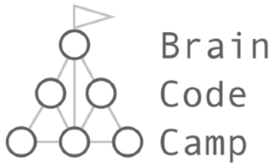


Sampling and Aliasing

Itthi Chatnuntawech

Brain Code Camp

The Sampling Theorem

Communication in the Presence of Noise*

CLAUDE E. SHANNON†, MEMBER, IRE

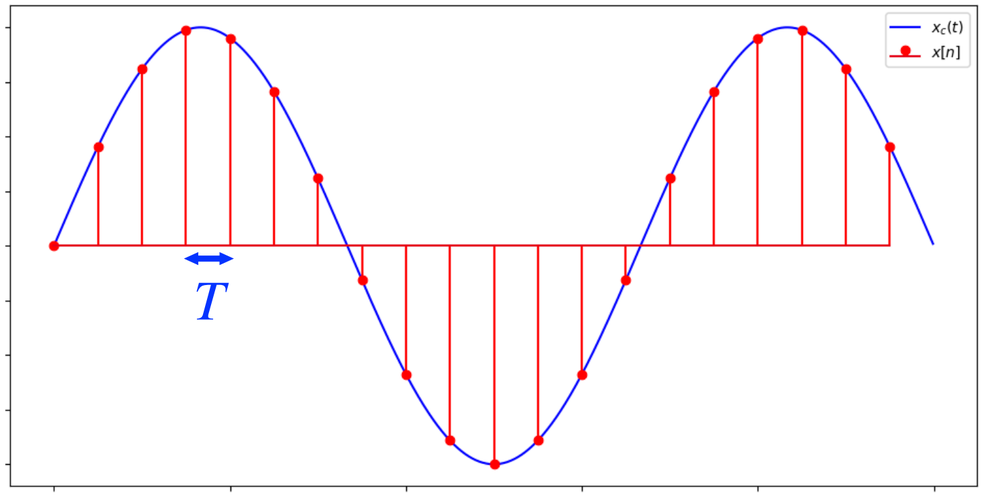
THEOREM 1: *If a function $f(t)$ contains no frequencies higher than W cps, it is completely determined by giving its ordinates at a series of points spaced $1/2W$ seconds apart.*

If a function $x_c(t)$ contains no frequencies higher than W Hz, then it can be completely determined from $x[n] = x_c(nT)$ if $T < \frac{1}{2W}$.

