 = u + (2 \times 3)$$

$$u = 4 \text{ m/s}$$

→ A particle is started from rest. Its displacement is  $s_1$  in first two seconds, and  $s_2$  in next two seconds, then  $s_1, s_2$

$$\Rightarrow s_1 = 3s_2$$

$$\frac{s_1}{s_2} = 3$$

$$\boxed{s_2 : s_1 = 1 : 3}$$

$$s_1 : s_2 : s_3 : s_4 = \underbrace{1 : 3}_{\text{in } 2 \text{ sec}} : \underbrace{5 : 7}_{\text{in } 2 \text{ sec}}$$

$$s_1 : s_2 = 4 : 12$$

$$\frac{s_1}{s_2} = \frac{4}{12}$$

$$s_1 = \frac{1}{3}s_2$$

$$\Rightarrow s_2 = 3s_1$$

$$s = ut + \frac{1}{2}at^2$$

$$s \propto t^2$$

$$\frac{s_1}{s_1 + s_2} = \frac{t_1^2}{t_1^2 + t_2^2}$$

$$\frac{s_1}{s_1 + s_2} = \left(\frac{2}{3}\right)^2 \\ = \frac{1}{4}$$

$$4s_1 = s_1 + s_2 \\ \Rightarrow s_2 = 3s_1$$

→ A train is started from rest and moves with an accel. of  $1 \text{ m/s}^2$  and man who is  $48 \text{ m}$  behind the train running uniformly with  $10 \text{ m/s}$ . The min. time to catch the train is \_\_\_\_\_

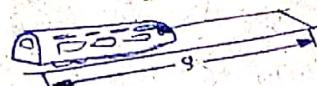
$$s = ut + \frac{1}{2}at^2$$

For train

$$s = \frac{1}{2}at^2 \quad \text{--- (1)}$$

For man

$$(s + 48) = v \times t \quad \text{--- (2)}$$



from (1) & (2)

~~s + 48~~

$$\left[ \frac{1}{2}at^2 + 48 = vt \right] \times 2$$

$$\frac{1}{2}t^2 + 48 = 2vt$$

$$t^2 + 96 = 2vt$$

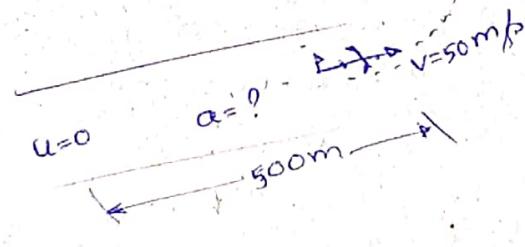
$$t^2 - 96t + 96 = 0$$

$$t = 88, t = 128$$

$t = 88$  min. time or  
time taken by man

$t = 128$  time in train

→ When an aeroplane requires  $15 \text{ m/s}$  in order to takeoff, the length of runway is  $500 \text{ m}$ .  $a = ?$



$$a = \frac{v}{t}$$

$$= 15$$

$$t = \frac{v}{a}$$

$$v^2 - u^2 = 2as$$

$$50^2 = a \times 500$$

$$\frac{2500}{1000} = 2.5 \text{ m/s}^2$$

$$= \frac{5}{2} \text{ m/s}^2$$

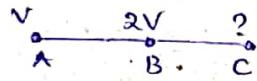
A body crosses points A & B with a velocities of 4 & 8 m/s its velocity at midpoint of A & B

$$\Rightarrow V = \sqrt{\frac{v^2 + u^2}{2}}$$

$$= \sqrt{\frac{16 + 64}{2}} = \sqrt{\frac{80}{2}} = \sqrt{40} \text{ m/s}$$

$\frac{10}{2} \left( \frac{80}{2} \right)$

A body has velocity at a point,  $\rightarrow$  after a disp. 'S', the velocity becomes  $2xV$  its velocity after another 'S' disp.



$$\Delta V = \sqrt{\frac{v^2 + u^2}{2}}$$

$$\Delta v = \dots$$

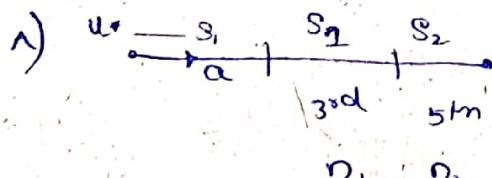
$$(\Delta V)^2 = \frac{v^2 + u^2}{2}$$

$$x^2 = 8v^2 - u^2$$

$$x = \sqrt{7v^2}$$

$\rightarrow$  A body is moving const. accl<sup>n</sup> its disp. 5m in

3<sup>rd</sup> sec & 9m in 5<sup>th</sup> sec  
its velocity after 7 sec



$$a = \frac{s_2 - s_1}{n_2 - n_1}$$

$$= \frac{9 - 5}{5 - 3} \Rightarrow \frac{4}{2} = 2 \text{ m/s}^2$$

$$s_n = u + \frac{1}{2} a(n - \gamma_2)$$

$$5 = u + \alpha(3 - \gamma_2)$$

$$u = 5 - (6 - 1) = 0$$

$$v = u + at$$

$$= 0 + 2 \times 7$$

$$= 14 \text{ m/s}$$

$\rightarrow$  A bullet of velocity 100m/s comes to rest within 10cm in a wooden block, the time taken is

$$u = 100 \text{ m/s}$$

$$v = 0$$

$$s = 10 \text{ cm}$$

$$s = \frac{(u+v)}{2} \times t$$

$$10 = \frac{(100+0)}{2} \times t$$

$$\frac{10}{50} = \frac{t}{sec}$$

$$0.2 = \frac{(100+0)}{2} \times t$$

$$0.2 = 50t$$

$$\frac{0.2}{50} = \frac{t}{sec}$$

$$0.2 = 100t$$

$$t = 0.002 \text{ sec}$$

$\rightarrow$  A bullet loses  $\frac{1}{20}$  th velocity when it passes through a plank then the min. no of planks required

$$N = \frac{n^2}{2n-1}$$

$$= \frac{400}{139} = 3 \frac{1}{139}$$

$$\cancel{+ 3.3}$$

$$= 10.2$$

$$= 11$$

A bullet loses half of the velocity when it passes 80cm in a wooden block the further distance travelled before it comes to \_\_\_\_\_

$$\begin{aligned}
 d &= N \times s \\
 &= \frac{(N^2)}{(N-1)} \times s = s \\
 &\quad \cancel{\text{---}} \quad \cancel{s} \\
 &= (N-1) \times s \\
 &= \left(\frac{4}{3}-1\right) \times 0.6 \\
 &= \frac{1}{3} \times 0.6 \\
 &= \frac{2}{10} \text{ m}
 \end{aligned}$$

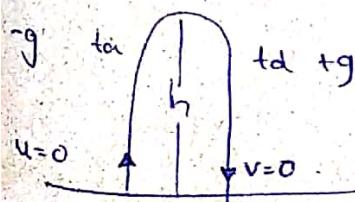
### Accel due to gravity

accel of a freely falling body due to gravit<sup>n</sup> force is called " due to gravity [g] "

- g is independent of mass of the body, but it changes from place to place
- For a freely falling body g is +ve
- + " " projected " g is -ve

CGS	MKS	FPS
1 9800 cm/s	1 9.8 m/s	32 ft/s

### Motion of a body under gravity



1. Time of ascent

$$V = u + at$$

$$0 = u - g t_a$$

$$t_a = u/g$$

2. Time of descent

$$V = u + g t$$

$$V = g t_d$$

$$t_d = V/g$$

3. Initial velocity

$$V^2 - u^2 = 2 a s$$

$$\Rightarrow 0 - u^2 = - g h$$

$$u = \sqrt{g h}$$

4. Final velocity

$$V^2 = a g h$$

$$V = \sqrt{a g h}$$

→ When air resistance is neglected then

$$V = u \quad , \quad t_d = t_a$$

→ When air resistance is considered then

$$V < u \quad ; \quad t_d > t_a$$

5. Time of flight

$$T = t_a + t_d$$

$$= \frac{u}{g} + \frac{V}{g} \quad [u = v]$$

$$= \frac{u + V}{g}$$

6. Max. height

$$u^2 = a g h$$

$$h = \frac{u^2}{a g}$$

$$h \propto u^2$$

7. Height of fall

$$S = u t^2 + \frac{1}{2} a t^2$$

$$h = 0 + \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2$$

$$h \propto t^2$$

For a freely falling body the disp. in successive seconds increase by  $g$

$$h = g \\ h_2 = h + g \\ h_n = h_{n-1} + g$$

For a freely falling body the disp. in  $n^{\text{th}}$  sec is  $h$ , then the next second disp. is  $h+g$  & previous disp. is  $h-g$

$$h-g$$

$$u=0$$

$$\left. \begin{array}{l} s = y_2 g t^2 \\ s_1 = 4.9 \end{array} \right\} h=g$$

$$\left. \begin{array}{l} s_2 = y_2 \times 9.8 \times 4 \\ = 14.7 \\ s_2 = s_1 + g \\ = 4.9 + 9 \end{array} \right\} h_2 = h_1 + g$$

$$\left. \begin{array}{l} s_3 = y_2 \times 9.8 \times 3^2 \\ = 24.5 \\ = s_2 + g \end{array} \right\} h_3 = h_2 + g$$

$$h_1 : h_2 : h_3 = 1 : 3 : 5$$

When a projected body has same velocities after  $x$  and  $y$  sec then the initial velocity is

$$\Rightarrow T_f = T_a + T_g$$

$$T = x + y$$

$$x+y = \frac{gu}{g}$$

$$u = \frac{g(x+y)}{2}$$



For a freely falling body

1. Velocity  $v = u + gt$

$$= 0 + 9.8 \times 1$$

$$= 9.8 \text{ m/s}$$

$$V \propto t = 1 : 2 : 3 : 4 : 5$$

After 1 second  $V_1 = 9.8 \text{ m/s}$

$$V_2 = 19.6 \text{ m/s}$$

$$\downarrow V_3 = 29.4 \text{ m/s} \dots$$

2. Displacement at  $S = ut + \frac{1}{2}gt^2$

$$S \propto t^2 \Rightarrow 1 : 4 : 9 : 16 : 25$$

$$S_1 = 0 + \frac{1}{2} \times 9.8 \times 1^2$$

$$= 4.9 \text{ m}$$

After 1 second  $S_1 = 4.9 \text{ m}$

$$S_2 = 19.6 \text{ m}$$

$$S_3 = 44.1 \text{ m}$$

$$\downarrow S_4 = 78.4 \text{ m}$$

$$S_5 = 122.5 \text{ m}$$

3. Disp. in successive seconds

$$S_n = ut + \frac{1}{2}gt^2 (at n-1)$$

$$S_n \propto n-1$$

$$S_1 = 0 + \frac{1}{2} \times 9.8(1) \\ = 4.9 \text{ m}$$

$$S_n \propto n-1 = 1 : 3 : 5 : 7 : 9$$

In the 1st second  $S_1 = 4.9 \text{ m}$

$$S_2 = 14.7 \text{ m}$$

$$S_3 = 24.5 \text{ m}$$

$$\downarrow S_4 = 34.3 \text{ m}$$

Displacement  
in successive seconds

Displacement  
in seconds

1st sec 4.9

1 sec

2nd sec 14.7

2 sec

3rd sec 24.5

3 sec

4th sec 34.3

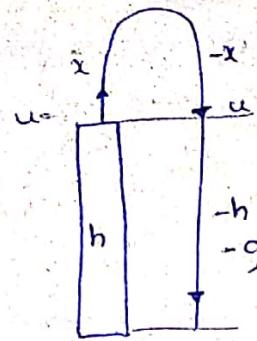
4 sec

When a body is projected up from the top of the tower which reaches the ground in the time  $t$  with a velocity  $v$ , then the height of tower.

$$\Rightarrow s = (x - x - h) \\ = ut + \frac{1}{2}(-g)t^2$$

$$-h = ut - \frac{1}{2}gt^2$$

$$h = \frac{1}{2}gt^2 - ut$$

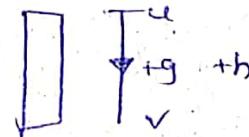


→ Height of tower

$$v^2 - u^2 = 2gh$$

$$v^2 - u^2 = 2gh$$

$$h = \frac{v^2 - u^2}{2g}$$



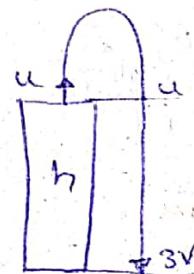
$$\rightarrow h = \frac{v^2 - u^2}{2g}$$

$$= \frac{9v^2 - v^2}{2g}$$

$$(u = v)$$

$$v = 3u$$

$$h = \frac{4v^2}{9}$$

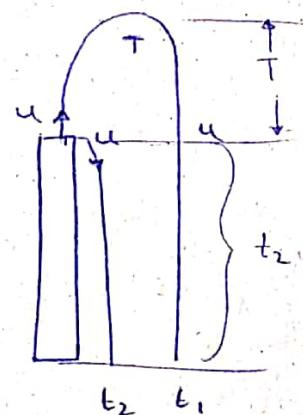


→ Special concept

$$t_1 - t_2 = T$$

$$t_1 - t_2 = \frac{2u}{g}$$

$$u = \frac{g(t_1 - t_2)}{2T}$$



→ The height of the balloon when the stone is dropped

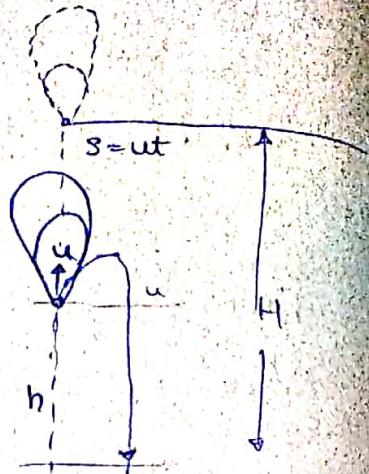
$$h = \frac{1}{2}gt^2 - ut$$

→ The height of the balloon when the stone touches the ground

$$H = h + s$$

$$= \left[ \frac{1}{2}gt^2 - ut \right] + [ut]$$

$$H = \frac{1}{2}gt^2$$



When a stone is dropped from the top of a tower & another stone is projected up from the ground, then the time taken by them to meet is

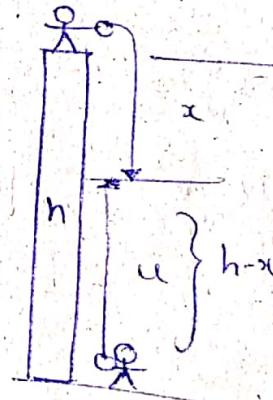
$$s = ut + \frac{1}{2}at^2$$

$$\text{I stone } x = \frac{1}{2}gt^2$$

$$\text{II stone } h - x = ut - \frac{1}{2}gt^2$$

$$h = ut$$

$$t = h/u$$



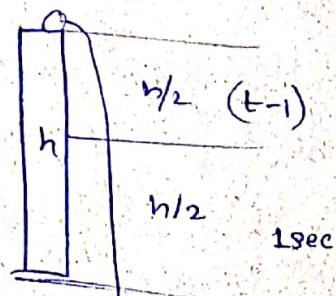
$$s = ut + \frac{1}{2}at^2$$

$$\rightarrow h = \frac{1}{2}gt^2$$

$$\rightarrow \frac{h}{2} = \frac{1}{2}g(t-1)^2$$

~~$$t =$$~~

$$h = g(t-1)^2$$



$$\frac{1}{2}gt^2 = g(t^2 + t - 2t)$$

$$t^2 = 2t^2 + 2 - 4t$$

$$t^2 - 4t + 2 = 0$$

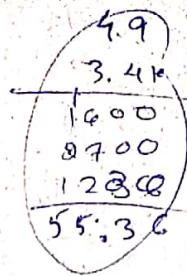
$$t = \frac{4 \pm \sqrt{16-8}}{2}$$

$$t = \frac{4(2 + \sqrt{2})}{2}$$

$$t = (2 + \sqrt{2}) \text{ sec}$$

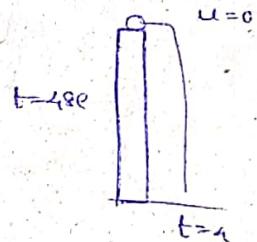
$$h = \frac{1}{2} \times (2 + \sqrt{2})^2 \times 9.8$$

$$h = 57.11 \text{ m}$$



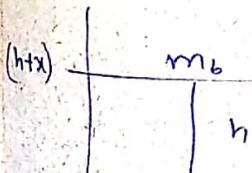
→ A stone is dropped from a top of the tower, the height

$$\begin{aligned} h &= \frac{1}{2}gt^2 \\ &= 4.9 \times 46 \times \frac{1}{2} \times 9.8 \times 1.6 \\ &= 43.84 \\ &= 78.4 \text{ m} \end{aligned}$$



→ Two bodies of mass of  $m_a, m_b$  are dropped from the heights of  $A, B$ . The ratio of time taken to reach the ground

$m_a$



$$S = ut + \frac{1}{2}gt^2$$

$$S \propto t^2$$

$$\sqrt{\frac{S_1}{S_2}} = \frac{t_1}{t_2}$$

$$t_a : t_b = \sqrt{A : B}$$



→ A stone is dropped from a height, it reaches the ground in 5 sec, after 3 sec it is stopped and allowed to fall again,  $T_2 = ?$

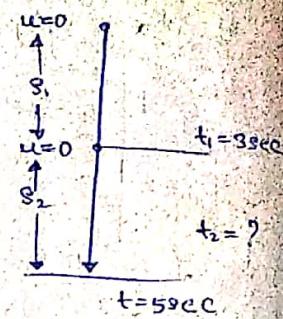
$$S = S_1 + S_2$$

$$\frac{1}{2}gt_2^2 = \frac{1}{2}gt_1^2 + \frac{1}{2}gt_2^2$$

$$t_2 = \sqrt{t_1^2 - t_1^2}$$

$$= \sqrt{5^2 - 9}$$

$$= 4\text{ sec}$$



→ Water drops are falling from a cap at an interval of  $\frac{1}{4}$  sec, the first drop does touch the ground the 5<sup>th</sup> drop is abt. to fall

Q. 1.

$$\text{Total time, } T = \frac{1}{4} \times 4$$

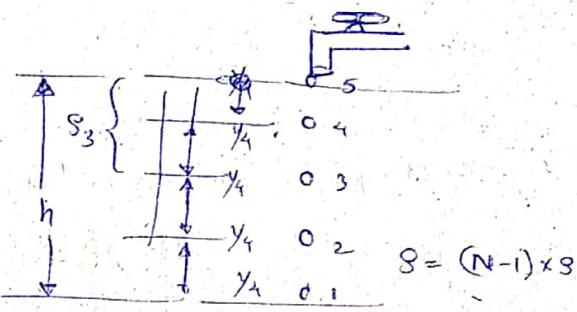
$$= 1\text{ sec}$$

$$S = \frac{1}{2} \times g \times t^2$$

$$= 4.9 \times 1$$

$$= 4.9$$

$$S_n = u + \frac{1}{2}g(2n-1) \quad \text{Time}$$



$$N = \frac{n^2}{2n-1}$$

Q. 2. Time taken for 3rd drop

$$t = 2 \times \frac{1}{4}$$

$$= 1.25 \text{ sec}$$

$$\text{distance dropped, } S_3 = \frac{1}{2} \times 4.9 \times (y_2)^2$$

$$= 1.225 \text{ m}$$

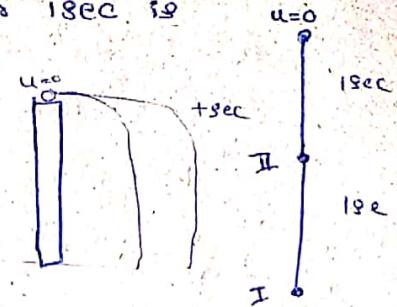
$$\text{Dist. remaining, } h_3 = 4.9 - 1.225$$

$$= 3.675 \text{ m}$$

→ A stone is dropped from a height after 1sec another stone is dropped. The dist. b/w them after another 1sec is

$$S = \frac{1}{2}gt^2$$

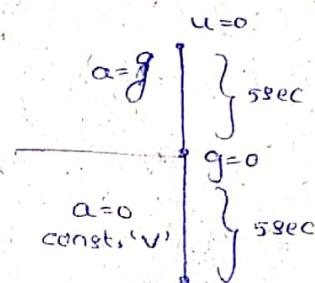
$$\begin{aligned} S_1 &= \frac{1}{2}g(1)^2 & S_2 &= \frac{1}{2}g \times (2)^2 \\ &= 4.9 \text{ m} & &= 9.8 \times 2 \\ & & &= 19.6 \\ & & & - \frac{4.9}{14.7} \\ S_2 - S_1 &= 14.7 \text{ m} \end{aligned}$$



→ A stone is dropped from a height after 5sec the gravity is disappear, its displacement in next 5sec

$$\begin{aligned} v &= u + gt \\ &= 0 + 9.8 \times 5 \\ &= 49 \text{ m/s} \end{aligned}$$

$$\begin{aligned} S &= v \times t \\ &= 49 \times 5 \\ &= 245 \text{ m} \\ &= 245 \text{ m} \end{aligned}$$



→ A stone is dropped from the top of a tower, after 1sec another stone is dropped from balcony which is 20m below the top. Both reaches the ground at once, the height

$$S_1 = S_2$$

$$(20+h) = h$$

~~$$h_1 = h_2 - 20$$~~

~~$$S_1 = \frac{1}{2}gt^2$$~~

~~$$(S_2 - 20) = \frac{1}{2}g(t-1)^2$$~~

~~$$S_2 = \frac{1}{2}gt^2$$~~

~~$$\frac{1}{2}gt^2 - 20 = \frac{1}{2}g(t-1)^2$$~~

~~$$t^2 - 2t + 20 = t^2 + 1 - 2t$$~~

~~$$-2t - 20 = 1$$~~

~~$$2t = 19/2$$~~

~~$$= 9.8$$~~

$$\frac{1}{2}[gt^2 + g - 2tg - gt^2] + 20 = 0$$

$$g - 2tg = -10$$

$$g(1 - 2t) = -10$$

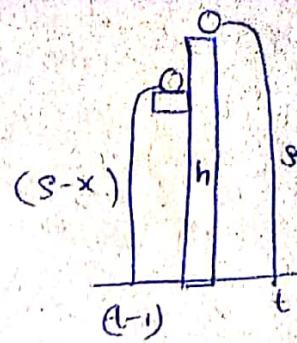
$$t = \frac{-1-1}{2}$$

$$s-x = s$$

$$\frac{1}{2}g(t-1)^2 = \frac{1}{2}gt^2$$

$$-20 = \frac{1}{2} \times 10 - 10t$$

$$t = 0.5 \text{ sec}$$



$$h = \frac{1}{2}gt^2$$

$$= \frac{1}{2}$$

$$= 31.25 \text{ m}$$

→ A stone from the top of the tower, it travels 24.5m in the last second, the height of tower is

$$24.5 = u + \frac{1}{2}a(2n-1)$$

$$\frac{24.5}{4.9} = u + 4.9(2n-1)$$

$$2n-1 = 5$$

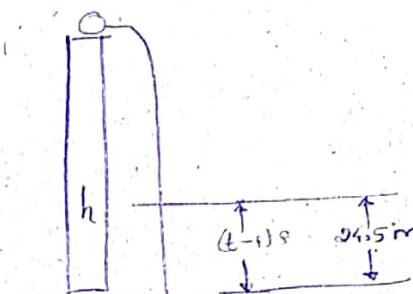
$$\therefore n = 3 \text{ sec}$$

$$\begin{array}{r} 24.5 \\ \hline 4.9 \\ \hline 4.9 \\ \hline 5 \\ \hline 44.1 \end{array}$$

$$h = \frac{1}{2}gt^2$$

$$= \frac{1}{2} \times 9.8 \times 3^2$$

$$= 44.1 \text{ m}$$



→ A ball is projected up which reaches the ground after 6sec. The find the height of tower after 4sec

$$v = u + gt$$

$$= 9.8 \times 2$$

$$= 19.6$$

$$T_f = \frac{ta + td}{2} = \frac{36}{9}$$

$$36 = \frac{8}{9} \times u$$

$$u = 30 \text{ m/s}$$

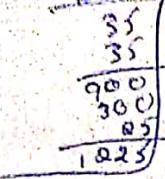
$$S = ut - \frac{1}{2}gt^2$$

$$= 30 \times 4 - \frac{1}{2} \times 9.8 \times 16$$

$$= 40 \text{ m}$$

736

In the above problem the max. height of the ball and its velocity after 4 sec.



$$H_{\max} = \frac{u^2}{2g}$$

$$= 45 \text{ m}$$

$$V = u - gt$$

$$= 30 - 9.8 \times 4$$

$$= -10 \text{ m/s}$$

~~at 8s~~  
45

→ A balloon rising uniformly with  $9.8 \text{ m/s}$  a stone is dropped from it which reaches the ground in 10 sec.

