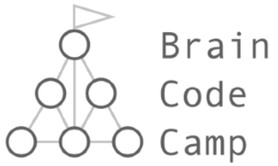

Short-Time Fourier Transform

Itthi Chatnuntawech

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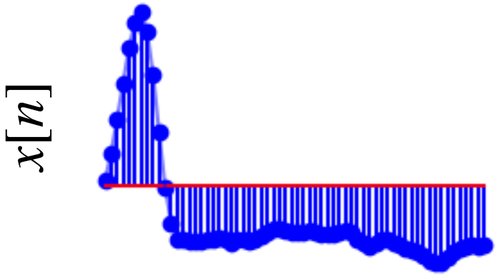
Discrete Fourier Transform (DFT)

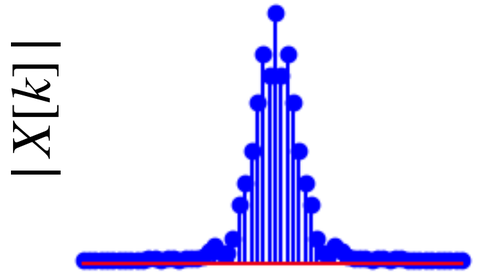
Time domain



$$x[n] \qquad X[k]$$

$$\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \quad DFT \quad \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

Frequency domain



scipy.fft.fft

```
scipy.fft.fft(x, n=None, axis=-1, norm=None, overwrite_x=False,
workers=None, *, plan=None)                                    [source]
```

Compute the 1-D discrete Fourier Transform.

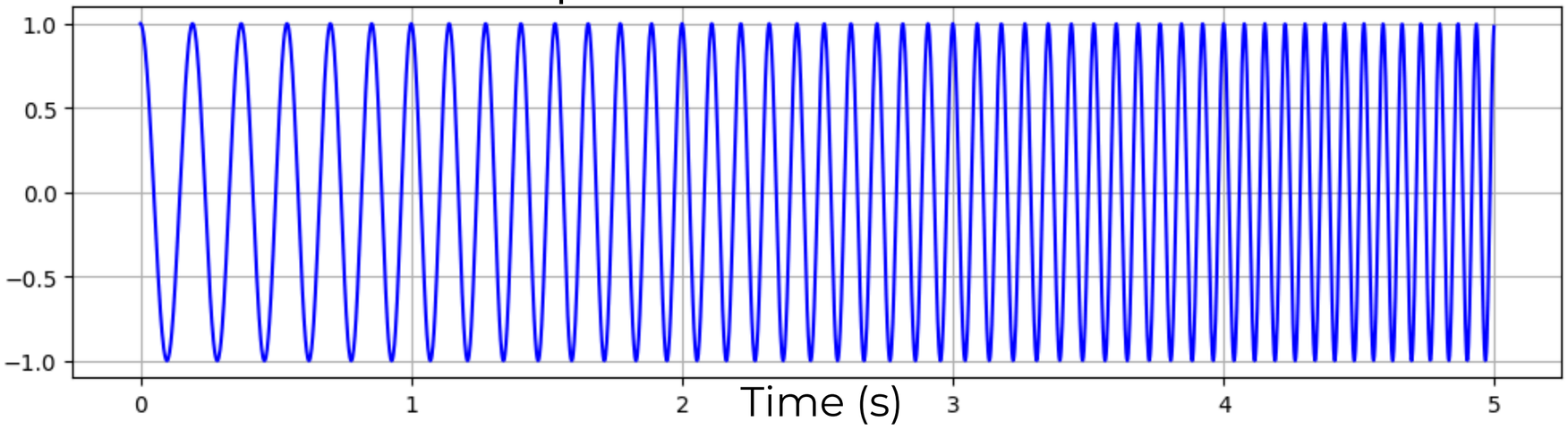
This function computes the 1-D *n*-point discrete Fourier Transform (DFT) with the efficient Fast Fourier Transform (FFT) algorithm [1].

Fast Fourier Transform (FFT) - An efficient algorithm that computes the DFT of a signal

