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Chapter 1

Mathematical Background

1.1 Introduction

An important part of every pattern recognition project handling large amounts of data, is the dimensionality reduction problem, that is, the transfer of the input data into a lower dimensional space, which will allow for more efficient processing and lower model error, as fewer parameters will have to be estimated. In order to develop algorithms for this task, one must be able to understand basic notions often arising in such context, e.g. Hilbert space, convergence, metric, manifolds. The purpose of this chapter is to offer a brief introduction to these concepts in connection with the broader topic of the project([Kre78]).

1.2 Metric Spaces

A *space* in the broader mathematical sense, is a set of elements X with some added structure.

Specifically, we will be working on *metric spaces*. Prior to defining a *metric space* we must first define what a *metric* is.

Definition 1. *A metric d on a set X (or distance function on X) is a function defined on* $X \times X$*, such that for all* $x,y,z \in X$ *,the following properties hold true:*

- *d is real-valued, finite and non-negative*
- $d(x,y) = 0$ *iff* $x = y$
- $d(x,y) = d(y,x)$ *(Symmetry)*
- $d(x,y) \leq d(x,z) + d(z,y)$ *(Triangle Inequality)*

Definition 2. *A metric space is a pair (X,d) where X is a set and d is a metric on X.*

Set X is also called the *underlying set* of (X,d).For the fixed points x,y the non-negative number *d(x,y)* is called the distance from x to y.

Examples of metric spaces are the well known $\mathbb{R}$ with the distance function

$$d(x, y) = |x - y|$$

, and the Euclidean space $\mathbb{R}^2$ with the Euclidean metric, defined by

$$d(x, y) = \sqrt{(x_1 - y_1) + (x_2 - y_2)}$$

for elements $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

Spaces and their structure play a vital role in dimensionality reduction.The goal of this process is to transfer the input space into a lower dimensional one, yet at the same time retain the original relations between input data. In order to do this, we need to make sure that the space we move to, has certain properties.We are especially interested in spaces where the triangular inequality is satisfied, so that we can take advantage of the convergence properties it offers to the space.This is the reason why we built on a metric space, adding properties and operations, to derive new spaces which offer the required structure.

We will now introduce some auxiliary concepts which will enable a smooth transition into further defining kinds of spaces.

Definition 3. *Given a point $x_0 \in X$ and a real number $r > 0$, we define three types of sets:*

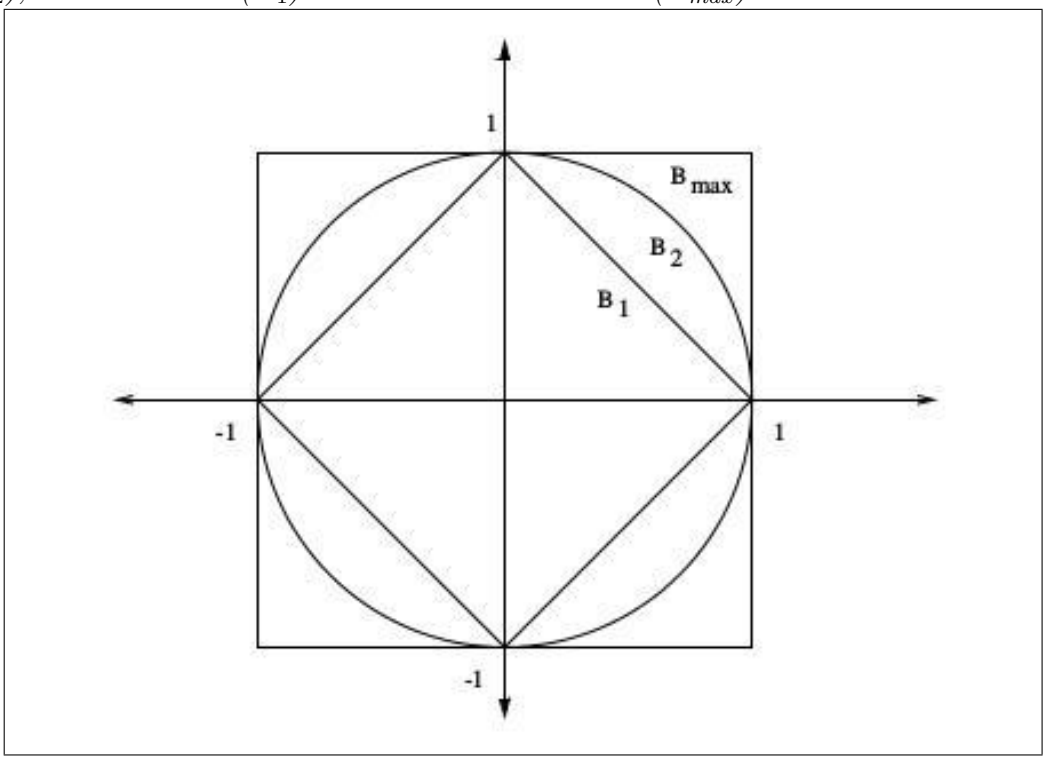
- *$B(x_0;r) = \{x \in X \mid d(x,x_0) < r\}$ (Open Ball)*
- *$B(x_0;r) = \{x \in X \mid d(x,x_0) \leq r\}$ (Closed Ball)*
- *$S(x_0;r) = \{x \in X \mid d(x,x_0) = r\}$ (Sphere)*

where point x_0 is called the center and r the radius.

Given the definition of a *ball*, we can define the notion of a neighbourhood of a point.

Definition 4. *An open ball $B(x_0, \varepsilon)$, $\varepsilon > 0$, is called an ε - neighbourhood of x_0. A neighbourhood of x_0 is any subset of X which contains an ε - neighbourhood of x_0.*

Figure 1.1: *The unit balls in the euclidean space defined using the euclidean norm (B_2), the sum norm (B_1) and the maximum norm (B_{max}).*



Since the process of dimensionality reduction is ,in its essence, a mapping from one space to another, we proceed by defining the notion of a continuous mapping.

Definition 5. *Let X = (X,d) and Y = (Y,$\bar{d}$) be metric spaces.A mapping $T : X \to Y$ is said to be continuous at a point $x_0 \in X$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that*

$$\bar{d}(Tx, Tx_0) \ < \ \varepsilon \ \forall \ x \ satisfying \ d(x, x_0) \ < \ \delta$$

T is said to be continuous if it is continuous at every point of X.

Having presented the ideas of a metric space and some of its subsets, as well as defined a continuous mapping, we proceed to remind the notions of convergence and completeness which will be used to move on to further spaces.

1.3 Convergence and completeness

Definition 6. *A sequence (x_n) in a metric space X = (X,d) is said to converge or to be convergent if there is a $x \in X$ such that*

$$\lim_{n\to\infty} d(x_n, x) = 0.$$

x is called the limit of x_n and we write

$$\lim_{n\to\infty} x_n = x \text{ or } x_n \to x.$$

It is obvious now why we need to be in a metric space to define the convergence of x_n: the metric d produces the sequence $a_n = d(x_n, x)$ whose convergence defines that of x_n.Furthermore, using the triangular inequality we can prove the following lemma:

Lemma 1. *Let X=(X,d) be a metric space.Then:*

- *A convergent sequence in X is bounded and its limit is unique.*
- *If $x_n \to x$ and $y_n \to y \in X$, then $d(x_n, y_n) \to d(x,y)$.*

Convergence plays an important role in the definition of completeness which will help us later to define Banach and Hilbert spaces (where we mostly work).

Definition 7. *Let x_n be a sequence in $\mathbb{R}$ or $\mathbb{C}$. x_n converges and we call it a Cauchy sequence if and only if it satisfies the Cauchy convergence criterion,that is, if and only if for every given $\varepsilon > 0$ there is a N=N(ε) such that*

$$|x_m - x_n| < \varepsilon \quad \forall m, n > N.$$

However, this is the case only for $\mathbb{R}$ and $\mathbb{C}$. Given that most pattern recognition tasks work on multi-dimensional spaces, we have to generalize this convergence property in such spaces.

Definition 8. *A sequence x_n in a metric space X = (X,d) is said to be Cauchy (or fundamental),if for every $\varepsilon > 0$ there is a N=N(ε) such that*

$$d(x_m, x_n) < \varepsilon \quad \forall m, n > N.$$

This generalization allows us to give the following definition of completeness:

Definition 9. *A metric space X = (X,d) is said to be complete if every Cauchy sequence in X converges (that is, it has a limit which is an element of X).*

It is important to notice that convergence is not a property of the sequence by itself, but it also depends on the metric space that the sequence lies: the limit of the sequence must be *in the space.*

(EXAMPLES NEEDED??)

1.4 Vector spaces and Banach spaces

As we explore the structure of spaces so that we reach a suitable one for our problem, we come across the notion of *vector spaces*.

Definition 10. *A vector space (or linear space) over a field K is a non-empty set X of elements x,y,... ,which are called vectors, together with two algebraic operations : vector addition and multiplication of vectors by scalars, that is by elements of K.*

Vector addition associates with every ordered pair (x,y) of vectors, a vector x+y, called the sum of x+y, in such a way that the following properties are satisfied:

$$x + y = y + x$$

$$x + (y + z) = (x + y) + z$$

Furthermore, there exists a vector θ, called the zero vector, and for every vector x there exists a vector -x, such that for all vectors we have:

$$x + \theta = x$$

$$x + (-x) = \theta$$

Vector multiplication by scalars associates with every vector x and scalar α a vector αx (or $x\alpha$) called the product of α and x in such a way that for all vectors x,y and scalars α, β the following hold:

$$\alpha(\beta x) = (\alpha\beta)x$$

$$1x = x$$

$$\alpha(x + y) = \alpha x + \alpha y$$

$$(\alpha + \beta)x = \alpha x + \beta x$$

A vector space X exactly as defined above, may or may not be a metric space. To make sure that a relation between the algebraic structure of X and the metric exists, and thus be able to combine algebraic and metric concepts, we have to define on X a metric d based on a *norm*.

Definition 11. *A norm on a vector space X (over $\mathbb{R}$ or $\mathbb{C}$) is a real-valued function on X whose value at a $x \in X$ is denoted by*

$$\|x\|$$

and which has the properties

$$\|x\| \geq 0$$

$$\|x\| = 0 \Rightarrow x = 0$$

$$\|\alpha x\| = |\alpha|\|x\|$$

$$\|x + y\| \leq \|x\| + \|y\|$$

where x,y $\in X$ *and* $\alpha \in K$.

A norm on X defines a metric d on X which is given by

$$d(x, y) = \|x - y\| \quad x, y \in X$$

and is called the metric induced by the norm.

A vector space X with a norm defined on it, is called a normed space and is denoted by X or $(X,\|\cdot\|)$.

If a normed space is complete in the metric defined by the norm, we call it a *Banach space*.

1.5 Inner product spaces and Hilbert spaces

In a vector space, vectors can be added and multiplicated giving the space its algebraic properties. By defining a norm on such a space, the concept of the length of a vector is generalised allowing us to define a metric and establish a relation between the algebraic and geometrical structure of the space.

To connect the above with pattern recognition, one should notice that when the problem is transfered to a lower dimensional space, it is often desirable to keep not only the length/magnitude relations between the data (as represented by the distance/norm for example) but also the angle between them.This reminds us of the inner product of a Euclidean space, which is what we will expand over normed spaces.

Definition 12. *An inner product on a vector space X is a mapping of* $X \times X$ *into the scalar field K of X; that is, with every pair of vectors x and y, there is associated a scalar which is written*

$$\langle x, y \rangle$$

and is called the inner product of x and y, such that for all vectors x,y,z and scalar α *the following properties hold:*

$$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$$

$$\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$$

$$\langle x, y \rangle = \overline{\langle y, x \rangle}$$

$$\langle x, x \rangle \geq 0$$

$$\langle x, x \rangle \iff x = 0$$

An inner product on X defines a norm on X given by

$$\|x\| = \sqrt{\langle x, x \rangle}$$

and a metric induced by the above norm given by

$$d(x, y) = \|x - y\| = \sqrt{\langle x - y, x - y \rangle}$$

A vector space X with an inner product defined on it, is called an *inner product space* (or *pre-Hilbert space*).An inner product space which is complete in the metric defined by the inner product, is called a *Hilbert space.*

It is apparent that an inner product space is a normed space and a Hilbert space is a Banach space.

An important theorem of Banach spaces, and thus Hilbert spaces, is the *Fixed point theorem* or *Contraction theorem.*Before we present the theorem and its useful proof we have to give the following definitions.

Definition 13. *A fixed point of a mapping T:X* $\to$ *X of a set X into itself, is a* $x \in$ *X which is mapped onto itself (it is kept "fixed" by T), that is,*

$$Tx = x$$

the image Tx coincides with x.

Definition 14. *Let X = (X,d) be a metric space. A mapping T:X* $\to$ *X is called a contraction on X if there is a positive real number* $\alpha < 1$ *such that for all x,y* $\in$ *X*

$$d(Tx, Ty) \leq \alpha d(x, y)$$

that is, the images of any points x,y are closer together that the points x,y.

The Banach fixed point theorem is an existence and uniqueness theorem for fixed points of certain mappings, and its proof gives a constructive procedure for getting closer and closer to the fixed point starting from an initial approximation.Thinking this idea in connection with pattern recognition one should think of the fixed point as the pattern to be found and the initial approximation as the observation available.

Theorem 1. ***Banach fixed point theorem.***

Consider a metric space X = (X,d) where X ≠ ∅. Suppose that X is complete and let T:X→ X be a contraction on X. Then T has precisely one fixed point.

Proof Idea: We construct a sequence (x_n) and show that it is Cauchy so that it converges in the complete space X.Then we prove that its limit X is a fixed point of T and T has no further fixed points.

