

# Accelerate Magnetic Resonance Imaging (MRI) Using Compressed Sensing and Deep Learning

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# Outline

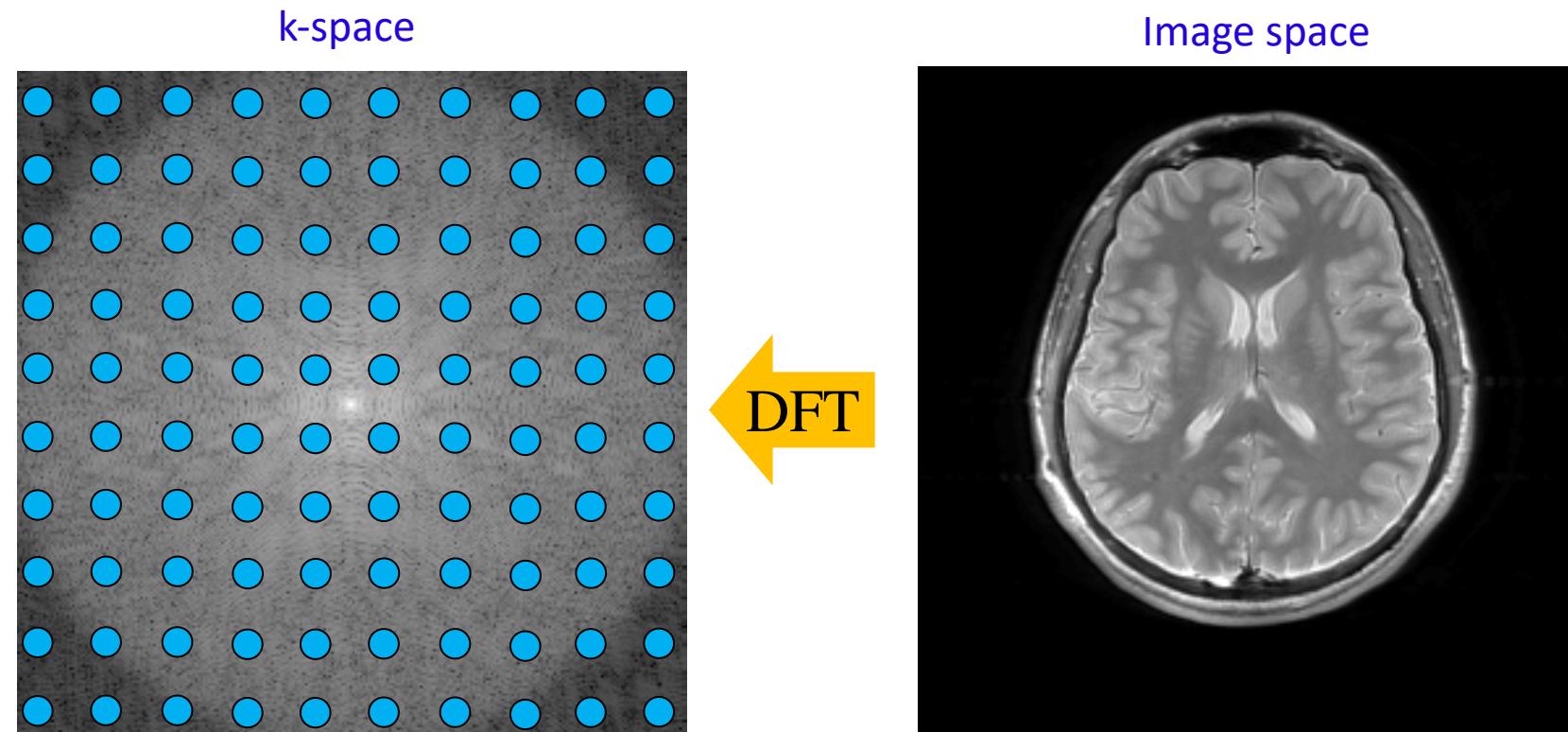
- MR Image Acquisition and Reconstruction
  - Imaging parameters
  - Reconstruction from accelerated scans
- Deep Learning for Accelerated MRI
  - Supervised learning
  - Experimental Results

# Outline

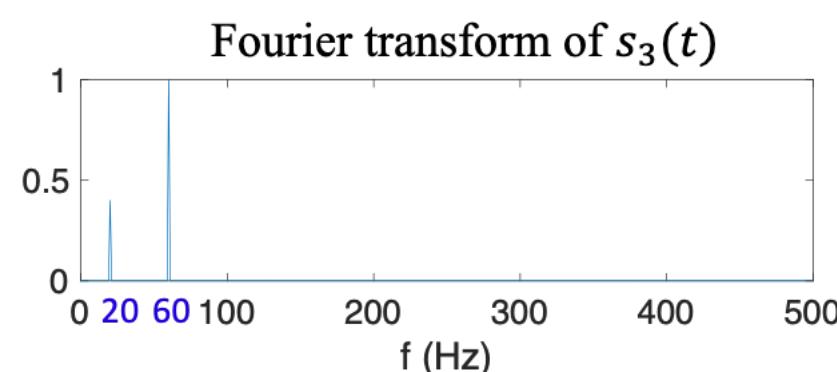
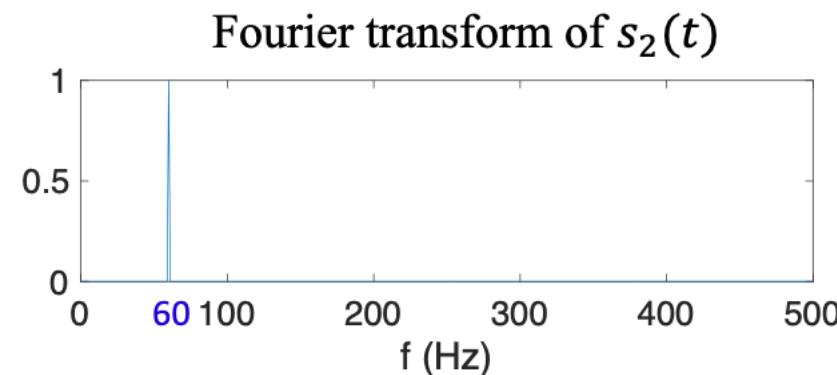
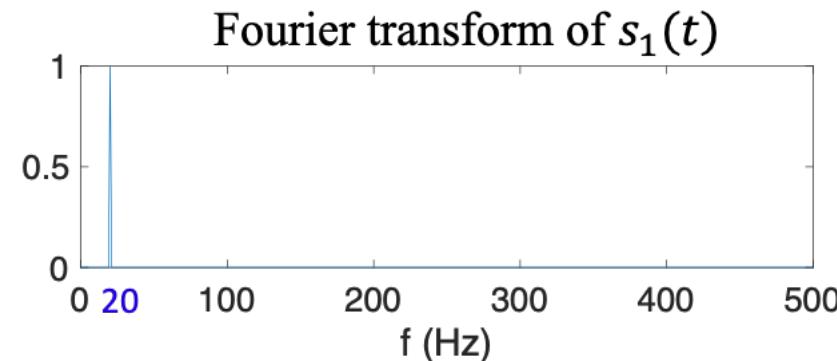
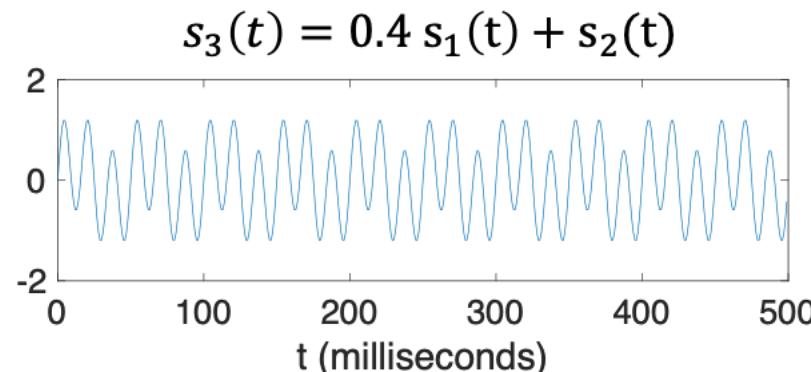
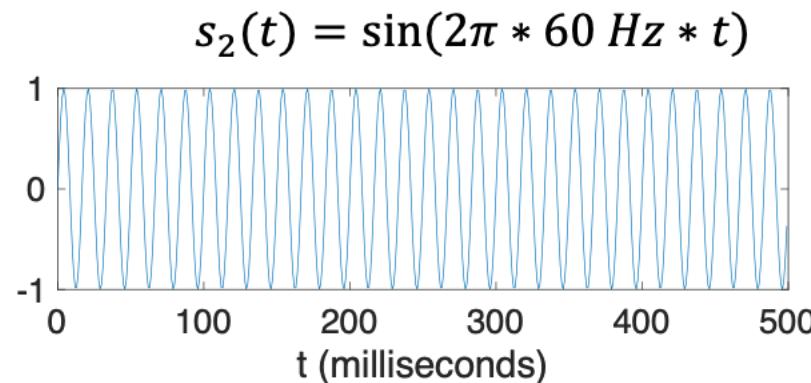
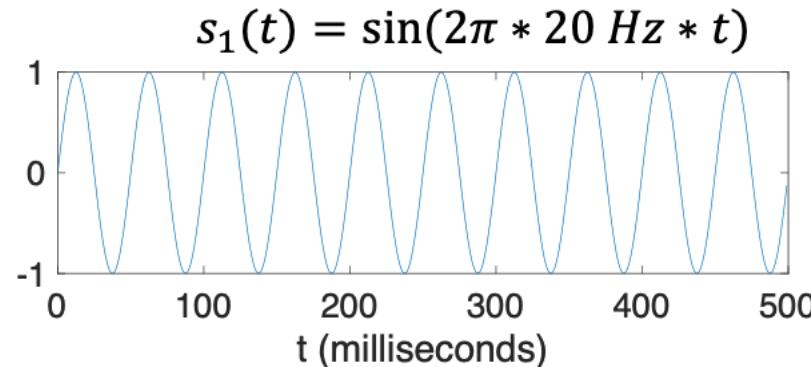
- **MR Image Acquisition and Reconstruction**
  - Imaging parameters
  - Reconstruction from accelerated scans
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  - Supervised learning
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# MR Image Acquisition and Reconstruction

- Raw data are collected in the Fourier domain (k-space)
  - The acquired data are the discrete Fourier transform (DFT) samples of the object being imaged
  - Using the conventional 2DFT acquisition, each line of k-space is acquired one after the other (one per **repetition time (TR)** of the readout sequence)

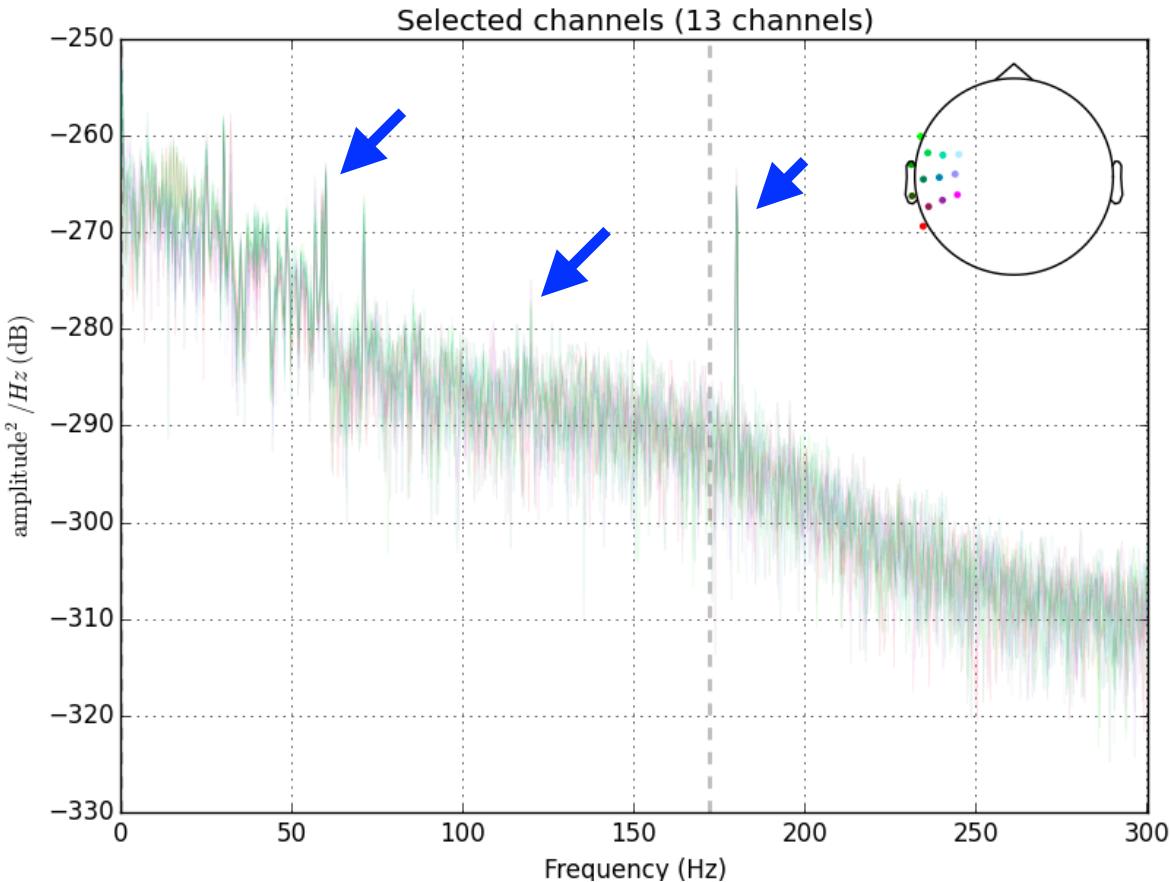


# Review: Fourier Transform

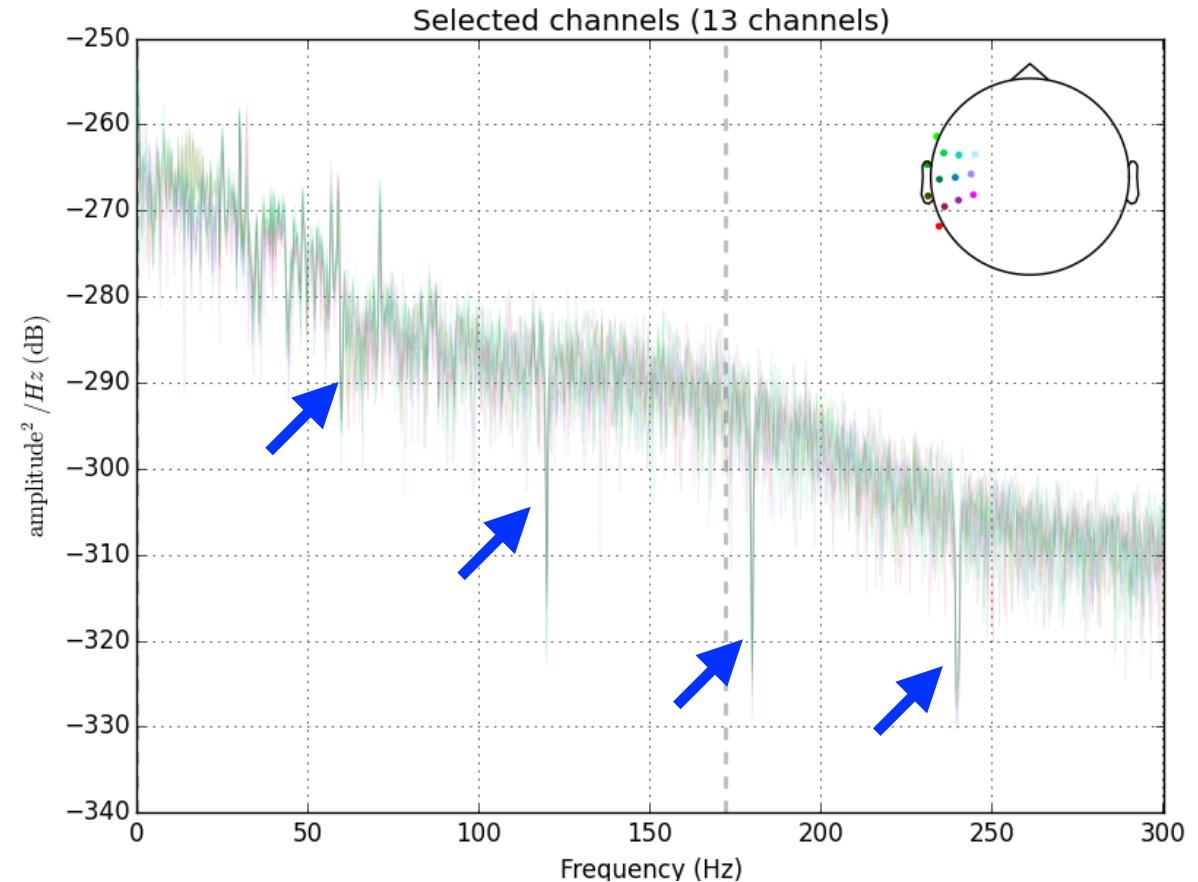


# Review: Application of Fourier Transform to EEG

The Fourier transform of the acquired EEG signals



The filtered signals at the integer multiples of 60 Hz

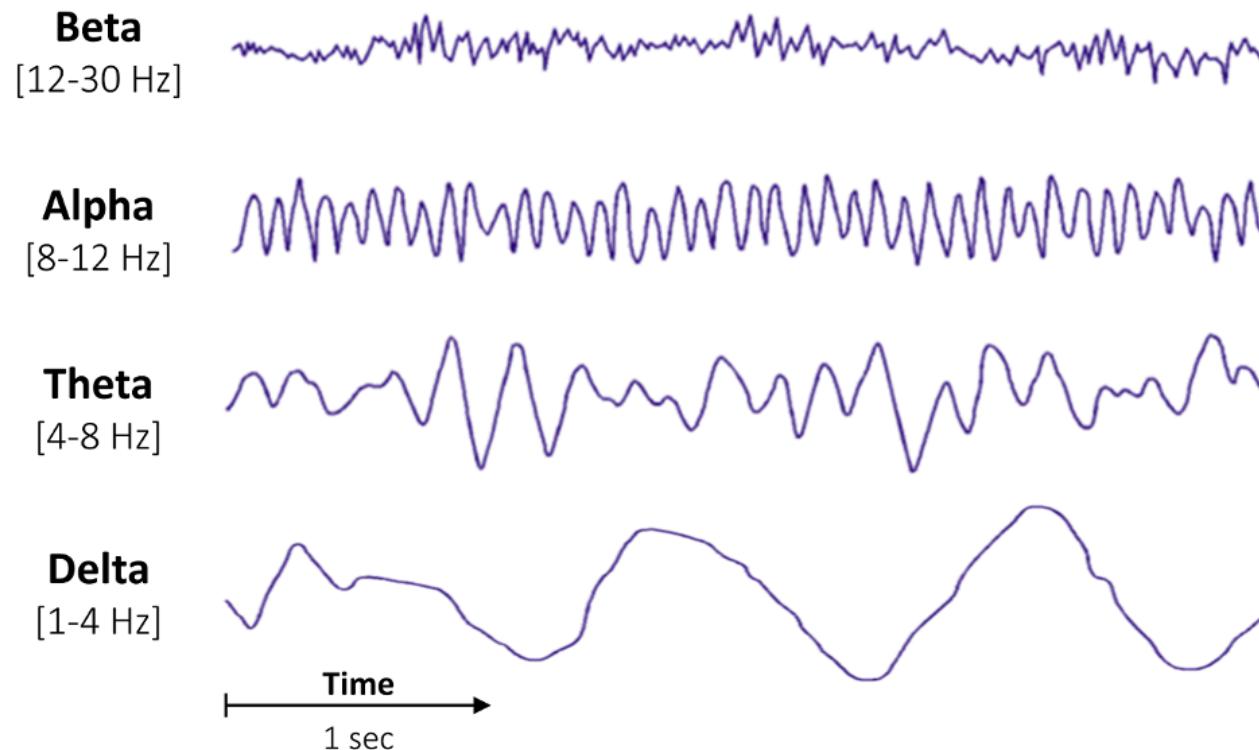


Power-line noise: potentially at the harmonic frequencies of 60 Hz (or 50 Hz)

MNE's Filtering and resampling data

# Review: Application of Fourier Transform to EEG

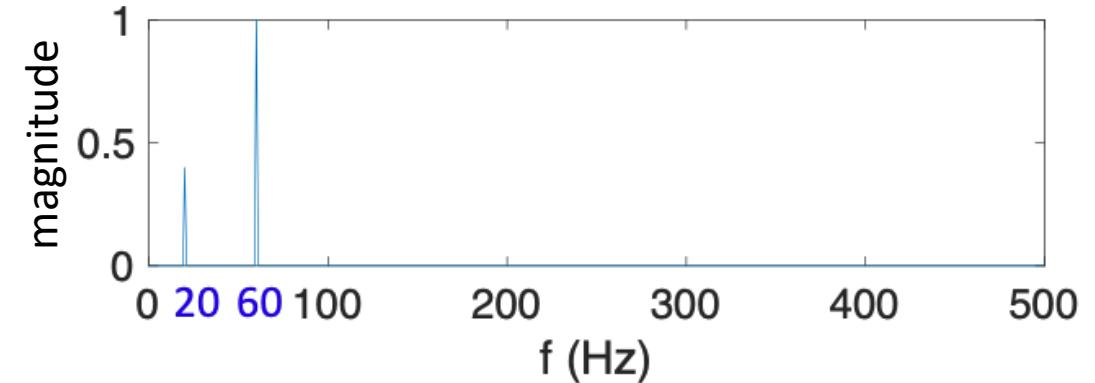
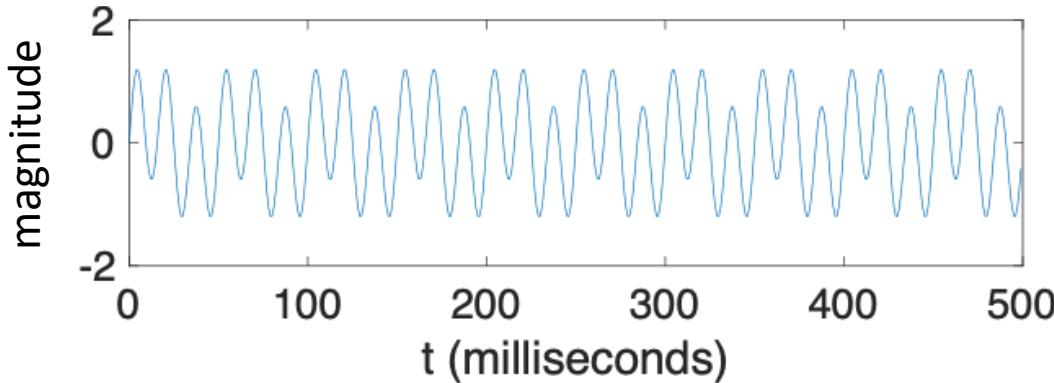
**Example of brain rhythm frequency bands associated with cognitive processes**



