University of Thessaly
Department of Mechanical Engineering

Diploma Thesis

PID CONTROL OF LAMBDA IN INTERNAL COMBUSTION ENGINES

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Abstract

Air pollution is an ever increasing problem with high impact on human tissues. One of its main contributors is the internal combustion engine (ICE). Taking that into consideration it is extremely important to reduce exhaust pollutants of ICEs. This can be achieved by the stoichiometric air-to-fuel ratio (lambda) control, which helps determine whether the air to fuel ratio of an ICE is either rich or lean. A PID controller is designed and tuned for this purpose. A dynamic model of a spark ignition (SI) engine is implemented in MATLAB in order to validate the proposed control scheme. To be more specific, the mean value engine model (MVEM) is applied, as it is considered to be an accurate and control-oriented dynamic model. This model describes four dynamic engine variables (states): manifold pressure, fuel flow into combustion chamber, crankshaft speed and lambda measured by sensor. The inputs in the MVEM are the injected fuel mass flow and the throttle plate angle. By combining the MVEM with the PID controller lambda control is achieved. As far as closed loop is concerned, the manipulated variable is the fuel mass flow into the combustion chamber, the controlled variable is lambda and the throttle plate angle is considered as a disturbance. Two simulations are created. The first one is a closed loop system for PID control of lambda with only one input, the injected fuel mass flow. The second one is the same closed loop system with the difference that in this case throttle plate angle is regarded as an input disturbance. While on the first simulation the PID ability to control lambda is very satisfactory, lambda response changes drastically when disturbance is inserted in the system.

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Standing on the shoulders of Giants

Contents

List of Figures

List of Tables

List of Symbols

α_0 closed throttle plate angle [deg]

α throttle plate angle [deg]

δ total delay in lambda sensor measurements [ms]

$\dot{m}_{ap}$ air mass flow entering the combustion chamber [kg/sec]

$\dot{m}_{at}$ air mass flow passing the throttle plate [kg/sec]

$\dot{m}_{ff}$ fuel film mass flow [kg/sec]

$\dot{m}_{fi}$ injected fuel mass flow rate [kg/sec]

$\dot{m}_f$ fuel flow into combustion chamber [kg/sec]

$\dot{P}_{man}$ manifold air pressure [bar]

λ_m lambda which is directly calculated from the MVEM

λ_{sensor} measured lambda by lambda sensor

τ_d time constant delay for the exhaust gas to reach the lambda sensor [ms]

τ_d time delay for the injected fuel to make power [sec]

τ_e time delay in measurements of lambda sensor [ms]

τ_f fuel film evaporation time constant [sec]

θ_{EVO} angle of crankshaft at which exhaust valve opens [deg]

AFR_{st} stoichiometric value of air to fuel ratio equals 14.86

H_u fuel heating value [kj/kg]

I total engine moment of inertia [kgm^2x$(\frac{\pi}{30})^2$x1000]

n crankshaft speed [krmp]

N_{cyl} number of engine's cylinders

N_i indicated efficiency

P_b engine load power [kW]

P_f engine mechanical friction loss power [kW]

P_p engine pumping loss power [kW]

R gas global constant [kj/kg K]

s slope of the normalized air charge

T_a ambient temperature [K]

T_i intake manifold temperature [K]

V_d engine displacement volume [l]

V_i intake manifold and port passage volume [m^3]

X ratio of injected fuel deposits in the intake manifold

y y-intercept of the normalized air charge

1 Introduction

1.1 Motivation

Internal Combustion Engines are one of the major causes of air pollution. By the late 1950s increased use of cars in major cities [10] [11] had resulted in serious concerns about air quality and human health. The three types of exhaust emissions are: hydrocarbons, oxides of nitrogen and carbon monoxide. A.J.Haägen-Smit showed that photochemical reactions among hydrocarbons and nitrogen oxides produce the many secondary pollutants that reduce visibility and cause eye and nose irritation in the Los Angeles area [12]. Furthermore, later studies showed that the above exhaust gas pollutants are cancerous gases which can also cause heart diseases and respiratory disorders.

Fortunately, the amounts of these pollutants could be considerably decreased by use of a three way catalytic converter. For a three way catalyst to work properly, air-to-fuel ratio (lambda) should by close to its stoichiometric value [13]. As a result, closed-loop control of air-to-fuel ratio is of high importance.

1.2 Research topic and Scope

Regarding all the above, the aim of this Thesis is to construct a closed-loop control scheme to sufficiently regulate air-to-fuel resulting in a fully functional catalytic converter. The first part of this work involves obtaining a proper dynamic model of an SI engine and using it to setup a corresponding simulation in MATLAB. The second part involves building the closed loop scheme, by implementing and tuning a PID controller.

1.3 Outline

Chapter 2 introduces some parts of the internal combustion engine required for understanding the need for controlling air-to-fuel ratio. These parts are the lambda sensor which determines the air-to-fuel ratio and the three way catalytic converter which converts gases and pollutants in the car exhaust into harmless compounds.

In Chapter 3, control terminology and basic concepts are discussed. The objective is to understand the importance of feedback loop in almost every dynamic system. Afterwards, the concept of a controller is introduced for the purpose of highlighting the PID controller and its use.

Chapter 4, thoroughly describes the case study of this Thesis. It starts with the dynamic model used for simulating an SI engine using the MATLAB software. This model is the mean value engine model (MVEM), which is described as the state of the art in the control oriented model of internal combustion engines. Thereafter, a closed-loop control is designed with a PID controller.

In Chapter 5, the performance of the closed-loop control is evaluated and the results from the simulations in Chapter 4 are discussed.

In Chapter 6, we reiterate and summarize on the findings from the previous chapters and highlight future work that can be done to further improve air-to-fuel ratio control.

2 Internal Combustion Engines

Our aim in this chapter is to describe some parts of the internal combustion engine which are relevant to this work. The first part to consider will be the lambda sensor and the second one will be the catalytic converter.

Both of them are extremely important for making the engine more efficient and decreasing the pollutants from the exhaust system.

2.1 Lambda Sensor

A *lambda* (or oxygen) *sensor* is a computerized apparatus that determines the proportion of oxygen (0_2) in a fluid being tested. The lambda sensor plays a significant role in the reduction of exhaust pollutants of Internal Combustion Engines (ICEs) [13] .

Oxygen sensors are so essential because they help resolve whether the air to fuel ratio (lambda) of an ICE is either rich or lean.

It is well known that the lambda sensors are located in the exhaust pipe, thus they can not directly measure the air or the fuel that is entering the engine. The sensor needs information from other sources so it can determine the air to fuel ratio. According to the sensor data, the fuel injection varies the fuel injector output so that the fuel is close to stoichiometrically burnt every time. This is referred to as closed-loop operation.

Lambda sensor helps electronic fuel injection to work properly. Another advantage of the lambda sensor is that it helps reduce the amounts of both unburnt fuel and oxides of nitrogen entering the atmosphere.

Unburnt fuel is emission from fuel that left the combustion chamber without being burnt, while oxides of nitrogen (NOx gases) are formed from very high temperatures in the combustion and are one of the main causes of smog and acid rain.

As mentioned earlier the sensor can not directly measure the air to fuel ratio. What is being measured is the difference between the concentration of 0_2 in the mixture leaving the combustion chamber and the concentration of oxygen in ambient air. Rich mixture means that the mixture leaving the combustion chamber contains little oxygen. The oxygen sensor will accordingly produce a voltage output, which the Engine Control Unit (ECU) senses and determines whether the fuel mixture is rich. If the mixture is lean, because of the oxygen excess, the oxygen sensor will generate a low voltage output.

2.1.1 History

The foundations of lambda technology reach back to the late 1960s, when Robert Bosch under the supervision of Dr. Günter Bauman developed the first functional oxygen sensor.

One of the earliest ordeals in the making of the sensors was that they had to be able to endure exhaust temperatures of up to 1000 degrees Celsius so the need for heat-resistant materials was crucial. In the next few years these materials were found thanks to the knowledge about manufacturing ceramics in spark plug production. The next step was to test several sensors for thermal rating and heat conductivity. At first the sensors tested lasted just two hours. After several years they finally achieved a hundred times of service life more than the first sensors. In the early 1980's oxygen sensor manufacturers, manufactured a model that was heatable and as a result it would function correctly half a minute after the engine was started.

The wideband sensor entered the market in the early nineties making the sensors more reliable and less expensive.

Figure 2.1: First generation lambda sensor, from the Bosch Archives collection, late 1970ies [1]

2.1.2 How it works

Internal combustion engines are one of the major causes of air pollution[13] [14] [15]. For this reason modern spark-ignited (SI) engines use lambda sensors and catalytic converters.

The first step of the closed-loop operation is getting information about the oxygen concentration. Then the information is sent to the engine management computer or *engine control unit* (ECU) and from there the amount of fuel injected into the engine is adjusted accordingly.

The essential goal is a compromise between power, fuel economy, and exhaust emissions, and in most of the times is made possible by an air to fuel ratio close to stoichiometric [1].

For these engine (such as those that burn gasoline or LPG, as opposed to diesel), the three types of emissions modern systems are concerned with are: hydrocarbons (which are released when the fuel is not burnt completely, such as when misfiring or running rich), oxides of nitrogen (they are very undesirable, because they react to the atmosphere in presence of sunlight to form ozone and causes photochemical smog) and carbon monoxide (CO is intermediate product of combustion which remains in the exhaust if there is not enough oxygen to convert to carbon dioxide, known as incomplete combustion)[16].

2.1.3 Failures of Lambda Sensor

There are several causes why a lambda sensor fails. The most common failure occurs through various contaminants that enter the exhaust. Some of these contaminants are silicates from internal engine coolant escape (because of a faulty head gasket or a fissure in a cylinder wall or combustion chamber) and excessive oil consumption (because of shabby rings or valve guides).

