Signal Averaging

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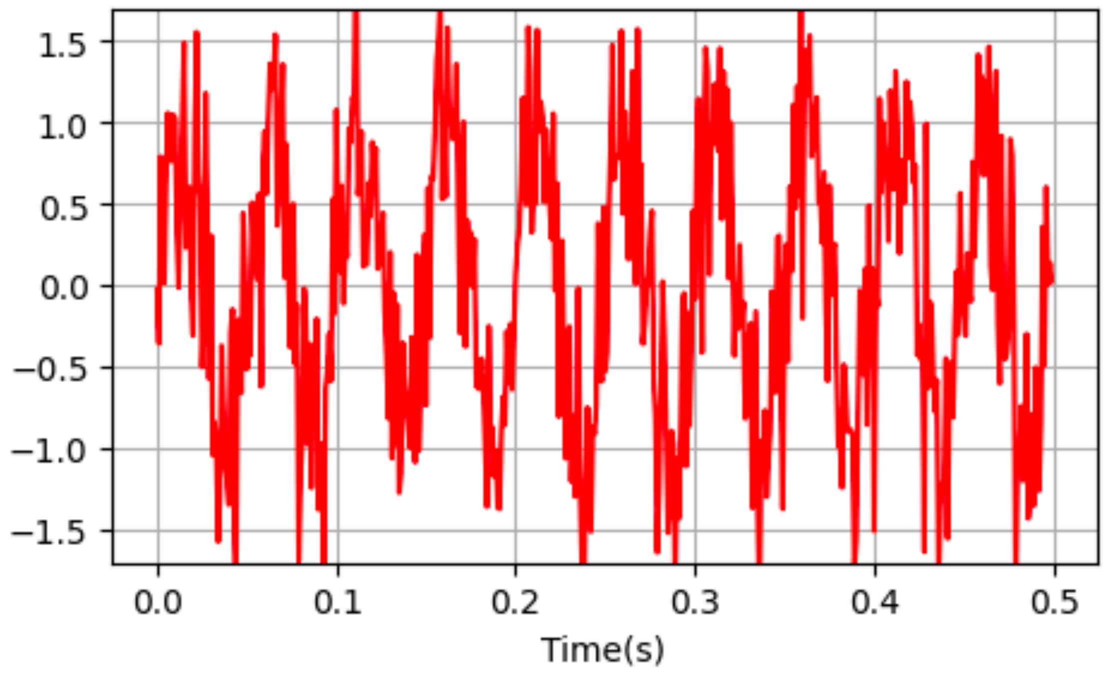
Signals and Noise

A simple model

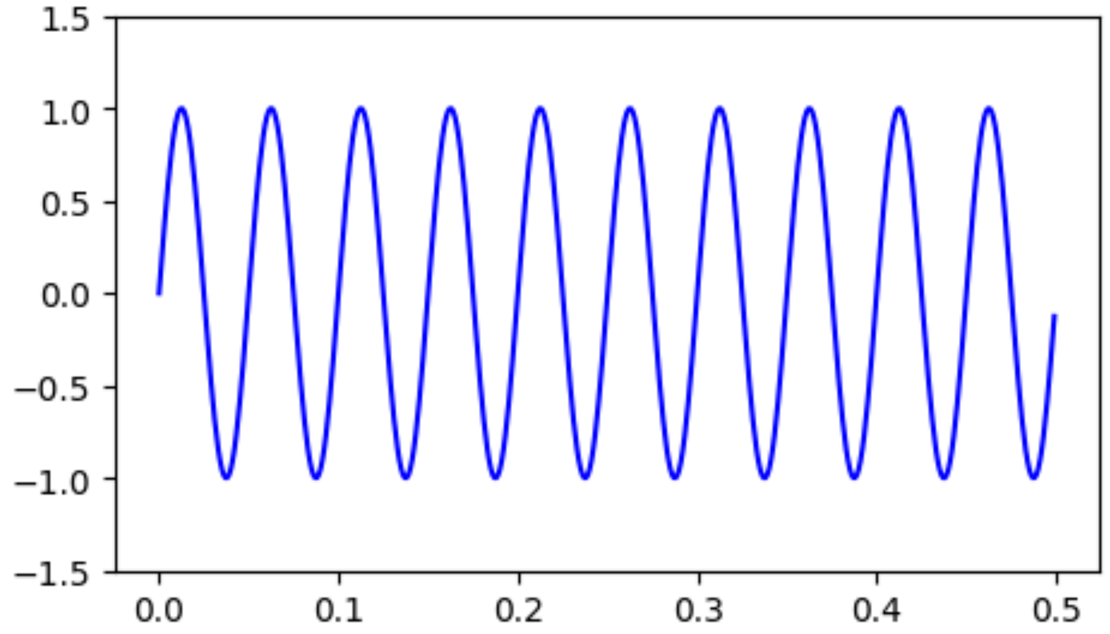
$$\underbrace{x}_{\text{Acquired signal}} = \underbrace{s}_{\text{Underlying signal}} + \underbrace{z}_{\text{Noise}}$$