$$f_s = \frac{1}{T} > 2W$$

The Nyquist rate - $2W$

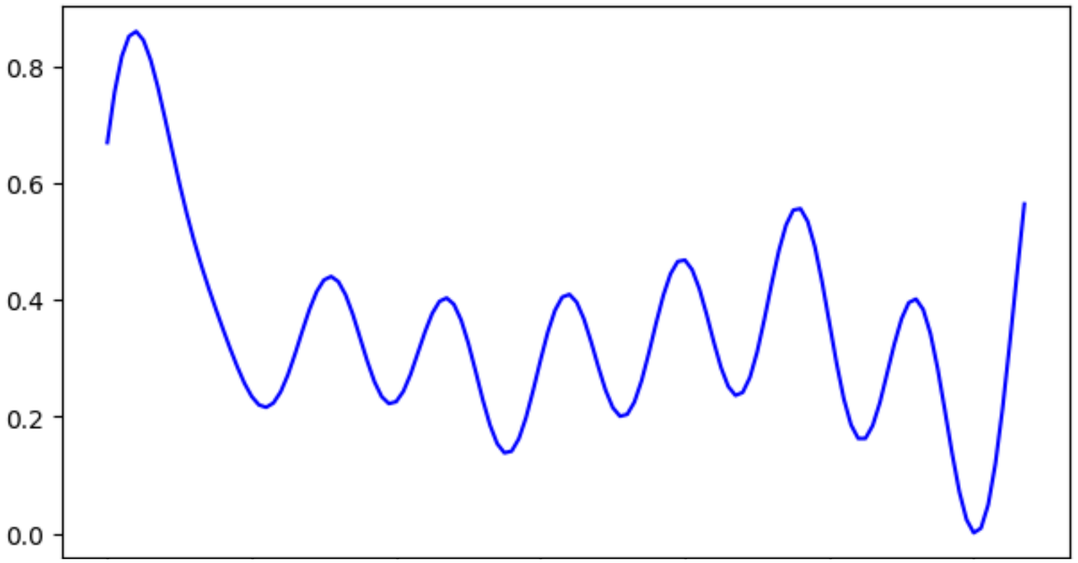
The Nyquist frequency - W



Brain Code Camp

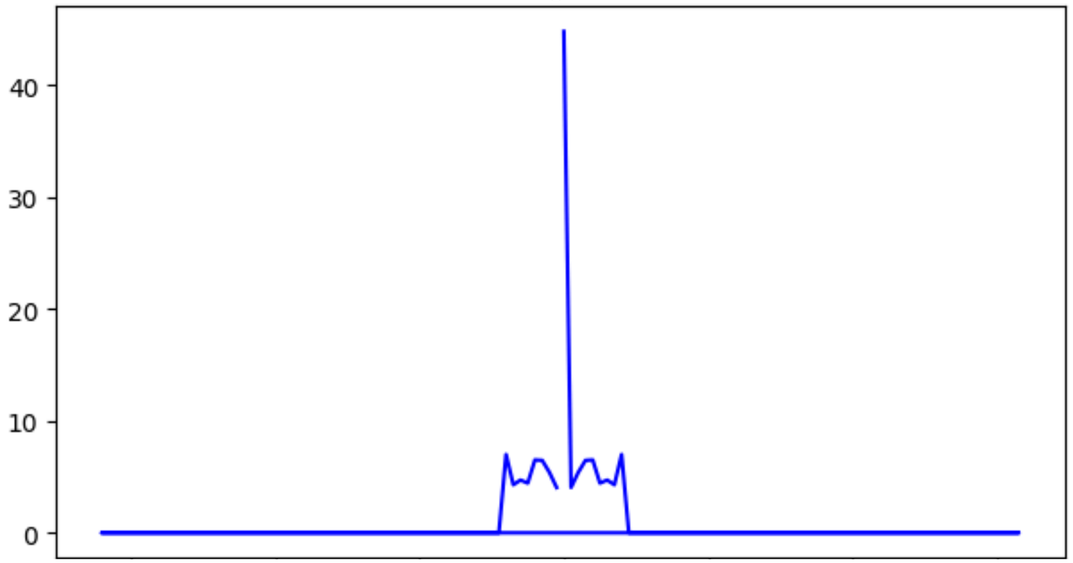
The Sampling Theorem

Time domain



Time

Frequency domain



-8 Hz $\quad 8$ Hz $\quad$ Frequency