Another cause is the failure of the heating element in the sensor. The heat inside the sensor and the temperature of the heating element results in the element eroding over time and eventually failure.

Also, using the wrong time of fuel or even the use of leaded fuels can cause the sensor to fail or damage the catalytic converter.

Last but not least, normal ageing is a notable cause of failure. The consequence of a damaged lambda sensor is air to fuel ratios that cannot correspond quickly to the time-changing engine conditions. Furthermore, a malfunctioning oxygen sensor will result in elevated fuel consumption as well as increased exhaust pollutants. Hopefully, most modern engine management systems have the ability to instantly detect a damaged lambda sensor [17].

2.1.4 Closed-Loop Operation

While the engine is under no load or during part-load conditions where the engine is maintaining the car at a constant velocity, the ECU is said to be operating in *closed-loop mode* [18]. This means that there is a feedback loop between the engine control unit and the lambda sensor. The purpose of the feedback loop is to keep the air to fuel ratio at its stoichiometric value.

In case of a lean mixture the ECU will try to increase the fuel efficiency, but at the same time there will be a minor rise in NOx emissions as well as way greater exhaust gas temperatures.

In case of a rich mixture the ECU will try to increase the power to a point, but at the same time there will be a reduction in fuel efficiency as well as a rise in unburnt hydrocarbons in the exhaust. The result will be overheating of the catalytic converter and thereafter failure.

[1] stoichiometric air to fuel ratio is 14.7 for spark-ignition engines

2.1.5 Open-Loop Operation

When an internal combustion engine is under high load, for example when it is accelerating or when it is decelerating the engine control unit is said to be in *open-loop mode* [18]. In this mode the output of the oxygen sensor is ignored and the engine control unit automatically enhances the mixture to preserve the engine. Whether the mixture is either rich or lean there will be no change in the fuel injector.

2.1.6 Types of Lambda Sensor

There are three main types of lambda sensor:

- The zirconium type which is also referred to as narrowband sensor, it is the most common and oldest type.
- The strontium titanate sensor which is most commonly encountered on vehicles using the Siemens engine management system.
- The planar or wideband sensor which is found in the latest generations of engines.

Zirconium Sensor

The sensor consists of a solid ceramic electrolyte (zirconium dioxide), coated with micro-porous platinum, which conducts oxygen ions at temperatures above 250 degrees celsius [19]. This arrangement acts like a tiny battery, and it is this (very small) voltage that is measured by the engine control unit using a high-impedance input. The sensor element consists of the inner and the outer layer which are isolated from each other and exposed to different gases. The inner layer is exposed to the ambient air, whereas the outer layer of the sensor is exposed to the exhaust pipe.

The sensor is right next to a heating element because it only starts to work properly when it reaches above 350 degrees Celsius. Although the exhaust gases are capable of heating the sensor up this much when the car is at idle, during cold start and other operating conditions where the exhaust gases are cooler the assistance of the heater is needed. Figure 2.2 shows the construction of a Zirconium Dioxide Sensor.

Strontium Titanate Sensor

Strontium Titanate is a ceramic semiconductor material. Its conductivity depends on the material temperature and oxygen partial pressure, which means that it does not generate its own voltage as the Zirconia type do. Instead, the resistance of the sensing element adjusts with regards to the 0_2 present in the exhaust emissions [20].

One motive strontium titanate sensors were originally used is because they are less susceptible to lead poisoning than the Zirconium types. They are, however, more susceptible to anti-freeze problems than the Zirconium types. Some of the advantages that led strotium titanate sensors to be widely used are faster response times and more compact packaging. As can be seen in Figure 2.3, the strontium titanate sensor has a planar structure.

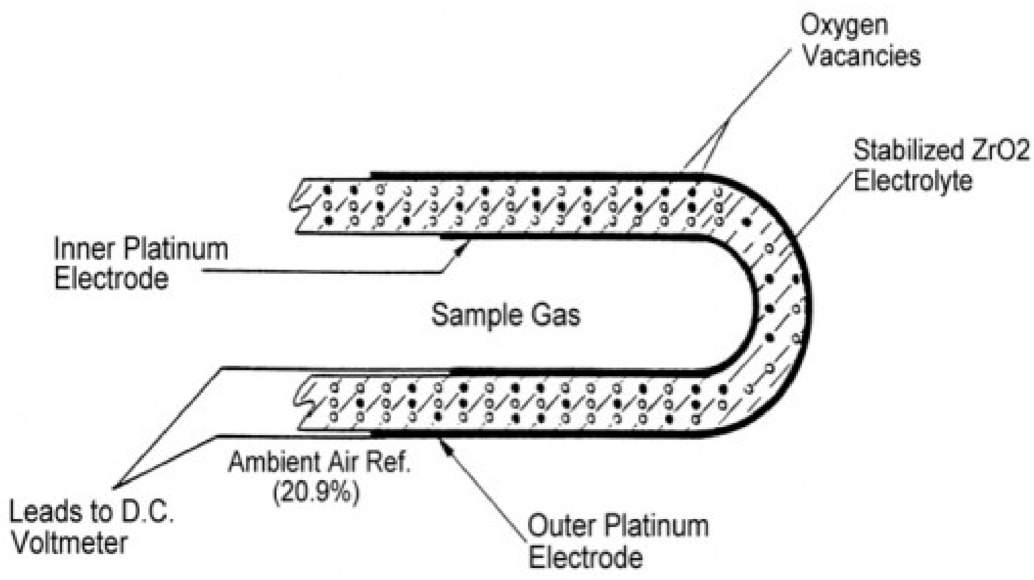

Figure 2.2: Enlarged cross sectional representation of the zirconia substrate [2]

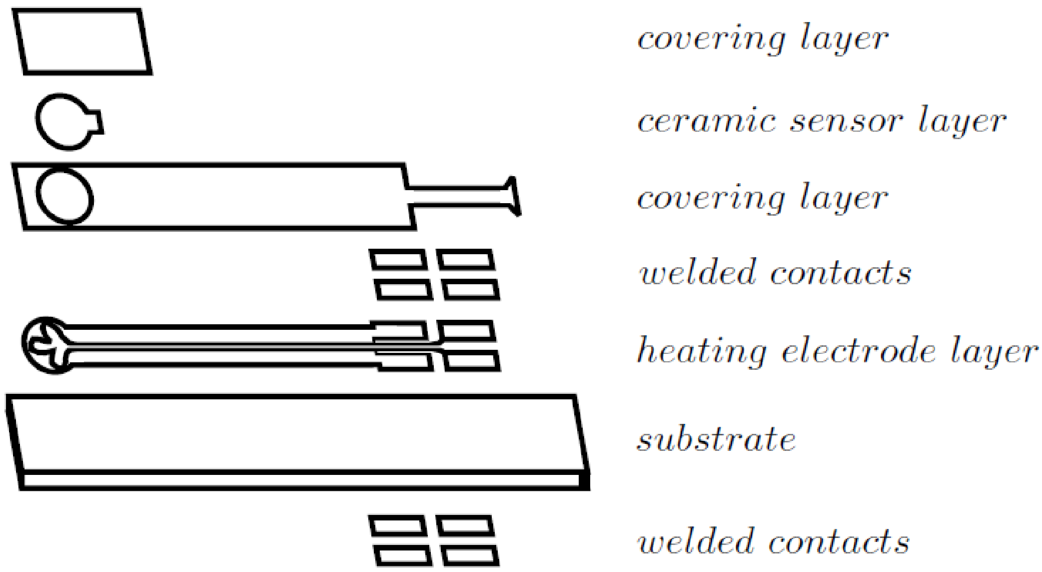

Figure 2.3: Planar structure of the strontium titanate sensor [3]

Wideband Sensor

Wideband Sensors are much better at metering exactly how much oxygen is in the exhaust pipe, rather than the simple shifting operation of the zirconium sensors. Wideband Sensors have become necessary due to strict regulations about gas emissions [21].

The only difference between the wideband sensor and the more common types is an extra internal system, a device which is called oxygen pump. Furthermore, it can measure a far wider range than a traditional sensor, but more importantly when it is within the range of interest (from lambda=0.9 to lambda=1.1) the response is fairly linear, meaning that we can determine the exact oxygen content of the exhaust gas.

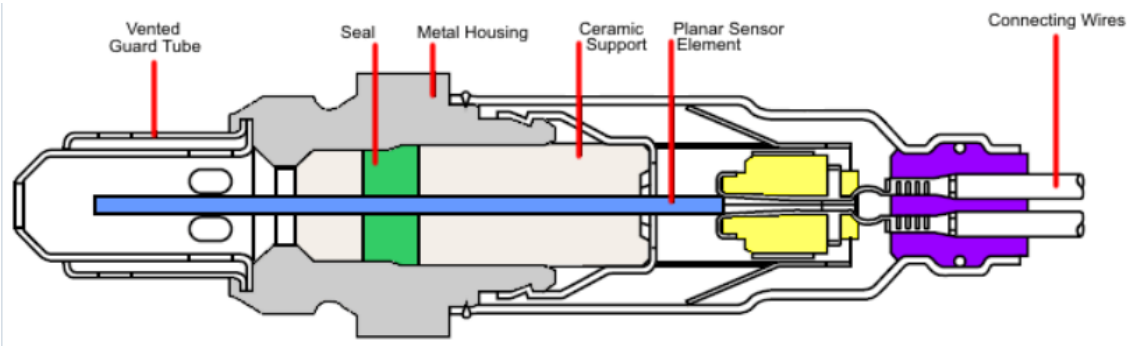

Figure 2.4: Cutway view of a wideband sensor [4]

2.2 Catalytic Converter

A catalytic converter is an exhaust emission control device that uses a catalyst to convert toxic gases and pollutants (hydrocarbons, carbon monoxide, nitrogen oxides) in the car exhaust into nontoxic admixture.