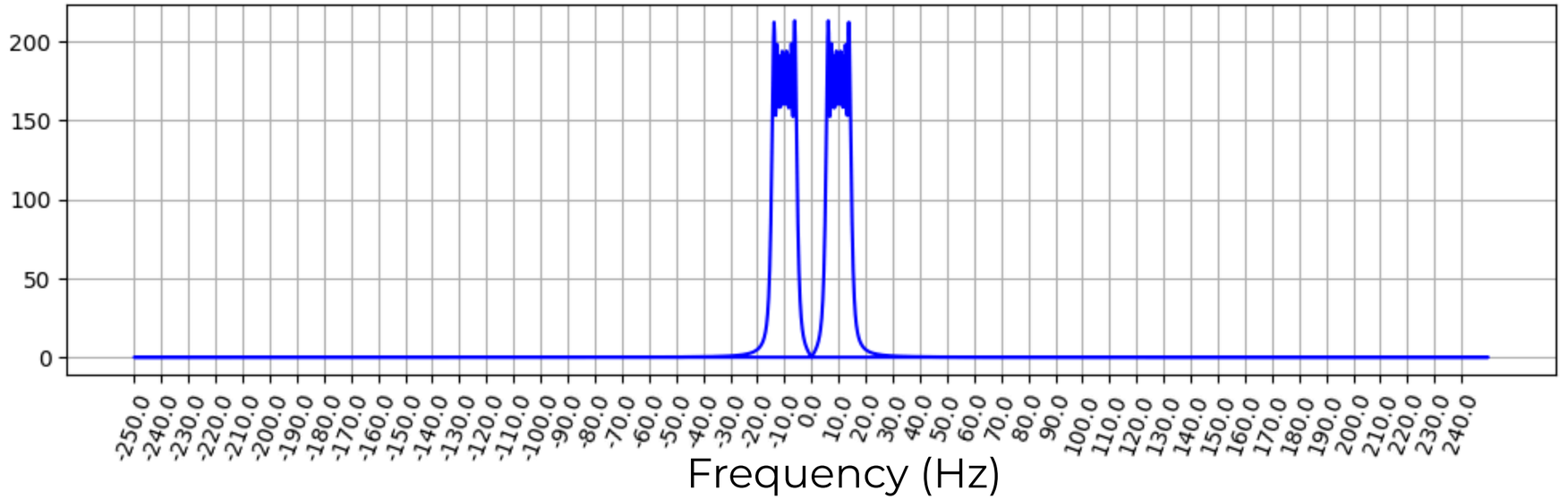
A Chirp Signal

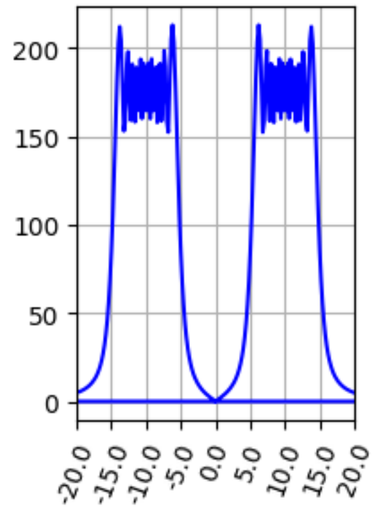
Frequencies: 5 Hz → 15 Hz

x



$|X[k]|$





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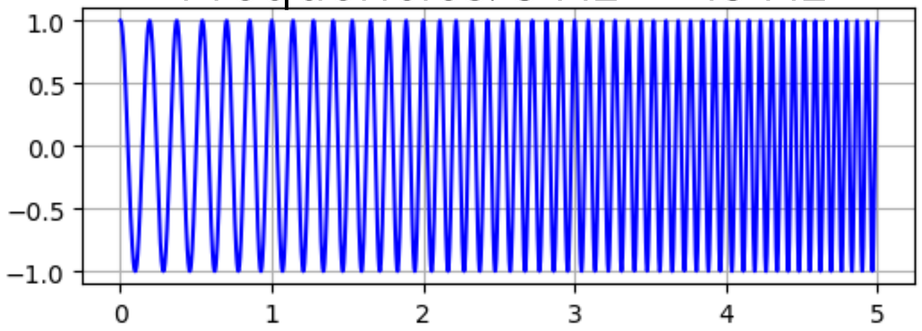
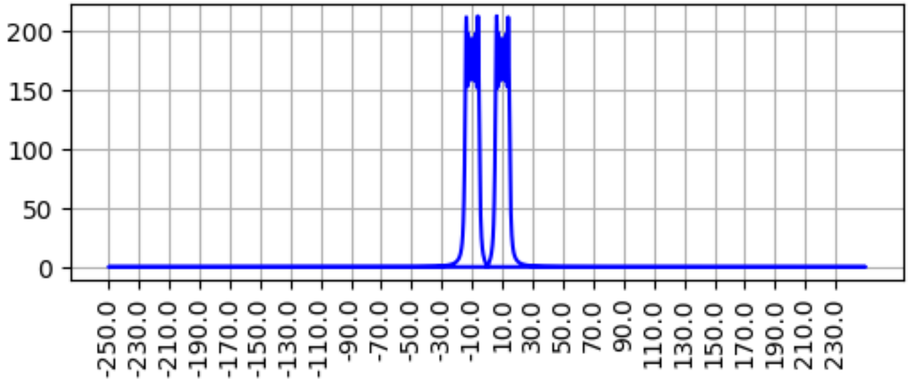
A Chirp Signal