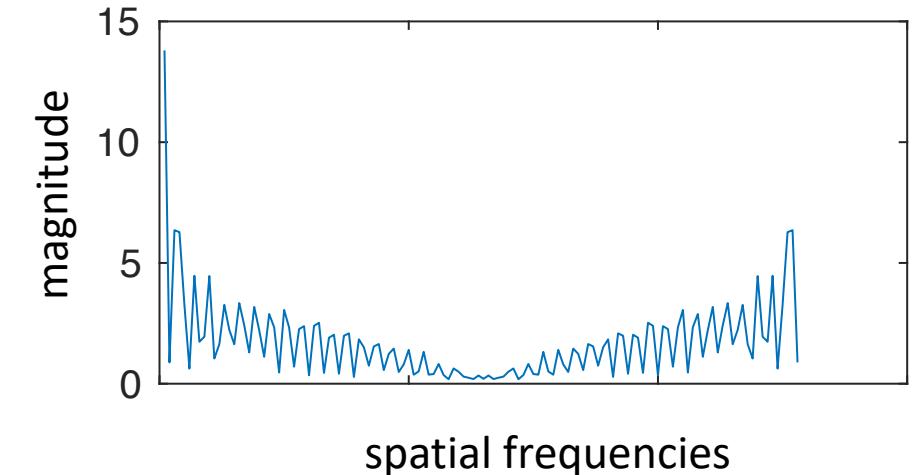
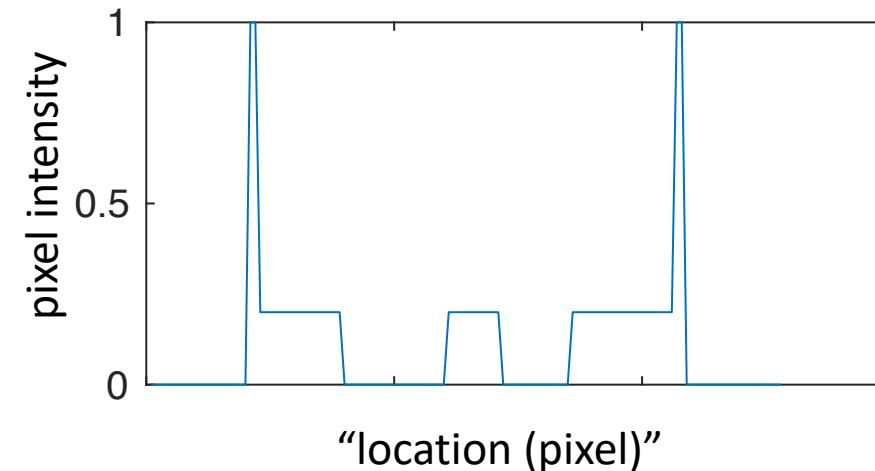
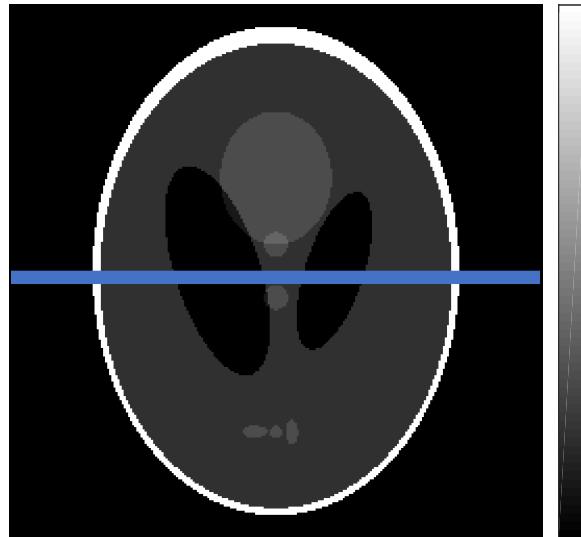
Compute the average bandpower of an EEG signal

# Review: Application of Fourier Transform to images

Fourier transform: time  $\leftrightarrow$  frequency

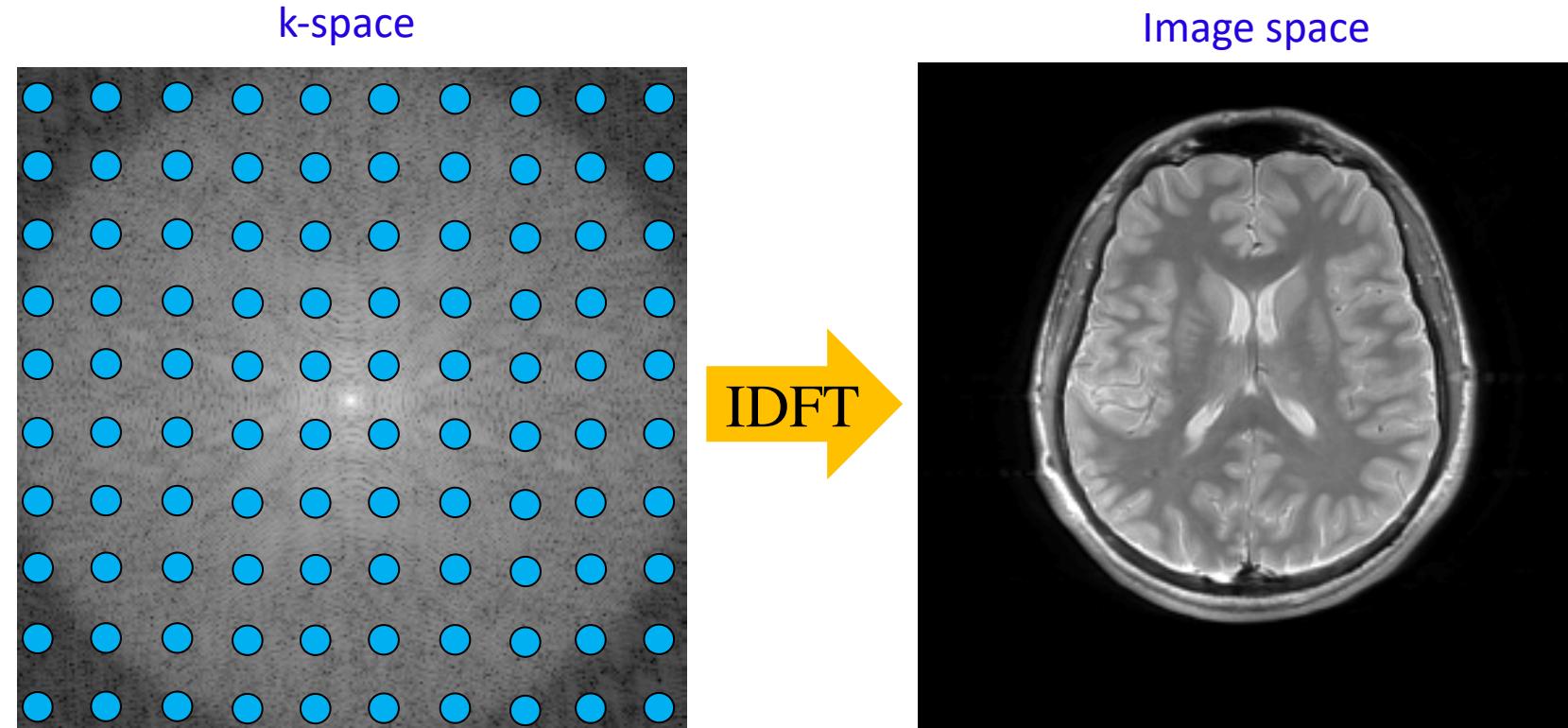


Fourier transform: location  $\leftrightarrow$  spatial frequency



# MR Image Acquisition and Reconstruction

- Raw data are collected in the Fourier domain (k-space)
  - The acquired data are the discrete Fourier transform (DFT) samples of the object being imaged
  - Using the conventional 2DFT acquisition, each line of k-space is acquired one after the other (one per **repetition time (TR)** of the readout sequence)
- If the sampling rate is high enough (“Nyquist”), the image can be reconstructed by applying the inverse DFT to the k-space data



k-space

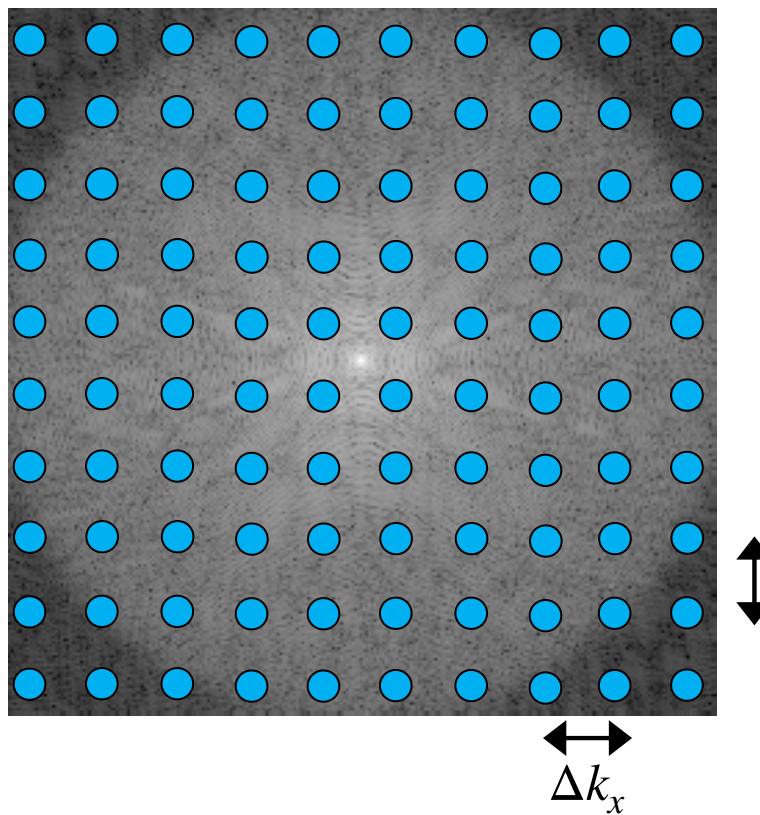
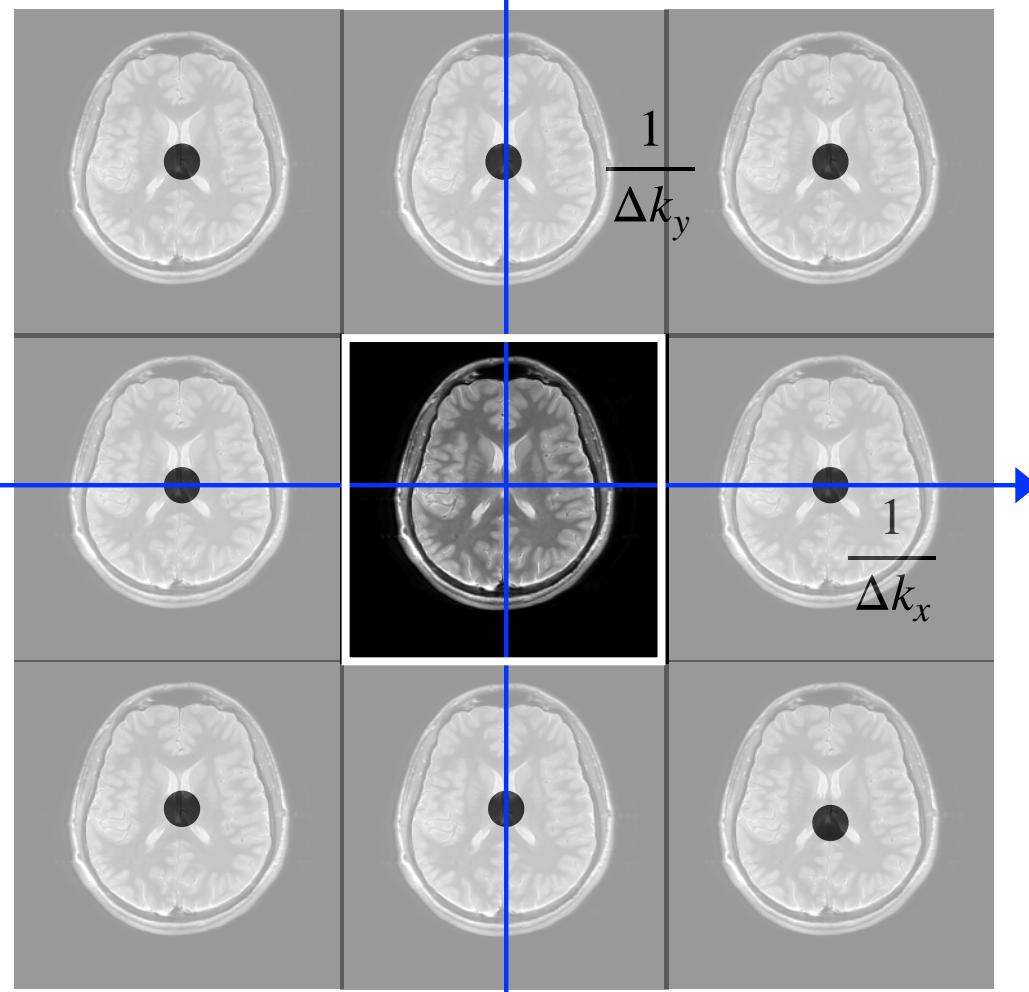
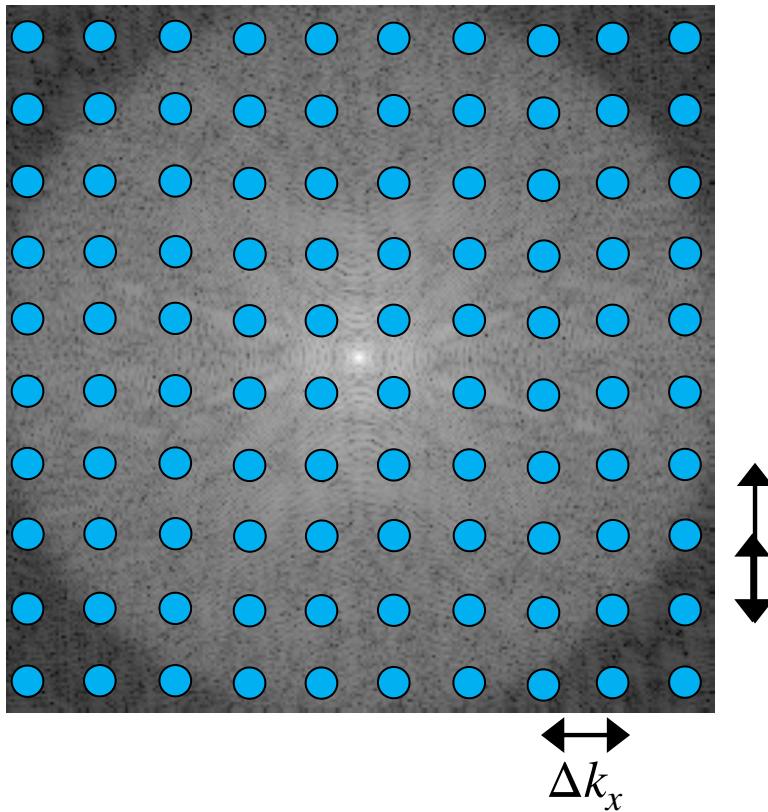


Image space

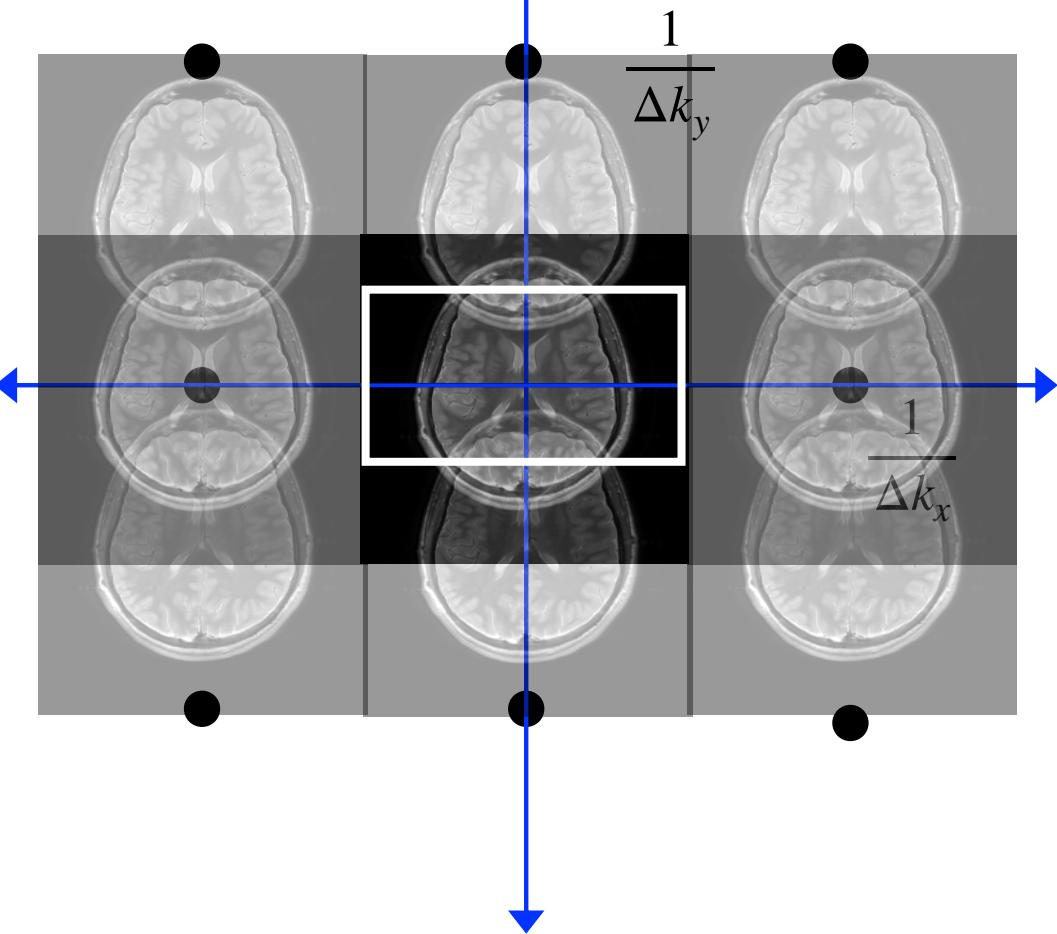


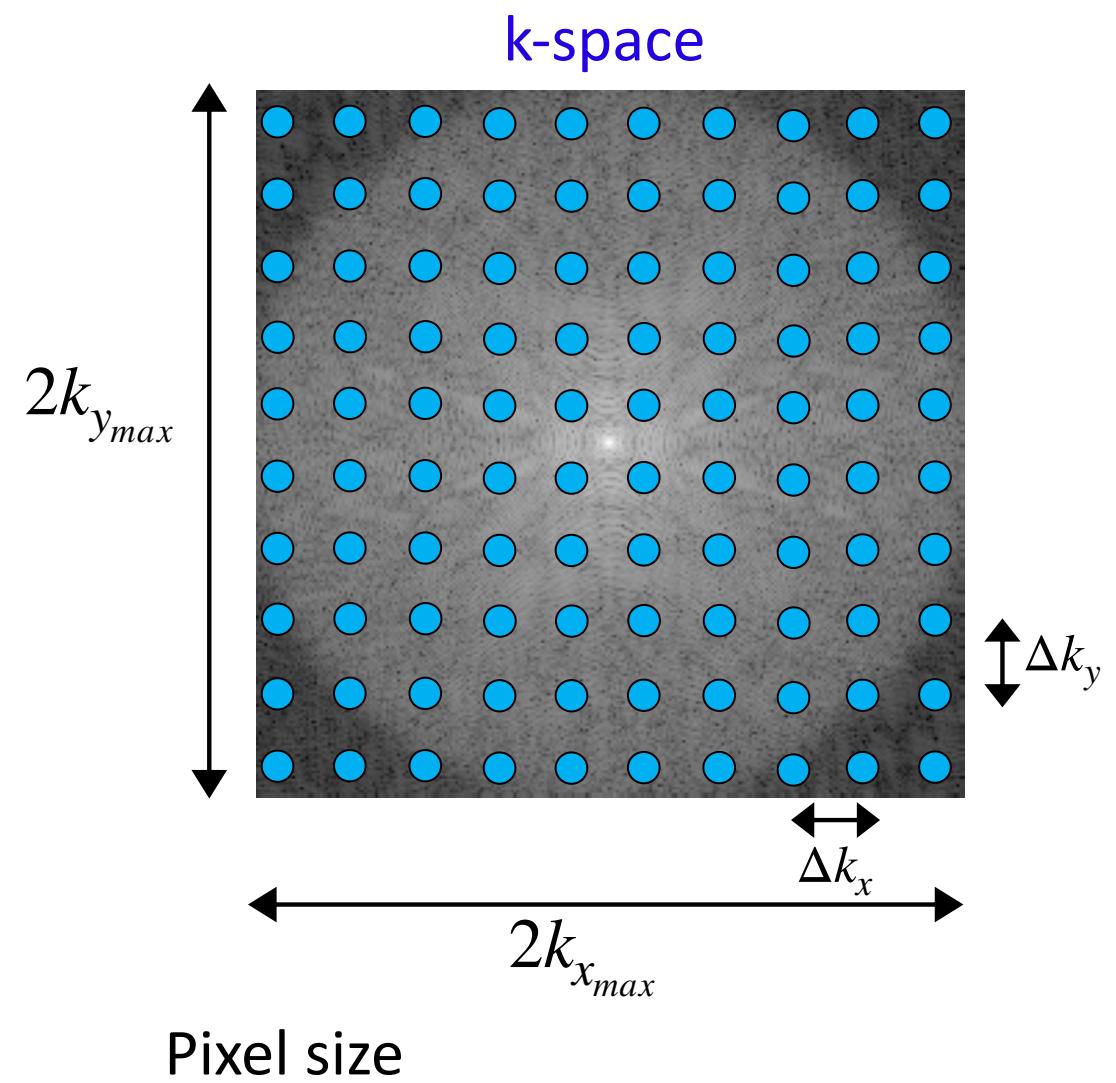
k-space



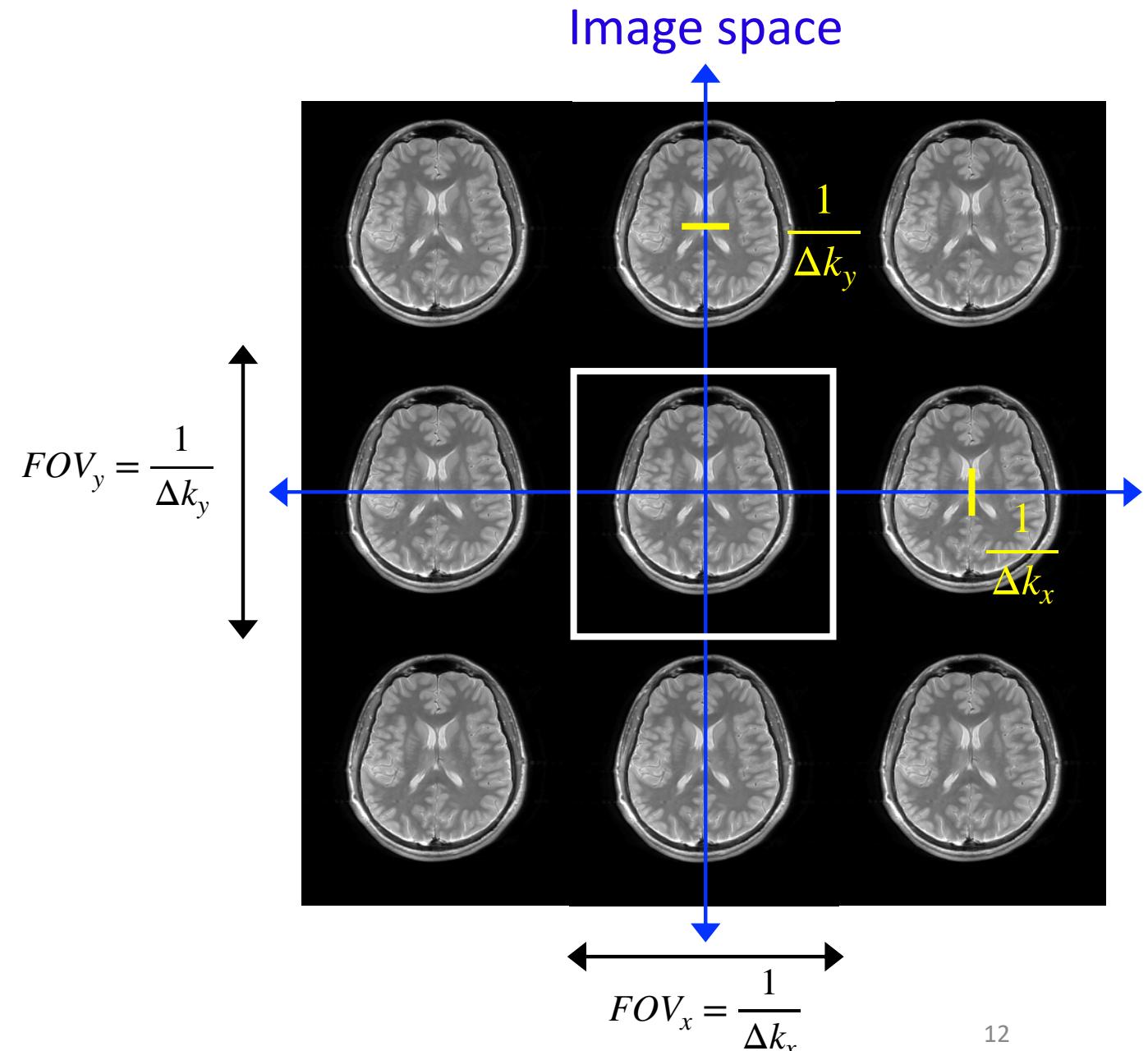
$$FOV_y = \frac{1}{\Delta k_y}$$

Image space



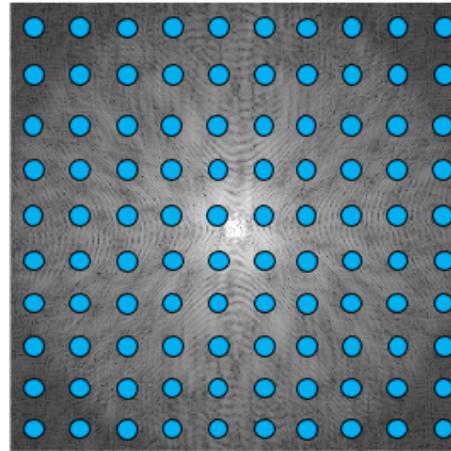


$$\Delta x \approx \frac{1}{2k_{x_{max}}} \quad \Delta y \approx \frac{1}{2k_{y_{max}}}$$

