Hydraulics And Pneumatic Systems

chapters Included

1) properties of fluid — 70 marks
2) Flow of Liquid
3) Flow through pipes
4) Impact of Jets
5) Water Turbines
6) Hydraulic pumps & Motors
7) Fluid power Hydraulics
8) Pneumatics
9) Hydro Pneumatic systems

chapter-1

Properties of Fluids

Fluid Mechanics
↓
(Eng Mechanics) (Behaviour of Bodies)

- Rigid
  - $\delta d = 0$
  - $e = 0$
- Resistant
  - solid
  - Fluid
    - Incompressible
      - Liquids
        - Hydraulics
    - Compressible
      - Gases
        - Pneumatics

Axial rigidity (AE) $\delta l = \frac{Pl}{AE}$

Torsional rigidity (GJ) $\frac{T}{\theta} = \frac{GJ}{l}$

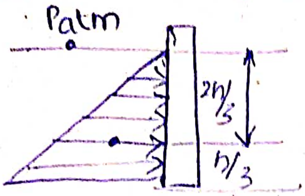
Bending/Flexural rigidity (EI) $y = \frac{Wl^3}{3EI}$

Fluid
- rest — Fluid statics
- motion
  - Fluid dynamics / Fluid kinetics
  - Fluid Kinematics (deals without considering force)

Fluidstatics

fluid at rest

Ex ① Construction of DAM



(h-depth)

h↑ P↑

P↑ linearly

shape - Right angle Δle

load - UVL

HYDROSTATIC PRESSURE DISTRIBUTION DIAGRAM

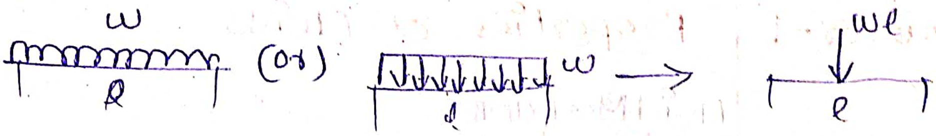
$$P = \frac{W}{A} = \frac{mg}{A} = \frac{\rho v g}{A} = \frac{\rho \cdot A \cdot h \cdot g}{A}$$

$$\frac{dP}{dz} = -\gamma$$

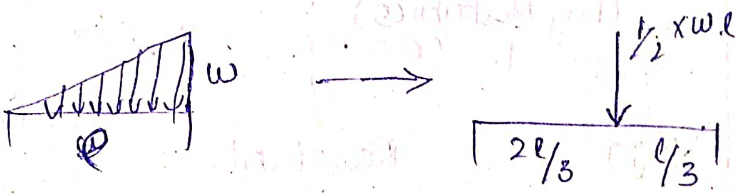
*** $\boxed{P = \rho g h}$ Hydrostatic law

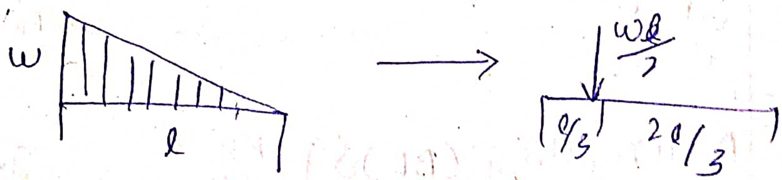
$\boxed{SF = \text{Area of loading diagram}}$

FOR UDL

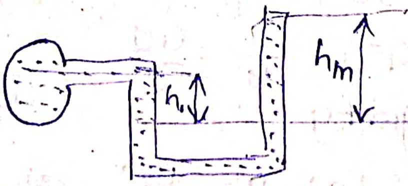


FOR UVL





Ex ② Manometers

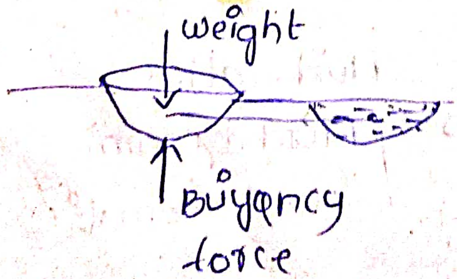


$\boxed{P + \rho g h_1 - \rho_m g h_m = 0}$

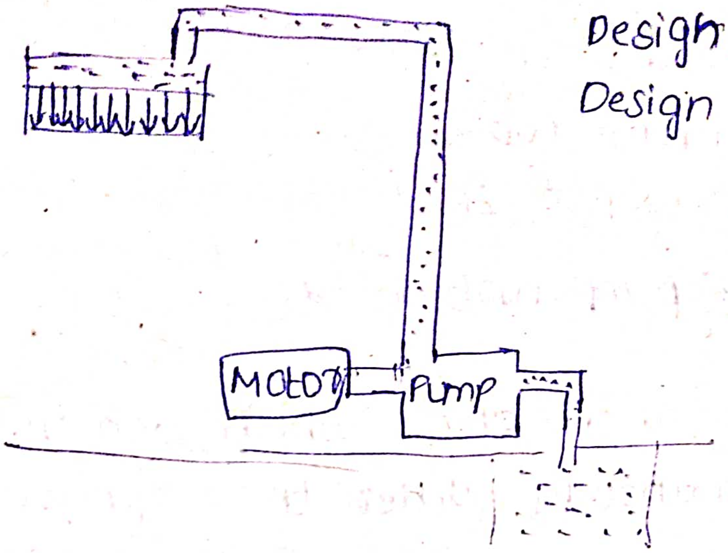
$P_{abs} = P_{atm} + P_g$

$P = P_{atm} - P_{vacc}$

Ex ③ Floating Bodies



Ex: (4) Overhead Tank

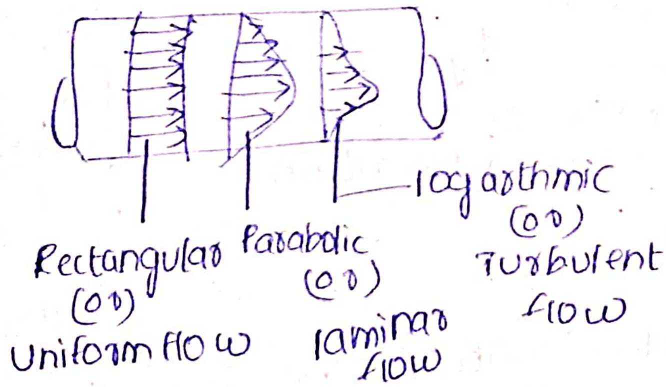


Design of vertical wall - UVL
Design of horizontal wall - UDL

Fluid Kinematics

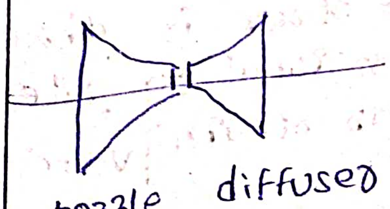
flow under motion without considering forces

Ex: (1) Flow through pipes



Ex: (2) Flow through nozzle & diffuser

$\boxed{a_1V_1 = a_2V_2}$



Ex:- (3) Branched pipe

$\boxed{Q_1 + Q_2 = Q_3 + Q_4}$

Hardy Cross Method / Iterative method to determine flow in pipe network system

ECET NOTE

Fluid Dynamics fluid under motion with consideration of forces

Ex :- (1) Pitot tube

(2) venturimeter

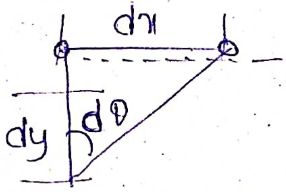
(3) pump, turbine etc.

Fluid fluid is a substance which deforms indefinitely or continously under the application of small

ECET

Tangential force, shear force, shear stress

Fluids cant resist shear forces



neglecting tan

$$d\theta = \frac{dx}{dy}$$

~~(Rate of~~ (Total deformation)

$$\frac{d\theta}{dt} = \frac{dx}{dt} \times \frac{1}{dy}$$

$$\boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

(Rate of deformation)

Note :- Rate of deformation is important than Total deformation itself.

$\tau \propto \frac{d\theta}{dt}$ [shear stress is directly proportional to rate of shear strain] velocity gradient

$\tau \propto \frac{du}{dy}$

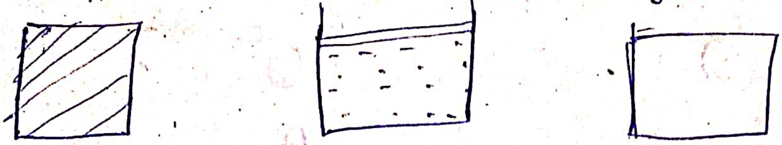
$$\boxed{\tau = \mu \frac{du}{dy}}$$

Newtons law of viscosity

μ = dynamic viscosity (or) absolute viscosity

Free surface (ECET)

Solid | liquid | gas



3 Free surfaces | F.S | NO F.S

$(\tau = 0)$

* The open surface available to atmosphere under equilibrium

ECET 2017
* The necessary and sufficient condition for a surface to be called as free surface ~~is~~ has shear stress equals to zero

Fluid Mechanics Branch of $\boxed{\text{continuum}}$ mechanics

↓

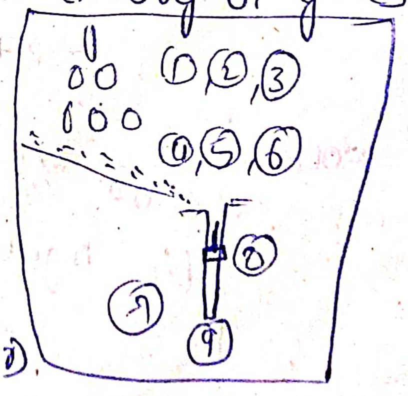
continuos, homogeneous without any voids, air gaps

Assumptions: Knudson number $= \dfrac{\text{Mean free Path}}{\text{characteristic length}}$

$\boxed{[K_n \leq 0.01]}$ continuum holds good

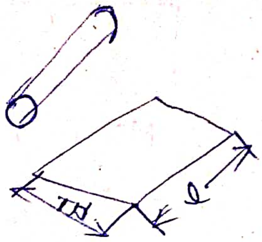
→ rarified Gases are not continuos and they are studied by using molecular theory of gases

Fluid properties



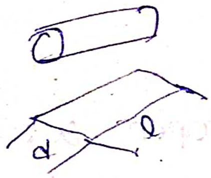
1. Mass - specific mass (or) mass density
2. weight - specific weight
3. gravity - specific gravity
4. flowability
5. ~~compress~~ viscosity (layer by layer)
6. surface tension
7. capilarity
8. compressibility $\alpha \dfrac{1}{\text{Bulk modulus}}$

Area



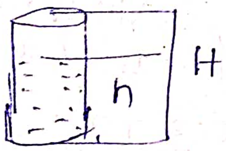
$A_{surface} = \pi d l$

$A_{cross\,section} = \frac{\pi}{4} d^2$
$= \pi r^2$



$A_{projected} = d \times l$

Volume



$V = A \times H$

$V_{cylinder} = \pi r^2 \times H$

$V_{wetted\ region} = \pi r^2 \times h$

$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cone} = \frac{1}{3}\pi r^2 h$

units — ft^3, $inch^3$, cm^3, mm^3, m^3, litre, gallon, TMC

TMC - Thousand miles per cubic feet

- 27×10^9 litres

Note :- $\frac{1}{3 \times 10^8}$ th part of second, distance travelled by light is 1metre

Mass

1kg = 1000 gm

1 pound = 1lbs = 453 gm = 0.453 kg

1 kg = 2.2 lbs = 2.2 pounds

weight

$W = mg$

specific mass (or) density

$\frac{m}{V} = \frac{1kg}{m^3} = 10^{-3} gm/cc$ [T↑ ρ↓ P↑ ρ↑]

specific weight weight density

$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g = \rho \times 9.81$ [N/m3]

specific gravity (or) relative density

$S = \frac{\gamma_{unknown}}{\gamma_{(Standard\ fluid)}} = \frac{\rho_{unk} \times \cancel{9.81}}{\rho_{stan.fluid} \times \cancel{9.81}} = \frac{\rho}{1000}$ → No units