Acquired signal



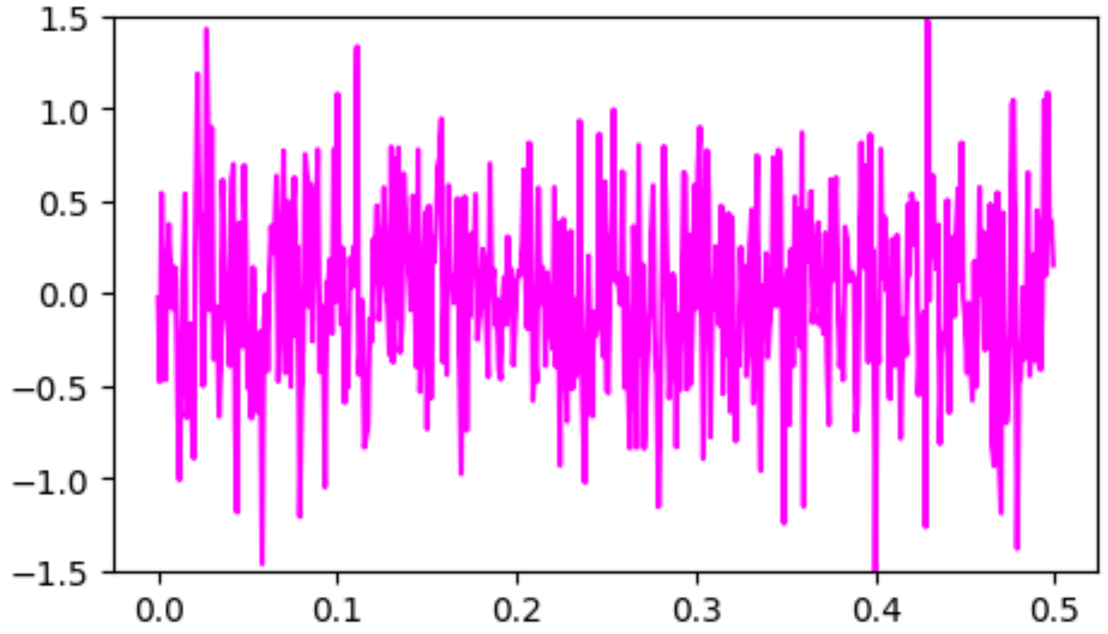
=

Clean signal

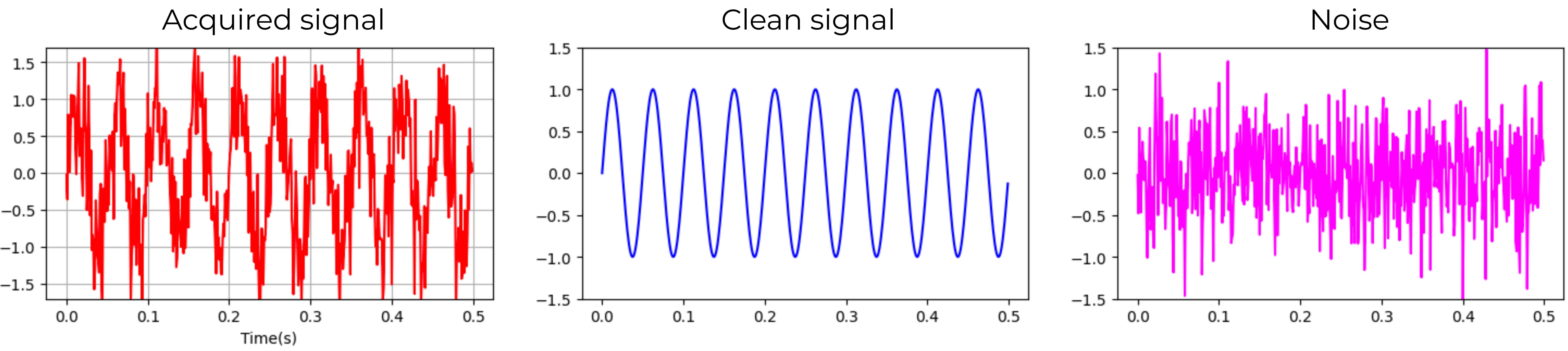


+

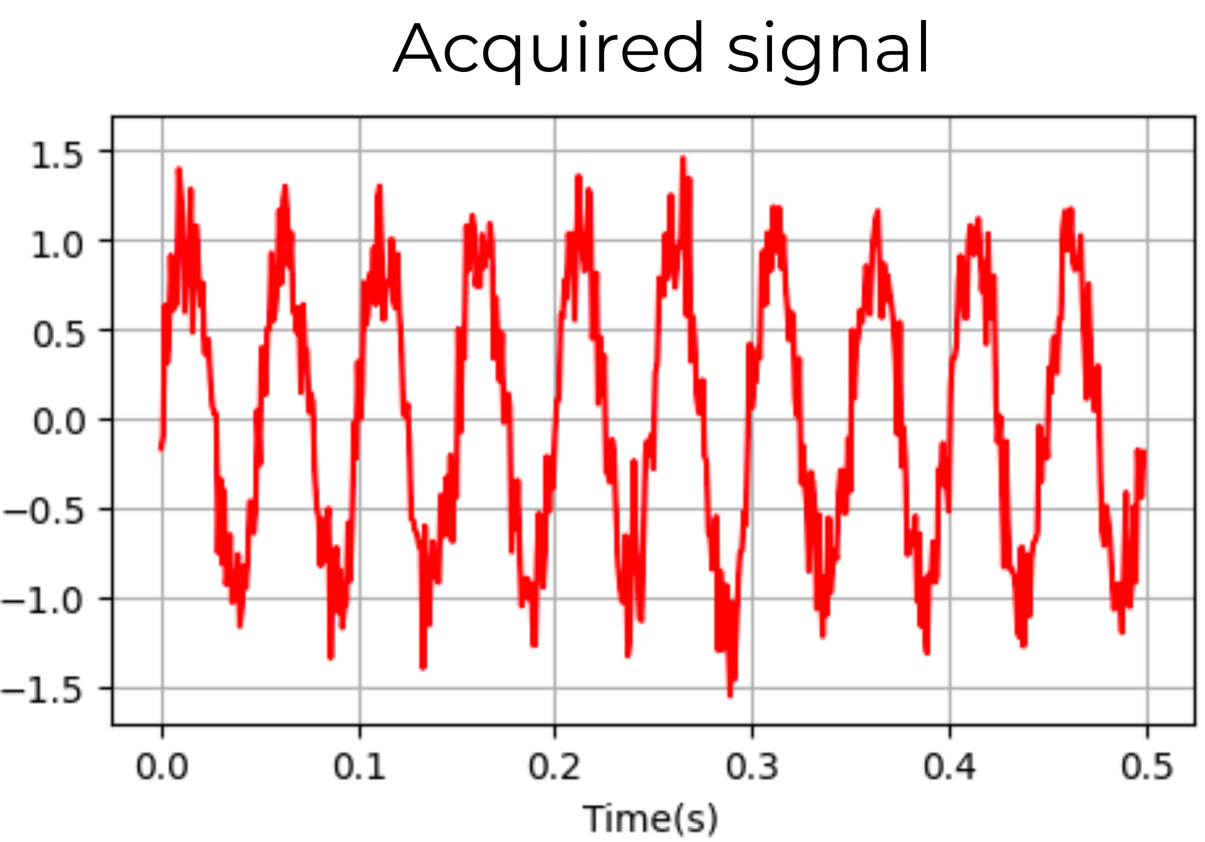
Noise

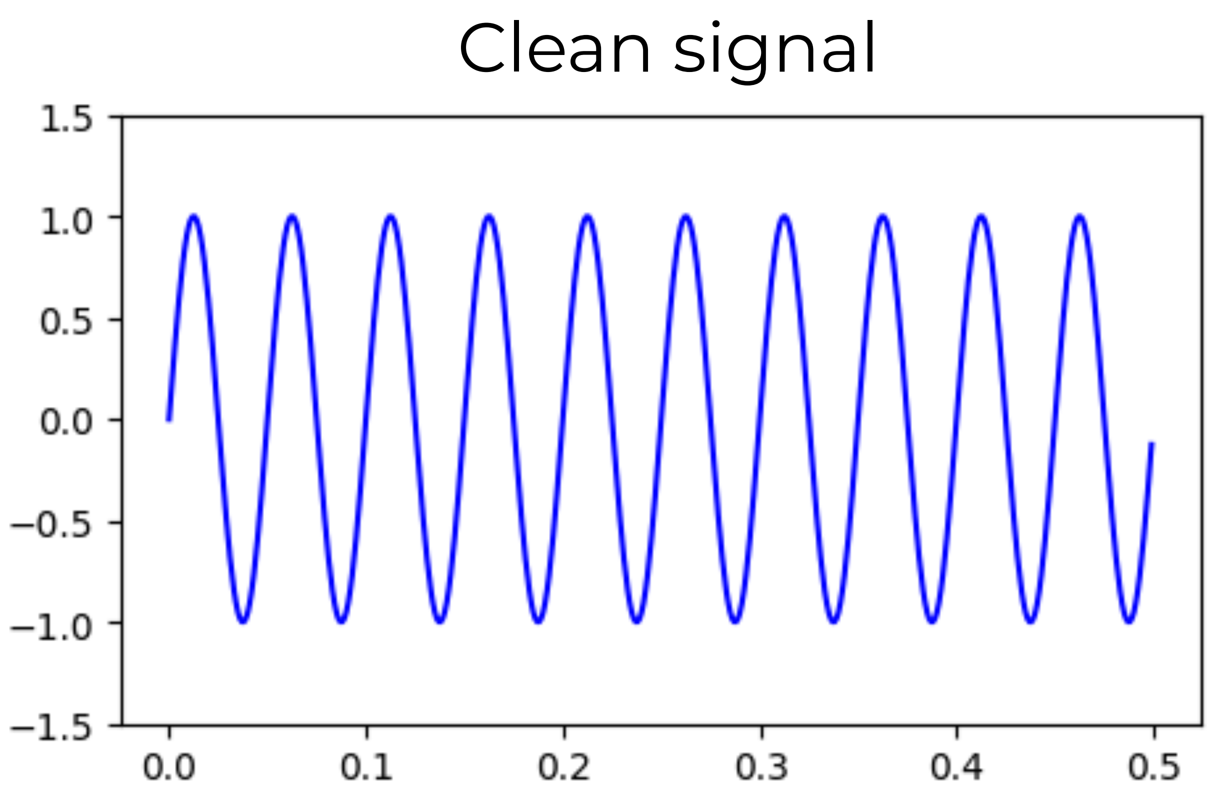


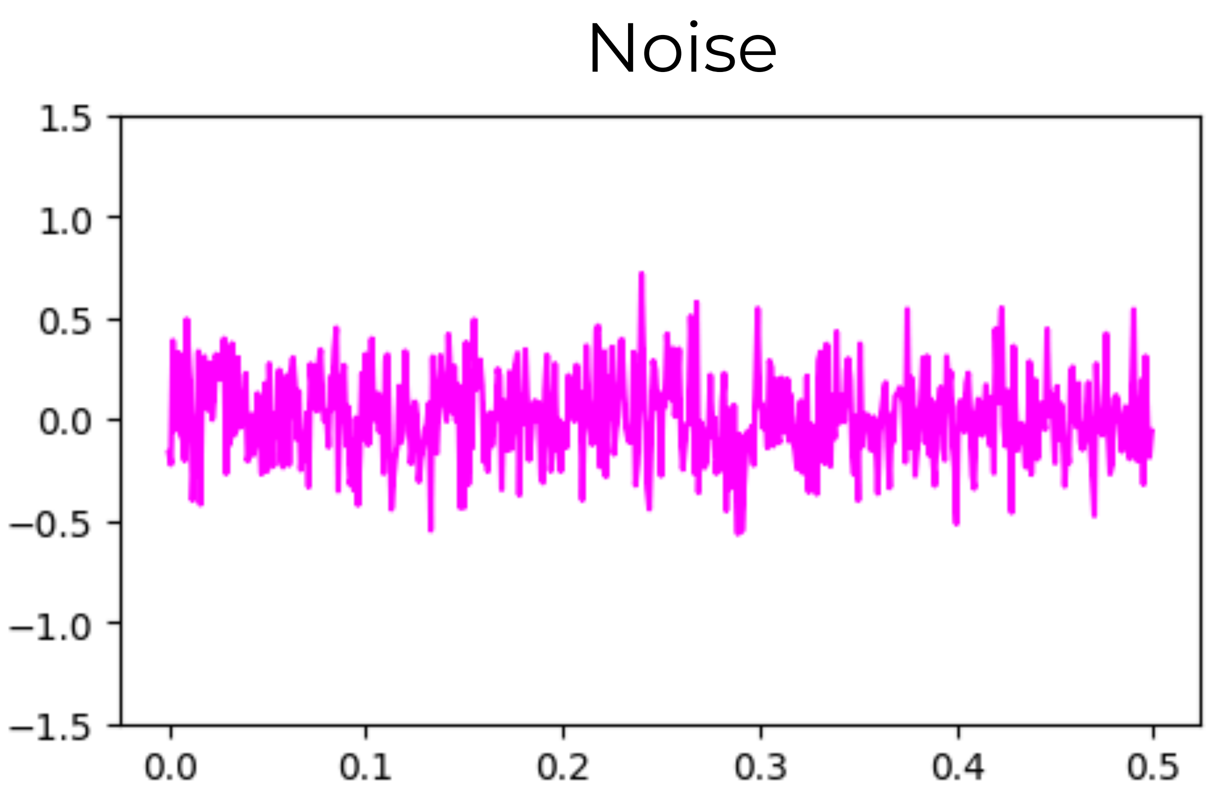
Signal Averaging



Signal Averaging



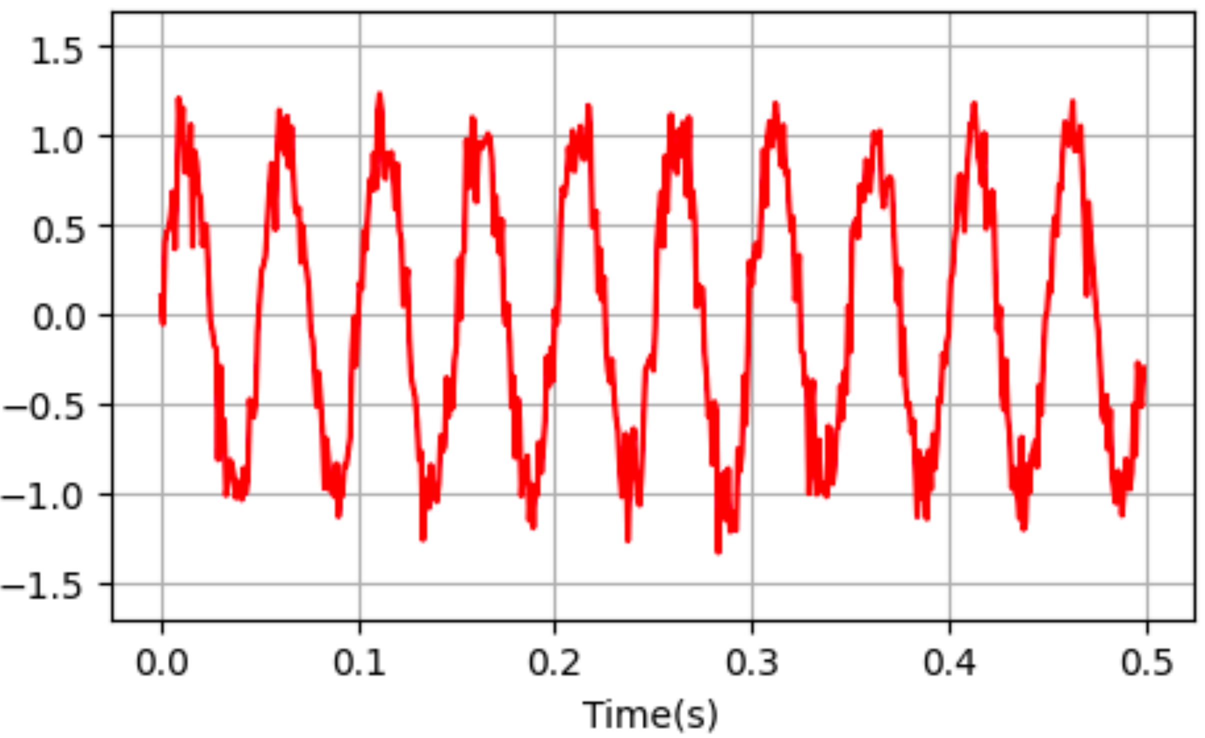




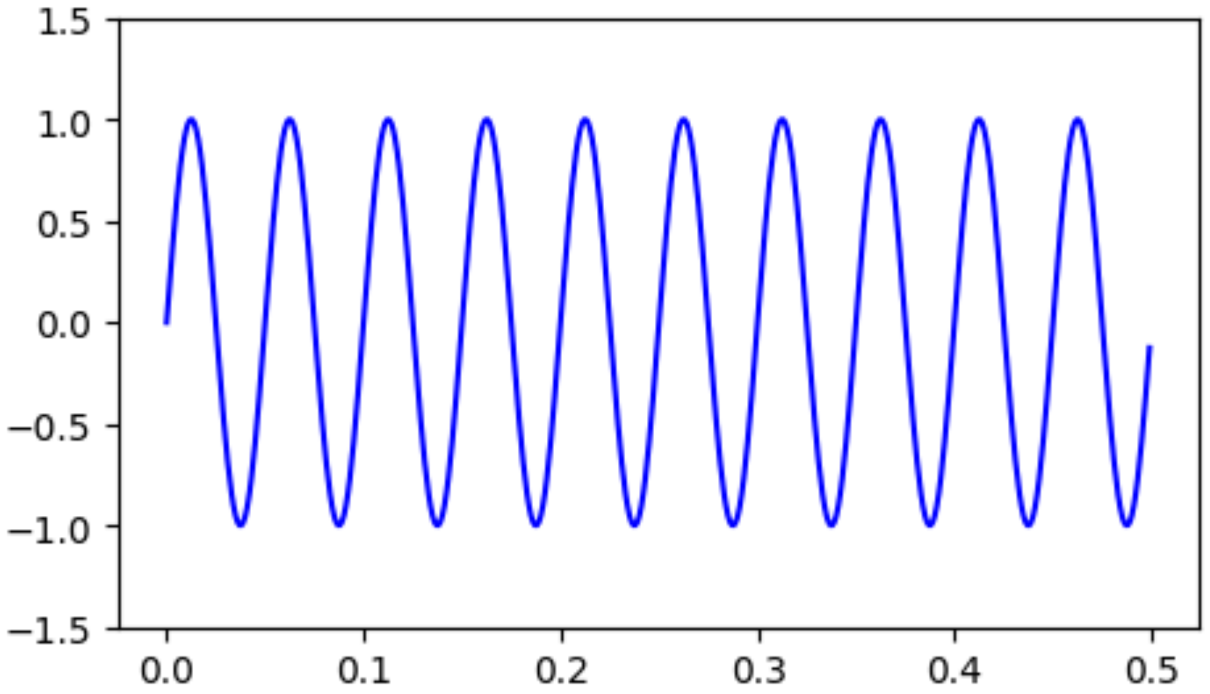