We choose any $x_0 \in$ X and define the iterative sequence (x_n) by

$$x_0\ ,\ x_1 = Tx_0\ ,\ x_2 = Tx_1 = T^2x_0\ ,...,\ x^n = T^nx_0,\ ... \tag{1.1}$$

which is the sequence of the images of x under repeated application of T. We now show that (x_n) is Cauchy. Since T is a contraction we have:

$$\begin{aligned} d(x_{m+1}, x_m) &= d(Tx_m, Tx_{m-1}) \\ &\leq \alpha d(x_m, x_{m-1}) = \alpha d(Tx_{m-1}, Tx_{m-2}) \\ &\leq \alpha^2 d(x_{m-1}, x_{m-2}) \\ ... &\leq \alpha^m d(x_1, x_0). \end{aligned}$$

Hence, using the triangular inequality we obtain for n>m:

$$\begin{aligned} d(x_m, x_n) &\leq d(x_m, x_{m+1}) + ... + d(x_{n-1}, x_n) \\ &\leq (\alpha^m + \alpha^{m+1} + ... + \alpha^{n-1}) d(x_0, x_1) \\ &= \alpha^m (\frac{1 - \alpha^{n-m}}{1 - \alpha}) d(x_1, x_0) \end{aligned}$$

Since 0< α <1, in the numerator we have 1 - $\alpha^{n-m} < 1$.Consequently,

$$d(x_m, x_n) \leq (\frac{\alpha^m}{1 - \alpha}) d(x_1, x_0)$$

On the right 0< α <1 and *d(*x_0, x_1*)* is fixed, so that we can make the right-hand side as small as we please by taking m sufficiently large (and $n > m$).This proves that (x_m) is Cauchy. Since X is complete, (x_m) converges, say $x_m \to x$. We now have to show that this limit x is a fixed point of the mapping T.

From the triangle inequality and the definition of contraction we have:

$$\begin{aligned} d(x, Tx) &\leq d(x, x_m) + d(x_m, Tx) \\ &\leq d(x, x_m) + \alpha d(x_{m-1}, x) \end{aligned}$$

The sum on the second line can be made smaller than any preassigned ε >0 because $x_m \to$ x.We draw the conclusion that *d(x,Tx)=0)* and consequently $x = Tx$, which means that x is a fixed point of T.

We now have to show that this fixed point is unique. Let there be a second fixed point $\bar{x}$.Then

$$d(x, \bar{x}) = d(Tx, T\bar{x}) \leq \alpha d(x, \bar{x})$$

which implies that $d(x,\bar{x})=0$ since $\alpha < 1$. Hence x = $\bar{x}$ and the proof of the theorem ends here.

1.6 Manifolds

An important notion that we will come across very often in the present work is the notion of a *manifold*.

An intuitive representation of a manifold is the following: suppose we have a set of data vectors in $\mathbb{R}^n$, and that a subset of their dimensions represents a certain feature of interest.If we focus on these dimensions only,"keeping" the rest "steady", we notice that the data vectors move along a certain path in $\mathbb{R}^n$. This path, or curve formed in the space, is the *manifold* of the feature in $\mathbb{R}^n$.A manifold has a locally Euclidean geometry,in a neighbourhood around each point, but on a global scale the relations between its elements are non metric.

1.7 Summary

In this chapter some basic ideas about spaces and their properties were presented.Starting from the metric and metric space, we moved on to the norm and vector space and based on these we defined the Hilbert space which is the closest generalisation of the well-konwn Euclidean space. Hilbert spaces provide us with the tools we need in our work: convergence, completeness, contractions etc.

Figure 1.2: *Illustration of a mapping T.*

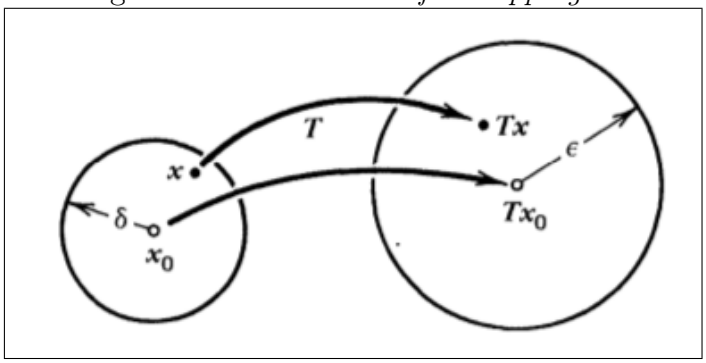
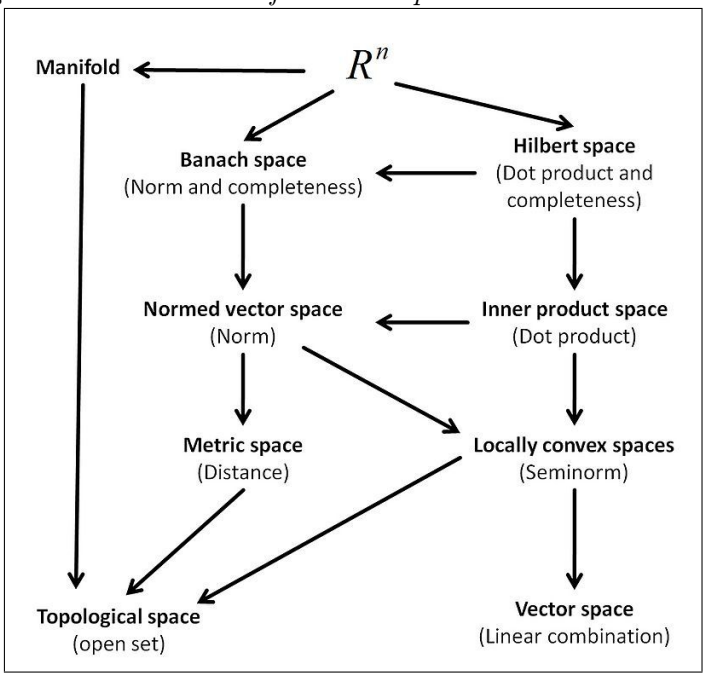


Figure 1.3: *Overview of various spaces and their relations*



Chapter 2

Automatic Speech Recognition

2.1 Introduction

The aim of the work on Automatic Speech Recognition (*ASR*) is to build systems that recognize spoken speech, that is, they are able to map acoustic signals to strings of words. In contrast to natural language processing, ASR does not try to determine the meaning behind speech or find the multiple meanings of words; it merely tries to recognize *which* words were spoken.

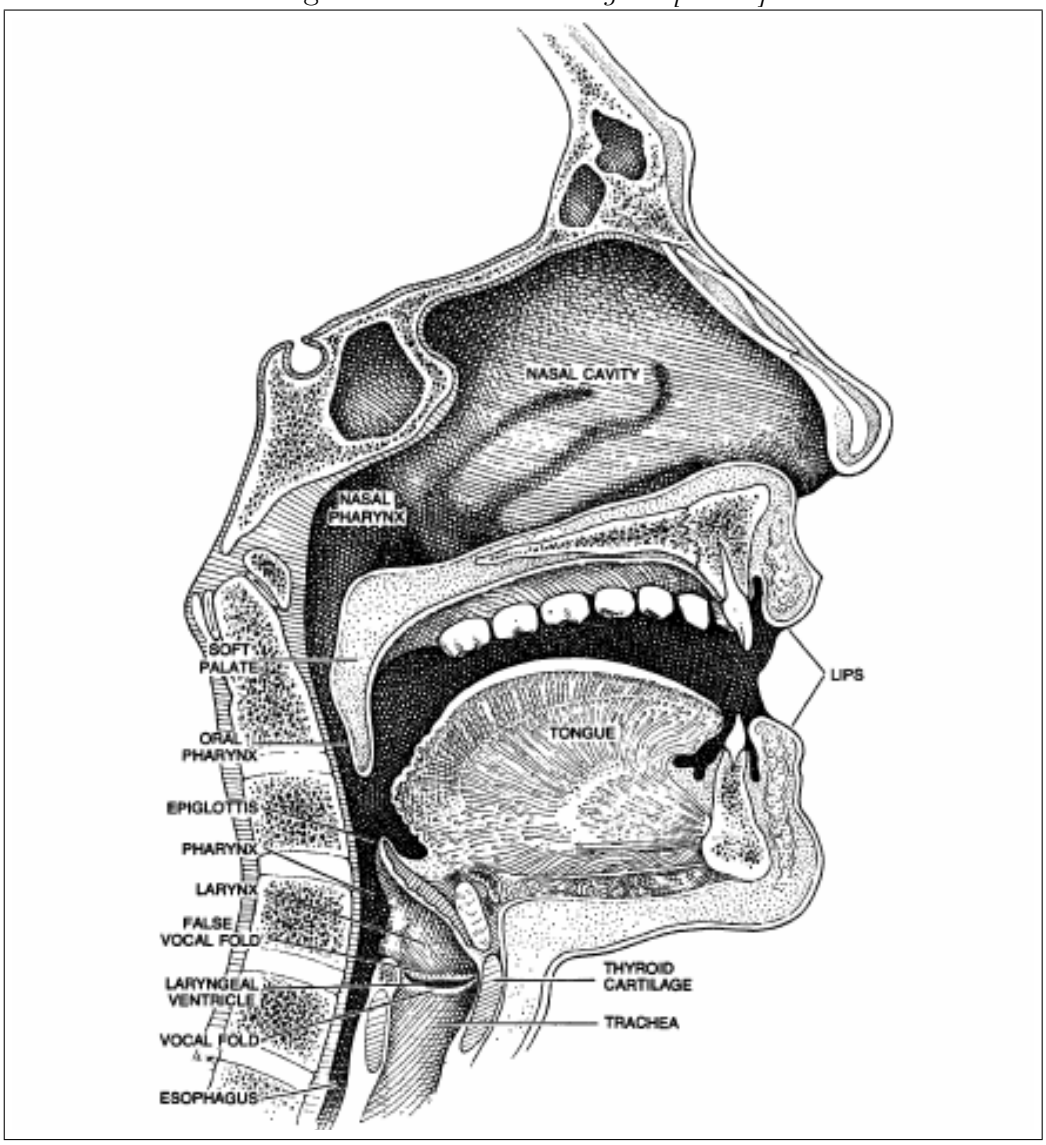
Research in ASR has come a long way in the last few years, which has allowed us to take advantage of it in multiple areas with very satisfying results: human-computer interaction (speech-only or multimodal interfaces), telephony (information passing, call routing) and dictation are examples where ASR systems can perform well irrespective of the speaker or their environment.

We will begin by providing a quick description of the way human speech is produced and move on to present the concept and mechanisms behind Automatic Speech Recognition.

2.2 Fundamentals of the speech production mechanism

[JM09] Sound is produced by the rapid movement of air from the lungs through the "windpipe", also called the *trachea* and out of the mouth or nose. As it flows through the trachea, it passes through the *larynx* (or *Adam's apple*) and there it affects the position of two small folds of muscle which are known as *vocal folds* or *vocal chords*. The possible movements of these muscles are either moving closer together or apart; the space between them is called the *glottis*. If the vocal folds are close together they will vibrate as air passes through the glottis and produce sounds

Figure 2.1: *The vocal organs.[JM09]*



that are known as *voiced* sounds. The vowels are examples of voiced sounds. On the other hand, if they are far apart they will not vibrate and the sounds produced are called *unvoiced*. Examples of unvoiced sounds are [p], [t] and [k].

After passing through the trachea and before exiting the body, air passes through the vocal tract, which consists of the *nasal* and *oral* tract. The vocal tract will act as a filter on the speech signal and the output of the filter will be the sound we will produce. The speech signal will vary according to what obstacles the air meets on its way out : the tongue, the lips or the teeth. These obstacles will define the vocal tract filter applied on the speech signal.

2.3 Design parameters of an ASR system

This section attempts to provide a short introduction into Automatic Speech Recognition and the most common architectures behind ASR systems[JM09].

However, before deciding on the architecture of a speech recognition system, one has to take into account its application domain.The most decisive parameters are:

- The vocabulary size,which is the number of distinct words that the system should be able to recognize. Few words imply relatively easy set-up and training of the system, whereas systems recognizing thousands of words, as in a broadcasting news vocabulary, are more complicated and harder to train.

- The fluency of the speech that the system will be asked to recognize.This includes whether the speech will be continuous or just isolated words as well as the speed and clarity of the speaker.Isolated word recognition systems, e.g. recognizing commands to a computer, are easier than ASR systems for continuous, conversational speech, e.g. a telephone conversation between humans.

- The environment where the recognition might have to take place. Systems designed to perform well in noisy environments with high distortion in the speech recorded are more demanding than systems that can clearly capture the speech for recognition in an isolated environment.

- The speaker variability. Speech recognition is easier if the system is expected to recognize the speech of a limited number of people. On the contrary, a general ASR system that should work with any speaker, regardless of sex, age, or accent is much more difficult to implement.

2.4 ASR system architecture

As mentioned in the introduction, the problem of ASR is, in principle, a structured sequence classification task, where a (relatively long) sequence of acoustic data is used to infer a (relatively short) sequence of the linguistic units such as words. Modern ASR systems use the model of a *noisy channel* to deal with the classification task. The idea behind this model is to think of the input signal as a distorted version of the corresponding words, which was produced as they passed through a noisy communications channel. If we manage to understand how the channel affected the signal, we can then match it to the original, noise-free set of words, by passing every acceptable -by the grammar of the language- sentence though the channel to get its distorted version and see which matches best the initial input signal.