Frequencies: 5 Hz → 15 Hz

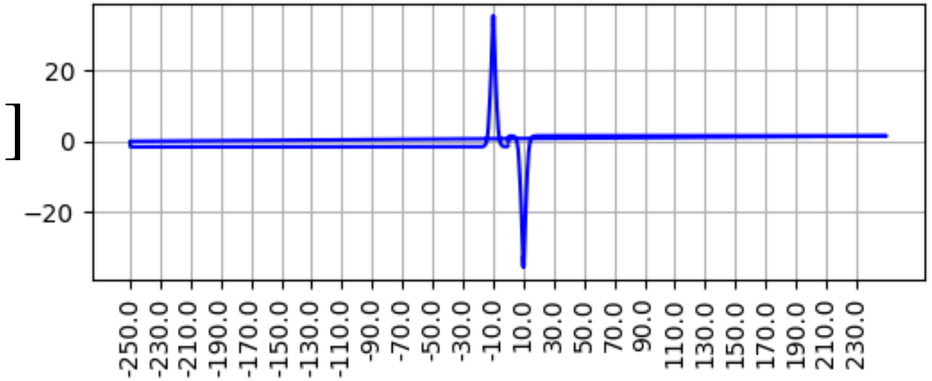
x_1



$|X_1[k]|$

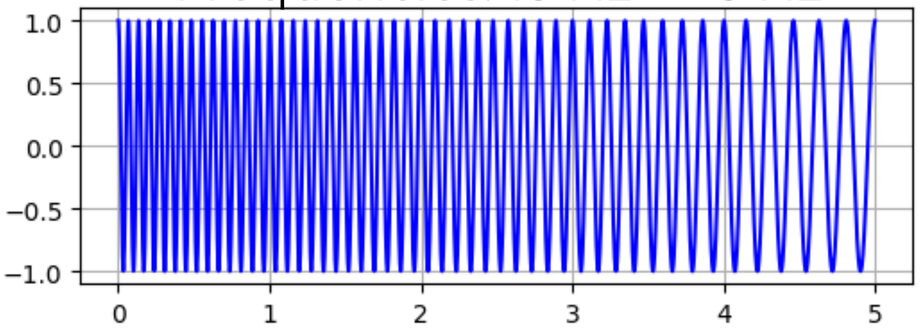


$\arg[X_1[k]]$

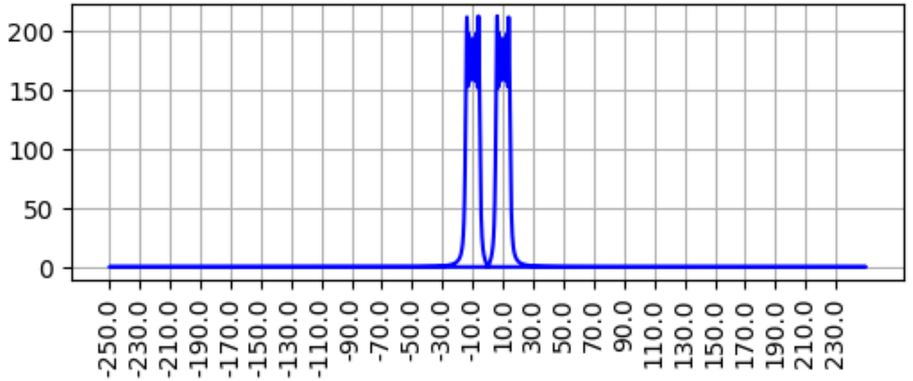


Frequencies: 15 Hz → 5 Hz

x_2



$|X_2[k]|$

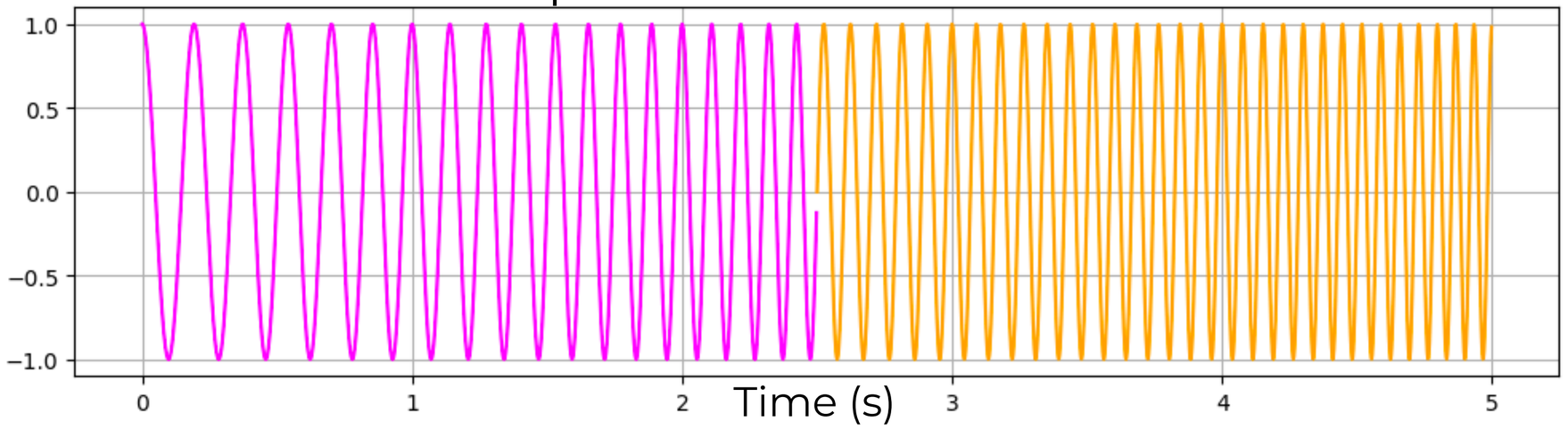