$\boxed{v \alpha \frac{1}{\rho}}$

	ρ Kg/m3	γ (N/m3)	S
Air	1.27	1.2×9.81	1.2×10⁻3
water	1000	$\boxed{9810 \simeq 10000}$	1 — ECET 2013
mercury	13,600	136000	13.6
wood	600	6000	0.6
concrete	2400	24000	2.4
Aluminium	2800	28000	2.8
Diamond	3000	30000	3
steel	7850	78500	7.85
Gold	19,500	195000	19.5
Platinum	24,500	245000	24.5

Note :- All are measured at 4°C — ECET

Trick to remember: Aluminium wood — concrete — Diamond — steel — Gold platinum

VISCOSITY
→ Property which comes into existence when fluid flows in layer by layer by transition
ECET
→ Resistance to flow of fluid
→ Also defined as shear stress required to produce unit rate of shear deformation/velocity gradient

~~Adhesion~~
COHESION
Property by virtue of which molecules remains attach with eachother

ADHESION
Property by virtue of which molecules attach with different substances

VISCOSITY
- Intermolecular cohesion [IMC]
- Intermolecular momentum exchange [IMME]

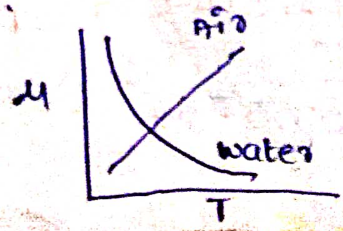
Note: for liquids: IMC > IMME
Temp ↑ viscosity ↓ (because cohesion decreases)
$\mu = Ae^{B/T}$

for gases: IMME > IMC
$\boxed{\text{Temp} \uparrow \text{ viscosity} \uparrow}$ ECET
$\mu \propto \sqrt{T}$

NEWTON'S LAW OF VISCOSITY
$\tau \propto \frac{d\theta}{dt}$ $\left[\frac{dv}{dy}\right.$ - velocity gradient$\left.\right]$

rate of angl. deformation (or) shear strain

ECET
$\boxed{\tau \propto \frac{du}{dy}}$ $\tau = \mu \frac{du}{dy}$



Ex :-

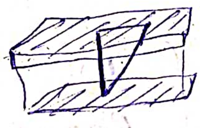
(If $v = 3y - 4y^2$); Find ~~viscosity~~ shear stress

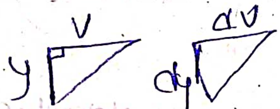
$$\tau = \mu \frac{dv}{dy}$$

$$= \mu \frac{d}{dy}(3y - 4y^2)$$

$$= \mu(3 - 8y)$$

$$= r\mu \quad (\text{If } \mu = 1)$$





$$\frac{dv}{dy} = \frac{V}{y}$$

$$\boxed{\tau = \frac{V}{y}\mu}$$

***

μ of water - 0.01p

1cp.

$$\frac{P}{A} = \mu \frac{V}{y}$$

$$\boxed{P = \mu A \frac{V}{y}}$$

ECET

CGS unit of μ

1poise = $1 \frac{\text{dyne}\cdot s}{cm^2}$

Units of dynamic viscosity (μ) mu

$$\tau = \mu \frac{du}{dy}$$

MKS unit of μ

$\frac{\text{KgF}\cdot s}{m^2} = 98.1 \frac{\text{dyne}\cdot s}{cm^2}$

$= 98.1$ Poise

$$\tau = \mu \frac{V}{y}$$

$$Pa = \mu \frac{m/s}{\cancel{m}}$$

1centipoise $= \frac{1}{100}$ Poise

ECET 2times

$$\mu = Pa\cdot s = \frac{N}{m^2}\cdot s = \frac{Kg}{m\cdot s} = \frac{10^3}{10^2}\frac{g}{cm\cdot s} = 10\,\text{poise}$$

$\boxed{1\text{poise} = 0.1\,Pa\cdot s}$

$ML^{-1}T^{-1}$ dimensional formula

$FL^{-2}T$

(ν (nu))

Kinematic viscosity

since its units do not contain mass

1 ~~Pas N~~ $\frac{\text{dynamic viscosity}}{\text{mass density}}$

[K.V also called as Momentum diffusivity]

$$\nu = \frac{\mu}{\rho} = \frac{\frac{Kg}{m\cdot s}}{\frac{Kg}{m^3}} = m^2/sec = 10^4 cm^2/sec$$

$= 10^4$ stoke

ν (nu)

$\boxed{1\text{stoke} = 10^{-4} m^2/sec}$

Types of Fluids

① Ideal Fluid

(i) Incompressible ($\rho = K$)

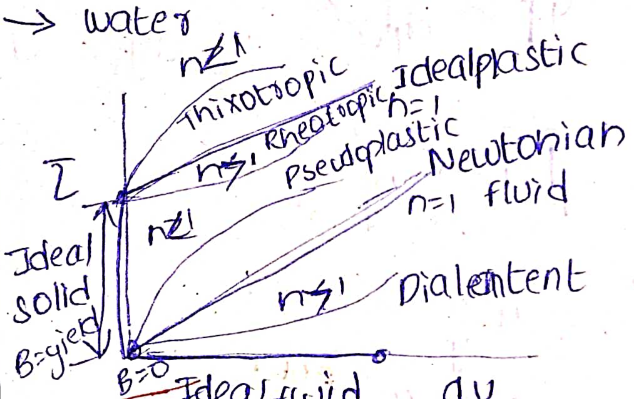
(ii) Frictionless (Non viscous) $\mu = 0$

[If $\mu = K$ newtonian fluid]

Fluid

- obeys (Newtonian fluid)
  - → Air
  - → water
- do not obeys (Non newtonian fluid)

Rheological diagram



A = μ

Thixotropic
Rheotropic
Pseudoplastic
Dialatent
All are Non newtonian fluid

$$\boxed{\tau = B + A\left(\frac{du}{dy}\right)^{n}}$$

ECET

① Newtonian ($n = 1$)

$B = 0$

$A = \mu$

$$\tau = B + A\left(\frac{du}{dy}\right)^{n}$$

Ex:- Air, water

$$= \mu \frac{du}{dy}$$

Time independent – [Dialent, pseudoplastic

② Dialatent ($n > 1$) (Shear thickening)

$B = 0$

$A = \mu$

$$\tau = \mu_{app}\left(\frac{du}{dy}\right)^{n > 1}$$

Ex:- Butter, rice starch, silica, Polyethylene glycol & Quick sand

cornstarch with water (oobleck)

$\mu_{app} = A\left(\frac{du}{dy}\right)^{n-1}$ +ve

③ Pseudoplastic (shear thinning)

$B=0 \quad A=\mu \quad n<1$

-ve $(n-1)$

$\mu_{app} = A\left(\frac{dy}{dy}\right)$

$\tau = \mu\left(\frac{du}{dy}\right)^{n<1}$

Herschel Bulkley Fluid

Ex:- Polymeric solution, Rubber, Milk, paints, paper pulp

④ Ideal plastic (Bingham plastic) - shear stress is more than yield

$B = \tau_{yield}$

$A = \mu$

$n = 1$

$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)$

$\mu_{app} = A\left(\frac{dy}{dy}\right)^0$

$= A$

slope $= A$

Ex:- Toothpaste, drilling mud, Sewage Sludge etc

[Time dependent viscosity fluids]

⑤ Thixotropic

$B = \tau_{yield} \quad A = \mu$

$n<1$

$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)^{n<1}$

Note:- Time ↑ μ ↑

Ex:- Printers Ink

~~nail polish~~

gypsum slurry

⑥ Rheotropic

$B = \tau_{yield} \quad A - \mu$

$n>1$

$\tau = \tau_{yield} + \mu\left(\frac{du}{dy}\right)^{n>1}$

Note Time ↑ μ ↓

Ex:- ~~Bentonite slurry~~ Bentonite slurry

drilling fluid

~~gypsum slurry~~

Surface Tension: It is a measure of liquid tendancy to take spherical shape caused by mutual attraction of liquid molecules (cohesion)

→ Surface tension is due to cohesion at surface of liquid per unit length

NOTE:- But capilarity is caused by both adhesion and cohesion

Surface tension

$\sigma = \frac{F}{L}$ (N/m)

Surface energy $\frac{W.D}{A} = J/m^2 = \frac{N}{m} \left(\frac{Nm}{m^2}\right)$

$[MT^{-2}]$ N/m - kg/sec^2

→ Surface tension can be calculated by Tensiometer / stalagmometer

Note → Specific gravity is measured by hydrometer

→ Viscousity is measured by viscometer, Ex:- Saybolt, Redwood

→ As the temperature increases surface tension (cohesion) decreases.

Applications: Soap Bubble, Rain water droplets, metal shots, mercury does not wet the glass etc

→ Due to cohesion surface tension causes pressure change across curved surfaces

Pressure difference

0 - 8 · in liquid Jet $\Delta P = \frac{2\sigma}{d} = \frac{\sigma}{r}$

0 - 4 in liquid droplet, Air Bubble $\Delta P = \frac{4\sigma}{d} = \frac{2\sigma}{r}$

0 - 2 in soap bubble (Hollow Bubble) $\Delta P = \frac{8\sigma}{d} = \frac{4\sigma}{r}$

Remember by its weight

$\boxed{\sigma_{water-air} = 0.073 N/m}$

Note: 1 bar = 100 kPa = 10^5 Pa

Ex:- $\sigma = 0.0075 \frac{kg}{m}$ of droplet of water having dia $= 0.075$ mm $\Delta P = ?$

$$\Delta P = \frac{4\sigma}{d} = \frac{4\times(0.075\times 10\ N/m)}{0.075\times 10^{-3}\ m}$$

$$= 4\ KN/m^2$$

$[1\ kg \simeq 10\ N \simeq 9.81\ N]$

(2) Excess pressure inside soap bubble of dia 1cm with $\sigma = 0.04\ N/m$ (surface tension)

$$\Delta P = \frac{8\sigma}{d}$$

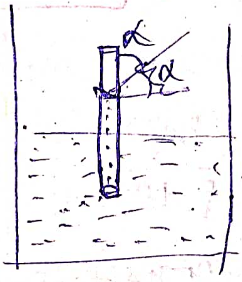
$$= \frac{8\times 0.04}{1\times 10^{-2}} = 32\ N/m^2$$

Capilarity

(<10mm tube) <5mm

Because for clear vision of ~~water~~ ~~bubble~~ fluid

capilary rise

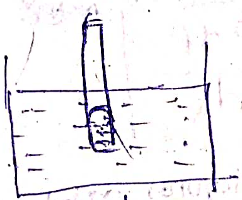


$\alpha = 0$

water

A>C convex downward

capillary depression/fall



approx $\alpha = 130^\circ$ (128° actual)

mercury

C>A

convex upward

α - neglected ≈ 0

$$\boxed{h = \left(\frac{4\sigma\cos(\alpha)}{\gamma d}\right)}$$

$$\boxed{h = \frac{30}{d} \text{ for water}}$$

ECET

Defination The phenomenon of rise (or) fall of a liquid surface relative to adjacent level of liquid in small diameter tubes (<10mm)
If (≥10mm) capillarity is neglected

Compressibility Ability to get compress under the action of normal force. It is denoted by 'β'

$$\beta \propto \frac{1}{\text{Bulkmodulous}}$$

$$K = \frac{-dP}{\frac{dv}{v}} \qquad \beta = \frac{\frac{-dv}{v}}{dp} \qquad \left[\frac{\text{direct stress}}{\text{volumetric strain}} = K\right]$$

Note:- υ (specific volume) $\propto \frac{1}{\rho\,(\text{density})}$

A gas is said to be incompressible if its mach no. < 0.2

$$dv = \frac{-1}{\rho^2}\cdot d\rho$$

$$\boxed{K = \frac{\rho\, dP}{d\rho}} \qquad \beta = \frac{\partial\rho}{\rho\, dP}$$