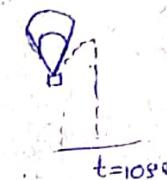
- Ans:

$$h = \frac{1}{2}gt^2 - ut$$

$$= \frac{1}{2} \times 9.8 \times 10^2$$

$$= 4.9 \times 100 = 98$$

$$= 490 - 98$$



$$h = 490 - 98$$

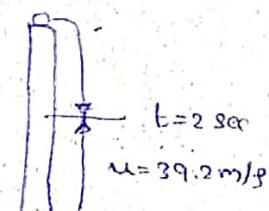
$$= 392 \text{ m}$$

→ When a stone is dropped from the top of the tower & another " " projected up " " the ground with  $39.2 \text{ m/s}$

$$h = ut$$

$$= 39.2 \times 2$$

$$= 78.4 \text{ m}$$



### → HORIZONTAL MOTION

- When a body is projected horizontally from the top of a tower, then it reaches the ground in parabolic path
- The horizontal velocity remains const.
- When a stone is dropped from the top of a tower & another stone is projected horizontally then both reaches the ground at the same time but with diff. velocities.

For a dropped (velocity) body,  $v = \sqrt{2gh} = gt$

$$\text{u } " \text{ projected } " " , v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{u^2 + 2gh}$$

$$\tan \alpha = \frac{v_y}{v_x}$$

$$\alpha = \tan^{-1} \left( \frac{v_y}{v_x} \right)$$

- When a bomb is dropped from aeroplane it seems to be falling in a parabolic path for the person on the ground.
- The bomb seems to be falling vertically down for the person in the aeroplane.

$$S = ut + \frac{1}{2}at^2$$

height of tower

$$h = \frac{1}{2}gt^2 \quad \text{--- (1)}$$

Range

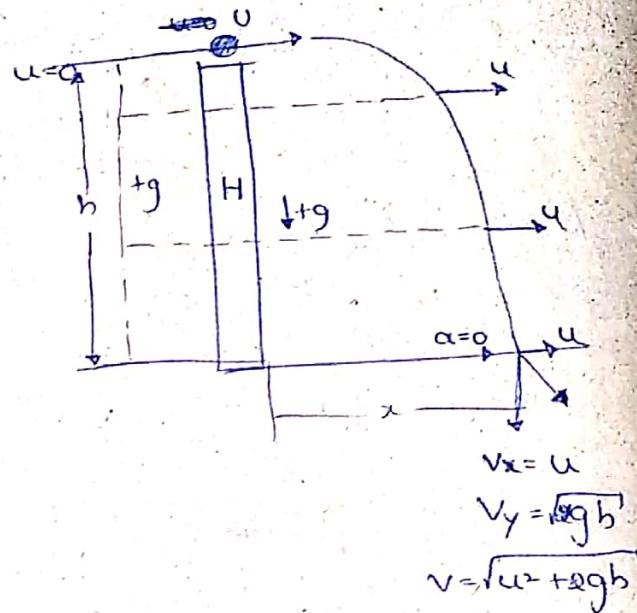
$$x = ut$$

Time of descent

$$t = \frac{x}{u} \quad \text{--- (2)}$$

from (1) & (2)

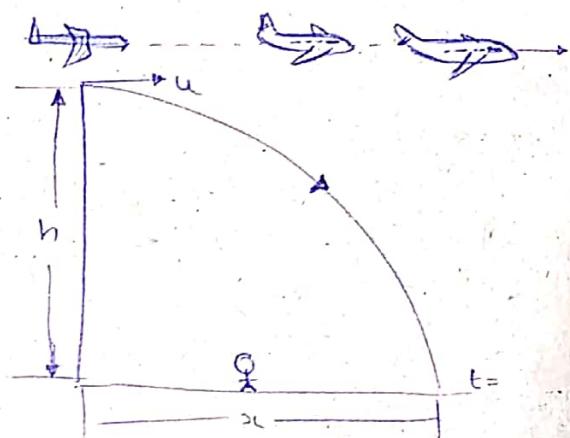
$$h = \frac{1}{2}g \frac{x^2}{u^2}$$



$$V_x = u$$

$$V_y = \sqrt{g h}$$

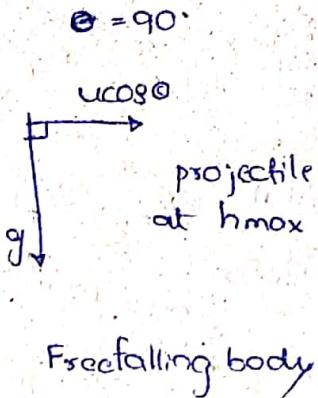
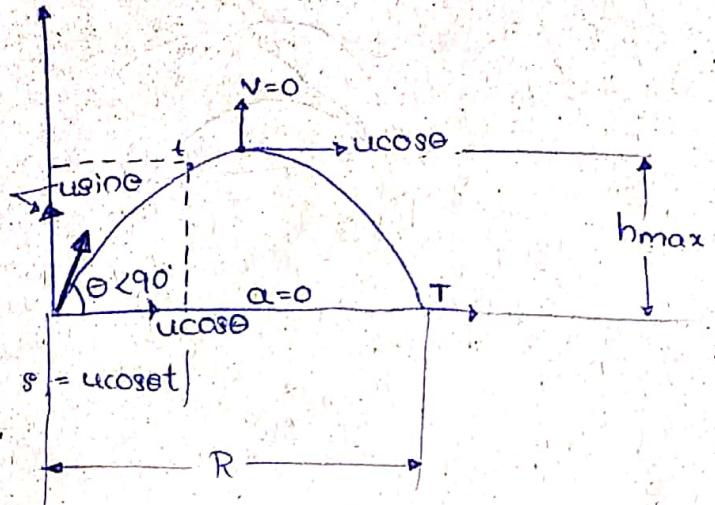
$$V = \sqrt{u^2 + g h}$$



### → OBlique PROJECTION

- When a body is projected with some initial velocity at a certain angle ~~with~~ ( $\theta < 90^\circ$ ) and allowed the body to move under gravity that projection is called oblique projection.

- The path of the projectile is parabola.
- The path is a straight line when it is observed from another projectile.
- The horizontal velocity ( $u \cos \theta$ ) remains constant but vertically ( $u \sin \theta$ ) changes due to gravitational force.
- The projectile has min. velocity at max. height ( $u \cos \theta$ )



Freefalling body

$$\begin{cases} \alpha, v \\ D = 0^\circ = \theta \\ g \end{cases}$$

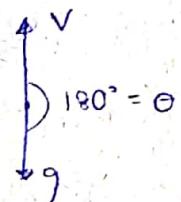
$$\text{Range, } R = u \cos \theta \times T$$

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Time of flight, } T = \frac{2u \sin \theta}{g}$$

$$R = u \cos \theta \times \frac{u \sin \theta}{g}$$

$$R = \frac{\sin \theta u^2}{g} \quad (\text{or}) \quad R = \frac{u^2 \tan \theta}{g \times (1 + \tan^2 \theta)}$$



→ For max,

$$\text{Range, } R = \sin 2\theta = 1 \quad ; \quad \theta = 45^\circ = \frac{u^2}{g}$$

$$\text{Height, } h = \sin^2 \theta = 1 \quad ; \quad \theta = 90^\circ = \frac{u^2}{2g} \quad u^2/2g$$

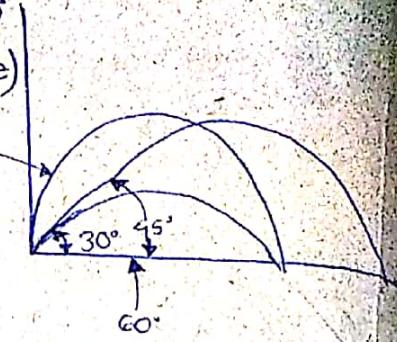
$$\text{Flight, } T = \sin \theta = 1 \quad ; \quad \theta = 90^\circ = u/g$$

$$\rightarrow S = ut + \frac{1}{2}at^2$$

$$\text{Horizontal disp. } x = u \cos \theta t$$

$$\text{Vertical } y, \quad y = u \sin \theta t - \frac{1}{2}gt^2$$

• Range is same for the angle of  $\theta$  &  $90^\circ - \theta$   
 " " " " " " " " " " " " " " " "  $(45^\circ - \theta) \approx (45^\circ + \theta)$



Relat'n b/w 'R' & 'h'

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$R = \frac{u^2 \sin \theta \cos \theta}{g}$$

$$\frac{h}{R} = \frac{\frac{u^2 \sin^2 \theta}{2g}}{\frac{u^2 \sin \theta \cos \theta}{g}} = \frac{\sin \theta}{2 \cos \theta}$$

$$\frac{h}{R} = \frac{\tan \theta}{2}$$

$$R \cdot \tan \theta = 4h$$

$$R = 4h \quad ; \quad h = R/4 \quad \{ \theta = 45^\circ \}$$

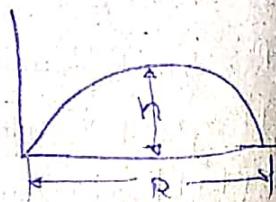
→ When the range of projectile is double to  $R_{\max}$ , then  
 Range is

$$R = 2H$$

$$R \tan \theta = 4h$$

$$2 \tan \theta = \frac{4h}{R}$$

$$\tan \theta = 2$$



$$R = \frac{u^2 \sin \theta}{g} \cdot 2$$

$$= \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{4u^2}{5g}$$

Relat<sup>n</sup> b/w h & T

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$(T = \frac{2 \pi \sin \theta}{g})^2$$

$$\frac{h}{T^2} = \frac{\frac{u^2 \sin^2 \theta}{2g}}{\frac{4 \pi^2 \sin^2 \theta}{g^2}}$$

$$\frac{h}{T^2} = \frac{g}{8}$$

$$g T^2 = 8h$$

$$h = \frac{g T^2}{8}$$

→ Relat<sup>n</sup> b/w R & T

R = u cos θ T

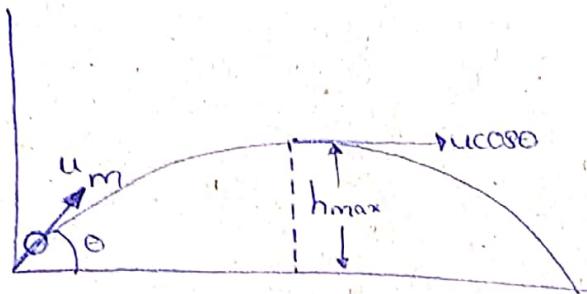
→ Energy of a projectile

At the ground

$$K.E = \frac{1}{2} m u^2$$

$$P.E = 0$$

$$T.E = \frac{1}{2} m u^2$$



At H<sub>max</sub>

$$1. K.E = \frac{1}{2} m v^2$$

$$\text{But } v = u \cos \theta$$

$$= \frac{1}{2} m u^2 \cos^2 \theta$$

2. Potential energy

$$P.E = mgh$$

$$\text{but } h = \frac{u^2 \sin^2 \theta}{2g}$$

$$P.E = \frac{m g u^2 \sin^2 \theta}{2g}$$

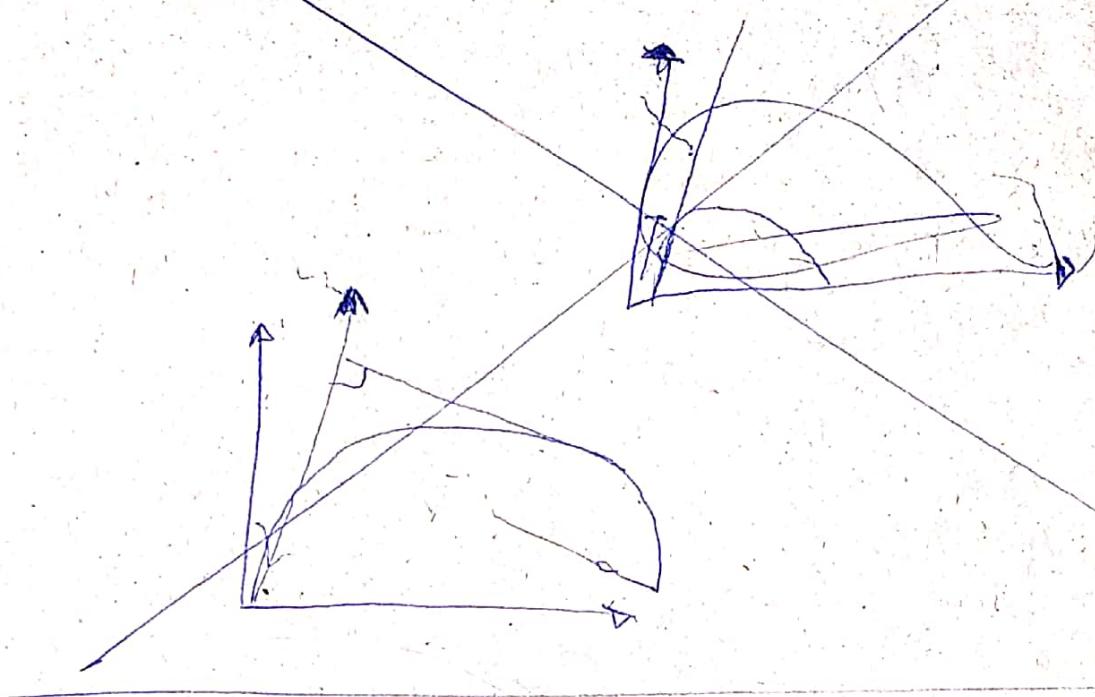
$$P.E = \frac{1}{2} m u^2 \sin^2 \theta$$

→ If θ = 45°

$$K.E = P.E = \frac{1}{2} \cdot \frac{1}{2} m u^2$$

$$= \frac{1}{2} T.E$$

→ When a body is projected with a velocity  $v$  and at angle  $\theta$ , the time taken by it to keep perpendicular to initial direction.

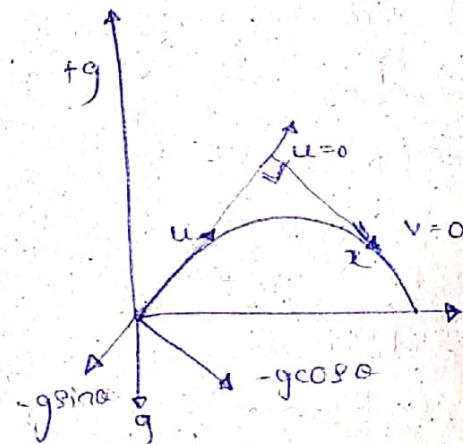


### 1. Time taken

$$v = u + at$$

$$0 = u - g \sin \theta t$$

$$\therefore t = \frac{u}{g \sin \theta}$$



### 2. Velocity

$$v = u + at$$

$$v = 0 - g \cos \theta \cdot \frac{u}{g \sin \theta}$$

$$v = -u \cot \theta$$

$$v = -u \cot \theta$$

An aeroplane is flying horizontally with 360 km/hr releases a bomb on to a target which is 200m away

$$\rightarrow h = \frac{1}{2} \times g \times \frac{x^2}{u^2}$$

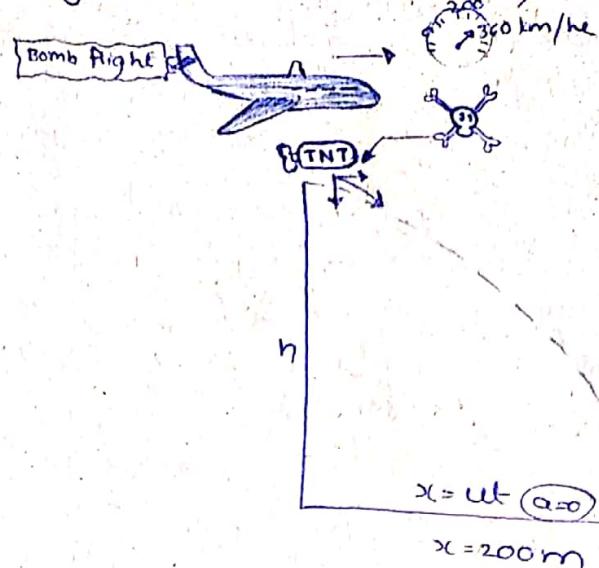
$$= \frac{1}{2} \times 9.8 \times \frac{200^2}{100 \text{ m/s} \times 100 \text{ m/s}}$$

$$= 19.6 \text{ m}$$

$$t = \frac{x}{u}$$

$$= \frac{200}{100}$$

$$= 2 \text{ sec}$$



→ A car jumps from the top of tower at height of 119.6m on to another tower height of 100m which are 30m apart.

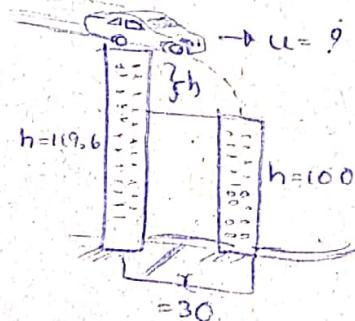
$$h = \frac{1}{2} \times g \times \frac{x^2}{u^2}$$

$$119.6 = \frac{1}{2} \times 9.8 \times \frac{900}{u^2}$$

$$u^2 = \frac{9.8 \times 900}{2 \times 119.6}$$

$$\approx 1225$$

$$\approx 35 \text{ m/s}$$



→ A stone is projected horizontal with 20m/s from the top of a tower, it hits the ground with a point which is joined makes 45°, then height.

$$h = \frac{1}{2} \times g \times \frac{x^2}{u^2}$$

$$h = \frac{1}{2} \times 9.8 \times \frac{h^2}{20^2}$$

$$\frac{400 \times 2}{196}$$

$$h = 80 \text{ m}$$

$$\tan 45^\circ = \frac{h}{x}$$

$$x = h$$



$$\cos 45^\circ = \frac{h}{u}$$

→ In the above problem, the velocity at ground is

$$v = \sqrt{u^2 + 2gh}$$

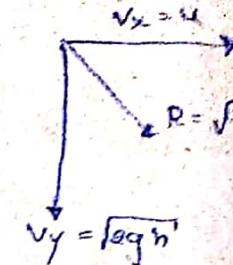
$$= \sqrt{20^2 + 2 \times 40}$$

$$\checkmark = 60$$

$$\sqrt{400 + 1600}$$

$$= \sqrt{20^2 + 400}$$

$$20\sqrt{1+4}$$



$$\sqrt{7} = 2.6457513$$

$$\sqrt{5} = 2.2360678$$

→ The horizontal & vertical disp. of projectile

$$x = 6t \quad \text{then } u = ?$$

$$y = 8t - 5t^2$$

$$x = u \cos \theta t$$

$$\Rightarrow u \cos \theta = 6$$

$$y = u \sin \theta t - \frac{1}{2}gt^2$$

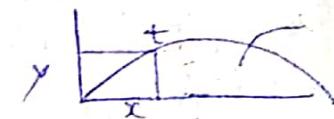
$$\Rightarrow u \sin \theta = 8$$

$$x^2 + y^2 \Rightarrow$$

$$u^2 \cos^2 \theta + u^2 \sin^2 \theta = 6^2 + 8^2$$

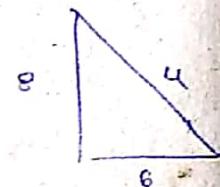
$$u^2(1) = 100$$

$$u = 10$$



$$\cos \theta = 6/u$$

$$\sin \theta = 8/u$$



$$\Rightarrow u = \sqrt{8^2 + 6^2}$$

$$= \sqrt{100}$$

$$= 10$$

$$\rightarrow \bar{u} = 3i + 4j \text{ m/s}$$

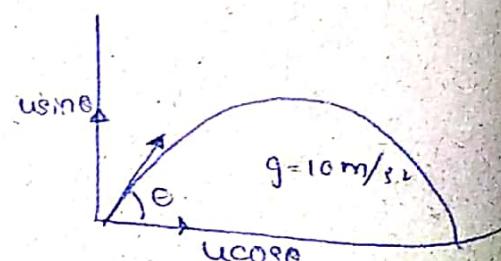
$$= u \cos \theta i + u \sin \theta j$$

∴

$$u \cos \theta = 3$$

$$u \sin \theta = 4$$

$$T = \frac{\omega \sin \theta}{g} = \frac{2 \times 4}{10} = 0.8s$$



$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{16}{20} \Rightarrow 0.8 \text{ m}$$

$$R = u \cos \theta t$$

$$= 8 \times 0.8$$

$$= 6.4 \text{ m}$$

→ The min velocity of a projectile eq/ to the half of the initial velocity, the angle of projection is \_\_\_\_\_

$$\cos \theta = \frac{1}{2} u$$

$$\theta = 60^\circ$$

→ The min velocity of a projectile is 8 m/s. It is in air for 6 seconds. its range is \_\_\_\_\_

$$u \cos \theta = 8$$

$$R = u \cos \theta \times t$$

$$= 8 \times 6$$

$$= 48 \text{ m}$$

→ The range ~~eq~~ 'R' is same for the angle of  $\theta + 90^\circ - \theta$ .  
the ratio of Time of flight

$$R = u \cos \theta \times T_1$$

$$R = u \sin \theta \times T_2$$

$$\frac{T_1}{T_2} = \frac{u \sin \theta}{u \cos \theta}$$

$$\frac{T_1}{T_2} = \tan \theta$$

$$T_1 = \frac{2u \sin \theta}{g}$$

$$T_2 = \frac{2u \sin(90^\circ - \theta)}{g}$$

$$\frac{T_1}{T_2} = \tan \theta$$

→ In the above the ratio of  $H_{\max}$  is

$$H_{\max 1} = \frac{u^2 \sin^2 \theta}{2g}$$

$$H_{\max 2} = \frac{u^2 \sin^2(90^\circ - \theta)}{2g}$$

$$H_1 : H_2 = \tan^2 \theta$$

→ " " " the relat' among R, h,  $h_2$

$$R = \frac{4h}{\tan \theta}$$

$$R \tan \theta = 4h \quad \text{--- (1)}$$

$$R \tan(90 - \theta) = 4h$$

$$\frac{R \tan}{\tan \theta} = 4h \quad \text{--- (2)}$$

$$(1) \times (2)$$

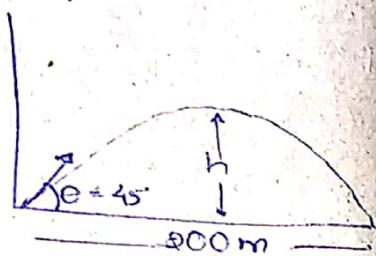
$$R^2 = 16h_1 h_2$$

$$R = 4\sqrt{h_1 h_2}$$

→ The range of projectile is 800m when it is projected with  $45^\circ$

$$\cancel{\frac{R \tan \theta}{\tan \theta}} = R \tan \theta = 4h$$

$$h = 50m$$



→ The range of projectile is 1.5km (1500m) when the angle of projection is  $15^\circ$ , the angle is made  $45^\circ$ , the range

$$\cancel{\frac{1500 \tan 15^\circ}{1500 \tan 45^\circ} = \frac{4h}{4h}}$$
$$\cancel{1500 (\tan 15^\circ - \tan 45^\circ) = 0}$$

$$R_1 = 1500m \quad \alpha_1 = 15^\circ$$

$$R_2 = x \quad \alpha_2 = 45^\circ$$

$$R = \frac{4h \sin 2\alpha}{g}$$

$$R \propto \sin 2\alpha$$

$$R / 1500 = \sin 2\alpha / 2$$

$$x = \sin 90^\circ / 2$$

$$x / \frac{1}{2} = 1500$$

$$x = 3000$$

$$= 30km$$

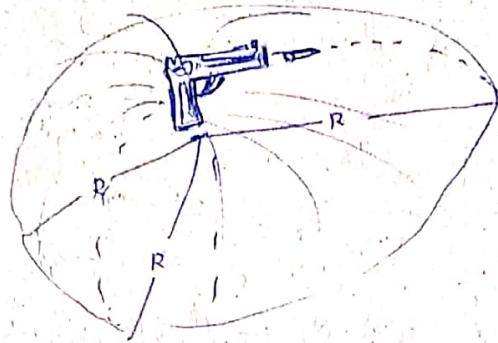
$$= 3km$$

→ A m/c gun fires the bullet in all directions with a velocity 'u', the max. area covered on the ground by the bullet is

$$R = u \cos \theta \rightarrow R_{\max} = \frac{u^2}{g}$$

$$\text{Area} = \pi u^2 \cos^2 \theta$$

$$\begin{aligned} \text{Area} &= \pi \left[ \frac{u^2}{g} \right]^2 \\ &= \frac{\pi u^4}{g^2} \end{aligned}$$