2.2.1 Exhaust gas emissions from SI Engines

Emissions from SI engines are strongly dependant by the air to fuel ratio lambda, hence it is really important to correlate them.

Figure 2.5 illustrates how the exhaust gases change according to different values of lambda.

Carbon monoxide and hydrocarbon concentrations decrease while lambda increases, whereas the nitrogen oxides concentrations increase gradually up to a maximum value (where lambda is exactly 1.07), and then decrease again. This behavior is described thoroughly by Manahan [22]. This occurs because in regions where lambda is greater than one the air to fuel mixture is lean and as a result the combustion may be incomplete, hence there will be more unburned fuel.

The behaviour of NOx is also interesting. When lambda in not close to one (either lower or higher), smaller temperatures are detected inside the combustion chamber. Therefore, as the highest temperatures are observed with stoichiometric combustion, the highest concentrations of NOx occur when lambda is close to one [23]. Last but not least, as we can see from the Figure for $\lambda = 1$ the emissions of HC, CO and NOx are relatively low, while for other values of lambda the emissions are getting considerably high.

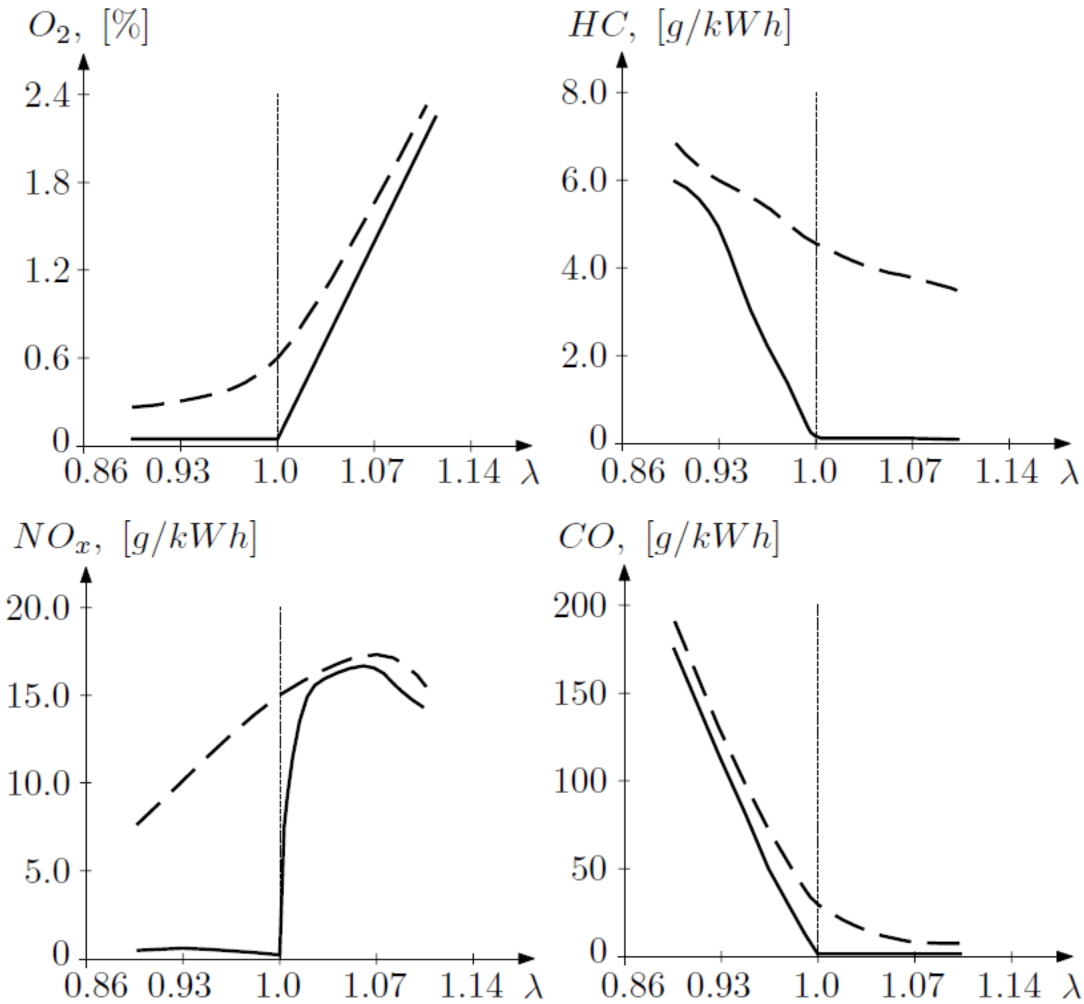

Figure 2.5: Measurement of exhaust gases: oxygen, hydrocarbon, nitrogen oxide and carbon monoxide. The concentration before the catalytic converter are indicated by dotted and the concentrations after the catalytic converter by straight lines [3].

2.2.2 The three-way catalyst

The most popular method used by vehicle manufacturers to reduce engine emissions is the three-way catalyst (catalytic converter). The 3-way catalytic converter is a complex apparatus that converts harmful gases in the engine exhaust to relatively harmless gases. It consists of two catalysts: one for the reduction of nitrogen oxides to nitrogen, and one for the oxidation of carbon monoxide to carbon dioxide as well as for the oxidation of unburnt hydrocarbons to carbon dioxide and water [24]. This procedure takes two stages to complete

1. **First stage**: The exhaust gases are sent over the reduction catalyst and the NOx are removed.

2. **Second stage**: The exhaust gases (free from NOx) are sent over the oxidation catalyst to be converted to less harmful gases.

The most important chemical reactions are listed below [3]:

Oxidation of HC **and** CO:

$$\begin{aligned} H_nC_m + \left(m + \frac{n}{4}\right) O_2 &\rightarrow mCO_2 + \frac{n}{2} H_2O \\ H_nC + 2H_2O &\rightarrow CO_2 + \left(2 + \frac{n}{2}\right) H_2 \\ CO + \frac{1}{2} O_2 &\rightarrow CO_2 \\ CO + H_2O &\rightarrow CO_2 + H_2 \end{aligned}$$

Reduction of NOx:

$$\begin{aligned} CO + NO &\rightarrow \frac{1}{2} N_2 + CO_2 \\ H_nC_m + 2\left(m + \frac{n}{4}\right) NO &\rightarrow \left(m + \frac{n}{4}\right) N_2 + \frac{n}{2} H_2O + mCO_2 \\ H_2 + NO &\rightarrow \frac{1}{2} N_2 + H_2O \end{aligned}$$

Other catalytic reactions:

$$\begin{aligned} SO_2 + \frac{1}{2} O_2 &\rightarrow SO_3 \\ SO_2 + 3H_2 &\rightarrow H_2S + 2H_2O \\ \frac{5}{2} H_2 + NO &\rightarrow NH_3 + H_2O \\ 2NH_3 + \frac{5}{2} O_2 &\rightarrow 2NO + 3H_2O \\ NH_3 + CH_4 &\rightarrow HCN + 3H_2 \\ H_2 + \frac{1}{2} O_2 &\rightarrow H_2O \end{aligned}$$

The conversion ratio depends on the air-fuel ratio and the converter volume. At idle speed engine operation, the conversion ratio is high, even if the catalyst would be already partially damaged. During transients, fluctuations in the air-fuel ratio occur, resulting in higher emissions. During the warm-up phase of the engine and the exhaust pipe, temperatures are too small for chemical reactions and the conversion rate is weak.

The three-way catalyst uses a specific catalyst formulation to reduce NOx and oxidize HC and CO all at the same time. It is designed to reduce all three major emissions by approximately 90 percent. As it can be seen from Figure 2.6 it is composed of a metal housing and the core has a honeycomb structure. The catalytic converter consists of precious metals such as platinum which supports more the oxidation of CO and HC, and rhodium which supports more the reduction of the nitrogen oxides NOx. Furthermore, to carry out the conversion of gases efficiently the catalytic converter must operate at a temperature of at least 426 degrees Celsius, for this reason the converter is mounted into the exhaust pipe.