↑Te Liquids: K ↓ β ↑ (generally β = 0) gases: K ↑ β ↓

for Incompressible fluids $\boxed{\rho = c}$ $\boxed{\beta = 0}$

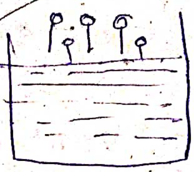
Pressure Normal force acting per unit area
[Pa/N/m²] 1MPa = 1N/mm²

atmospheric pressure
P_a = 0.760 m of Hg
= 101.325 kPa
= 1.01325 bar
= 10.3 m of water
= 14.7 psi [1bar = 10^5 Pa]

$$P_{abs} = P_{atm} + P_{guage}$$

$$P_{abs} = P_{atm} - P_{vaccum}$$

vapour pressure In a closed vessel at constant temperature liquid molecules break away from liquid surface and exert pressure over the top surface that is vapour pressure open vessel



(At K temp)

saturated humidity

Benzene = 24.47
Mercury = 0.173

vapour pressure
↑ high
↓ low

[solubility of gases in liquids ∝ vapour pressure]

CAVITATION

cavitation in fluid flow occurs when pressure of flow is <u>reduced below vapour pressure</u> ECET 2015

cavitation occurs due to <u>low vapour pressure</u> ECET 2013

→ cavitation is formation of vapour bubbles that is when pressure drops below saturation pressure (vapour pressure)

→ vapour bubble collapses as it swept from low pressure to high pressure and generates destructive pressure waves

→ cavitation leads to noise, vibration, erosion etc

→ It is observed in turbine runner exits, syphon

Boiling: $P_{constant}$ $T\uparrow$ T_{sat} liq → vapour

cavitation: $T_{constant}$ $P\downarrow$ P_{vapour} liq → vapour

	Water	Air
μ	0.001 Pa·sec	18×10^{-6} Pa·sec
ν	10^{-6}	15×10^{-6} m^2/s
K	2×10^{9} Pa	1 atm
σ	0.073 N/m	

→ 1 centistoke

0.01 poise
1 centipoise

CAVITATION occurs at
(i) suction side of centrifugal pump
(ii) exit of reaction rotor exit ~~Pump~~ Turbine

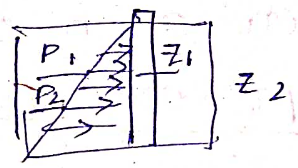
HYDROSTATICS

→ Variation of pressure in vertical direction above the datum in static fluid is proportional to its height from the datum [i.e depth]

→ $P = \frac{W}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h = \gamma h$

$dP = -\gamma\, dz$

$$\boxed{\frac{dP}{dz} = -\gamma}$$



$P_2 = z_2 \gamma$

$P_1 = z_1 \gamma$

$P_2 - P_1 = -\gamma(z_2 - z_1)$

$dP = -\gamma\, dz$

$\frac{dP}{dz} = -\gamma$

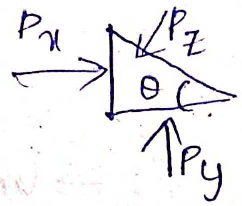
(−) Indicates that is depth
Since measuring downwards

γ - Specific weight

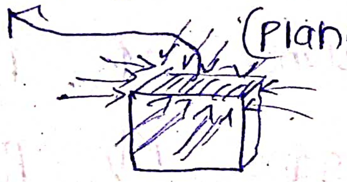
→ Hydrostatic law

Pascal's Law

It states that at any point in the fluid at rest or uniform motion, intensity of pressure exerted equally in all directions



$P_x = P_y = P_z$ (Idependent of θ (Plane))



conditions for pascal's law validity :-

① – Shear stress is zero

Note ② It is not valid under real fluid in non uniform motion

ECE 2015

Pressure at 3m below is 39.72 KN/m², then unit weight

a) 0.46 KN/m² c) 14 KN/m²

b) 4.6 KN/m² d) 1.4 KN/m²

$P = \rho g h$

$P = \omega h$

Pressure Measurement Devices

$P_{absolute} = P_{atm} + P_{guage}$

$P_{absolute} = P_{atm} - P_{vaccum}$

ECET

Absolute pressure at a point 3m below the clear water surface is measured as 125.5 KPa, $P_{atm} = 101 KN/m^2$, $P_g =$ ____ KPa

a) 24.4 c) 101.0 125.5 − 101 = 24.4

b) 48.0 d) 226.5

(Torricelli) tubes inverted

① Barometer — It is utilized for the measurement of local atmospheric pressure

② Piezometer — It is utilized for the measurement of atmospheric & positive guage pressure

We Cant measure gas pressure → It is not utilized for high pressure of fluids (more hieghts of piezometer are difficult to manufacture

③ Manometer — Balancing of high density fluid against high pressurised fluid is called Manometric principle

→ Property of manometric fluid is high specific gravity, high specific weight, low vapour pressure

→ Mercury has low vapour pressure and high mass density

Simple Utube Manometer

*** Differential Utube Manometer

ECET 2012 2017

- It measures both positive and negative guage pressure
- It is used for the measurement of pressure differences in between two pipe section

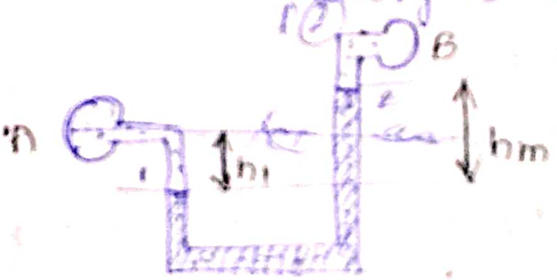
→ Manometric liquid is heavier than liquid for which pressure difference is to be measured

③ $g \uparrow$ -ve $g \downarrow$ +ve

$P = \rho g h$

④ $S = \frac{\rho_B}{\rho_{std}} = \frac{\delta}{\delta_{stand}}$

section — mercury is ending
— mercury is intersecting



$$P_A + \rho_1 g h_1 = P_B + \rho_2 g h_2 + \rho_m g h_m$$

$$P_A - P_B = \rho_2 g h_2 + \rho_m g h_m - \rho_1 g h_1$$

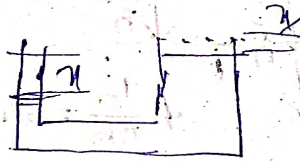
$$\frac{P_A}{\delta_{stan}} - \frac{P_B}{\delta_{stand}} = \frac{\delta_2 h_2}{\delta_{stan}} + \frac{\delta_m h_m}{\delta_{stan}} - \frac{\delta_1 h_1}{\delta_{stand}}$$

$$\boxed{h_A - h_B = S_2 h_2 + S_m h_m - S_1 h_1}$$

Inverted U tube Differential Manometer → It consists of lighter fluid as manometric fluid and measure low pressure differences

Note:- For high pressure measurement, out limb can be inclined, area increased to demagnify the pressure measurement

~~Are~~ lowerpressure – area↓



$V\downarrow = V\uparrow$

$A_1 h_1 = 3A_1 h_2$

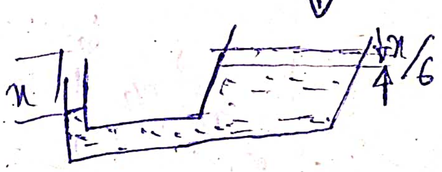
$h_2 = \frac{h_1}{3}$

$x_2 = \frac{x_1}{3}$

For ex:- if $\alpha = 30^\circ$

Input x = output $x \sin\alpha$

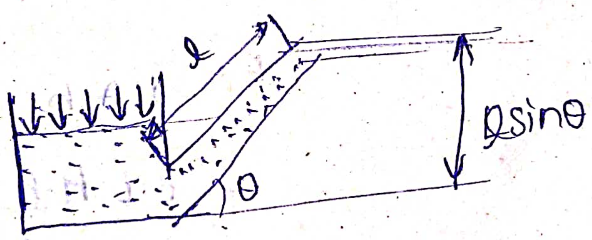
$x_2 = \frac{x}{2}$ $(\alpha = 30^\circ)$



$x = \frac{x}{3} \sin\alpha$

$x_2 = \frac{x_1}{6}$

Inclined Manometer



→ Accurate low pressure

→ $P = \rho g h$

$P = \gamma l \sin\theta$

Sensitivity of I-M $= \frac{1}{\sin\theta}$

Total Pressure

$\text{Force} = P \times A = \gamma \bar{h} \cdot A$

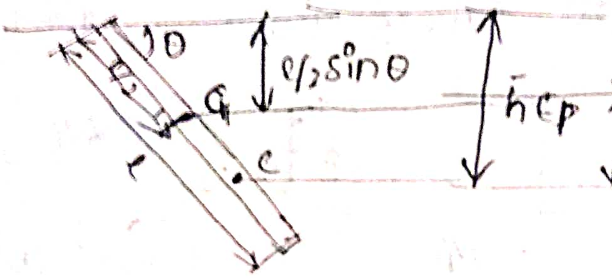
$F = \gamma \cdot A \cdot \bar{h}$

Centre of pressure

Point of application of Total pressure

$$\bar{h}_{c.p} = \bar{h} + \frac{I_G}{A\bar{h}} \sin^2\theta$$

Inclined Body



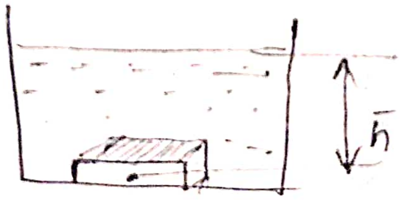
$\frac{I_G}{A\bar{h}} \sin^2\theta$

$\bar{F} = \gamma A \bar{h}$

$= \gamma l B \frac{L}{2} \sin\theta$

centre of pressure (c) lies always below the centre of gravity

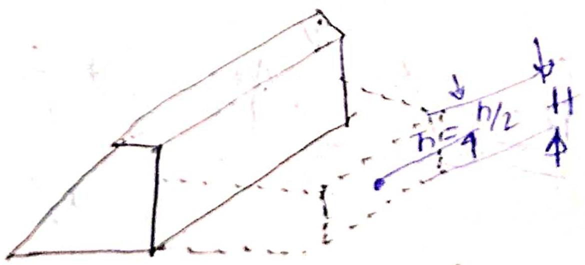
Horizontal



$\bar{h} = \bar{h}_{c.p} \; (\theta = 0 \; ; \; \frac{I_G}{A\bar{h}} \sin^2\theta = 0)$

$F = \gamma B L \bar{h}$

Vertical



$A = LH$

$I_G = \frac{LH^3}{12}$

$F = \gamma \cdot A \cdot \bar{h}$

$= \gamma \cdot L \cdot H \cdot \frac{H}{2}$

$= \frac{\gamma H^2 L}{2}$

$$\boxed{\frac{F}{L} = \frac{\gamma H^2}{2}}$$

$$\bar{h}_{cp} = \bar{h} + \frac{I_G}{A\bar{h}}\sin^2\theta$$

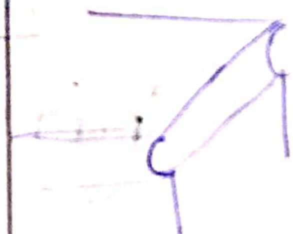
$$\bar{h}_{cp} = \bar{h} + \frac{I_G}{A\bar{h}} \quad (\theta = 1)$$

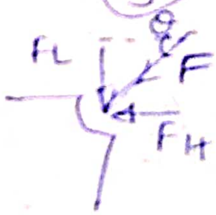
$$\bar{h}_{cp} - \bar{h} = \frac{I_G}{A\bar{h}}$$

$$h_{cp} = \frac{H}{2} - \frac{\frac{LH^3}{12}}{LH \times \frac{H}{2}}$$

$$= \frac{H}{2} + \frac{H}{6} = \frac{2H}{3}$$

curved surface





F_H - Total pressure water
F_l = weight of ~~fluid~~ from curved surface to free surface

$$F_H = \gamma A\bar{h}$$

$$= \gamma R l \frac{R}{2}$$

$$F_H = \frac{\gamma R^2 \cdot l}{2} \Rightarrow \boxed{\frac{F_H}{L}} = \boxed{\frac{\gamma R^2}{2}}$$