→ A ball is projected with [20m/s] at angle of  $60^\circ$ , the velocity of boy who runs to catch the ball when it falls on the ground is

$$u = 20 \text{ m/s}$$

$$u \cos \theta = 10 \text{ m/s}$$

NOTE:



→ A ball is thrown from one player to another player its max. height is, Time = 2sec

$$T_f = \frac{u \sin \theta}{g} = 2 \quad H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$u \sin \theta = g T$$

$$\begin{aligned} H_{\max} &\Rightarrow \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \frac{g}{2} \\ &= 4.9 \text{ m} \end{aligned}$$

$$g T^2 = 8h$$

$$g \times 4 = \frac{3}{2} h$$

$$\begin{aligned} h &= g/2 \\ &= 4.9 \end{aligned}$$

→ The max. Range of a projectile is numerically equal to initial velocity.

$$R_{\max} \rightarrow \frac{u^2 \sin 2\theta}{g} = u$$

$$\rightarrow \frac{u^2}{g} = x$$

$$u = g$$

$$u = 9.8 \text{ m/s}$$

→ A body is projected with an initial velocity  $u$  at an angle  $\theta$ . After one second, it is at an angle of  $45^\circ$ . After 2 seconds, it is horizontal.

$$T_f = 2 + 2 = 4 \text{ sec}$$

$$T = \frac{u \sin \theta}{g}$$

$$4 = \frac{u \sin \theta}{10}$$

$$u \sin \theta = 40$$

$$\tan 45^\circ = \frac{V_y}{V_x}$$

$$1 = \frac{V_y}{V_x}$$

$$V_y = V_x$$

$$V_y = u \cos \theta$$

$$V_y = u \sin \theta - g t \quad [u - gt]$$

$$V_y = u \sin \theta - (10 \times 1)$$

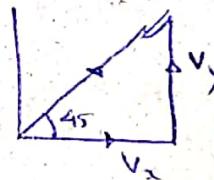
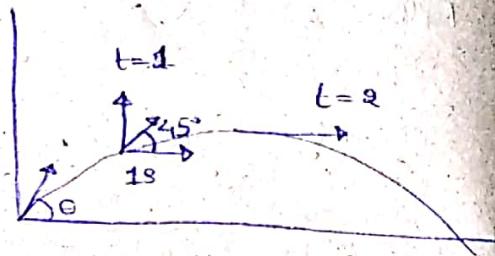
$$V_y = 40 - 10$$

$$V_y = u \cos \theta = 10 = V_x$$

$$u \sin \theta = 40$$

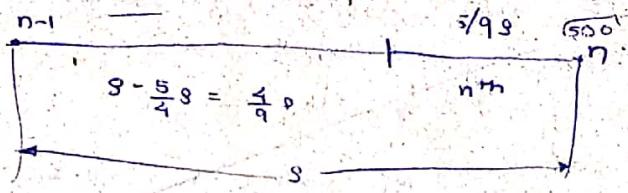
$$u \cos \theta = 10$$

$$u = 10\sqrt{5}$$



$$S = \frac{1}{2} \alpha n^2$$

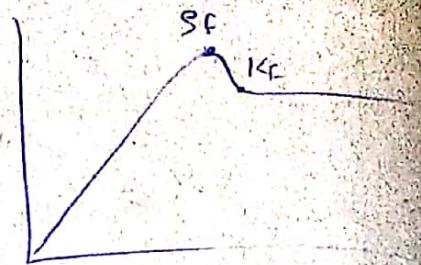
$$\frac{4}{9} S = \frac{1}{2} \alpha (n-1)^2$$



## Friction:

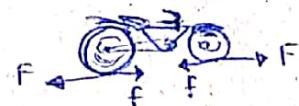
~~Friction is an external force which acts b/w two surfaces & which opposes the motion of body.~~

~~When a man is skating on ground, it acts in the direction of motion of man.~~



- Friction is an external force which acts b/w two surfaces and which opposes the motion of body.
- When a man is skating on ground, the friction acts in direction of motion of man.
- For a cycle

In forward direction for the back wheel & backward direction for front wheel



- When the pedalling is stopped, the friction acts in the backward direction for both the wheels

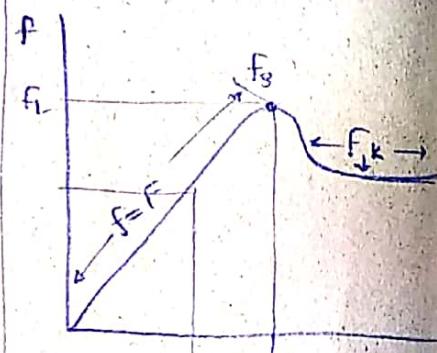
→ Friction is 3 types :

1. Static friction
2. Kinetic friction
3. Rolling friction

→ 1. Static friction is a self-adjusting force

• The max. static friction is called limiting friction [limiting eq/bm state (or) ready to ready to start].

• The min. ext. force required to start the body is limiting friction



→ Advantages of friction

1. Without friction we can't hold the things
2. " " " m/c can't be run

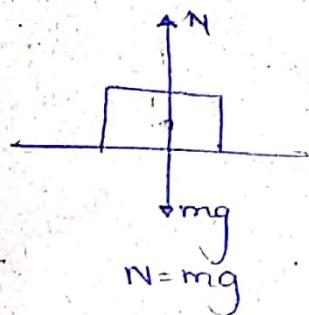
→ Disadvantages

1. η of engine ↓
2. Heat & wear takes place in the m/c

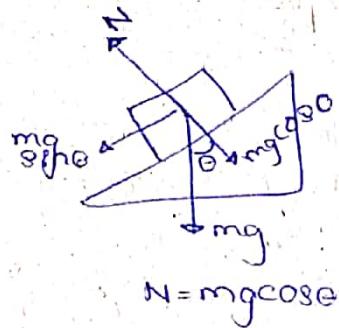
→ Methods to reduce friction

1. Polishing
2. Using lubricating oils
3. ball bearings
4. Using anti-friction mtl [steel]
5. Streamlining shape of a vehicle [aerodynamic shape]

- When the surface is moderately polished
- When the surface is heavily polished the friction increases due to adhesive force
- Normal react<sup>n</sup>: The force applied by the surface on the body in  $\perp^{\circ}$  to the surface is called Normal react<sup>n</sup>



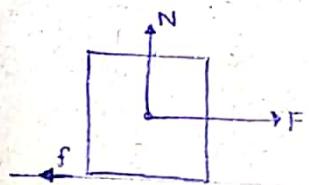
Horizontal



Inclined surface

→  $f \propto N$

→  $f$  ~~is~~ independent of area of contact except Rolling friction



$f \propto N$

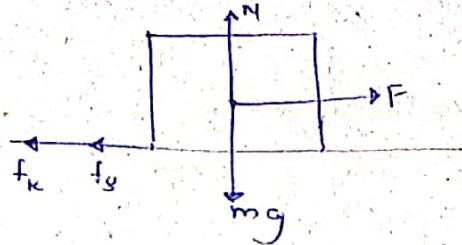
$f = \mu N$

$\mu = \frac{f}{N}$

$\mu_s > \mu_k > \mu_r$	
$F_s = \mu_s N$	$F_k = \mu_k N$
	$F_r = \mu_r N$

If the Normal react<sup>n</sup> doubles, the friction becomes double but coefficient of friction remains same (depends on nature of the surface).

→ Motion of a body on a horizontal surface



At rest  $F = f_s$  [Self adjusting force]

The min. ext. force required to start the body

$$F = f_s = f_g = \mu_s N = \mu_s mg = 400N$$

The min. force required to move the body uniformly

$$F = f_k = \mu_k N = \mu_k mg = 300N$$

IF  $F > F_k$  then the body moves with accn

Static



Net Force

$$F = f_g$$

$$F - f_g = ma$$

$$\boxed{a = \frac{F - f_g}{m}}$$

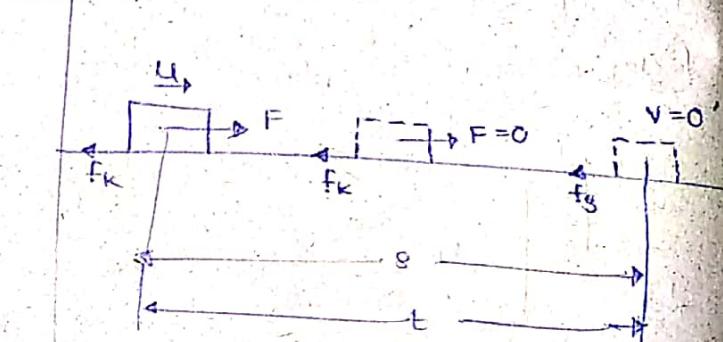
(accn)

$$F = 350, f_g = 300$$

$$m = 10$$

$$\text{then } a = 5 \text{ m/s}^2$$

Kinetic



Net Force

$$0 - f_k = ma$$

$$f_k - \mu_k mg = ma$$

$$\boxed{a = -\mu_k g} \quad (\text{Retardation})$$

For stopping dist.

$$v^2 - u^2 = 2as$$

$$0 - u^2 = 2(-\mu_k g)s$$

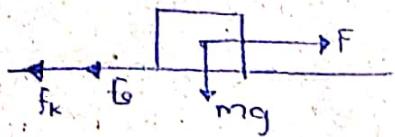
$$\boxed{s = \frac{u^2}{2\mu_k g}}$$

Time taken,  $v = u + at$

$$\Rightarrow u - \mu_k g t = 0$$

$$\boxed{t = \frac{u}{\mu_k g}}$$

→ When a body is started with a force & the same force is continued, then the accn<sup>2</sup> of the body is



$$F = f_g = \mu_s mg = f_L$$

$$f_k = \mu_k mg$$

Net force

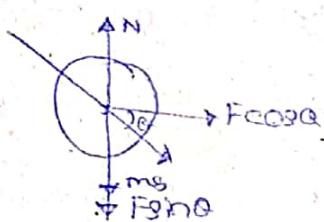
$$F - f_k = ma$$

$$N_s mg - N_k mg = ma$$

$$a = g(\mu_s - \mu_k)$$

→ It is easy to pull a solid than pushing it. bcoz while pulling net W +  $\uparrow \alpha f_g$   
" pushing " " "  $\uparrow \alpha f_k$

• Pushing

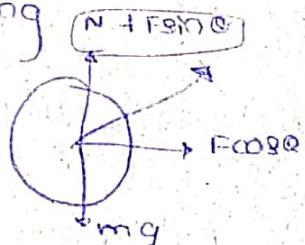


$$N = mg + F\sin\theta$$

$$f_R = \mu R N$$

$$f_R = \mu_R [mg + F\sin\theta]$$

• Pulling



$$N + F\sin\theta = mg$$

$$N = mg - F\sin\theta$$

$$f_R = \mu_R [mg - F\sin\theta]$$

Angle of friction :

The angle b/w contact force & normal reactn is called  
[Resultant of  $N$  &  $f_s$ ]

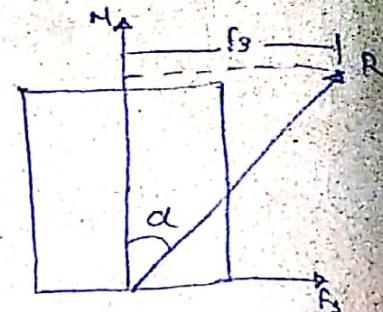
angle of friction or limiting angle

$R$  = contact force

$$\tan \alpha = \frac{f_s}{N}$$

$$\text{but } \frac{f_s}{N} = \mu_s$$

$$\tan \alpha = \mu_s$$



Contact force

$$R = \sqrt{N^2 + f_s^2}$$

$$R = N \sqrt{1 + \mu_s^2}$$

→ A block of 10kg is placed on a horizontal surface ( $\mu_s = 0.5$ )  
the min. force is required to start the block.

$$F = \mu_s \times m \times g$$

$$= 0.5 \times 10 \times 9.81 \quad [\text{kg} \times \text{m/s}^2]$$

$$= 49.5 \text{ N}$$

→ A block of mass is 20kg is dragged on a horizontal surface  $\mu_k = 0.2$  with a force of 59.2N If  $a = ?$

~~$$a = g(\mu_s - \mu_k)$$~~

~~$$= 9.8$$~~

$$\mu_s = \frac{F}{m \times g}$$

$$= \frac{59.2}{20 \times 9.81}$$

$$F - \mu_k mg = ma$$

$$a = \left( \frac{F - \mu_k mg}{m} \right)$$

$$= \frac{[59.2 - (0.2 \times 20 \times 9.81)]}{20}$$

$$= 1 \text{ m/s}^2$$

$$\frac{59.2 - 39.2}{20} = 1$$

→ A block of mass 10kg is dragged on a horizontal surface of  $\mu = 0.2$  with  $a = 2\text{m/s}^2$ . The work done in 5 seconds is (1)

$$A) N = F \times s$$

$$= \mu mg \times v \times t$$

$$= 0.2 \times 10 \times 9.8 \times 2 \times 5$$

$$= 0.2 \times 9.8 \times 100$$

$$= 9.8 \times 50$$

$$= 196\text{J}$$

→ A block of mass = 20kg is placed on a horizontal surface of  $\mu_s = 0.5$ . A force of 20N is applied on the body. The static friction is

$$m = 20\text{kg} \quad \mu_s = 0.5 \quad F = 20\text{N}$$

$$f_s = f_L = \mu_s mg$$

$$= 0.5 \times 20 \times 9.8$$

$$= 9.8 \times 10$$

$$= 98\text{N}$$

$$F < F_L \quad [\text{Rest}]$$

$$F = f = 20\text{N}$$

A car is moving on's path of radius 100m &  $\mu_s = 0.2$ , the  $v_{max}$  to move safely and angular velocity.

$$\text{Ans} \quad r = 100\text{m}$$

$$\mu_s = 0.2$$

C.F.F. = friction

$$\frac{\cancel{m}v^2}{r} = \mu_s mg$$

$$N = \sqrt{\mu_s \times g}$$

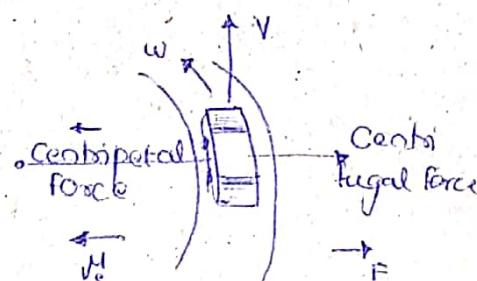
Linear velocity

$$\text{but } v = r\omega$$

$$r\omega = \sqrt{\mu_s \times g}$$

$$\omega = \sqrt{\frac{\mu_s g}{r}}$$

Angular velocity



$$V = \sqrt{0.2 \times 100 \times 9.8} \\ = \sqrt{196} \\ = 14 \text{ m/s}$$

$$\omega = \sqrt{\frac{0.2 \times 9.8}{100}}$$

$$= \sqrt{\frac{19.6}{100}}$$

$$\omega = \sqrt{\frac{14}{100}} = 0.14 \text{ rad/s}$$

→ A ball is moving with 9.8 m/s on a horizontal surface of  $\mu_k = 0.2$ . It comes to rest after a distance of —

$$u = 9.8 \text{ m/s}$$

$$\mu_k = 0.2$$

$$\Rightarrow s = \frac{u^2}{2\mu_k g}$$

$$= \frac{9.8 \times 9.8}{2 \times 0.2 \times 9.8}$$

$$= 24.5 \text{ m}$$

$$\Rightarrow t = \frac{u}{\mu_k g}$$

$$= \frac{9.8}{0.2 \times 9.8}$$

$$= 5 \text{ s}$$

→ On a horizontal surface  $\mu_s = 0.5$ ,  $\mu_k = 0.3$ . A block is started with a force and same force continued, then its  $a =$  —

$$\mu_s = 0.5$$

$$\mu_k = 0.3$$

$$F = f_s = f_k = mg \times \mu_s$$

$$f_k = \mu_k mg$$

Net force

$$F - f_k = ma$$

$$\mu_s mg - \mu_k mg = ma$$

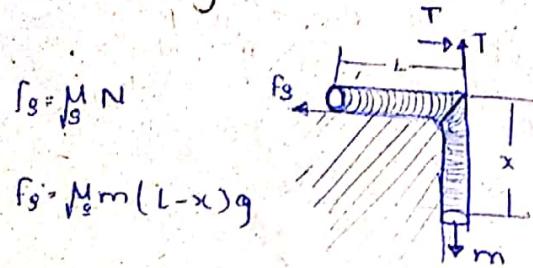
$$a = (0.5 - 0.3) \times 9.8$$

$$= 0.2 \times 9.8$$

$$= 1.96 \text{ m/s}^2$$

A rope of length 'L' is placed on a horizontal table,  $\mu_s$

The max. length of the rope can be hanged over the edge without slipping is



$$m_x g = T = f_s = M_s (L-x) g$$

$$x = \frac{M_s}{M_s + M_0} (L-x)$$

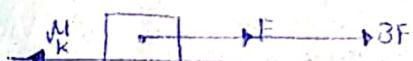
~~$$\frac{x}{L-x} = \frac{\mu_s}{\mu_s + \mu_0}$$~~

$$x = \frac{\mu_s L}{\mu_s + \mu_0} = \frac{M_s}{M_s + M_0} L$$

$$x(1 + \mu_0) = M_s L$$

$$x = \frac{M_s L}{1 + \mu_0}$$

A body of mass  $m_1$  is dragged uniformly by a force  $F$ , the force is made  $3F$ , then its  $a = ?$



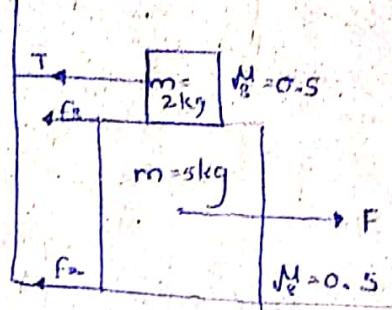
$$F = f_k$$

$$3F - f_k = ma$$

$$3F - 2F = ma$$

$$a = \frac{2F}{m}$$

Force required to move block  $m$ ,



$$F = f_k + F_2$$

$$f_1 = \mu_s m g = 0.5 \times (5+2) 9.8$$

$$f_2 = \mu_s m g = 0.5 \times (2) 9.8$$

$$F = 0.5 \times 9.8 (7+2)$$

$$= 16 \times 0.5 \times 9.8$$

$$= 44.1 \text{ N}$$

Angle of repose! The min. angle of inclination required by a body to be at limiting eq<sup>bm</sup> is called angle of repose.

$$\Rightarrow m g \sin \alpha = f_s$$

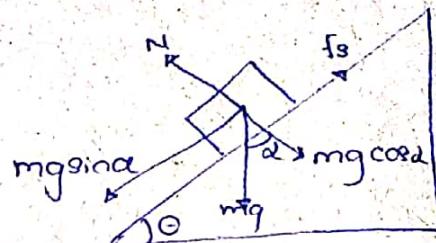
$$m g \cos \alpha = N$$

$$\boxed{① / ②}$$

$$\tan \alpha = F_s / N$$

$$\boxed{\tan \alpha = \mu_s} \quad \text{eq}^{\text{bm}}$$

$$\boxed{\tan \alpha = \mu_k} \quad \begin{matrix} \text{sliding} \\ \text{uniformly} \end{matrix}$$



$F_s = N$

Sliding

uniformly

→ Angle of repose = angle of friction

$\theta < \alpha \rightarrow$  the body will be at rest,  $f_s = m g \sin \theta$  (self adjusting)

$\theta = \alpha \rightarrow$  The body will be at limiting eq<sup>bm</sup>, or slides uniformly

$\theta > \alpha \rightarrow$  the body slides with accn

Motion of a body on an inclined rough surface:

i. Downward

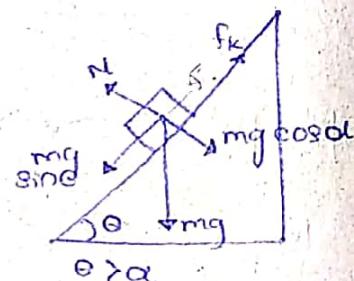
Net force

$$m g \sin \theta - f_k = m a$$

$$\text{but } f_k = \mu_k m g \cos \theta$$

$$m g \sin \theta - \mu_k m g \cos \theta = m a$$

$$\text{Accn: } \boxed{a = g [\sin \theta - \mu_k \cos \theta]}$$



$\theta > \alpha$

$$\boxed{a = g \sin \theta} \quad \text{- on smooth surface } \mu_k = 0$$

$$\boxed{\alpha = 0} \quad \text{if } \theta = 0^\circ$$

$$\boxed{\alpha = 90^\circ} \quad \text{if } \theta = 90^\circ$$

Time of descent

$$s = ut + \frac{1}{2} a t^2$$

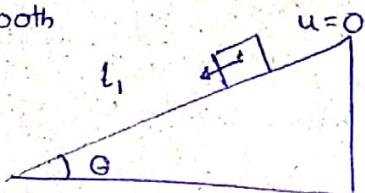
$$t = \sqrt{\frac{2s}{g}} \quad \Rightarrow \quad \cancel{s}$$

$$t = \sqrt{\frac{2s}{g(\sin\theta - \mu_k \cos\theta)}} \quad (1)$$

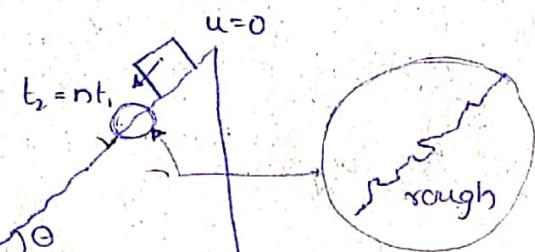
$$t = \sqrt{\frac{2s}{g\sin\theta}} \quad (\mu_k = 0) \quad (2)$$

When a body takes  $n$  times greater time to slide down on a inclined rough surface and on smooth surface, then  $\mu_k$  —

smooth



$$\theta = 0$$



$$t_1 = \sqrt{\frac{2s}{g\sin\theta}} \quad (1)$$

$$t_2 = nt_1 = \sqrt{\frac{2s}{g(\sin\theta - \mu_k \cos\theta)}} \quad (2)$$

$$(1) \% (2)$$

$$\left[ \frac{t_1}{nt_1} \right]^2 = \frac{\sin\theta - \mu_k \cos\theta}{\sin\theta}$$

$$\frac{1}{n^2} = 1 - \mu_k \frac{\cos\theta}{\sin\theta}$$

$$\mu_k = \left( 1 - \frac{1}{n^2} \right) \times \tan\theta$$

$$\mu_k = 1 - \frac{1}{n^2}$$

If  $\tan\theta = 45^\circ$

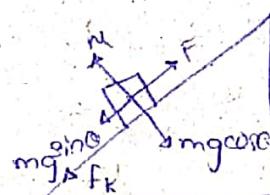
## 2. Upward

Max. force required to move

up the body uniformly

$$F = mgs\sin\theta + \mu_k mgs\cos\theta$$

$$F = mg(\sin\theta + \mu_k \cos\theta)$$



$$\text{If } F > mgs\sin\theta + \mu_k mgs\cos\theta$$

then body moves with accn

→ When body moved up & released ( $F=0$ )