5 averages

Signal Averaging

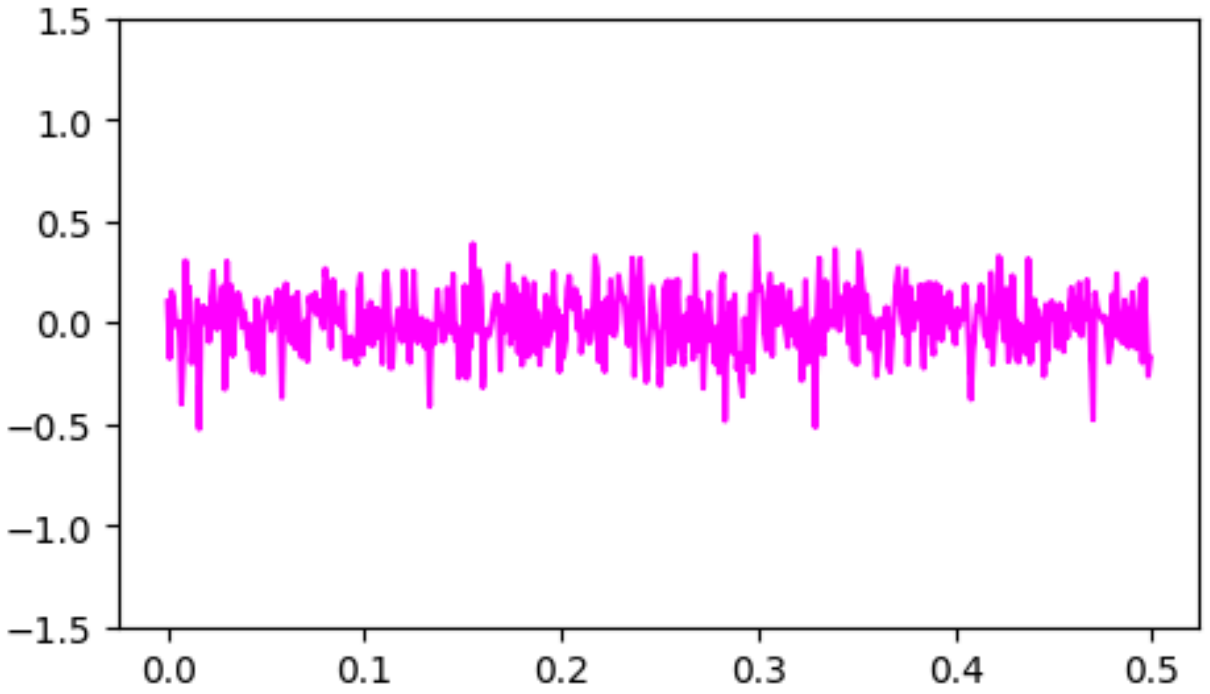
Acquired signal



Clean signal

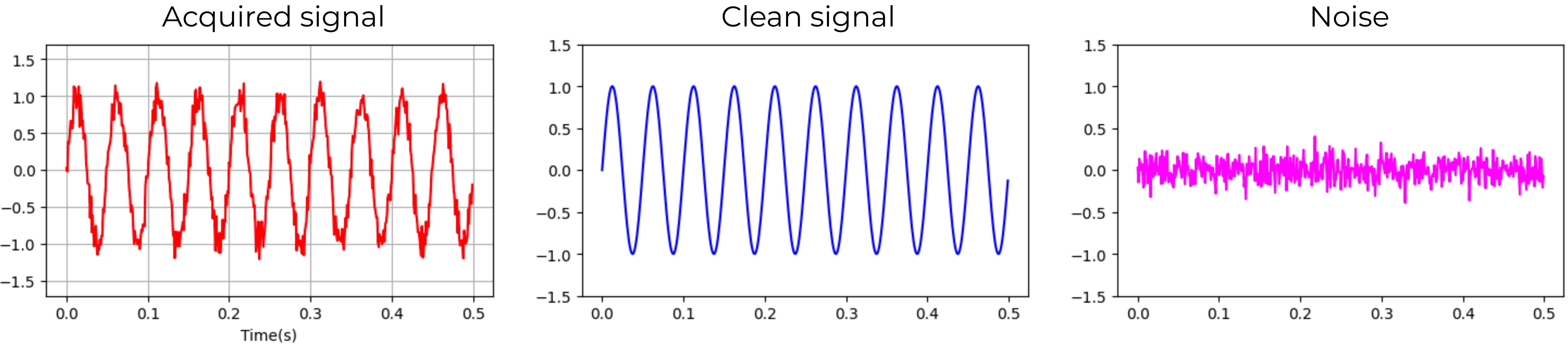


Noise



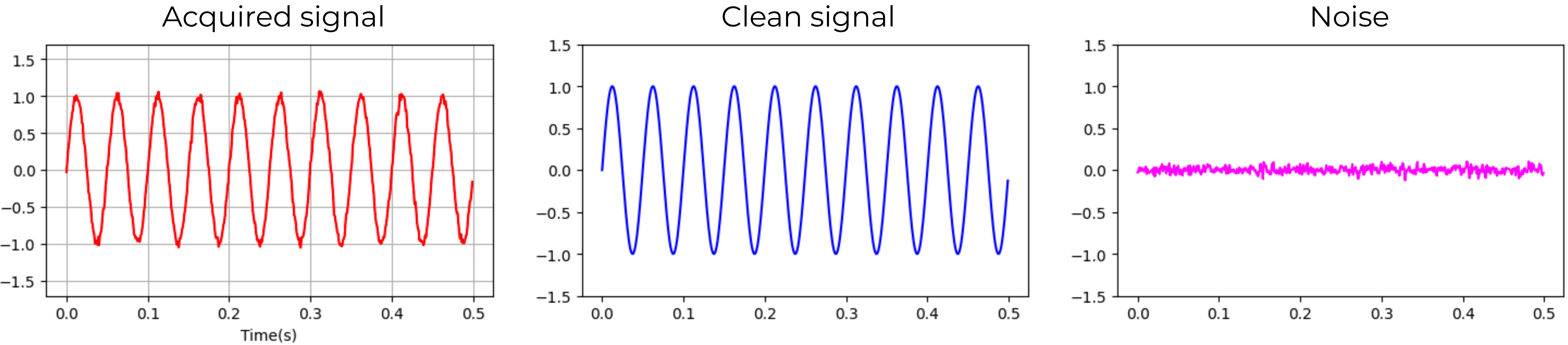
10 averages

Signal Averaging



15 averages

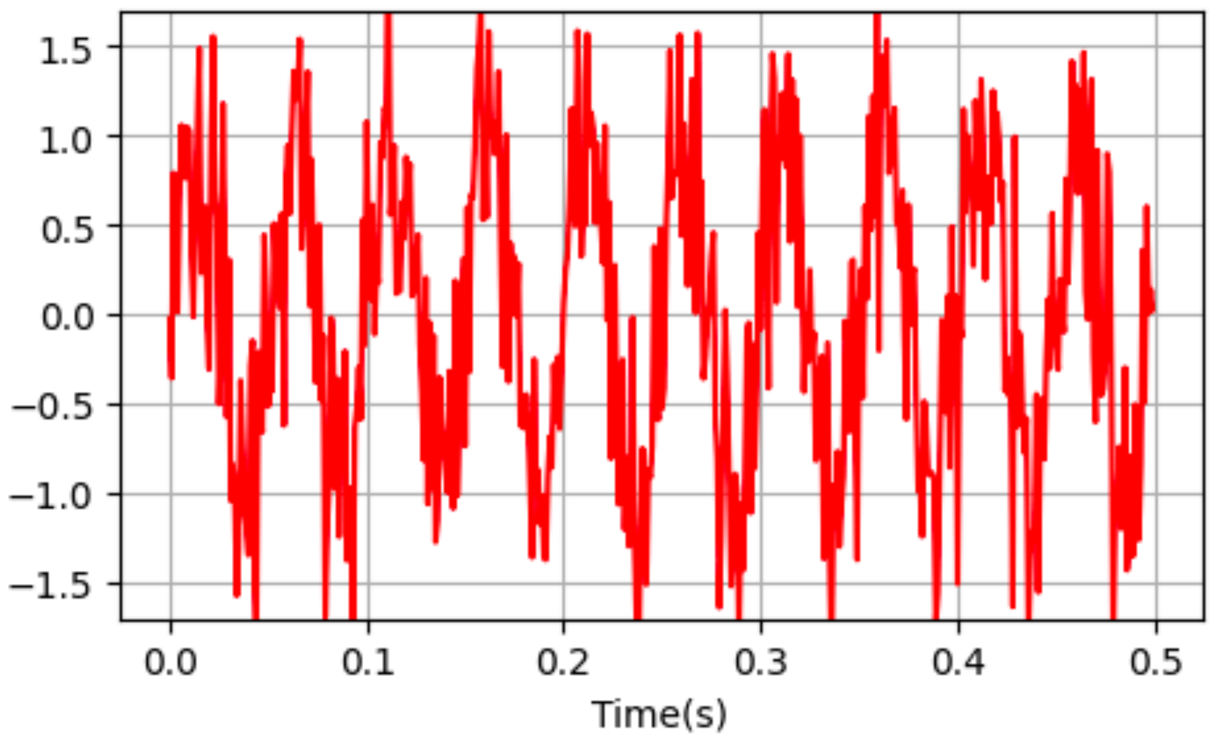
Signal Averaging



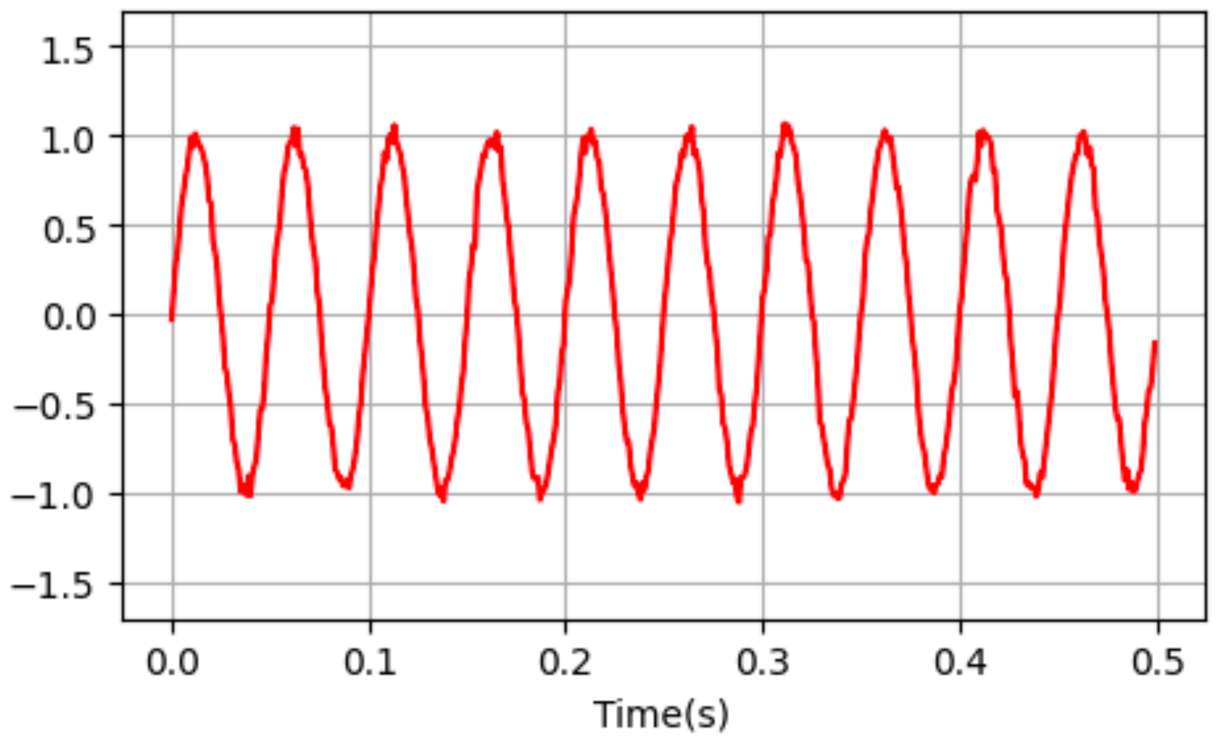
200 averages

Signal Averaging

