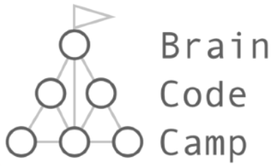
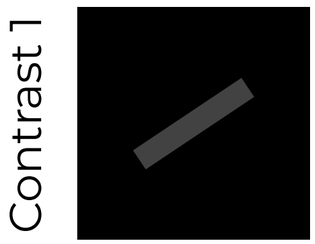


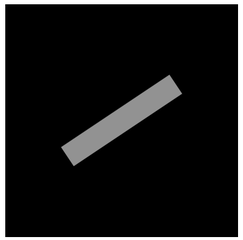
Simple Linear Models

Itthi Chatnuntawech

Spike Count Prediction in 2D

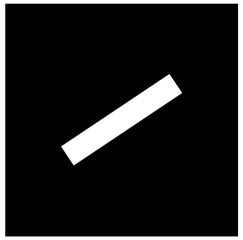


Contrast 1



Contrast 2

⋮



Contrast n

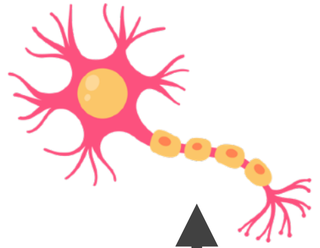
spike count

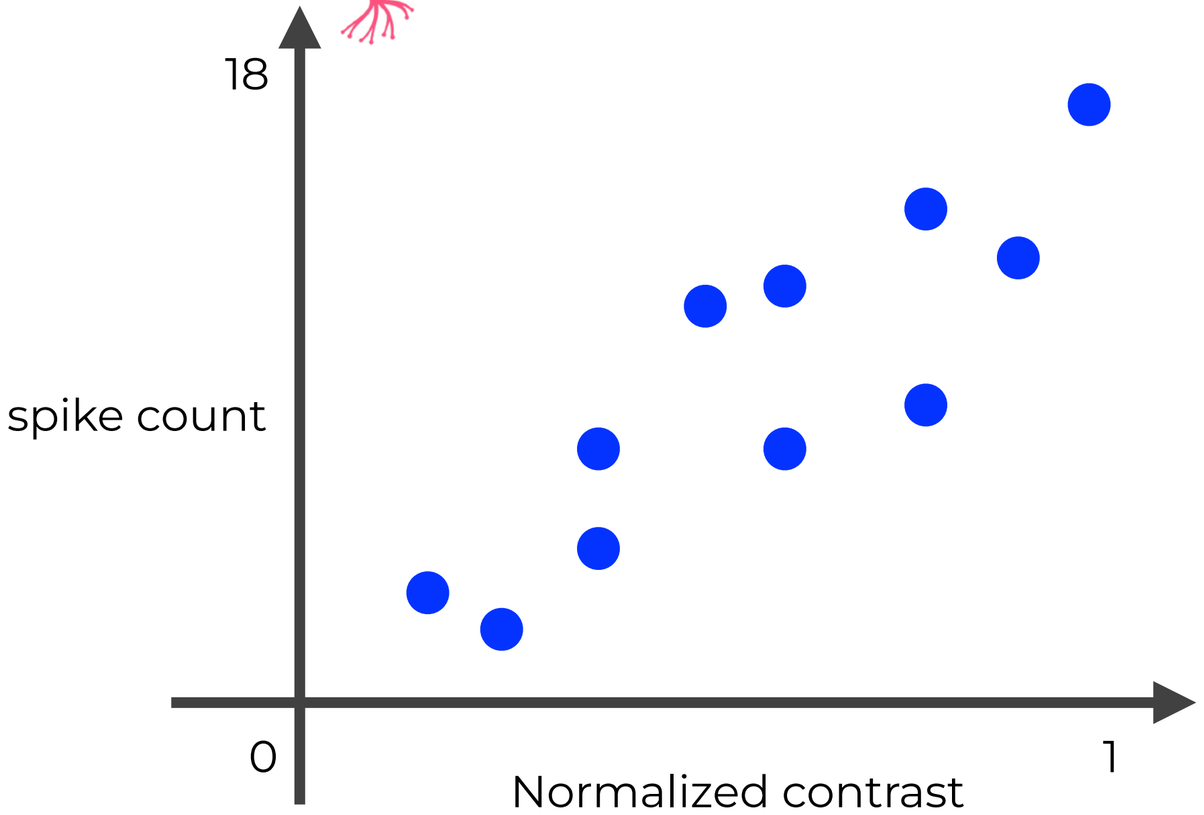
2

9

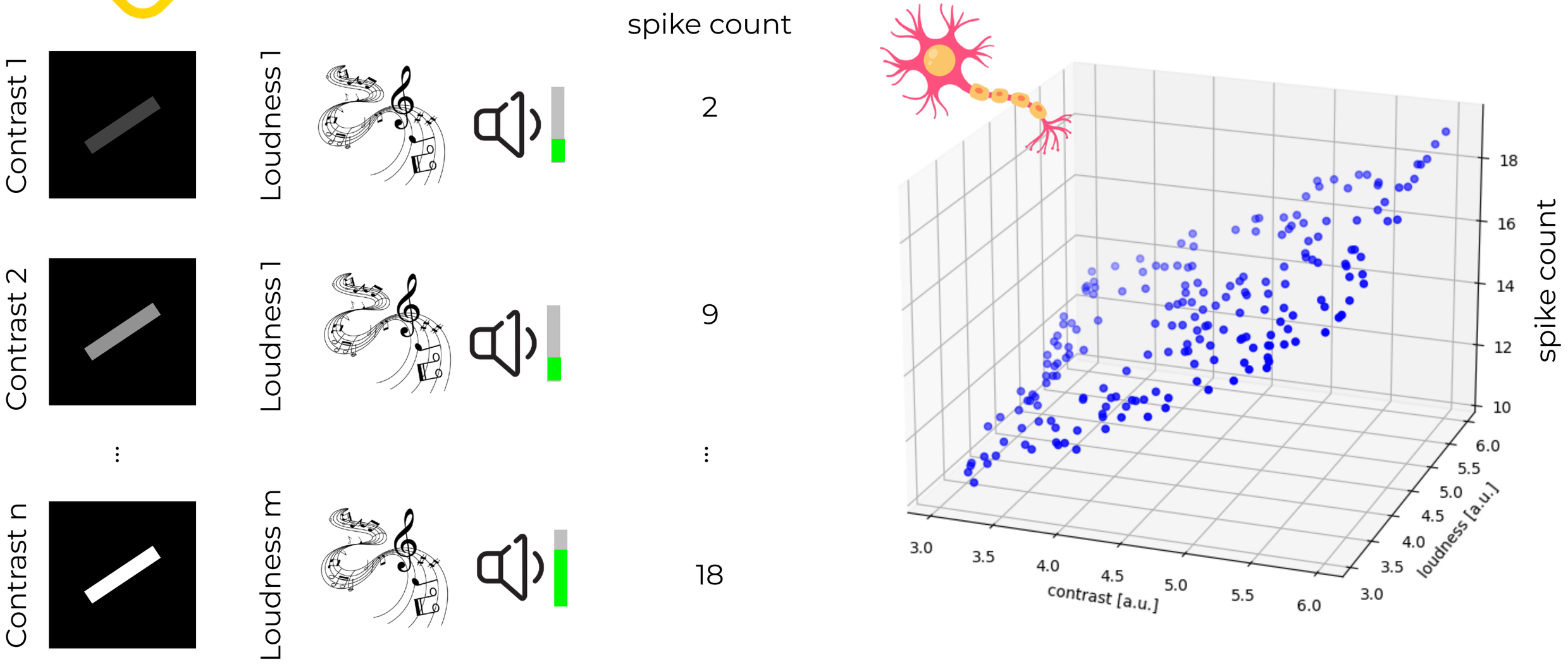
⋮

18





Spike Count Prediction in 3D

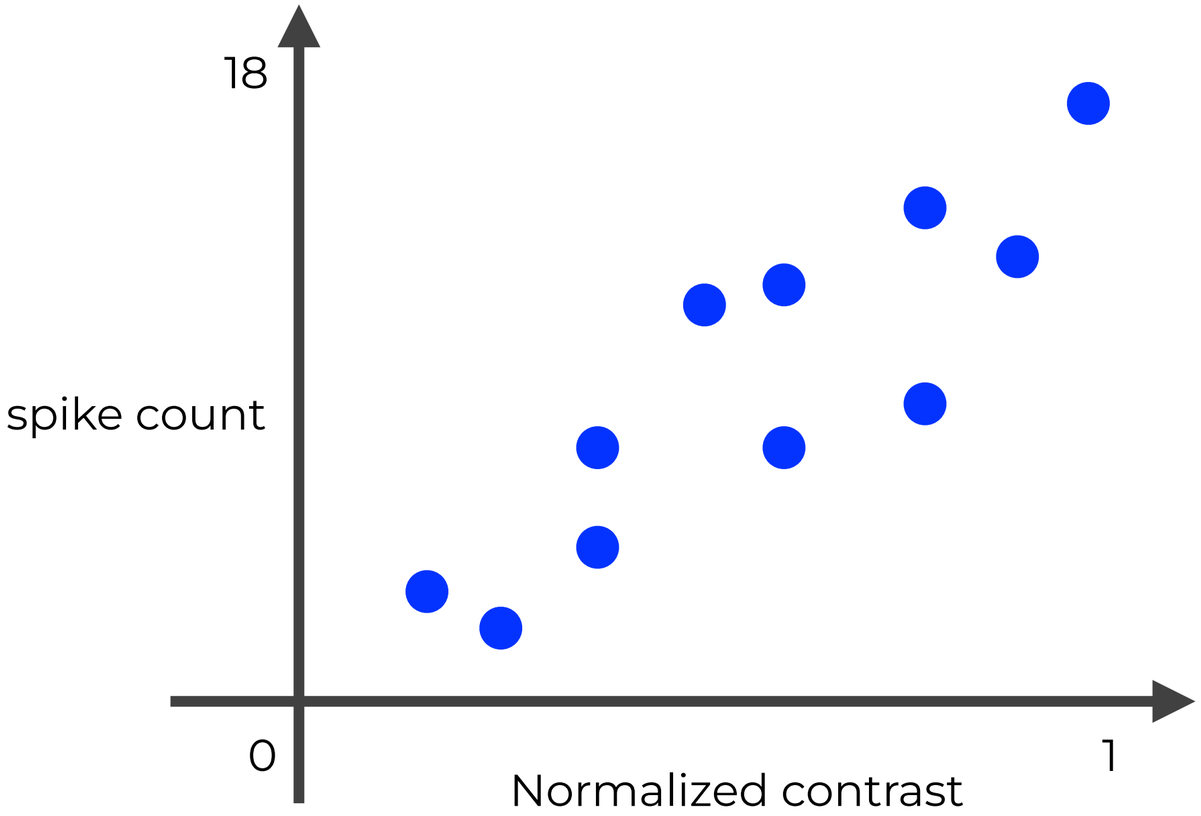


Spike Count Prediction