Net force

$$0 - mgs\sin\theta + \mu_k mgs\cos\theta = ma$$

$$a = -g(\sin\theta + \mu_k \cos\theta)$$

On smooth surface,  $\mu_k = 0$

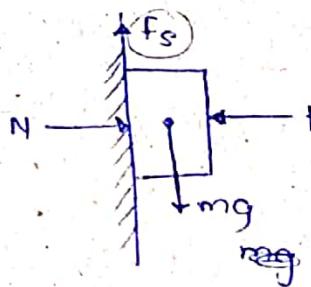
$$a = -g \sin \theta$$

$$a = 0 \quad \text{if } \theta = 0^\circ$$

$$a = -g \quad \text{if } \theta = 90^\circ$$

When a block is pressed against a wall

1. At eq. bm.



$$f_s = mg$$

$$\mu_s N = mg$$

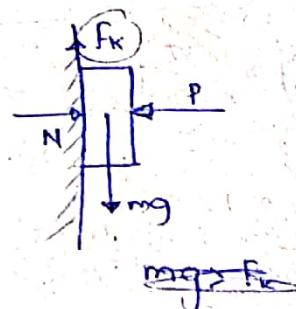
$$\mu_s P = mg$$

$$[N = P]$$

$$g = \frac{\mu_s P}{m}$$

$$P = \frac{mg}{\mu_s}$$

2. When slides down



$$mg - f_k = ma$$

$$a = \frac{mg - \mu_k P}{m}$$

A block is sliding on an inclined smooth surface of  $30^\circ$ . accn is \_\_\_\_\_

ANSWER

$$a = g \sin \theta$$

$$= 9.8 \times \sin 30$$

$$= 4.9$$

A block is started from rest on a smooth inclined plane. It travels ~~12.5 m~~ in 3rd sec, the angle of inclination is \_\_\_\_\_

ANSWER

$$S_n = u + a(n - \frac{1}{2})$$

$$a = g \sin \theta$$

$$12.25 = 9.8 \sin \theta \left( 3 - \frac{1}{2} \right)$$

~~12.25~~  
12.25  
~~9.8~~  
9.8

$$12.25 = 4.9 \times 5 \sin \theta$$

$$12.25 = 24.5 \sin \theta$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

A block is started from an inclined smooth surface, it takes 4 seconds to reach the ground, the time taken to cover 1st  $\frac{1}{4}$  dist. is \_\_\_\_\_

~~12.25~~  
12.25  
~~g sine~~  
g sine

$$S = ut + \frac{1}{2}at^2$$

$$S \propto t^2$$

$$\frac{S_1}{S_2/4} = \frac{t_1^2}{t_2^2}$$

$$t_2 = \sqrt{4^2/3}$$

$$t_2 = 2 \text{ seconds}$$

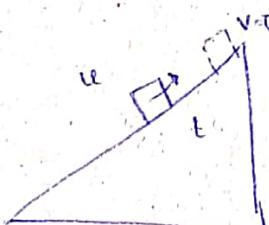
A block is pushed up an incline of smooth surface of  $30^\circ$  with a velocity of  $20 \text{ m/s}$ , its time of descent is \_\_\_\_\_

$$x = u + at$$

$$x = u - g \sin \theta t \quad [a = -g \sin \theta]$$

$$t = \frac{x - u}{-g \sin \theta}$$

$$t = \frac{u}{g \sin \theta}$$



3

A block is sliding down uniformly on an inclined surface of  $30^\circ$ . The angle is made so then the acc? is

$$\cancel{\alpha} = 30^\circ \quad \theta = 60^\circ$$

$$\tan 30^\circ = \mu_k$$

$$\frac{1}{\sqrt{3}} = \mu_k$$

$$\Rightarrow a = g(\sin \theta - \mu_k \cos \theta)$$

$$= 9.8 \left( \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}} \times \frac{1}{2} \right)$$

$$\frac{9.8}{2} \left[ \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2 \times 3} \right]$$

$$4.9 \left( \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} \right)$$

$$4.9 \times \frac{2\sqrt{3}}{3}$$

$$= \frac{9.8 \sqrt{3} \times \sqrt{3}}{3\sqrt{3}}$$

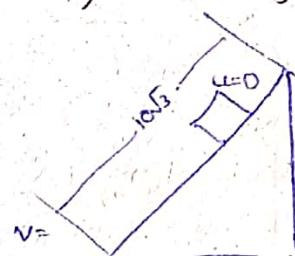
$$= \frac{9.8}{\sqrt{3}} \text{ m/s}^2$$

In the above problem, the length of inclination is  $10\sqrt{3}$  m  
the block is started from rest, its velocity at the ground is

$$v = \sqrt{2 \times \frac{9.8}{\sqrt{3}} \times 10\sqrt{3}}$$

$$= \sqrt{98}$$

$$= 14 \text{ m/s}$$



A body is started from rest on an inclined surface of an angle  $\theta$ . The lower half is rough, the body reaches the ground and stops, then  $\mu_k =$

$$v^2 - u^2 = 2as \quad [\text{for first half dist.}]$$

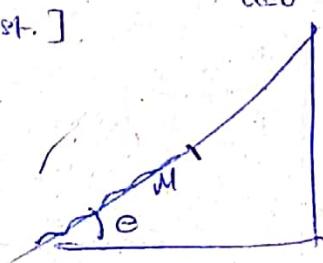
$u=0$

$$v^2 = 2g \sin \theta \times s$$

For second half dist.

$$v^2 = u^2$$

$$v^2 - u^2 = das$$



$$v=0$$

$$-u^2 = \alpha s$$

$$-\alpha g \sin\theta = \alpha(g \sin\theta - \mu_k \cos\theta) \times s$$

$$-\sin\theta = \sin\theta - \mu_k \cos\theta$$

$$\mu_k \cos\theta = \frac{\sin\theta + \sin\theta}{\cos\theta}$$

$$= \frac{2\sin\theta}{\cos\theta}$$

$$\mu_k = 2\tan\theta$$

A body going takes double the time to slide down on an inclined rough surface of  $45^\circ$ , then on smooth surface of same angle, then  $\mu_k$  is \_\_\_\_\_

$$t_1 = t_2$$

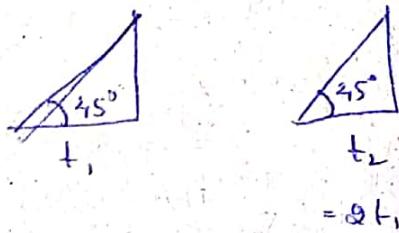
$$t_1 = \alpha t_2$$

$$\mu_k = \tan\theta \left(1 - \frac{1}{n}\right)$$

$$\tan\theta \left(1 - \frac{1}{2}\right)$$

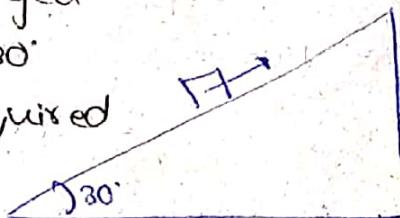
$$= \frac{3}{4} \tan\theta$$

$$\mu_k = \frac{3}{4} = 0.75$$



A block of mass  $10\text{kg}$  is dragged up on an incline surface of  $30^\circ$

If  $\mu_k = \frac{1}{5}$ , the min. force is required



$$F = mg \sin\theta + f_g$$

$$= mg \sin\theta + \mu_k mg \cos\theta$$

$$= mg$$

$$F = mg \sin\theta + f_g$$

$$= mg \sin\theta + \mu_k mg \cos\theta$$

$$= mg(\sin\theta + \mu_k \cos\theta)$$

$$10 \times 9.8 \times \left[ \frac{1}{2} + \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \right]$$

$$F = 98 \text{ N}$$

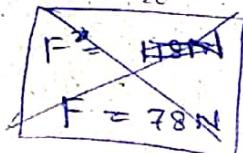
In the above problem the force required to move up the body with an accn of  $2 \text{ m/s}^2$  is \_\_\_\_\_

rat.  $F = mg(\sin\theta + \mu_k \cos\theta)$

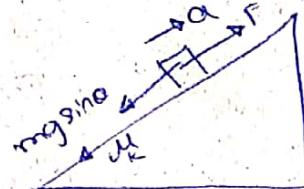
$$F = mg(\sin\theta) + mg\mu_k \cos\theta + ma$$

$$= m[g(\sin\theta + \mu_k \cos\theta) + a]$$

$$= \frac{98}{2} + (10 \times 2) = 98 + 20$$



$$118 \text{ N}$$



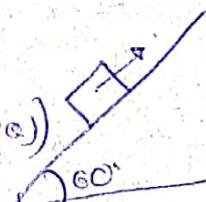
The force required to move up a body on an inclined surface of  $60^\circ$  is double to the force required prevent the body from sliding down then  $\mu_k$  -

$$\begin{matrix} F & = & \alpha F \\ \text{upward} & & = \text{static} \end{matrix}$$

$$mg(\sin\theta + \mu_k \cos\theta) = \alpha(-mg(\sin\theta + \mu_k \cos\theta))$$

$$\frac{\sqrt{3}}{2} + \mu_k \frac{1}{2} = -2 \times \left[ \frac{\sqrt{3}}{2} + \mu_k \frac{1}{2} \right]$$

$$\frac{\sqrt{3}}{2} + \mu_k \frac{1}{2} - 2 \frac{\sqrt{3}}{2} - \mu_k \frac{1}{2} = 0$$



$$\frac{\mu_k}{2} + \mu_k = \sqrt{3} - \frac{\sqrt{3}}{2}$$

$$\frac{3\mu_k}{2} = \frac{\sqrt{3}}{2}$$

$$\therefore \mu_k = \frac{1}{\sqrt{3}}$$

If time of descent on inclined surface of  $60^\circ$  is double to the time of ascent, then  $\mu_k = ?$

$$\frac{d}{2} = \alpha t_a$$

$$d = 2t_a$$

$$td = \alpha t_a$$

$$\sqrt{\frac{2s}{g(\sin\theta - \mu_k \cos\theta)}} = \sqrt{\frac{2s}{g(\sin\theta + \mu_k \cos\theta)}}$$

$$\frac{1}{2} \times \frac{2g}{g(\sin\theta - \mu \cos\theta)} \times \frac{1}{2} \times g(\sin\theta + \mu \cos\theta) = 0$$

$$\begin{aligned} \sin\theta + \mu \cos\theta &= 0 \\ \sin\theta - \mu \cos\theta &\\ \mu &= -\tan\theta \end{aligned}$$

$$\left. \begin{aligned} s &= \cancel{Nt} - \frac{1}{2} at^2 \\ &= -\frac{1}{2}(g(\sin\theta - \mu \cos\theta)) \\ s &= \frac{1}{2}(g(\sin\theta + \mu \cos\theta)) \end{aligned} \right\}$$

$$\sin\theta + \mu \cos\theta = 4(g(\sin\theta - \mu \cos\theta))$$

$$\begin{aligned} N &= \frac{3g \sin\theta}{5 \cos\theta} \\ &= \frac{3}{5} \tan\theta \\ &= \frac{3}{5} \frac{\sqrt{3}}{\sqrt{5}} \\ &= \frac{3\sqrt{3}}{5} \end{aligned}$$

$$t = \sqrt{\frac{2s}{g(\sin\theta + \mu \cos\theta)}}$$

A block of  $\text{2kg}$  is pressed against a wall of  $\mu_s = 0.5$ . The min force required for eqbm. and static friction.

$$\begin{aligned} \text{Ans} \quad 1. \quad f_x &= mg \\ &= 2 \times 9.8 \\ &= (1.96 \times 10) \text{ N} \\ &= 19.6 \text{ N} \end{aligned}$$

$$\begin{aligned} 2. \quad P &=? \\ P &= mg/\mu_s \\ &= 19.6/0.5 \\ &= 39.2 \\ P &= 39.2 \text{ N} \end{aligned}$$

# Work, Power & Energy

h

Force is a physical quantity which can change (or) try to change the state of body.

## Force units

Inertia units. ( $F=ma$ )

CGS -  $\text{g cm}/\text{s}^2$  - dynes

MKS -  $\text{kg. m}/\text{s}^2$  - newtons

FPS -  $\text{lb. ft}/\text{s}^2$  - poundals

1 lb<sub>in</sub> = pound

Gravitational units  
( $W=mg$ )

grav. wt = 980 dynes

kg wt = 9.8 N

lb wt = 32 poundal

$$1 \text{ N} = 10^5 \text{ dynes}$$

$$1 \text{ poundal} = 13,800 \text{ dynes}$$

## Newton's law of Motion:

1st law: Every body continues in state of rest (or) uniform motion in a straight-line unless a external force applied on it.

Every body ~~continues~~ continues in a state same state due to inertia.

Inertia is the property of body which opposes the change in the state.

Inertia of three types

Inertia of rest      Inertia of motion      Inertia of direct

Mass is the measurement of inertia.

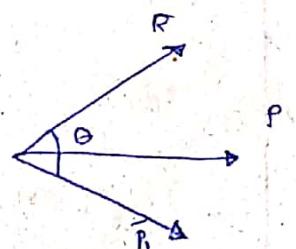
Newton's first law also called as law of inertia.

Momentum: The product of mass and ~~and~~ velocity ~~projectile~~ is called momentum.

units =  $\text{kg m/s}$

$$\bar{P} = m \bar{v}$$

$\bar{P}$  is vector



$$P = \sqrt{P_1^2 + P_2^2 + 2P_1 P_2 \cos\theta}$$

change in motion

~~ΔP~~

$$\Delta \vec{P} = \vec{P}_2 - \vec{P}_1$$

$$\Delta P = \sqrt{P_1^2 + P_2^2 + 2P_1 P_2 \cos\theta}$$

$$\Delta P = \sqrt{P_1^2 + P_2^2 - 2P_1 P_2 \cos\theta}$$

$$\text{If } \theta = 0^\circ$$

$$\vec{P}_1 = \vec{P}_2$$

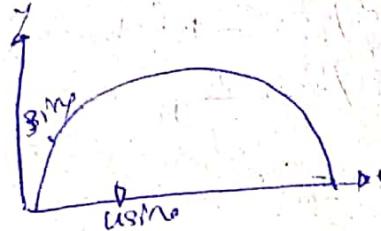
$$\Delta P = \Delta p = P_2 - P_1$$

$$\text{if } \theta = 180^\circ$$

$$\Delta P = P_2 + P_1$$

~~$$\Delta P = 2mv \sin\theta$$~~

$$\Delta P = 2mv \sin\theta$$



Second law:

$$F \rightarrow O \rightarrow \vec{P}_1 = mu$$

$$O \rightarrow \vec{P}_2 = mv$$

$$F \propto \frac{\vec{P}_2 - \vec{P}_1}{t}$$

$$F = \frac{dP}{dt}$$

$$F \propto \frac{dp}{dt}$$

$$\text{but } p = mv$$

$$F = k \frac{dp}{dt}$$

$$F = \frac{d}{dt}(mv)$$

$$[k=1]$$

$$= \frac{dmv}{dt} + m\frac{dv}{dt}$$

If  $m \neq 0$

$$h = 4$$

① If  $m$  is const. ( $dm=0$ )

$$\cos \frac{\sqrt{3}}{4}$$

$$F = m \frac{dv}{dt}$$

$$\frac{\sqrt{3}}{2}$$

$$\text{but } \frac{dv}{dt} = a$$

$$F = ma$$

② If  $v$  is const. ( $dv=0$ )

$$F = \frac{dm}{dt} v$$

Second law: The rate of change in momentum is a applied force & also follows the direction of the future.

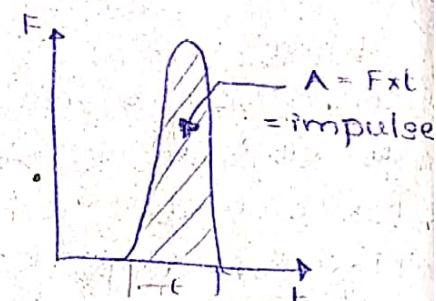
Impulse: When a large amount of force in very short interval of time, the change in momentum is called impulse.

$$F = \frac{dp}{dt}$$

$$F = \frac{mv - mu}{t}$$

$$I = Fxt \\ = mv - mu$$

units : N.s (or) kgm/s



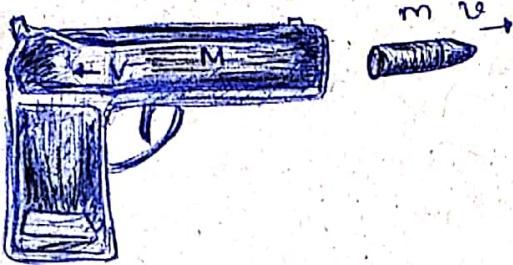
Third law: For "react" there is opp. & eq "react".

Rocket & jet engines works on third law of motion (or) law of conservation of linear momentum

law of momentum

Unless the ext. force applied on the sys. the total momentum of the sys. remains const.

Recoil of gun:



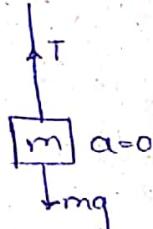
$$\text{initial } \oplus \quad \text{final } \oplus \\ m_1 u_1 + M_1 V_1 = m_2 u_2 + M_2 V_2$$

$$0 = m_2 u_2 + M_2 V_2$$

$$V_2 = -\frac{m_2 u_2}{M}$$

### Tension Concepts:

Rest or uniform velocity

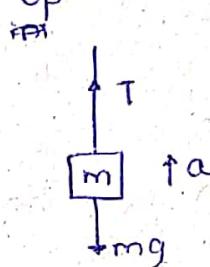


No force

$$T - mg = ma$$

$$T = mg$$

Upwards



Net force

$$T - mg = ma$$

$$T = mg + ma$$

Downwards



Net force

$$mg - T = ma$$

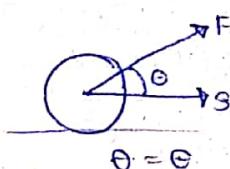
$$T = mg - ma$$

### Work:

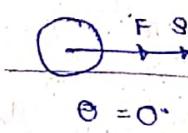
When a force is applied on a body to move through a certain disp. through in a direction of force, the work is said to be done.

• Work is dot product of force & disp.

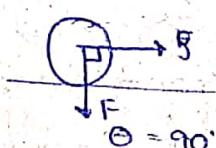
• " " a scalar



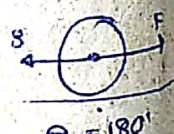
$$W = F \cdot s \cos \theta$$



$$W = F \cdot s$$



$$W = 0$$



$$W = -F \cdot s$$

$$\left. \begin{array}{l} \text{when } F=0 \\ S=0 \\ \theta=90^\circ \end{array} \right\} \rightarrow W \cdot D = 0$$

units

$$W = F \cdot s$$

(5)

CGS

dyne

erg

MKS

newton

Joule

FPS

Poundal ft

H-poundal

$$1 \text{ Joule} = 10^7 \text{ erg}$$

$$1 \text{ ft poundal} = 4.2 \times 10^5 \text{ erg}$$

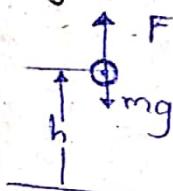
When a man is carrying a load on his head and moves on a level road, then W.D by the gravitational force is '0'.

When an electron is rotating around a nucleus, then the W.D by the electrostatic force is '0'.

When a pendulum is oscillating, then the workdone tension is zero.

### Types of work

Work done by lifting a body

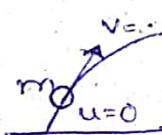


$$W = F \cdot s$$

$$\boxed{W = mgh}$$

W.D by gravity

$$\boxed{W.D = -mgh}$$

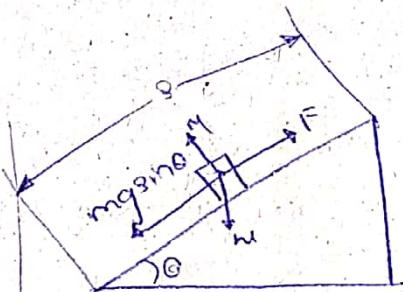


$$W = F \cdot s$$

$$= mas$$

$$= m \left( \frac{v^2 - u^2}{2s} \right) \times s$$

$$\boxed{W = \frac{1}{2}mv^2}$$



$$W = F \cdot s$$

$$\boxed{W = mgs \sin \theta}$$

+ if  $f_s$  adds

$$\boxed{W = mg(\sin \theta + \mu_k \cos \theta) \times s}$$

Work done on a moving body



$$W.D. = F \cdot s$$

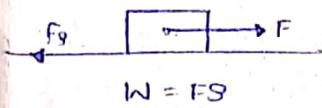
$$= mas$$

$$= m \left( \frac{v^2 - u^2}{2s} \right) s$$

$$= \frac{mv^2}{2} - \frac{mu^2}{2}$$

W.D. = change in K.E.

W.D against friction



$$W = FS$$

$$\text{but } W.D. = F S$$

$$\text{but } F = f_g = M g$$

$$f_k = M g$$

$$\boxed{W.D. = M g S}$$

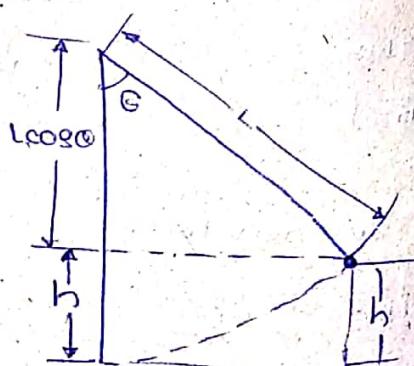
Sp

W.D. to pull a pendulum aside

$$W = mgh$$

$$\text{but } h = L - L \cos \theta$$

$$W = mgL(1 - \cos \theta)$$



W.D to compress (or) to stretch a spring

Force const. of the spring

$$k = \frac{F}{x}$$

$$F = kx$$

$$S = dx$$

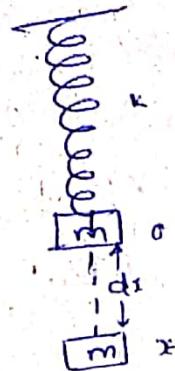
$$W = \int F S$$

$$= \int_0^x kx \cdot dx$$

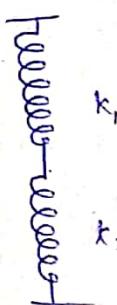
$$= k \left[ \frac{x^2}{2} \right]$$

$$W = \frac{1}{2} k x^2$$

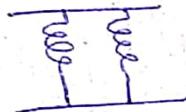
$$\rightarrow W = \int_{x_1}^{x_2} kx \cdot dx$$
$$= \frac{1}{2} k \left[ \frac{x_2^2 - x_1^2}{2} \right]$$



Spring in series



Parallel series



Spring in cut into parts

$$k_a = \frac{k(a+b)}{a}$$

$$k_b = \frac{k(a+b)}{b}$$

For two eq. parts

$$k' = 2k$$

For \$n\$ eq. parts

$$k' = nk$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = \frac{k_1 \times k_2}{k_1 + k_2}$$

Series

$$\frac{1}{K_{\text{eff}}} = \frac{1}{K_1} + \frac{1}{K_2}$$

Parallel

$$K_{\text{eff}} \Rightarrow K_{\text{eff}} = \text{const.}$$

$$K_{\text{eff}} = k_1, k_2 = k_1, k_2$$

$$\rightarrow K = k_1 + k_2$$

Cut into parts

$K_{\text{eff}} = \text{const.}$

$$K_{\text{eff}}(a+b) = K_a a = K_b b$$

$$K_a = \frac{K(a+b)}{a}$$

$$K_b = \frac{K(a+b)}{b}$$

Power: The rate of work done is called power and is a scalar.