Figure 2.2: *The noisy channel model. The decoder searches through all possible sentences and finds the one that after passing through the channel best matches the initial input signal[JM09].*

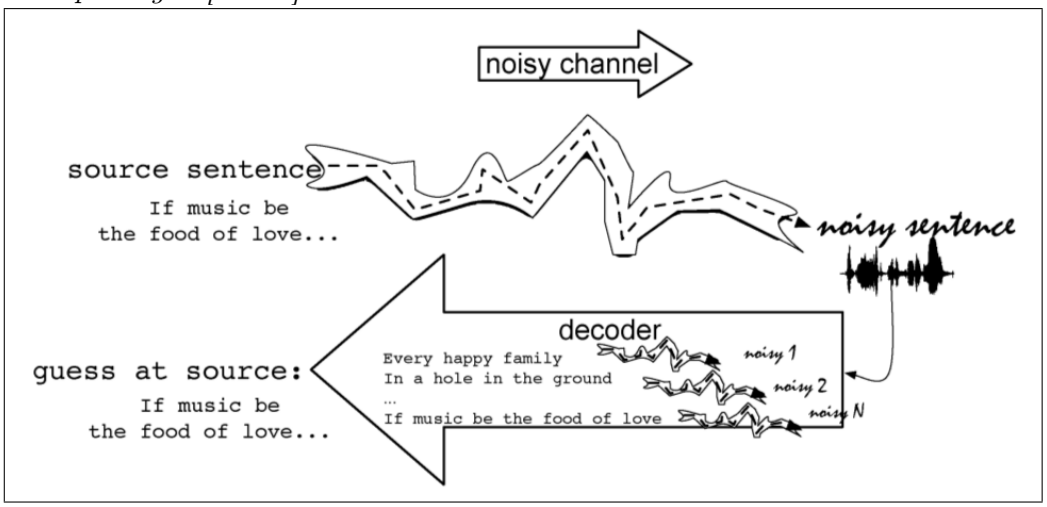
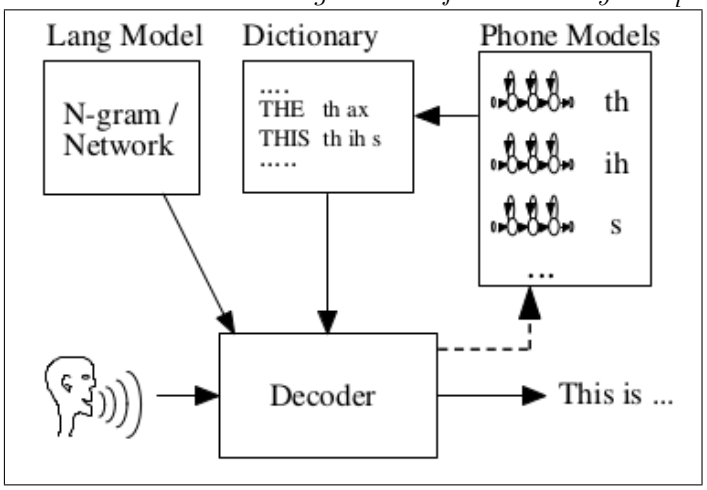


Figure 2.3: *The main building blocks of an ASR system[You02].*



In order to achieve the modelling of the noisy channel and the subsequent decoding of a new acoustic signal, we will need to have at our disposal the following tools: the prior probability of each sentence of the language, the probability of words being the concatenation of certain speech units and the probability of these speech units being realised as acoustic or spectral features, which are drawn from the input signal.

These tools define the main components of a modern automatic speech recognition system, which will be presented below, following the formulation of the computational/mathematical problem of Automatic Speech Recognition.

2.4.1 Computational formulation of ASR

We will now use mathematical notation and probability theory to answer the basic question in ASR: ''What is the most likely sentence Ŵ out of all sentences belonging to language $\mathbb{L}$ given an acoustic signal O? ''.

The acoustic input signal O is a sequence of observations o_i,

$$O = o_1, o_2, o_3, ...o_t$$

each one representing features of a specific part of the input speech, which is usually split into overlapping parts of a certain duration (*frames*). In the same way, if we treat each sentence of the language as a string of words,

$$W = w_1, w_2, w_3, ...w_n$$

we can write the answer to the ASR question in the following way:

$$\hat{W} = \underset{W \in \mathbb{L}}{\operatorname{argmax}} P(W|O)$$

, which we cannot compute directly. However, if we apply Bayes' rule, the equation takes the following form:

$$\hat{W} = \underset{W \in \mathbb{L}}{\operatorname{argmax}} \frac{P(O|W)P(W)}{P(O)}$$

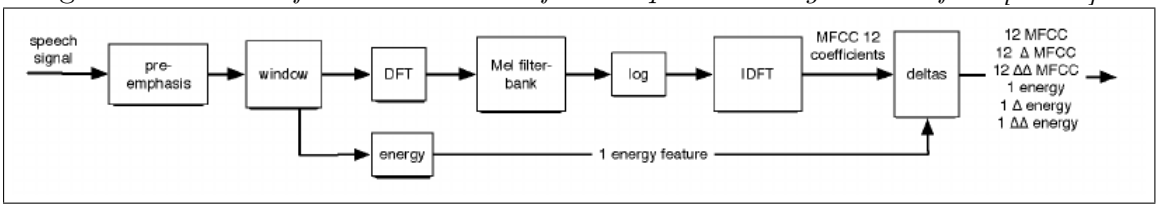
We can simplify the computation even further, if we consider that the probability of the observation in the denominator, *P(O)*, does not affect the maximization with respect to *W*, since the observation signal stays the same as we search over the sentences space. Consequently, the answer to our problem can be computed by the following form:

$$\hat{W} = \underset{W \in \mathbb{L}}{\operatorname{argmax}} P(O|W)P(W)$$

The two probabilities on the right hand side of the equation, represent the tools we need to address the recognition problem, as we have mentioned above: *P(W)*, which is the prior probability of each sentence of the language, is computed by the *language model*, whereas $P(O|W)$, which includes the probability of words being the concatenation of certain speech units and the probability of these speech units being realized as certain features, is calculated by the *acoustic model*.

Having presented the computational formulation of the automatic speech recognition problem, we can move on to present the required steps to be taken and models to be constructed in order to build a speech recognition system.

Figure 2.4: *MFCC feature extraction from a quantized digital waveform[JM09].*



2.4.2 Feature Extraction

[JM09] The first issue we have to address is how and in what format do we "insert" the speech waveform into our system. This process, known as *feature extraction* results in the extraction from the speech waveform of a sequence of acoustic *feature vectors*, each of them representing the information included in a small time window of the signal. These feature vectors are further preprocessed and finally presented as the input to the ASR system.

Mel Frequency Cepstral Coefficients

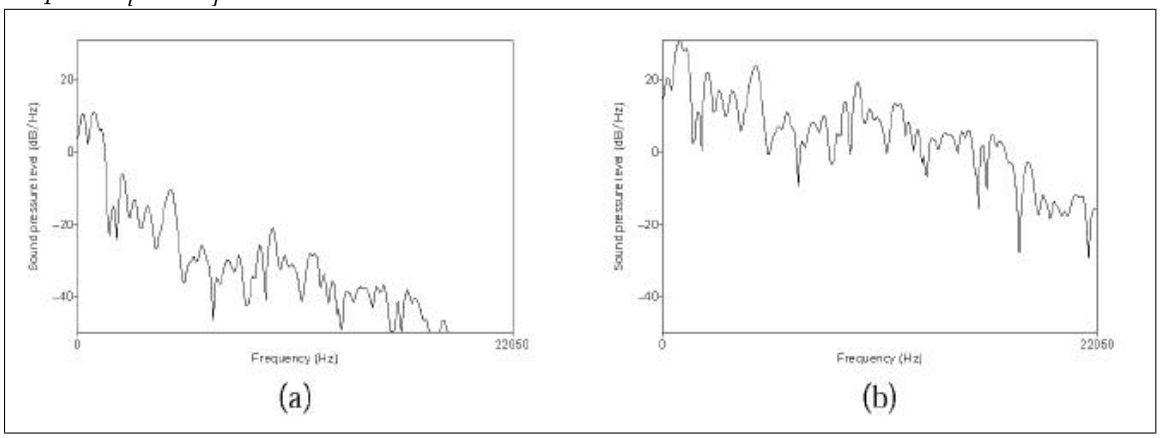
Although research in feature extraction moves towards using the raw waveform directly as input to the system, so far the most common feature representation in speech recognition systems is the Mel Frequency Cepstral Coefficients (*MFCC*).The steps involved in extracting the MFCC feature vectors follow the analog to digital conversion of the speech signal (sampling and quantization) and are outlined in the next paragraphs:

Pre-emphasis In the pre-emphasis step we want to amplify the amount of energy in the high frequencies of the signal. If we take a look at the spectrum of a vowel we will note a drop in energy as we move on to higher frequencies; this is known as *spectral tilt* and is due to the nature of the glottal pulse. **(EXPLAIN STH????)** Amplifying the energy of higher frequencies will improve phone detection accuracy as it will provide more information to the acoustic model coming from the boosted higher formants. In essence, pre-emphasis is applying to the signal a first order, high-pass filter whose equation is:

$$y[n] = x[n] - \alpha x[n-1],\ 0.9 \leq \alpha \leq 1.0$$

Windowing Given that the feature vectors we want to extract will be used to train phone classifiers, i.e. the acoustic model, we want them to able to capture the spectral properties corresponding to these fundamental speech units.Consequently, since speech is a non-stationary signal - its statistical properties are not constant

Figure 2.5: *Part of the spectrum of vowel [aa] before (a) and after (b) pre-emphasis[JM09].*



across time - we extract the feature vectors from a small window of the speech signal that corresponds to a phone or subphone, where the signal can be considered stationary - its statistical properties stay constant across time.

The windowing process comes down to applying a filter to the signal that is non-zero inside some region and zero elsewhere, moving this filter along the speech signal and extracting segments of the signal (or *frames*).The window is characterized by its *width* (in milliseconds, also called *frame size*), the *overlap* between successive windows (usually a percentage of the width) and its *shape*, e.g. rectangular, Hamming etc. The Hamming window is usually preferred over the rectangular as it gradually reduces the signal values at the boundaries of the window towards zero, thus avoiding discontinuities which cause problems during the next step of feature extraction (Fourier analysis):

$$w_h amming[n] = \begin{cases} 0.54 - 0.46\cos(\frac{2\pi n}{L}),\ 0 \leq n \leq L-1 \\ 0\ otherwise \end{cases}$$

The application of the filter is an element-wise multiplication of the signal values at each time step n with the values of the window:

$$y[n] = w[n]s[n]$$

where w is the window and s the signal.

Fourier Analysis The next step in the feature extraction process is to acquire the spectral information included in the windowed segments. The tool to extract such information, e.g. the amount of energy included in different discrete frequency bands of each segment, is the Discrete Fourier Transform.Given a part of a signal, the DFT will produce a complex number representing the magnitude and phase of

Figure 2.6: *Rectangular and Hamming windows and their effect on the signal[JM09].*

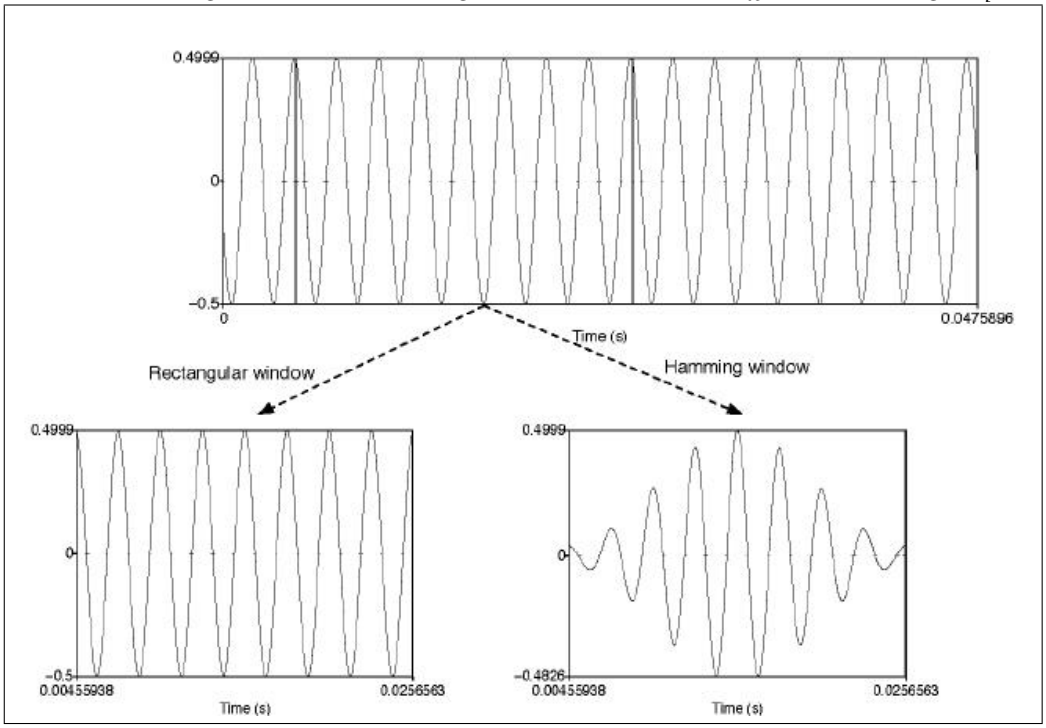


Figure 2.7: *Windowing process with a rectangular window.After a figure by Brian Pellom[JM09].*