$$y = w_0 + w_1 x$$

1 feature

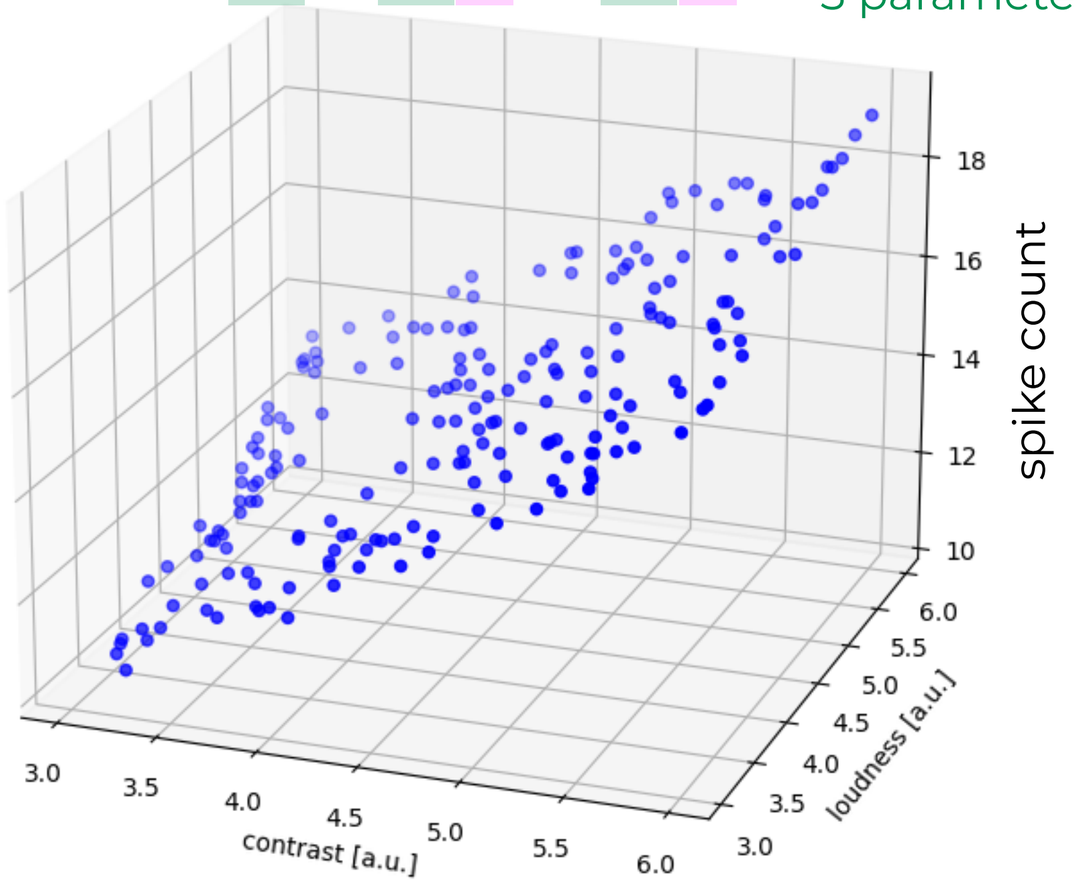
2 parameters



$$y = w_0 + w_1 x_1 + w_2 x_2$$

2 features

3 parameters

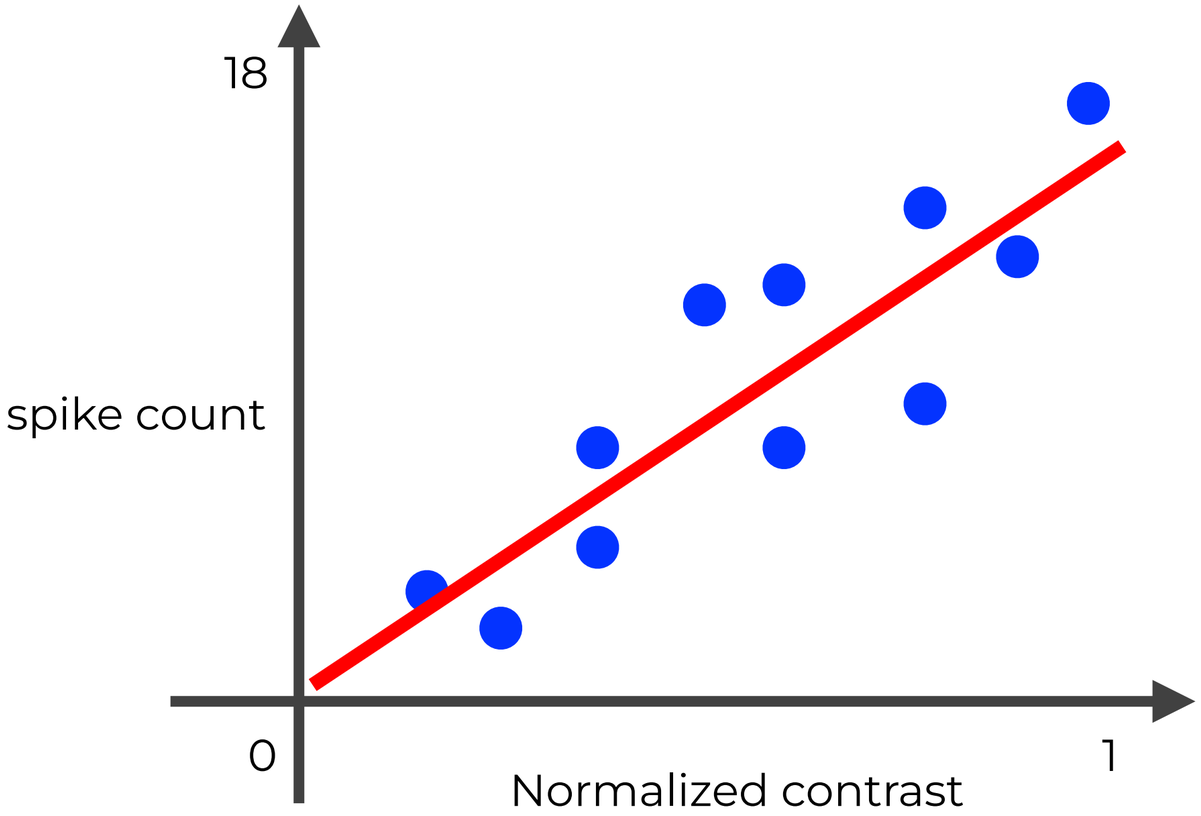


Spike Count Prediction

$$\hat{y} = \hat{w}_0 + \hat{w}_1 x$$

1 feature