Units

$$P = W/t$$

CGS

MKS

IPPS

erg/s

Joules/s  
(watt)

ft-pounds

- HP

$$1 \text{ calorie} = 10^3 \text{ joules/s}$$

$$1 \text{ H.P.} = 736 \text{ watts (metric)}$$

$$= 746 \text{ watts}$$

$$= 550 \times 32 \text{ ft-pounds/s}$$

$$1 \text{ metric HP} = 735.5 \text{ watts}$$

→ Powers of a m/c gun:

$$P = \frac{W}{t} = \frac{1}{2} n \cdot m v^2 / t$$

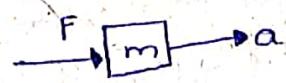
$n$  = no. of bullets fired

$m$  = mass of bullet

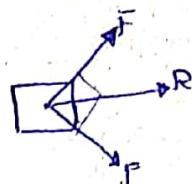
$$\text{efficiency } \eta = \frac{P_c}{P_r}$$

A force  $F$  is applied on a body, it moves with a accl<sup>n</sup> 'a'. Two forces of  $F$  each acts in  $\perp^{\text{re}}$ , then accl<sup>n</sup> of a body is

$$F = ma$$



$$R = \sqrt{F_1^2 + F_2^2} \\ = \sqrt{2} F$$



$$F \alpha a$$

$$\frac{F_1}{F_2} = \frac{a_1}{a_2}$$

$$\frac{F}{\sqrt{2} F} = \frac{a_1}{a_2}$$

$$a_2 = \sqrt{2} a$$

A body of mass 'm' kilogram is acted by a force, it moves through a disp.  $x$ ;  $t = \sqrt{x} - 3$  the force applied is ~~222~~  $\frac{2}{2}$  second is W.D

$$t = \sqrt{x} - 3$$

$$\sqrt{x} = t + 3$$

$$x = t^2 + 9 + 6t$$

$$\text{velocity. } \frac{dx}{dt} = 2t + 6 \text{ m/s}$$

$$\text{accn. } \frac{dv}{dt} = 2 \text{ m/s}^2$$

$$F = m \cdot a$$

$$= 1 \cdot 2$$

$$= 2 \text{ N}$$

$$W.D = F \cdot S$$

$$= 2 \text{ N} \times (1 + 9 + 12)$$

$$= 50 \text{ Nm}$$

$$= 50 \text{ J}$$

$$W.D = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 1 \times (4+6)^2$$

$$= \frac{100}{2} = 50 \text{ J}$$

Q. Second law: The rate of change in momentum of a particle due to applied force

Momentum of a particle is

$$P = t^2 + t + 5 \text{ (kg m/s)}$$

$$F = \frac{dp}{dt}$$

$$= \alpha t + \alpha \quad (t=2)$$

$$F = \alpha \times 2 + \alpha$$

$$= 6N$$

Mass of rocket is 5000kg is fixed vertically, the mass of the fuel to be burned per second to overcome the gravitational force to move with a velocity of 1000m/s<sup>2</sup>

$$F = \frac{dp}{dt}$$

$$= \frac{d(mv)}{dt}$$

$$= \frac{dm}{dt} v + m \frac{dv}{dt}$$

$$F = Mg$$

$$Mg = \frac{dm}{dt} v$$

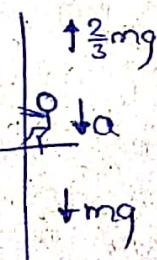
$$5000 \times 9.8 = \frac{dm}{dt} \times 1000$$

$$\frac{dm}{dt} = 49 \text{ m/s}$$

A man is sliding down along a rope whose breaking strength is  $\frac{2}{3}$  of its weight. The min. accn of man to slide down safely is

Net force

$$mg - \frac{2}{3}mg = ma$$



$$\frac{2}{3}mg = ma$$

$$a = \frac{2}{3}g$$

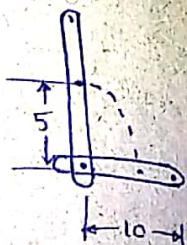
An uniform rod of mass 8kg and length 10m is lying on the ground, the W.D to make it vertical

$$W.D = mgh$$

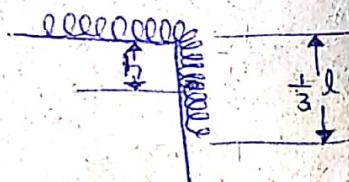
$$= 8 \text{ kg} \times 9.8 \times 5$$

$$= 400 \times 9.8 \times 40$$

$$= 329 \text{ J}$$



An uniform change of mass, length 'l' is placed on a table with  $\frac{1}{3}$  length hanging over the edge, the W.D to pull the hanging part to back on to the table is



$$W.D = mgh$$

~~$$= 8 \times 9.8 \times$$~~

$$= \frac{m}{3} \times 9.8 \times \frac{l}{6}$$

$$W = \frac{mgl}{18}$$

A water bucket has mass 2kg is hinged to a rope of mass 1kg and length 10m is lifted, then the W.D to be lifted the bucket with load

$$W = W_1 + W_2$$

$$= m_1 g h_1 + m_2 g h_2$$

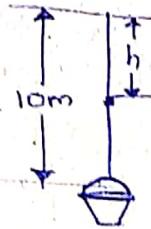
$$= 2 \times 9.8 \times 10$$

+

$$1 \times 9.8 \times 5$$

$$= 196 + 49$$

$$W = 245 \text{ J}$$



$$\begin{aligned} W.D &= mgh + \frac{1}{2}mv^2 \\ &= m(9.8 \times 10) + \frac{1}{2} \times 100 \\ &= 10(98 + 50) \\ &= 1480 \text{ J} \end{aligned}$$

A force acts on a body at an angle of  $60^\circ$  with horizontal, the W.D is 500J to move the body through 10m

$$\frac{500}{10}$$

$$\cos 60^\circ$$

$$W = FS \cos \theta$$

$$500 = F \times 10 \times \cos 60^\circ$$

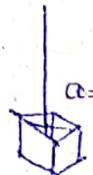
$$500 = F \times 5$$

$$F = 100 \text{ N}$$

A block of mass 10kg is lifted from rest with an accn' of  $1.2 \text{ m/s}^2$ , the W.D in 1st sec is

$$S = \frac{1}{2}at^2$$

$$\begin{aligned} F &= mg + ma \\ &= m(g+a) \end{aligned}$$



$$W.D = m(g+a) \cdot \frac{1}{2}at^2$$

$$= \underline{\underline{m}}$$

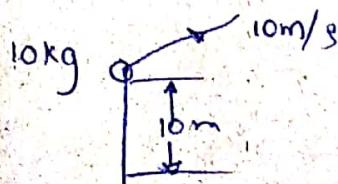
$$10(9.8 + 1.2) + \frac{1}{2} 1.2 \times \underline{\underline{4}}$$

$$= 10 \times 11.2 + 8.4$$

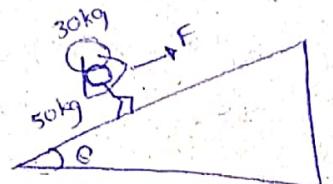
$$= 114.4 \text{ J}$$

$$= 864 \text{ J}$$

A body of mass 10kg is lifted through 10m and projected with  $10 \text{ m/s}$  the W.D is



A man of mass 50kg carries a load of 30kg on his head and moves up on an incline surface raising it in 10, the W.D to move 10m is



$$\sin \theta = \frac{1}{10}$$

$$W = F \times S$$

$$= mg \sin \theta S$$

$$= 80 \times 9.8 \times \frac{1}{10} \times 10$$

$$= \cancel{800}$$

$$= 784 \text{ J}$$

$$\begin{aligned} &80 \times 10 \\ &\frac{800}{4} \\ &= 200 \end{aligned}$$

$$\begin{aligned} &80 \times 10 \\ &\frac{800}{16} \\ &= 50 \end{aligned}$$

A block of mass 10kg is dragged on a horizontal surface of  $\mu = 0.2$  with a velocity of 2m/s. The work done in 5sec.

~~W=F×s~~

$$F = \mu N$$

$$= \mu mg$$

$$= 0.2 \times 10 \times 9.8$$

$$= 9.8 \times 0.2$$

$$= 19.6 \text{ N}$$

$$W.D = \bar{F} \cdot \bar{s}$$

$$= 19.6 \times 10$$

$$= 196$$

$$S = v \times t$$

$$= 2 \times 5$$

$$= 10$$

20cm is

$$W = \frac{1}{2} k x^2$$

$$= \frac{1}{2} \times 400 \times 0.04 \times 2$$

$$= 8 \text{ Nm}$$

200

W.D

$$x_1 = 20 \text{ cm}$$

$$x_2 = 20 + 20$$

$$= 40 \text{ cm}$$

$$= \frac{1}{2} k (x_2^2 - x_1^2)$$

$$= \frac{1}{2} \times 400 \left( \frac{16}{0.02^2} - \frac{4}{0.01^2} \right)$$

$$= 200 \times 12$$

$$= 2400 \text{ J}$$

$$W_2 = 3W_1 \quad \text{for same disp. extra}$$

A body of mass 2kg is moving with 4m/s, its velocity is doubled by applying a force, the W.D by force is —

$$m = 2 \text{ kg} ; u = 4 \text{ m/s} ; W = ? \\ v = 8 \text{ m/s}$$

$$\Delta K.E = W.D = \frac{1}{2} m(v^2 - u^2)$$

$$= \frac{1}{2} \times 2(64 - 16)$$

$$= 48 \text{ J}$$

A pendulum of mass 2kg and length 2m is pulled aside through an angle of 60°.

W.D =

$$= \theta = mgl(1 - \cos\theta)$$

$$W.D = 2 \times 9.8 \times 2 \left(1 - \frac{1}{2}\right)$$

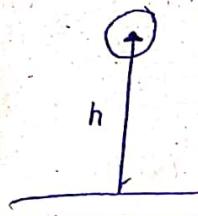
$$W.D = 19.6 \text{ J}$$

A spring of  $K = 400 \text{ N/m}$  is stretched through 20cm (0.2)m

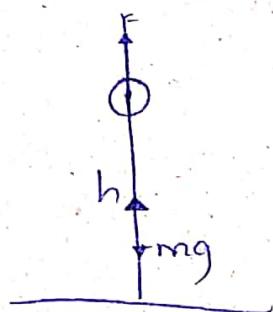
The W.D is

" " to stretch another

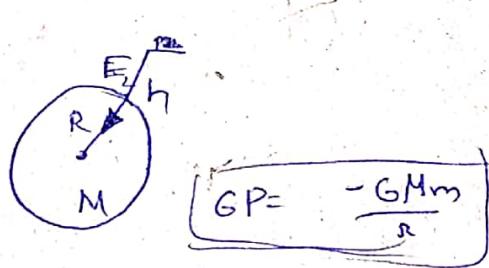
→ Potential energy:



Potential energy: The energy possessed by a body by virtue of position is called potential energy.



$$P = mgh$$



Flying object possesses both PE & KE

$$E_1 = -\frac{GMm}{R}$$

$$E_2 = -\frac{GMm}{R+h}$$

$$\Delta E = E_2 - E_1$$

$$= -\frac{GMm}{R+h} + \frac{GMm}{R}$$

$$\Delta E = GMm \left[ \frac{1}{R+h} + \frac{1}{R} \right]$$

$$= \frac{GMm}{R} \left[ \frac{h}{R+h} \right]$$

~~$$GMm \left( \frac{1}{R+h} + \frac{1}{R} \right)$$~~

~~cancel~~

$$\Rightarrow g = \frac{GM}{R^2} \Rightarrow \frac{GM}{R} = gR$$

$$\Delta E = mgh \left( \frac{1}{R+h} \right)$$

$$\Delta E = mgh \left( \frac{1}{R+rh} \right)$$

$$\Delta E = \frac{mgh}{1 + \frac{r}{h}}$$

IF  $h = R$

$$\Delta E = \frac{1}{2} M G R$$

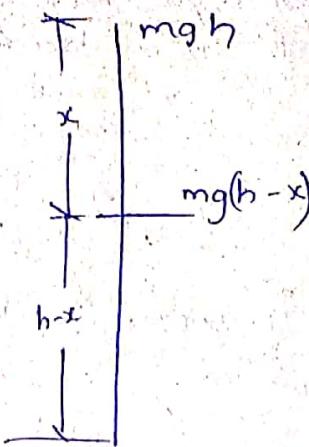
$G$  = Gravitational const.

$$= 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$M$  = mass of the earth

$R$  = Radius of the earth

$R = 6400 \text{ km}$



→ for a freely falling body, the loss P.E converts into K.E

$$K.E = mgh - mg(h-x)$$

$$= mgx$$

• When two protons brought closer, the P.E of the sys. ↑  
(Work is done on the sys.)

• When a proton and the electron brought closer, then  
P.E of the sys. ↓ (Work is done by the sys.)

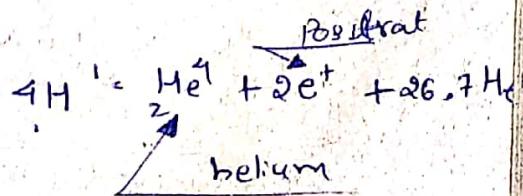
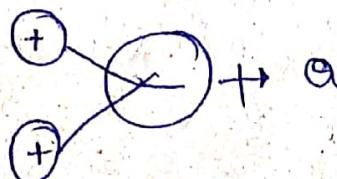
### → Sources of energy

1. Source of energy which cannot be renewed, is called  
conventional source of energy. Ex: - Wood, (Rahul), coal  
petroleum etc.

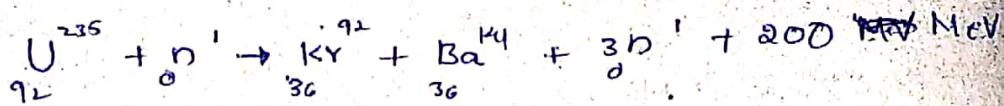
2. Source of energy which can be renewed.  
nonconventional " " " Ex: solar, hydro, wind energy  
etc.

Solar energy will be produced by the sun due to nuclear fusion react".

Fusion reaction requires high temperatures to overcome  
the coulombic ~~attractive~~ repulsive forces.



- In hydrogen bomb, nuclear fusion is the main reaction.
- In atom bomb, nuclear fission is the main reaction.
- " " " " uncontrolled chain reaction takes place



In power reactors, the controlled chain reaction takes place.

$\rightarrow$  A body of mass  $3\text{kg}$  is moving with a velocity of  $v = \bar{v}$ .  
 $3i + 4j \text{ m/s}$ , then  $K.E =$

$$\bar{v} = 3i + 4j$$

$$K.E = \frac{1}{2}mv^2$$

$$v = \sqrt{5}$$

$$v^2 = 25$$

$\rightarrow$  The K.E of a body is  $5\text{J}$ , its mass & velocity are double the K.E.

$$5\text{J} = \frac{1}{2} \times m \times v^2$$

$$K.E = \frac{1}{2} \times 2m \times 3v^2$$

$$= 16 \text{ J}$$

$$\frac{8}{E} = \frac{m_1 v_1^2}{2m_2 v_2^2}$$

$$E = 16\text{ J}$$

A

Ie  $\rightarrow$  The K.E of a body is double to K.E of a man whose mass is double to mass of body. The ratio of Velocities of man & body

$$\frac{K.E_2}{K.E_1} = 2$$

$$K.E = \frac{1}{2}mv^2$$

$$\frac{V_1}{V_2} = \sqrt{\frac{4}{1}}$$

$$\frac{E_1}{m_1} = \frac{E_2}{m_2}$$

$$= 2 : 1$$

$$V_2 : V_1 = 1 : 2$$

$$\frac{\partial E}{m_1} = \frac{E_2}{2m_1}$$

(Q) A gives B flowing with a velocity of 2m/s, the KF of water of 1m³ is \_\_\_\_\_

$$P = \frac{m}{v} ; m = P \times v \\ = 1000 \times 1 \\ = 10^3 \text{ kg}$$

$$K.E = \frac{1}{2} m v^2 \\ = \frac{1}{2} \times 10^3 \times 2^2 \\ = 2 \times 10^3 \text{ J}$$

→ Two metal spheres of same metals and radii 1:2 are moving with velocities of 2:1. The ratio of K.E

$$K.E_1 : K.E_2 \\ \frac{1}{2} m_1 v_1^2 : \frac{1}{2} m_2 v_2^2$$

$$\cancel{\frac{1}{2}} \times \cancel{\frac{4}{3}} \pi r^3 \times \rho \times v^2 : \cancel{\frac{1}{2}} \cancel{\frac{4}{3}} \pi r^3 \times \rho \times v^2$$

$$r^3 \cdot v^2 : r^3 \cdot v^2$$

$$1^3 \cdot 2^2 : (2)^3 \cdot 1^2$$

$$4 : 8$$

$$1 : 2$$

→ A shell of mass 5kg which is at rest explodes into two parts in which 1 part of 3kg moves with 20 m/s

Law of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$u = 0 = 3 \times 20 + 2v$$

$$-60 = 2v$$

$$v_2 = -30 \text{ m/s}$$

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 2 \times 900$$

$$= 900 \text{ J}$$

$$K.E \propto \frac{1}{m}$$

$$\therefore \frac{E_1}{E_2} = \frac{m_2}{m_1}$$

$$\cancel{\frac{1}{2} \times 3 \times 20 \times 20}{\cancel{\frac{10}{2} \times \frac{10}{3}}} = \frac{1}{3}$$

$$E_2 = 900 \text{ J}$$

→ A projected up with a K.E then at height 'h' the K.E & P.E are eq., the  $h_{max}$

~~height~~

$$P.E = K.E$$

$$mgh = \frac{1}{2}mv^2$$

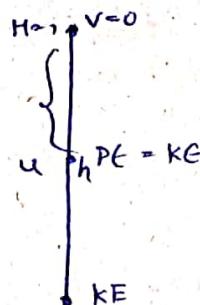
$$v^2 = egh$$

$$2as = v^2 - u^2$$

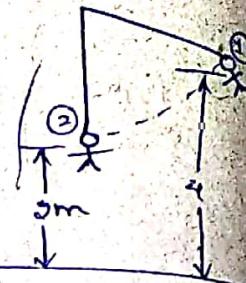
$$\cancel{mgS} = \cancel{-vgh} b$$

$$S = h$$

$$\Rightarrow H = h + h \\ = 2h$$



→ A boy in a swing has  $H_{max}$  4m & min height 2m above the ground, the max. velocity the spring scoring is



$$\Delta KE = \Delta PE$$

$$\frac{1}{2}mv^2 = mgh_1 - mgh_2$$

$$\frac{v^2}{2}$$

$$v^2 = 4 \times g \Rightarrow v = \sqrt{m/g} \times \sqrt{9.81} \\ = \sqrt{9.81}$$

→ A stone is projected which reaches  $h_{max}$  of 100m, the height at which the K.E becomes  $\frac{3}{4}$  of initial value is

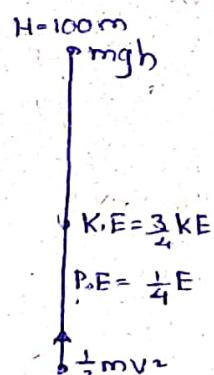
$$K.E = \frac{3}{4} K.E$$

$$P.E = \frac{1}{4} E$$

$$mgh = \frac{1}{4} \times mgH$$

$$h = \frac{1}{4} \times 100$$

$$= 25m$$



→ A body is moving with a velocity of 1m/sec at a height of 3m, its velocity at a height of 2m is

$$\cancel{\frac{1}{2}mv^2} + mgh = \cancel{\frac{1}{2}mv^2} + mgh$$

$$\frac{1}{2} + 9.81 \times 3 = \frac{1}{2}v^2 + 9.81 \times 2$$

$$\frac{1}{2}(v^2 - 1^2) = 9.81(3 - 2)$$

$$= 9.81 \times 2$$

$$= 19.6 + 1$$

$$V = \sqrt{20.6}$$

→ A ball is dropped from a height of 20m it rebounces to height of 15m, the % of loss of energy is

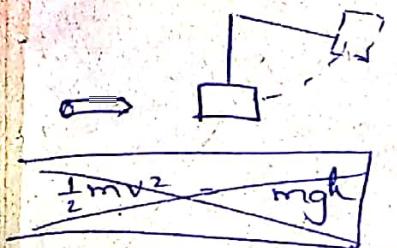
is

% of loss of energy

$$= \frac{mgh_1 - mgh_2}{mgh_1} \times 100$$

$$= \frac{20 - 15}{20} \times 100 = 25\%$$

→ A bullet of mass 10g moving with 100 m/s embedded into a suspended wooden block of 990g, the sys. rises to a height of \_\_\_\_\_



$$\cancel{\frac{1}{2}mv_2^2 = mgh}$$

$$\cancel{m_2v_2 + m_1v_1 = M_1v_1 + M_2v_2}$$

$$0 \quad 10 \times 100 = (M_1 + M_2)v_2$$

$$v_2 = \frac{1000}{1000}$$

$$= 1$$

$$\text{Then } K.E = P.E$$

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2} \times 10^2 = 9.81 h$$

$$h = \frac{1}{20}$$

$$h = 0.05 \text{ m}$$

$$= 5 \text{ cm}$$

The motion of particle

## Simple Harmonic Motion (SHM)

The motion of a particle in a straight line above or below the mean position is called SHM.

- This is a periodic motion
- In this motion acc<sup>n</sup> & disp. but opp. in direction

$$\rightarrow a \propto -x$$

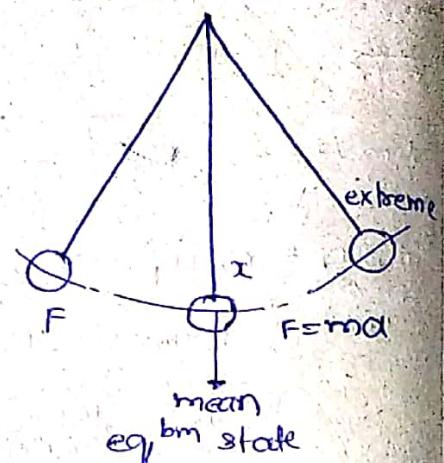
$$\rightarrow a = -kx$$

$$\frac{d^2x}{dt^2} + kx = 0$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d^2x}{dt^2}$$



→ The force acting on a particle is always directed towards the mean position.

→ Phase: The angle subtended by the particle in SHM is called phase.

• It denotes the angle of position & direction of vibration of the particle.

• The initial phase is called epoch.