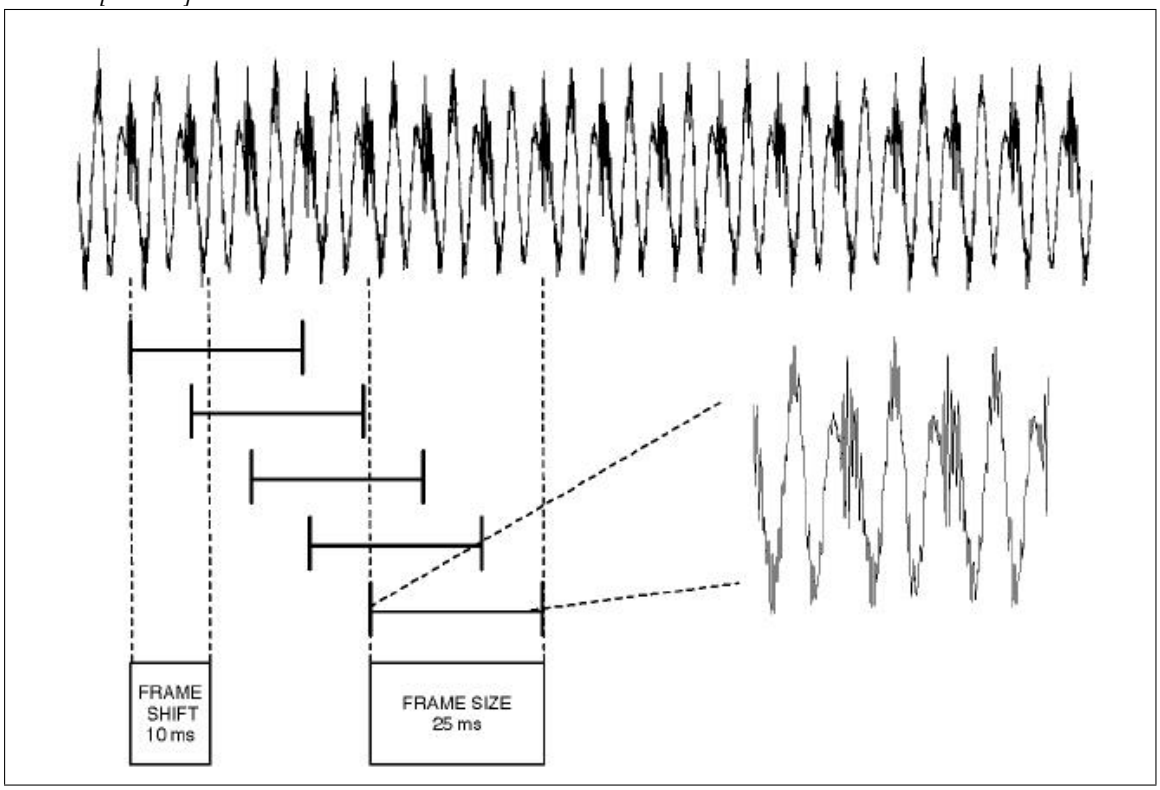
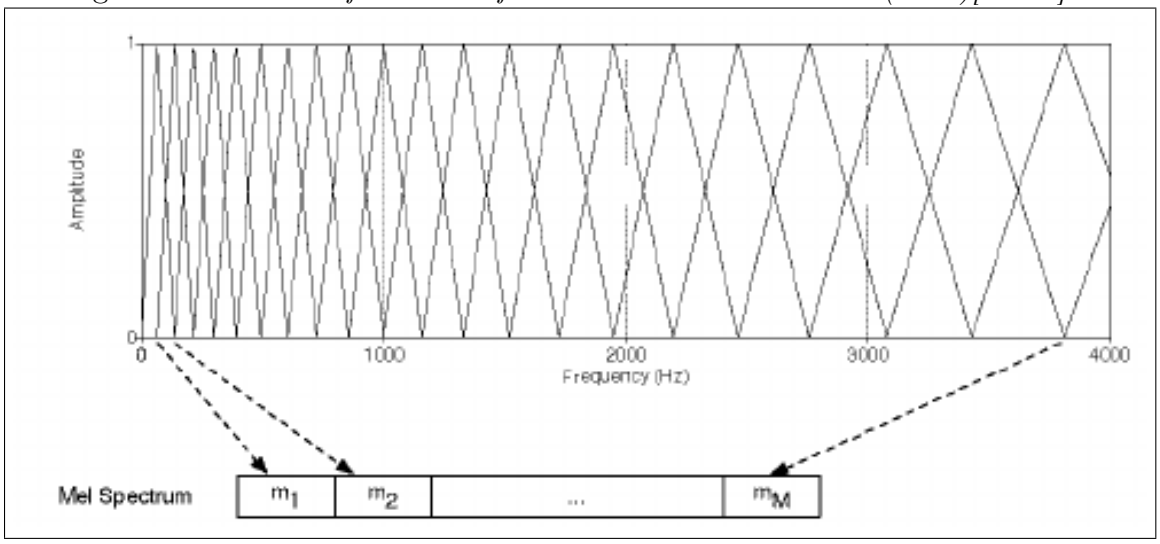


Figure 2.8: *The mel filterbank after Davis and Mermelstein (1980)[JM09].*



each frequency component of the corresponding segment.

Mel Filterbank Research has shown that the human hearing is not equally sensitive at all frequency bands - it is less sensitive at higher frequencies (above 1000 Hz).Moreover, humans are less sensitive to differences in amplitude at high amplitudes than at low amplitudes. Consequently, if we model this information and introduce it into the ASR system, we will improve its performance.

The way we take advantage of this information during the feature extraction process is by passing the DFT of the windowed signal parts through an array of triangular filters (a *filterbank*), which collect energy from each frequency band. These filters have their center frequencies spread on a *mel* scale, that is, 10 of them are spaced linearly below 1000 Hz and the remaining filters of the bank are spaced logarithmically above 1000 Hz. A *mel* is a unit of pitch defined so that pairs of sounds which are perceptually equidistant in pitch are separated by an equal number of mels[SVN37].The mapping between frequency in Hz and the mel scale is described by the relation:

$$mel(f) = 1127 \ln(1 + \frac{f}{700})$$

Finally, having passed the DFT of the signal through the filterbank, the final step is to take the logarithm of the mel spectrum values, which is a form of normalization to make the feature estimates less sensitive to variations in input.

The Cepstrum The next step in the MFCC feature extraction process is the computation of the *cepstrum*.As mentioned before, the speech waveform is created when a glottal source waveform is passed through the vocal tract which acts as a filter. As we have already mentioned, the shape of the vocal tract will determine the

Figure 2.9: *Effect of vocal tract on source signal.Tomi H. Kinnunen, Speech Technology Workshop*

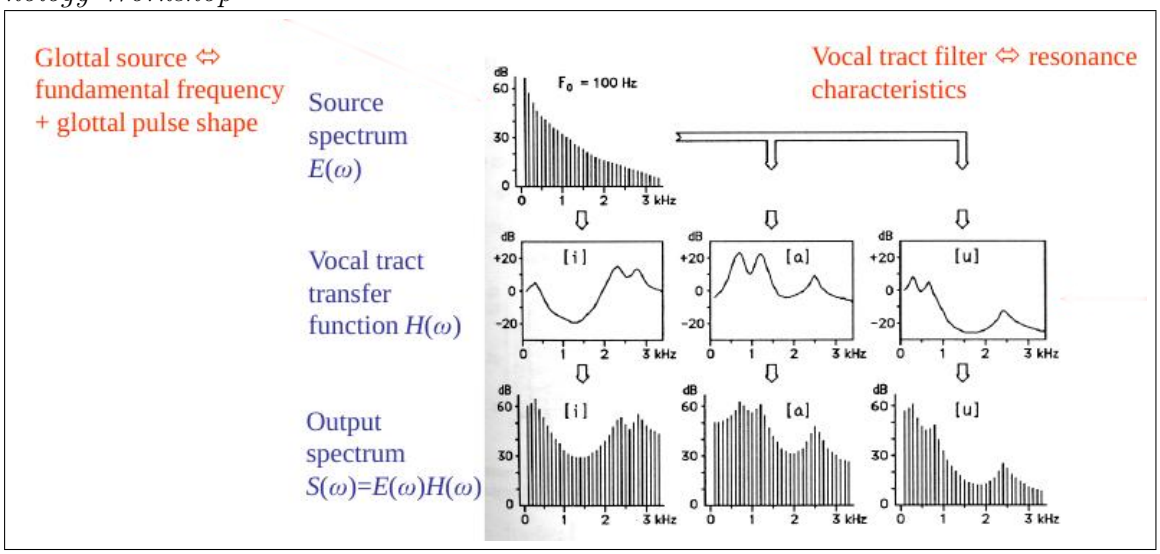
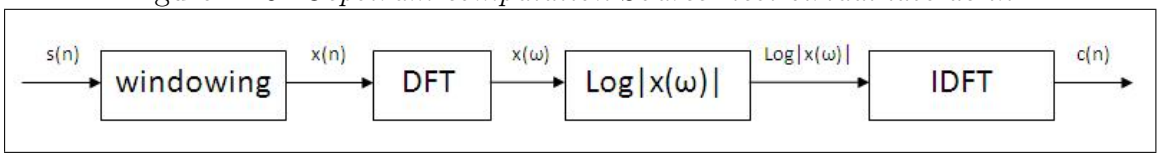


Figure 2.10: *Cepstrum computation.Source: test.virtual-labs.ac.in*

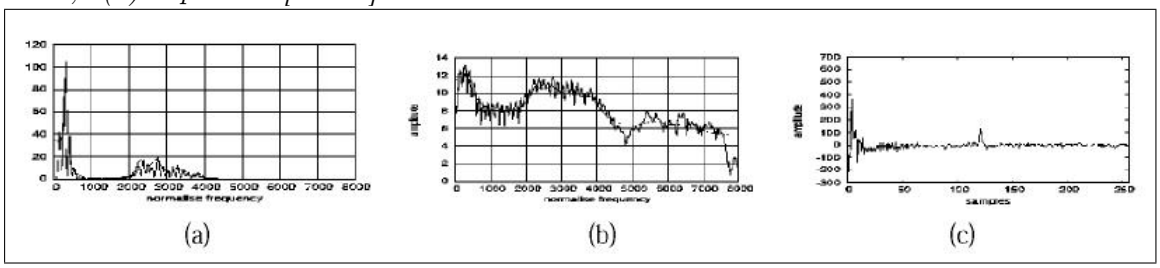


outcome of this filtering process, i.e. the sound, the phone that will be produced.

Therefore we aim to have some information about the vocal tract inserted into the feature vectors. The cepstrum provides us with a way of separating the glottal source from the vocal tract filter.

The peaks at lower values on the x-axis in the cepstrum of Figure 9 (c) correspond to the vocal tract characteristics whereas peaks at higher values correspond to the glottal source. Therefore, since we need information about the way a phone was produced, i.e. about the vocal tract shape, for the feature vectors we want to extract, we will keep a few of the first cepstral values (usually 12). Furthermore, an

Figure 2.11: *Cepstrum example: (a) magnitude spectrum, (b) log magnitude spectrum, (c) cepstrum.[JM09].*



important property of the cepstral coefficients is that their variance is uncorrelated, contrary to spectral coefficients, which are correlated at different frequency bands. This is extremely important for acoustic models based on Gaussian Mixture Models, as it allows us to keep the number of their parameters low.

Feature vectors The cepstral coefficients extracted from the previous process are just a part of the feature vectors. They are further enhanced by adding a few extra coefficients that provide more information helpful towards determining speech units such as phones.

The first extra piece of information we include by adding one more coefficient is the energy of the frame, which is defined as:

$$Energy = \sum_{t=t_1}^{t_2} s^2[t],$$

where s is the speech signal and t_1, t_2 are the boundaries of the frame.

Energy is useful for phone detection since it correlates with phone identity: higher energy reveals the presence of e.g. a vowel whereas very low energy could identify a pause in speech.

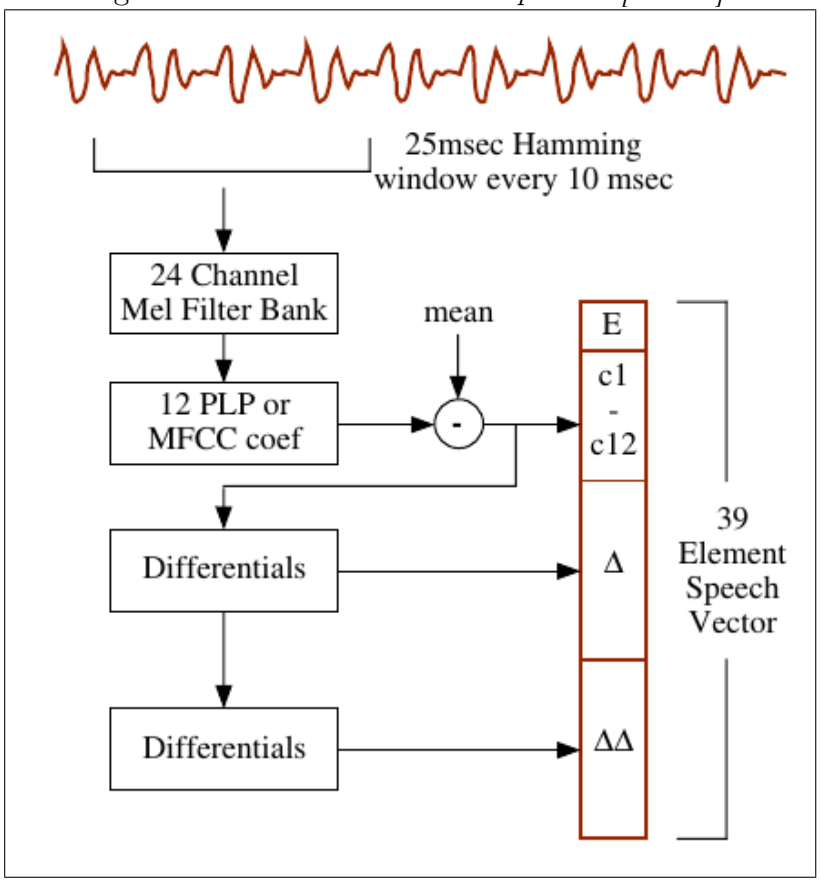
Considering that speech properties change from frame to frame it is reasonable to expect that capturing these changes would provide more information about the nature of the speech signal. This motivated the use of *delta* and *delta-delta* coefficients for each one of the cepstral coefficients and the energy. The *delta* coefficients capture the change of the corresponding feature between successive frames whereas the *delta-delta* coefficients capture the change of the *delta* features between successive frames. The simplest way to compute these *delta* and *delta-delta* coefficients is by taking the difference of the corresponding features between successive frames:

$$d(t) = \frac{c(t+1) - c(t-1)}{2},$$

where c is the cepstral feature with delta coefficient d at time t.