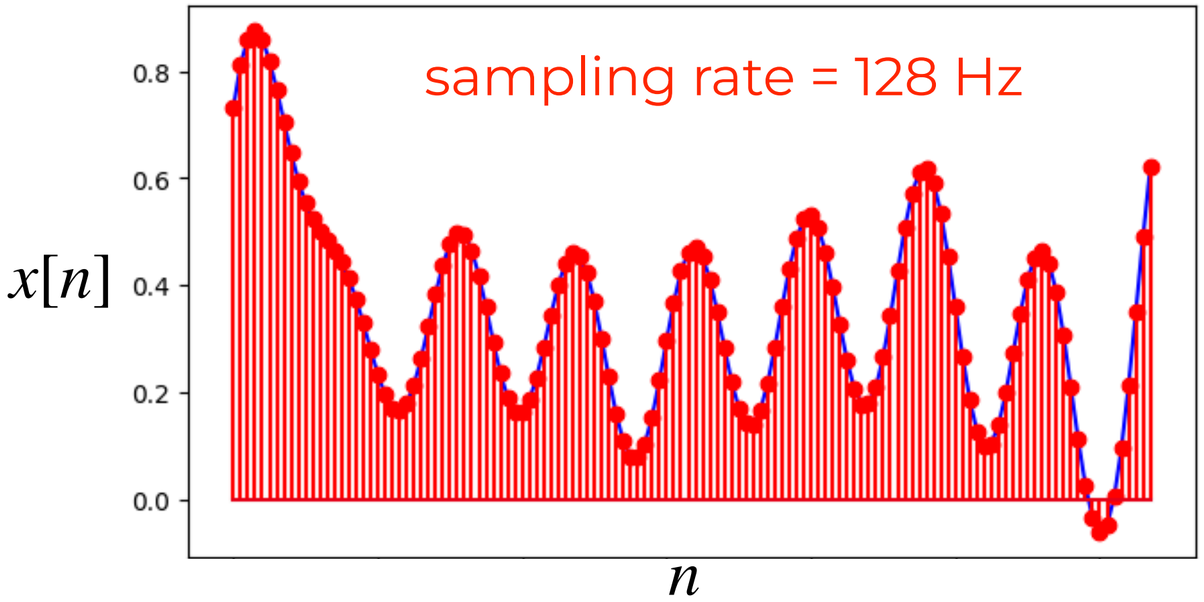
Looking at the magnitude plot in the frequency domain, we can see that this signal has non-zero values between -8 Hz and 8 Hz (inclusive)

$$f_s = \frac{1}{T} > 2W \text{ and } W = 8 \quad \Rightarrow \quad f_s > 16 \text{ Hz}$$