• The disp max of a particle in SHM is called amplitude [R=A]

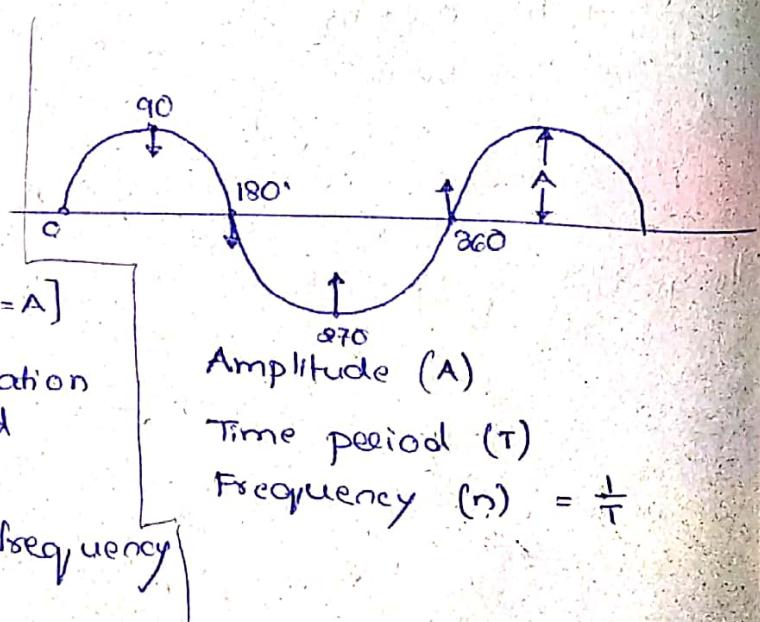
• The time for one vibration is called time period

• No. of vibrations in 1 second is called frequency

$$n = \frac{1}{T}$$

• The rate of phase is called angular velocity [ang. frequency]

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$$



# Disp. of a particle in SHM

on  $x$ -axis

$$\cos \theta = \frac{x}{A}$$

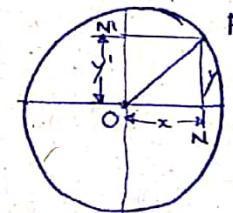
$$x = A \cos \theta$$

$$x = A \cos \omega t$$

$$\omega = \theta / t$$

$$x = A \cos \omega t$$

$$y = A \sin \omega t$$

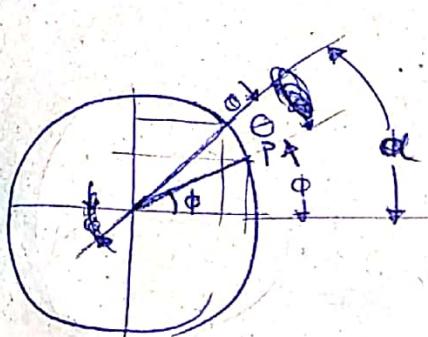


on  $y$ -axis

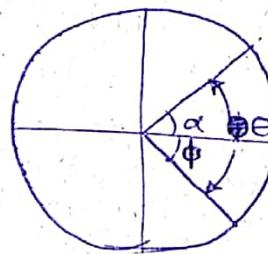
$$\sin \theta = \frac{y}{A}$$

$$y = A \sin \theta$$

$$y = A \sin \omega t$$



$$\alpha = \theta + \phi$$



$$\alpha = \theta - \phi$$

$$y = A \sin \alpha$$

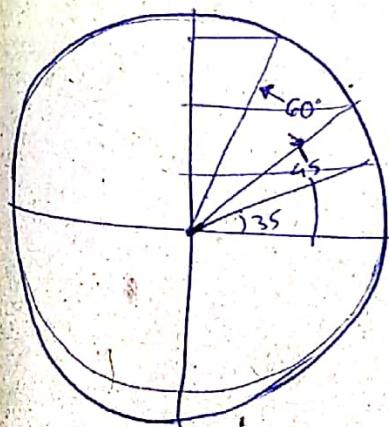
$$\text{but } \alpha = \theta \pm \phi$$

$$y = A \sin (\theta \pm \phi)$$

$$\text{but } \theta = \omega t$$

$$y = A \sin(\omega t \pm \phi) \quad \text{Transverse wave}$$

$$x = A \cos (\omega t \pm \phi) \quad \text{longitudinal wave}$$



$$\theta = 0^\circ \quad y = A \sin \theta = 0$$

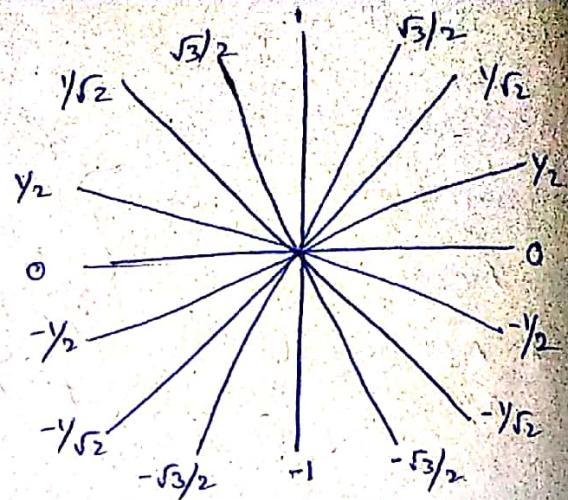
$$\theta = 30^\circ \quad y = A \sin 30^\circ = y_2 A$$

$$= \frac{1}{2} A$$

$$= \sqrt{3}/2 A$$

$$= A$$

Sine values



→ Velocity of a particle in SHM

Disp.

$$y = A \sin \omega t$$

$$\text{velocity, } v = \frac{dy}{dt}$$

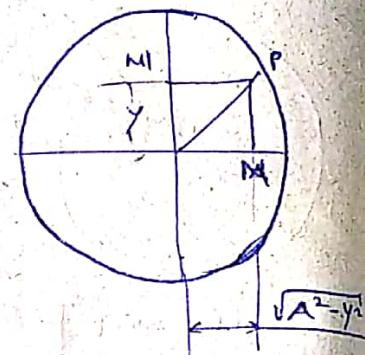
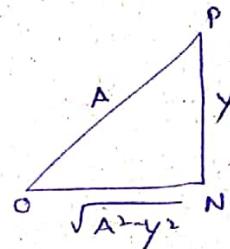
$$= \frac{d(A \sin \omega t)}{dt}$$

$$= \omega A \cos \omega t$$

$$= A \cos \omega t \cdot \left[ \frac{\omega}{\omega} \right]$$

$$v = \omega A \cdot \frac{\sqrt{A^2 - y^2}}{A}$$

$$v = \sqrt{A^2 - y^2} \cdot \omega$$



$$v = \omega A \quad \text{when } y = 0 \quad \text{at mean position}$$

$$v = 0 \quad \text{when } y = A \quad \text{at extreme position}$$

→ Accel. of a particle in SHM

$$\text{Velocity, } v = A \omega \sin \omega t$$

$$a = \frac{dv}{dt} = \frac{d(A \omega \sin \omega t)}{dt}$$

$$= -A \omega^2 \sin \omega t$$

$$= -\omega^2 A \sin \omega t$$

$$a = -\omega^2 y$$

$$a = -\omega^2 y$$

$$\begin{aligned} a = 0 & \quad y = 0 \quad \text{at mean position} \\ a = -\omega^2 y & \quad y = A \quad \text{at extreme } II \end{aligned}$$

→ Time period

$$a = \omega^2 y$$

$$\omega^2 = a/y$$

$$\omega = \sqrt{a/y}$$

$$\omega = \frac{\omega \pi}{T}$$

$$\frac{\omega \pi}{T} = \sqrt{a/y}$$

$$T = \frac{2\pi}{\sqrt{a/y}} = \frac{2\pi \sqrt{y}}{\sqrt{a}}$$

→ Force acting on a particle

$$F = ma$$

$$\boxed{F = -m\omega^2 y} \quad \{a = -\omega^2 y\}$$

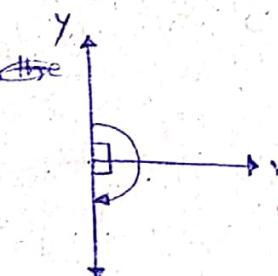
$$F \propto -y$$

$$\frac{F_1}{F_2} = \frac{y_1}{y_2}$$

→ The phase diff. b/w velocity-disp. is  $90^\circ$

↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
II	I										

- accn  
disp -  $180^\circ$



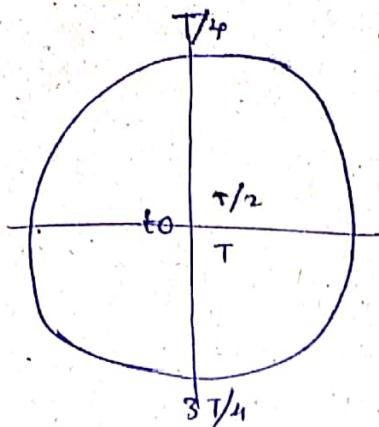
$$y = A \sin \omega t$$

$$V = A \omega \cos \omega t$$

$$V = A \omega \sin (\omega t + 90^\circ)$$

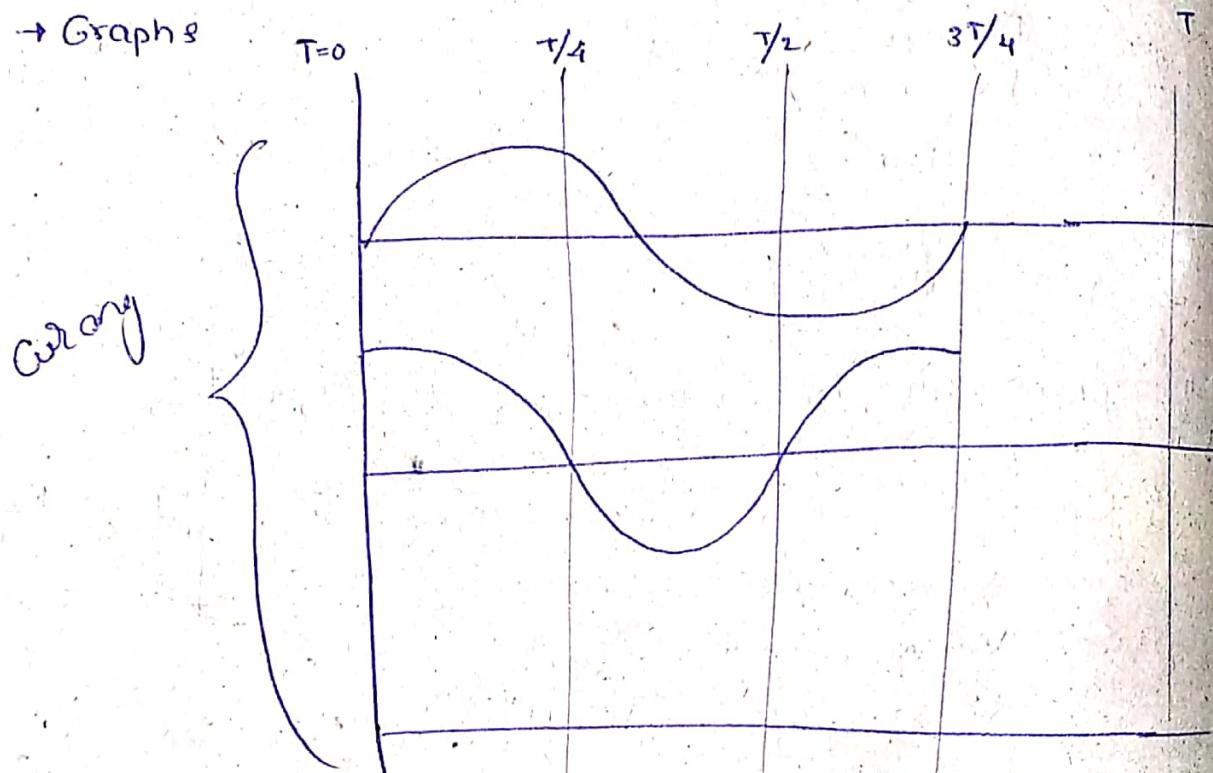
$$\begin{aligned} \delta &= 90^\circ + \omega t - \omega t \\ &= 90^\circ \end{aligned}$$

# Graphical representation

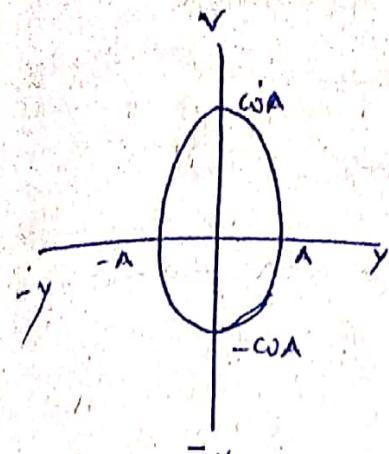


time	$y$	$v$	$a$	$\theta$
0	0	$\omega A$	0	0°
$T/4$	$A$	0	$-\omega^2 A$	90°
$T/2$	0	$-\omega A$	0	180°
$3T/4$	$-A$	0	$\omega^2 A$	270°
$T$	0	$\omega A$	0	360°

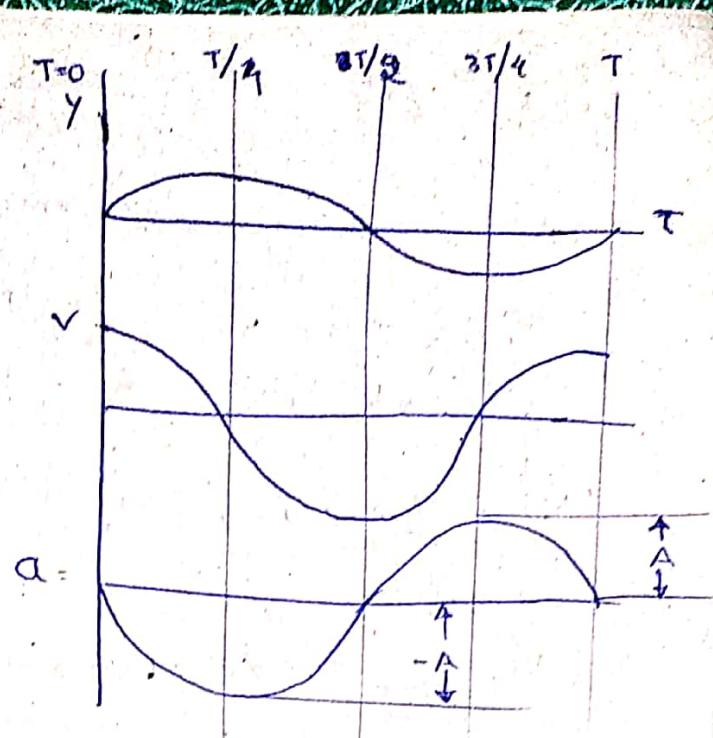
→ Graphs



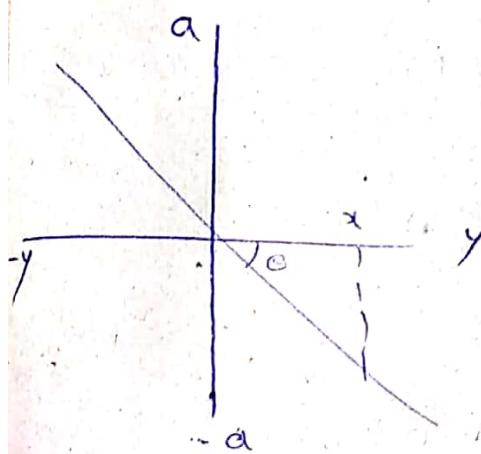
Elliptical curve



v-y graph



a-y graph



Straight line with  
-ve slope

→ Angle of slope

$$\tan \alpha = -\frac{a}{y} = -\frac{(-\omega^2 y)}{y}$$

$$\tan \alpha = \omega^2$$

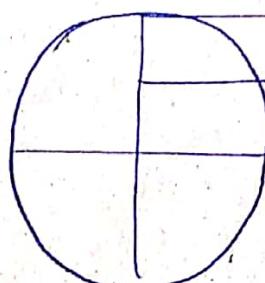
$$\omega = \sqrt{\tan \alpha}$$

$$\frac{2\pi}{T} = \sqrt{\tan \alpha}$$

$$T = \frac{2\pi}{\sqrt{\tan \alpha}}$$

$$T = 2\pi \quad [\alpha = 45^\circ]$$

→ Energy of a particle in SHM



PE = max, KE = 0

KE = 50%, PE = 40%

KE = max, PE = 0

$\rightarrow K.E$

$\rightarrow P.E$

At a displacement

$$K.E = \frac{1}{2} m v^2$$

$$\Rightarrow v = \omega \sqrt{A^2 - y^2}$$

$$K.E = \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$P.E = T.E - K.E$$

$$= \frac{1}{2} m \omega^2 A^2 - \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$P.E = \frac{1}{2} m \omega^2 y^2$$

(y=0) At mean

(y=0)

$$T.E = K.E = \frac{1}{2} m \omega^2 A^2$$

$$P.E = 0$$

At extreme

(y=A)

(y=A)

$$K.E = 0$$

$$T.E = P.E = \frac{1}{2} m \omega^2 A^2$$

$$T.E \propto A^2$$

$\rightarrow$  When frequency of vibration is 'n', then the frequency of change in K.E (or) P.E is 'an'

→ Time period of a particle is 1.8 seconds, the time taken for the disp. = A/2 from mean is \_\_\_\_\_  
 $T = 1.8 \text{ sec}$

$$y = A \sin \omega t$$

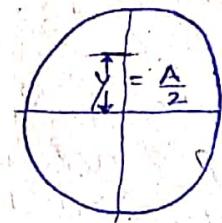
$$\frac{A}{2} = A \sin \omega t$$

$$\sin \omega t = \frac{1}{2}$$

$$\left(\frac{\theta\pi}{T}\right)t = \frac{\pi}{6} + \frac{\pi}{2k}$$

$$\frac{\theta\pi}{2k} \times t = \frac{\pi}{6}$$

$$t = 1.8 \text{ second.}$$



$$\left(\frac{\theta\pi}{T}\right) \times$$

$$y = A \sin \omega t, \quad A/\sqrt{2}$$

$$t = 3, \quad 1.5$$

$$t = \frac{\pi/4 \times 1.8}{2\pi}$$

$$t = 3$$

Time period of a particle is 3.14 seconds and amplitude is 5cm, its velocity at a disp. of 3cm is \_\_\_\_\_

$$\begin{aligned} v &= \omega \sqrt{A^2 - y^2} \\ &= \frac{\theta\pi}{T} \left[ \sqrt{5^2 - 3^2} \right] \\ &= \frac{8\pi}{3.14} \\ &= 8 \text{ cm/s} \end{aligned}$$

$$v_{\max} = \omega \times A$$

$$\begin{aligned} &= \frac{\theta\pi}{3.14} \times 5 \\ &= \frac{25}{9} \\ &= 10 \text{ cm/s} \end{aligned}$$

The  $v_{\max}$  is 10, its velocity at a disp. of half of the Amplitude is \_\_\_\_\_

$$v_{\max} = \omega y, \quad v = \omega A$$

$$v = \omega \sqrt{A^2 - y^2}$$

$$\begin{aligned} &\text{circled } \frac{62.5}{12} \\ &= \omega \sqrt{A^2 - \frac{A^2}{4}} \\ &= \omega A \frac{\sqrt{3}}{2} \end{aligned}$$

$$\boxed{v = \frac{\sqrt{3}}{4} \omega A}$$

→ Amplitude of a particle is 5cm → The  $\omega_{\max}$  and  $V_{\max}$  are numerically eq,  $T = ?$

the ratio of velocities and the disp.s of. 3cm and 4cm is

$$A = 5\text{ cm}, V_1 : V_2 = ? \quad y_1 = 3$$

$$\frac{V}{y} = \frac{\cos \theta}{\sin \theta}$$

$$y_2 = 4$$

$$\omega \propto \propto = \omega T$$

$$\omega = 1$$

$$\frac{\partial \pi}{T} = 1$$

$$T = \frac{\partial \pi}{\omega}$$

$$V = \omega \sqrt{A^2 - y^2}$$

$$\frac{V_1}{V_2} = \frac{\cos \theta}{\sin \theta} \sqrt{\frac{5^2 - 3^2}{5^2 - 4^2}}$$

$$\frac{V_1}{V_2} = \frac{4}{3}$$

→ A particle of mass 8kg in S.H.M with an Amplitude of 5m and time period 3.14sec, it K.E at a disp. of 3m

→ Time period of a particle is 3.14 second, it accn at a disp. of 3cm is

$$a = -\omega^2 y$$

$$= -\omega^2 \cdot 3$$

$$\omega = \frac{\partial \pi}{T}$$

$$\omega^2 = \frac{4\pi^2}{T^2}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{\omega} \times 3$$

$$\frac{4\pi^2}{3.14^2} \times 3$$

$$a = -6\text{ m/s}^2$$

$$-128$$

$$= \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$= \frac{1}{2} \times 8 \times \frac{4\pi^2}{3.14^2} (5^2 - 3^2)$$

$$= 4 \cdot 16$$

$$K.E = 64 \text{ KJ}$$

$$P.E = \frac{1}{2} m \omega^2 y$$

$$= 36 \text{ J}$$

$$T.E = K.E + P.E$$

$$= 100 \text{ J}$$

$$\rightarrow T = 8.8, y =$$

$$K.E = P.E \text{ is}$$

$$\frac{1}{2} m \omega^2 (A^2 - y^2) = \frac{1}{2} m \omega^2 y^2$$

$$A^2 = 8y^2$$

$$y = \frac{A}{\sqrt{2}}$$

$$\cancel{y = \frac{A}{\sqrt{2}}}$$

$$\frac{A}{\sqrt{2}} = A \sin \omega t$$

A → The accn of a particle is  $25 \text{ cm/s}^2$  at a disp of 4cm,  $T = ?$

$$25 = -\frac{4\pi^2}{T^2} \times 4$$

$$T^2 = \frac{-16\pi^2}{25}$$

$$T = \frac{-4\pi}{5}$$

$$= \frac{4\pi}{5}$$

$$\frac{1}{T} = \sin \omega t$$

$$\frac{1}{T} = \frac{\omega t}{\pi}$$

$$t = 18 \text{ sec}$$

→ The % of K.E in the total energy at a disp. of  $\frac{A}{2}$

$$\frac{\Delta K.E}{K.E} / / / / /$$

$$\frac{K.E}{T.E}$$

$$\frac{\frac{1}{2}m\omega^2(A^2 - y^2)}{\frac{1}{2}m\omega^2 A^2} \times 100$$

$$(1 - \frac{y^2}{A^2}) \cdot \times 100$$

$$\frac{y}{A}$$

$$\frac{A}{2A} = \frac{1}{2}$$

$$= 1 - \frac{Ax}{4Ax}$$

$$= 1 - \frac{1}{4}$$

$$= 3/4 \times 100$$

$$= 75\%$$

}  $\times 100$

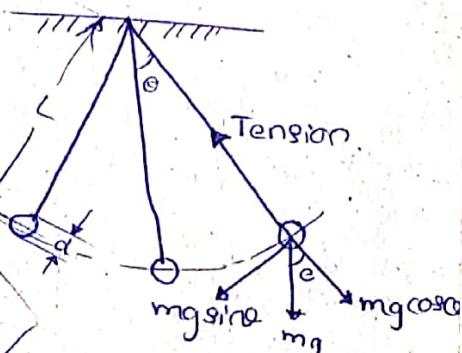
→ Time period of a simple pendulum :

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$T \propto \sqrt{L}$$

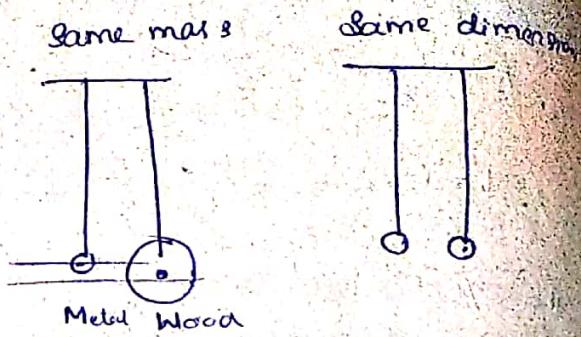
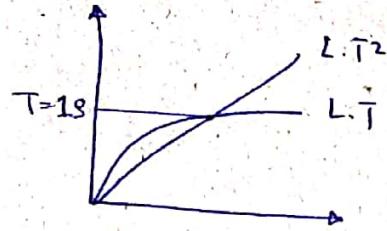
$$T \propto \frac{1}{\sqrt{g}}$$

Time period is independent of mass



$L$  = length of the pendulum

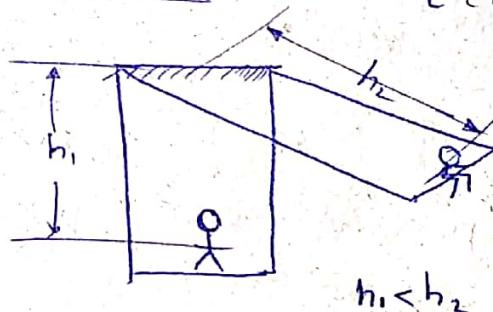
$g$  = accn due to gravity.