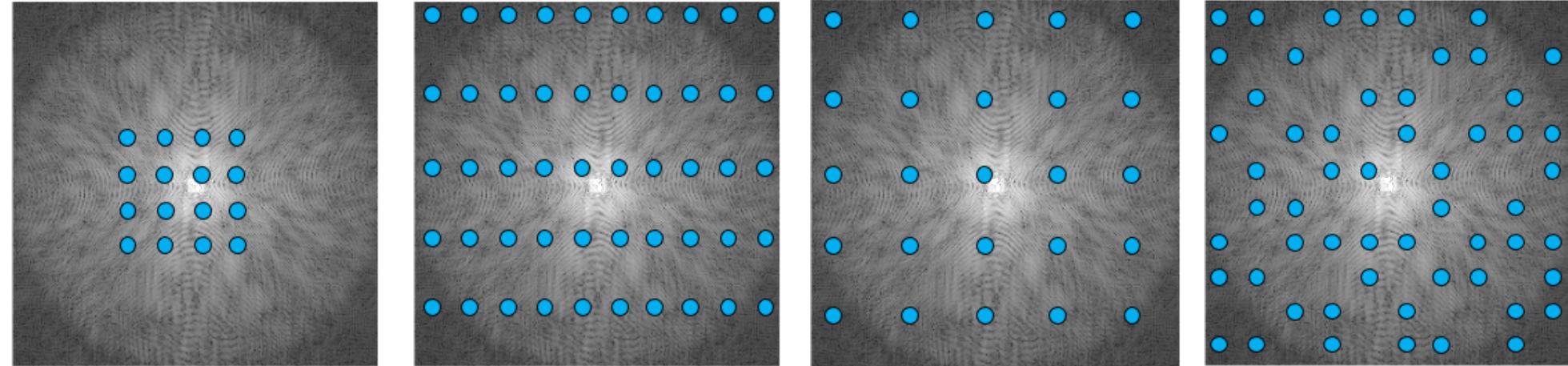


# Accelerated MRI – Collect fewer k-space samples

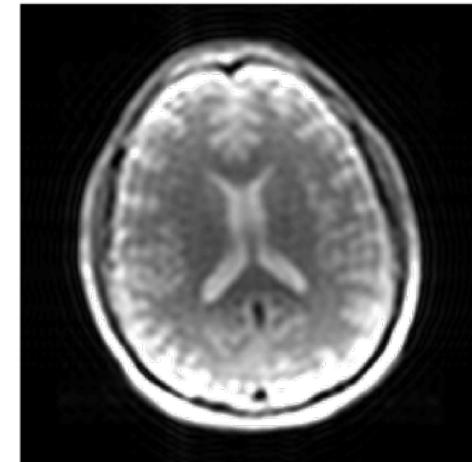
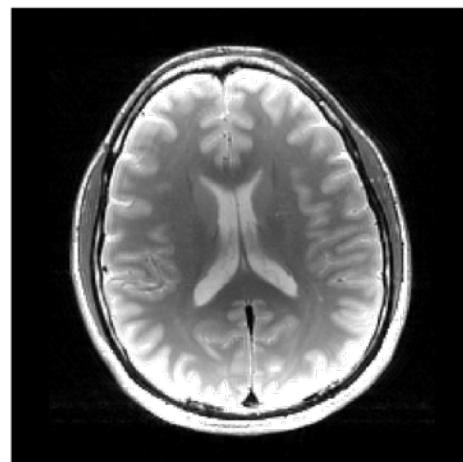
Fully sampled  
acquisition



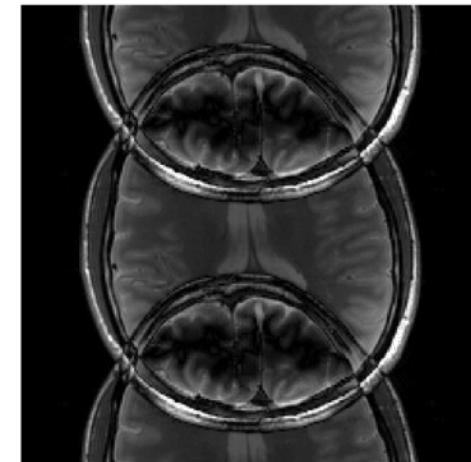
Accelerated acquisition



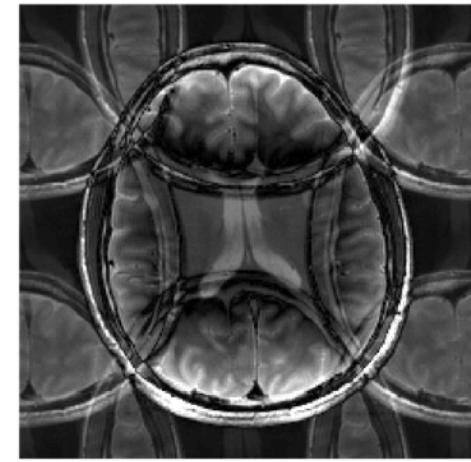
Direct application of 2D inverse Fourier transform to the acquired data



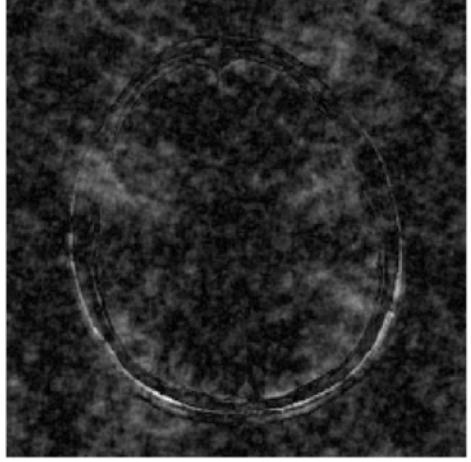
blurry



Artifact (one direction)

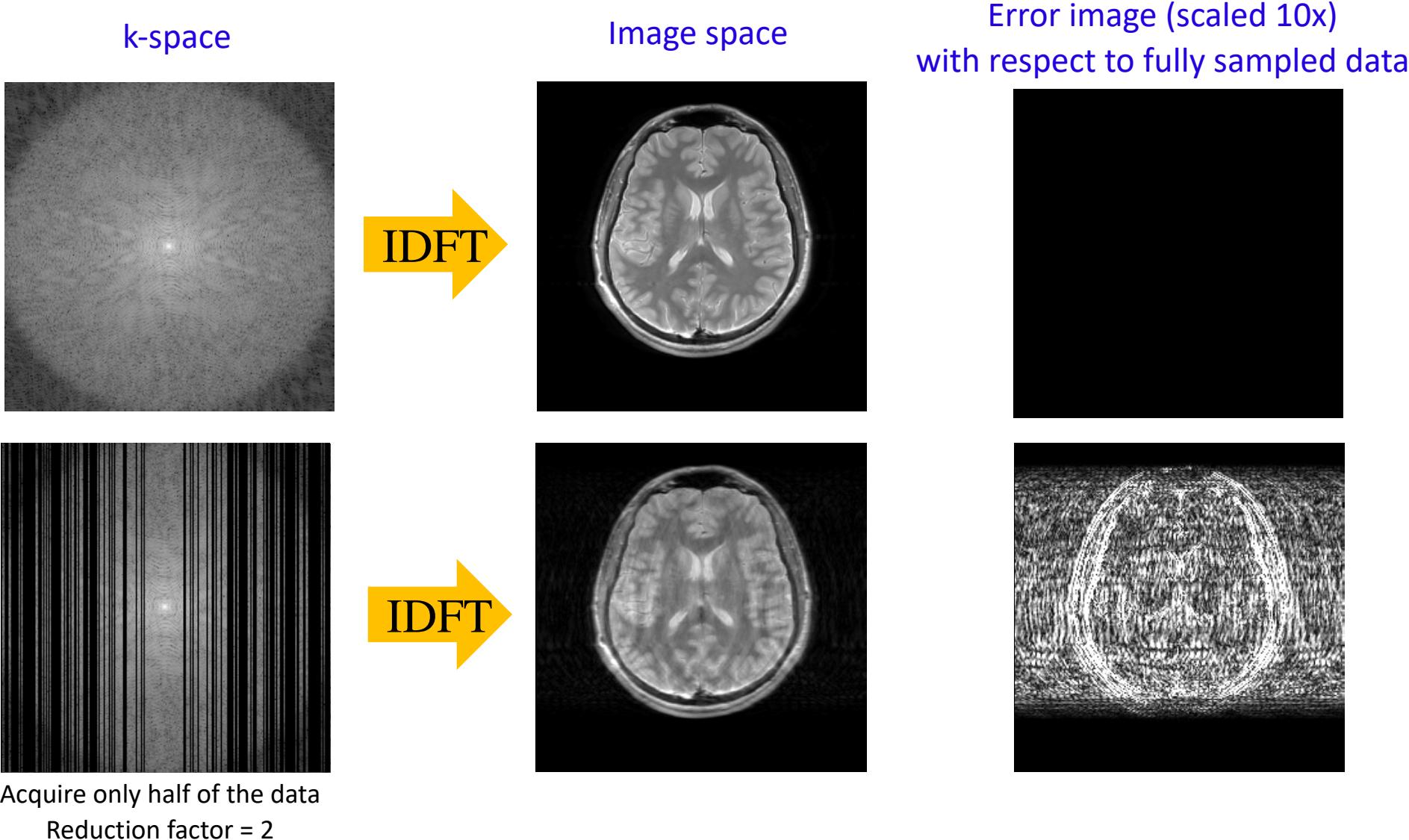


Artifact (two directions)

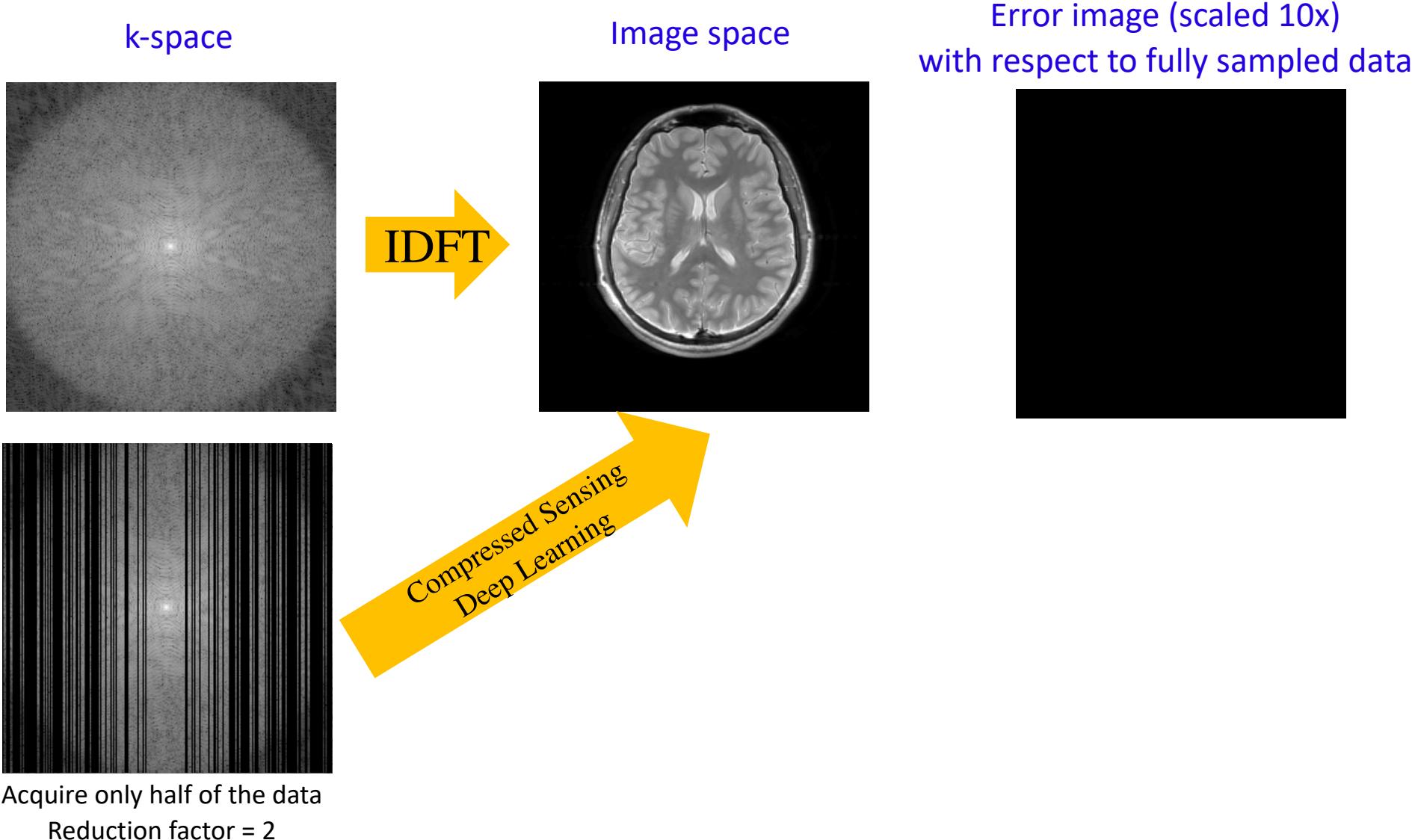


Noise-like artifact

# MR Image Reconstruction from Accelerated Scans



# MR Image Reconstruction from Accelerated Scans



# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|f(x) - y\|_2^2 + \lambda R(x)$$

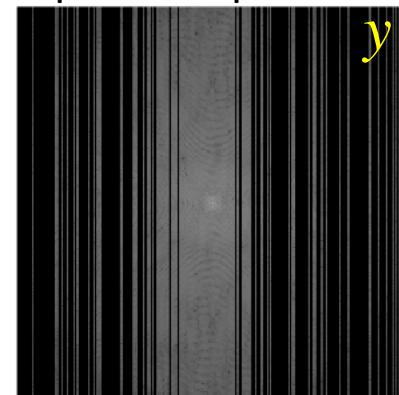
$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
$M$	: Undersampling mask
$y$	: Observed k-space data
$R$	: Regularization term
$\lambda$	: Regularization parameter

# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|f(x) - \mathbf{y}\|_2^2 + \lambda R(x)$$

Acquired k-space data



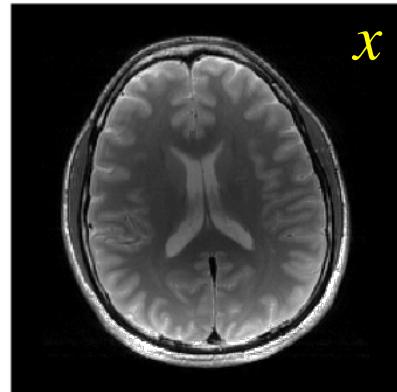
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|f(\textcolor{blue}{x}) - y\|_2^2 + \lambda R(x)$$

What you want to reconstruct



$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
$M$	: Undersampling mask
$y$	: Observed k-space data
$R$	: Regularization term
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|f(x) - y\|_2^2 + \lambda R(x)$$

Our hand-designed  
function to approximate  
the acquisition process

$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
$M$	: Undersampling mask
$y$	: Observed k-space data
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|MFx - y\|_2^2 + \lambda R(x)$$

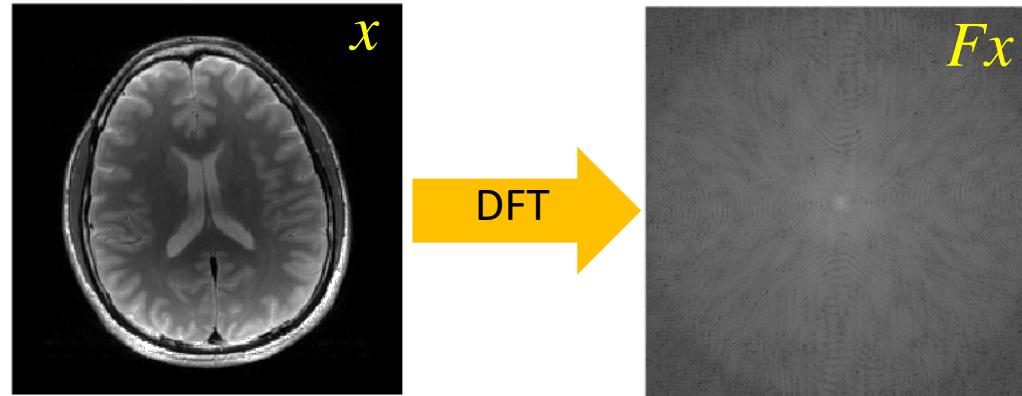
Our hand-designed  
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|M\mathbf{F}x - y\|_2^2 + \lambda R(x)$$



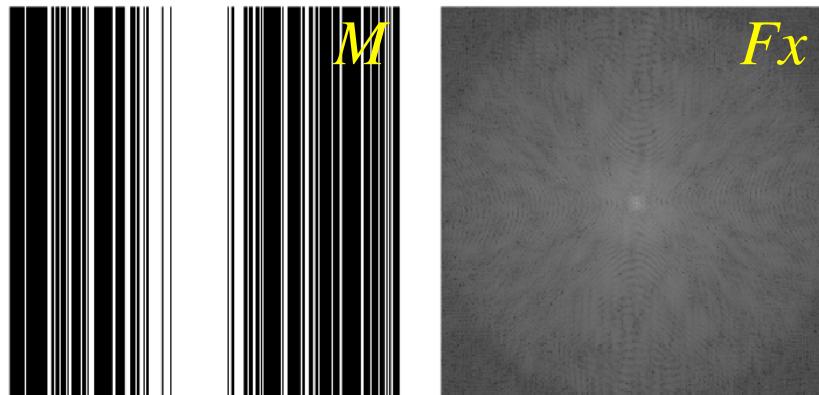
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$\lambda$	: Regularization parameter

# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \| \mathbf{M} \mathbf{F} x - y \|_2^2 + \lambda R(x)$$

Pixel-wise multiplication



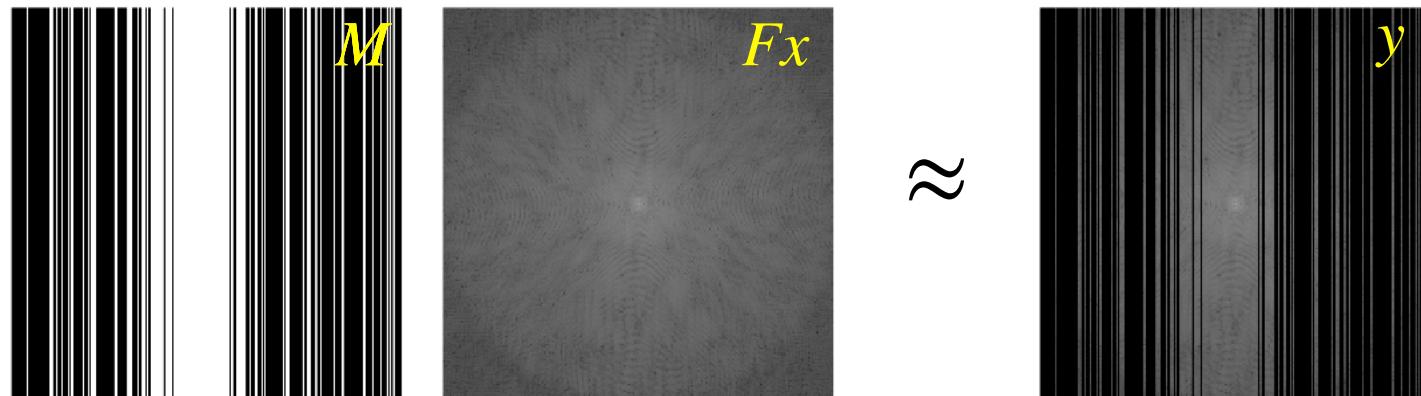
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|MFx - y\|_2^2 + \lambda R(x)$$

Data consistency → keep improving  $x$  until you get  $MFx \approx y$



$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|MFx - y\|_2^2 + \lambda R(x)$$

Regularization

Incorporate prior knowledge on  $x$

$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
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# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

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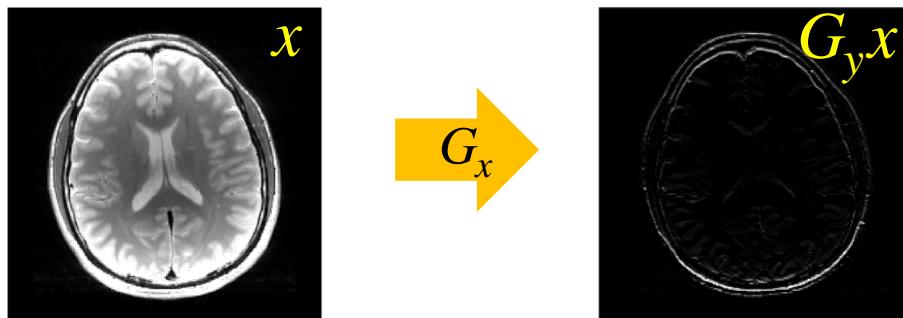
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$M$	: Undersampling mask
$y$	: Observed k-space data
$R$	: Regularization term
$\lambda$	: Regularization parameter

# Compressed Sensing

- Solve the following optimization problem to reconstruct the data

$$\hat{x} = \arg \min_x \frac{1}{2} \|Mx - y\|_2^2 + \lambda \|Gx\|_1$$

$G$ : Gradient operator discretized using finite differences



$x$	: Image
$F$	: Fully sampled discrete Fourier transform (DFT)
$M$	: Undersampling mask
$y$	: Observed k-space data
$R$	: Regularization term
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# Compressed Sensing

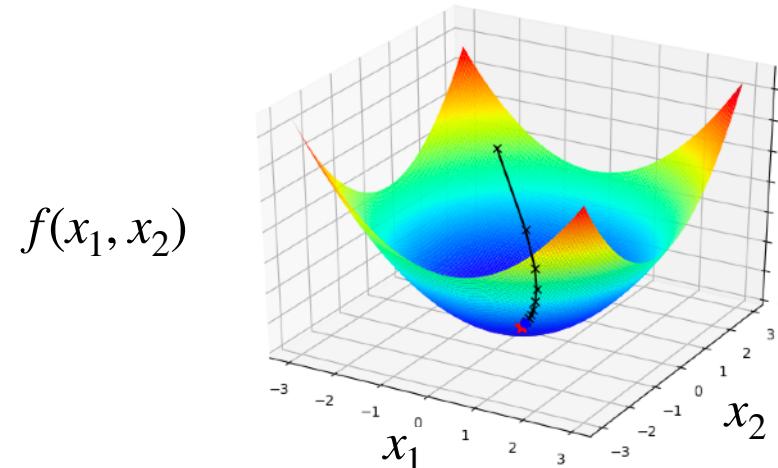
$$\min_x \frac{1}{2} \|MFx - y\|_2^2 + \lambda \|Gx\|_1$$