This concludes the construction of the feature vectors.As mentioned before, we usually pick 12 cepstral coefficients plus an energy coefficient, thus we will have 39-dimensional feature vectors: 13 + 13 *delta* +13 *delta-delta* coefficients.It is also common to concatenate the cepstral coefficients to produce higher dimensional vectors.This way we manage to include contextual information in the feature vectors.To deal with computational problems arising from the increase in the number of dimensions, various dimensionality reduction techniques are used, from simple ones such as Linear Discriminant Analysis or Principal Component Analysis, to more sophisticated such as techniques aiming to discover the lower dimensional manifold on which the feature vectors lie.

Figure 2.12: *Feature extraction process.[You02].*



2.4.3 The Language Model

The language model (LM) expresses how likely a given string of words is, taken into consideration certain linguistic constraints. In order to do this, we build on the idea of predicting the next word in a sequence of words, which is formalized with probabilistic models called *N-gram models*.

N-grams

An *N-gram* is a sequence of N words, e.g. a 2-gram or bigram is a sequence of two words, a 3-gram or trigram is a sequence of three words etc. An *N-gram model* is a probabilistic model which computes the N^{th} word of a sequence of N words given the previous N-1.

The power of N-grams becomes evident in areas such as speech or handwriting recognition, machine translation, spelling correction and natural language processing tasks. What all these areas have in common is that they might have to deal with noisy or ambiguous input. N-grams can deal with ambiguity by assigning a higher likelihood to word sequences that are valid according to the language constraints.

Since these models are capable of assigning a conditional probability to the next possible word in a group, we can exploit them to compute the joint probability of a sequence of words,i.e. a sentence, which is what we were aiming to from the beginning.

Suppose we have a sequence of n words $W = w_1, w_2, w_3, ..., w_n$. Then, the probability *P(W)* can be computed in the following way:

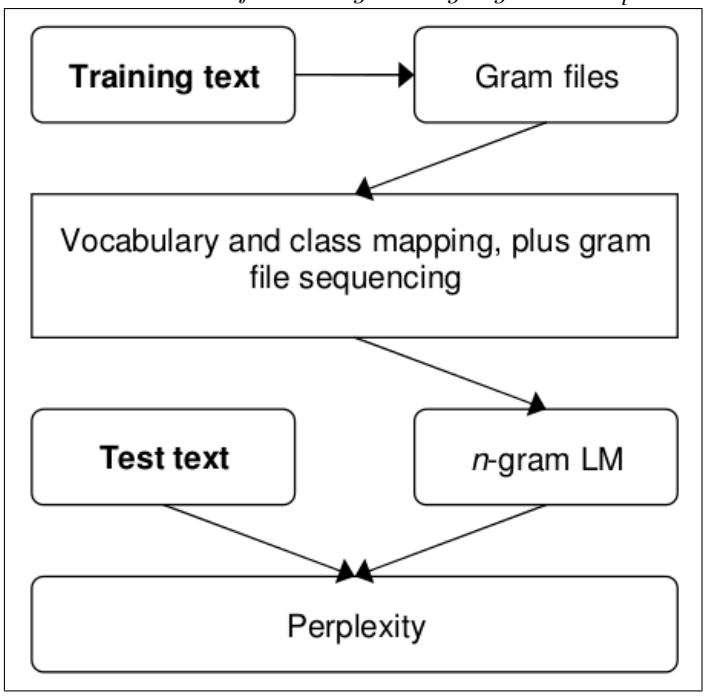
$$P(W) = P(w_1, w_2, w_3, ..., w_n) = \prod_{i=1}^{n} P(w_i|w_1, w_2, w_3, ..., w_{i-1})$$

As this computation for every possible word sequence in the language is very difficult if not impossible, we make the assumption that the i^{th} word depends only on the previous N-1 words (its *history*). At this point we can take advantage of the intuition behind the N-gram model:

$$P(W) = \prod_{i=1}^{n} P(w_i|w_1, w_2, w_3, ..., w_{i-1}) \approx \prod_{i=1}^{n} P(w_i|w_{i-N+1}, ..., w_{i-1})$$

to approximately compute the probability of the sequence. The expressions on the two sides would be exactly equal for sufficiently high n and if the language were ergodic, that is, the probability of any word could be estimated from sufficient history independent of the starting conditions.

Figure 2.13: *Process of building a language model.[YEK+02].*



Building a language model based on N-grams

[YEK+02]

Considering the intuition of N-grams and assuming that the probability of an N-gram occurring in an unknown text can be estimated from its frequency in a given training text, we can build language models based on N-grams.

The construction of such an LM can be broken down into three stages:

- Collect and store the N-grams of the training text (*corpus*)
- Possibly map some words into classes, e.g. out-of-vocabulary class mapping ???????????
- Count the N-grams and compute the N-gram probabilities

The last step of computing the probabilities is based on maximum likelihood estimation:

$$\hat{P}(w_i|w_{i-N+1},...,w_{i-1}) = \frac{C(w_{i-N+1},...,w_i)}{C(w_{i-N+1},...,w_{i-1})}$$

where *C(.)* is the count of a given word sequence extracted from the training text.

When building a language model based on N-grams, there are several factors one has to take into account before they decide on N. Resources constraints (e.g. storage) and size of the *vocabulary* (i.e. the set of distinct words in the language) will play a major role in deciding on N, as the number of parameters of the model grows

exponentially with N : $|V|^N$, where V is the vocabulary. However, storage needs because of parameters' size are lower than one would expect, because not all N-word combinations are acceptable/valid sequences in the language. What increases storage and computational needs however, is the large training sets required, so that our model estimates parameters with a minimum acceptable degree of confidence. Apart from processing, also acquiring these training sets might be difficult, especially for domain-specific applications, where training sets have to be specifically constructed.

Data sparsity and smoothing

[JM09] However, since the training set will always be finite, one can never have a sufficient number of N-grams for every valid N word sequence of the language. This issue faced when developing LMs is known as *data sparsity* and the technique used to deal with it is called *smoothing*. Smoothing aims to increase the robustness of the language model by redistributing the probability mass assigned by the maximum likelihood estimates: it removes some of it from higher counts of N-grams and assigns it to very low or zero counts, in order to ''smooth'' the distribution.

Laplace Smoothing. *Laplace* or *add-one* smoothing is the simplest form of smoothing : we just add one to all the counts. Before we compute the ML probabilities, one has to to take into account the extra $|V|$ ''words'' that we added, in order to maintain the sum of all probabilities equal to 1. As this last step is necessary and because Laplace smoothing does not perform well, it is more convenient to use an *adjusted count*

$$c_i^* = (c_i + 1)\frac{K}{K+V}$$

where c_i is the original count and K is the number of word tokens.

Discounting. Instead of adding the same amount of probability mass to all N-gram probabilities, a different approach would be to remove some mass out of the higher counts and assign it to lower ones. We can therefore define a *relative discount* as the ratio of the new counts and the originals:

$$d_c = \frac{c^*}{c}$$

One algorithm applying discounting, is the *Good-Turing discounting*.The intuition behind it is that we use the MLE of N-grams occurring c+1 times in the training set, to define the MLE of N-grams occurring c times. The new smooth count c^* is thus defined as:

$$c^* = (c+1)\frac{N_{c+1}}{N_c}$$

where N_i is the number of different N-grams occurring i times in the training set.

Back-off and interpolation. Discounting techniques allow us to distribute some probability mass equally to the unseen events. However, we can distribute it fairer if we take into consideration information from lower or higher order N-grams. This is the idea of *back-off* smoothing. In particular, *Katz back-off* always resorts to the (i-1)-gram if the i-gram has zero counts (i starting from our originally selected N). From a different point of view, we back-off to lower order N-grams only if we have zero evidence for a higher order one. On the other hand, *interpolation* deals with zero counts by summing estimates of all N-grams using weights (e.g. interpolate the estimates of unigrams,bigrams and trigrams).

Language model evaluation

Given the numerous applications of N-grams and LMs as well as their inherent drawbacks (static and finite vocabulary, finite training sets, more N-grams than can ever be collected and utilized), it is evident that we must have a way to compare LMs and evaluate their performance.

The most obvious way to compare two different language models would be to use them in our application and see which gives the best results. However, this way is expensive and time consuming as it is based on training and evaluating systems using huge speech datasets.

Another way to compare two LMs independent of the application, is to use the *perplexity* metric. The perplexity of an LM on a test set is a function of the probability that the LM assigns on it and is defined as:

$$PP(W) = P(w_1 w_2 w_3 ... w_N)^{-\frac{1}{N}}$$

where $W = w_1 w_2 w_3 ... w_N$ is the test set.

The best LM would be the one that has the minimum perplexity, since that would mean that it maximizes the test set probability, i.e. it better predicts the details of the test set. Another way to look at perplexity is as the weighted average branching factor of the language, that is, the number of best possible words following a word sequence. The smaller that number is, the better the work of the LM on coping with the ambiguity of the language.

Finally, when comparing language models using the perplexity metric, one should take care to use the *same vocabulary* for both LMs and evaluate them on the same test set which will be presented to the system for the first time during evaluation.

Recent advances in language modeling

In a paper presented recently [BDVJ03], a novel approach to building language models was presented which takes advantage of neural networks. The motivation

behind using NNs to build language models is based on the following problems associated with N-gram models:

- a huge amount of training data is needed to train LMs which will still have limited context capabilities (1-2 words)
- N-gram models ignore word similarity, which makes generalizing difficult

To deal with these issues becomes even more important when one considers that language models are probabilistic models using discrete random variables (words) which largely increases the amount of free parameters they need.

The writers suggest using distributed representations of words (i.e. real-valued feature vectors) which allows them 1) to identify similarities between words, since similar words will have similar feature vectors and 2) to exploit the smooth probability function modeled by a neural network in order to generalize: in this way, each vector representing a word will be able to provide information about a huge number of similar words, i.e. its "neighbors" in the feature space.In their work they present a neural network which simultaneously learns its parameters and the feature vectors associated with each word in the training set, and most importantly, the number of parameters it uses scales linearly with the vocabulary and context size. Due to the high computational cost of the training (higher than N-gram based models), they use parallel methods to efficiently train the model.

In recent years the use of neural networks for language modeling has included using recurrent neural networks which can take advantage of arbitrarily long contexts for each word (like humans do), something that was not possible with feedforward NNs [MKB+10].

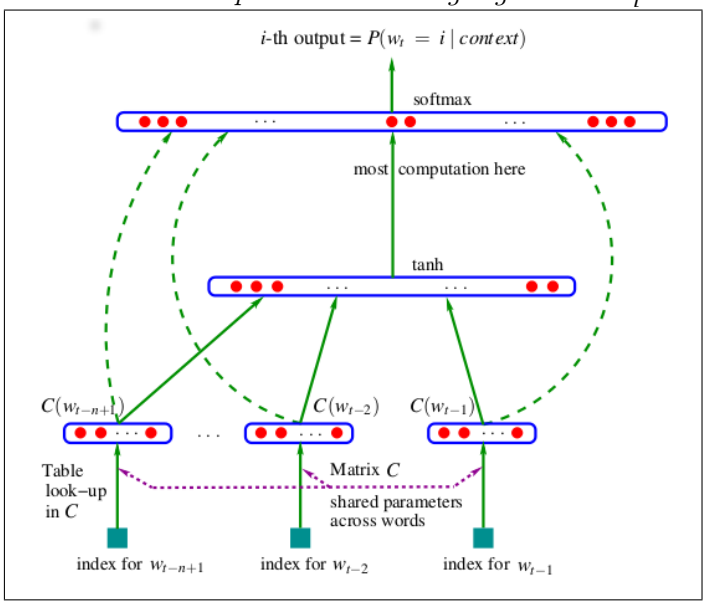
2.4.4 The Acoustic Model

[You02]

According to the computational formulation of the ASR problem, we need the likelihood of the observed data (i.e. the acoustic signal) given the word sequence $P(O|W)$.However, it would be impractical and inefficient if we tried to compute this likelihood by building a separate model for each word in the language, since sub-word units are shared among different words. Instead, as mentioned before, the *acoustic model* calculates the probability of words being the concatenation of certain speech units and combines it with the probability of these speech units being realized as certain features, to produce the desired likelihood for a word.

We will first present some basic notions in acoustic modeling and then we will go into more details about this important part of the system.

Figure 2.14: *A neural probabilistic language model .[BDVJ03].*



Phones

The basic unit of speech analysis we use, is the *phone* which is the smallest identifiable unit we find in a stream of speech. The sequence of phones that constitutes each word in the training dataset is determined by a *pronouncing dictionary*. Using a phone sequence to represent each word makes it easy to add new words in the dataset just by adding them and their phone sequence to the dictionary.