$\arg[X_2[k]]$

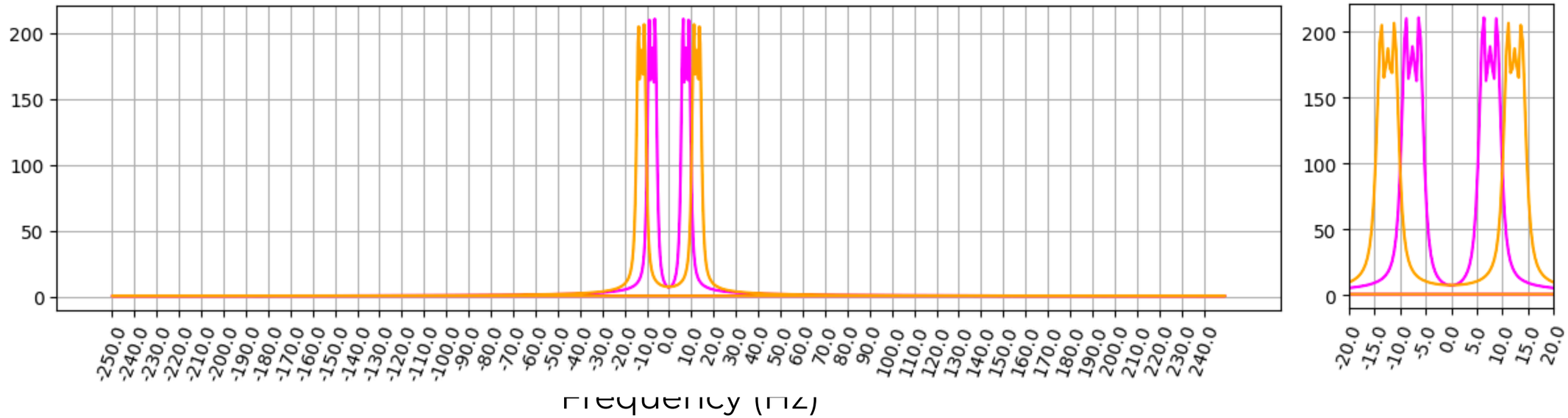
A Chirp Signal

Frequencies: 5 Hz → 15 Hz

x

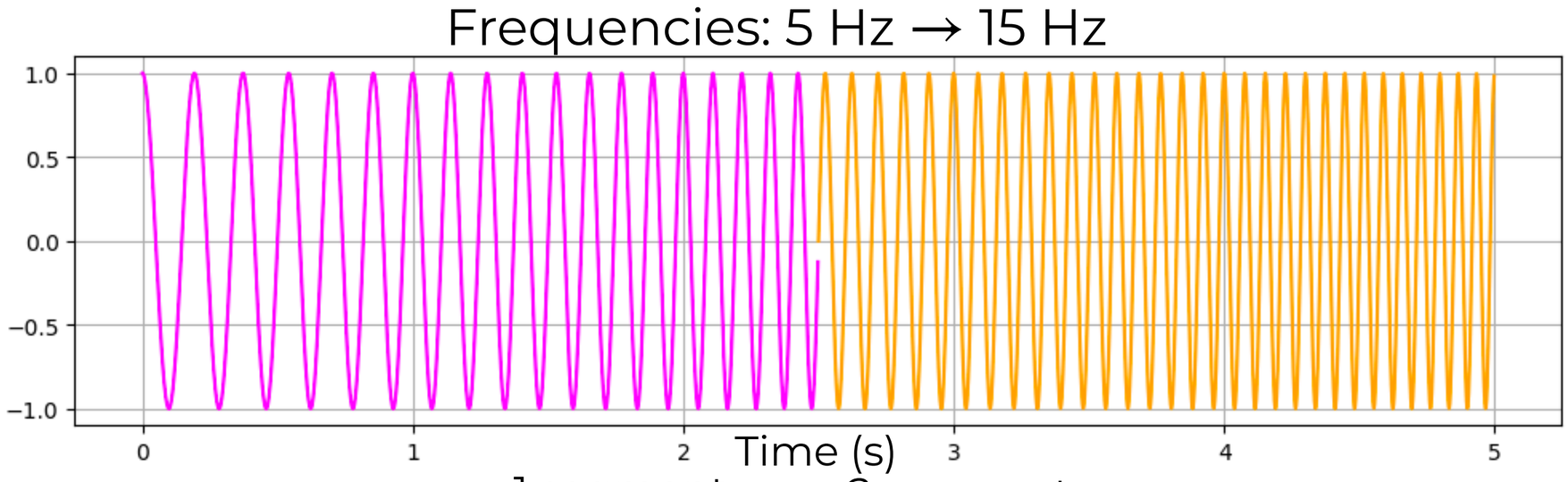


$|X[k]|$

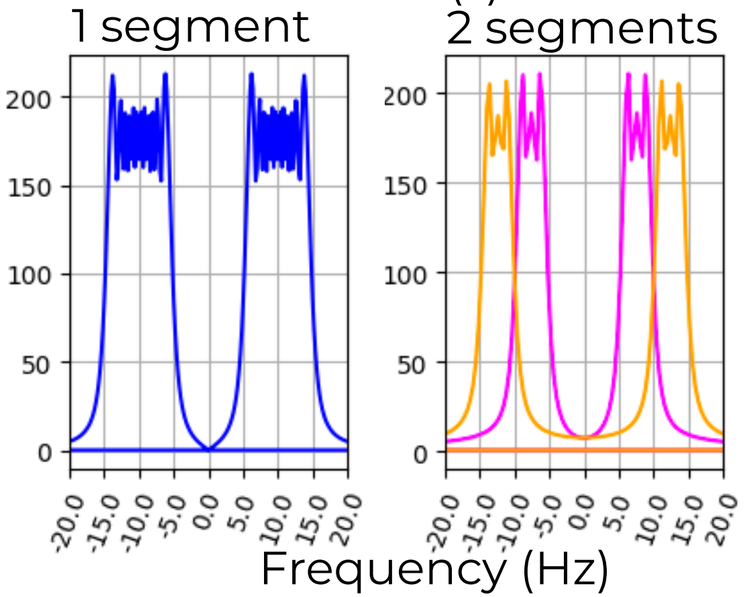


A Chirp Signal

x



$|X[k]|$



Short-Time Fourier Transform (STFT)