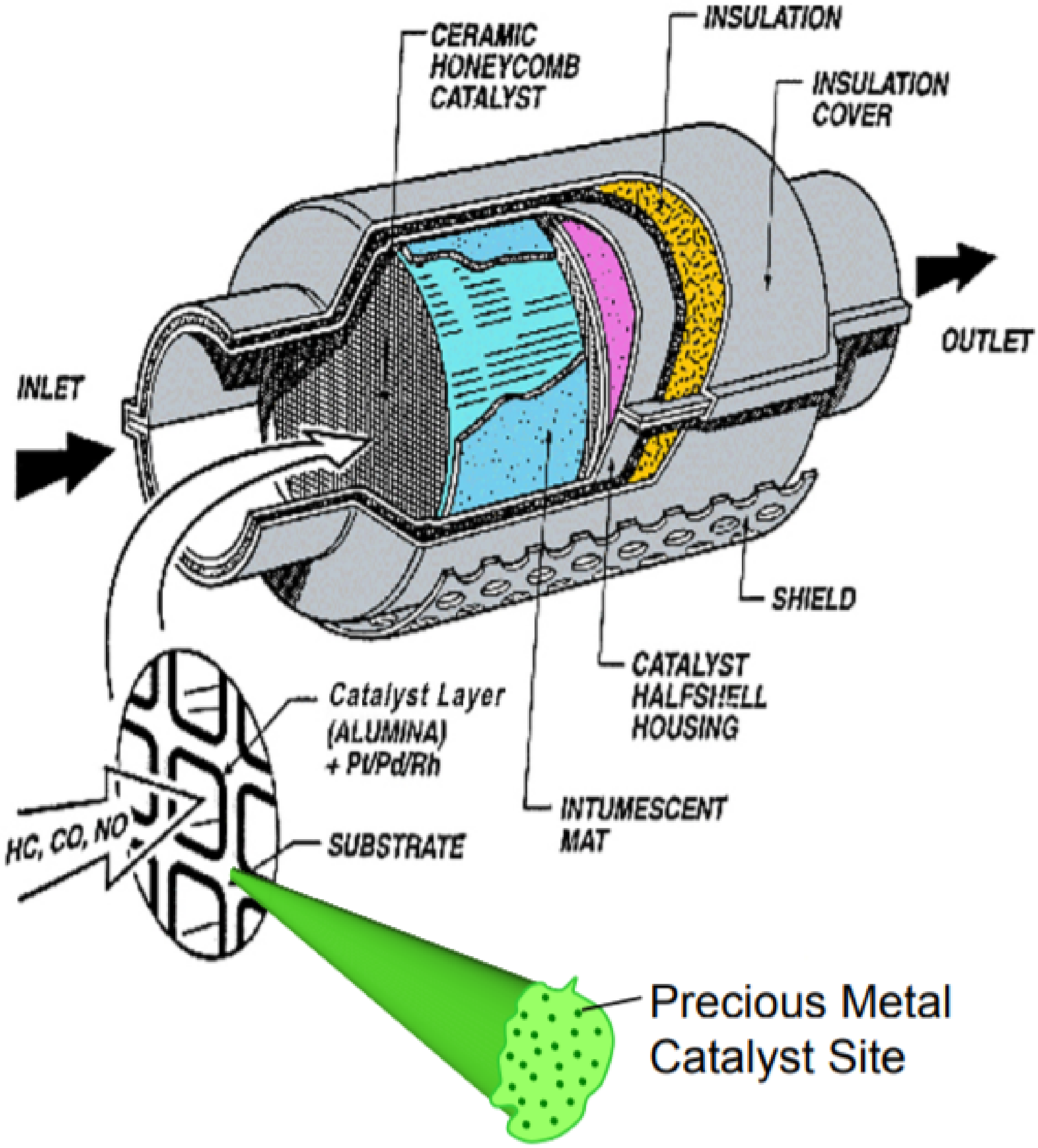

Figure 2.6: Three-way catalytic converter with ceramic substrates [5].

There are two possibilities concerning the catalytic converter. Either the mixture running through is rich or lean. When the mixture is lean, there is excess oxygen and as a result the reactions support the oxidation of CO and HC. On the other hand, when the mixture is rich, there is more fuel than needed, thus the reactions support the reduction of NOx. That is the reason why the catalytic converter can not be 100 percent efficient.

2.2.3 Failures of the three-way Catalyst

The three-way catalyst is designed to last for more than 15 years, but they can become contaminated, clogged, overheated or physically damaged.

Catalyst poisoning occurs when the three-way catalyst is exposed to contaminants that coat the working surfaces. One potential contaminant is leaded gas. Others contaminants include engine coolant, which can enter the exhaust pipe in case of an engine leak due to a faulty cylinder head gasket, or engine oil that leaks in the exhaust system, due to piston rings that lose their ability to properly seal. If the pollutants have a low boiling point there is a possibility that the catalyst poisoning can be reversed by running the engine under a very large cargo, resulting in high exhaust temperatures.

Another problem that may occur is overheating. Three-way catalyst can overheat due to big loads of unburned fuel caused by spark plugs that do not fire or a faulty exhaust valve. Overheating can harm or even destroy the honeycomb-like structure that is needed for the catalytic converter to function efficiently.

3 Control

Control theory and control engineering deal with dynamic systems such as internal combustion engines, aircrafts, industrial manufacturing, conveyor belts, reading and writing from hard drives, other processes such as keeping satellites in place above the earth, sun tracking control of solar collectors and last but not least several biological functions.

The objective is to sufficiently control the aforementioned systems. This includes a system with good stability, robustness to exogenous disturbances and minimum oscillations. To achieve this, a controller and a feedback action are necessary.

3.1 Feedback Loop

Feedback is a process occurred when the output of a system is returned back to a point as an input. This process generates a chain of cause and effect called the *feedback*.

Karl Astrom, one of the most prolific contributors to control theory, states that *the magic of feedback is that it can create a system that performs well from components that perform poorly* [25].

A feedback control system at its basic form has three components: a system that needs to be controlled which is called *plant*, a controller to alter the input signal of the plant and a feedback loop.

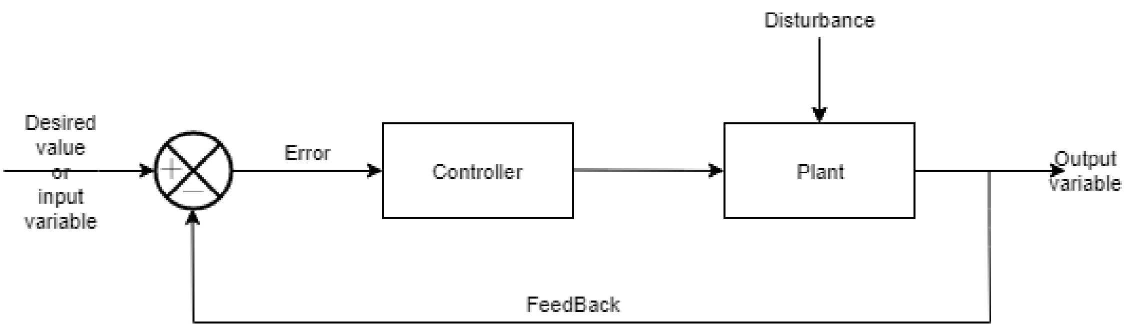

Figure 3.1: A schematic diagram of a general automatic control system

The elements of a closed loop system are illustrated in Figure 4.1. Below, we describe these elements and the information, or signals [1], that flow between elements:

- The dependent variable, called **output** is the quantity or signal of the controlled system that is directly measured and controlled.

[1] a signal is a function that conveys information about a phenomenon

- The independent variable, called **input** or **desired value** is the value that the output variable needs to converge with.
- The output **signal of the feedback** is a signal of the controlled output which is sent back to the input to be compared with the desired value.
- The **error** is the difference between the desired value and the controlled output.
- The **disturbance** is any exogenous change that affects the output signal.

3.1.1 Control Properties

Stability

A stable system is a system that produces a bounded output for a given bounded input [26]. Generally for a stable system oscillations must die out as early as possible or steady state[2] should be reached fast.

Stability is typically the first property considered in constructing control systems since unstable systems are not often used.

In figure 3.4 there are illustrated some examples of stable and unstable systems. If the output variable converges, the system is said to be stable. Otherwise, it is an unstable system or it is marginally stable.

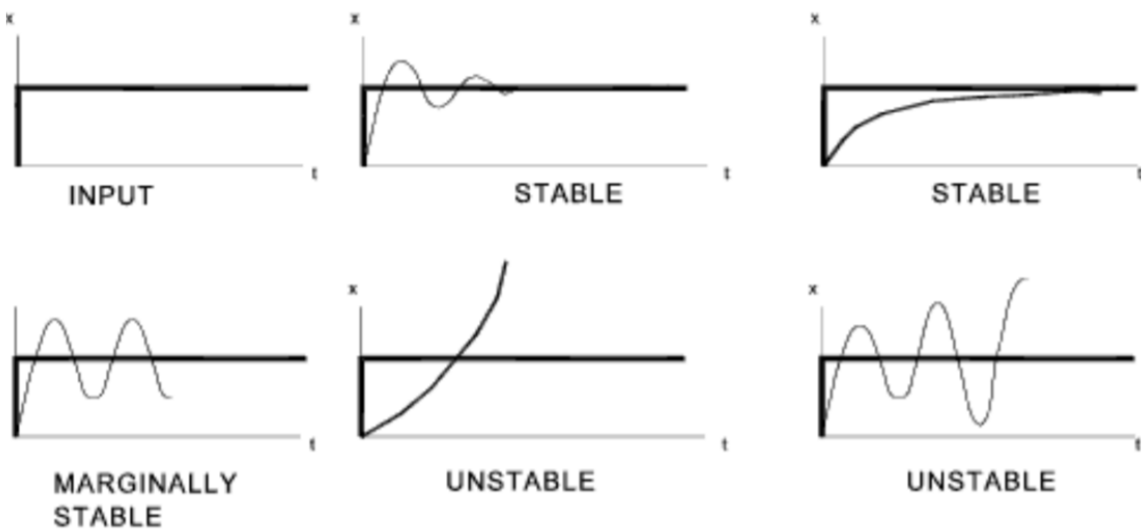

Figure 3.2: Stability of different systems [6]

Accuracy

The control system is accurate if the output signal converges (or reaches sufficiently close) to the desired value, or more generally if an actual controlled process approaches the desired process.

[2]when the state variable is unchanging in time

Robustness

Most of the times, control systems are subjected to disturbances which are unwanted signals that alter the desired behaviour. With the use of a feedback loop the sensitivity of a control system to these disturbances can be reduced.

Robustness is the ability of a closed loop control system to function properly when subjected to exogenous disturbances. More specifically as [27] reports robustness is the ability of a closed loop system to be able to sufficiently ignore exogenous noise.

Miscellaneous

If a control system converges quickly to its desired state it is said to have short **settling time**. As defined by Tay, Mareels and Moore in [28] settling time is the time required for the response curve to reach and stay within a range of certain percentage (usually 5% or 2%) of the final value.