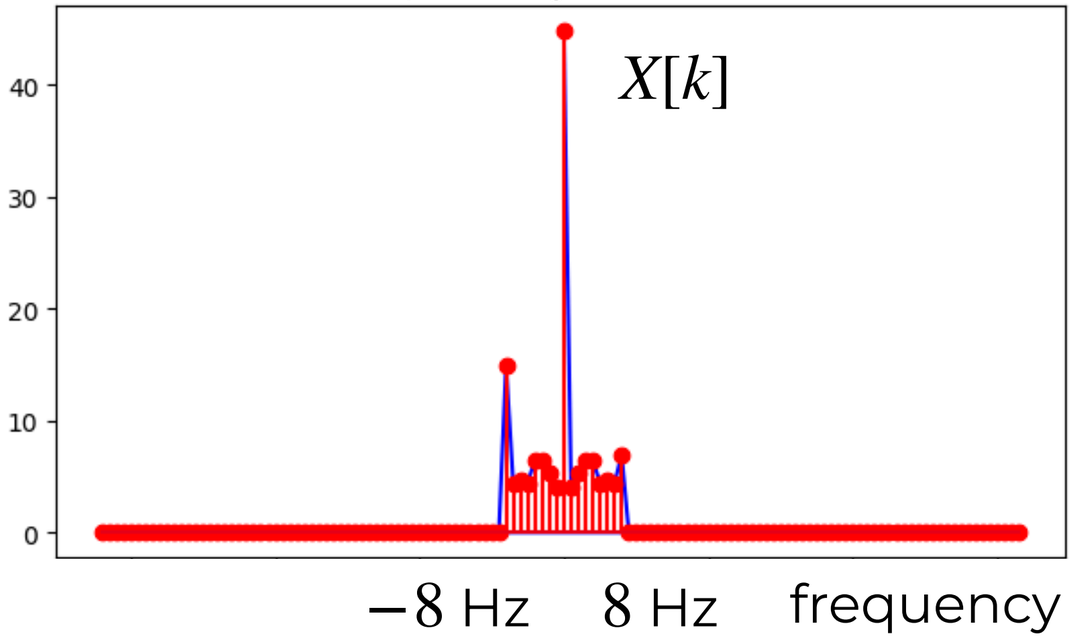
Brain Code Camp

Downsampling and Aliasing

Time domain



Frequency domain



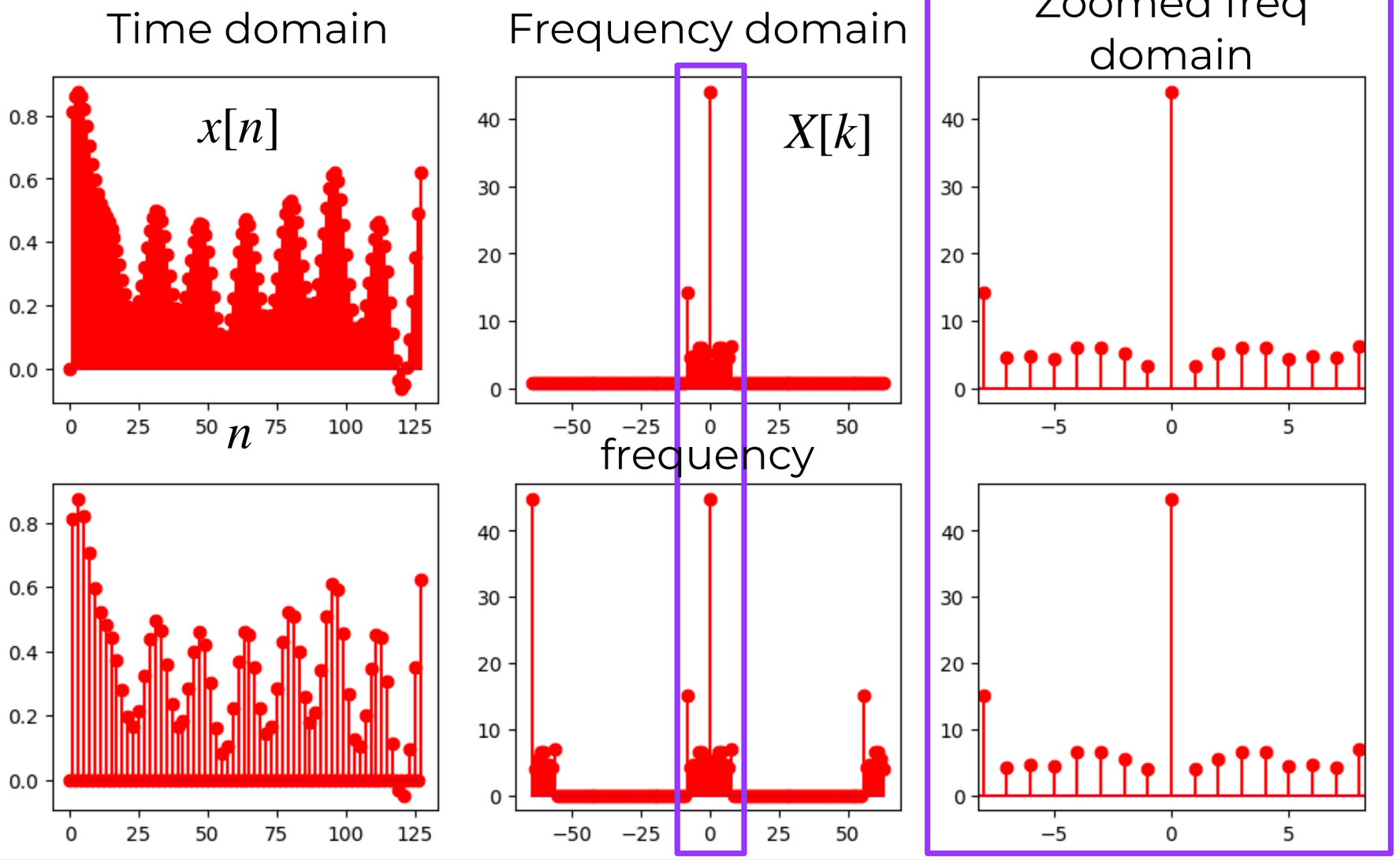
Downsampling and Aliasing

The Nyquist rate = 16 Hz

Use $f_s > 16$ Hz.