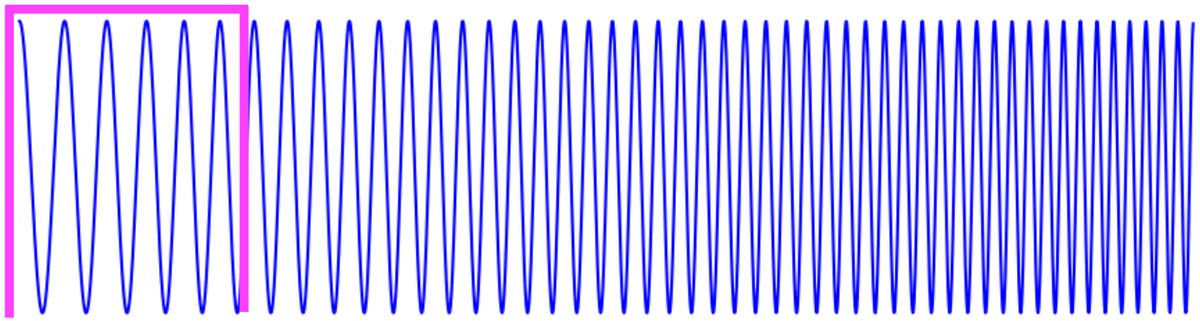
The short-time Fourier transform (the time-dependent Fourier transform) of $x[n]$ is denoted as

discrete continuous

$$X[n,\lambda) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\lambda m}$$

The discrete-time Fourier transform (DTFT) of the cropped signal

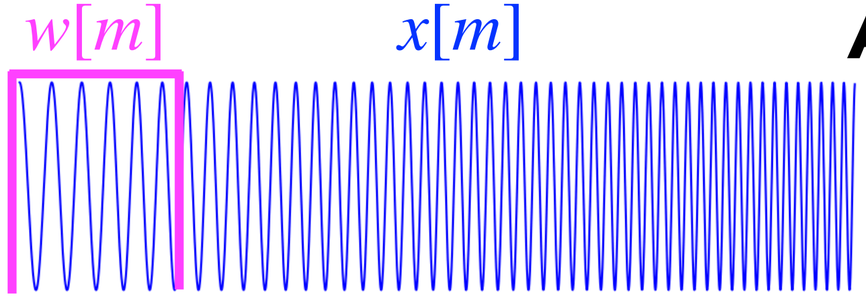
$$x[n+m]w[m]$$



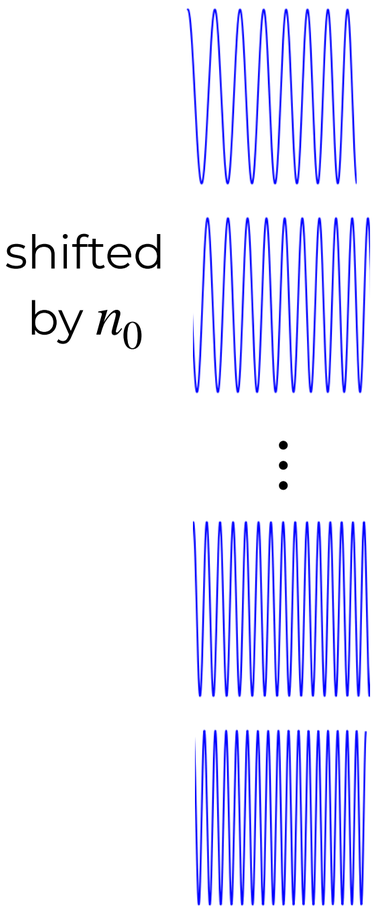
Oppenheim, Alan V. Discrete-time signal processing. Pearson Education India, 1999.

A Discrete Version

(DTFT → DFT)