Context-dependent phones

Given that there are thousands of words in a language, but just tens of phones (e.g. 44 in the English language) the computational and storage gain acquired from the use of phones as the basis of the acoustic model becomes immediately apparent. However, contextual effects like co-articulation cause large variations in the way that different sounds are produced even if in principle they correspond to the same phone. Hence, to achieve good phonetic discrimination, we build *context-dependent* phone models, with the most common being the *triphone*: for each phone there is a different model for every unique pair of left and right neighboring phones.

There are two dominant triphone models:

- *cross-word triphones*, which include phones of the previous and following words in the first and last triphones of the word of interest:

$$ten\ pots \rightarrow sil\ {}_{sil}t_e\ {}_te_n\ {}_en_p\ {}_np_o\ {}_po_t\ {}_ot_s\ {}_ts_{sil}\ sil$$

The advantage of this approach is that they model co-articulation across word boundaries, but on the other hand, they complicate the decoding process since the phone models of each word depend on the following and preceding words as well.

- *word-internal triphones*, which explicitly encode word boundaries, thus making decoding easier:

$$ten\ pots \rightarrow sil\ {}_{sil}t_e\ {}_te_n\ {}_en_-\ {}_-p_o\ {}_po_t\ {}_ot_s\ {}_ts_{sil}\ sil$$

State of the art systems use mostly cross word triphones because of their ability to model contextual effects.

As a consequence, the number of distinct triphones greatly increases and the number of parameters for such systems can grow up to hundreds of millions, while at the same time we have too little training data in our disposal. In addition, we might have unseen triphones appearing in evaluation tasks. To deal with these problems we have developed smoothing techniques, just as was the case with language models.

Smoothing techniques

Back-off and interpolation. When too little data is available for the training of a context-dependent model of a particular order, one can instead use a model of lower order at the expense of some inaccuracy in the modeling of the context: e.g. use a biphone or a context-independent phone (monophone) when we cannot use a triphone. In order to implement a more robust model one can use a weighted combination of models with various levels of context dependency (*interpolation*).

Parameter tying. An alternative that offers a greater degree of flexibility while maintaining the high level of context-dependency in the model, is the technique of *parameter-tying*, in which parameters of context-dependent phone models that are acoustically indistinguishable are tied together, to facilitate training in case there is little training data available.Before tying parameters together one has to apply some form of clustering to build the groups of phone models that will share their parameters.In practice, the most commonly used clustering technique is the *phonetic desicion tree*, where a binary tree is build for each phone model and its leaves contain the parameters to be shared.

Acoustic Modeling with Gaussian Mixture and Hidden Markov models

State-of-the-art ASR systems use Hidden Markov Models to represent each phone in conjunction with Gaussian Mixture Models to determine the probability that an

acoustic observation was produced by a certain phone. The high representational capabilities and ease of training of these models are what has made them prevalent in ASR. We will present the GMM/HMM acoustic model and in the next chapters we will examine their most recent competitor, that is , deep neural networks.

Gaussian Mixture Models.[You02][YD14]

Multivariate Gaussian random variables and Mixture Models.

Given that the feature vector corresponding to an acoustic observation is multi-dimensional (usually 39-dimensional as was presented above), we will have to treat it as a multivariate random variable and use a multivariate distribution to assign a probability to it.The reason why we choose the Gaussian distribution is not only its desirable computational properties but also its ability to approximate many real-world data (owing to the law of large numbers), such as speech features.

Supposing that $\boldsymbol{\Sigma}$ is the co-variance matrix, $\boldsymbol{\mu}$ is the mean vector and d is the number of dimensions of the feature vector, the multivariate Gaussian distribution is defined as:

$$p(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{d/2}\,|\boldsymbol{\Sigma}|^{1/2}} \exp(-1/2(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\vec{x}-\boldsymbol{\mu}) = \mathcal{N}(x|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

However, because of the inherent multimodality of the speech features, a single Gaussian distribution is insufficient to describe them. Therefore, we use a *mixture* of Gaussian distributions:

$$p(\boldsymbol{x}) = \sum_{i=1}^{M} c_i \mathcal{N}(x|\boldsymbol{\mu_i}, \boldsymbol{\Sigma_i})$$

where c_i are the positive mixture weights and $\sum_{i=1}^{M} c_i = 1$.Usually, the number of the mixture components is chosen according to the nature of the problem and the information we have about the data.The variability and multimodality of the features may be due to multiple latent causes; provided we can identify these underlying causes we can match each one to the corresponding mixture component in the distribution.

As mentioned earlier, the Gaussian distribution is favorable both for its modeling and its computational properties. GMMs can model complex, multimodal distributions to any required level of accuracy and they can be trained using standard maximum likelihood approaches.Their attractiveness is also due to research into GMM training having come up with approaches to optimize the trade-off between their modeling effectiveness and the amount of training time and data needed. For

example, we have the ability to reduce the number of free parameters (from M $\times d^2$ down to M) while still achieving high performance, if instead of using full co-variance matrices we opt for diagonal Σ or even use the same matrix for all mixture components. The use of diagonal co-variance matrices has been thought to impose uncorrelatedness among features, but, given that a mixture of Gaussians with diagonal Σ can at least effectively describe the correlations modeled by a single full co-variance Gaussian, this thought has been misleading.

Specifically for speech recognition, a number of ways has been proposed to improve recognition accuracy of a GMM system.We can discriminatively train the system after the generative maximum-likelihood training, so that we maximize the probability of generating the observed speech features in the training data, or augment the input speech features with bottleneck features acquired using neural networks (the latter will be examined later on in this project).

The set of free parameters to be estimated for a Gaussian-mixture distribution is denoted by $\boldsymbol{\Theta}$ and consists of : $\{c_i, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i\}$. In order to acquire the parameters we rely on maximum likelihood methods and in particular on the Expectation-Maximization (EM) algorithm [DLR77]. The EM algorithm is used to find locally maximum likelihood parameter estimates of statistical models when the equations cannot be solved directly.It is especially useful for models involving latent variables apart from the parameters-to-be-estimated and the observable data. A GMM can be treated as such a model if we assume that each observable data point has a corresponding hidden data point specifying the component of the mixture that each point belongs to.Furthermore, the EM algorithm provides us with closed-form expressions for the computation of the estimates in the M-step:

$$c_i^{(j+1)} = \frac{1}{N}\sum_{t=1}^{N} h_i^{(j)}(t),$$

$$\boldsymbol{\mu}_i^{(j+1)} = \frac{\sum_{t=1}^{N} h_i^{(j)}(t) x^{(t)}}{\sum_{t=1}^{N} h_i^{(j)}(t)},$$

$$\boldsymbol{\Sigma}_i^{j+1} = \frac{\sum_{t=1}^{N} h_i^{(j)}(t)[\boldsymbol{x}^{(t)} - \boldsymbol{\mu}_i^{(j)}][\boldsymbol{x}^{(t)} - \boldsymbol{\mu}_i^{(j)}]^T}{\sum_{t=1}^{N} h_i^{(j)}(t)}$$

where the posterior probabilities, i.e. the "latent" variables corresponding to the mixture components, computed in the E-step are:

$$h_i^{(j)}(t) = \frac{c_i^{(j)} \mathcal{N}(\boldsymbol{x}^{(t)} | \boldsymbol{\mu}_i^{(j)}, \boldsymbol{\Sigma}_i^{(j)})}{\sum_{m=1}^{n} c_m^{(j)} \mathcal{N}(\boldsymbol{x}^{(t)} | \boldsymbol{\mu}_m^{(j)}, \boldsymbol{\Sigma}_m^{(j)})}$$

The last equation computes the conditional probability for a given data point $\boldsymbol{x}^{(t)}, t = 1, ..., N$ being generated from mixture component i using the current (denoted by j) parameter estimate.

Despite the ease of training of GMMs, they have two serious drawbacks when it comes to speech recognition systems.The first one is that they cannot model the sequence information contained in speech features. To balance their inability, we combine GMMs with more general models able to capture sequence information: the Hidden Markov Models, which will be presented next. The second disadvantage, is that, in spite of their huge modeling capabilities, GMMs are statistically inefficient for modeling data lying on or near a nonlinear manifold in the data space; this is the case however for speech features, despite their seemingly high dimensionality.To deal with this matter, there are a number of techniques we can apply to extract the lower dimensional manifold of the features. We will go into more details about manifolds and speech features in the following chapter.

Hidden Markov Models and acoustic modeling.[YD14]

As we have already mentioned, mixture-of-Gaussian random variables (single- or multidimensional) lack a "temporal" dimension, which would make the length of the random vectors variable, in order to follow the length of the speech sequence we intent to model. Therefore, although Gaussian mixture models are appropriate for short-term sound patterns, we will need to introduce a new model appropriate for sequences of speech acoustic vectors.

Extending the notion of the random variable to the discrete-time random sequence will provide us with the necessary tool to model acoustic vector sequences of variable length. A discrete-time random sequence is a collection with variable length, consisting of random variables indexed by uniformly spaced discrete times.We will focus on the most commonly used class of random sequences which is the Markov sequences.

Markov sequences

The concept of *state* is a key point in Markov sequences. If we think of a system functioning as a Markov sequence generating random variables, then the configuration of the system at each time step is defined by a specific *state* of the sequence. If the state of the Markov sequence is confined to be discrete, then the Markov sequence is called a *Markov chain* and the possible values of each discrete state

constitute the discrete state space. When each discrete state value is generalized to be a new random variable (either discrete or continuous) the Markov chain is generalized to the *Hidden Markov Sequence*, also called *Hidden Markov Model* when it characterizes statistical properties of real-world data sequences.The Hidden Markov Model is the tool that we will use to model speech units used, such as sub-phones.

Markov chains

A Markov chain is a discrete-time Markov sequence.Its state space is of discrete nature, finite and each element of the space is associated with a state in the chain:

$$q_t \in s^{(j)}, j \ = \ 1, 2, ..., N$$

where q_t symbols a state.

A Markov chain denoted by $q_1^T = q_1, q_2, ..., 1_T$, is completely characterized by the initial state distribution probabilities (*priors*) and the transition probabilities defined by:

$$P(q_t = s^{(j)} | q_{t-1} = s^{(i)}) \doteq a_{ij}(t), \quad i, j \ = \ 1, 2, ..., N$$

Given the transition probabilities of a Markov chain, the state-occupation probability

$$p_j \doteq P[q_t = s^{(j)}]$$

can be recursively computed by

$$p_i(t) = \sum_{j=1}^{N} a_{ij} p_j(t-1), \quad \forall i$$

Hidden Markov Models[RJ86][Rab89][DHS00]

If the states of a Markov chain are *emitting*, that is, they are able to generate observational output variables, then we call the chain an observable Markov sequence. However, since there is an one-to-one correspondence between the output of the chain and the states, the model is inadequate to describe real-world informational sources such as sequences of speech features.To overcome this limitation, we will add randomness to the Markov chain by associating each state with an observation probability distribution, thus creating the hidden Markov sequence. It is called hidden because the underlying Markov chain is no longer directly observable but it can be observed only through a separate random function characterized by the observation probability distributions, which overlap across the states.

A Hidden Markov Model (*HMM*) is characterized by:

- N, the number of states in the model
- K, the number of distinct observation symbols per state

- The transition probabilities, $\mathbf{A} = [\alpha_{ij}]$, i,j = 1,2,...,N where

$$\alpha_{ij} = P(q_t = j|q_{t-1} = i),\ i, j\ =\ 1, 2, ..., N$$

- The initial Markov chain prior probabilities

$$\pi\ =\ [\pi_i],\ i\ =\ 1, 2, ..., N$$

  where π_i = P(q_1=i)

- The observation probability distribution, P($\boldsymbol{o}_t|s^{(i)}$), i = 1,2,...,N.

  If $\boldsymbol{o}_t$ is discrete, the distribution associated with each state gives the probabilities of symbolic observation $\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_k$:

$$b_i(k) = P[\boldsymbol{o}_t = \boldsymbol{v}_k|q_t = i],\quad i\ =\ 1, 2, ..., N.$$

  If the observation probability distribution is continuous, then the parameters $\boldsymbol{\Theta}_i$ in the p.d.f. characterize state i in the HMM.The most common p.d.f used in speech processing is, as we have seen, the multivariate mixture of Gaussian distributions:

$$b_i(\boldsymbol{o}_t) = \sum_{m=1}^{M} c_{i,m}\mathcal{N}(\boldsymbol{o}_t|\boldsymbol{\mu_{i,m}}, \boldsymbol{\Sigma_{i,m}})$$

  with $\boldsymbol{\Theta}_i = \{c_{i,m}, \boldsymbol{\mu}_{i,m}, \boldsymbol{\Sigma}_{i,m}\}$

Given these parameters one could consider the HMM as a generative model producing a sequence of observational data, $\boldsymbol{o_t},\ t\ =\ 1, 2, ...T$. According to this perspective, the data at each time t is generated from the model according to:

$$\boldsymbol{o_t} = \boldsymbol{\mu_i} + \boldsymbol{r_t}(\boldsymbol{\Sigma_i})$$

where state i at a given time t is determined by the evolution of the Markov chain characterized by α_{ij} and

$$\boldsymbol{r_t}(\boldsymbol{\Sigma_i}) = \mathcal{N}(0, \boldsymbol{\Sigma_i})$$

is a zero-mean, independent and identically distributed (*IID*) residual sequence.Given that $\boldsymbol{\mu_i}$ is constant, the observation $\boldsymbol{o_t}$ is also IID given the state. Consequently, the HMM would produce locally stationary sequences making it appropriate to model sub-phone units.A procedure to generate sequences of observations from an HMM is described in the figure 2.15.