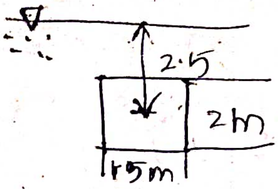
$$F_L = w = mg = \rho \cdot v \cdot g$$

$\frac{1}{4}$th of circle

$$= \rho \times \frac{\pi}{4} \times r^2 \times l \cdot g$$

$$F_R = \sqrt{F_L^2 + F_H^2} \qquad \theta = \tan^{-1}\left[\frac{F_V}{F_H}\right]$$

2015 ECET



Total pressure

$= \gamma A \bar{h}$

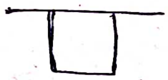
$= \cancel{7.5\times1}\ 10^4 \times 2 \times \frac{3}{2} \times \frac{5}{2} = 7.5\times10^4$ (10^3 N kN)

~~a)~~ 73.5 kN c) 7.35 kN

b) 0.735 kN d) 735 kN

~~Trapezium~~

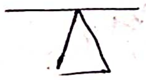
Depth of centre of pressure



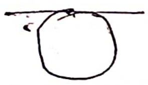
$\bar{h}_{cp} = \frac{2h}{3}$

ECET 2015

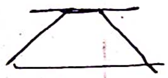
$\bar{h}_{cp} = \frac{h}{2}$



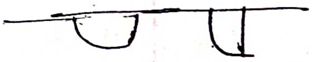
$\bar{h}_{cp} = \frac{3h}{4}$



$\bar{h}_{cp} = \frac{5D}{8}$

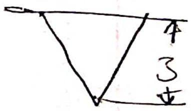


$\bar{h}_{cp} - \frac{h}{2}\left(\frac{a+3b}{a+2b}\right)$



$\bar{h}_{cp} = \frac{3\pi D}{32}$

ECET 2016

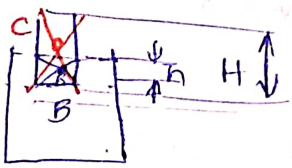


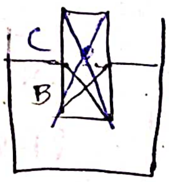
$\bar{h}_{cp} = \frac{3}{2}$

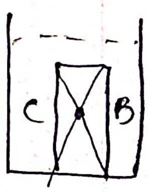
$= 1.5$

Buoyancy

Archimedes principle







→ centre of gravity of displaced volume of water is called as centre of buoyancy

$W_{Body} = W_{water\ displaced} = F_{buoyancy}$

$$W_{Body} = F_{Buoyancy}$$

$$m_{body} \times g = W_{liq}$$

$$m_{body} \times g = m_{liq} \times g$$

$$\rho_B V_B = \rho_L \times V_L$$

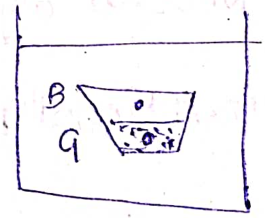
$$\rho_B A H = \rho_L A h$$

$$\frac{\rho_B H_B}{\rho_{stan}} = \frac{\rho_L h_L}{\rho_{stand}}$$

$$\boxed{S_1 H_1 = S_2 h_2}$$

Special case

For a submerged body, sometimes centre of gravity can be below the centre of buoyancy

→ 

Based on mass, centre of gravity decides and for ~~hollow parts having~~ centre of buoyancy it depends on volume of water displaced

→ when a body is immersed fully (or) partially it is lifted up by a force equals to weight of liquid displaced by the body

→ This statement is archimedes principle

→ Tendency of liquid to uplift a submerged body due to upthrust of body is called Buoyancy

ECET2013

An oil of specific gravity is 0.75 and height of oil at a point is 32m corresponding height of water

$$s_1 h_1 = s_2 h_2 \quad (s_2 = 1 \text{ for water})$$

$$\frac{0.75 \times 32}{1} = h_2 = 24m$$

ECET2012

$h_1 = 8m$
$s_1 = 0.8$

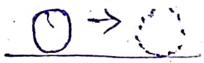
$(S = 0.8)$ head $=$ $\underline{6.4m}$ water head

$(s_1 h = s_2 h_2) \rightarrow s_2 = 1$

stability

- stable equilibrium

- unstable equilibrium

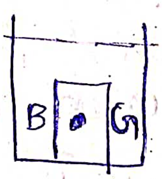


- Neutral equilibrium

- conditionally stable

SUBMERGED BODIES

<u>submarines</u>

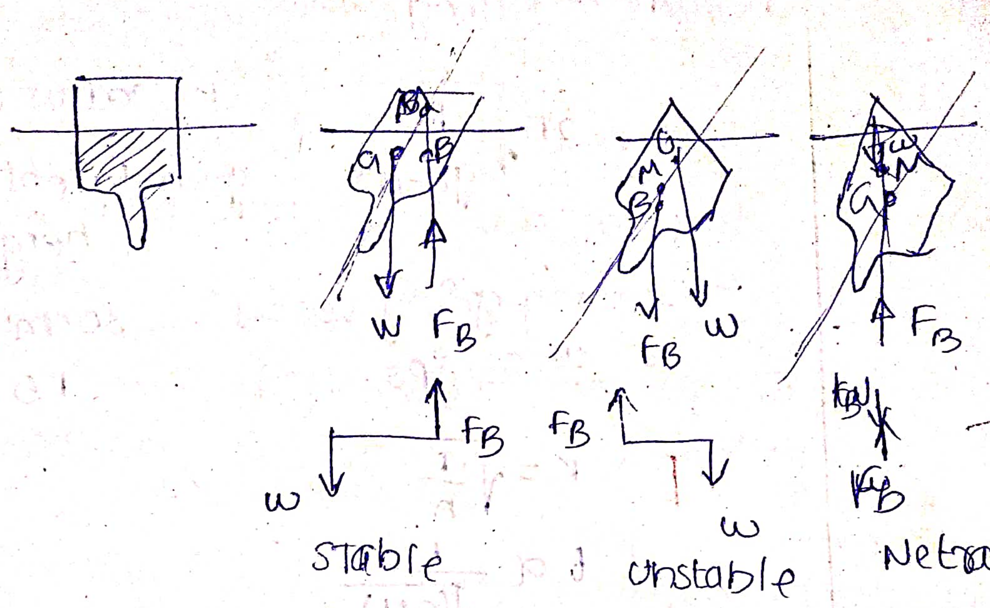


B is above G
Body is stable

G is above B
Body is not stable

Neutral eqll
B & G are coincident

Floating Bodies



Metacentre (M) :- point of intersection of (axis of body & line of action of buoyancy force)

→ Body oscillation centre is metacentre

GM +ve - stable
GM -ve - unstable
GM 0 ~~neutral~~ - neutral

$GM = BM - BG$ (Metacentre)

$GM = \frac{I_{min}}{V_{wet}} - \left[\frac{H}{2} - \frac{h}{2}\right]$ (depth)

$GM_{pitching} > GM_{rolling}$

Equilibrium	stable	Unstable	Neutral
Submerged	B is above G	B is below G	B & G are coincident
Floating	M is above G GM = +ve	M is below G GM -ve	M & G are coincident GM = 0

Period of rolling

$$t = 2\pi\sqrt{\frac{K^2}{g(GM)}}$$

K- radius of gyration

g - metacentric height

* ocean going vessels 30cm - 1.25m
* warships 1.5m

$$K = \sqrt{\frac{I}{A}}$$

$$t \propto \frac{1}{\sqrt{(GM)}}$$

FLUID KINEMATICS

Fluid kinematics → It is a science which deals with behaviour of fluid under motion without considering forces.

→ It studies displacement, velocity, angular motion, deformation, acceleration etc

Types of Flows

① Steady flow :- If the parameters (velocity, ρ, P, μ, t etc) does not change with time.

$$\left.\frac{d(\text{Property})}{dt}\right]_{\text{space fixed}} = 0 \qquad \frac{du}{dt}, \frac{d\rho}{dt}, \frac{dP}{dt}, \frac{d\mu}{dt}, \frac{dT}{dT} = 0$$

Note constant (rate/discharge) corresponds to study flow

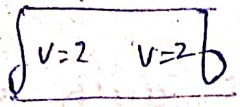
② Unsteady flow :- flow parameters changes with time Ex:- opening of valve (discharge varying)

Ex:- Opening a valve

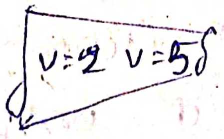
③ uniform flow :- Flow parameters at any point both in magnitude and direction from point to point does not changes at any given instant

$$\left.\frac{d(\text{prop})}{ds}\right]_{\text{time fixed}} = 0 \qquad \frac{dv}{ds} = 0$$

④ Non uniform flow : fluid parameters vary with each point to point dia is constant (Uniform)



dia. varies (Non uniform)



Steady uniform	Flow through long pipe of constant dia at constant rate
Steady nonuniform	Flow through tapering pipe at constant rate
Nonsteady uniform	Flow through pipe of constant dia at changing rate
Nonsteady non uniform	Flow through pipe of tapering dia at changing rate

Flow patterns — ① Flow line

Based on langragian approach

a) Path line :- line traced by a single fluid particle

b) Stream line :- tracing of motion of different fluid particles (imaginary curve line tangent to velocity vector at every point in a given instant

~~c) streak line~~ :-

~~② stream tube~~

→ D.e of stream line $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$ (3-D)

$$\boxed{\begin{aligned} vdx &= udy \\ vdx - udy &= 0 \\ udy - vdx &= 0 \end{aligned}} \leftarrow \frac{dx}{u} = \frac{dy}{v} \quad (2\text{-}D)$$

Not in case of path line {
→ stream line do not intersect itself
→ No two stream line can intersect each other

→ St. line spacing $\propto \frac{1}{\text{velocity}}$

stream line space ↓ velocity ↑ K.E ↑ (Ex:- Nozzle)

Imaginary line (t = c)

ECET 2016 NOTE

© streak line :- line joining locus of particles which had earlier passed through a choosen point

→ Streak line is traced by joining no. of path lines at any given instant

Under steady flow; streamline & pathline & streak line are same under steady flow

② stream tube :- An imaginary tube formed by group of stream lines passing through small closed curves

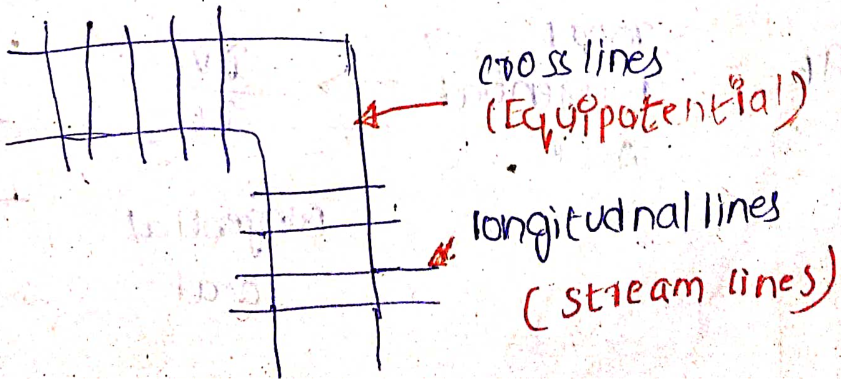
③ Flow Net :- It is a grid obtained by a series of longitudnal (stream lines) and cross lines (equipotential lines).