- The simple pendulum does not oscillate in the spring but oscillates in the vacuum.
  - $L/T^2$  &  $L/T$  graph intersects at  $T = 1s$ .
  - The pendulum which has time period 1 second is called seconds pendulum.
  - When an iron pendulum and a wooden pendulum of same mass are oscillates, the time period of wooden pendulum is more.
  - When they have same sides, then the time period will be same.
- When a body who is standing in a swing suddenly sits then the time period of the swing

Increases, length increases

$$[T \propto \sqrt{L}]$$



→ Time period for considerable length

$$T = 2\pi \sqrt{\frac{1}{g\left(\frac{1}{L} + \frac{1}{R}\right)}}$$

R = radius of earth

$$R = 6400 \text{ km}$$

IF  $L = \alpha$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

$$T = 2\pi \sqrt{\frac{6400 \times 1000}{9.8}}$$

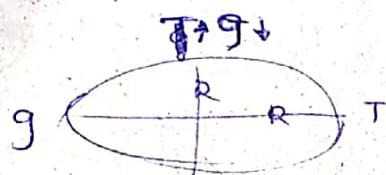
$$T = 84 \text{ min}$$

⑥ IF  $L = R$

$$T = 2\pi \sqrt{\frac{R}{2g}}$$

$$T = 59 \text{ min}$$

When a pendulum is carried from equator to poles its time period decreases  $g \uparrow T \downarrow$



$$g = \frac{GM}{R^2}$$

$$g \propto \frac{1}{R^2}$$

$$g \uparrow R \downarrow$$

$$g \uparrow T \downarrow$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$\frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}}$$

$$g_m = \frac{1}{6} g_e$$

$$T_m = \sqrt{6} T_e$$

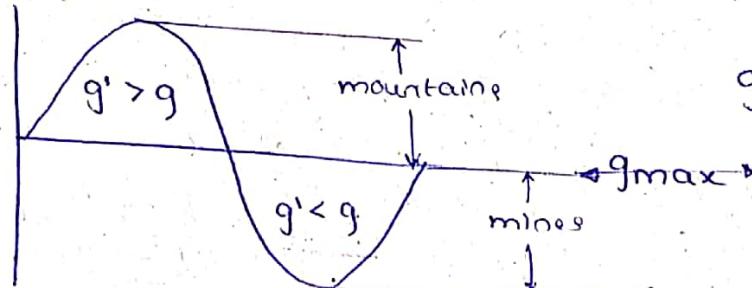
~~When a pendulum is carried from Earth to Moon, its time period increases~~

When a pendulum is carried from Earth to Moon, its time period increases



$$g' = \frac{GM}{(R+h)^2} \quad , \quad g = \frac{GM}{R^2}$$

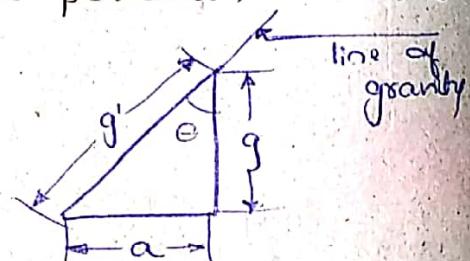
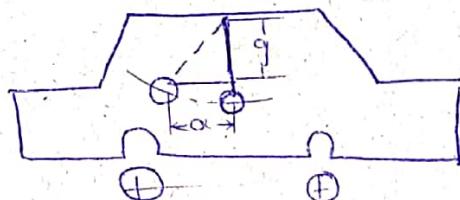
→ When a hydrogen balloon is released on the moon, it falls down with accn with  $\frac{1}{6}$  gravity on earth.



$$g' = g \left( \frac{R}{R+h} \right)^2$$

$$g' = g \left( 1 - \frac{h}{R} \right)$$

When a pendulum is suspended in a vehicle with some accn. then the time of period of the pendulum decreases



$$g' = \sqrt{g^2 + a^2}$$

$$g' \uparrow \quad T \uparrow$$

Angle of pendulum

$$\tan \theta = \frac{a}{g}$$

$$\theta = \tan^{-1}(a/g)$$

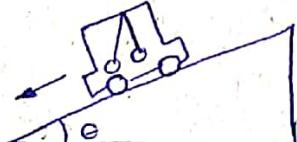
A

le

H

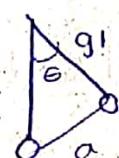
W

When a pendulum is sliding down on a incline surface then the time period of pendulum increases



$$g' = g \cos \theta$$

$$g' = \sqrt{g^2 - a^2}$$

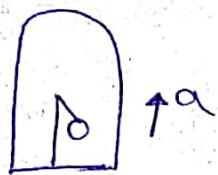


$$T = 2\pi \sqrt{\frac{L}{g \cos \theta}}$$

Time period of the pendulum decrease when it is placed in a lift which is moving up w.e.t with some accln.

$$g' = g + a$$

$$g \uparrow T \downarrow$$



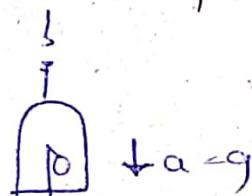
Time period of the pendulum increases when the lift moves down with some accln.

$$g \downarrow T \uparrow$$

$$g' = g - a$$

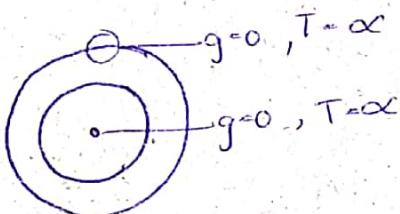


In a freely falling lift at the centre of earth and in a orbiting satellite, the time period of pendulum is infinity.



$$g' = g - a \\ = 0$$

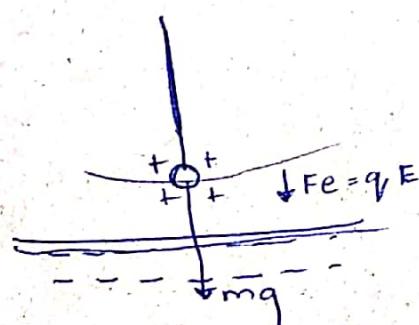
$$g = 0, T = \infty$$



1. When a pos +vely charged pendulum is allowed to oscillate on a -vely charged plate, Then time period decrease.

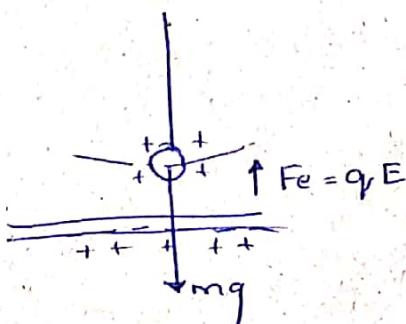
2. When a +vely charged pendulum is allowed to oscillate on a +vely charged plate, Then time period increases.

2. When a +vely charged pendulum is allowed to oscillate on a +vely charged plate, Then time period increases.



$$g' = g + \frac{qE}{m}$$

$$g \uparrow T \uparrow$$

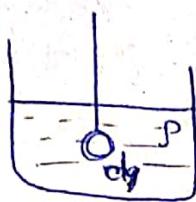


$$g' = g - \frac{qE}{m}$$

$$g \downarrow T \uparrow$$

When a pendulum is allowed to oscillate in water, then its time period increases

Sp. gravity or relative density



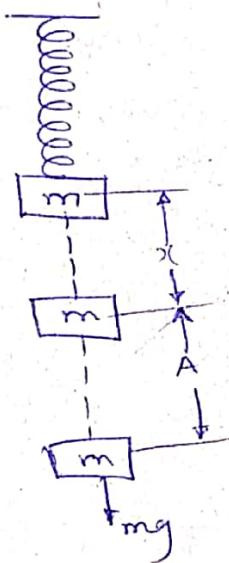
$$\frac{\rho g}{m(g - g')} = \frac{d}{l}$$

$$\rho g = dg - dg'$$

$$g' = \frac{g(d - \rho)}{d}$$

$$g \uparrow T \uparrow$$

Time period of a loaded spring



Force const.  $K = \frac{F}{x}$

$$P.E. = W.D.$$

$$\frac{1}{2} m \omega^2 y^2 = \frac{1}{2} k y^2$$

$$k = m \omega^2$$

$$\omega = \sqrt{k/m}$$

$$\rightarrow \text{when } \omega = \frac{2\pi}{T}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

→ for suspended load

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$\text{but } K = \frac{F}{x} = \frac{mg}{x}$$

$$T = 2\pi \sqrt{\frac{m}{mg/x}}$$

$$T = 2\pi \sqrt{x/g}$$

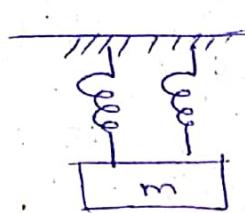
Series



$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

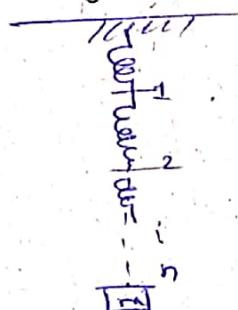
$$K = \frac{K_1 K_2}{K_1 + K_2}$$

Parallel



$$K = K_1 + K_2$$

spring cut into 'n' parts



$$K = nk$$

$$T \propto \frac{1}{\sqrt{K}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{nk}{K}} = \sqrt{n}$$

$$T_2 = T_1 / \sqrt{n}$$

Time period of a pendulum of length 9.8 m is

$$T = 2\pi \sqrt{\frac{L}{g}} \quad L = 9.8$$

$$= 2\pi$$

length of seconds pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$L = 1 \text{ m}$$

length on moon is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$L \propto g$$

$$\frac{L_m}{L_e} = \frac{g_m}{g_e}$$

$$\% (\frac{11}{10} - 1) \times 100$$

$$= \frac{1}{10} \times 100 \\ = 10$$

$$\frac{L_m}{L_e} = \frac{16g_e}{g_e}$$

$$L_m = \frac{1}{6} m$$

decrease in length

$$\Delta L = L_m - L_e \\ = 1 - \frac{1}{6} \\ = \frac{5}{6} m$$

length of a second pendulum  
is doubled, then time period

is \_\_\_\_\_

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$T \propto \sqrt{L}$

$$\frac{T_1^2}{T_2^2} = \frac{L_1}{2L_2}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{L_1}{2L_2}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\frac{2}{T_2} = \sqrt{\frac{L_1}{2L_2}}$$

$$T_2 = 2\sqrt{2} s$$

$$\frac{dT}{T}$$

$$\log T = \frac{1}{2} \log L$$

$$\frac{dT}{T}$$

$$\frac{1}{T} \cdot \frac{dT}{dL} = \frac{1}{2} \times \frac{1}{L}$$

$$\frac{dT}{T} = \frac{1}{2} \times \frac{dL}{L}$$

$$\frac{dT}{T} \times 100 = \frac{1}{2} \frac{dL}{L} \times 100$$

$$100 \times \frac{dT}{T} = \frac{1}{2} \times [1 \times 100]$$

$$\frac{dT}{T} = \frac{1}{2}$$

$$= 0.5$$

A second pendulum is placed on a planet of double the radius of earth, then Time period becomes \_\_\_\_\_

[same mass]

$$g = \frac{GM}{R^2} - k$$

$$g \propto \frac{1}{R^2}$$

$$\frac{g_1}{g_2} = \left(\frac{R_2}{R_1}\right)^2$$

$$\frac{g_1}{g_2} = \left(\frac{2R_1}{R_1}\right)^2$$

$$\left(\frac{T_2}{T_1} - 1\right) \times 100 = ?$$

$$\left(\frac{T_2}{T_1} - 1\right) \times 100 = ?$$

$$\frac{T_2}{T_1} = \sqrt{\frac{121}{100}}$$

$$= \frac{11}{10}$$

$$g_1 = 4g_2$$

$$T_1 \propto \frac{1}{\sqrt{g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$$

$$\frac{g}{T_2} = \sqrt{\frac{g_2}{4g_1}}$$

$$T_2 = 4 \text{ seconds}$$

$$g = \frac{4}{3}\pi R P$$

$$\text{Time on earth} = 6s$$

$$\text{Time on moon} = ?$$

$$T_m = \sqrt{6} T_e \\ = 6\sqrt{6}$$

A second pendulum is placed in a lift which is moving up with an accn' of  $3g$ , time period becomes \_\_\_\_\_

$$T_1 = 2s \quad g_1 = g$$

$$T_2 = ? \quad g_2 = g + a \\ = g + 3g \\ = 4g$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$$

$$\frac{g}{T_2} = \sqrt{\frac{4g}{g}}$$

$$T_2 = 1 \text{ second}$$

A pendulum is suspended in a vehicle which is moving with accn' of  $g/\sqrt{3}$ , the angle made by pendulum is \_\_\_\_\_

$$\theta = \tan^{-1}(a/g)$$

$$= \frac{1g}{\sqrt{3}g}$$

$$\Rightarrow 30^\circ \leftarrow$$

$$g' = \sqrt{g^2 + a^2}$$

$$= \sqrt{g^2 + \frac{g^2}{3}}$$

$$= \frac{4g^2}{3}$$

$$g' = \frac{2g}{\sqrt{3}}$$

A pendulum of sp. gravity '4' has a length 'L' 9.8m, it is oscillated in a cage, Time period \_\_\_\_\_

$$\text{Sp. gravity} = 4 = \frac{d}{P}$$

$$\frac{P}{d} = \frac{1}{4}$$

$$g' = g \left[ 1 - \frac{P}{d} \right]$$

$$= g \left[ 1 - \frac{1}{4} \right]$$

$$= \frac{3}{4}g$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$\sim 2\pi \sqrt{\frac{9.8}{3/4g}} = \frac{4\pi}{\sqrt{3}} \text{ seconds}$$

A spring of  $K = 100 \text{ N/m}$  is loaded with a mass of 4kg

Time period is \_\_\_\_\_

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \times \sqrt{\frac{4}{100}}$$

$$= 2\pi \times \frac{2}{10}$$

$$= \frac{4\pi}{10}$$

$$= \frac{2\pi}{5}$$

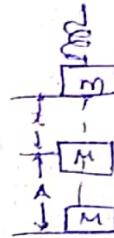
A spring is loaded with a mass which it stretches by 20cm

It is further pulled to 5cm and released,  $T = ?$

$$T = 2\pi \sqrt{\frac{x}{g}}$$

$$= 2\pi \times \sqrt{\frac{20}{9.8 \times 10^2}}$$

$$\cancel{= \frac{14\sqrt{5}}{3} \text{ sec}}$$



$$= 2\pi \times \sqrt{\frac{20}{980}}$$

$$\cancel{= \frac{12\pi}{3+5}}$$

$$\cancel{= \frac{2\pi}{7}}$$

In above problem mass suspended is 2kg. Energy of vibration is  $\Delta = 5 \text{ cm}$

$$E = \frac{1}{2} m \omega^2 A^2$$

$$= \frac{1}{2} \times 2 \times \frac{2\pi}{T} \times (5 \times 10^{-2})^2$$

$$= \frac{2}{2} \times 25 \times 10^{-4}$$

$$= 49 \times 25 \times 10^{-4} \text{ J}$$

Time period of spring increases from 3s to 4s, the mass suspended increased by 1kg  
the initial mass is \_\_\_\_\_

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T_1 = 3 \quad m_1 = 1$$

$$T_2 = 4 \quad m_2 = m_1 + 1$$

$$\frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{4}{3} = \sqrt{\frac{m+1}{1}}$$

$$\frac{16}{9} = \frac{m}{m+1}$$

$$\cancel{m+1}$$

$$9m + 9 = 16m$$

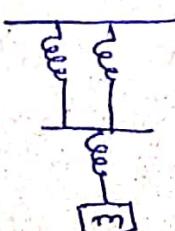
$$\cancel{7m = 9}$$

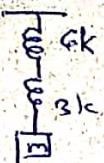
$$m = 9/7 \text{ kg}$$

Time period of a spring is 2sec, it cut into 3eq. parts then two parts are kept in parallel and 1 in series,  $T = ?$

$$T_1 = 2 \text{ sec} \quad k_1 = k_3$$

$$T_2 = ? \quad k_2 = ?$$





$$K = 3k + 3k \\ = GK$$

$$\frac{1}{K_1 + K_2} = \left( \frac{6k + 3k}{18k^2} \right)^{-1}$$

$$K_2 = \frac{18k^2}{9k}$$

$$K_2 = 2k$$

$$\frac{T_1}{T_2} = \frac{k_2}{k_1}$$

$$\frac{\frac{g}{T_2}}{\frac{g}{T_1}} = \sqrt{\frac{2k}{k}}$$

$$T_2 = \frac{2}{\sqrt{2}}$$

$$= \sqrt{2} \text{ seconds}$$

In the above problem the spring is cut into 4 equal parts.  
Then Time period of one part

is

$$T_1 = 2 \quad k_1 = k$$

$$T_2 = \quad k_2 = 4k$$

$$\frac{T_2}{T_1} = \sqrt{\frac{k_1}{k_2}}$$

$$\frac{T_2}{2} = \sqrt{\frac{1}{4}}$$

$$T_2 = 1 \text{ second}$$

# Sound (Acoustics)

It is a form of energy which will be produced when an body vibrated in an elastic medium. The range b/w  $20\text{Hz} - 20\text{kHz}$  is called audible range.

$$V = n \lambda$$

$$\lambda = \frac{V}{n}$$

Infrasonic	$< 20\text{Hz}$	ultrasonic	$> 20\text{kHz}$
$\lambda = \frac{340}{20}$	$= 17\text{m}$	$\lambda = \frac{340}{20\text{kHz}}$	$= 0.0017\text{m}$

At room temp.  $V = 340\text{m/s}$   
 "  $0^\circ \text{C}$  " "  $= 332\text{m/s}$   
 min  $V = 330\text{m/s}$   
 In Vacuum = 0

The range below  $20\text{Hz}$  is infrasonics, which it is heard by insects, rats, snakes,

The range above  $20\text{kHz}$  is ultrasonic, "

heard by dog ( $50\text{kHz}$ ), bats ( $100\text{kHz}$ )

- Ultrasonics are used to purify the liqu. like milk, in scanning, in sonar [sound navigation and ranging] & to find the velocity of moving objects
- The ratio b/w the Velocity of obj.  $V_o$  & Velocity of sound  $V_s$  is called mach number

$$\rightarrow \text{Mach. no.} = \frac{V_o}{V_s}$$

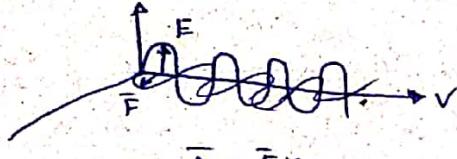
$V_o < V_s$  — subsonic

$V_o = V_s$  — sonic

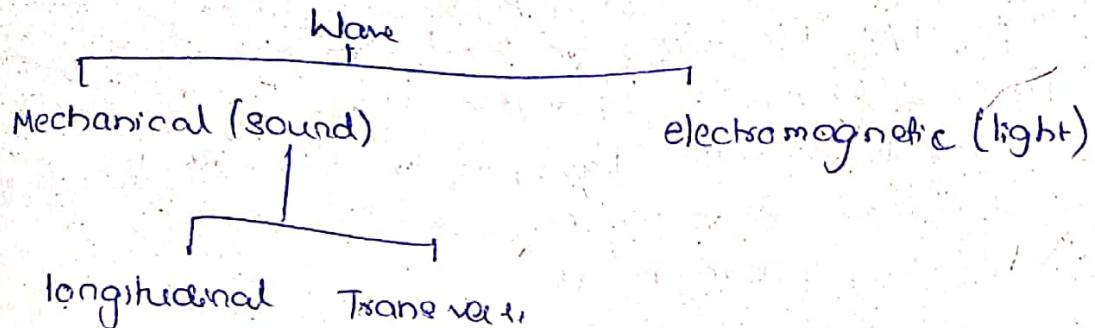
$V_o > V_s$  — supersonic

$$1 \text{ mach} = 340\text{m/s}$$

Wave motion: The disturbance created in a elastic medium travels from particle to particle is called wave motion

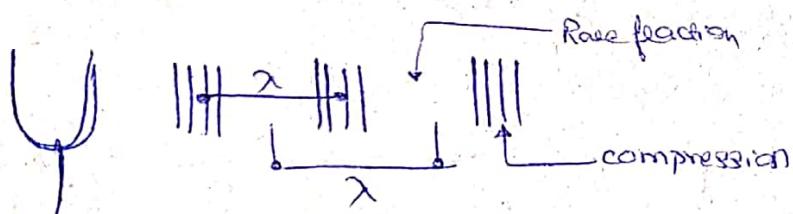


$$\vec{v} = \vec{F} \times \vec{E}$$



**Longitudinal wave:** The wave in which the direction of vibration of particle, the direction of propagation of wave are parallel to each other is called longitudinal wave.

- This wave contains compression and rarefaction.
- The dist. b/w two successive compression or rarefaction is called is eq to wave length ( $\lambda$ ) and phase difference is  $2\pi$  radians.
- Sound travels in gases only in the form of longitudinal mechanical wave.
- Longitudinal wave cannot be polarised.



longitudinal wave

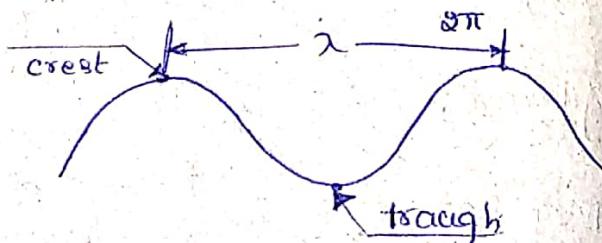
$$x = A \cos(\omega t \pm \phi)$$

## Transverse wave

- The wave in which the direction of vibration of particle & the direction of propagation wave are  $\perp$  to each other is called transverse wave.
- This wave contains crest and trough.
- The dist. b/w two successive crests or troughs = wavelength & phase difference is  $2\pi$  radians.
- Sound travels in solids, liq's, in the form of longitudinal as well as transverse waves.
- Transverse waves can be polarised.

Transverse wave

$$y = A \sin(\omega t \pm \phi)$$



Velocity of sound:

$$V = \sqrt{\frac{E}{\rho}} \quad [\text{Newton's law}]$$

E = elastic modulus

$\rho$  = density of elastic medium

$$\overline{V_s > V_l > V_g}$$

V in solids  $\rightarrow$  V in liquid  $\rightarrow$  V in gases.

5000 m/s

1500 m/s

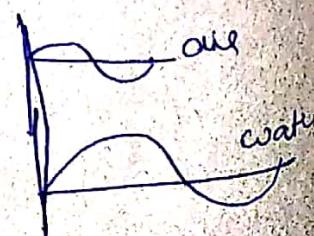
340 m/s

$$\left. \begin{aligned} V &= n \lambda \\ V &\propto ? \end{aligned} \right\}$$

When sound passes from rare to denser medium, its velocity  $\uparrow$   
frequency remains const.

→ When sound passes from rare to denser medium, its Velocity  $\uparrow$

→ When



## Velocity

1. In solids

$$V = \sqrt{\frac{Y}{d}}$$

}  $Y$  = young's modulus

2. In liq.

$$V = \sqrt{\frac{k}{d}}$$

$k$  = bulk's modulus

3. In gases

$$V = \sqrt{\frac{\gamma p}{d}}$$

$\gamma p$  = Adiabatic modulus

For isothermal,  $k = p$

for Adiabatic,  $k = \gamma p$

In Gases:

Effect on temp

$$V = \sqrt{\frac{\gamma p}{d}}$$

$$V = \sqrt{\frac{\gamma t}{m}}$$

$$PV = \gamma t$$

$$P \times \left(\frac{m}{d}\right) = \gamma t \quad \left\{ d = \frac{m}{V} \right\}$$

$$\boxed{V \propto \sqrt{t}}$$

At 819°C the velocity of sound is double to the velocity

at 0°C

$$\frac{V_1}{V_2} = \sqrt{\frac{t_1}{t_2}}$$

Effect on pressure

At const. temp.

Gas follows Boyle's law

$$PV = \text{const.}$$

$$\frac{Pm}{d} = \text{const.}$$

$$\frac{P}{d} = \frac{\text{const.}}{m}$$

$$V = \sqrt{\frac{\gamma \text{const.}}{m}}$$

$$V = \text{const.}$$

$V$  is const.

Velocity of sound in gases is independent of pressure at const. temp.

### 3. Effect on density

$$V = \sqrt{\frac{\gamma p}{d}} \text{ at const. temp.}$$

$\gamma, p$  - are const.

$$V \propto \frac{1}{\sqrt{d}}$$

[density  $\propto$  mol. wt]

Echo:

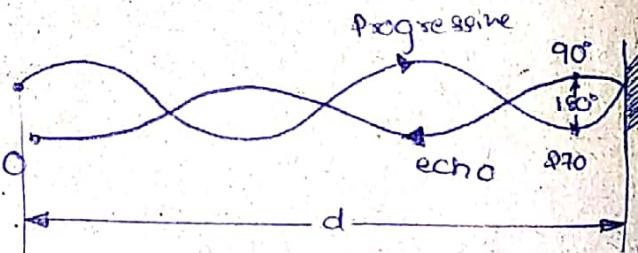
When a sound is produced, which is heard after a time gap due to reflection of sound, then the reflected wave is called echo wave, the progressive wave

p forst

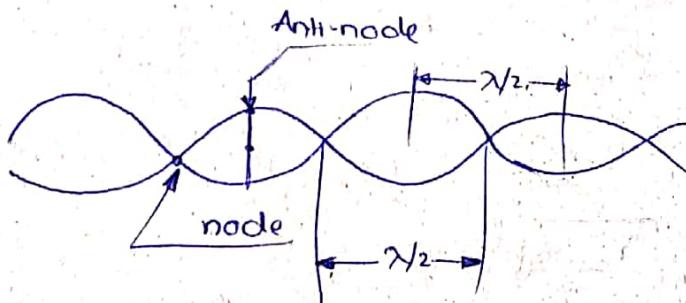
The intensity of echo wave is always < than the progressive wave

- The min. time required to recognise two sound waves separately is  $\frac{1}{10}$  seconds.
- The min. dist. required to absorb the echo is 16.5m

$$\begin{aligned} V &= \frac{2d}{t} \\ d &= Vt/2 \\ &= \frac{330 \times 1}{2 \times 10} \\ &= 16.5 \text{ m} \end{aligned}$$



- The phase difference b/w progressive wave and echo wave at the reflecting surface is 180°
- At the reflecting surface always node takes place



### Beats:

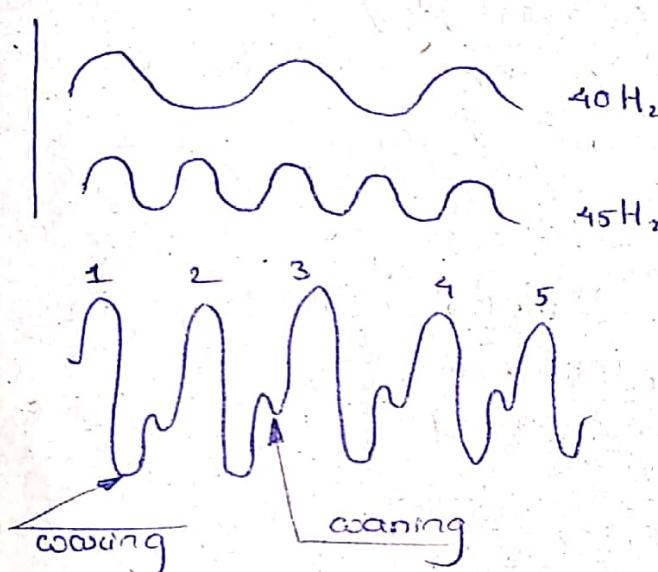
When two sound waves are slightly different frequencies propagated together, then they combine and causes the variation in the intensity [effect of waxing and waning] called beats.