- Gradient descent (an iterative method)

$$x = \arg \min_x f(x)$$

- Update equation for iteration  $k$

$$x_{k+1} = x_k - \alpha \nabla f(x_k)$$



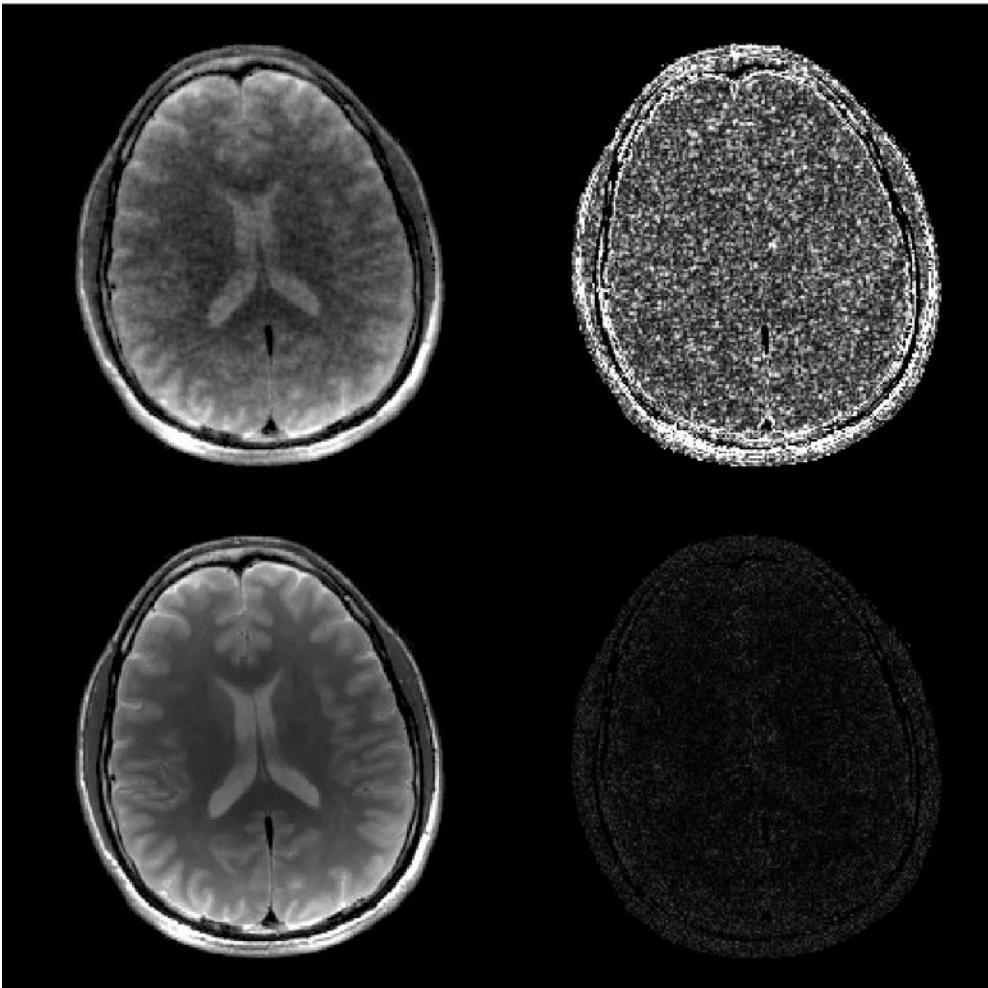
# Compressed Sensing

Reduction  
factor = 2

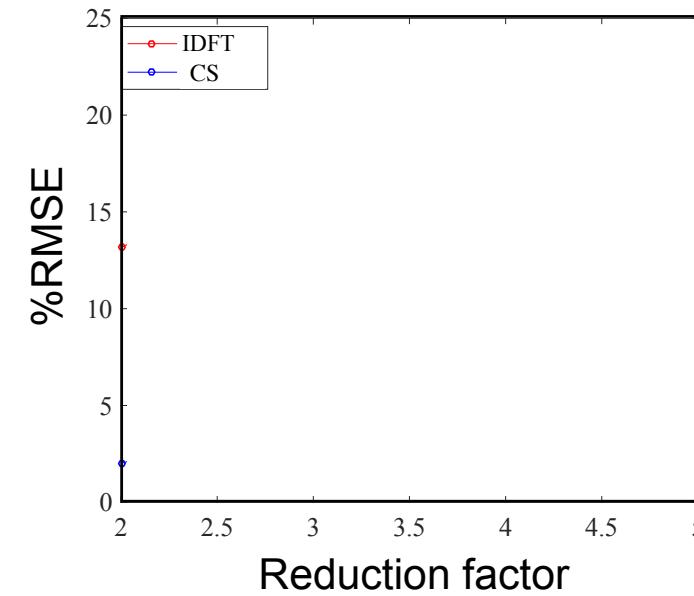
Reconstructed  
Image

Error  
(scaled 10x)