ECET 2016

→ Stream lines and Equipotential lines are perpendicular to each other

→ It is a graphical technique for 2d ~~steady~~ of steady irrotational flow

→ It is used when mathematical relation and potential function are either not available (or) difficult to use



Velocity of Fluid Flow

$$\bar{U} = u_x\hat{i} + u_y\hat{j} + U_z\hat{k}$$

If ~~$u_x = u_y = u$~~ $u_x = u$

$u_y = v$

$u_z = w$

$$|\bar{u}| = \sqrt{u^2+v^2+w^2}$$

Acceleration

$$\bar{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|\bar{a}| = \sqrt{a_x^2+a_y^2+a_z^2}$$

ECET

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \underbrace{\frac{\partial w}{\partial t}}_{\text{time accl}} + \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{space accl}}$$

local (or) temporal acceleration — Convective accleration

Normal a_N — Total Normal Accel $\longrightarrow \underbrace{\frac{\partial v_n}{\partial t}}_{\text{Normal local}} + \underbrace{\frac{v^2}{r}}_{\text{Normal convective}}$

Tangential

a_t — Total Tangential Accl $\longrightarrow \underbrace{\frac{\partial v_t}{\partial t}}_{\text{tangential local}} + \underbrace{v\frac{\partial v}{\partial s}}_{\text{tangential connective}}$

→ In steady flow, local accl. is zero $\left(\frac{\partial v}{\partial t}=0\right)$

→ stream lines are equidistant (constant velocity) and uniform flow tangential convective accleration becomes zero.

→ If the stream lines are straight and parallel, then convective accl^n is zero $(r=\infty)$ ← Infinity

Discharge (or) Flow rate (or) Volumetric flow rate

$$Q = \frac{\text{Volume}}{\text{time}} = \frac{A \cdot S}{t} = A \cdot V$$

units :- m^3/sec (or) lit^3/sec

For mass flow rate

$$\dot{m} = \frac{m}{t} = \frac{\rho \times V}{t} = \rho Q = \rho A V$$

Continuity Equation → ECET 2017

It is based on law of conservation of mass

→ Time rate change of mass in a fixed volume is equals to net rate of flow of mass across the surface

$$\dot{m}_1 = \dot{m}_2$$

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

$$\rho_1 Q_1 = \rho_2 Q_2$$

$$\rho_1 Q_1 \times \frac{g}{g} = \rho_2 Q_2 \times \frac{g}{g}$$

$$\boxed{\frac{\omega_1 Q_1}{g} = \frac{\omega_2 Q_2}{g}}$$

For incompressible fluids $[\rho_1 = \rho_2]$ - liquids

$$\boxed{A_1 V_1 = A_2 V_2}$$

Assumption steady, incompressible, one dimensional, uniform flow

NOTE :- Divergance from continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

$$\left[\nabla - \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right]$$

for steady compressible

$$\frac{\partial \rho}{\partial t} = 0 \; ; \; \boxed{\nabla(\rho v) = 0}$$

~~for steady~~ $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

For steady incompressible

$$\nabla(v) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad - 3D$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0} \quad - \cancel{3D}$$

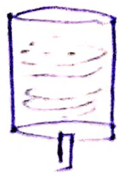
$$\boxed{\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}} \quad - 2D$$

Rotational and Irrotational flows — If the fluid particle while moving in the direction of flow rotate about their mass centre is R.F

Ex:- liquid in rotating tank where velocity of each particle varies ∝ to distance from centre
washing machines, juicers

<u>forced vertex</u>

Rotating drum



$v \propto r$

$\frac{v}{r} = c$

velecity maximum at maximum radius and minimum at centre

<u>free vertex</u>

In free vertex

$vr = c$

velecoity maximum at centre

— for Irrotational flow, curl of velocity vector is zero

$\cancel{\Delta V = 0}$ $\cancel{\nabla V = 0}$

$\nabla \times V = 0$

delioperator

Angular velocity (3d matrix)

~~Abc~~ $\omega = \frac{1}{2}\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$

$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$

$\omega_y = -\frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right] = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$

$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

Since $\omega = 0$ for irrotational flows

$\omega_x = \omega_y = \omega_z = 0$

so

so

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

→ Flow is said to be irrotational when $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

→ for fluid of flow of large velocity then flow is irrotational ~~flo~~

→ For fluids such as air & water having low viscosity flow in region away from boundary is irrotational flow

→ In case of rapidly converging, acceleration flows flow may be treated as irrotational

Circulation Line integral of tangential velocity vector around a closed loop is circulation

$$\Gamma = \int_0^{2\pi} v_t \, ds$$

$$= \int_0^{2\pi} R\omega R \, d\theta$$

$$= \omega R^2 \int_0^{2\pi} d\theta$$

$$\boxed{\Gamma = 2\pi\omega R^2}$$

Vorticity Ω :

$$\Omega = \frac{\text{circulation}}{\text{Area}}$$

$$= \frac{\int \omega R^2 d\theta}{\int \frac{R^2 d\theta}{2}} = 2\omega_z$$

$$= \cancel{2}\left[\frac{1}{\cancel{2}}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial z}\right)\right] = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

If it is an irrotational flow then $\boxed{\Omega = 0}$

$$\frac{\Gamma}{A} = 0 \quad ; \quad \nabla \times V = 0$$

$$\boxed{\Omega = \frac{\Gamma}{A} = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla \times V}$$

for irrotational flows $= 0$

Shear strain when torque is applied to the fluid particles fluid in rotation subjected to shear

→ shear strain is the average of velocity gradient

$$\gamma_{xy} = \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

$$\gamma_{yz} = \frac{1}{2}\left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]$$

$$\gamma_{zx} = \frac{1}{2}\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]$$

Stream function & Potential function (ψ) It is a scalar function of space and time such that its partial derivative w.r.t any direction gives the velocity component at right angles (opposite direction to other velocity component)

STREAM FUNCTION

stream function [ψ]

$$\frac{\partial \psi}{\partial x} = v \qquad \frac{\partial \psi}{\partial y} = -u$$

$$d\psi = \frac{\partial u}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0$$

$$v\,dx - u\,dy = 0$$

$$\boxed{v\,dx = u\,dy} \longrightarrow \text{stream line eq}^n$$

Slope of stream line

$$\frac{dy}{dx} = \frac{v}{u}$$

*** → stream function can not defined for 3d incompressible flows

→ laplace equation for stream function

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0} \longrightarrow \text{Irrotational flow}$$

→ Difference between the stream function of 2 stream line equals to discharge per unit width

$$\frac{Q}{\text{unit width}} = |\psi_2 - \psi_1|$$

Potential ~~line~~ function

[ϕ] - Potential function is a scalar function of space and time whose negative partial derivative w.r.t any direction will give velocity component in that direction only

$\phi - [x, y]$

$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$

$-u\,dx - v\,dy = 0$

$-u\,dx = v\,dy$

$$\boxed{m_2 = \frac{dy}{dx} = \frac{-u}{v}}$$

$-\frac{\partial \phi}{\partial x} = u$

$-\frac{\partial \phi}{\partial y} = v$

$-\frac{\partial \phi}{\partial z} = w$

~~$m_1 \times m_2 =$~~

product of slopes < streamline $m_1 = \frac{v}{u}$; equipotential line $m_2 = \frac{-u}{v}$

$m_1 \times m_2 = \frac{v}{u} \times \frac{-u}{v} = -1$

then lines are perpendicular [ECET]

→ stream lines and equipotential lines are perpendicular to each other [except at stagnation points] where local velocity = 0

→ velocity potential exists only for irrotational flows, any function (ϕ) that satisfies laplace equation is a case of irrotational flows

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

FLUID DYNAMICS

→ It is the study of fluid motion by considering forces, causing fluid flow

→ A fluid in motion is subjected to various forces

① Body, Volume force → $F \propto V$

Ex:- weight, centrifugal force

② Surface & Area force → $F \propto A$

Ex:- Pressure force, shear force, compressibility

③ Linear & length force → $F \propto l$

Ex:- surface tension force

G P V T S C

G – gravity
P – Pressure
V – Viscous force
T – Turbulency
S – Surface tension
C – compressibility

Equations of form.

Newton → $F = F_g + F_p + F_v + F_t + F_s + F_c$

Reynolds → $F = F_g + F_p + F_v + F_t$

ECET — Navier Stokes → $F = F_g + F_p + F_v$

Eulers → $F = F_g + F_p$

Bernoullis Equation

Integration of eulers equation — stead, incompressible, Non viscous — imp

Stedy Ideal Fluid

ECET 2016

① Pressure head $\left(h = \frac{P}{\gamma}\right)$

② Kinetic head $= \frac{k.e}{weight} = \frac{\frac{1}{2}mv^2}{mg} = \frac{V^2}{2g}$

③ potential head $= \frac{P.e}{weight} = \frac{mgz}{mg} = Z$

Bernoullis theorm

$$T.e = \cancel{\frac{V^2}{2g}}\ \frac{P}{\gamma} + \frac{V^2}{2g} + Z$$

$$T.e_1 = Te_2$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

Units

① $\frac{P}{\gamma} + \frac{V^2}{2g} + Z \quad - \quad \frac{N\text{-}m}{N} = J/N$

② $\times g = \frac{P}{\rho g} \times g + \frac{V^2}{2} + gZ \quad - \quad \frac{N/m^2}{kg/m^3} = \frac{Nm}{kg} = \frac{J}{kg}$

③ $\times \rho$

$P + \frac{V^2\rho}{2} + \rho g Z \quad - \quad \frac{N}{m^2} \times \frac{m}{m} - J/m$

Note:- It is based on law of conservation of energy

→ It is valid for Incompressible fluids - liquids

Bernoullis eqn valid for compressible

$$\int \frac{dP}{\rho} + v\,du + g\,dz$$

$$\frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant}$$

B.E for Non uniform flows

→ Kinetic energy correction factor

$$\alpha = \frac{1}{A v^3} \int v^3 dA$$

$\alpha = 1$ — uniform

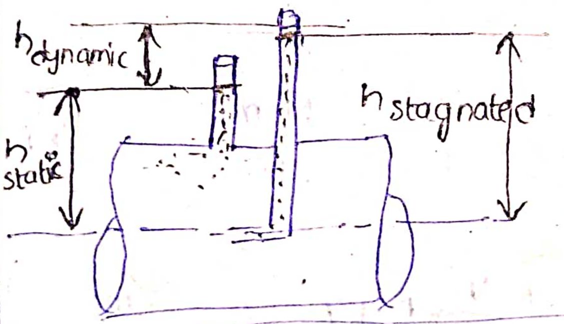
$= 2$ — laminar

$= 1.01 - 1.2$ — turbulent

Applications of Bernoullis theorem

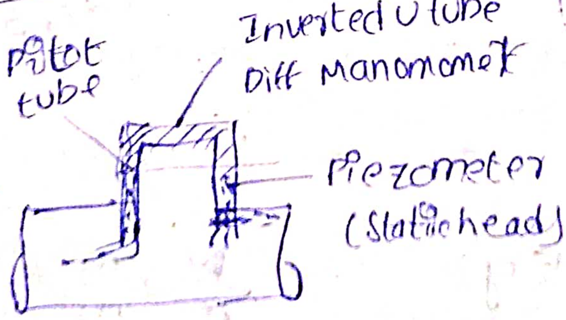
① Pitot tube

It is a L shaped tube used for calculating velocity of fluids



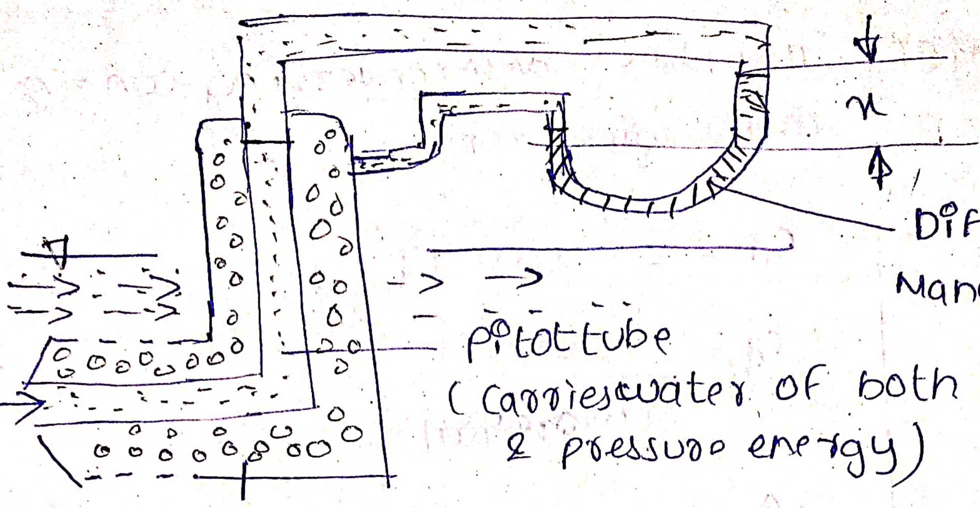
$h_{stagnated} = h_{static} + h_{dynamic}$