$x[n+m]w[m]$



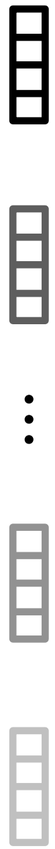
DFT → $X[0,k]$

DFT → $X[n_0,k]$

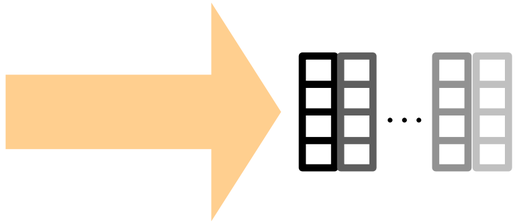
DFT →

DFT →

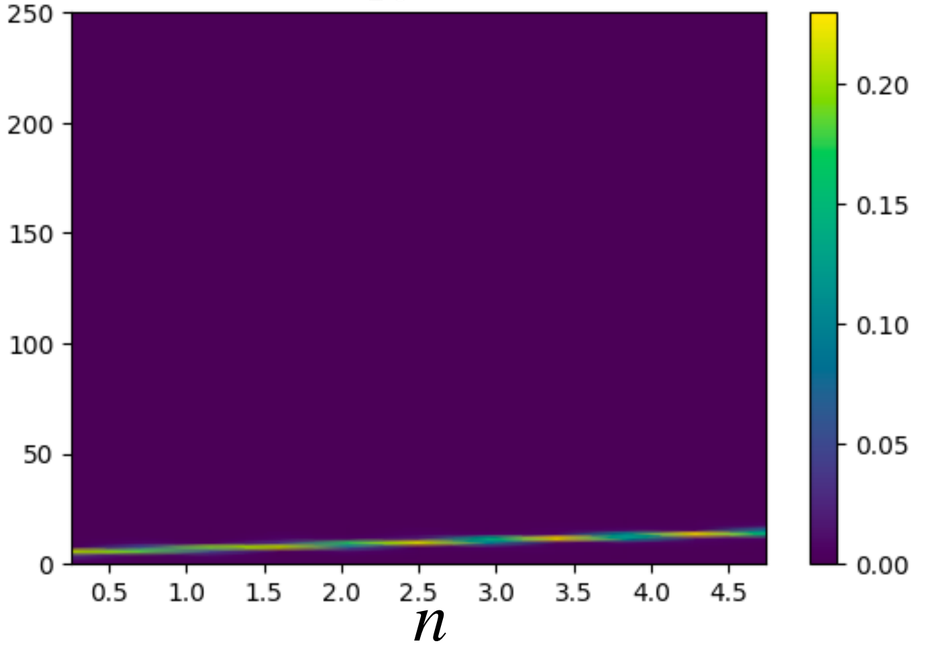
$X[n,k]$



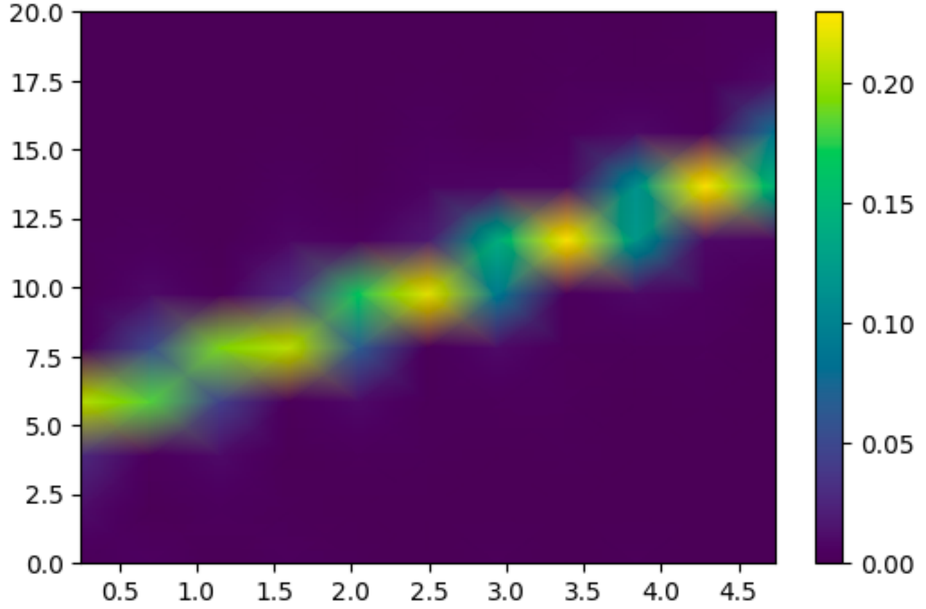