In a control system **overshoot** occurs when the output signal exceeds the desired input. As defined by Katsuhiko Ogata in [29] overshoot is the maximum peak value of the response curve measured from the desired response of the system.

3.2 PID Controller

In general, the task of a controller is to maintain a desired system performance while coping with possible system disturbances. In the case of the PID controller, to achieve a desired performance, the user needs to select carefully the amount of each control action: proportional, integral, and derivative [30].

The controller can either be used with all its parts or with only the P, PI or PD terms. The proportional part reacts to the present control errors, the integral part sums up previous control errors and the derivative part predicts future control errors by using the derivative of the control error.

The controller should be adjusted properly or else desired performance will not be achieved. For example if the three gains of a PID controller are not chosen correctly, oscillations and instability can occur.

Based on a survey of over eleven thousand controllers in the refining, chemicals and pulp and paper industries, 97 percent of regulatory controllers utilize PID feedback [31].

3.2.1 P term

The proportional term of a PID controller depends only on the difference between the desired input value and the output signal, which is also known as the error signal. Thus, a significant usage of the P controller is to decrease the steady state error of the system [32].

It also defines the ratio of output signal to the error signal. As the proportional gain factor increases, the steady state error of the system decreases. However, despite the reduction, P control can never manage to eliminate the steady state error of the system.

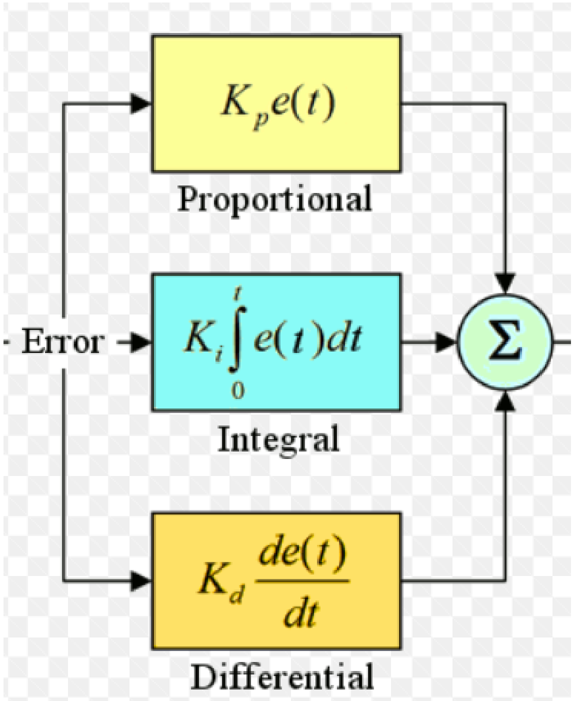

Figure 3.3: A block diagram of a PID controller [7]

As we increase the proportional gain, it provides smaller amplitude and phase margin. Also, raising the P term will boost the speed of the control system response (shorter settling time). This controller is used by itself when the control system can withstand a constant steady state error.

It can be easily concluded that applying P controller decreases the rise time, but if the P term is too big, oscillations will occur to the output signal. Further increase of P gain will result in larger oscillations and the system may become unstable.

Transient Response

Advantages of using P controller to control a second order plant:

- Increasing gain decreases rise time.
- Increasing gain decreases steady state error.

Disadvantages of using P controller to control a second order plant:

- Increasing gain increases percent overshoot and oscillations.
- Steady state is never zero if only P controller is used.

3.2.2 I term

Due to limitation of the Proportional controller where the desired input value cannot be equal to the output variable, there is a need to eliminate the steady state error. With the help of Integral Controller zero steady state error[3] can be achieved [32].

[3] Steady-State error is the final difference between the output variable and the desired value

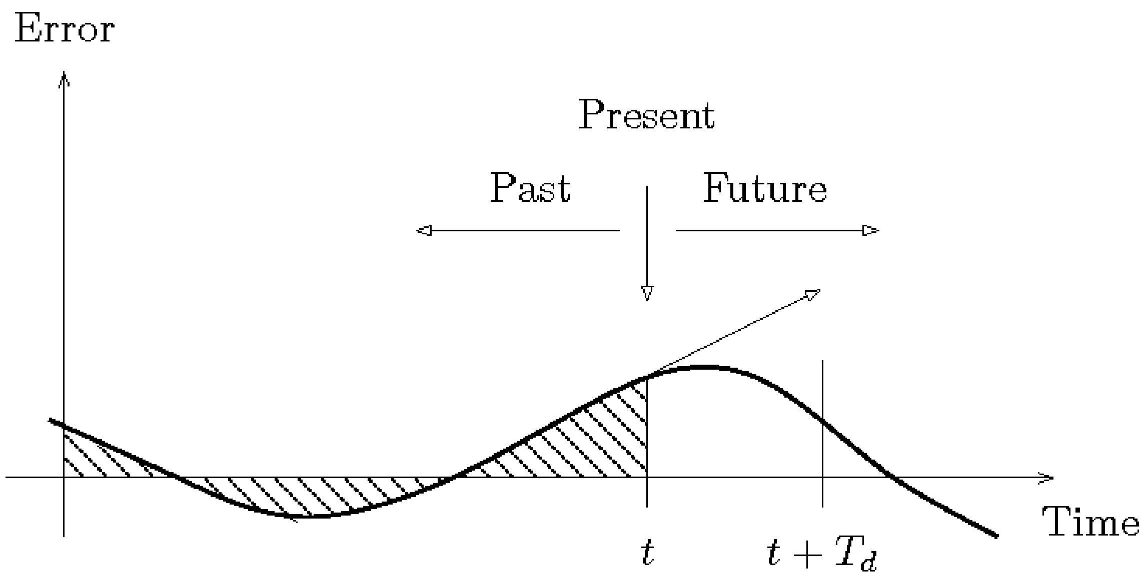

Figure 3.4: A PID controller takes control action based on past, present and prediction of future control errors [8]

The Integral Controller integrates the error signal over a period of time. This behaviour will continue to go on. If the error becomes zero the output of the I term will remain constant.

Transient Response

Integral action eliminates steady state error. However, it has very poor transient response. Using integral action increases the oscillations in the output of the closed loop systems, as well as it limits the speed of response and affects stability of the system.

3.2.3 D term

The aim of using Derivative controller is to increase the stability of the control system by controlling it more successfully thanks to the ability to predict the future error of the system response. Derivative control is the reaction to the rate of change of error according to time, multiplied by derivative constant. Because of the function of Derivative control sudden change in the value of the error signal must be avoided. In order to do so, the derivative is taken from the output signal of the state variable instead of the error signal [32].

In conclusion, increasing the derivative gain parameter will cause the control system to react more sufficiently to changes in the error signal and will reduce oscillations. On the other hand, most practical control systems use very small derivative gain, because the higher the derivative action the less robust the system becomes to exogenous disturbances. If the feedback signal is oscillating, the derivative action can make the whole system unstable. Thus, it is important to state that derivative controller is never used without P controller.

Transient Response

Derivative controller is usually used to improve transient response of the closed loop system. Derivative term decreases oscillations, but it increases high frequency noise.

3.2.4 PID Tuning

Tuning a control loop is setting the optional control gains for P, I and D to their optimum values in order to get an ideal response from a control system.

The first requirement that should be met is the stability of the control system. Beyond that, different systems have different requirement and different behaviours are expected.

Although the PID controller has only three terns, it is not easy to find good values for them, because the criteria might not be compatible with each other.

PID tuning is mostly a heuristic concept but the existence of many objectives to be met such as no oscillation, good stability and low rise time and settling time makes this process harder. For example sometimes, systems might have nonlinearity problem which means that while the parameters works properly for full load conditions, they might not work as effective for no load conditions. Also, if the PID parameters are not well tuned, the state variable may become unstable causing oscillations that result in mechanical failure.

For a system to work properly, the output signal should be stable, as well as it should not oscillate. However, there are case where some oscillations are acceptable.

Trial and Error Method

One of the many methods for tuning a PID controller is the trial and error method. To tune a PID controller using this method one should completely understand the action of each term.

In this method, the Integral and Derivative terms are set to zero first, and the proportional gain is increased until the system starts oscillating. As the proportional gain is increased, rise time and settling time are decreasing, until a threshold where the system becomes unstable. At the time proportional term is tuned to make the system faster and the closed loop system oscillates at a constant rate, the integral term is adjusted so that the oscillations will be gradually reduced. The integral controller minimizes the steady state error, but at the same time boosts overshoot. At this point in time proportional and derivative gains have been tuned. The parameter remained to be tuned is the derivative one. Derivative gain is increased until the system reaches quickly its desired value. Increasing derivative term decreases the overshoot of the output signal.

4 Control of Lambda in Internal Combustion Engines

The control of lambda in internal combustion engines has proven to be of high importance to both the efficiency of the vehicle and the environment. In this chapter we discuss a case study of controlling lambda using a PID controller. The simulation model used to describe dynamic engine variable responses is the so-called Mean Value Engine Model (MVEM), and the whole simulation is running in MATLAB software. The simulated engine is a 1275cc British Leyland mapped at DTU in 1990 [33].

4.1 Mean Value Engine Model

Mean Value Engine Models (MVEMs) are dynamic simulation models which seek to predict the mean values of the engine variables dynamically in time. This chapter presents a nonlinear four state (four differential equation) dynamic model of an SI engine. State variables are those which are determined by integrating the differential equations which are used to describe the engine [34].

In the engine model to be presented the state variables are the manifold pressure the crank shaft speed the fuel flow into the combustion chamber and the measured lambda by sensor which is considered as the fourth state of the MVEM.