$h_d = h_t - h_s$



$$h = x\left(1 - \frac{S_m}{S_0}\right)$$

$$V = \sqrt{2 g h_{dyn}}$$



[PITOT STATIC TUBE]

→ If the velocity of flow at a point equals to zero, it is known as stagnation point

→ Pitot static tube measures both static and stagnation pressure head

→ It consists of two concentric tubes with ~~Angular Sp~~ Angular Space (hollow)

→ Outer tube has holes drilled perpendicular to flow direction which provides liquid static head

→ Inner tube works as normal pitot tube

→ A differential connecting these tubes is called as prandle pitot tube ($C_v = 1$)

Venturimeter: It consists of ① converging cone ② Throat ③ diverging cone

$C_d = 0.98$ to 0.99

$$C_d = \frac{Q_{actual}}{Q_{theoritical}}$$

$$Q_{act} = C_d \cdot \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$$

where

$h = x\left[\frac{S_m}{S_o} - 1\right]$ where ($S_m > S_o$)

length of convergent cone $= 2.7 \times (D - d)$

D - dia of pipe

d - dia of throat

Angle involved in convergent cone $= 21° \pm 1°$

" divergention $= 5° - 15°$

preferable $6° - 7°$

efficient $6°$

→ If ~~<6°~~ angle is $< 6°$ length of divergent increases

→ If angle is $> 7°$ turbulency increases

Dia of throat $= \frac{1}{2}$ (to) $\frac{3}{4}$ D

length of divergent is greater than length of convergent to avoid turbulency & seperation of fluid

$$C_d = \sqrt{1 - \frac{h_L}{h}}$$

$$C_d^2 = 1 - \frac{h_L}{h}$$

$$\frac{h_L}{h} = 1 - C_d^2 \Rightarrow h_L = h[1 - C_d^2]$$

→ If ~~the~~ venturimeter is inclined from horizontal position, its reading of discharge remains same

Oriffice Meter :-

→ It consists of plate having sharp edge circular hole called oriffice

→ Dia of hole = 0.5D

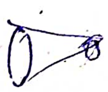
Plate thickness = 0.05D

$$Q_{act} = C_d \cdot Q_{theo}$$

$$= C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

→ Oriffice meter is cheaper than venturimeter

Nozzle Meter :- (Discharge)



→ when compared to venturimeter, diverged hence greater dipersion of energy

→ Equation of discharge is same as venturimeter

Rotameter :- (To measure discharge)

→ It is a variable area flow rate measuring device

→ It consists of glass tube inside which a float positions and reads calibrated discharge

→ It involves gravity, pressure, buoyancy

NOTE :- In a forced vertex height of Paraboloid (or) height of fluid raised is $\frac{v^2}{2g}$

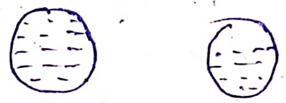
$$v^2 = 2gh$$

$$h = \frac{v^2}{2g}$$

$$h = \frac{r^2\omega^2}{2g}$$

Flow through pipes

→ Pipe is used to convey fluid through confined passage in a closed section



Pressurised flow

Open channel flow -

Reynolds No :- $\frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho A V^2}{\frac{\mu A V}{L}}$

$$= \frac{\rho V L}{\mu} = \frac{\rho V D_H}{\mu}$$

Froude No :- $\sqrt{\frac{\text{Interia F}}{\text{gravity F}}}$

Eulers No :- $\sqrt{\frac{\text{Interia F}}{\text{Pressure F}}}$

Webers No :- $\sqrt{\frac{I_F}{\text{surface tension}}}$

Mach No :- $\sqrt{\frac{I_F}{\text{Elastic force}}}$

→ As per ~~Rey~~ Laminar & Turbulent flow, Reynold's number is the criteria for identification of type of flow

$\boxed{R_n = \frac{\rho V D}{\mu}}$ ρ - density, V - velocity, μ - dynamic viscosity

< 2000

2000 < R_n < 4000 - transition semi turbulent → unstable flow

> 4000 - turbulent

⟹ 2800 is lower critical R.N

Loses in Pipes :-

~~(I) Losses in pipes :~~

(1) Laminar ⟷ hagen poisuelle equation

$Q = aV$

$= \frac{\pi}{4} d^2 \times V$ $\boxed{V = \frac{4Q}{\pi d^2}}$

$\boxed{h_l = \frac{32 \mu V L}{\gamma D^2}} = \frac{32 \mu L}{\gamma D^2} \times \frac{4Q}{\pi d^2}$

$\boxed{h = \frac{128 \mu Q L}{\pi \gamma D^2}}$

$P = \frac{W}{t} = \frac{F \times S}{t} = \frac{W \times S}{t} = \frac{\gamma (\text{volume}) \times S}{t}$

$\boxed{P_{loss} = \gamma Q h_L}$

$P = \gamma Q h_L = \gamma Q \left[\frac{128 \mu Q L}{\pi \gamma D^4}\right]$

$= \frac{128 \mu Q^2 L}{\pi D^4}$

→ Due to viscosity

Majorlosses (friction)

① Darcy's Weisbacheq's

$$h_f = \frac{4f'lv^2}{2gd}$$

f' (coefficient of friction)

f (friction factor)

$f = 4f'$

$$h_f = \frac{4f'lQ^2}{2gd \times \left(\left(\frac{\pi}{4}\right)^2 \times d^2\right)^2}$$

$$= \frac{f'lQ^2}{d^5} \times \frac{4}{9.81 \times 2 \times \frac{\pi^2}{16}}$$

$$= \frac{f'lQ^2}{d^5} \times 0.33$$

$$= \frac{f'lQ^2}{d^5} \times \frac{1}{3} = \frac{f'lQ^2}{3d^5}$$

(or)

$$= \frac{flQ^2}{12d^5}$$

$$\left.\begin{array}{l} f = \frac{64}{R_e} \\ f' = \frac{16}{R_e} \end{array}\right\} \text{laminar flow}$$

$$\left.\begin{array}{l} f = \frac{0.316}{R_e^{1/4}} \\ f' = \frac{0.075}{R_e^{1/4}} \end{array}\right\} \text{turbulent flow}$$

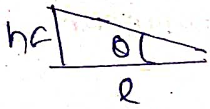
2) Chezy's equation:-

$$V = C\sqrt{mi}$$

m - hydraulic mean depth

$$m = \frac{A}{P} = \frac{\frac{\pi}{4}\times d^2}{\pi d} = \frac{d}{4}$$

$$i = \frac{h_f}{l}$$



$$V^2 = C^2 m i$$

C - chezy's constant

~~dz~~

$$i = \frac{V^2}{C^2 m} = \frac{4V^2}{C^2 d}$$

$$\frac{h_f}{l} = \frac{4V^2}{C^2 d} \Rightarrow \boxed{h_f = \frac{4lV^2}{C^2 d}}$$

Relation between C & f', f

$$h_f = h_f$$

$$\frac{4f'lV^2}{2gd} = \frac{4V^2 l}{C^2 d}$$

$$\frac{f'}{2gd} = \frac{1}{C^2}$$

$$C = \sqrt{\frac{2g}{f'}} = \sqrt{\frac{2g}{f}}$$

(4) Mannings equation

$$V = M m^{2/3} i^{1/2} \quad [M - \text{manning constant}]$$

$$V = \frac{1}{n} r^{2/3} s^{1/2}$$

→ Relation between chezy's & Mannings constant

$$\cancel{C m^{1/2}} \quad V = V$$

$$C m^{1/2} i^{1/2} = M m^{2/3} i^{1/2}$$

$$C \cancel{m^{1/2}} \cancel{i^{1/2}} = M \cancel{m^{1/2}} m^{1/6} \cancel{i^{1/2}}$$

$$C = M m^{1/6}$$

Power :- $wQ(H - h_f)$ — friction head

Maximum :- $H = 3h_f \Rightarrow h_f = \frac{H}{3}$ — gross Head

Efficiency :-

$$\frac{P_{\text{with friction}}}{P_{\text{without friction}}} = \frac{wQ[H - h_f]}{Q[H]}$$

$$\eta = \cancel{HH} \frac{H - h_f}{H}$$

$$= \frac{3h_f - h_f}{3h_f} = \frac{2h_f}{3h_f} = \frac{2}{3} \times 100$$

$$= 66.6\%$$

FLUID MACHINES

Impulse Momentum Equation

It is Based on law of conservation of momentum. This equation is applicable to control volume analysis. [volume fixed]

It states that

(P-momentum-M)

$$\boxed{F = \frac{dM}{dt} = \frac{dP}{dt} = \frac{d}{dt}mV}$$

[Momentum equation]

$$\boxed{F \times dt = d(m \times V)}$$ - Impulse momentum equation

$~~MT^{-1}~~$.

Units = N sec - $ML^{1}T^{-1}$

Momentum correction factor

$$\beta = \frac{1}{AU^2}\int u^2 dA$$

$\beta = 1$ — uniform

$\beta = 1.33$ — laminar

$\beta = 1.02 - 1.05$ — turbulent

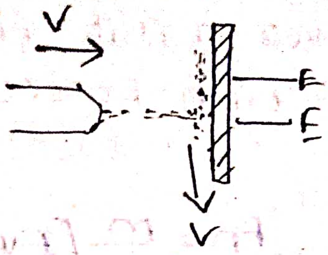
Mass flow rate

$$\dot{m} = \frac{m}{t} = \frac{\rho \cdot \text{volume}}{\text{time}} = \rho \times Q = \rho \times A \times V$$

$$\boxed{\dot{m} = \rho AV}$$

NOTE:- $F = m \times a$ [mass of fluid x acceleration = Inertia force = momentum flux]

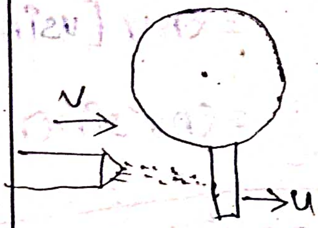
Flat fixed plate



$$F = \frac{\dot{m}}{t}[I \cdot V - F \cdot V]$$

$$= \rho AV(V-0) = \boxed{\rho AV^2}$$

Flat moving vertical plate [single plate]