1. Constructive interference causes waxing effect

2. Destructive " " waning "

The no. of beats heard in 1 second is called Beat frequency

The maxi. no. of beats " " " " is 10



### Constructive interference

Constructive interference



$$A = A_1 + A_2$$

Waxing

Destructive interference



$$A = A_1 - A_2$$

Waning

## Beat Frequency

$$N = n_1 \sim n_2 \quad | \quad (n_2 - n_1)$$

Time period,

$$T = \frac{1}{n}$$

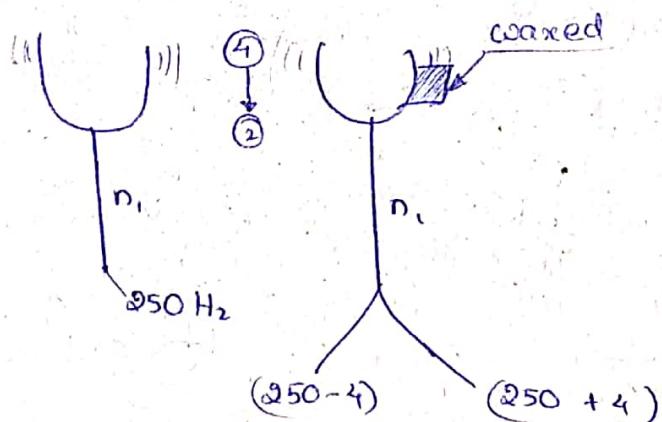
Max  $\rightarrow$  Max }  $T$

Min  $\rightarrow$  Min

Max  $\rightarrow$  Min  $(T/2)$

=

- When the frequency ratio is 1:1, they are set to be in unison.
- " " " " " in 1:2 " " " " " octave
- When a tuning fork is waxed, its frequency  $\downarrow$  (individual)
- " " " " " filed, " " " " "  $\uparrow$  ( " )
- When a tuning fork is sounded with another fork and beats are heard, when the second force is waxed then beats are decreased. The frequency of second force is



$$\begin{array}{l} N = 4 \leftarrow 246 \\ \quad \quad \quad 5 \quad 245 \\ \quad \quad \quad 6 \quad 244 \\ \quad \quad \quad 7 \quad 243 \end{array}$$

$$\begin{array}{l} 254 \rightarrow N = 4 \\ 253 \quad \quad \quad 3 \\ 252 \quad \quad \quad 2 \quad \checkmark \\ 251 \end{array}$$

(0.8)

$$N = n_2 - n_1$$

$$4 = n_2 - 250$$

$$n_2 = 254 \text{ Hz}$$

Fork

Beats are decreased

blaxed

$$N = n_2 - n_1$$

filed

$$N = n_1 - n_2$$

Beats are increased

$$N = n_1 - n_2$$

$$N = n_2 - n_1$$

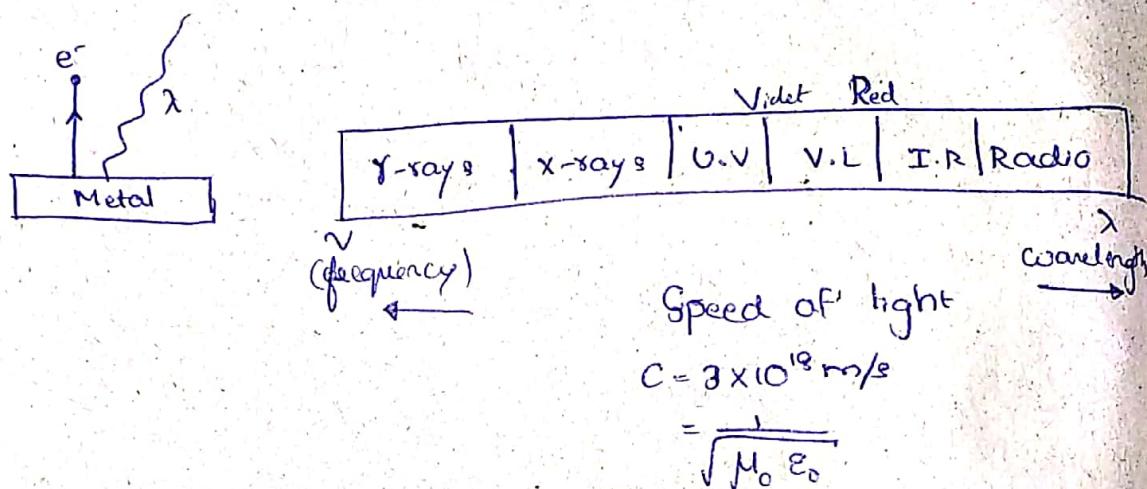
Beats cannot be heard due to echo, when the source and observer at rest.

Beats can be heard due to echo, when the source and observer has relative motion.

## Modern Physics

**Photoelectric effect:** When a metal surface is exposed to electromagnetic radiation of suitable wave length ( $\lambda$ ). Then electrons will be emitted from the surface of the metal. This effect is called photoelectric effect.

- Photo electric effect was discovered by Hertz and developed by Einstein
- The electrons emitted are called photo electrons.

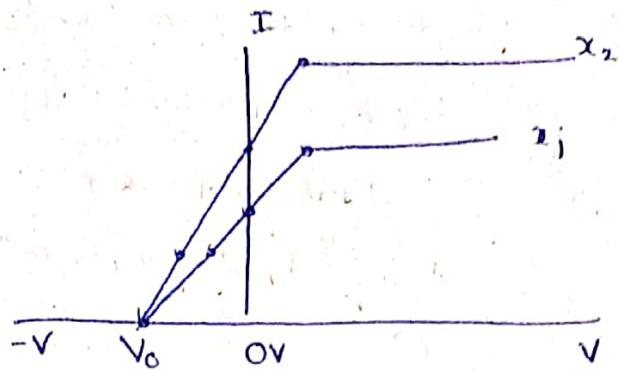
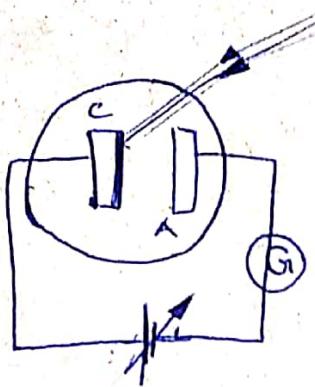


$$\text{Scattering power, } S \propto \frac{1}{\lambda^4}$$

- Visible light ranges from 4000 Å - 7200 Å being strong

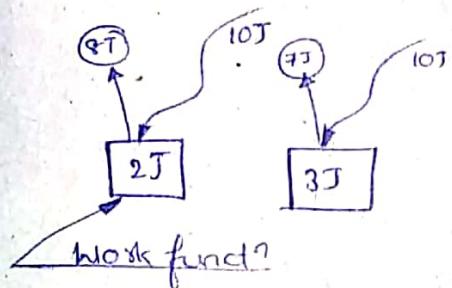
- Photoelectric effect is a instantaneous process.
- The rate of emission of photoelectrons depends on intensity of incident light & independent of frequency.
- The Kinetic energy of photoelectron depends on the frequency of incident light photon and independent of intensity.
- The min. frequency of incident light photon required by the substance to emit photoelectron is called threshold frequency ( $\nu_0$ )

# Relation b/w V & I



$\left\{ \begin{array}{l} V_0 = \text{cut off potential} \\ \text{stopping potential} \end{array} \right.$

$$eV_0 = \frac{1}{2}mv^2$$



- All the photoelectrons will not have the same K.E.
- Free electrons do not cause photoelectric effect.
- The min. -ve potential required at which no photo-electric current, does not pass in the circuit, is called cut-off potential or stopping potential  $[V_0]$ .
- At cut-off potential the max. K.E. of photoelectron becomes eq to electrical energy  $eV_0 = \frac{1}{2}mv^2$
- Cutoff potential depends on the frequency of incident light photon and workfunc<sup>n</sup> of the substance
- Cut-off potential is independent of intensity of incident light.

→ Electron volt [ev]

$$\text{Energy} = \text{charge} \times \text{potential}$$

$$\left. \begin{array}{l} 1 \\ e^- \end{array} \right\} \left. \begin{array}{l} 1 \\ V \end{array} \right\}$$

- The energy attained by an electron when it passes through one volt potential is called electron volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 1.6 \times 10^{-19} \times 10^6 \text{ J}$$

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

## Energy of photon

$$E = h\nu$$

$h$  = plank's consta

$$= 6.625 \times 10^{-34} \text{ Js}$$

$\nu$  = frequency of incident photon

$$\text{but } C = \lambda A$$

$C$  = velocity of light

$$= 3 \times 10^8 \text{ m/s}$$

$$V = \frac{C}{\lambda}$$

$$E = \frac{hc}{\lambda}$$

$$\bullet 1 \text{ Angstrom unit (A}^\circ\text{)} = 10^{-10} \text{ m.}$$

$$E \propto \frac{1}{\lambda}$$

$$\bullet 1 \text{ nm} = 10^{-9} \text{ m}$$

$$310 \text{ nm} = 3100 \text{ A}^\circ$$

\*\*\*

$$E_{\text{in eV}} = \frac{12400}{\lambda \text{ in A}^\circ}$$

Work function incident light

The min. energy of photon required by substance just to emit the photoelectrons is called work function

$$W = E_0 = h\nu_0$$

$\nu_0$  = threshold frequency

$$\text{But } C = \nu_0 \lambda_0$$

$\lambda_0$  = cut off wave length

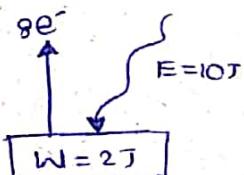
$$\nu_0 = \frac{C}{\lambda_0}$$

$$W = \frac{h.c}{\lambda_0}$$

$$W_{\text{in eV}} = \frac{12400}{\lambda_0 \text{ in } \text{Å}}$$

### Einstein's Photoelectric function

The max. K.E. of photoelectron is = diff. b/w the energy of incident light photon & the work function of the substance.



$$\text{K.E.} = E - W$$

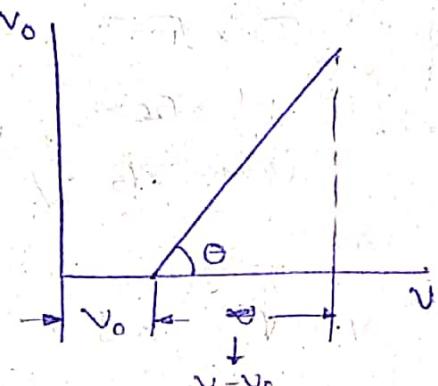
$$\frac{1}{2}mv^2 = h\nu - h\nu_0$$

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

~~$\nu_0$~~ .

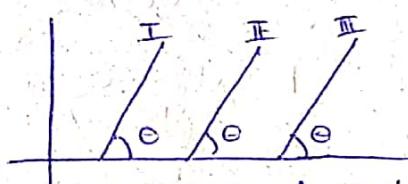
$$e\nu_0 = h(\nu - \nu_0)$$

$$\frac{\nu_0}{\nu - \nu_0} = \frac{h}{e} = \text{const.}$$



$$\tan \theta = \frac{\nu_0}{\nu - \nu_0} = \frac{h}{e}$$

= const.



$\theta$  is same for each metal

- Photoelectric is a device which converts the light energy into electrical energy
- Photoelectric cells are used in calculators & satellites
- In fire alarms & burglar alarms
- In automatic opening & closing of doors
- In street lights as the timer
- In cinematography to reproduce the sound track printed on the films

Cutoff potential of electron is

9.1 volt? If velocity is

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$\frac{1}{2}mv^2 = ev_0$$

$$v^2 \times \frac{1}{2} \times 9.1 \times 10^{-31} = 9.1 \times 1.6 \times 10^{-19}$$

$$\left\{ \begin{aligned} v &= \sqrt{\frac{9.1}{9.1} \times \frac{9.1}{10^{-31}}} \\ &= \sqrt{2 \times 10^{31}} \\ &= \sqrt{20} \times \sqrt{10^{30}} \\ &= 10^{15} \times \sqrt{20} \text{ m/s} \\ &= 9 \times 10^{15} \times \sqrt{5} \text{ m/s} \end{aligned} \right.$$

$$v = \sqrt{3.2} \times 10^6 \text{ m/s}$$

Wavelength of a photon is

400 nm its energy is

$$\lambda = 400 \text{ nm}$$

$$= 4000 \text{ Å}$$

$$E = \frac{1.8400}{4000 \text{ Å}}$$

$$= 31/10$$

$$E = 3.1 \text{ eV}$$

Workfunction of a metal is

3.3 eV, the threshold frequency

is

$$W = h\nu_0$$

$$3.3 \text{ eV} = 6.6 \times 10^{-34} \times \nu_0$$

$$\text{eV} = \text{J}/\text{kg}$$

$$3.3 \times 1.6 \times 10^{-19} \text{ J} = 6.6 \times 10^{-34} \times \nu_0$$

$$\nu_0 = \frac{3.3 \times 1.6 \times 10^{-34}}{6.6 \times 10^{-19}}$$
$$= 0.8 \times 10^{15} \text{ Hz}$$

$$C = \lambda_0 V_0$$

$$3 \times 10^8 = \lambda_0 \times 8 \times 10^4$$

$$\lambda_0 = \frac{3 \times 10^8}{8 \times 10^4}$$

$$= 0.375 \times 10^{-6} \text{ m}$$

$$\lambda_0 = 375 \times 10^{-9} \text{ m}$$

$$\lambda_0 = 375 \text{ nm}$$

Work function of a metal is  
1.6 eV wavelength of incident  
photon is 310 nm the K.E  
of photo electron & cutoff  
potential are

$$K.E = E - W$$

$$= \frac{1.8400}{3100 \text{ Å}} \rightarrow 1.6 \text{ eV}$$

$$= 4 - 1.6$$

$$= 2.5 \text{ eV}$$

$$eV_0 = \frac{1}{2}mv^2 \quad K.E$$

$$1.6 \times 10^{-19} \times V_0 = 2.5 \times 1.6 \times 10^{-19}$$

$$eV_0 = 2.5 \text{ eV}$$

$$V_0 = 2.5 \text{ volt}$$

$$K.E \text{ in eV} = V_0 \text{ in volt}$$

Light is a form of energy which cause a sense of vision.

Light

Light has properties of types

1. Reflection



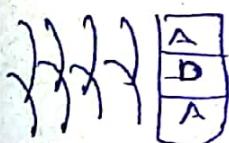
2. Diffraction



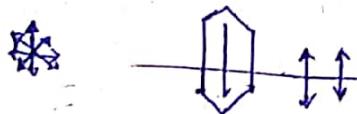
3. Refraction



4. Interference



5. Refraction

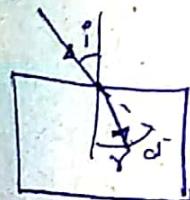


Refraction:

When a light passes from one medium to another medium due to change in the density of the medium called refraction.

When the light passes from rarer to denser medium it bends towards the normal. The angle between the incident ray and the refracted ray is called angle of refraction.

Refraction



$$^a\mu_2 = \frac{\sin i}{\sin r} \quad [\text{Snell's law}]$$

$$^a\mu_{\text{air}} = 1$$

$$^a\mu_{\text{water}} = \frac{4}{3} = 1.33$$

$$^a\mu_{\text{glass}} = \frac{3}{2} = 1.5$$

$$^a\mu_{\text{diamond}} = 2.4$$

Angle of deviation

$$d = i - r$$

$$\textcircled{1} \quad \mu \propto \frac{1}{v}$$

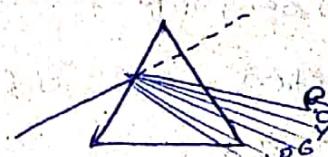
$v$  = velocity of light

$$\frac{\mu_m}{\mu_a} = \frac{v_a}{v_m}$$

$$\textcircled{1} \quad \boxed{\mu_m = \frac{v_a}{v_m}}$$

$$\textcircled{2} \quad \mu \propto \frac{1}{\lambda}$$

$\lambda$  = wavelength



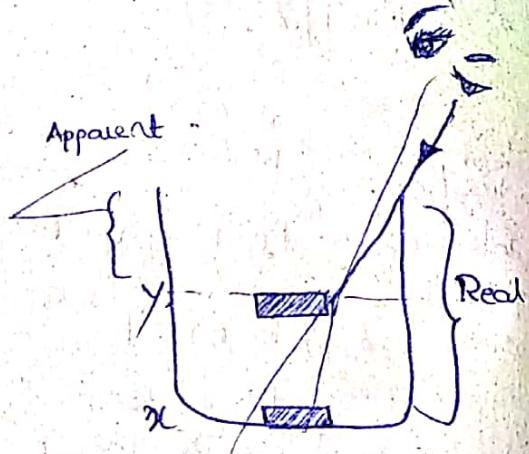
$$\boxed{v \propto \lambda}$$

$$\boxed{\mu \propto \frac{1}{\lambda}}$$

→ Refractive index of light

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\mu = \frac{x-z}{y-z}$$



Relative refractive index

$$\omega \mu_g = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$= 1.125$$

- When the light passes from air to water, it bends towards the normal from its path normally.

• Critical angle:

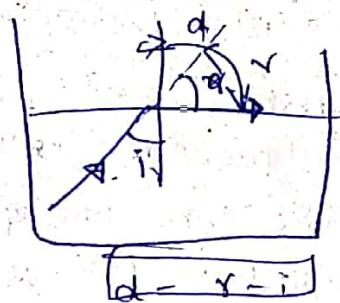


$$M_2 = \frac{\sin i}{\sin r}$$

$$M_1 = \frac{\sin r}{\sin i}$$

$$= \frac{\sin \alpha}{\sin i}$$

$$M = \frac{1}{\sin i}$$



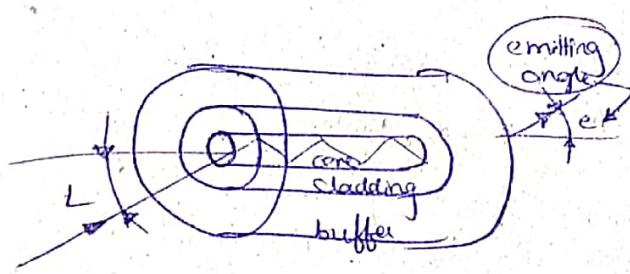
When a light passes from denser to rarer medium & with the angle of incidence of which

The angle of refraction becomes ~~> 90°~~ 90° called ~~becomes~~ ~~and that~~

90° is called critical angle

Total internal reflection (TIR):  
When the light ray tried from decrease

- Optical Fibre is a narrow tube of dia.  $2\mu$  to  $150\mu$  [ $\mu$ =micron]
- It is made with high quality glass or silicon.
- Optical fibre works on the ~~principle~~ principle of TIR
- The fibre tube through which the light pulses passes is called core ( $1.7 - \mu_1$ )
- The core is coated with less density material called cladding ( $\mu_2 = 1.6$ )
- To protect the cladding, a plastic coating is given called buffer
- The angle of incidence at which the light enters the core, called launching angle.



launching angle

$$L = \sin^{-1} \sqrt{\mu_1^2 - \mu_2^2}$$

$$\sqrt{\mu_1^2 - \mu_2^2} = NA$$

NA = Numerical aperture

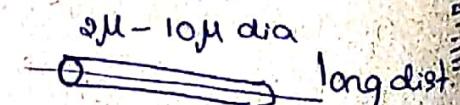
Optical fibre is two types:

1. Step index fibre in which the refractive index of core and cladding has step profile.

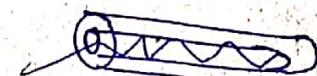
It is of two types

① Single mode

② Multi mode



$10\mu - 50\mu$



Graded index fibre: in which multiple core sys. is present. It is normally multimode.



USES

$$d = 50\mu - 150\mu$$

- Optical fibre are used for telecommunication
- In endoscopic
- To send secret msgs. in defence
- As a sensor in the factories to measure temp., pressure

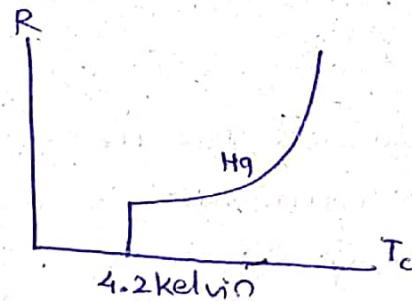
→ The substance for which the resistance become zero before reaching the abs. zero kelvin is called super conductor.

Ex:- Hg, tin with mixed, lead oxides etc.

- Superconductivity was discovered by Kammerlingh Onnes
- The temp. at which the substance exhibits the superconductivity is called critical temp. ( $T_c$ )
- Super conductivity was first observed in mercury at 4.2 kelvin temp.



Ordinary conductor



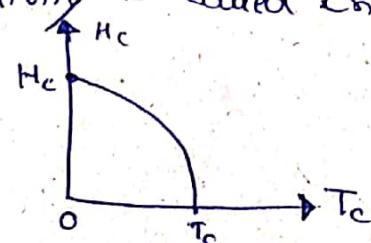
Superconductor

→ Critical field

The min. intensity of magnetic field required to destroy the superconductivity is called Critical field

→  $H_c$  is max. in 0K

→  $H_c$  is zero in  $T_c$



## Critical Current ( $I_c$ )

- The max. current that can be passed through the superconductor is called critical current.

$$H_c = \frac{I_c}{2\pi r}$$

$A/m$

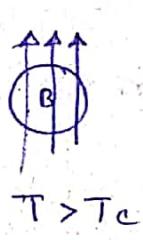


$r$  = radius of the superconductor

- Superconductor has zero resistance and pure diamagnetism  $\Theta$

- Meissner's effect:

When the temp. of the substance is less than Critical temp., then the magnetic field in substance will be evacuated.



- Superconductors are used

- In power transmission as cables
- In switching circuits
- In measuring instruments like galvanometer
- In mechanical transportation
- To generate high intensity magnetic field

$$1) \quad i = 60^\circ \quad r = 30^\circ \\ \mu = ?$$

$$\mu = \frac{\sin i}{\sin r}$$

$$= \sqrt{3} = 1.73$$

$$2) \quad \mu_g = 1.5 \quad V_g = ?$$

$$\mu \propto \frac{1}{V}$$

$$\frac{\mu_a}{\mu_g} = \frac{V_g}{V_a}$$

~~$$\mu_g = \frac{1}{1.5} = \frac{V_g}{3 \times 10^8}$$~~

$$V_g = 2 \times 10^8$$

$$3) \quad \mu_g = 1.414 = \sqrt{2}$$

$$c = ?$$

$$\mu = \frac{1}{\sin c}$$

$$\sin c = \frac{1}{\mu}$$

$$c = \frac{1}{1.414} < \frac{1}{\sqrt{2}}$$

$$c = 45^\circ$$