process and concatenate



$20\log_{10}|X[n,k]|$



zoomed-in view



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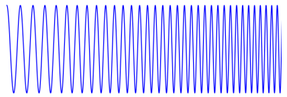
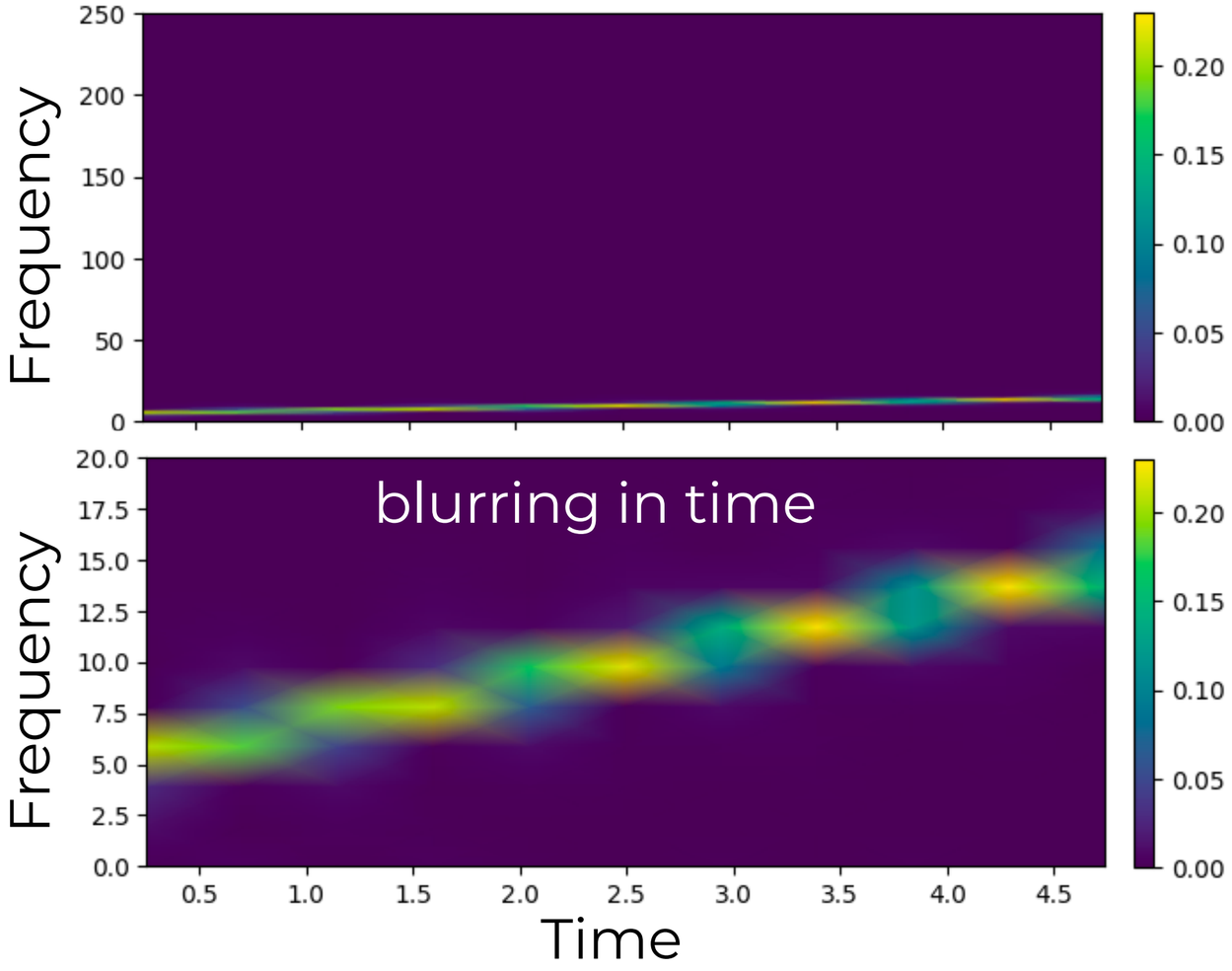
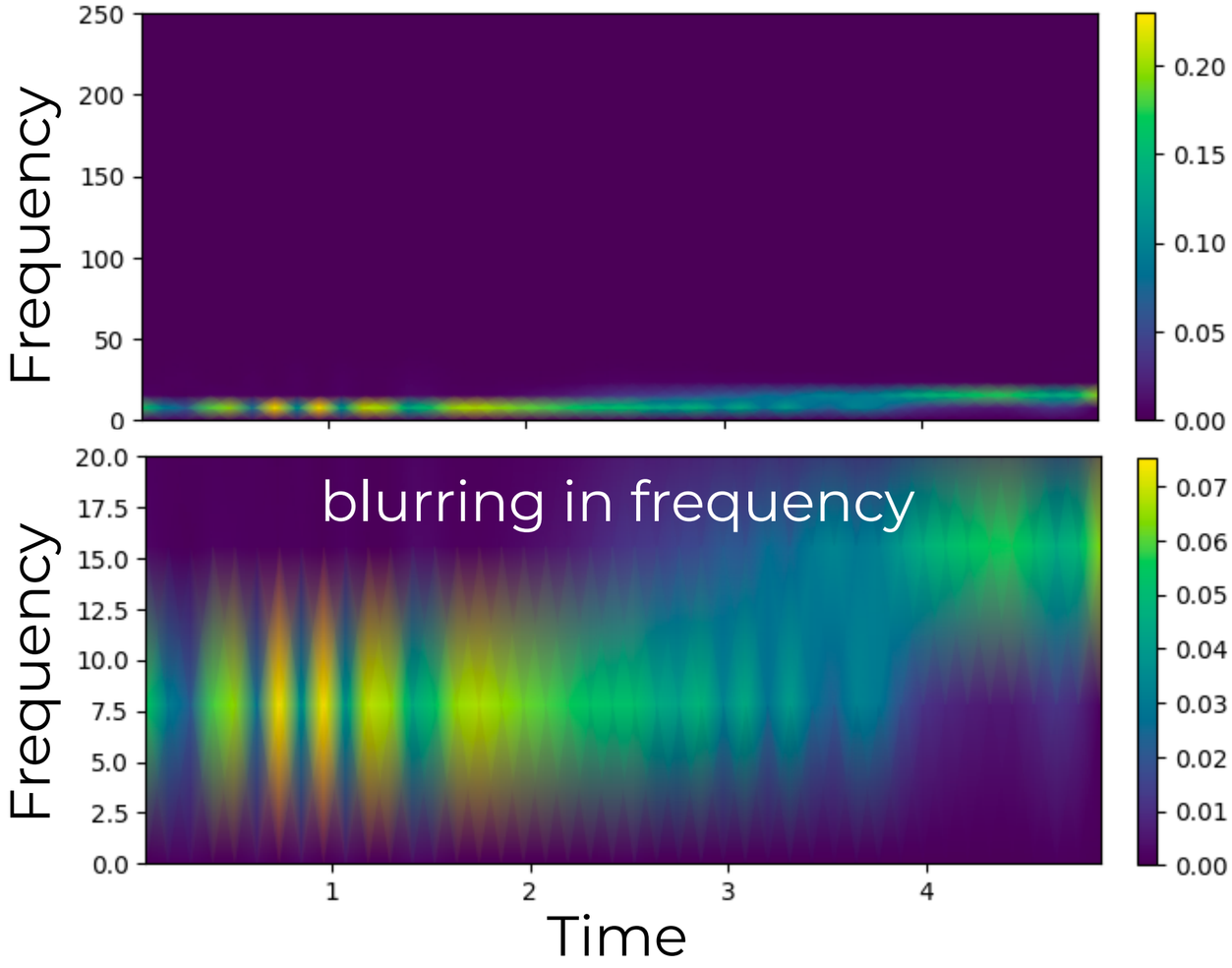
Spectrograms