single measurement



200 averages



Optional: Signal-to-Noise Ratio (SNR)

The ratio of the power of a signal to the power of background noise

$$SNR = \frac{P_{signal}}{P_{noise}}$$

signal power

noise power

Note: There is also an alternative definition of SNR - the ratio of mean to standard deviation

$$SNR = \frac{\mu}{\sigma}$$

signal mean

noise standard deviation

Optional: SNR for Averaged Signal

measurement i $\quad x_i = s + z_i$

zero-mean

$$SNR_i = \frac{P_{signal}}{P_{noise}} = \frac{E[s^2]}{E[z_i^2]} \overset{\downarrow}{=} \frac{E[s^2]}{Var(z_i)} = \frac{E[s^2]}{\sigma^2}$$

Assumptions

- Deterministic signal s
  - $E[s^2]$ remains the same in replicate measurements
- Random noise z_i
  - Independent and identically distributed (iid)
  - Mean of zero: $E[z_i] = 0$
  - Constant variance: $Var(z_i) = \sigma^2$
- Signal and noise are uncorrelated

Brain Code Camp

Optional: SNR for Averaged Signal

Measurement i $\quad x_i = s + z_i \rightarrow SNR_i = \frac{E[s^2]}{\sigma^2}$

Averaging N measurements $\quad \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{N}\sum_{i=1}^{N}(s+z_i) = s + \frac{1}{N}\sum_{i=1}^{N} z_i$