The three basic problems for an HMM.

Given the HMM model as presented above, there are three main problems associated with it that apply to real-world problems:

Figure 2.15: *Generate observation sequence from an HMM .[YD14]*

procedure DRAWFROMHMM(A, π, $P(\mathbf{o}_t|s^{(i)})$)
▷ A is the transition probability
▷ π is the initial state occupation probability
▷ $P(\mathbf{o}_t|s^{(i)})$ is the observation probability given a state (either Eq. 3.7 if discrete or Eq. 3.8 if continuous
Select an initial state $q_1 = s^{(i)}$ by drawing from the discrete distribution π
for $t \leftarrow 1; t \leq T; t \leftarrow t+1$ **do**
Draw an observation $\mathbf{o}_t$ based on $P(\mathbf{o}_t|s^{(i)})$
Make a Markov-chain transition from the current state $q_t = s^{(i)}$ to a new state $q_{t+1} = s^{(j)}$ according to the transition probability a_{ij}, and assign $i \leftarrow j$.
end for
end procedure

- The evaluation problem: Suppose we have an HMM $(A_{ij}, b_{ik}, \Theta_i)$.How do we determine the probability that a given sequence of observations was generated by that model?

- The decoding problem: Suppose we have an HMM $(A_{ij}, b_{ik}, \Theta_i)$.How do we determine the most likely hidden state sequence that led to the generation of a given observation sequence?

- The parameter estimation problem: Given the basic structure of an HMM (number of states and number of distinct observation symbols) as well as a set of training observations, how do we determine the parameters $(A_{ij}, b_{ik}, \Theta_i)$?

The evaluation problem. Let $\boldsymbol{q}_1^T$ be a finite length sequence of states in a Gaussian-mixture HMM and $P(\boldsymbol{o}_1^T, \boldsymbol{q}_1^T)$ be the joint likelihood of the observation sequence $\boldsymbol{o}_1^T$ and the state sequence $\boldsymbol{q}_1^T$.

Then, $P(\boldsymbol{o}_1^T|\boldsymbol{q}_1^T)$ denotes the likelihood that the observation sequence $\boldsymbol{o}_1^T$ is generated by the model conditioned on the state sequence $\boldsymbol{q}_1^T$ and is in the form of:

$$\prod_{i=1}^{T} b_i(\boldsymbol{o}_t)$$

whereas the probability of state sequence $\boldsymbol{q}_1^T$ is the product of transition probabilities:

$$P(\boldsymbol{q}_1^T) = \pi_{q_1} \prod_{t=1}^{T-1} a_{q_t q_{t+1}}$$

The joint likelihood $P(\boldsymbol{o}_1^T, \boldsymbol{q}_1^T)$ can be obtained as:

$$P(\boldsymbol{o}_1^T, \boldsymbol{q}_1^T) = P(\boldsymbol{o}_1^T|\boldsymbol{q}_1^T)\boldsymbol{q}_1^T$$

Figure 2.16: *The Forward algorithm for HMM probability evaluation.[JM09]*

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
  $forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

Since the hidden state sequence $\boldsymbol{q}_1^T$ is not known, we will have to sum over all possible state sequences in order to compute the desired probability:

$$P(\boldsymbol{o}_1^T) = \sum_{\boldsymbol{q}_1^T} P(\boldsymbol{o}_1^T, \boldsymbol{q}_1^T)$$

The amount of this computation though, is exponential in the length T of the observation sequence. However, an efficient algorithm (linear complexity in T) to evaluate the above expression has been found, based on the principle of optimality (dynamic programming) [Bel03].The algorithm, known as *Forward algorithm* is described in figure 2.16

The decoding problem. The decoding problem consists of finding the most probable sequence of HMM hidden states given a sequence of observations.It is essentially a path-finding optimization problem that will be dealt with using again the dynamic programming paradigm.In fact, the decoding algorithm, also known as the *Viterbi algorithm* (figure 2.17) is very similar to the *Forward* algorithm presented above.

The Viterbi algorithm returns the maximum joint likelihood of the observation and state sequence as well as the corresponding state transition path.The optimal path for a left-to-right HMM, i.e. an HMM where transitions are only allowed in the forward direction, is equivalent to the information required to determine the optimal segmentation of the HMM states to match the observation sequence.

The parameter estimation problem. The goal in HMM training, is to extract the model parameters so as to minimize the empirical risk with respect to the joint likelihood loss, involving a sequence of acoustic data and their corresponding linguistic labels. To estimate the parameters of an HMM model given training data, we will apply the Expectation-Maximization algorithm, also known as *Baum-Welch algorithm* in the context of HMM training.First however, we will introduce the *Backward algorithm*, which is a part of the EM computation for HMMs.

Figure 2.17: *The Viterbi algorithm for HMM decoding.[YD14]*

function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi*$[N+2,T]$
for each state s **from** 1 **to** N **do** ; initialization step
  $viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
  $backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**
    $viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$
    $backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$
$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

Figure 2.18: *The Backward algorithm for HMM decoding.[DHS00]*

initialize $\omega(T), t = T, a_{ij}, b_{jk}$, visible sequence V^T
  for $t \leftarrow t-1$;
    $\beta_j(t) \leftarrow \sum_{i=1}^{c} \beta_i(t+1) a_{ij} b_{jk} v(t+1)$
  until $t = 1$
return $P(V^T) \leftarrow \beta_i(0)$ for the known initial state
end

The *Backward algorithm* (figure 2.18) is very similar to the *Forward* but now we are moving backwards, that is, the algorithm computes

$$\beta_t(i) = P(\boldsymbol{o}_{t+1}^T | q_t = i), \ t = 1, ..., T-1$$

The EM algorithm uses both the Forward and Backward algorithms in the expectation E-step in order to obtain:

- the posterior state transition probabilities in the HMM

$$\xi_t(i,j) = \frac{\alpha_t(i)\beta_{t+1}(j)\alpha_{ij} \exp N_{t+1}(j)}{P(\boldsymbol{o}_1^T|\theta_0)}, \quad t = 1, ..., T-1$$

  where $N_t(i)$ is the logarithm of the Gaussian p.d.f. associated with state i,

- the posterior state occupancy probabilities

$$\gamma_t(i) = \sum_{j=1}^{N} \xi_t(i,j)$$

In the maximization M-step the parameters are computed using the current estimates of ξ and γ:

$$\hat{\alpha}_i j = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

$$\hat{\Sigma}_i = \frac{\sum_{t=1}^{T} \gamma_t(i)(\boldsymbol{o}_t - \hat{\boldsymbol{\mu}}_i)(\boldsymbol{o}_t - \hat{\boldsymbol{\mu}}_i)^T}{\sum_{t=1}^{T} \gamma_t(i)}$$

$$\hat{\mu}_i = \frac{\sum_{t=1}^{T} \gamma_t(i)\boldsymbol{o}_t}{\sum_{t=1}^{T} \gamma_t(i)}$$

for each state i.

HMMs in speech modeling and recognition.[YD14][ST04] We have already seen that HMMs can be used as generative models to produce sequences of observations. They are able to produce sequences of variable length, which is of utmost importance for speech modeling and recognition, and they have proven to be good models for the statistical distribution of sequence data of speech acoustics. Consequently, HMMs have become very popular in the ASR community.

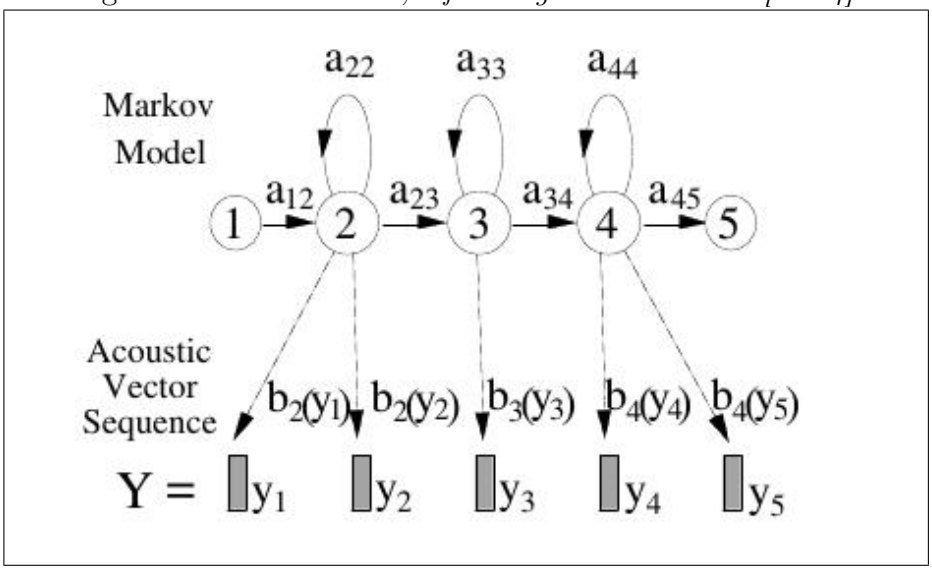
GMM/HMMs in ASR. As we have seen, a GMM/HMM is a statistical model that describes two dependent random processes, an observable and a hidden, where the observable is assumed to be generated by a hidden state according to a Gaussian mixture p.d.f.

In the context of speech, we think of a single HMM as a system generating acoustic features of a modeled speech unit, which can be a word, a syllable, a single phone , or, usually a context-dependent phone. The motivation behind choosing context-dependent phones, and therefore states, stems from the effort to reduce output variability of speech feature vectors associated with each state, leading to more detailed generative modeling. However this leads to an expansion of the state space, which we have seen how it is dealt with, at the beginning of the chapter.

The most common HMM model is the three-state left-to-right HMM (figure 2.19). The number of states is chosen based on the behavior of the vocal tract. It goes through three states when uttering a phone : changing from the previous phone, steady pronunciation of the current phone and changing to the next phone.

However, despite their advantages, HMMs have been found to have several weaknesses. The temporal independence of speech data conditioned on the HMM states and the lack of proven correlation between acoustic features and ways in which

Figure 2.19: *Three-state, left-to-right HMM model.[ST04]*



speech sounds are produced (e.g. speaking rate and style) have motivated the replacement of GMMs associated with each state by more realistic, temporally correlated dynamic systems containing hidden, continuous-valued dynamic structure (e.g. [Bil03]).

2.4.5 The Decoder

[MPR01] The components described above are finally combined in the decoder which will find the most likely word sequence given a sequence of feature vectors. In order to define a common framework for the representation and use of the aforementioned models in LVCSR, we represent them by Weighted Finite State Transducers; an approach which provides significant algorithmic and engineering benefits.

A *finite-state transducer* is a finite automaton whose state transitions are labeled with both input and output symbols.Consequently, a path through the transducer encodes a mapping from an input to an output symbol sequence. *Weighted finite-state transducers (WFSTs)* , in addition to input/output symbols, have weights on the transitions which accumulate along paths to compute the total cost of a mapping from an input to an output symbol sequence.Thus, seeing the components of an ASR system as WFSTs and mathematical operations on them, allows for generalizing and efficiently implementing many of the common processing methods in speech recognition. We will briefly present the WFST framework in speech recognition, yet first we will introduce some necessary notation and algorithms.

Figure 2.20: *Composition algorithm .[MPR01].*

```
WEIGHTED-COMPOSITION(T_1, T_2)
 1  Q ← I_1 × I_2
 2  S ← I_1 × I_2
 3  while S ≠ ∅ do
 4       (q_1, q_2) ← HEAD(S)
 5       DEQUEUE(S)
 6       if (q_1, q_2) ∈ I_1 × I_2 then
 7            I ← I ∪ {(q_1, q_2)}
 8            λ(q_1, q_2) ← λ_1(q_1) ⊗ λ_2(q_2)
 9       if (q_1, q_2) ∈ F_1 × F_2 then
10            F ← F ∪ {(q_1, q_2)}
11            ρ(q_1, q_2) ← ρ_1(q_1) ⊗ ρ_2(q_2)
12       for each (e_1, e_2) ∈ E[q_1] × E[q_2] such that o[e_1] = i[e_2] do
13            if (n[e_1], n[e_2]) ∉ Q then
14                 Q ← Q ∪ {(n[e_1], n[e_2])}
15                 ENQUEUE(S, (n[e_1], n[e_2]))
16            E ← E ∪ {((q_1, q_2), i[e_1], o[e_2], w[e_1] ⊗ w[e_2], (n[e_1], n[e_2]))}
17  return T
```

Notation and Algorithms

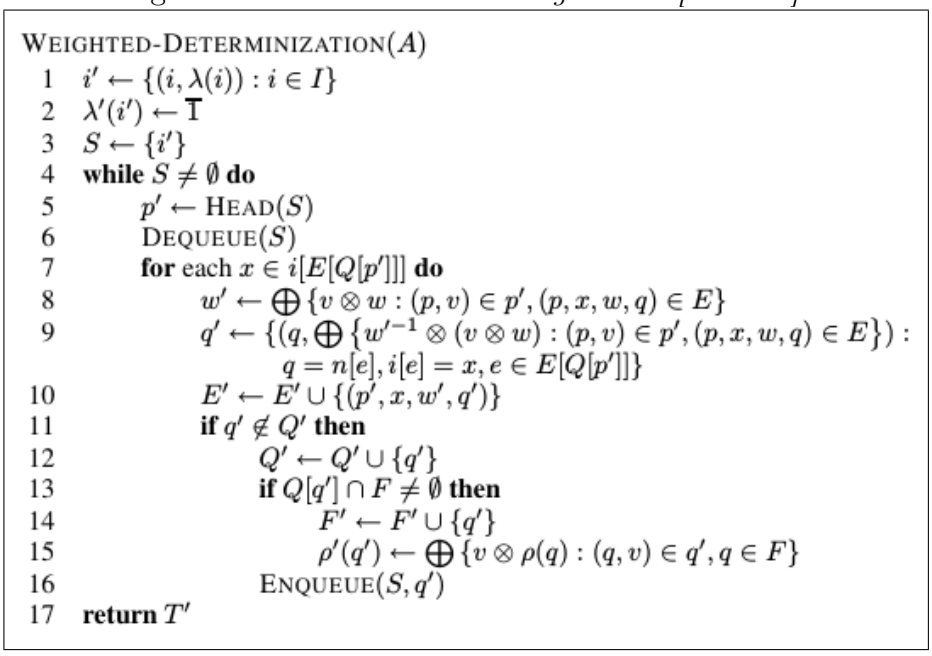
- A *semi-ring* $(\mathbb{K}, \bigoplus, \bigodot, \bar{0}, \bar{1})$ is defined by a set of values $\mathbb{K}$, two binary operations of addition ($\bigoplus$) and multiplication ($\bigodot$) and two designated values $\bar{0}$ and $\bar{1}$. The addition operation is associative, commutative and has $\bar{0}$ as the identity element. The multiplication operation is associative, has $\bar{1}$ as the identity element, is distributive with respect to addition and has $\bar{0}$ as the annihilator element: $\forall a \in \mathbb{K}, a \bigodot \bar{0} = \bar{0} \bigodot a = \bar{0}$. If $\bigodot$ is also commutative the semi-ring is called *commutative*, which will be the case for all the semi-rings mentioned later.