IDFT  
Compressed  
Sensing

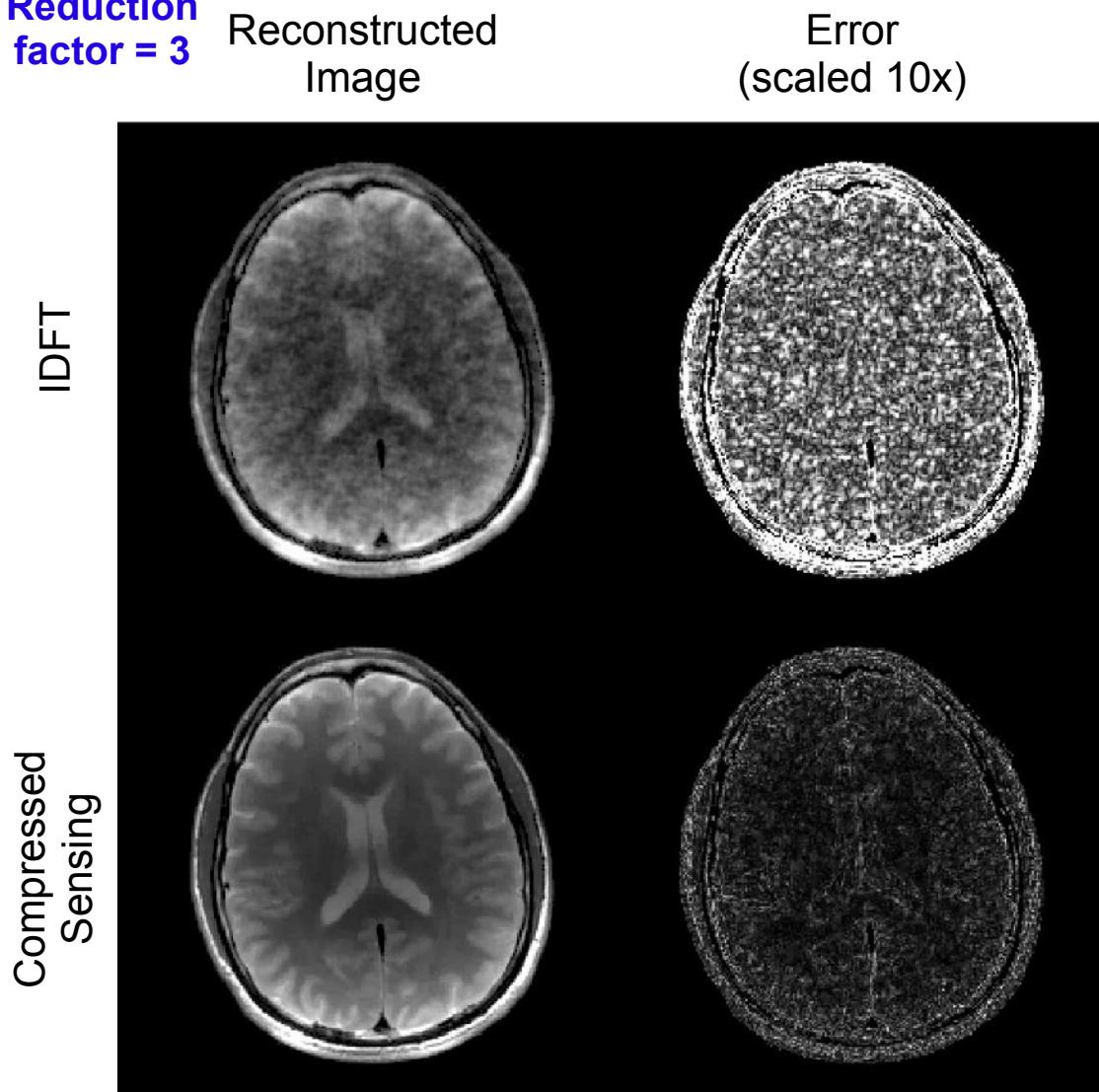


$$\hat{x} = \arg \min_x \frac{1}{2} \|M\hat{F}x - y\|_2^2 + \lambda \|Gx\|_1$$

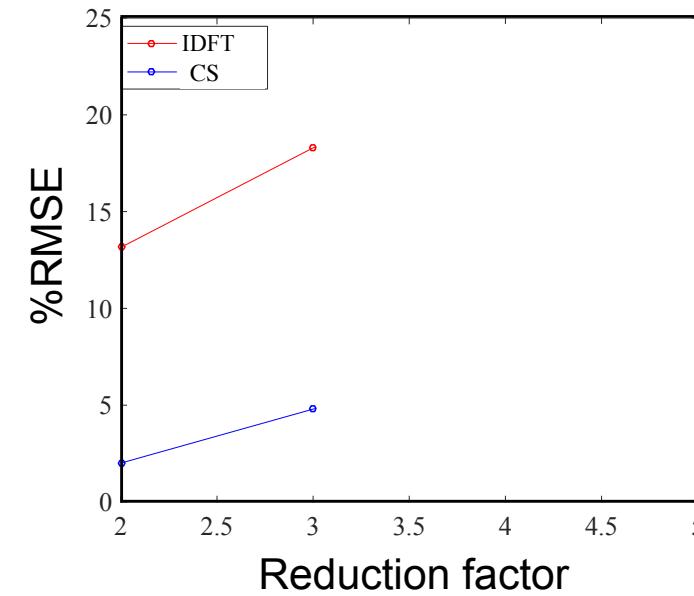


# Compressed Sensing

Reduction  
factor = 3

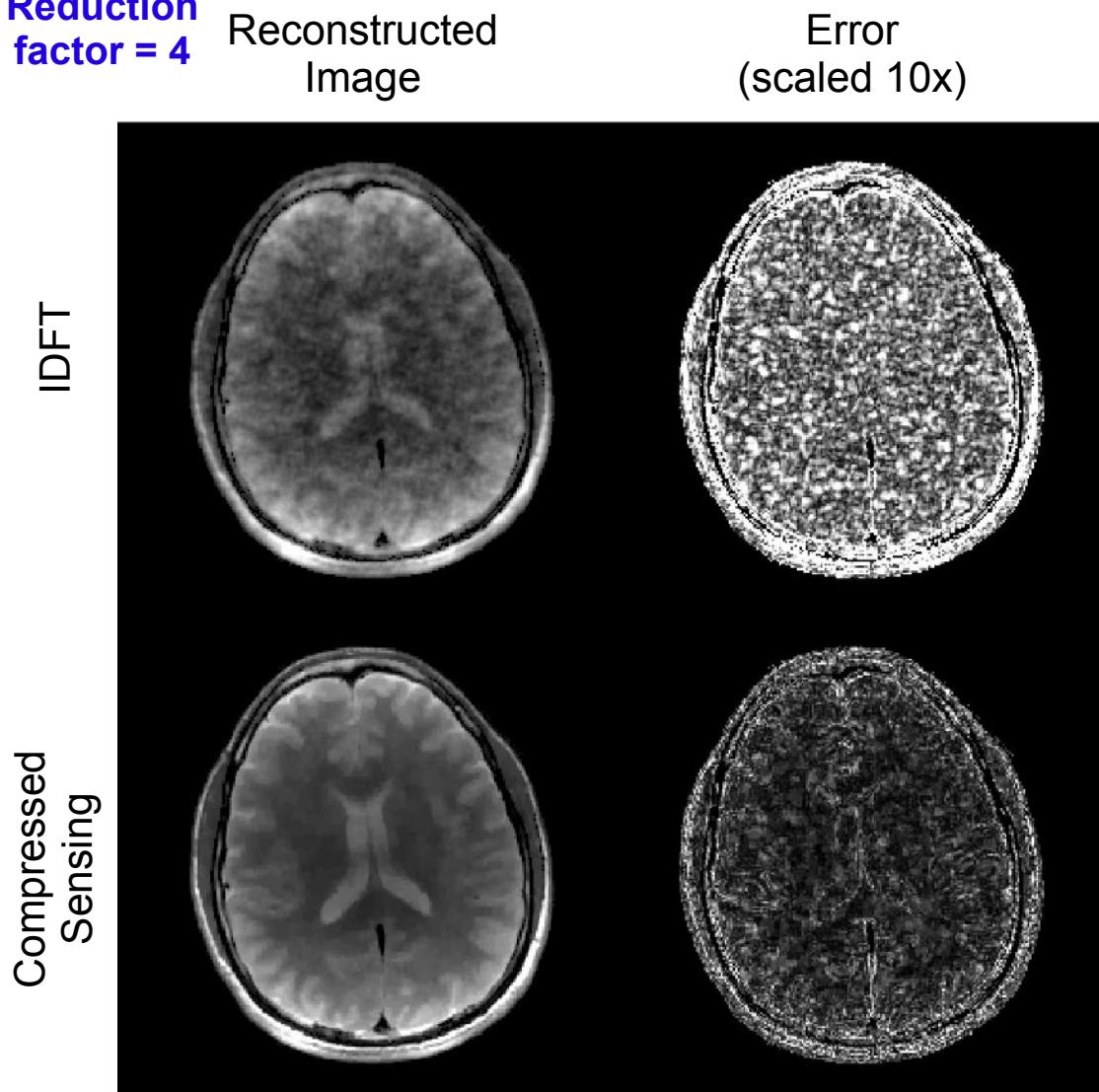


$$\hat{x} = \arg \min_x \frac{1}{2} \|M\hat{F}x - y\|_2^2 + \lambda \|Gx\|_1$$

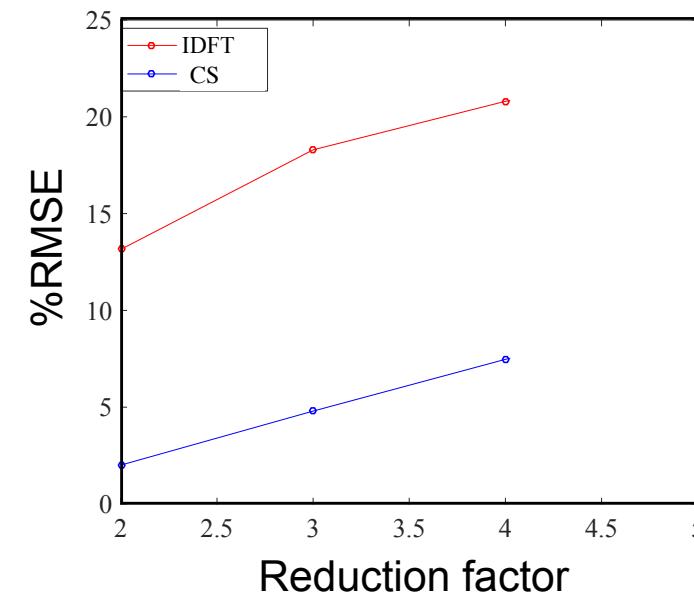


# Compressed Sensing

Reduction  
factor = 4

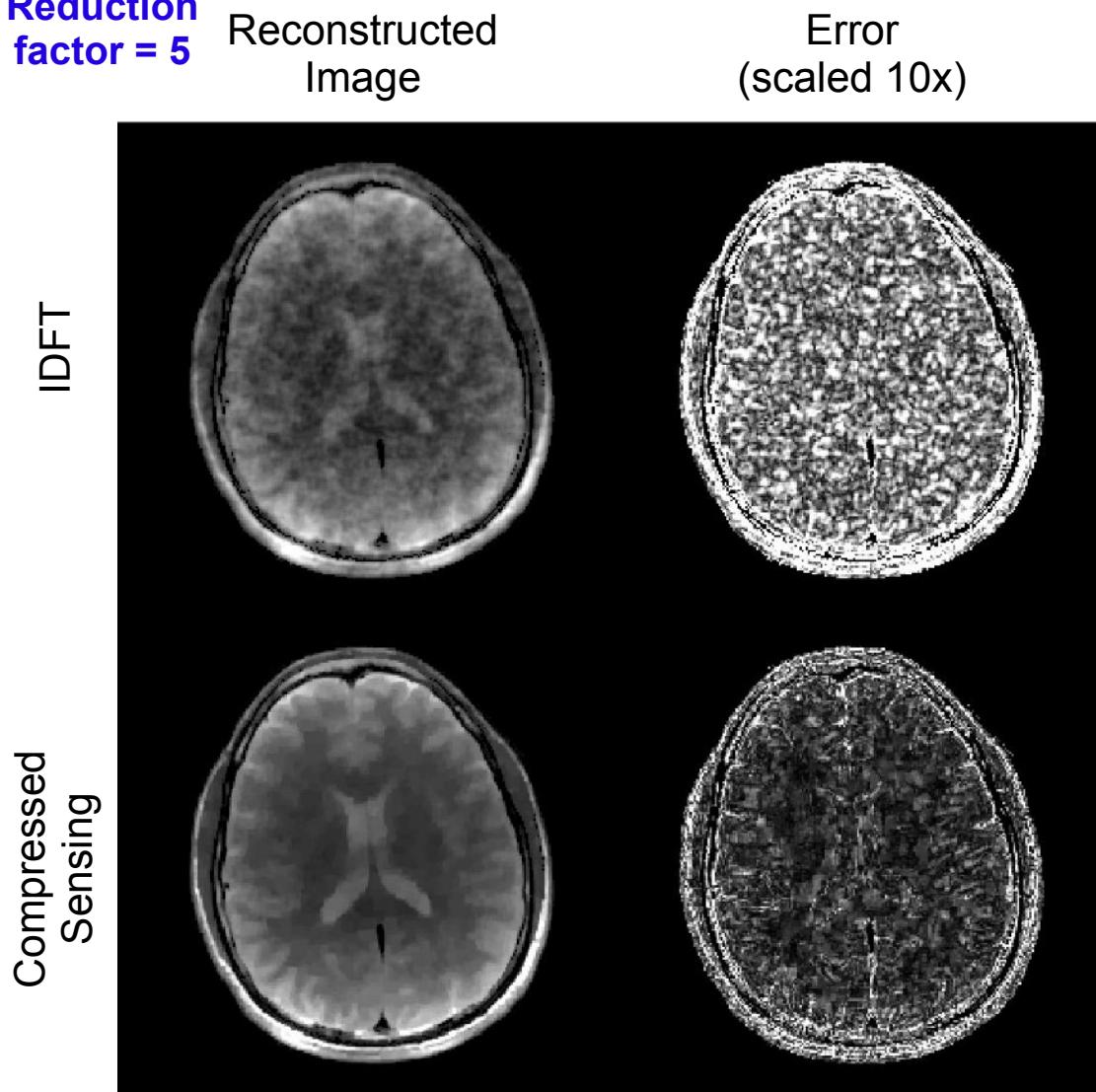


$$\hat{x} = \arg \min_x \frac{1}{2} \|M\hat{F}x - y\|_2^2 + \lambda \|Gx\|_1$$

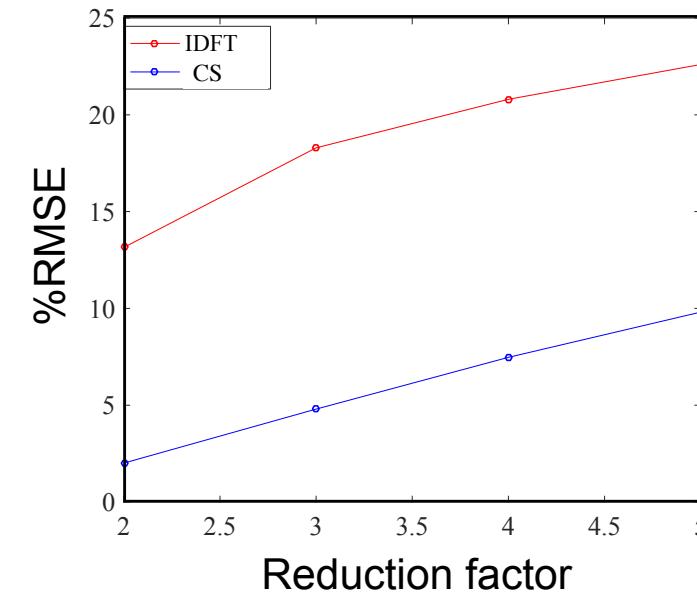


# Compressed Sensing

Reduction  
factor = 5



$$\hat{x} = \arg \min_x \frac{1}{2} \|M\hat{F}x - y\|_2^2 + \lambda \|Gx\|_1$$



# Compressed Sensing

$$\hat{x} = \arg \min_x \frac{1}{2} \|M F x - y\|_2^2 + \lambda \|G x\|_1$$

Reduction factor = 1

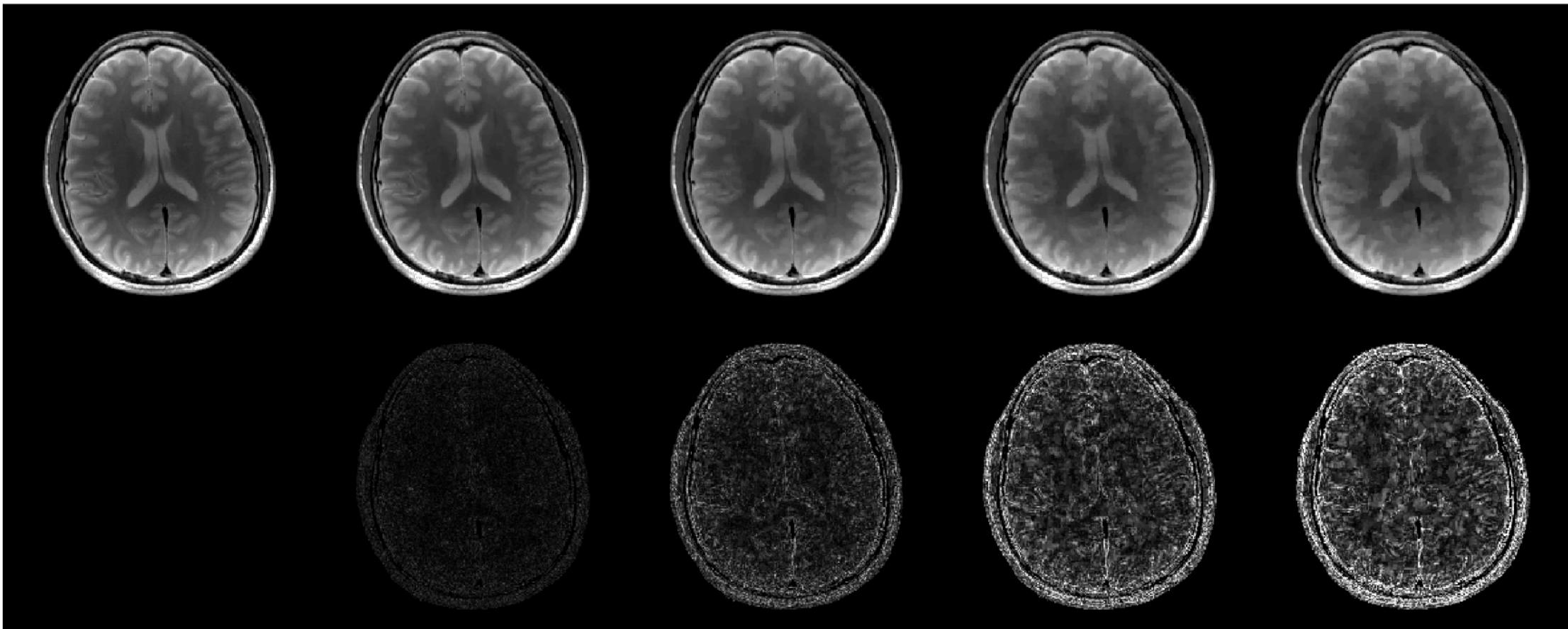
Reduction factor = 2

Reduction factor = 3

Reduction factor = 4

Reduction factor = 5

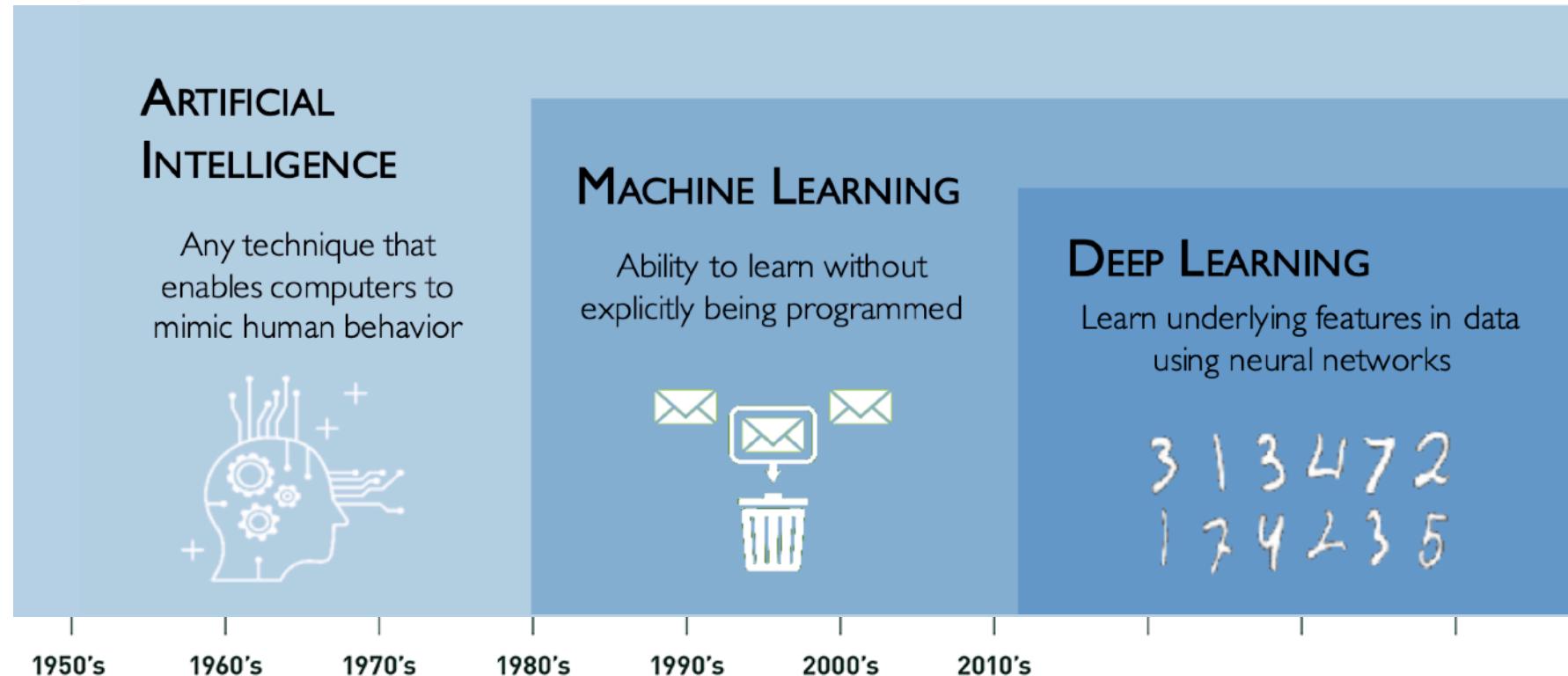
Reconstructed Image



Error (scaled 10x)

# Outline

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Fasting blood  
sugar test

$x$  mg/dL

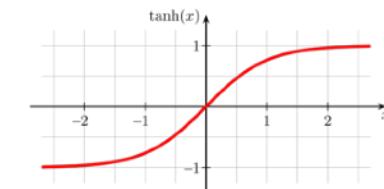
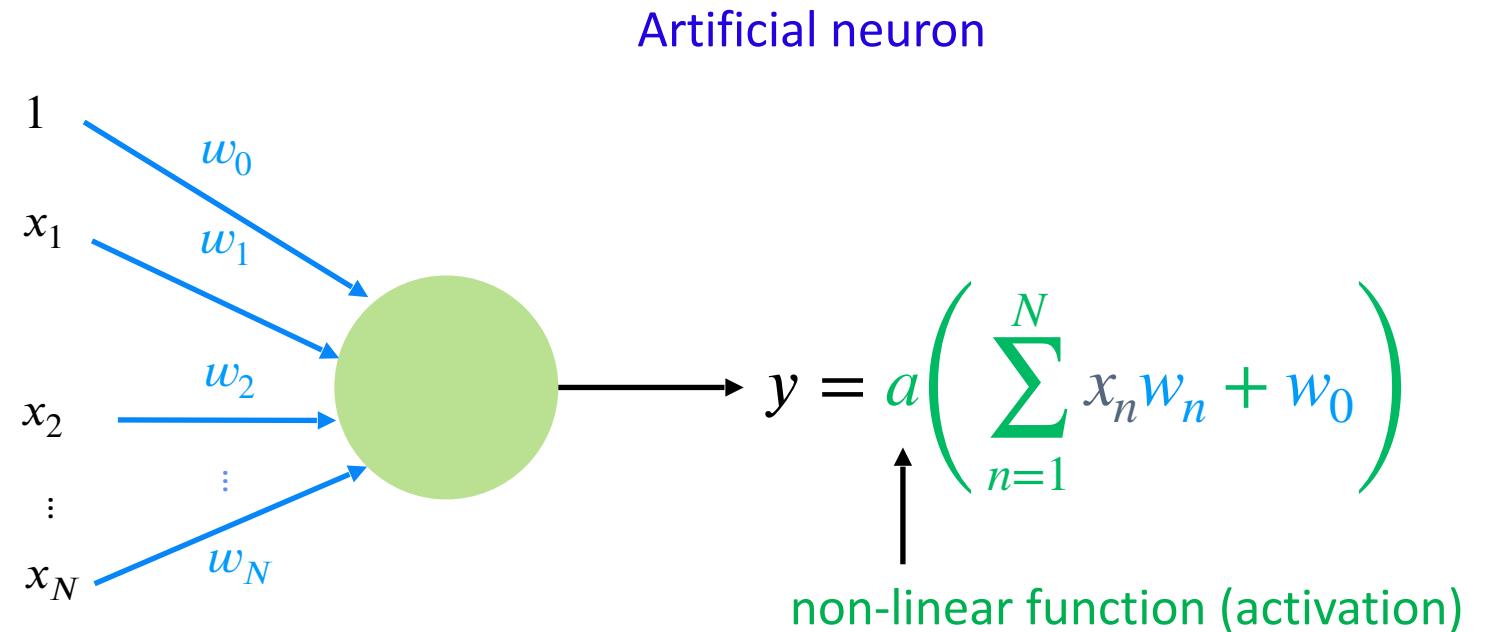
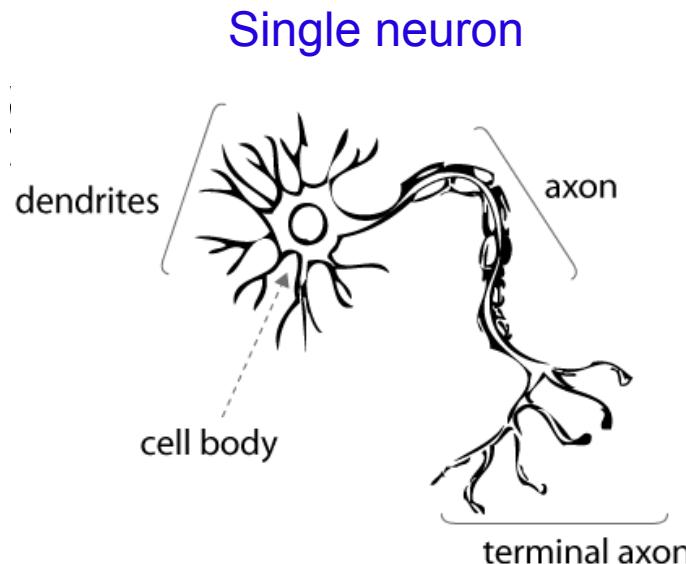
normal if  $x < 100$

prediabetes if  $100 \leq x \leq 125$

diabetes if  $125 < x$

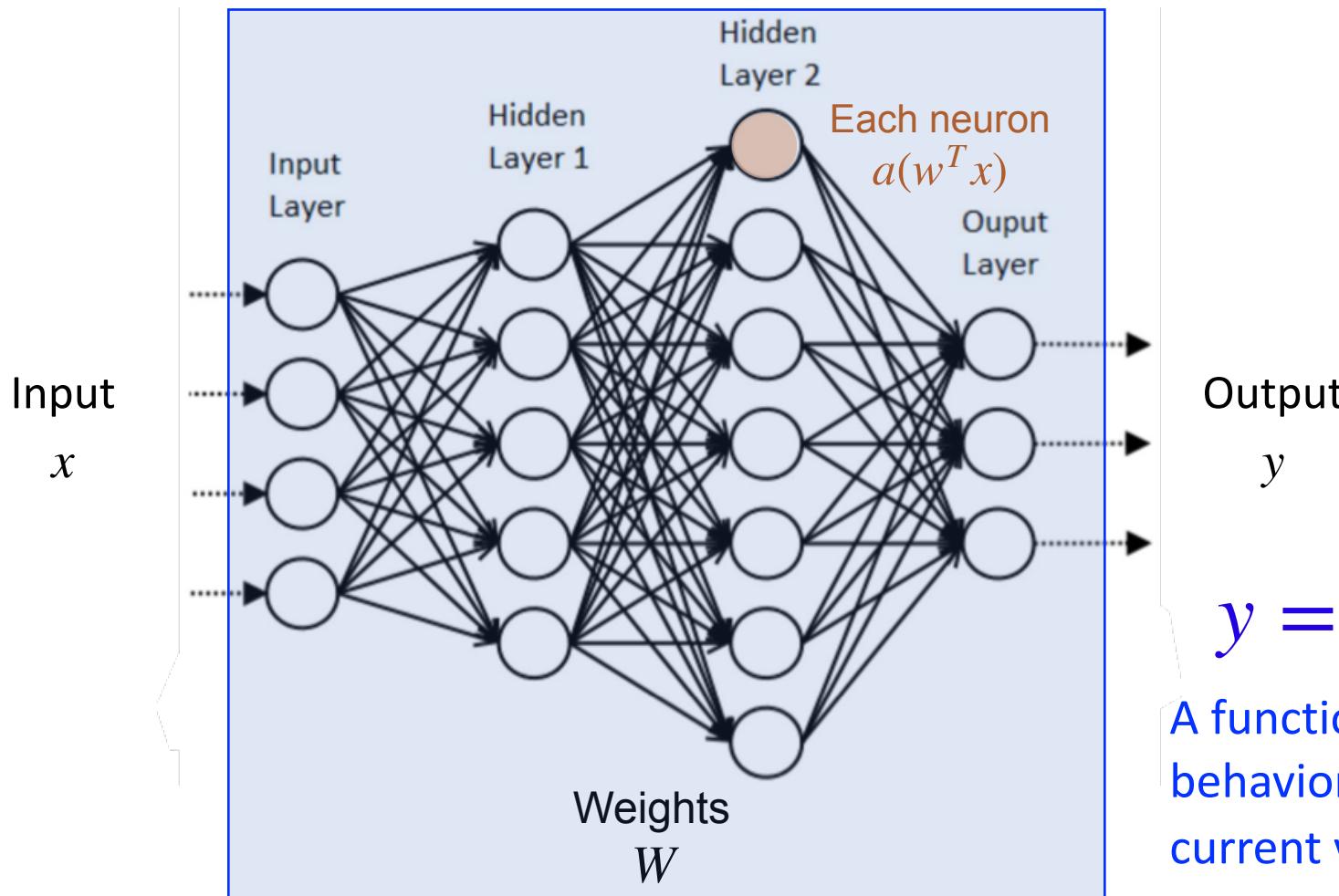
# Deep Learning

- Deep learning is a subfield of machine learning that stems from artificial neural networks (ANN)



# Deep Learning

- An artificial neural network with **many hidden layers** is called a **deep** artificial neural network (ANN)



# Deep Learning

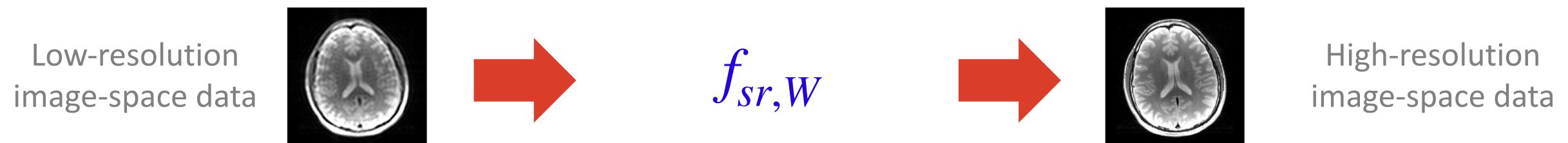
- We can use a deep artificial neural network to approximate any functions by modifying its weight
  - MRI reconstruction function



- MRI segmentation function

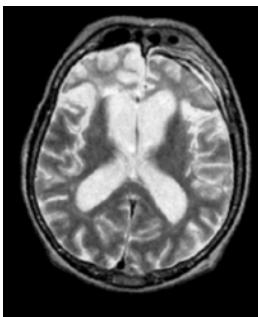
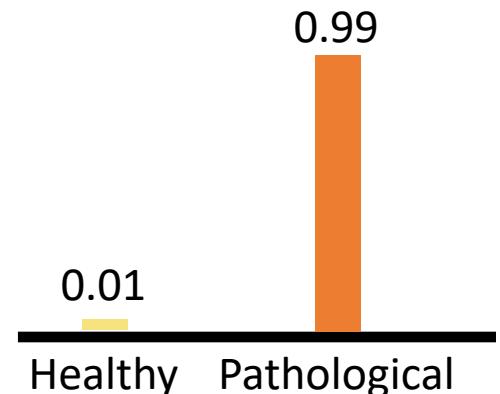
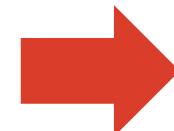


- MRI super-resolution function

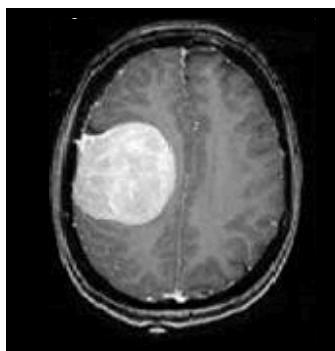
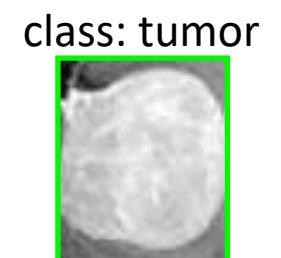
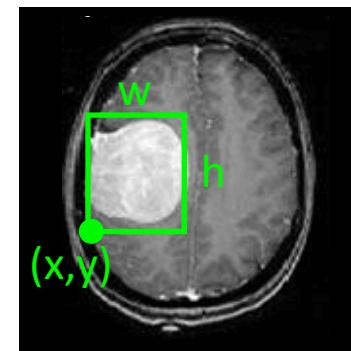


# Deep Learning

- We can use a deep artificial neural network to approximate any functions by modifying its weight
  - MR image classification function

 $f_{classif, W}$ 

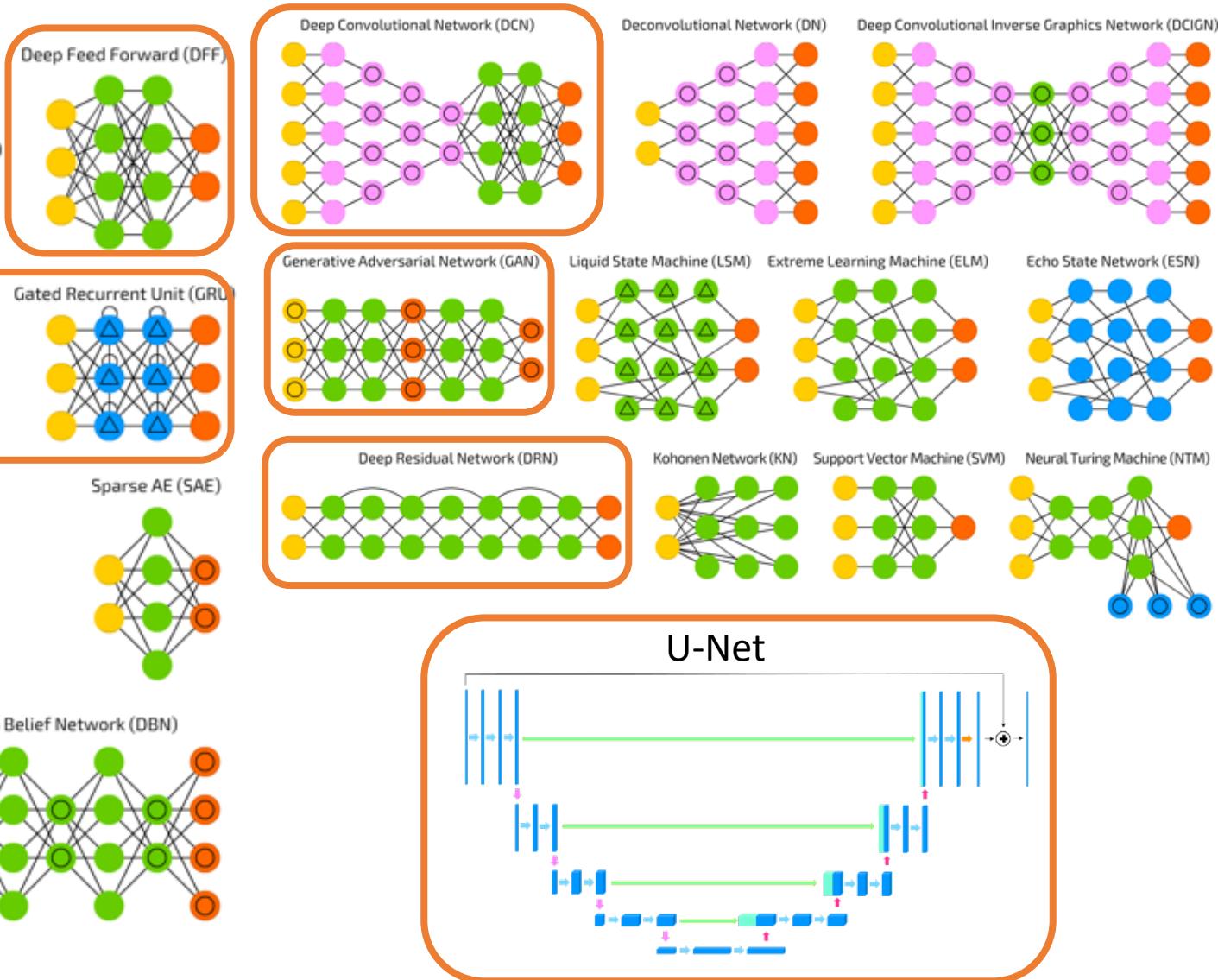
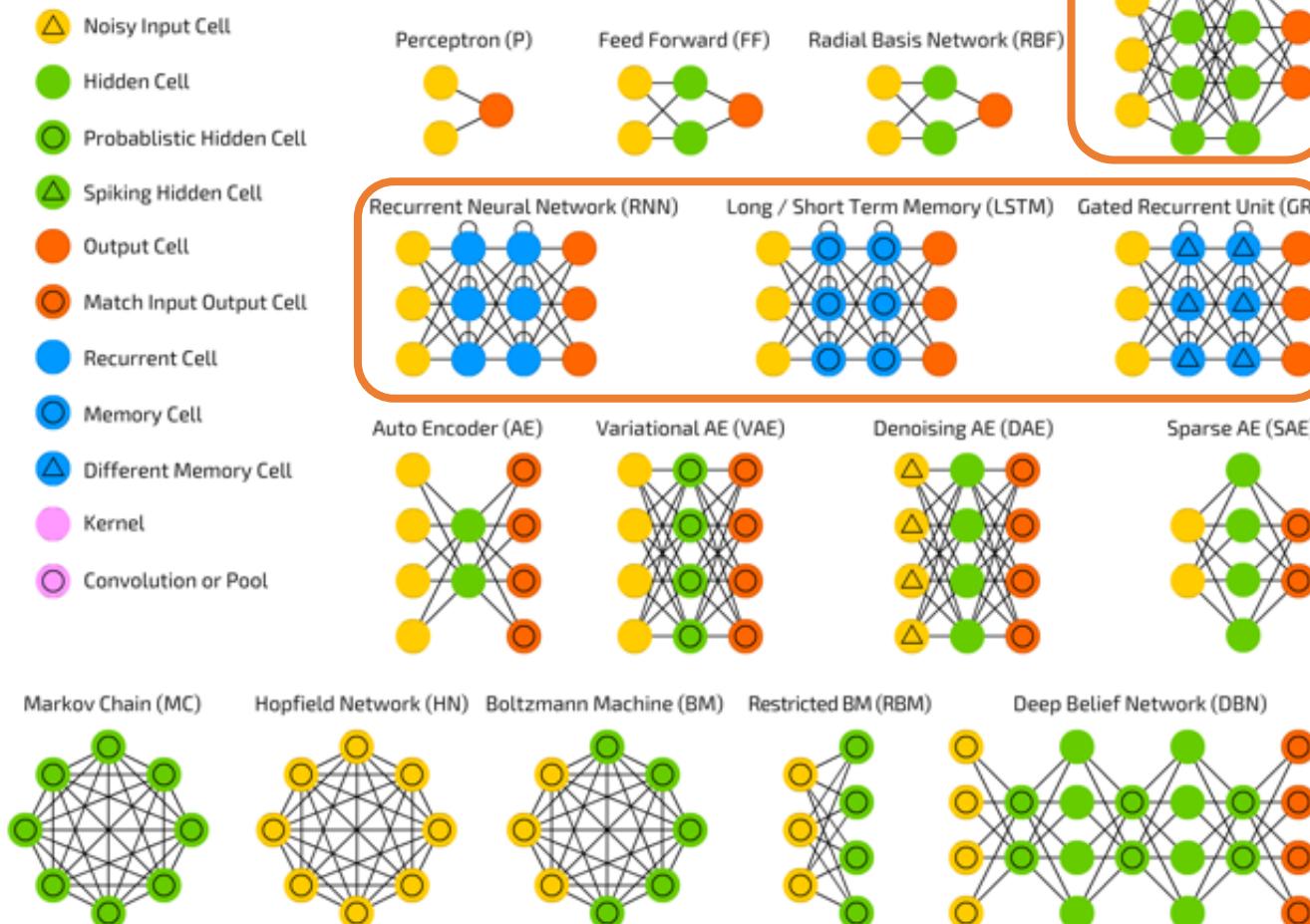
- Detection function

 $f_{detection, W}$ 

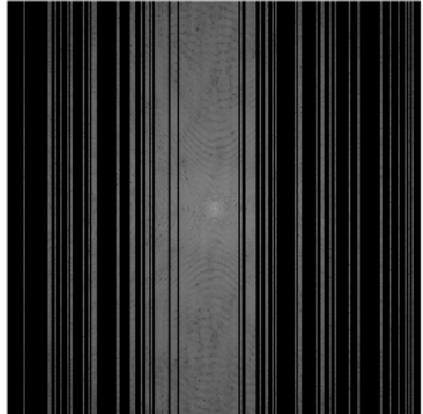
# A mostly complete chart of Neural Networks