sampling rate = 128 Hz

sampling rate = 64 Hz



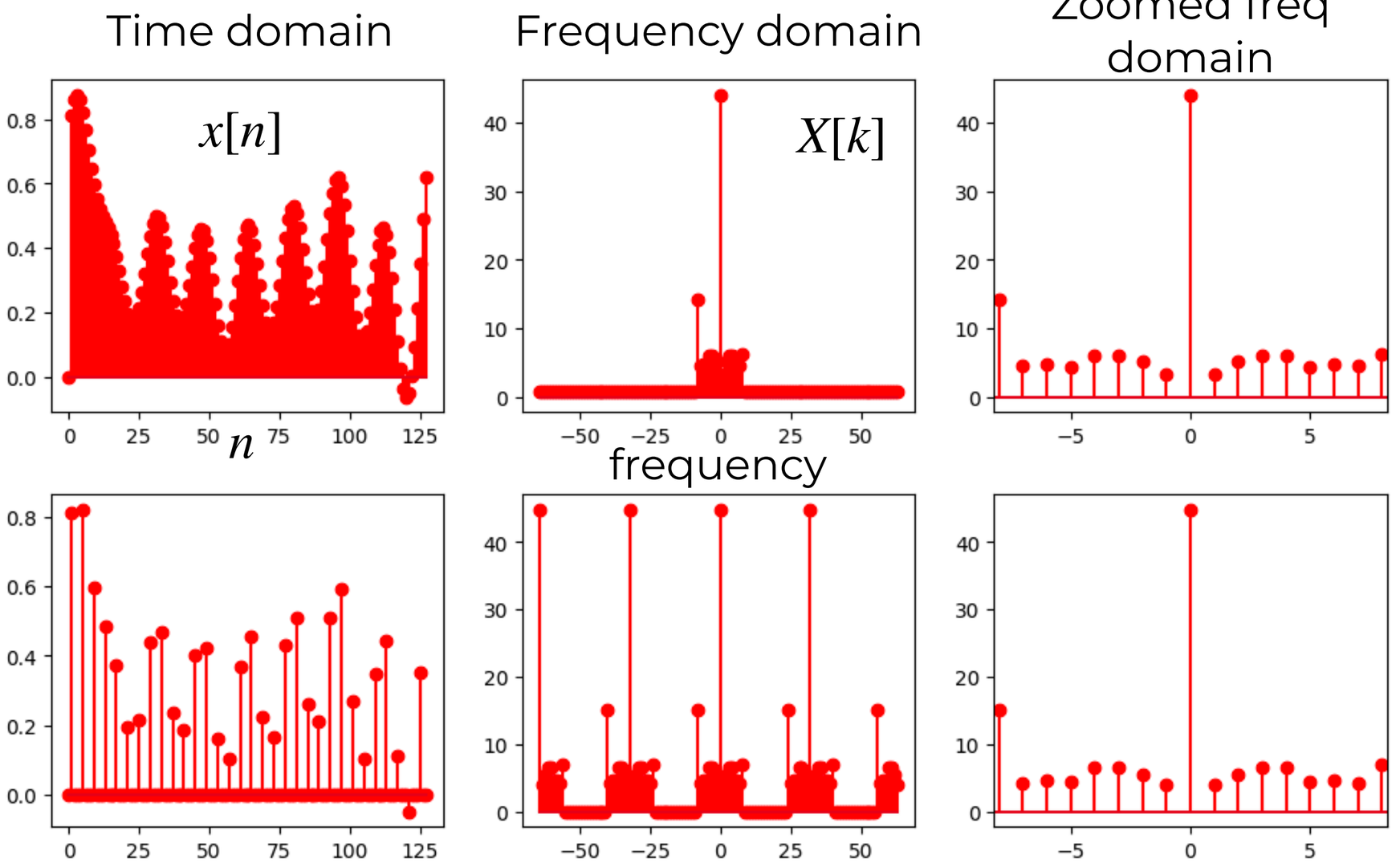
Downsampling and Aliasing

The Nyquist rate = 16 Hz

Use $f_s > 16$ Hz.

sampling rate = 128 Hz

sampling rate = 32 Hz



Downsampling and Aliasing

The Nyquist rate = 16 Hz

Use $f_s > 16$ Hz.