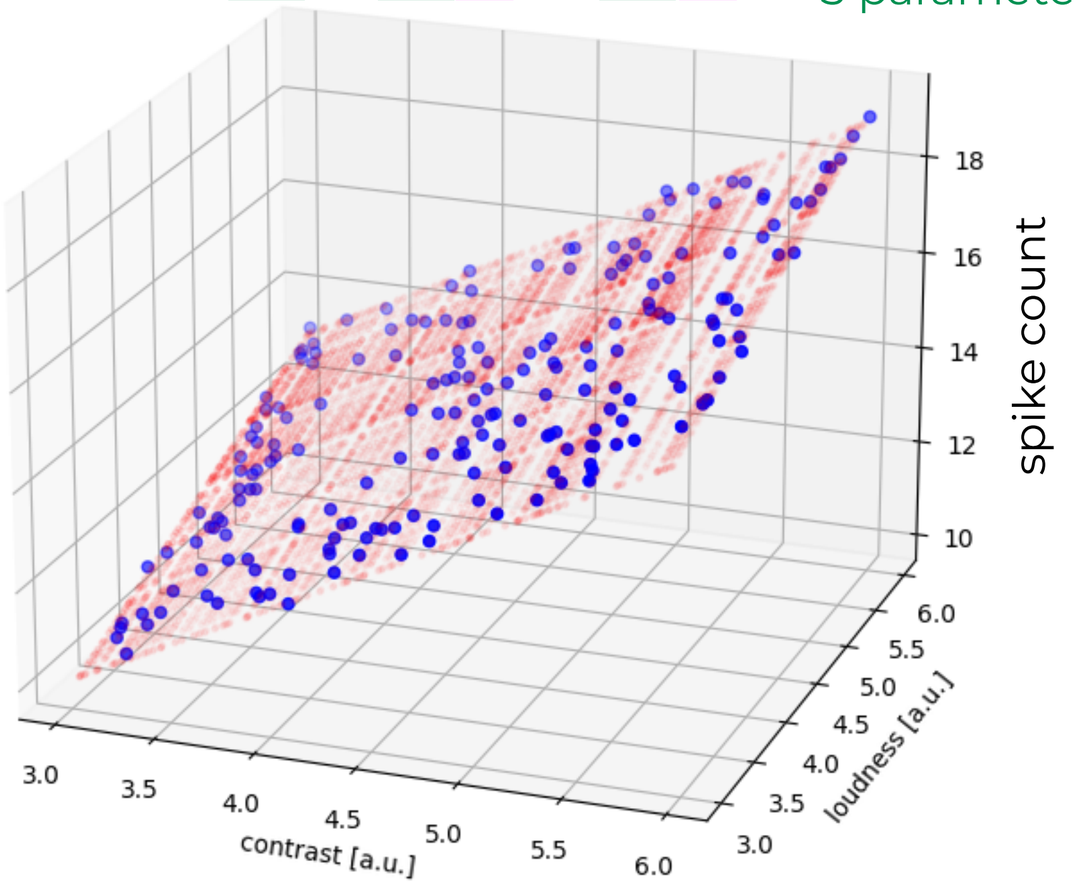
2 parameters



$$\hat{y} = \hat{w}_0 + \hat{w}_1 x_1 + \hat{w}_2 x_2$$

2 features

3 parameters



Multiple Linear Regression

feature 1 of sample i

feature 2 of sample i

$$MSE(Y, \hat{Y}) = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - (\hat{w}_0 + \hat{w}_1 x_{i1} + \hat{w}_2 x_{i2})\right)^2$$

Find $\hat{w}_0, \hat{w}_1$ and $\hat{w}_2$ that minimize $MSE(Y, \hat{Y})$

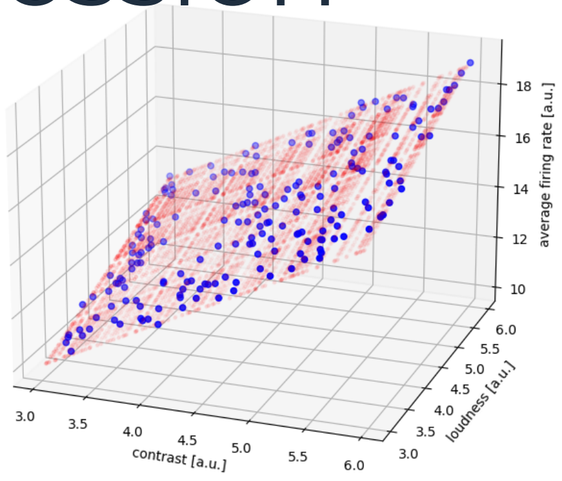
$$\min_{\hat{w}_0,\hat{w}_1,\hat{w}_2} MSE(Y, \hat{Y}) = \min_{\hat{w}_0,\hat{w}_1,\hat{w}_2} \frac{1}{n}\sum_{i=1}^{n}\left(y_i - (\hat{w}_0 + \hat{w}_1 x_{i1} + \hat{w}_2 x_{i2})\right)^2$$

sklearn.linear_model.LinearRegression

class sklearn.linear_model.**LinearRegression**(*, *fit_intercept=True*, *copy_X=True*, *n_jobs=None*, *positive=False*)

[source]

