Discrete Fourier Transform (DFT)

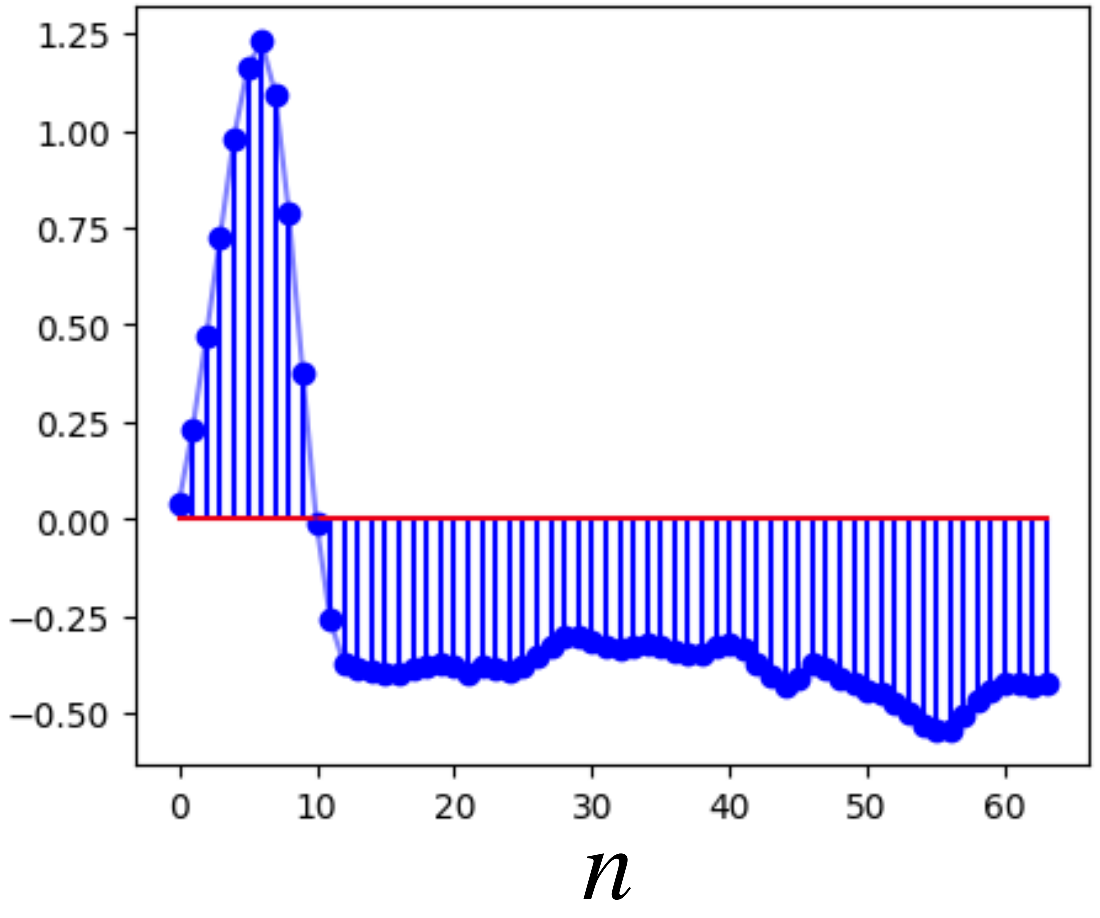
Itthi Chatnuntawech

Signal Representation

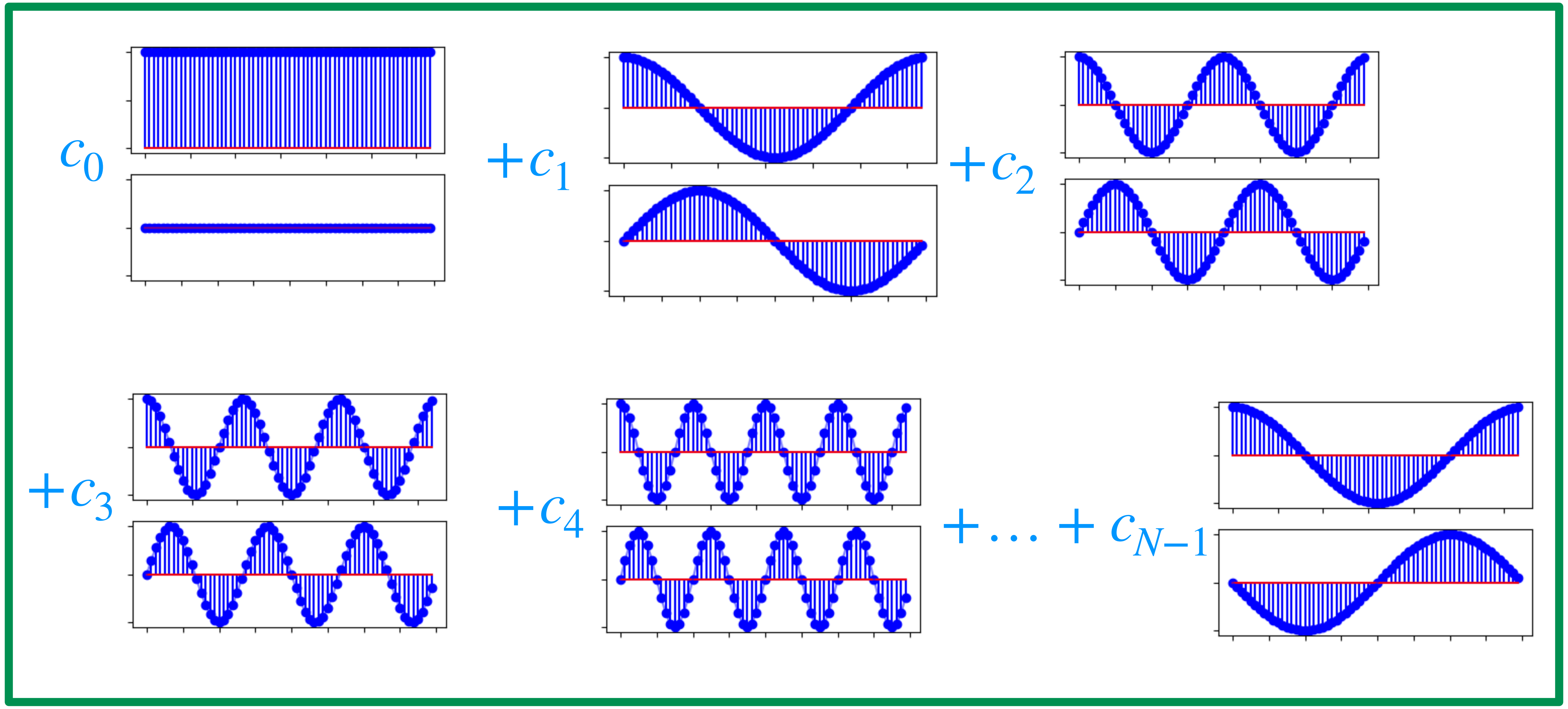
finite-length sequences

Most practical finite-duration, discrete-time signals can be written as a weighted sum of harmonically related complex exponentials