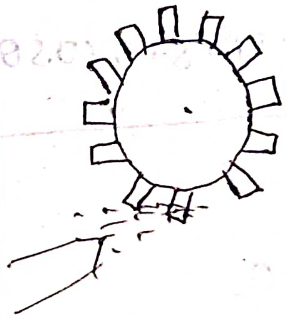
$$F = \frac{m}{t}[I \cdot V - F \cdot V]$$

$$\cancel{= \rho AV[V-u}$$

$$= \rho A(V-u)[(V-u) - 0]$$

$$\boxed{F = \rho A(V-u)^2}$$

Series of moving plate



$$F = \frac{m}{t}[I \cdot V - F \cdot V]$$

$$= \rho AV(V-u-0)$$

$$\boxed{F = \rho AV(V-u)} \rightarrow \frac{\pi DN}{60}$$

$$\boxed{P = \frac{W \cdot d}{sec} = \rho AV(V-u) \cdot u}$$

$$\eta = \frac{\frac{W \cdot D}{sec}}{\frac{K \cdot E}{sec}} = \frac{O \cdot P}{I \cdot P}$$

when a jet of water strikes vertical hinged plate

$$\sin\theta = \frac{F}{W} = \frac{\rho a V^2}{W}$$

$$= \frac{\rho AV(V-u)u}{\frac{1}{2}(\rho AV)V^2}$$

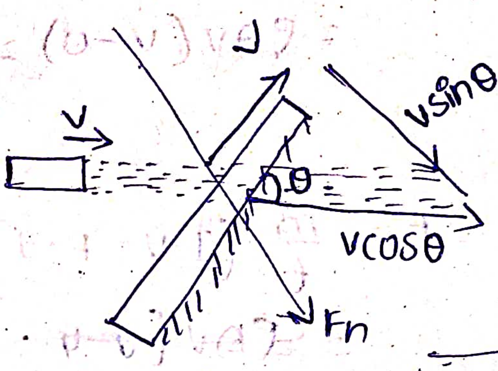
$$= \frac{2[V-u]u}{V^2}$$

$$\boxed{\eta_{max} \text{ condition } V = 2u} \text{ or } \eta = \frac{V}{2}$$

$$\boxed{\eta_{max} - 50\%}$$

peripheral velocity would be half of the velocity of jet

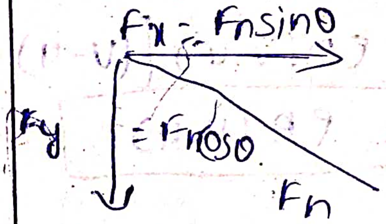
CASE 4:- Force exerted over a flat fixed inclined plate 'θ' with horizontal



$$F_n = \frac{m}{t}[I.V - F.V]$$

$$= \rho A V [v\sin\theta - 0]$$

$$= \rho A v^2 \sin\theta$$



$$\boxed{\begin{aligned} F_n &= \rho A v^2 \sin\theta \\ F_x &= \rho A v^2 \sin^2\theta \\ F_y &= \rho A v^2 \sin\theta \cdot \cos\theta \end{aligned}}$$

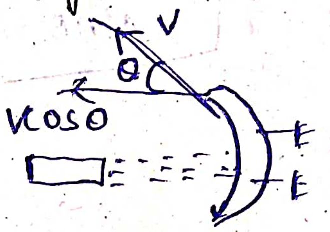
for moving plate

$$F_n = \rho A (v-u)^2 \sin\theta$$

$$F_x = \rho A (v-u)^2 \sin^2\theta$$

$$F_y = \rho A (v-u) \sin\theta \cdot \cos\theta$$

case 5:- Force exerted over a smooth fixed symmetrical curved vane fixed at centre



$$F = \frac{m}{t}[I.V - F.V]$$

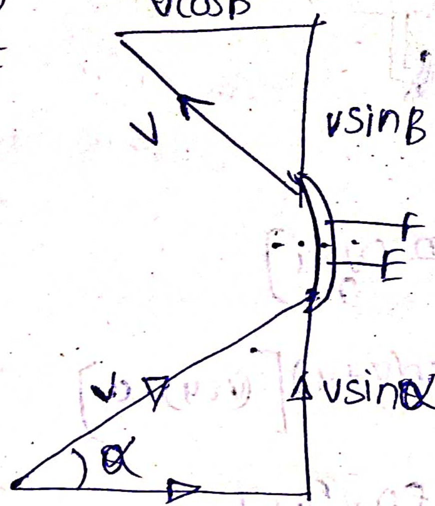
$$= \rho A v [v - (-v\cos\theta)]$$

$$= \rho A v^2 (1 + \cos\theta)$$

For moving plate

$$\boxed{= \rho A (v-u)^2 (1 + \cos\theta)}$$

a) Force of jet impinging on a fixed plate in the direction perpendicular to motion of jet



$F_x = \rho AV(V\cos\alpha + V\cos\beta)$

$= \rho AV^2(\cos\alpha + \cos\beta)$

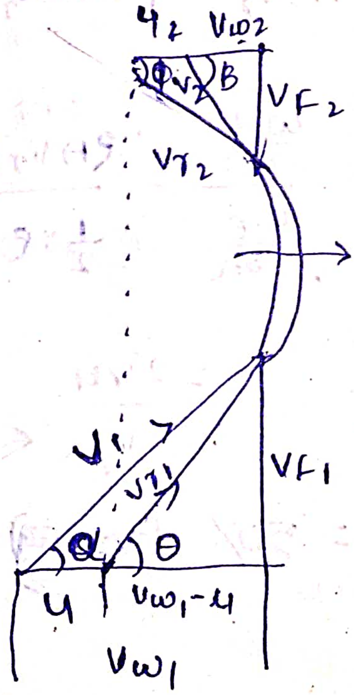
$F_y = \rho AV(V\sin\alpha - V\sin\beta)$

$= \rho AV^2(\sin\alpha - \sin\beta)$

$F_R =$

Unsymmetrical moving curved vane jet strikes at tips

b)



$F_x = \frac{m}{t}[I.V - F.V]$

$= \rho AV[(V_{w1}-u) - (V_{w2}+u_2)]$

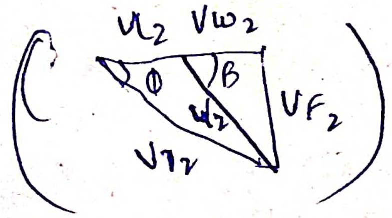
$\boxed{= \rho AV_{r1}[V_{w1} + V_{w2}]}$

$\frac{W.D}{sec} = \rho AV[V_{w1} \pm V_{w2}] \times u$

β = +ve

= -ve

= 0



$\beta < 90^\circ$ +ve

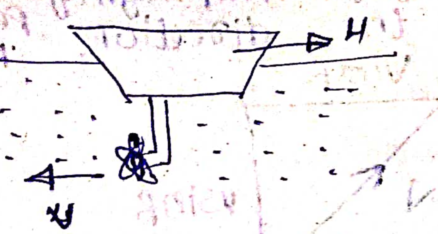
$\beta > 90^\circ$ -ve

$\beta = 90^\circ$ $V_{w2} = 0$

Force of jet on semicircular vane

$F = 2\rho aV^2$

Boat case



$\overleftarrow{V_r}$ $\xrightarrow{u}$

$\overrightarrow{V}$ $\overleftarrow{u}$

(Reaction forces)

so $V_r = V + u$

$$F = \frac{m}{t}[V_r - u]$$

$$= \rho A(V+u)[(V+u) - u]$$

$$= \rho A V_r [V]$$

$$\frac{W \cdot d}{sec} = \rho A V_r V \cdot u$$

$$\frac{K \cdot e}{sec} = \frac{1}{2} \cdot \rho A \cdot V_r^3 \quad \left[m \times V_r^2 \right]$$

$$\eta = \frac{\frac{K \cdot e}{sec}}{W \cdot d} \quad \frac{\frac{W \cdot d}{sec}}{\frac{K \cdot e}{sec}} = \frac{\rho A V_r [V] \times u}{\frac{1}{2} \times \rho \cdot A \cdot V_r^3}$$

$$= \frac{2Vu}{V_r^2}$$

$$\eta = 50\% \quad [\cancel{V+u=V}] \quad [V = u]$$

TURBO MACHINERY

→ A machine which converts water (hydraulic energy) energy into mechanical energy is called Turbine. This energy is furthur converted into electrical energy which is the cheapest power generation

Penstock	It is a pipe of larger dia which carries water from resorvoier to turbine
Headrace	water surface in the storage resorvoier
Forebay & Surgetank [works by water hammer]	→ Storage reservoier at the head of penstock is forebay → purpose is to temporary store water → cut through portion from land → Tanks kept nearest power house, temporary storing of water are surge tanks
Tail race	channel which carries water away from power house after it has passed through turbine
Classification	Based on Action :- Inlet energy only in k.e 1) Impulse :- Available energy is only in the form nozzle
ECET 2013	Important component of Turbine is Nozzle

Ex:- peltonwheel, Turgo, Girard, Banky, Jonval
generally used for high heads low discharges
Also called as Free jet turbine

→ Inlet is both K.E & Pressure Energy

2) Reaction Turbine:- Available energy is converted into K.E but maximum remains in the form of P.E

Ex:- Francis, Kaplan (Adjustable blade), Propeller (fixed blade), Fourneyon, Thomson

ECET 2017 Old Francis is an reaction turbine and has inward flow

→ used for low heads & high discharge

Based on flow:-

1) Tangential flow:- when flow is tangentially

Ex:- Pelton wheel, Turgo Impulse

2) Radial — Inward (centrifugal)  Old Francis, Thompson, Guvard

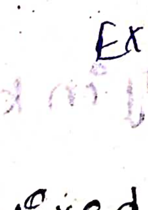
Outward (centripetal)  $\boxed{\text{Fourneyon}}$ ECET

3) Axial flow:- fluid flows parallel to axis of shaft

Ex:- Kaplan Turbine, propeller turbine, Jonval

4) Mixed flow:- Radial flow + Axial flow.

Ex:- Modern francis turbine



> 250m ~~1000m~~	60-250m	< 60m
High head	~~medium~~ Head	Low ~~Medium~~ head
low Q	medium Q	high Q
single jet	300	high N_s ECET 2015
Pelton wheel	Modern francis	Kaplan → $\boxed{300-900}$
		Propeller $\boxed{300-1000}$

specific speed $\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}}$

ECET 2018 2times

Ascending order of sp. speed

$\boxed{\text{Peltonwheel} < \text{Francis} < \text{kaplan} < \text{Propellor}}$

$N_u = \frac{N}{\sqrt{H}}$ $Q_u = \frac{Q}{\sqrt{H}}$ $P_u = \frac{P}{H^{3/2}}$

Driving (or) Motive force in turbine is <u>change in Momentum</u> (2014)

Efficiency of Turbine :

$\eta_h = \frac{R.P}{W.P} = \frac{\text{Power developed by runner}}{\text{Power supplied at inlet of turbine}}$

water horse power ↑

$= \frac{\cancel{WHP}}{\cancel{\frac{w(Q+\Delta Q)H}{75}}}$

$\eta_m = \frac{S.P}{R.P} = \frac{\text{Power at impeller}}{SHP}$ $\frac{\cancel{R}\,H.P}{SHP}$

$\eta_o = \frac{S.P}{W.P} = \frac{S.P}{R.P} \times \frac{R.P}{W.P}$

$\eta_o = \eta_h \times \eta_m = \eta_b \times \eta_v \times \eta_m$

ohm

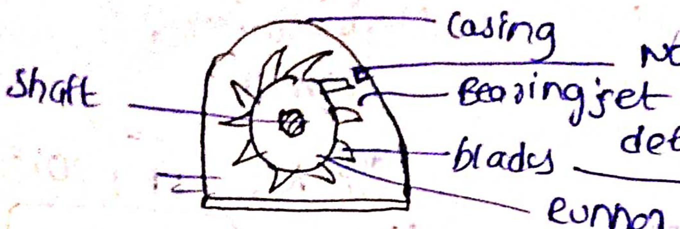
→ as of

$\eta_s = \eta_b \times \eta_v$

sbv

$\boxed{\eta_{h\,max} = \frac{1+\cos\phi}{2}}$

PELTON WHEEL TURBINE:-



double hemispherical
ellipsoidal blades

No. of blade

No. of buckets = 16 [18 to ...] also by Tygun form

m - Lunguss emperical formula

$D \Rightarrow \left(\text{velocity} : \frac{\pi D N}{60} \right)$ (u)

No. of jets $\quad Q_{total} = n \times a \times v \Rightarrow n = \frac{Q_{total}}{a v}$

$a = \frac{\pi}{4} d^2$

Actual velocity of jet : $V_{act} = C_v \sqrt{2gH}$; $u_{act} = \phi \sqrt{2gH}$

$C_v = 0.98 - 0.99$

$\phi = K_u = 0.73 - 0.48$

Jet ratio = 12

$= \frac{\text{dia of wheel}}{\text{dia of jet}}$

$\frac{W.D}{sec} = \rho A V [V - u][1 + \cos\phi] \times u$

ϕ : exit blade angle

$\eta = \frac{2(V-u)(1+\cos\phi) \times u}{V^2}$

Flow ratio $= \frac{V \text{ at inlet}}{V \text{ by } \sqrt{2gH}}$

Speed ratio $= \frac{u}{\sqrt{2gH}}$

[0.45 to 0.50]