- A WFST $T = (\mathcal{A}, \mathcal{B}, \mathcal{Q}, \mathcal{I}, \mathcal{F}, \mathcal{E}, \lambda, \rho)$ over a semi-ring $\mathbb{K}$ is specified by a finite input alphabet $\mathcal{A}$, a finite output alphabet $\mathcal{B}$, a finite set of states $\mathcal{Q}$, a set of initial states $\mathcal{I} \subseteq \mathcal{Q}$, a set of final states $\mathcal{F} \subseteq \mathcal{Q}$, a finite set of transitions $\mathcal{E} \subseteq Q \times (\mathcal{A} \cup \epsilon) \times (\mathcal{B} \cup \epsilon) \times \mathcal{K} \times \mathcal{Q}$, an initial state weight assignment $\lambda : \mathcal{I} \to \mathcal{K}$ and a final state weight assignment $\rho : \mathcal{F} \to \mathcal{K}$. $\mathcal{E}[q]$ denotes the sum of the number of states and transitions of $\mathcal{T}$.

Based on the notation just introduced we will present the operations on WFSTs that will be used in speech recognition applications.

- *Composition* is the basic operation that allows us to create complex WFSTs from simpler ones, thus putting together all the fundamental components of an ASR system.

Figure 2.21: *Determinization algorithm .[MPR01].*



- *Determinization* removes non-determinancy from the WFST by ensuring that each state has no more than a single output transition for a given input label.Not every WFST is determinizable, however, there is a *pre-determinization* algorithm that can be used to make determinizable an arbitrary WFST over the tropical semi-ring $((\mathcal{R} \cup \{-\infty, +\infty\}, min, +, +\infty, 0))$ by inserting transitions labeled with special symbols.The determinization operation is particularly important in ASR considering the redundancy found in e.g. the WFST representing the pronunciation lexicon.A deterministic lexicon WFST will contain at most one path for any input string, thus less time and space will be needed to process the string (figure 2.21).

- *Minimization* transforms a WFST to an equivalent one with the fewest possible states and transitions which saves both space and time during its processing. Before minimising the transducer, a form of re-weighting (*weight-pushing*) is performed to redistribute weight among transitions as well as to improve search operations.

WFST models in speech recognition

There are four principal models of weighted finite-state transducers that are used in speech recognition:

- G : the word level grammar

- L : the pronunciation lexicon
- C : the context-dependency transducer
- H : the HMM transducer

The word level *grammar transducer G* has a state w_i for each word and transitions are added according to the N-gram model used in the grammar. For example, a bigram grammar has a transition from state w_1 to w_2 for every bigram w_1w_2 seen in the training corpus.The label of the transition is then w_2 and the weight is the negative logarithm of the transition probability $(-\log(\hat{p}(w_2|w_1)))$.To deal with unseen N-grams while keeping the complexity of constructing the WFST low, we introduce a back-off state b.An unseen bigram w_1w_3 is then represented as two transitions: an ϵ transition w_1b with weight $-\log(\beta(w_1))$ and a transition bw_3 with weight $-\log(\hat{p}(w_3))$. Because ϵ transitions introduce non-determinism in the WFST, we can treat ϵ labels as normal symbols during determinization, thus keeping the number of transitions low - otherwise transitions become quadratic with respect to the size of the vocabulary after determinization.

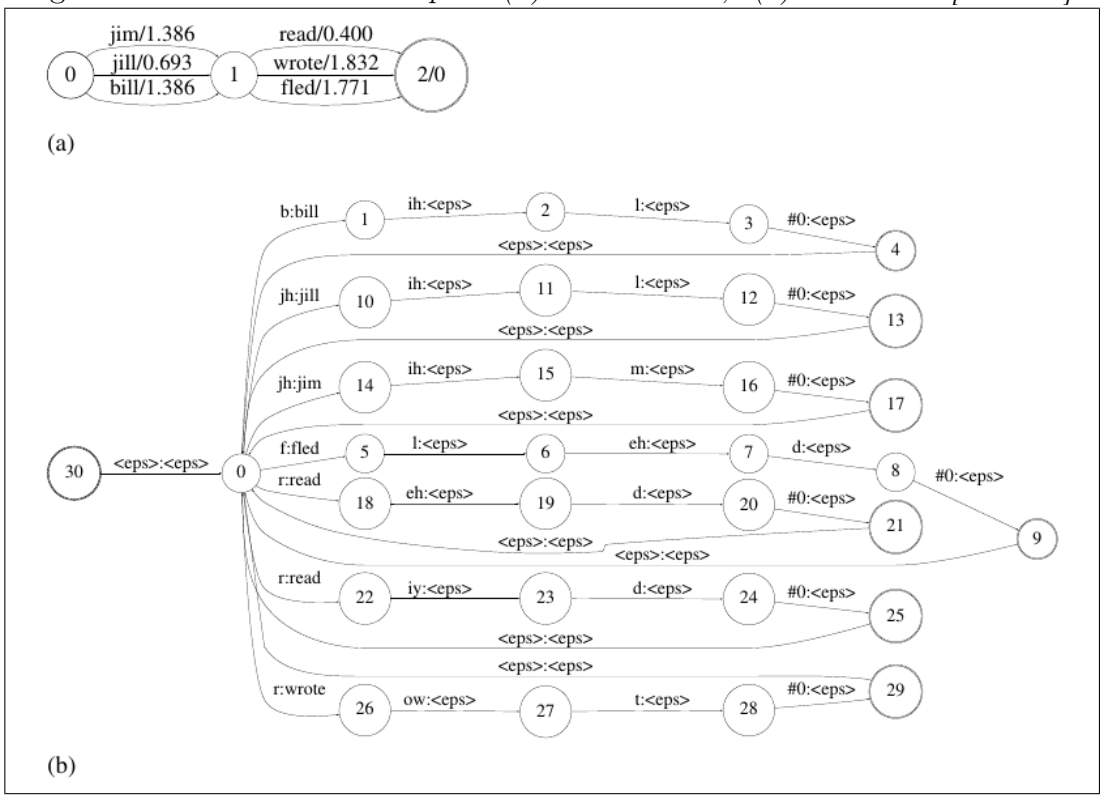
The *pronunciation lexicon transducer L* is the Kleene closure of the union of individual word pronunciations.It is easy to see that L is, in general, not determizable, considering the existence of homophones and the fact that the first word of the output string might be impossible to determine before the entire phone string is scanned. To make L determinizable we add a number of disambiguation symbols ($\#_i$) as well as a symbol ($\#_0$) to mark the end of the phonetic transcription of each word. The new lexicon transducer after the addition of these symbols is determinizable and is denoted by $\bar{L}$.

The *context dependency transducer* represents a mapping from context independent phones to context dependent units.The transducer has a state for every pair of phones with label (a, b) ,where a is the past and b is the future phone, and transitions marked as *a:phone/left context_ right context*.To apply the context dependent triphone models often used in ASR in the WFST framework, we need to be able to compose a context dependency transducer with the lexicon transducer introduced earlier.In order to make this composition feasible we first invert the context dependency transducer (interchange input and output labels) and create the transducer C which maps from context dependent triphones to context independent phones.

The final transducer that is used in the decoding process is the *Hidden Markov Models transducer H* which is the closure of the union of the individual HMMs of the acoustic model.

Applying the composition operation on the transducers presented here will output the decoding transducer which we will further optimize to help decoding and

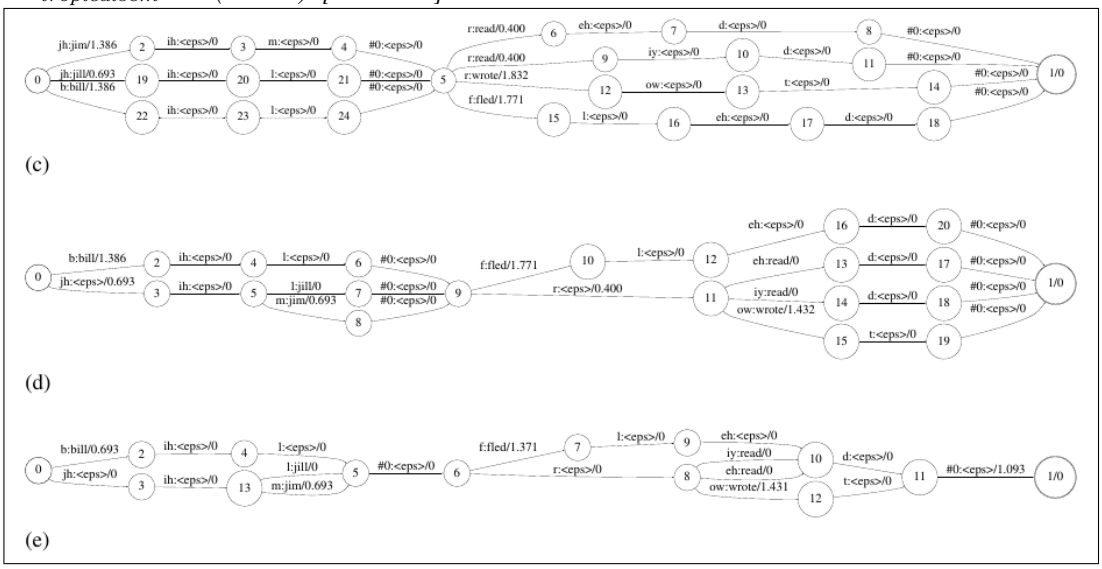
Figure 2.22: *Transducer example: (a) Grammar G, (b) Lexicon $\bar{L}$ [MPR01].*



make it as efficient as possible.First, composing the lexicon and grammar transducers gives a new transducer that maps from phones to word strings restricted to the grammar ($L \circ G$).The resulting transducer is then composed with the context dependency transducer C and the resulting transducer maps from context dependent phones to word strings restricted to the grammar ($C \circ L \circ G$).Finally, $C \circ L \circ G$ is composed with the HMM transducer H and the outcome is a transducer that maps from the identifiers of context-dependent HMM states to word strings restricted to G $(H \circ C \circ L \circ G)$.

After the construction of the final transducer we have to determinize and minimize it so that we have the optimal transducer for recognition.By determinizing it we eliminate redundant paths which reduces recognition time.Moreover, if the determinization is applied after each composition stage during the construction of the graph, the composition operations that follow are performed more efficiently and the total size of the transducer is reduced.Once we have determinized the transducer we can further optimize it by minimization and the weight pushing process that precedes it. As far as weigh pushing is concerned, provided we are using the -log semi-ring, we can have large efficiency gains during the Viterbi beam search, and make sure that all weights leaving a state sum up to one (which is desirable for a language model).

Figure 2.23: *Transducer example: (c)* $\bar{L}\circ G$*, (d)*$\bar{L}\circ G$ *determinized, (e)* $min_{tropicalsem}$ *det(*$\bar{L}\circ G$*) [MPR01].*



Given the *HCLG* transducer and an utterance of N frames, how do we decode it, namely how do we find the most likely word sequence and its corresponding state-level alignment?

The first step is to construct an acceptor U of the utterance, which is a WFST with identical input and output symbols. The acceptor has $N+1$ states with an arc for each (time,context-dependent HMM state) combination and weights on these arcs the scaled negated acoustic log likelihoods.Following the construction of U, we compose it with the decoding WFST and get a new WFST S:

$$S = U \circ HCLG$$

The decoding problem now reduces to finding the best path through S.The input symbol sequence of the best path is the state-level alignment and the output sequence is the corresponding sentence [PHB+12].

2.4.6 Evaluation

The performance of a large vocabulary continuous speech recognition system is evaluated based on the Word Error Rate (WER) metric. There are three types of possible errors when recognizing continuous speech: (a) substitution errors, i.e. the wrong word is recognized, (b) word deletions, namely the presence of a word is not recognized at all and (c) word insertions, meaning that an extra word is recognized.If we define the number of words in the text speech as N and denote with $C(\cdot)$ the

number of errors of each type, then WER is defined as:

$$WER = \frac{C(substitutions) + C(deletions) + C(insertions)}{N}$$

ADD STATE OF THE ART WER + MENTION SYSTEM AND DATASET

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