Averaged signal $\quad s \rightarrow$ P_{signal} remains the same

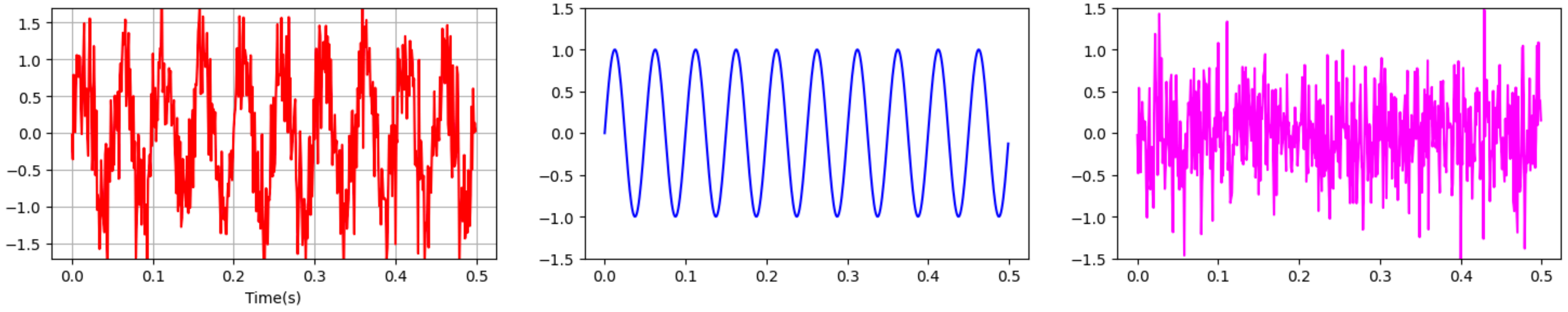
Averaged noise $\quad E\left[\frac{1}{N}\sum_{i=1}^{N} z_i\right] = \frac{1}{N}\sum_{i=1}^{N} E[z_i] = 0$

$$Var\left(\frac{1}{N}\sum_{i=1}^{N} z_i\right) = \frac{Var\left(\sum_{i=1}^{N} z_i\right)}{N^2} \overset{\text{iid}}{=} \frac{N\sigma^2}{N^2} = \frac{\sigma^2}{N}$$