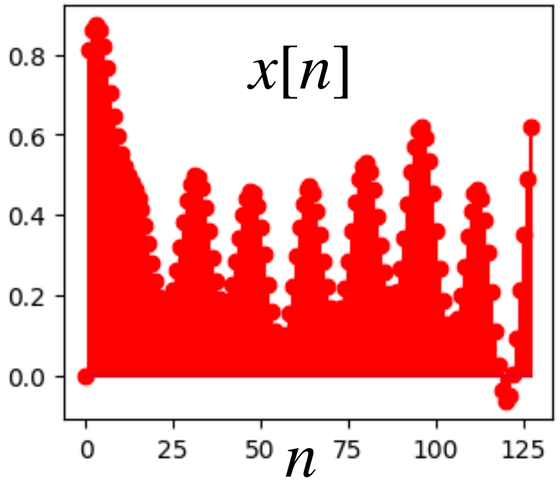
sampling rate = 128 Hz

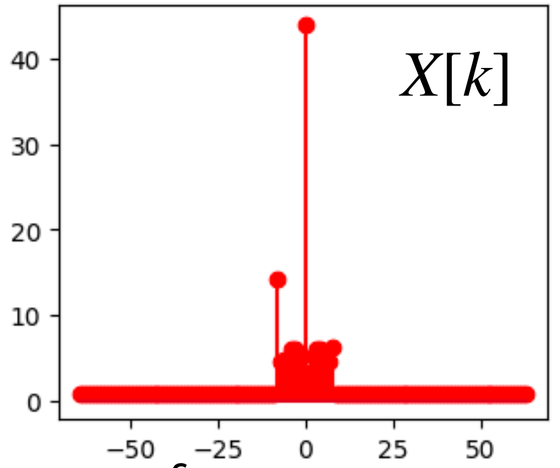
sampling rate = 16 Hz

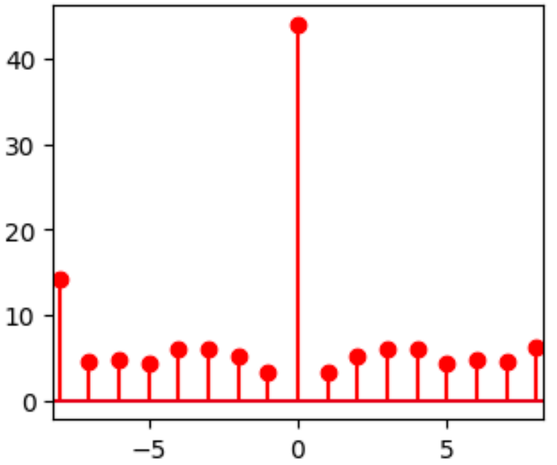
Time domain

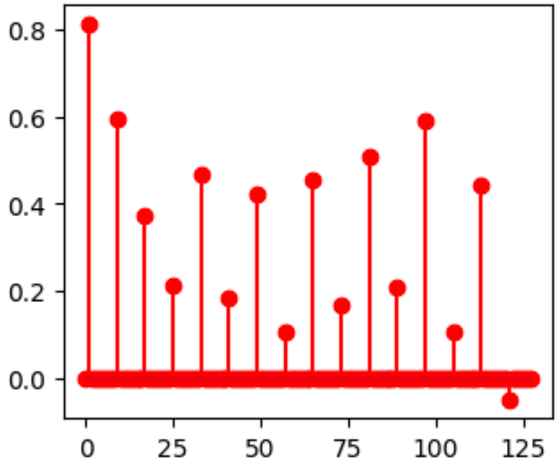
Frequency domain

Zoomed freq domain

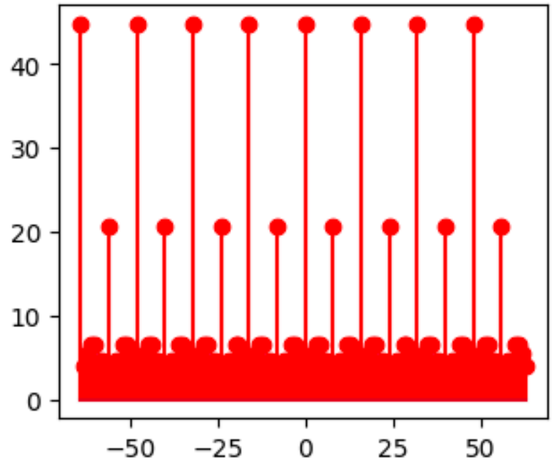


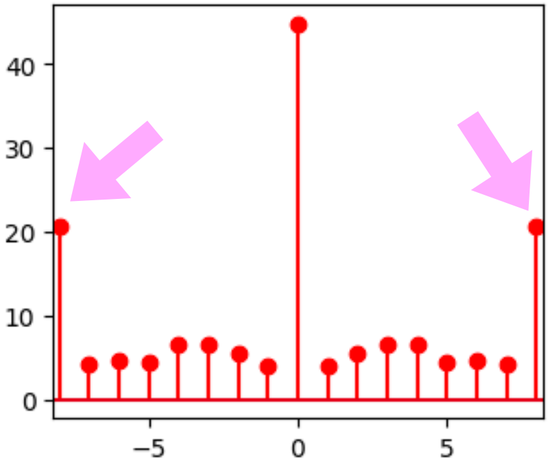






frequency





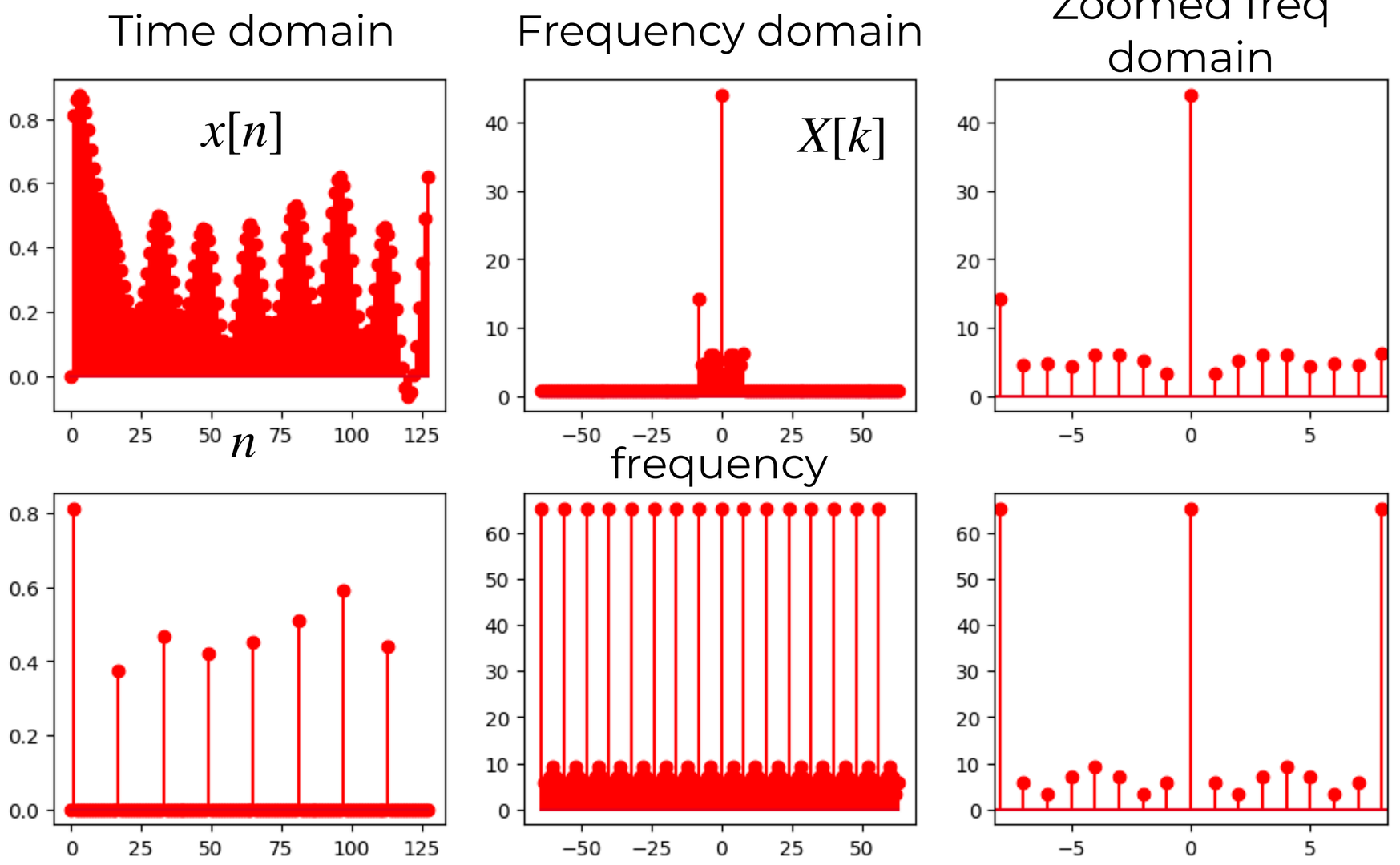
Downsampling and Aliasing

The Nyquist rate = 16 Hz