Multiple Linear Regression



Data

$$\boxed{(x_{11}, x_{12}, y_1)}, (x_{21}, x_{22}, y_2), \ldots, \boxed{(x_{n1}, x_{n2}, y_n)}$$

$\mathbf{X}$ $\quad$ $\mathbf{y}$

sample 1

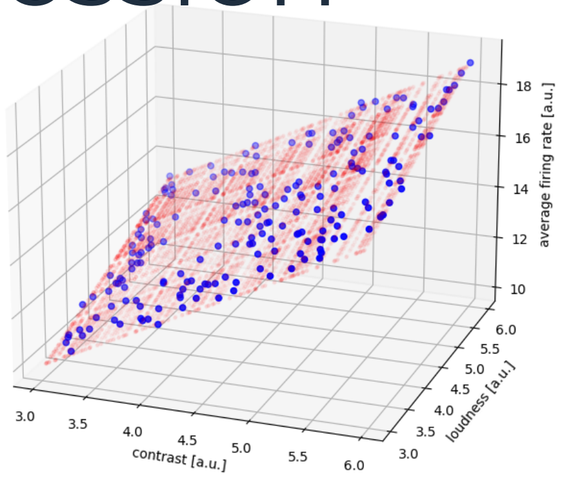
sample n

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix} \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

feature 1 $\quad$ feature 2

shape = (n, 2) $\quad$ shape = (n, 1)

Multiple Linear Regression



Data

$$(x_{11}, x_{12}, y_1), (x_{21}, x_{22}, y_2), \ldots, (x_{n1}, x_{n2}, y_n)$$

$\mathbf{X}$

sample 1

sample n

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix}$$

feature 1 feature 2

shape = (n, 2)

$\mathbf{y}$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

shape = (n, 1)

```python
# Import a necessary module
from sklearn.linear_model import LinearRegression

# Create the model
model_linear = LinearRegression()

# Train the model
model_linear.fit(X, y)

# Make prediction
y_hat = model_linear.predict(x)
```

Multiple Linear Regression

Let consider p features and n samples

Each datapoint is represented by a point in (p+1)-dimensional space

$$y = w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_p x_p$$

Find $\hat{w}_0, \hat{w}_1, \ldots, \hat{w}_p$ that minimize $MSE(Y, \hat{Y})$

feature 1 of sample i

feature p of sample i

$$\min_{\hat{w}_0,\ldots,\hat{w}_p} MSE(Y, \hat{Y}) = \min_{\hat{w}_0,\ldots,\hat{w}_p} \frac{1}{n} \sum_{i=1}^{n} \left(y_i - (\hat{w}_0 + \hat{w}_1 x_{i1} + \ldots + \hat{w}_p x_{ip}) \right)^2$$

sklearn.linear_model.LinearRegression

class sklearn.linear_model.**LinearRegression**(*, *fit_intercept=True*, *copy_X=True*, *n_jobs=None*, *positive=False*)

[source]

Multiple Linear Regression

Data

$\mathbf{X}$

$$\begin{bmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix}$$

feature 1 ... feature p

shape = (n, p)

$\mathbf{y}$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

shape = (n, 1)

```
# Import a necessary module
from sklearn.linear_model import LinearRegression

# Create the model
model_linear = LinearRegression()

# Train the model
model_linear.fit(X, y)

# Make prediction
y_hat = model_linear.predict(x)
```

Brain Code Camp

Multiple Linear Regression

```
# Import a necessary module
from sklearn.linear_model import LinearRegression

# Create the model
model_linear = LinearRegression()

# Train the model
model_linear.fit(X, y)

# Make prediction
y_hat = model_linear.predict(x)
```

1 feature

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

shape = (n, 1)

2 features

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix}$$

feature 1, feature 2

shape = (n, 2)

p features

$$\mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix}$$

feature 1, feature p

shape = (n, p)

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

shape = (n, 1)