$\rightarrow$ P_{noise} gets reduced by a factor of N

Optional: SNR for Averaged Signal

Measurement i $\quad x_i = s + z_i \quad \Rightarrow \quad SNR_i = \frac{E[s^2]}{\sigma^2}$

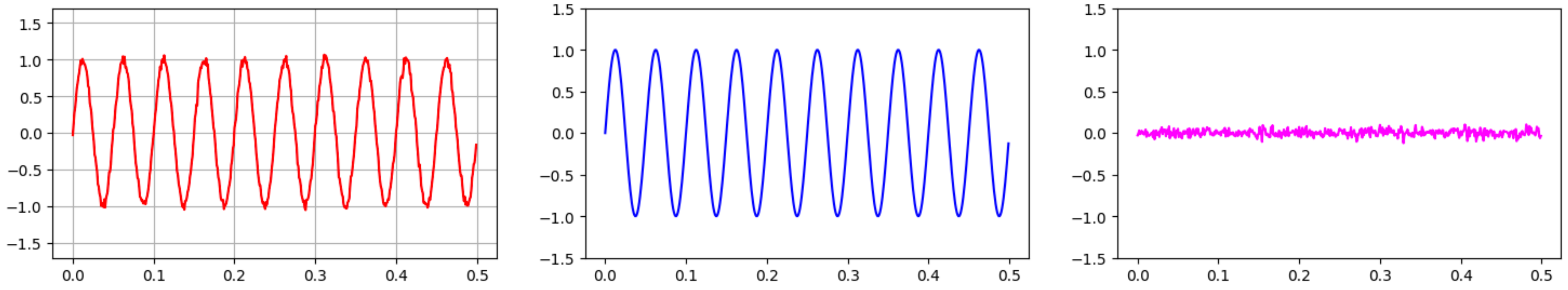


results from previous slide

Averaging N measurements

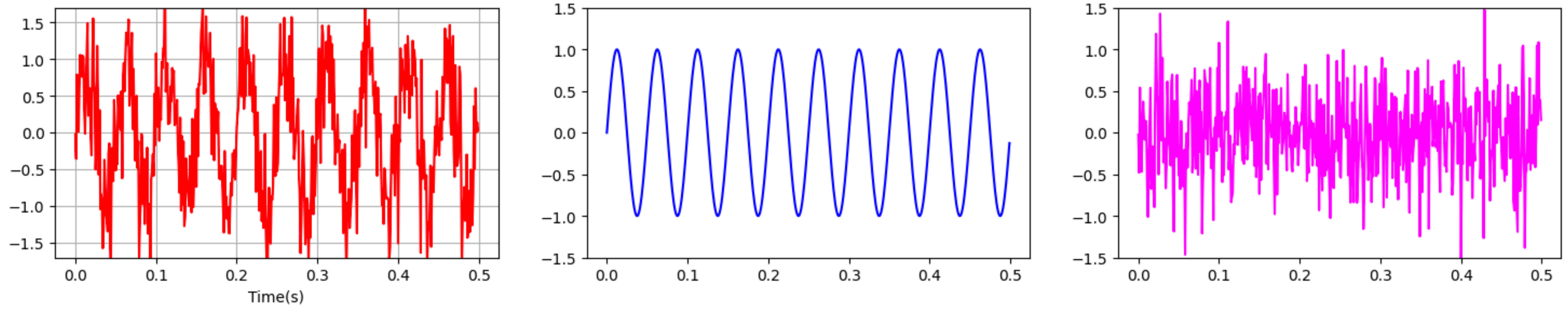
$$SNR_{avg} = \frac{P_{signal}}{P_{noise}} = \frac{E[s^2]}{\sigma^2/N} = N \times \frac{E[s^2]}{\sigma^2} = N \times SNR_i$$

N times higher SNR!



Optional: SNR for Averaged Signal

Measurement i $\qquad x_i = s + z_i \;\Rightarrow\; SNR_i = \frac{E[s^2]}{\sigma^2}$



results from previous slide

Averaging N measurements

$$SNR_{avg} = \frac{P_{signal}}{P_{noise}} = \frac{E[s^2]}{\sigma^2/N} = N \times \frac{E[s^2]}{\sigma^2} = N \times SNR_i$$

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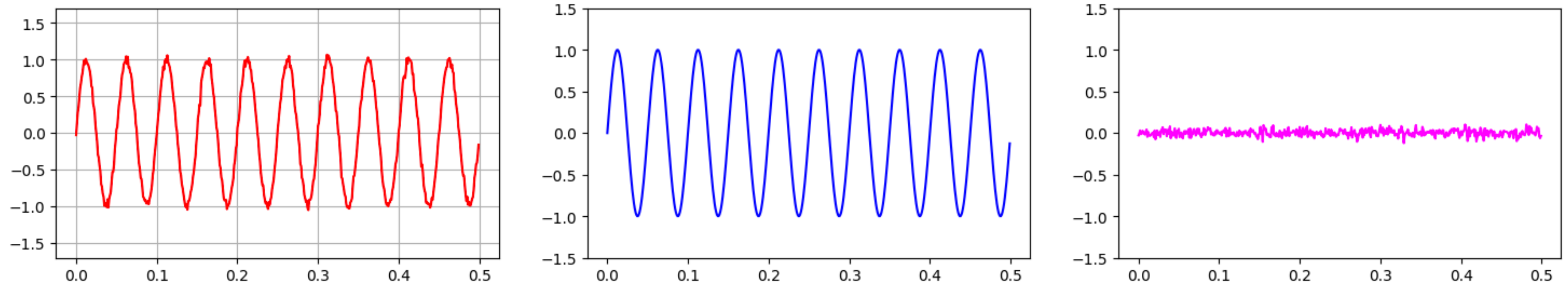

*For $SNR = \frac{\mu}{\sigma}$, averaging N measurements result in $\sqrt{N}$ times higher SNR.