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- Backfed Input Cell
- Input Cell
- △ Noisy Input Cell
- Hidden Cell
- Probabilistic Hidden Cell
- △ Spiking Hidden Cell
- Output Cell
- Match Input Output Cell
- Recurrent Cell
- Memory Cell
- △ Different Memory Cell
- Kernel
- Convolution or Pool



# Deep Learning for MR Image Reconstruction

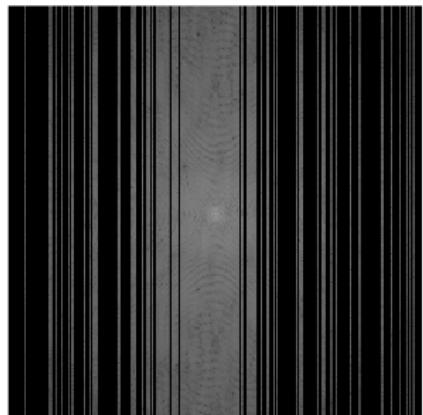
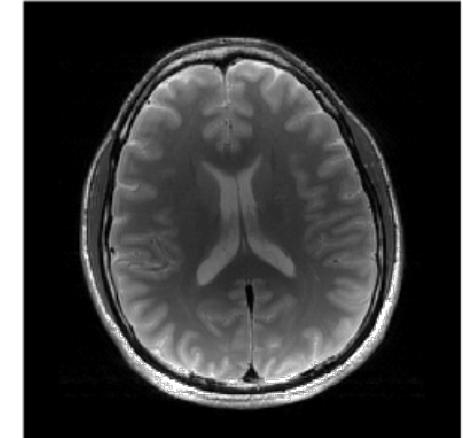


Underlying Process

$g_{recon}$

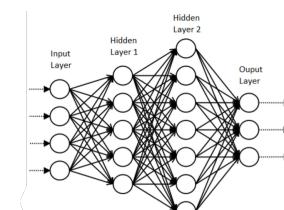
An *unknown* highly complicated function that includes (but not limited to)

- Inverse Fourier transform
- Artifact removal
- Magnetic field map correction

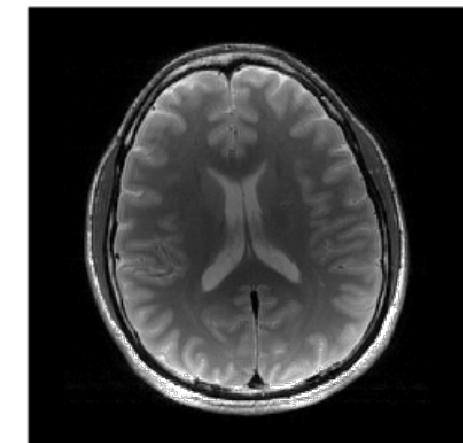


Deep Learning

$f_W$

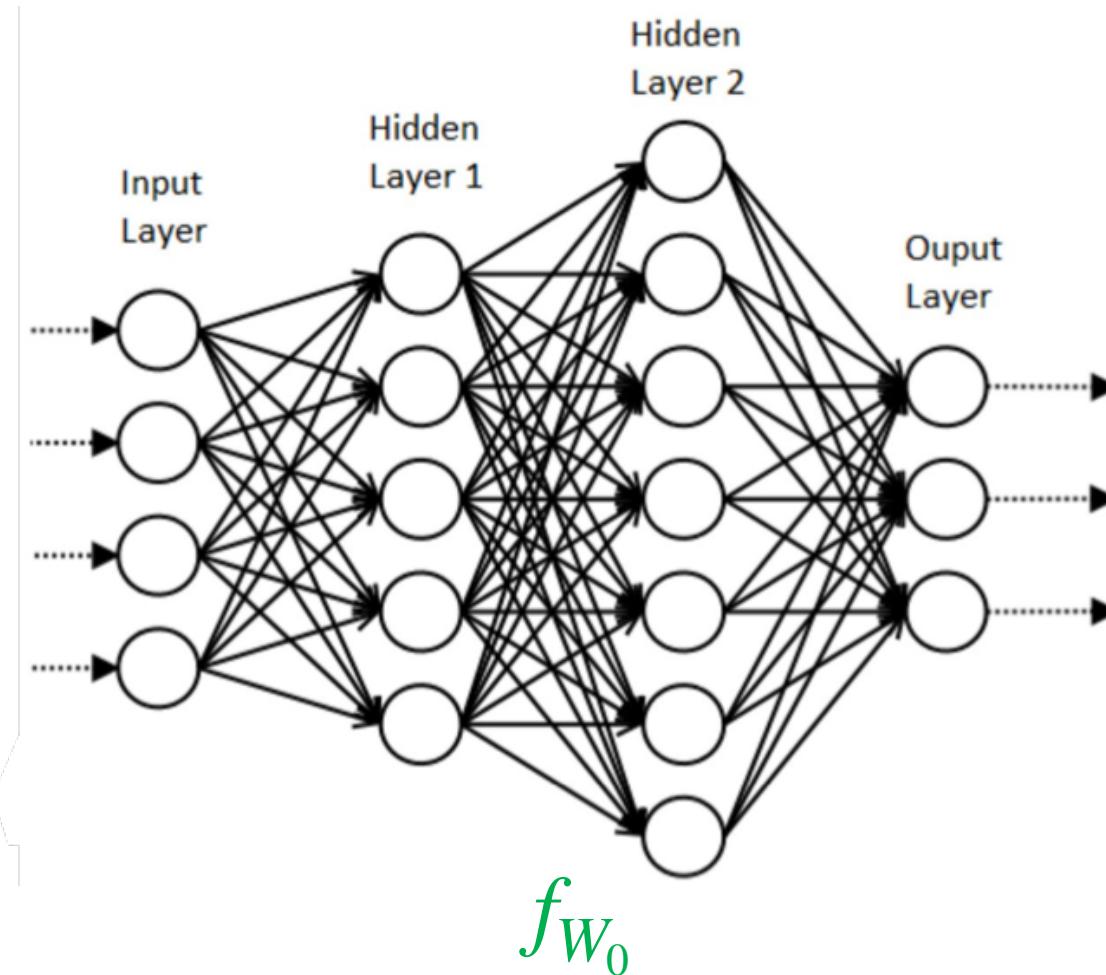


With model training, we could obtain  
 $f_W \approx g_{recon}$



# Supervised Model Training

Step 1: Create a neural network with some initial weight  $f_{W_0}$

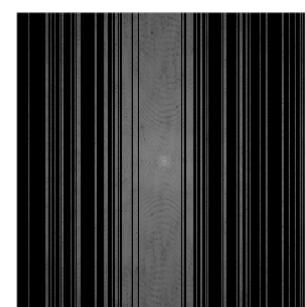
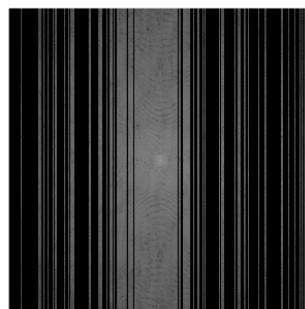
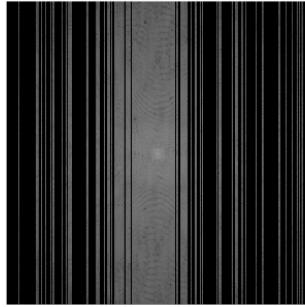


*Keywords: dataset preparation, target, labels, ground truth, true, input-output pairs*

# Supervised Model Training

Prepared input

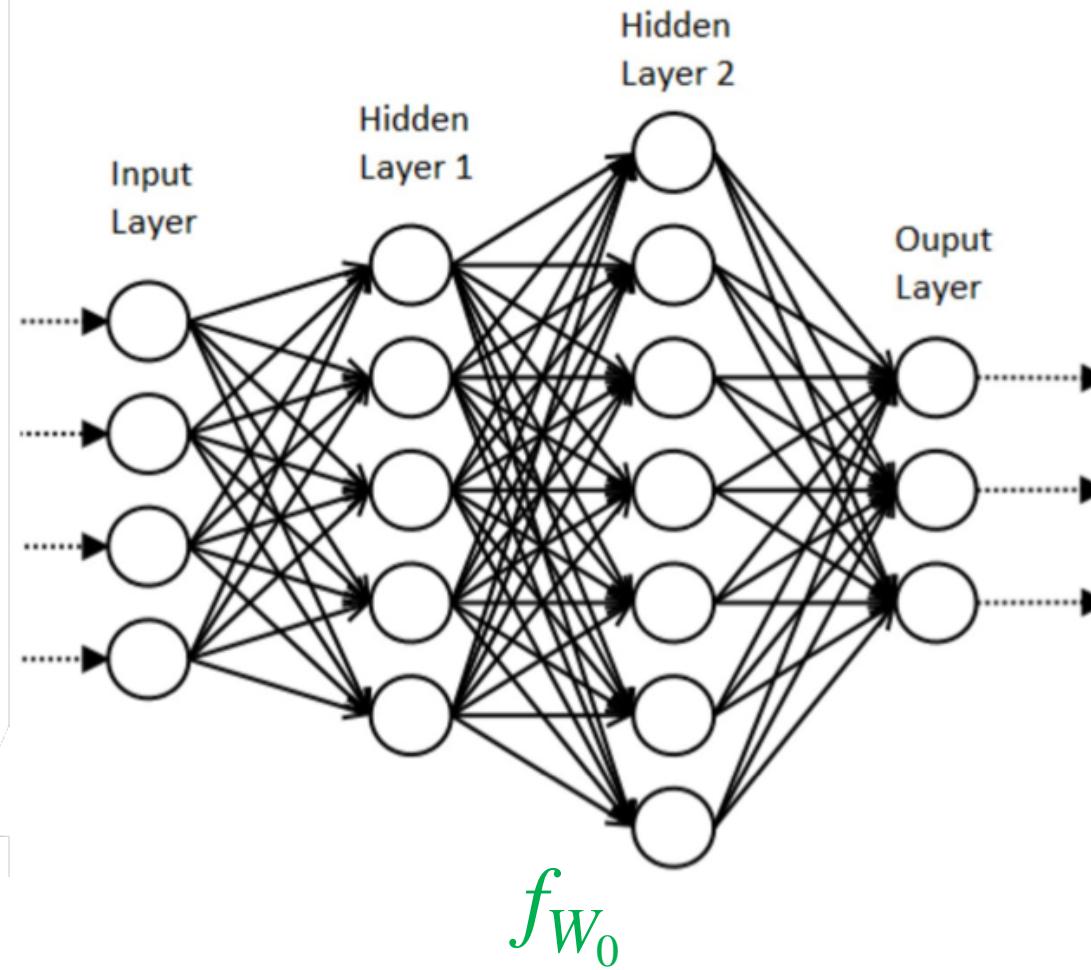
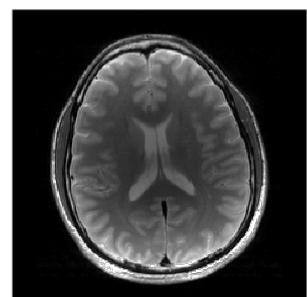
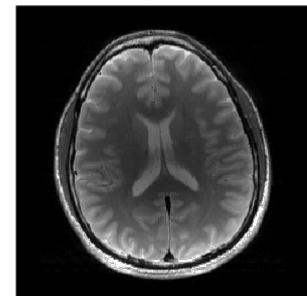
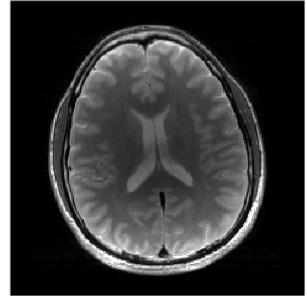
$x$



Step 2: Prepare a dataset which is a collection of input-output pairs

Prepared output

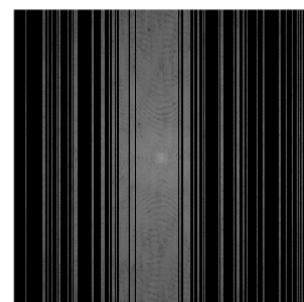
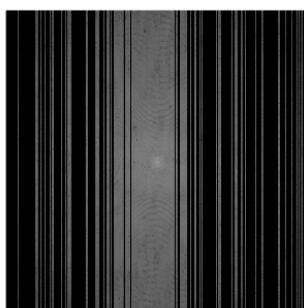
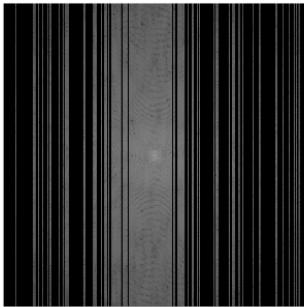
$y$



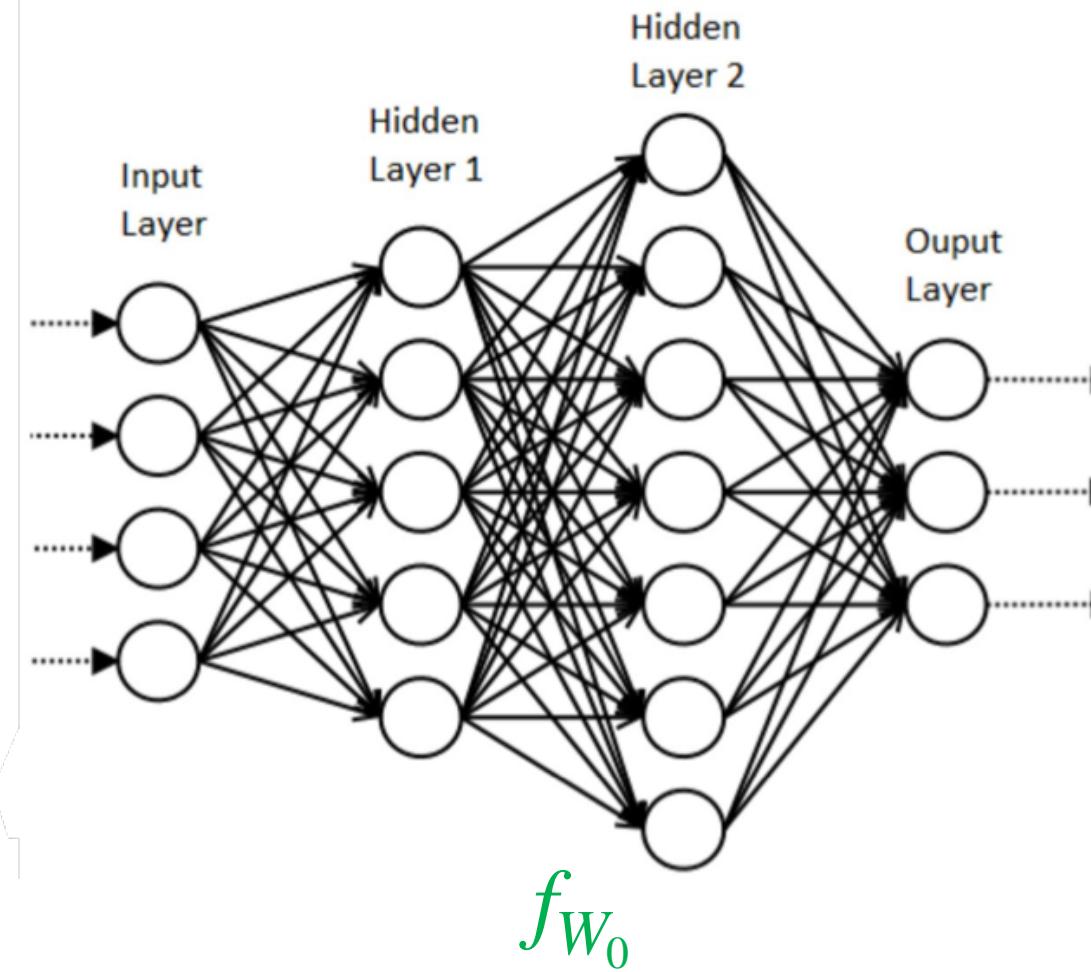
# Supervised Model Training

Prepared input

$x$

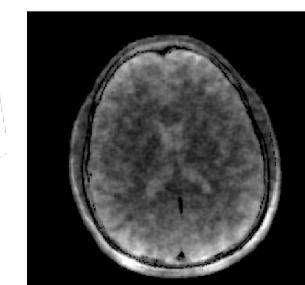
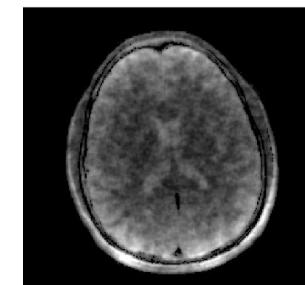
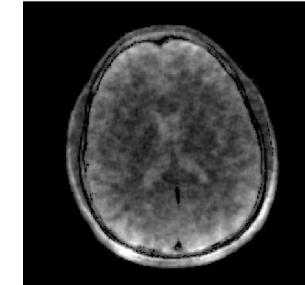


Step 3: Pass the prepared input to the network



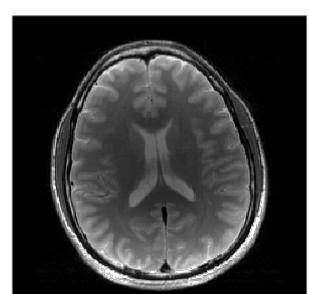
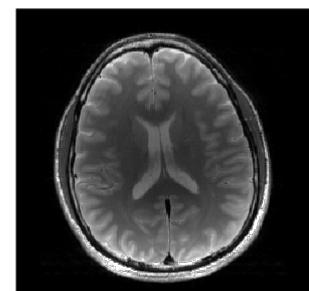
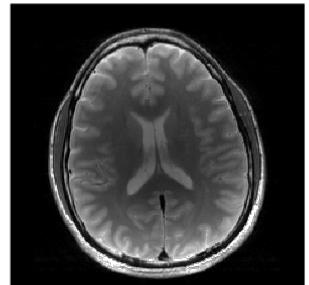
Estimated output

$$\hat{y} = f(x)$$



Prepared output

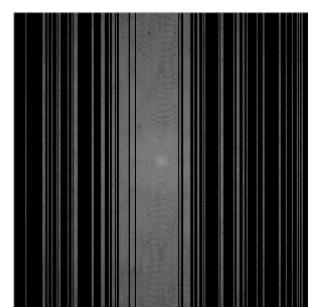
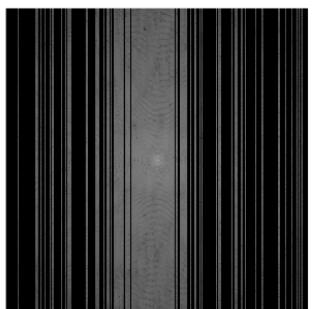
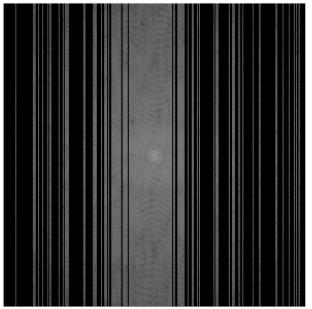
$y$



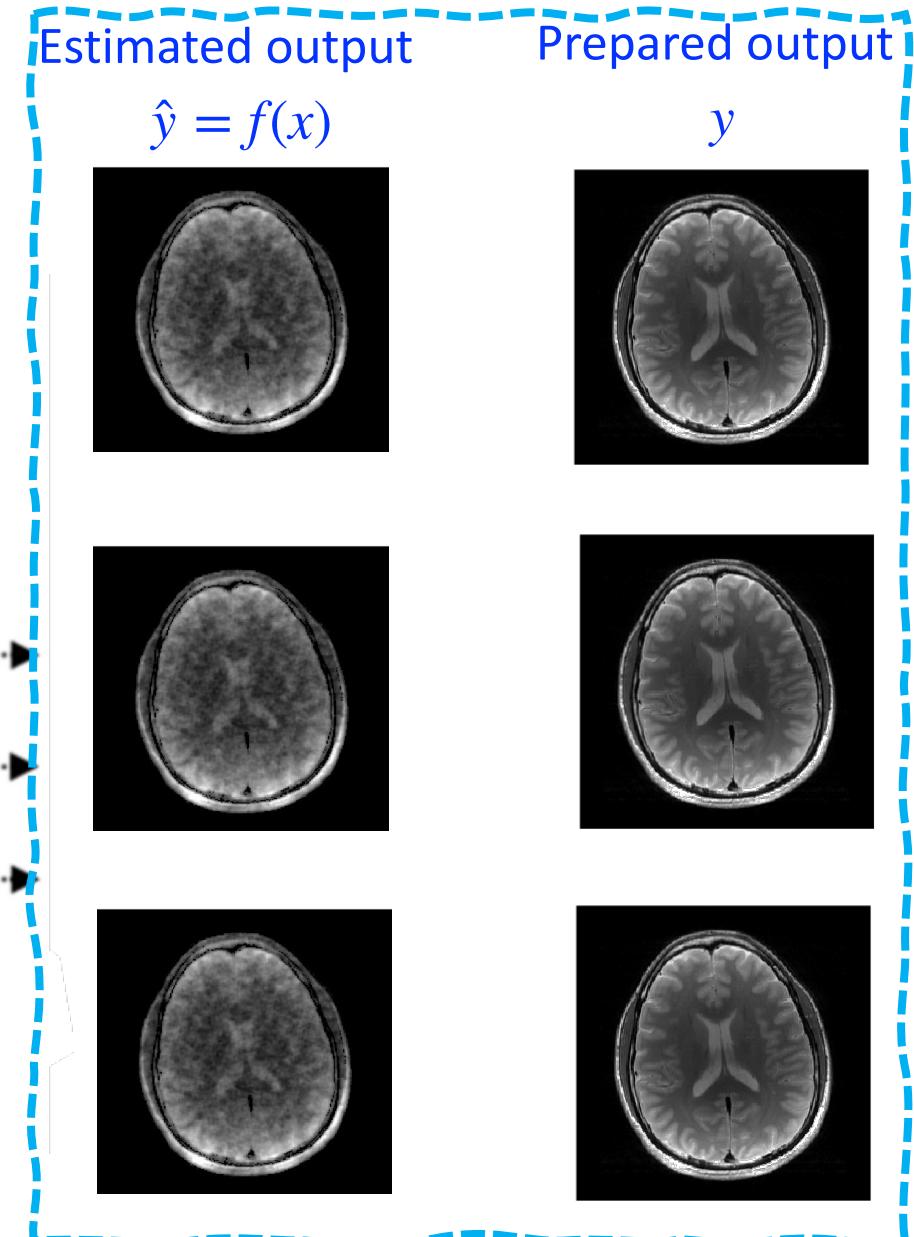
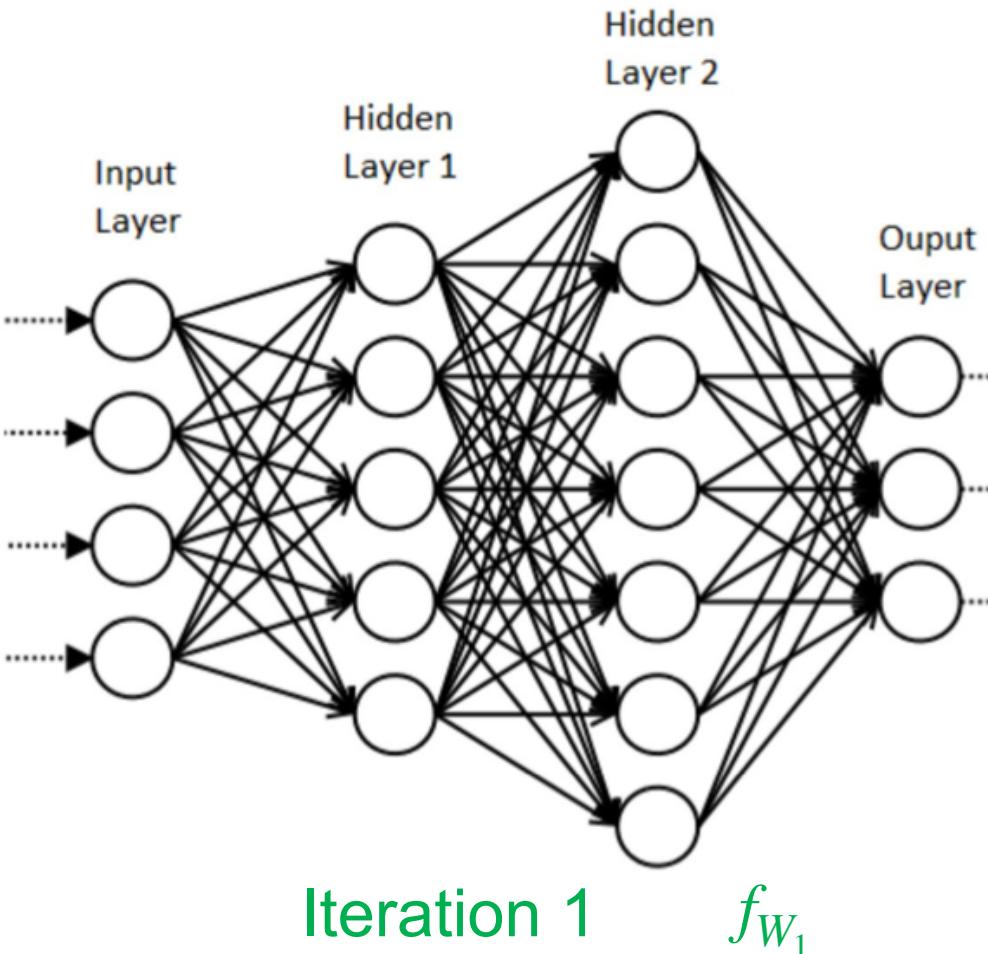
# Supervised Model Training