Th engine input variables are those which can be adjusted external to the engine with the purpose of controlling it. For this certain model the inputs are the injected fuel mass flow and the throttle plate angle, which in this case is regarded as a disturbance.

4.1.1 State Equations

As Hendricks and Sorenson say in [34], an SI engine can be modelled from considerations of where the energy is concentrated in the engine. The fuel flow together with the air flow constitute the input energy flow to the engine while the energy output is the load power plus the heating and frictional and pumping losses. This suggests that a mean value engine model can be built up of three dynamic subsystems.

1. Intake manifold air mass flow.

2. Crankshaft and loading.

3. fuel vapor and fuel film.

To sufficiently construct a working MVEM the fourth state (the measured lambda by sensor) is needed.

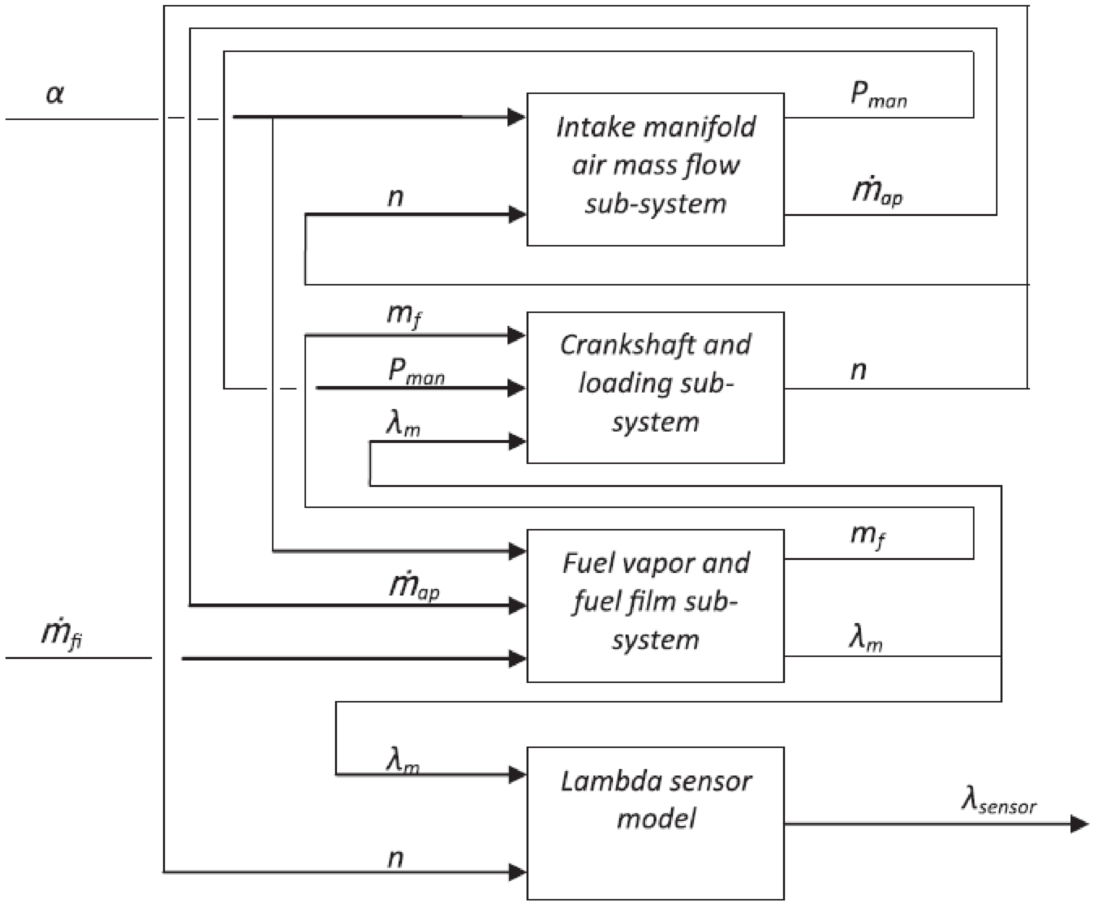

Figure 4.1: Block diagram of mean value model of engine [9]

4.1.1.1 Intake manifold air mass flow sub-system

In the derivation of the manifold pressure state equation the procedure is to apply conservation of mass on the intake manifold which is considered as a control volume and assume air as an ideal gas. This results in the first state equation as

$$\dot{P_{man}} = \frac{RTi}{Vi}(-\dot{m_{ap}} + \dot{m_{at}}) \tag{4.1}$$

In eq. (3.1) $\dot{m_{ap}}$ is a function of crank shaft speed and manifold pressure. Hendricks has obtained the following modified dynamics for $\dot{m_{ap}}$ [35]

$$\dot{m_{ap}} = \frac{Vd}{120RTi}(sP_{man} - y)n \tag{4.2}$$

where s and y are cosidered as the constants for typical 4-stroke SI engines according to [35].

$\dot{m_{at}}$ is a function of the throttle plate angle and the manifold pressure which can is defined, according to [35], as follows

$$\dot{m_{at}} = \dot{m_{at1}} \frac{Pa}{\sqrt{Ta}} \beta_1(\alpha)\beta_2(P_{man}) \tag{4.3}$$

where $\dot{m_{at1}}$ is a fitting constant and $\beta_1(\alpha)$, $\beta_2(P_{man})$ are considered as:

$$\beta_1(\alpha) = 1 - \alpha_1 cos(\alpha_0 + \alpha_2 cos(\alpha)) \tag{4.4}$$

$$\beta_2(P_{man}) = \begin{cases} \frac{1}{P_n}\sqrt{P_{man}^{P_1} - P_{man}^{P_2}}, & \text{if } P_r \geq P_c \\ 1, & \text{if } P_r < P_c \end{cases} \tag{4.5}$$

where a_0 is the closed throttle plate angle, P_c is a constant scalar, $P_r = \dfrac{P_{man}}{P_a}$ and P_a is the pressure just in front of the throttle plate.

4.1.1.2 Crankshaft and loading sub-system

The crankshaft state space equation is obtained through the energy conservation principle for the crankshaft rotation. As explained from Hendricks in [34], in order to avoid modelling the cooling and exhaust system losses, the thermal efficiency of the engine is inserted as a multiplier of the fuel mass flow.

$$\dot{n} = \frac{-1}{In}[P_f(n) + P_p(n, x_1) + P_b(n)] + \frac{1}{In} H_u N_i(n, P_{man}, \lambda)\dot{m_f}(t - t_d) \tag{4.6}$$

where the loss powers and thermal efficiency, as well as a complete MVEM for the 1275cc engine, will be thoroughly discussed in Appendix to give an idea of the order of magnitude of the different model parameters in a convenient system of units.

4.1.1.3 Fuel vapor and fuel film sub-system

From all the injected fuel in the intake manifold, the fraction of the fuel flow which strikes the manifold and becomes fuel film after reaching the cold air in the manifold is defined as X, while that which becomes fuel vapor is 1-X. The condensed fuel film gradually vaporizes to be mixed with the fuel vapor and makes $\dot{m_f}$ to enter the combustion chamber. Accordingly, the dynamics of the fuel vapor and fuel film sub-system could be summarized as

$$\ddot{m_f} = \frac{1}{\tau_f}[(1 - X)\ddot{m_{fi}} + X\dot{m_{fi}} - \dot{m_{ff}}] \tag{4.7}$$

$$\ddot{m_{ff}} = X\dot{m_{fi}} - \dot{m_{ff}} \tag{4.8}$$

where τ_f is the fuel film evaporation time constant.

4.1.1.4 Lambda sensor model

The fourth state space equation determines the dynamics of a first order lambda sesnor which is summarized as follows [9]

$$\dot{\lambda_{sensor}} = \frac{1}{\tau_e}(-\lambda_{sensor} + \lambda m(t - \delta) \tag{4.9}$$

where λ_{sensor} is the measured lambda by lambda sesnor, and τ_e defines the time delay in the measurements made by the sensor. λm is the value of lambda which is directly calculated from the mean value engine model and is defined as follows

$$\lambda m = \frac{\dot{m_{ap}}}{AFR_{st}\dot{m}_f} \tag{4.10}$$

In eq. (3.9) δ is the total delay of the lambda sensor defined as

$$\delta = (\frac{\theta_{EVO}}{720})(\frac{120}{N_{cyl}n}) + t_d \tag{4.11}$$

where θ_{EVO} is the angle of crankshaft at which the exhaust valve opens and N_{cyl} is the number of cylinders.

4.2 Simulation

The simulation of the mean value engine model was performed using the MATLAB software. The differential equations describing the MVEM are nonlinear delay differential equations with general delays and for this reason, the *ddesd*[1] solver has been used.

4.2.1 ddesd Solver

The ddesd solver, in its main form, has four arguments.

1. ddefun: Function handle that evaluates the right side of the differential equations.
2. delays: Function handle that returns a column vector of delays.
3. history: Function of t such that $y = history(t)$ returns the solution $y(t)$ for $t \leq t_0$ as a column vector.
4. tspan: Interval of integration.

4.2.1.1 ddefun

In this function handle every equation of the MVEM, both differential, and algebraic, was written in the form: $dydt = ddefun(t, y, Z)$, where t corresponds to the current t, y is a column vector that approximates y_t and $Z(:, j)$ approximates $y(d(j))$ for delay $d(j)$ given as component j of delays(t, y). The output is a column vector corresponding to $f(t, y(t), y(d(1), ..., y(d(k)))$.