$\eta_{max} \Rightarrow V = 2u$

$\eta_{max} = \frac{1 + \cos\phi}{2}$

$Q = \frac{\pi}{4} d^2 \sqrt{2gh}$

$\phi = \ldots$

Bucket dimensions

Depth - 0.8d to 1.2d
Radial height - 2d to 3d
Axial width - 3d to 5d

Reaction turbine :-

spiral casing - distribution flow ~~No hy~~
stay ring - supports non supporting parts
Guide mechanism - guide vanes are supported [Air foil type]
Draft tube - K.e to P.e

Types of draft tube :- (d.t) :- Pipe of increasing cross sectional area which connects the runner exit to tail race.

conical divergent   Moody spreading   Elbow type

$$\eta_{d.t} = \frac{V_1^2 - V_2^2}{V_1^2}$$

straight divergent draft tube should not be more than 8°

$$\eta_{d.t} = \frac{\left[\frac{V_1^2}{2} - \frac{V_2^2}{2}\right] - h_L}{\left(\frac{V_1^2}{2}\right)}$$

→ It is only provided for francis Turbine, Kaplan Turbine

→ Runner is a one which differs Reaction & Impulse turbine

$Q_{francis} = \pi D B V_f$

$Q_{kaplan} = \frac{\pi}{4}(D_0^2 - D_b^2)$

Specific speed [speed of turbine working under unit head & develops unit horsepower]

Modern Francis Turbine

No. of blade = ~~15~~ 15 to 20

$$W \cdot D = \rho A V [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$u_1 = \frac{\pi D_1 N_1}{60} \qquad u_2 = \frac{\pi D_2 N_2}{60}$$

$$n = \frac{\text{breadth}}{\text{depth}} = 0.1 - 0.45$$

$$Q = \pi D B V_f$$

Flow ratio ~~fluorido~~ $\psi = \frac{V_F}{\sqrt{2gH}}$ $[0.15 - 0.3]$

speed ratio $\phi = \frac{u}{\sqrt{2gH}}$ $[0.3 - 0.6]$

$[0.6 - 0.9]$ sometimes

Specific speed :- $\frac{\text{Unit discharge}}{\text{Unit head}}$ = ~~develops unit H.P and dis~~

	in HP	in kW
Pelton	10-35	8.5-30
Francis	60-300	50-250
Kaplan	300-1000	250-850

$$\boxed{M^{1/2} L^{-1/4} T^{-5/2}}$$

$$\boxed{F^{1/2} L^{-3/4} T^{-3/2}}$$

→ Also specific speed defined as working under unit head & develops unit H.P

→ Unit discharge is discharge of turbine when head on turbine is unity

Unit power- (head = 1) same

PUMPS

→ pumps are the devices which convert ~~to~~ mechanical to hydraulic energy (pressure energy)

Pumps

- Positive displacement
  - Reciprocating
    1) Single Acting / Double acting
    2) Single cylinder / Multi
    3) with Airvessel / without
  - Rotary
    - Gear pump
    - Vane pump
    - Lyshom pump
    - Screw pump
- Non positive displacement
  - Rotodynamic
    1. centrifugal
    2. Axial

→ W.D) centrifugal pump

$$-(W.D) \text{ of inward radial flow} \over \text{weight}$$

$$= \frac{-[\cancel{\rho A V}]\,[\cancel{V_{w_1} u_1}^{0} - V_{w_2} u_2]}{\cancel{m} \cdot g}$$

$\alpha = 90^\circ$

$V_{w_1} = 0$

$$\boxed{W.D)_{centrifugal} = \frac{V_{w_2} U_2}{g}}$$ 2015

→ Action of centrifugal pump is same as that of reverse reaction turbine

→ Impellar of centrifugal pump may have volute casing (pressure energy increases)

→ In casing of centrifugal pump, k.e converts into p.e before it leaves the casing

Manometric Head :- [Total head at oulet - inlet]

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

W.D = Hm + impellor loss

$$\boxed{H_m = W.D - \text{impellor loss}}$$

→ Actual head against which pump has to work

→ In casing of centrifugal pump, Pressure energy is directly proportional slope of divergent cone

→ Discharge of centrifugal pump is similar to francis turbine

$$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$$

shaft power

→ S.P. $S.P = \frac{WQH_m}{\eta_o}$ — Pump

$S.P = \eta_o \, WQH_m$ - turbine

→ Multi stage centrifugal pump are needed to give high head

Series connection centrifugal axial — high discharge, low head

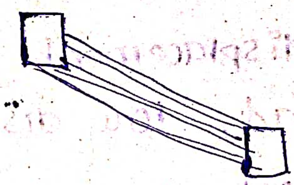
Parallel connection Reciprocating — high head, low discharge

$$\eta_o = \eta_{man} \times \eta_m$$

Pipes in parallel (High discharge)

$Q = Q_1 + Q_2 + Q_3$

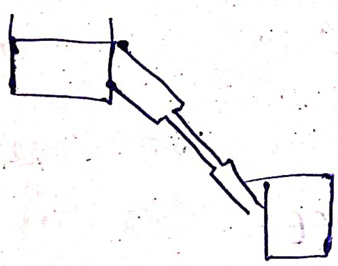
$[Q_1 = Q_2 = Q_3 = Q]$



$h_f = h_{f_1} + h_{f_2} + h_{f_3}$

$\frac{flv^2}{2gd}$ $[h_{f_1} = h_{f_2} = h_{f_3}]$

Pipes in series :- High head



$Q_1 = Q_2 = Q_3$

$h_f = h_{f_1} + h_{f_2} + h_{f_3}$

$\frac{f l Q^2}{3d^5} = \frac{f l_1 Q^2}{3d_1^5} = \frac{f l_2 Q^2}{3d_2^5} = \frac{f l_3 Q^2}{3d_3^5}$

$\frac{l}{d^5} = \frac{l_1}{d_1^5} = \frac{l_2}{d_2^5} = \frac{l_3}{d_3^5}$

* Maximum efficiency of pump obtained when blades are bent backward

* centrifugal pump will start delivering liquid only when pressure raised in impellar equals to manometric head

* 1) Cavitation causes noise and vibration
  2) It reduces discharge, power output, η
  3) Cavitatation in turbines can be avoided to a greater extent by
     A) Installing the turbine below tailrace level
     B) Using stainless steel to runner.
     C) Providing highly polished plate
     D) All of the Abov

It occurs when
- High runner speed
- High temp
- Less available NSPH

Reciprocating pump

→ positive displacement pump
→ High head, low discharge

$\boxed{Q = \frac{ALN}{60}}$ for single acting

→ Negative slip $Q_{act} > Q_{th}$

Reason:- → Suction pipe is long
→ delivery pipe is short
→ pump is running at high speed

→ Theoritical power required to lift water in pump is $\gamma Q[h_s + h_d]$

$\gamma Q h_f$ - actual

ECET → Air vessels are used to reduce net acceleration head (for uniform velocity)

→ Discharge for centrifugal ~~compressor~~ pump

$Q = A \times V = A \cdot \frac{\pi D N}{60}$

$Q \propto N$

→ Power of centrifugal pump is $\propto$ to N^3

$\cancel{P = \gamma Q h}$

$P = \gamma Q h$ $\left(h \propto \frac{V^2}{2g} \propto N^2\right)$

$= \gamma \cdot N \cdot N^2 N^3$ $\boxed{h \propto N^2}$

$P \propto N^3$

Turbine $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

Pump $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$: $\frac{\text{Head developed is unity}}{\text{Discharg is } 1m^3}$

Net positive suction Head :-

→ It is defined as net inlet head and head corresponding to vapour pressure of liquid

→ whenever pressure forms below vapour pressure then cavitation occurs

Priming :- Escaping of all air particles with manual pouring of water (domestic pumps) or with vaccum pumps for higher capacity units

→ A hydraulic accumulator is a device to store sufficient energy incase of machines which work intemittently : To suplement energy from normal source

→ A power transmission system represents

1. pump
2. Hydraulic accumulator :- stores energy
3. Intensifier
4. coupling :- transmitting same torque to the driven shaft

$[P_{water} \times V_{accu}]$

→ Torque convertor :-

(i) supplies energy to fluid

(ii) extracts energy from fluid

→ which of the following is not a Positive displacement

a) Jet pump
b) Lobe pump
c) Reciprocating pump
d) vane pump

→ which one is not a rotary pump

1) diaphagrm pump
2) screw
3) lobe
4) sliding (vane)

→ Hydraulic ram is a pump which works on the principle of water hammers

→ In food processing industry, thick syrup is to be pumped by Lobe Pump

→ Pump which raise oil (or) water by buoyancy of an aerated coloumn of oil (or) water in a submerged tube is called Air lift pump

→ Hydraulic Ram is a device which can lift small stream of water to large heights

→ screw pump helps in pumping viscous

Short Notes & IMP Points

stationary plate		moving plate
for vertical plate	$F_x = \rho a v^2$	$= \rho a(v-u)^2$
inclined plate	$= \rho a v^2 \sin^2\theta$	$= \rho a (v-u)^2 \sin^2\theta$
curved plate & jet strikes at centre	$= \rho a v^2(1+\cos\theta)$	$= \rho a (v-u)^2(1+\cos\theta)$
curved plate jet strikes at tips	$= 2\rho a v^2 \cos\theta$	$\cancel{= 2\rho a(v-u)\cos\theta}$

$$\boxed{\sin\theta = \frac{\rho a v^2}{W}}$$

jet strikes on curved moving plate

$$F_x = \rho a V_{r_1}(V_{w_1} \pm V_{w_2})$$

$$W = \rho a V_{r_1}(V_{w_1} \pm V_{w_2}) \times u$$

$$W(\text{per kg}) = \frac{1}{g}(V_{w_1} \pm V_{w_2})$$

→ for series of vanes $\eta = \frac{2u(v-u)}{v^2}$; If $u = \frac{V}{2}$ $\eta_{max} = 50\%$

→ Gross head :- Difference between head race level & tail race level when no water is flowing - static head [Hg]

Net Head [H] = $H_g - H_f$ where $H_f = \frac{4fLV^2}{2gd}$

→ work saved by fitting air vessels in single acting reciprocating pump is 84.8% and double acting is 39.2%

air vessels: used to increase speed; continuous supply at uniform rate

operating characteristic curve	- constant speed - (Governor)	constant at
Main characteristic curve	- main head	
Mushel curve	- efficiency	

→ power developed by pelton wheel = Power by n jets by 1 jet

→ Separating pressure; 2.5m ; at the end of suction stroke or delivery stroke

US

→ In Pumps Speed ratio $= \frac{u_2}{\sqrt{2gh}}$ (0.95 to 1.25)

Flow ratio $= \frac{V_f}{\sqrt{2gh}}$ (0.1 to 0.25)

→ For optimum η $V_{w_1}=0$ & pump will start pumping only when pressure head will be greater than (or) equal to manometric head

For pumps	at constant speed	at same impeller dia
discharge -	$Q \propto D^3$	$Q \propto N$
head -	$H \propto D^2$	$H \propto N^2$
horsepower -	$H.P \propto D^5$	$HP \propto N^3$

IMP POINTS

1) For starting an axial flow pump its delivery valve should be open

High specific speed

2) η centrifugal pump would be maximum when its blades are bent backward

3) Centrifugal pump started with its delivery valve kept fully closed

4) optimum value of vane exit angle is 20-25°

5) volute casing - spiral vortex flow ; Irrigation applications - Axial flow pump

6) Indicator diagram of a reciprocating pump is a graph between pressure in cylinder vs swept volume

~~7) Impulse turbine - low speed~~

7) change in load of turbine is adjusted by relative velocity of flow at inlet

8) For Pelton wheel $\frac{\text{width of bucket}}{\text{dia of jet}} = 5$ $\frac{\text{depth of bucket}}{\text{dia of jet}} = 1.2$

Discharge

pelton wheel $= \pi D B V_f$

Francis $= \frac{\pi}{4} d^2 \times \sqrt{2gh}$

Kaplan $= \frac{\pi}{4}(D_o^2 - D_b^2)$

9) Run way speed - No load speed with no governor mechanism

spouting velocity - ideal velocity