Prepared input

$x$

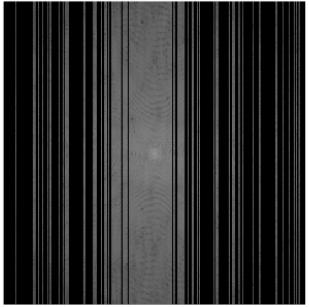


Step 4: Compare  $\hat{y}$  to  $y$  and modify the weights of the neural network to make  $\hat{y}$  approach  $y$  using the backpropagation algorithm

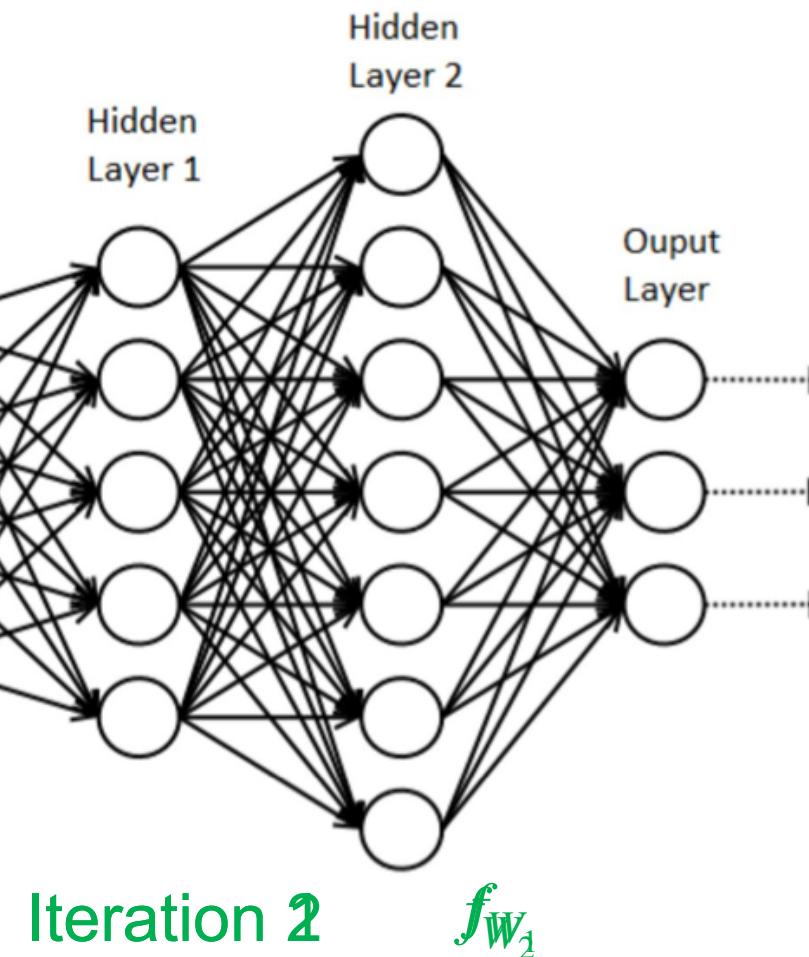


# Supervised Model Training

Prepared input  
 $x$

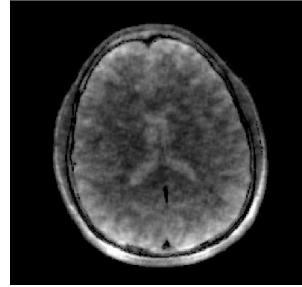


Repeat steps 3 and 4 to continuously improve the weights of the neural network



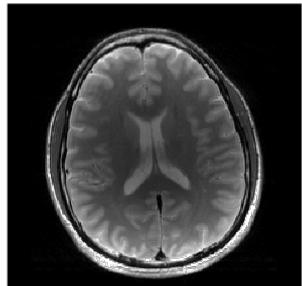
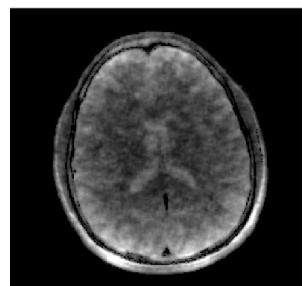
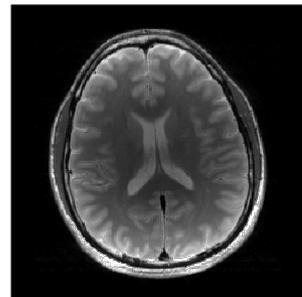
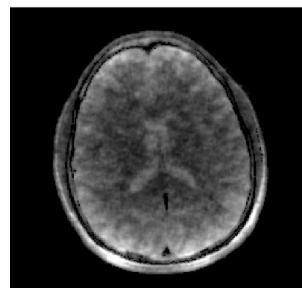
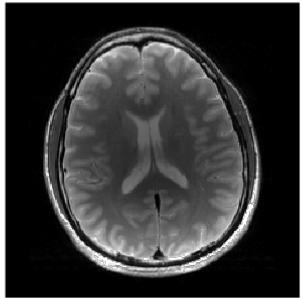
Estimated output

$$\hat{y} = f(x)$$



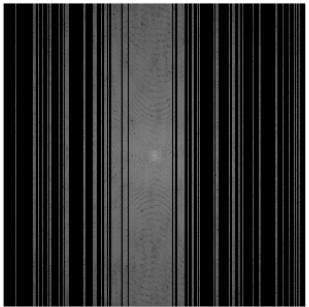
Prepared output

$$y$$

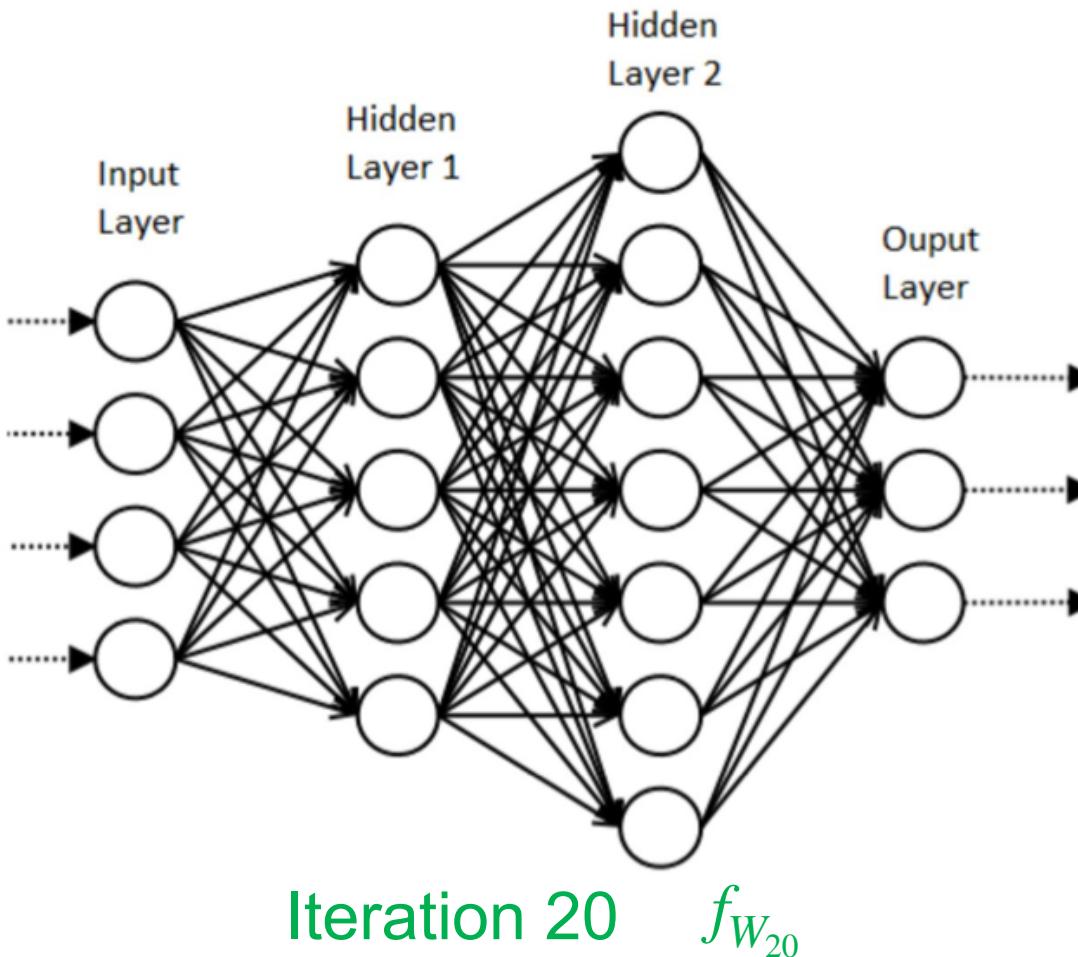


# Supervised Model Training

Prepared input  
 $x$

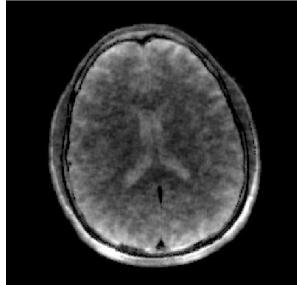


Repeat steps 3 and 4 to continuously improve the weights of the neural network



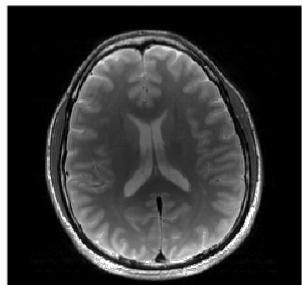
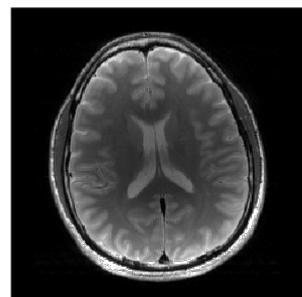
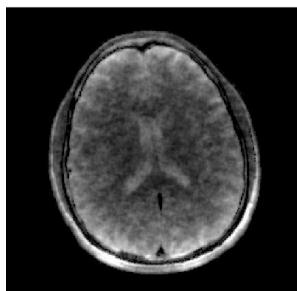
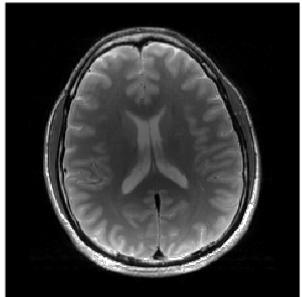
Estimated output

$$\hat{y} = f(x)$$



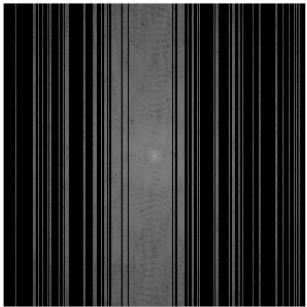
Prepared output

$$y$$

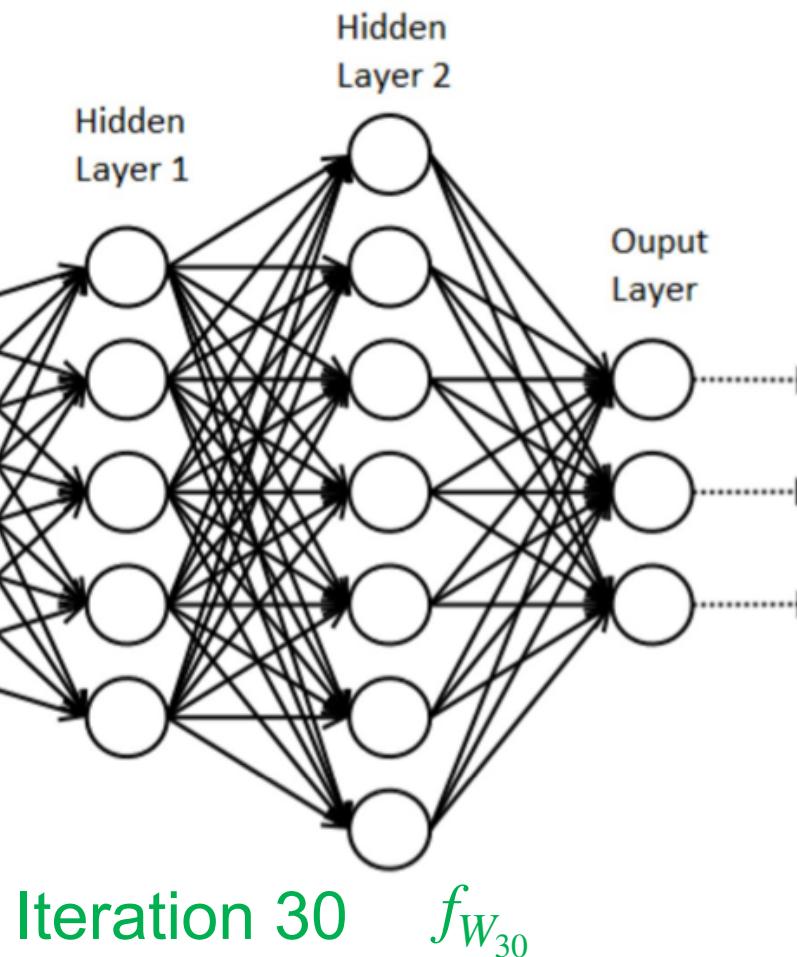


# Supervised Model Training

Prepared input  
 $x$

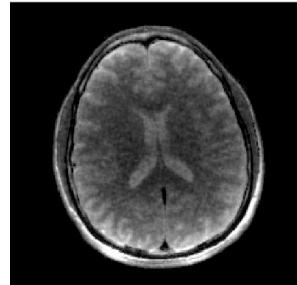


Repeat steps 3 and 4 to continuously improve the weights of the neural network



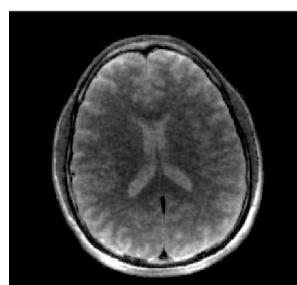
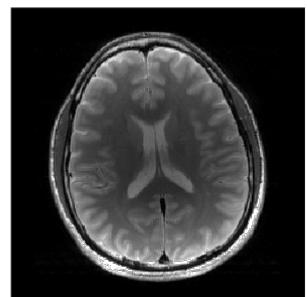
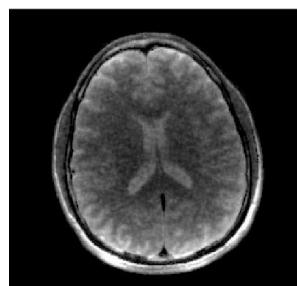
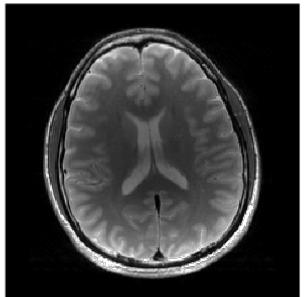
Estimated output

$$\hat{y} = f(x)$$



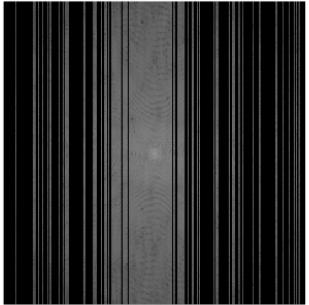
Prepared output

$$y$$

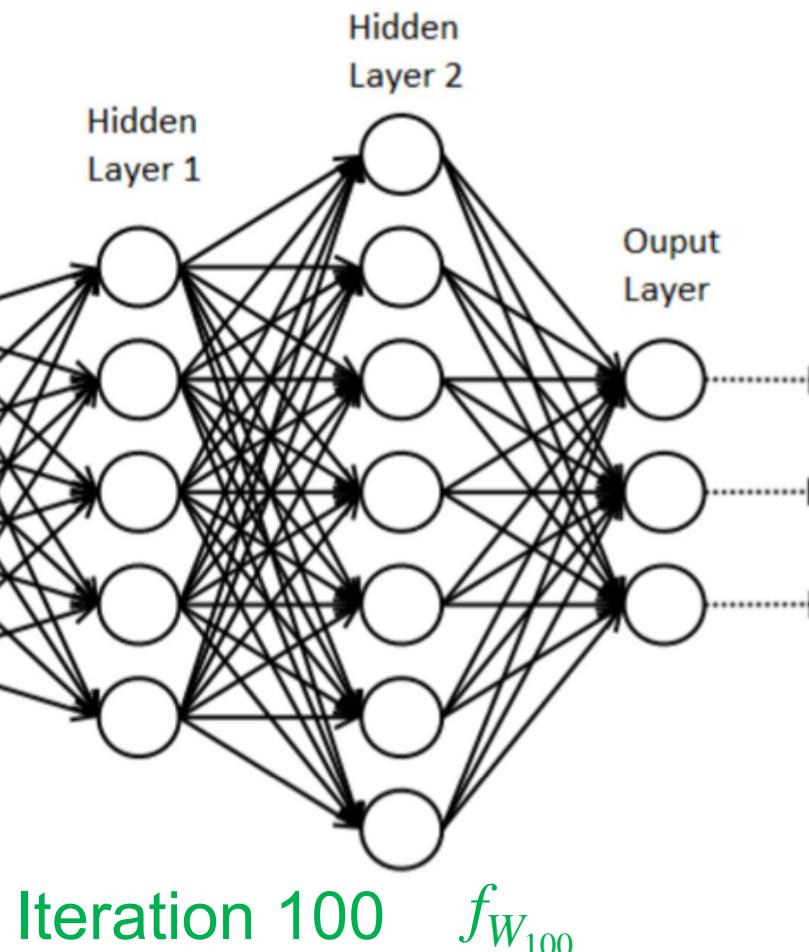


# Supervised Model Training

Prepared input  
 $x$

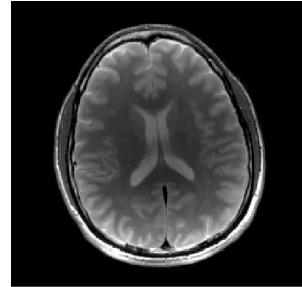


Repeat steps 3 and 4 to continuously improve the weights of the neural network

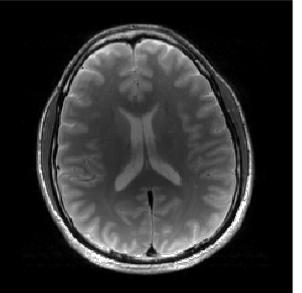


Estimated output

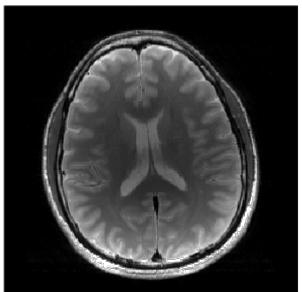
$$\hat{y} = f(x)$$



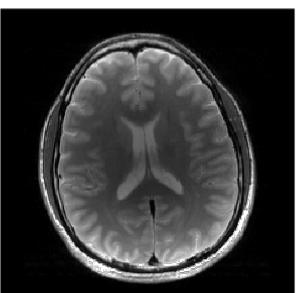
Prepared output  
 $y$



≈



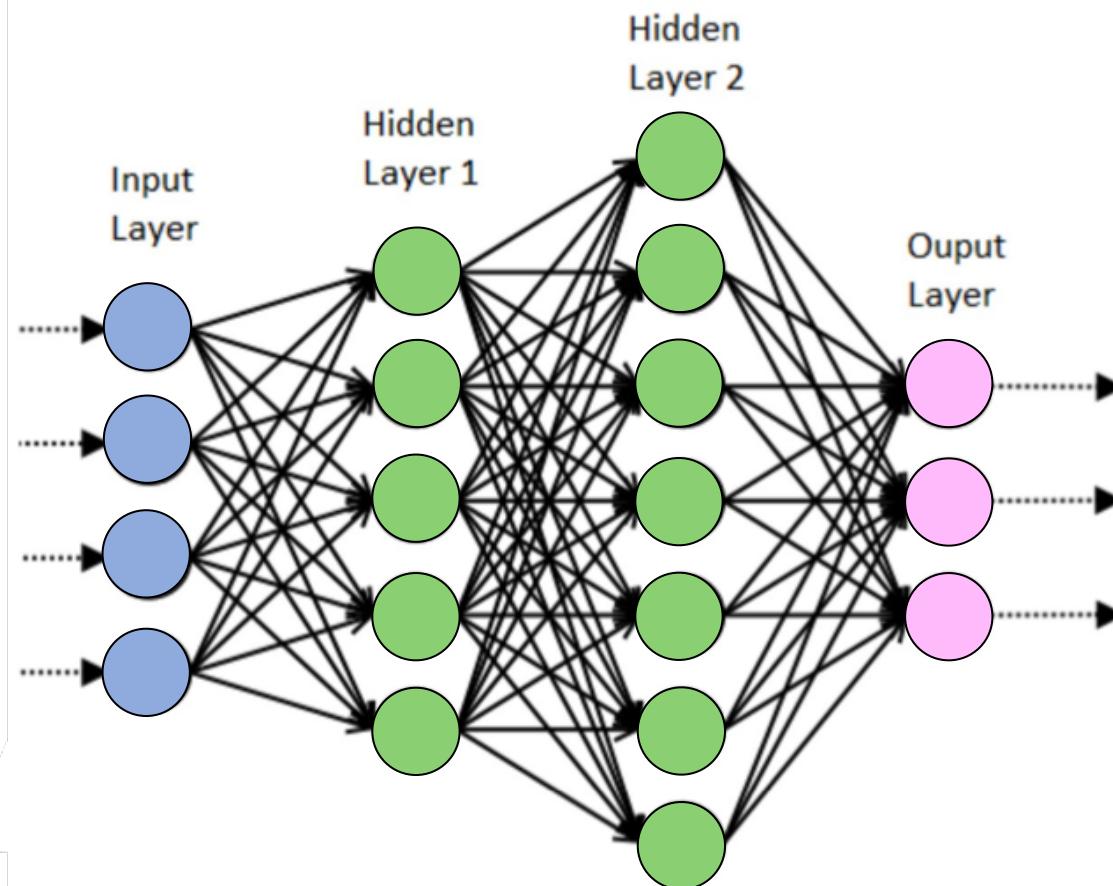
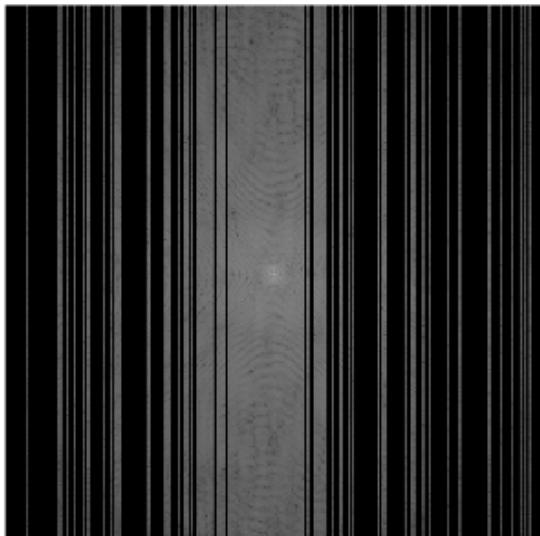
≈