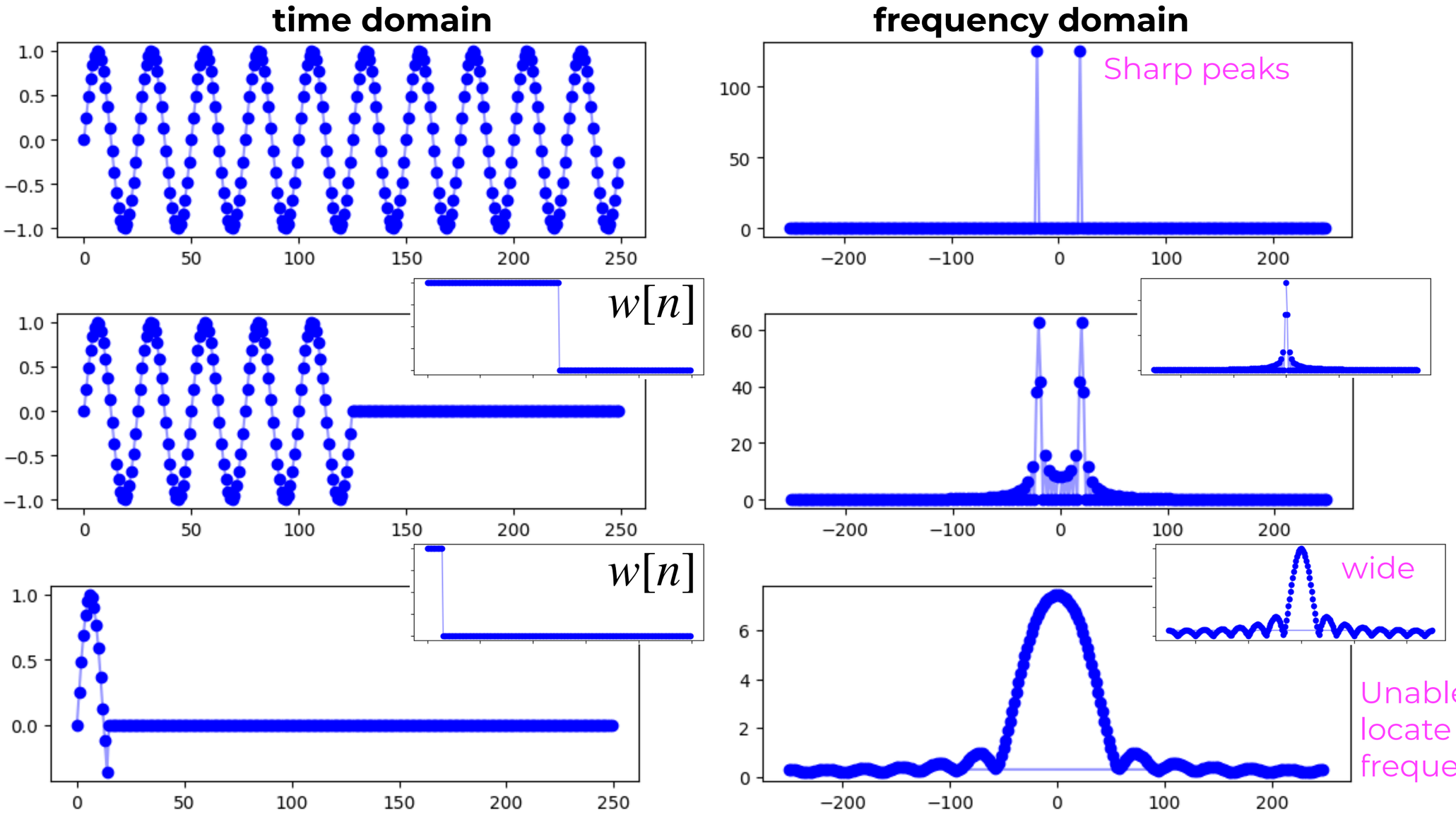
window length = 256

window length = 64





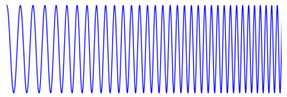
The Inverse Relationship Between Time and Frequency



Brain Code Camp

Spectrograms

window length = 256



window length = 64