[1]Full documentation: `https://www.mathworks.com/help/matlab/ref/ddesd.html`

4.2.1.2 delays

The delays evaluated by this function handle are those from the crankshaft and loading differential equation (τ_d) and the lamdba sensor differential equation (δ).

$$t_d = \frac{1}{2}\frac{60}{N_{cyl}n}$$

$$\delta = (\frac{\theta_{EVO}}{720})(\frac{120}{N_{cyl}n}) + t_d$$

4.2.1.3 history

The history of the differential equations was filled in with the rationale of fully warm conditions and is presented in Table 4.1.

Table 4.1: History of the state equations.

State	History
intake manifold pressure	0.7 [bar]
crankshaft speed	3 [krmp]
fuel flow into combustion chamber	0.001 [kg/sec]
measured lamdba by lambda sensor	0.8

4.3 Closed-loop Control

Once the ddesd solver has its four arguments, one can obtain the change of all state variables against time, but for certain fixed inputs.

To sufficiently control lambda a closed-loop is needed. In this case study a PID controller is used to regulate lambda. Injected fuel mass flow $\dot{m}_{fi}$ is the manipulated variable and the throttle plate angle α is considered as disturbance.

That being said, a PID controller is constructed to control lambda by alternating the value of the injected fuel mass flow. A schematic of the closed-loop control is illustrated in Figure 4.2.

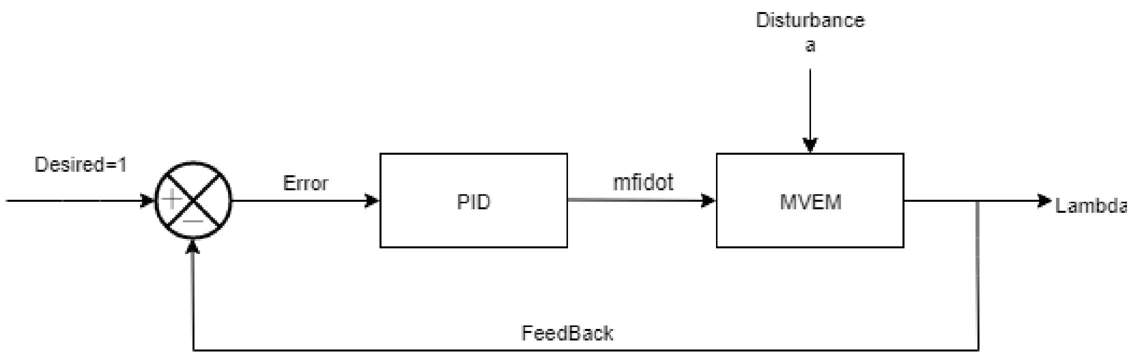

Figure 4.2: Schematic of closed-loop control using a PID controller

4.3.1 PID Controller

The PID controller source code used in this case study was implemented by Mohammad Saadeh [36]. After making some alternations with the scope of adjusting it to the case study, a closed-loop control with the MVEM for plant and the PID for controller was completed.

4.3.1.1 PID tuning

To sufficiently control lambda the PID contoller needed to be tuned. The tuning method used in this case study is the trial and error method.

1. Integral and derivative gains air set to zero.
2. Proportional gain is increased until a steady oscillation is obtained. With the increase of P gain the system responses faster but it does not become unstable.
3. The resulting proportion value is the current value divided by 1.5.
4. Next the integral coefficient is increased until the current oscillations gradually reduce. The scope of the integral controller is to reduce the steady state error, thus integral gain is increased even more until steady state error is minimal. After all these changes an increase in overshoot is occured.
5. Finaly, derivative action is increased until, again, the oscillations are at a constant rate. At this state the closed-loop control reacts quickly to the desired value.

5 Results and Discussion

In chapter 4 we described the mean value engine model for simulating an SI engine using MATLAB software. We proceed to connect the MVEM with a PID controller and finally tune the controller using the trial and error method.

In this chapter we will first present and discuss the results of controlling lambda with a PID controller when using a constant value for the throttle plate angle. Then the results when treating the throttle plate angle as a disturbance variable will be presented and discussed.

5.1 Constant throttle plate angle

For this simulation the following considerations are made:

- Throttle plate angle α is considered as constant and equal to 25 degrees.
- The single input $\dot{m}_{fi}$ is the output signal of the PID controller.
- The history of the MVEM is considered as presented in section 3.2.1.3
- Desired input value is set to 1.
- Simulation time is set to 10 seconds.

As one can see from Figure 5.1 the results are satisfactory. First and foremost the steady state error is zero. Furthermore, the system response is very quick. The lambda value stabilizes at around 0.7 seconds which is much less than the time that the engine needs to warm up. One should also note that overshoot is not remarkable, it reaches up to 1.155 which is very close to the stoichiometric value of lambda. Last but not least, oscillations are almost zero, which means that the system response is really adequate.

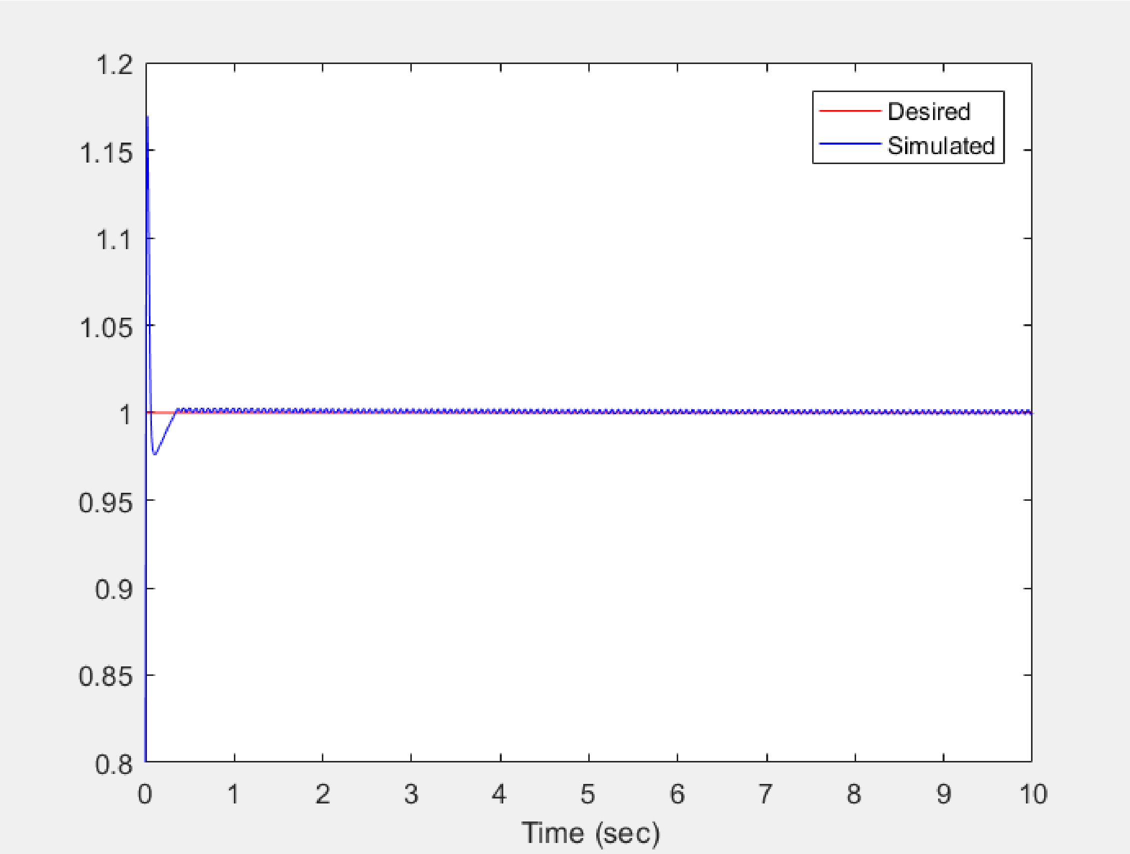

Figure 5.1: Lambda signal for constant throttle plate angle

5.2 Disturbance throttle plate angle

For this simulation the following considerations are made:

- Throttle plate angle α is considered as noise disturbance .
- The input m_{fi} is the output signal of the PID controller.
- The history of the MVEM is considered as presented in section 3.2.1.3
- Desired input value is set to 1.
- Simulation time is set to 20 seconds.

As one can see from Figure 5.2 the results are very different from the previous case. First and foremost, in this case the lambda signal does not converge to the desired value which is equivalent to one. Another noteworthy statement is that in this case overshoot reaches up to 1.8 which is a value that is quite far from the stoichiometric one, which is one. Last but not least, one can observe that oscillations are quite remarkable and as a result affect the stability of the system.

Even though the results on the first sight seem unsatisfactory, they are not. The fact that lambda does not converge is only due to the disturbance which changes at each time step. Furthermore, one should take into consideration the simulated model on the SI engine. The mean value engine model is a nonlinear four state dynamic model and as one knows PID controllers cannot effectively control nonlinear models. Last but not least, studies have shown that classic controllers, such as PI systems, could not result in robust control of lambda against disturbances [9]. A nonlinear controller could perform better, but it wouldn't converge either in the presence of disturbances.