≈

# Test the Trained Model

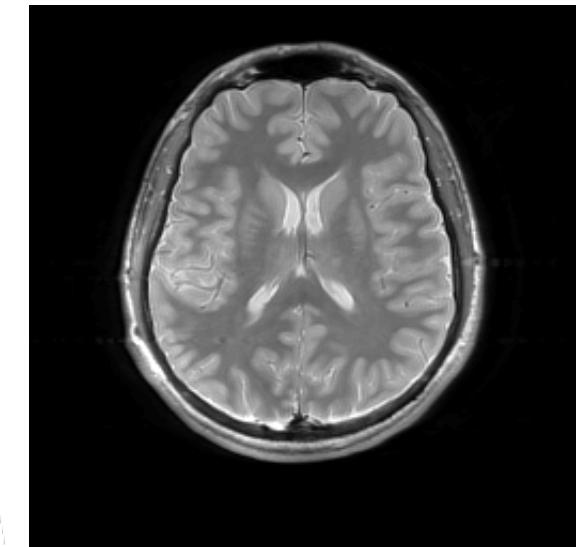
Acquired undersampled k-space data that you wish to reconstruct



Trained model

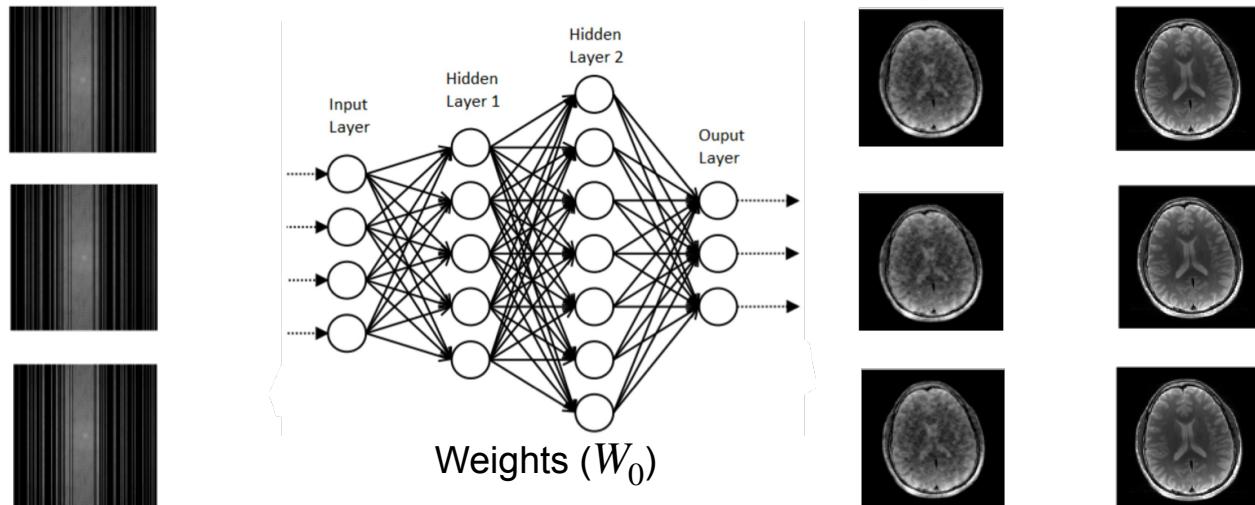
$$f_{W^*}$$

Reconstructed image

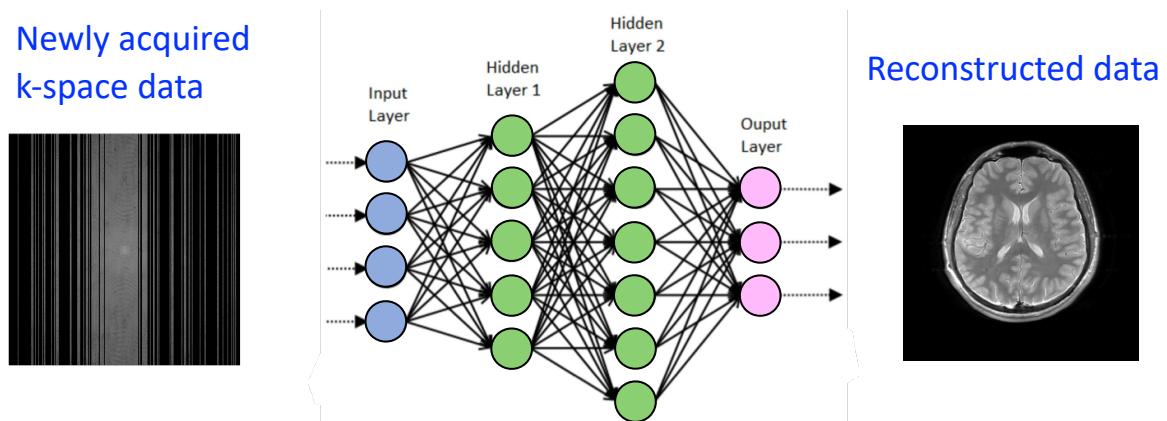


# Accelerated MRI Using Deep Learning

- Training phase: Optimize the weights of a deep neural network

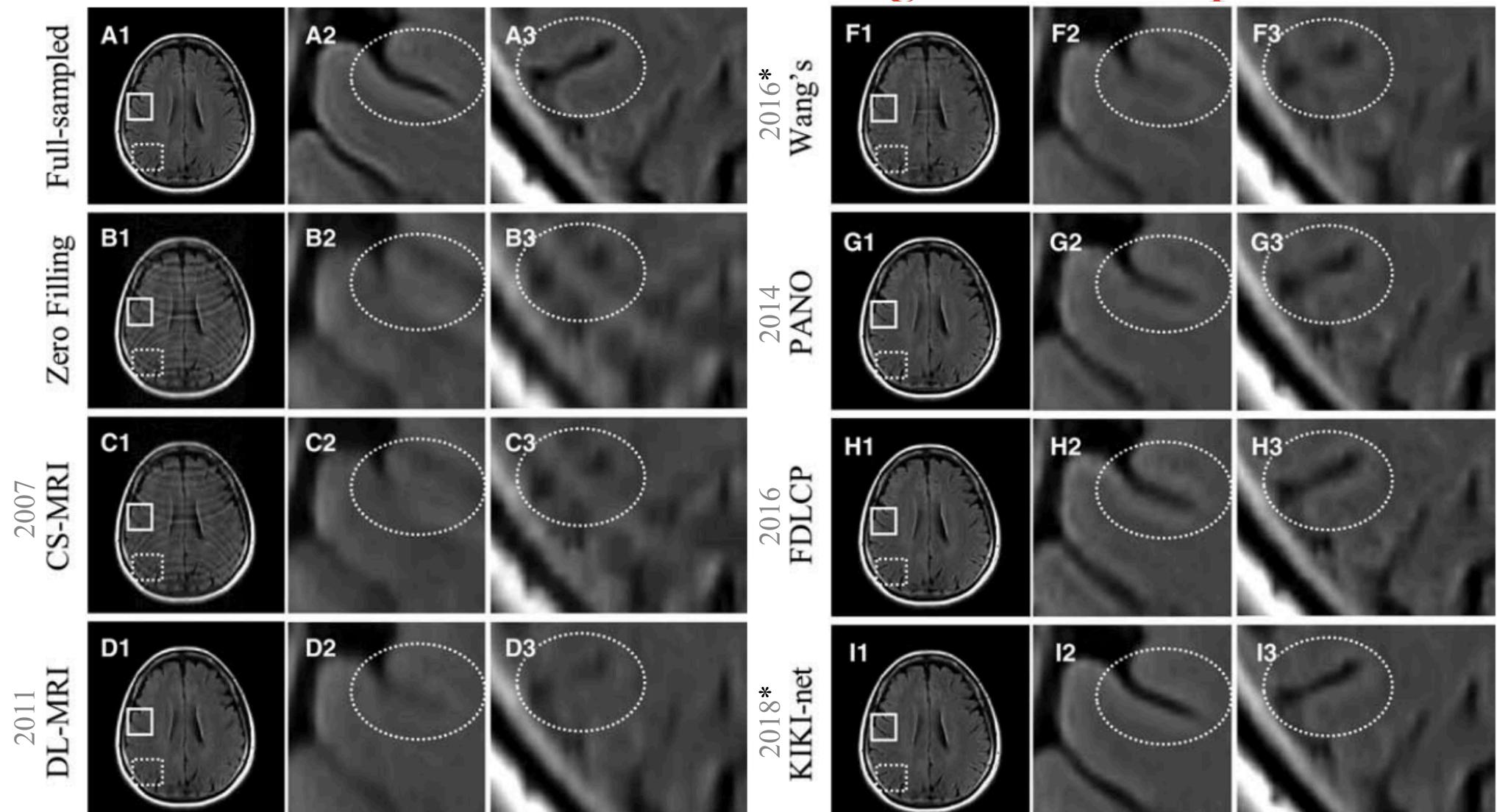
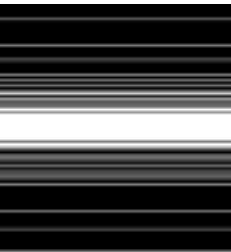


- Test phase: Reconstruct new data using the trained deep neural network



# Cross-domain CNNs for Reconstructing Undersampled MRI

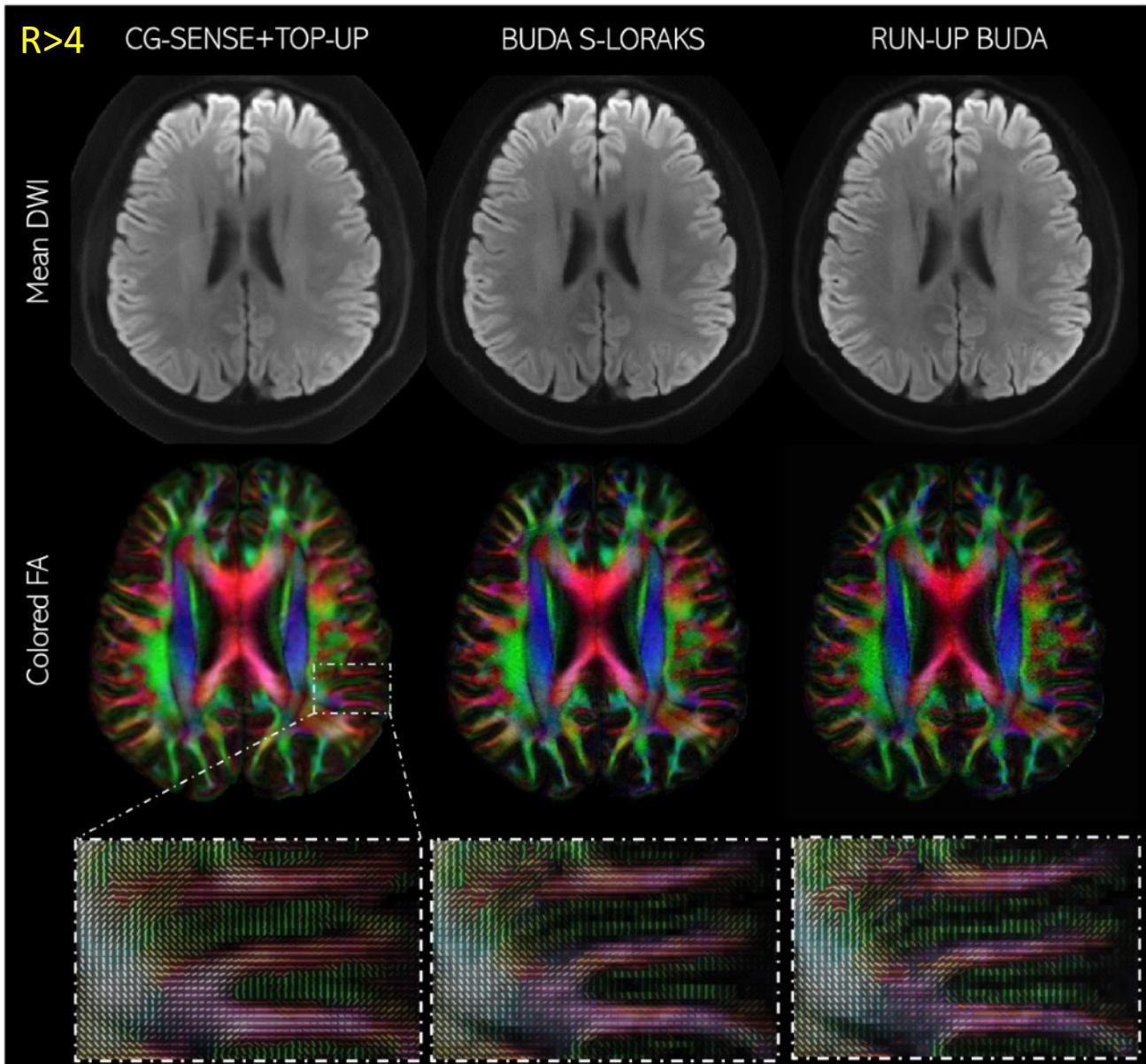
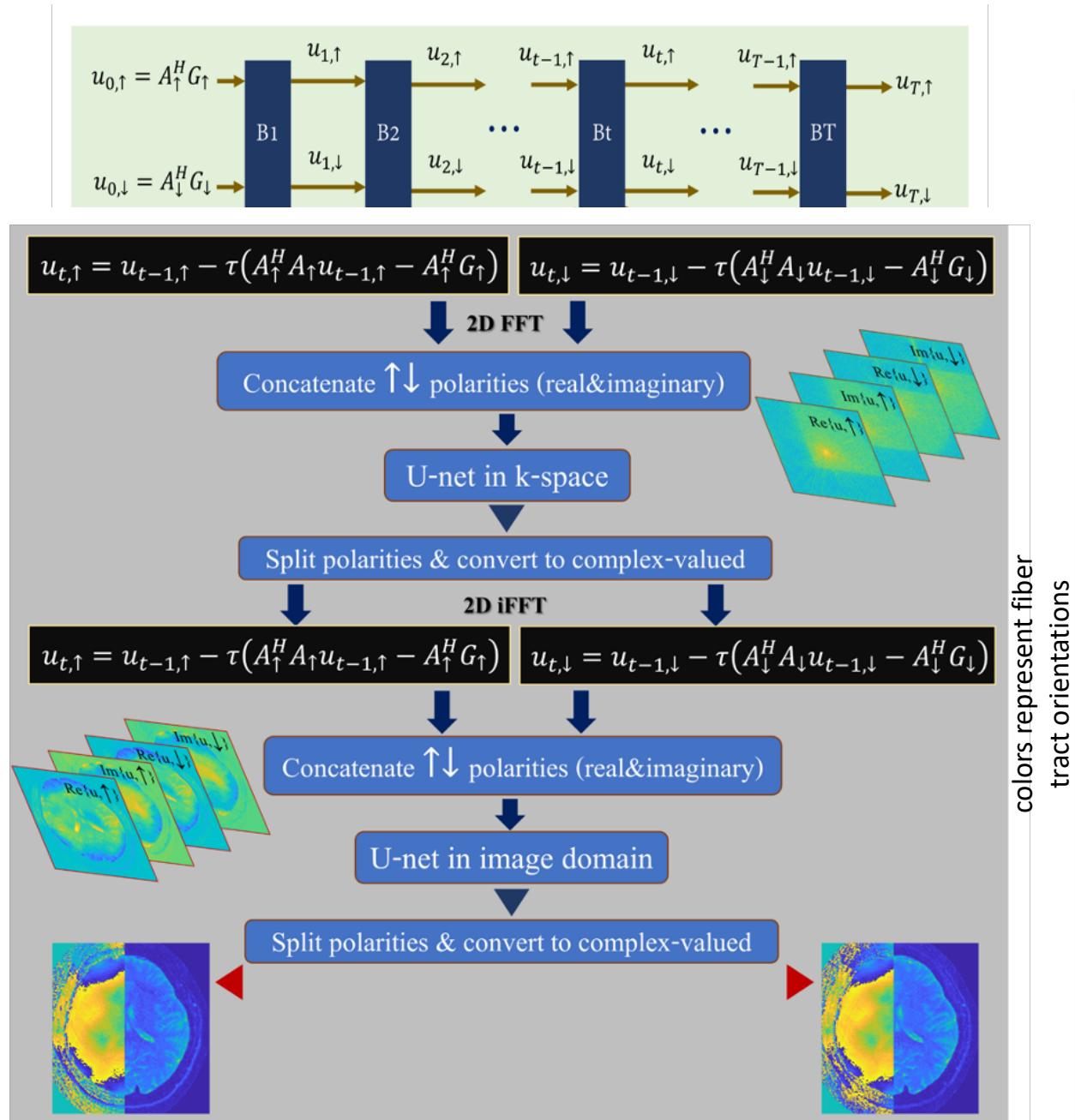
$T_2$ -FLAIR  
(R=4)



Eo, Taejoon, et al. "KIKI-net: cross-domain convolutional neural networks for reconstructing undersampled magnetic resonance images." Magnetic resonance in medicine 80.5 (2018): 2188-2201.

\*deep learning based methods

88x faster recon time



# Deep Learning

- We can use a deep artificial neural network to approximate any functions by modifying its weight
  - MRI reconstruction function



- MRI segmentation function

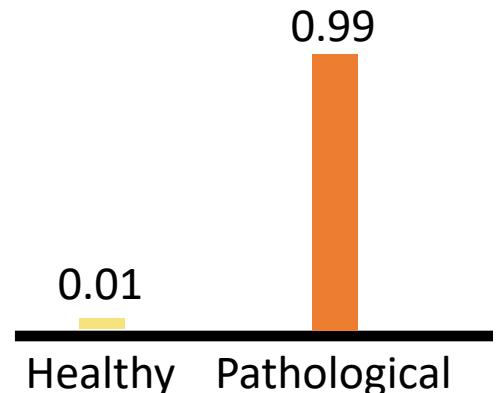
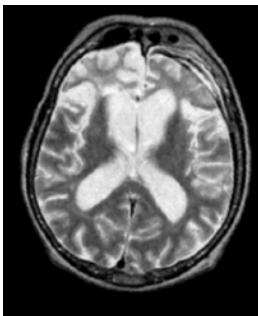


- MRI super-resolution function

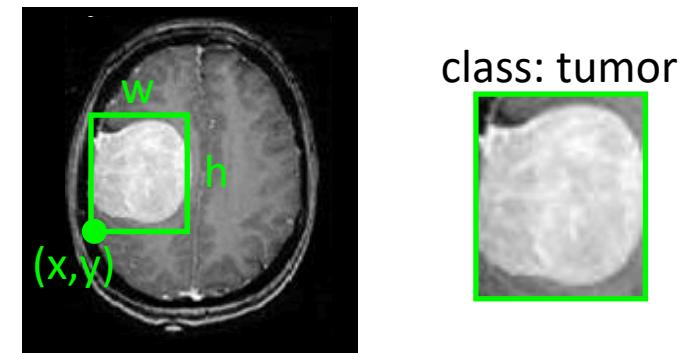
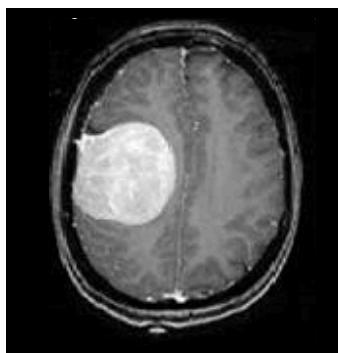


# Deep Learning

- We can use a deep artificial neural network to approximate any functions by modifying its weight
  - MR image classification function



- Detection function



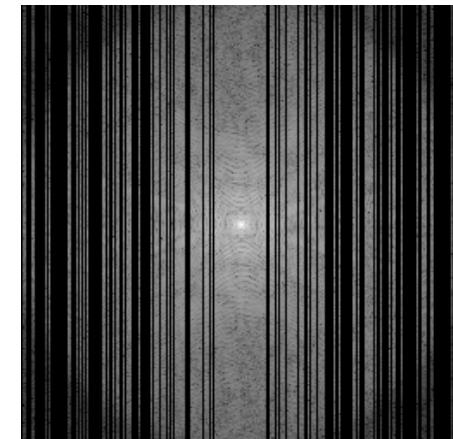
Alam, Sadia, et al. "An efficient image processing technique for brain tumor detection from MRI images." 2019 IEEE Asia-Pacific Conference on Computer Science and Data Engineering (CSDE). IEEE, 2019.

# Summary

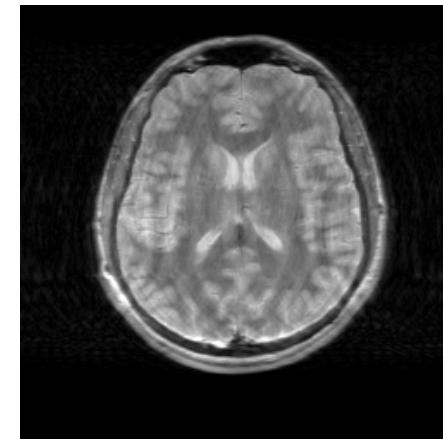
- **Magnetic Resonance Imaging (MRI)**

- Data acquired in the Fourier transform domain (k-space)
  - If the sampling rate is high enough, the inverse DFT can be directly applied to recover the data
- Data acquisition time can be reduced by collecting fewer k-space samples
  - Applying the inverse DFT to the undersampled k-space data leads to reconstruction with artifacts
    - Need more sophisticated approaches to reconstruct data: compressed sensing and deep learning

2x faster acquisition



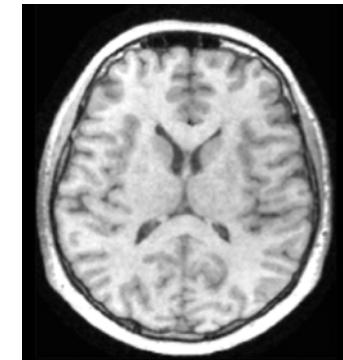
IDFT



# Summary

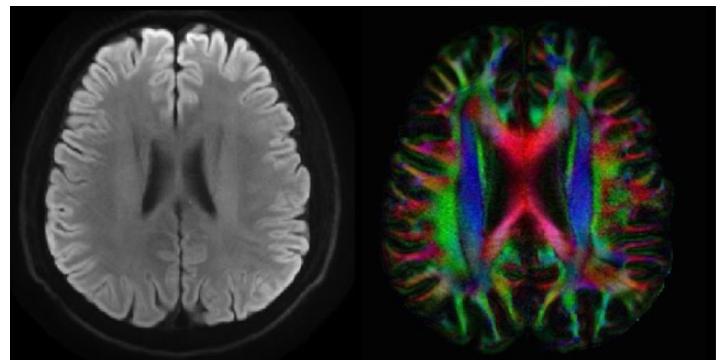
- Deep Learning for MRI
  - With lots of training data, supervised deep learning is an attractive approach for MR image reconstruction, analysis, quantification, and diagnosis
  - Other types of learning have recently gained in importance
- Current Challenges
  - Robustness
    - Uncontrollable factors
    - Adversarial attack
  - How to use data more efficiently
  - Explainable models

9x faster acquisition

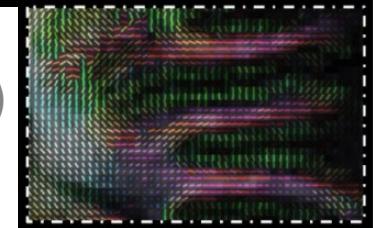


Cho, Jaejin, et al.  
ISMRM (2021)

88x faster reconstruction



Yarach, Utan, et  
al. ISMRM (2022)



# Acknowledgments



MGH/HST Athinoula A. Martinos  
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Health Sciences & Technology



**Massachusetts  
Institute of  
Technology**