Use $f_s > 16$ Hz.

sampling rate = 128 Hz

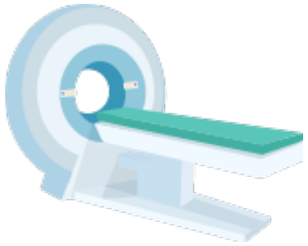
sampling rate = 8 Hz



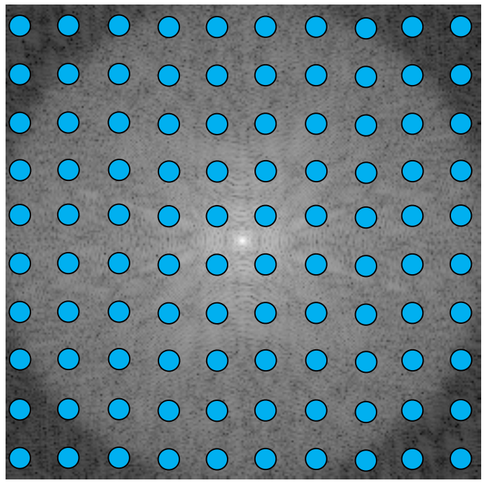
MRI Acquisition and Reconstruction

- The acquired data are the DFT samples of the object being imaged
- If the sampling rate is high enough, the image can be reconstructed by applying the inverse DFT to the k-space data

MRI scanner

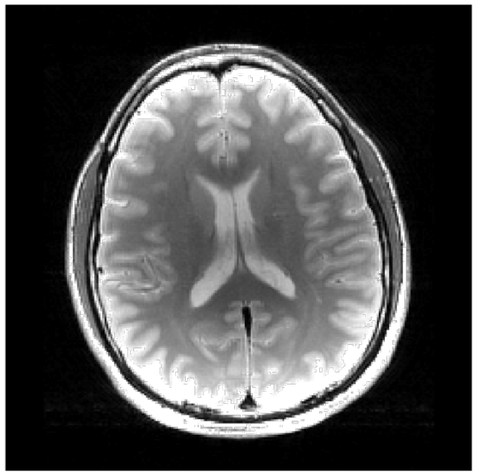


Acquired data
(k-space)