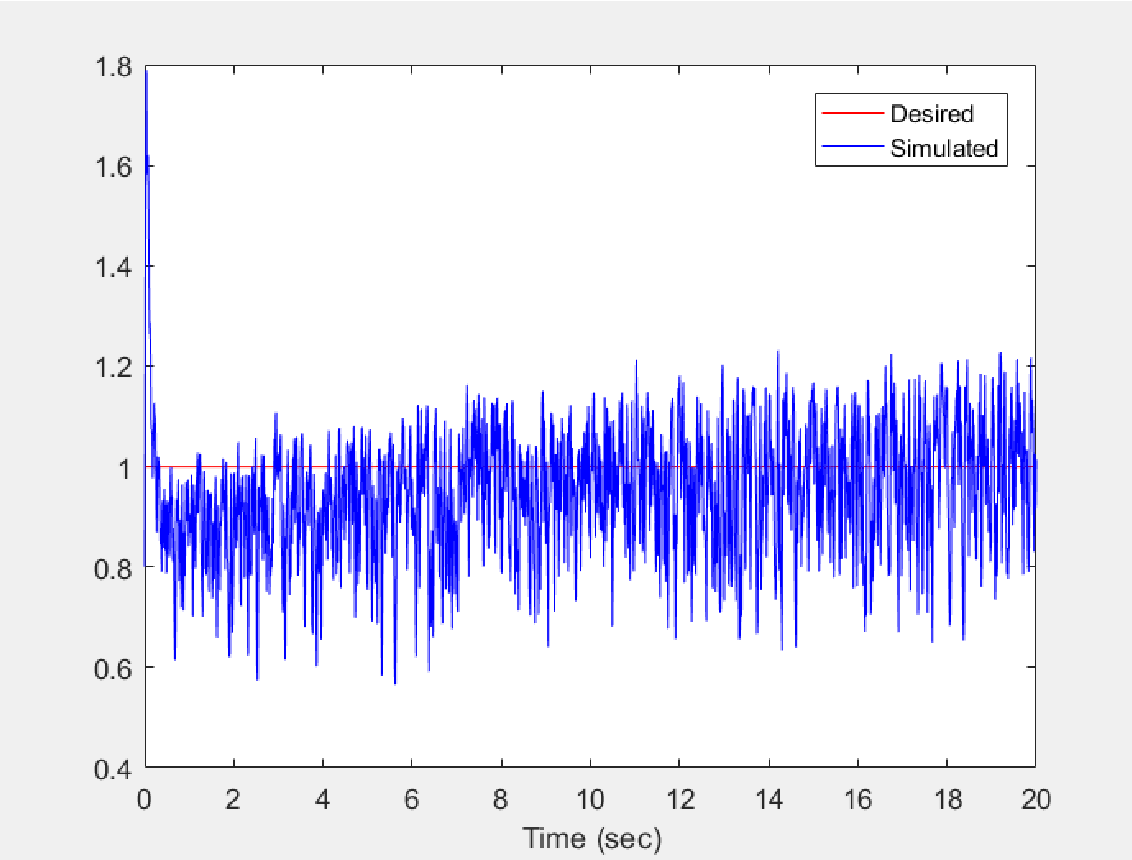

Figure 5.2: Lambda signal for throttle plate angle as disturbance

6 Conclusion and Future Work

6.1 Conclusion

The main objective of this Diploma Thesis was to control lambda in internal combustion engines using a PID controller. Firstly, we discussed about some parts of internal combustion engines which are of high importance about minimizing exhaust pollutants and gas emissions. Furthermore, we explained the functionality and applications of feedback control and highlighted the importance of the PID controller. Last but not least, we conducted a case study about controlling lambda in internal combustion engines using a PID controller.

The case study was conducted completely in MATLAB software. The engine simulated in MATLAB was a 1275cc British Leyland engine which had been mapped in Laboratory for Energetics, Technical University of Denmark. The simulation model was the mean value engine model, which is a nonlinear four state dynamic model of an SI engine. The controller used was a PID controller which had been tuned using the trial and error method.

Finally, we illustrated the results from two simulations. In the first simulation the input of the MVEM was the injected fuel mass flow, while in the second simulation throttle plate angle was added as noise disturbance. The air-to-fuel ratio (lambda) diagrams against time were pretty different for the two simulations. More specifically, while on the first simulation the PID ability to control lambda is very satisfactory, when noise disturbance was added to the system, oscillations occurred and steady state error was different from zero.

6.2 Future Work

Results can get better for both simulations. The PID controller, could not result in robust control of lambda against exogenous disturbances and modelling uncertainties [37]. If one wants to control lambda within 1 % of its stoichiometric value, other control techniques should be applied.

Nowadays, microcomputers have become all the more powerful and they can be applied in a lot of different situations. Electronic Control Units (ECUs) of engines can implement intelligent and adaptive control algorithms of lambda. The performance of these controllers is improved compared to classic control schemes, such as the PID controller. These controllers are robust to exogenous disturbances and model uncertainties as well as, they have significantly better transient and steady state performances.

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Appendices

A Complete MVEM for the mapped engine

The intention behind the MVEM is that it should as far as possible be physically based and have a form which can be easily understood and used by engine specialists.

The units in the MVEM have been selected to be convenient for calculations on engines with data given in the conventional form rather than for purely physical significance. With that being said, one can conclude that, the units used in the model are as shown in Table A.1

Table A.1: Units in the Mean Value Engine Model.

Variable	**Unit**
throttle angle	degrees
crankshaft speed	rpm/1000
engine displacement volume	litres
fuel mass flow	kg/sec
port and throttle air mass flows	kg/sec
pressures	bar
powers	kW
manifold volume	m^3
temperatures	Kelvin
moment of inertia	kg m^2

A.1 Equations of the MVEM

As mentioned in Chapter 3, in a general MVEM for SI engines include three nonlinear differential equations and a fourth one, which is the measured lambda by sensor, to describe four different dynamic engine subsystems. In order to use the differential equations, it is necessary to find algebraic functions for the instantaneous internal engine variables in terms of the engine state variable.

In this section all these differential and algebraic equations will be formed with the parameters for the mapped engine. [1]

Intake manifold air mass flow sub-system

For the intake manifold the parameters used in the equations are the following:

- $T_i = 308$ K
- $V_i = 614.6 * 10^{-6} m^3$

[1] the 1275cc British Leyland engine

- $R = 287 * 10^{-5}$
- $V_d = 1.275$ litres
- $s = 0.952$
- $y = 0.0793$
- $P_a = 1.013$ bar
- $T_a = 297$ K
- $\dot{m_{at1}} = 7.32$
- $\alpha_1 = 1$
- $\alpha_0 = 5.4$ deg
- $\alpha_2 = 0.4087$
- $P_n = 0.7404$
- $P_1 = 0.4408$
- $P_2 = 2.3143$
- $P_c = 0.4125$

The equations used for these parameters are thoroughly described in Section 3.1.1.1

Crankshaft and loading sub-system

The crankshaft speed state equation can be written

$$\dot{n} = \frac{-1}{In}[P_f(n) + P_p(n, x_1) + P_b(n)] + \frac{1}{In} H_u N_i(n, P_{man}, \lambda)\dot{m}_f(t - t_d) \tag{A.1}$$

For the crankshaft speed the parameters used in the equations are the following:

- $N_{cyl} = 4$
- $I = I_{ac}(\frac{\pi}{30})^2 1000$ †
- $I_{ac} = 0.48 kg * m^2$
- $H_u = 4.3 * 10^4$ kJ/kg
- $k_b = 0.22 kW/krmp^3$

†the scaling of I is necessary because the crankshaft speed state equation is written in terms n rather than the angular velocity for convenience

In eq (A.1) $t_d = \frac{1}{2}\frac{60}{N_{cyl}n}$

The algebraic function for both the loss powers and thermal efficiency are described below.

The loss powers can currently only be written as empirical regression expressions because of the lack of physical theory for friction, thus

$$P_f(n) = n(1.673 + 0.272n + 0.0135n^2) \tag{A.2}$$

$$P_p(n, \dot{P}_{man}) = n(-0.969 + 0.206n)\dot{P}_{man} \tag{A.3}$$

It has been found convenient to express the load power as the function

$$P_b(n) = k_b n^3 \tag{A.4}$$

where k_b is the loading parameter. It is adjusted in such a way that the engine is loaded to the desired power level at a given operating point.

The thermal efficiency has several contribution and is thus written as a product of three terms

$$N_i(\lambda, n, \dot{P}_{man}) = N_{in}(n)N_{ip}(\dot{P}_{man})N_{i\lambda}(\lambda) \tag{A.5}$$

These terms are given in order of decreasing importance.

$$N_{in}(n) = 0.558(1 - 0.392n^{-0.360}) \tag{A.6}$$

$$N_{ip}(\dot{P}_{man}) = 0.9301 + 0.2154\dot{P}_{man} - 0.1657\dot{P}_{man}^{\,2} \tag{A.7}$$

$$N_{i\lambda}(\lambda) = \begin{cases} -0.0205 + 1.741\lambda - 0.745\lambda^2, & \text{if } \lambda > 1 \\ -1.299 + 3.599\lambda - 1.332\lambda^2, & \text{if } \lambda \leq 1 \end{cases} \tag{A.8}$$

Fuel vapor and fuel film sub-system

The model for the the fuel dynamics is due to Aquino [38]. This model keeps track of the fuel mass in the intake manifold instead of the fuel flow. The single time constant version of this model can be written

$$\ddot{m}_f = \frac{1}{\tau_f}[(1 - X)\ddot{m}_{fi} + X\dot{m}_{fi} - \dot{m}_{ff}] \tag{A.9}$$

$$\ddot{m}_{ff} = X\dot{m}_{fi} - \dot{m}_{ff} \tag{A.10}$$

where τ_f is approximately constant at given coolant temperature and is on the order of 1 sec for a throttle body injection system.

Using step fuel pulse identification techniques originally reported in 1990 [34], the X function for the mapped engine is

$$X(\alpha) = 0.1 + 8.89 * 10^{-3}\alpha \tag{A.11}$$

Lambda sensor model

For the lambda sensor model the parameters used in the equations are the following:

- $\tau_e = 0.01$ sec
- $AFR_{st} = 14.86$
- $\theta_{EVO} = 180 - 40 = 140$ degrees

The equations used for these parameters are thoroughly described in Section 3.1.1.4