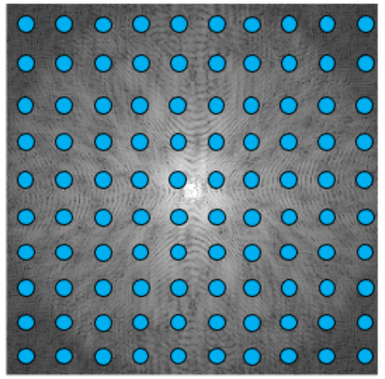
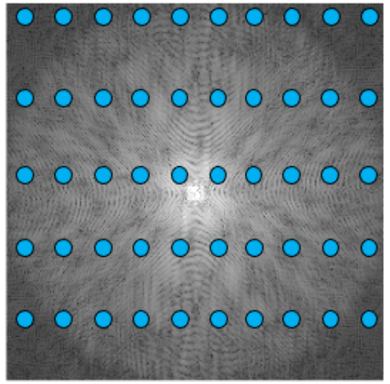
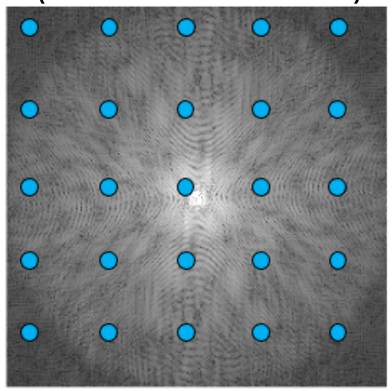
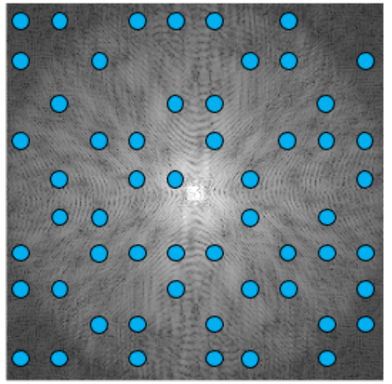
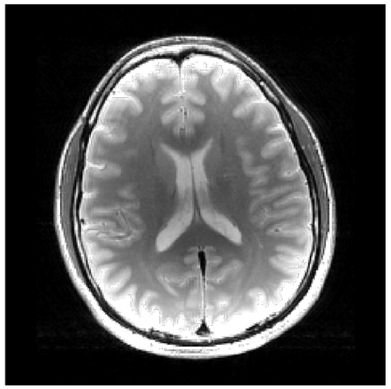
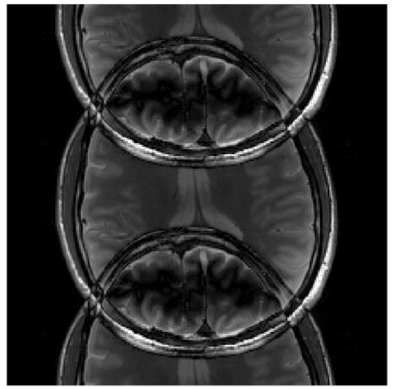
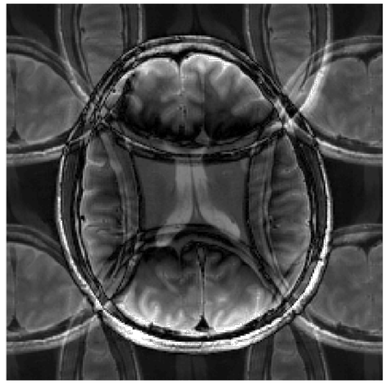
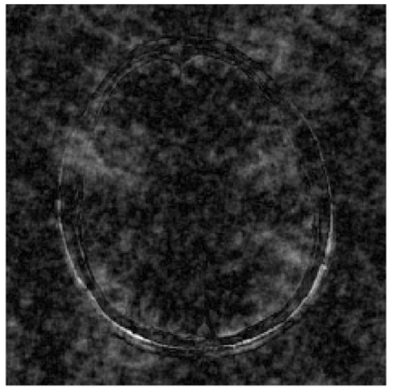


2D-IDFT

Reconstructed data
(image-space)



Undersampling

	Fully sampled acquisition	Uniformly undersampled (one direction)	Uniformly undersampled (two directions)	Randomly undersampled
Acquired data (k-space)				
	2D-IDFT	2D-IDFT	2D-IDFT	2D-IDFT
Reconstructed data (image-space)				
		Artifact (one direction)	Artifact (two directions)	Noise-like artifact