$x[n]$ of length N

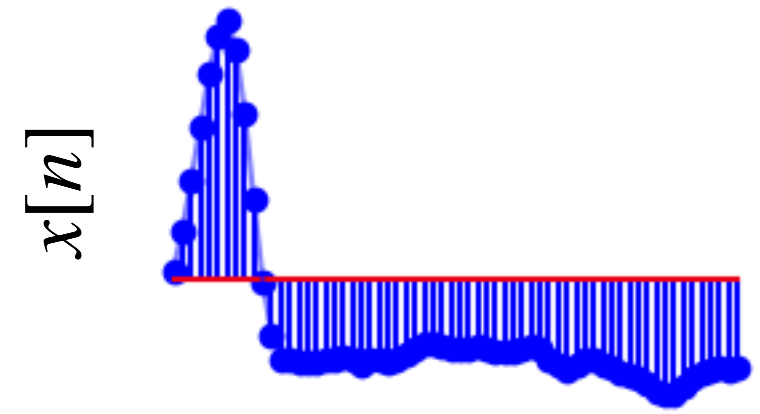


$\approx$



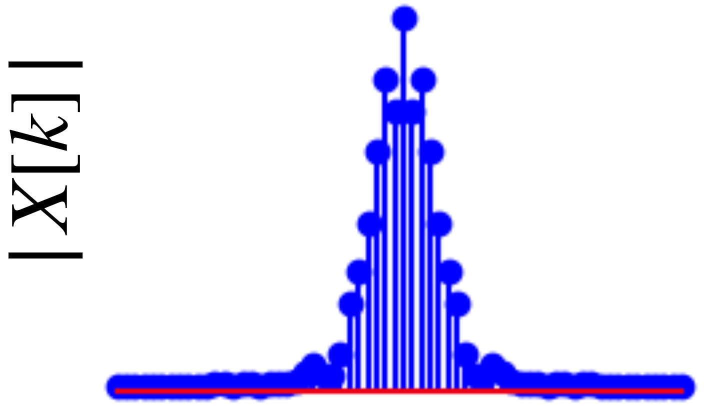
Discrete Fourier Transform (DFT)

Time domain



$$\underbrace{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}}_{}\overset{x[n]}{} \quad \overset{DFT}{\longleftrightarrow} \quad \overset{X[k]}{} \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

Frequency domain



$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j(\frac{2\pi}{N})kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]e^{-j(\frac{2\pi}{N})kn}$$

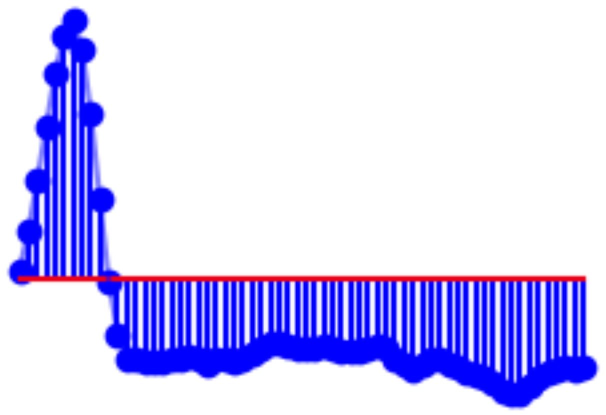
Oppenheim, Alan V. Discrete-time signal processing. Pearson Education India, 1999.

Oppenheim, Alan V., et al. Signals and systems. Vol. 2. Upper Saddle River, NJ: Prentice hall, 1997.

Brain Code Camp

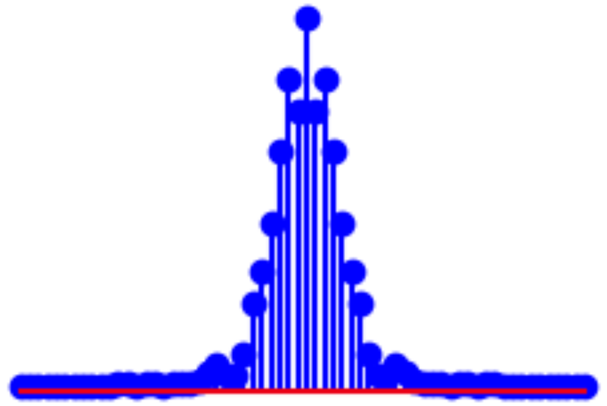
Discrete Fourier Transform (DFT)

Time domain



$$\underbrace{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}}_{x[n]} \quad \xleftrightarrow{DFT} \quad \underbrace{\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}}_{X[k]}$$

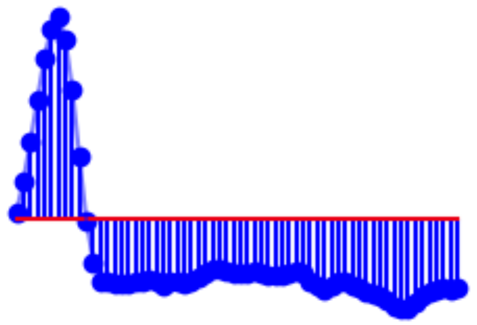
Frequency domain



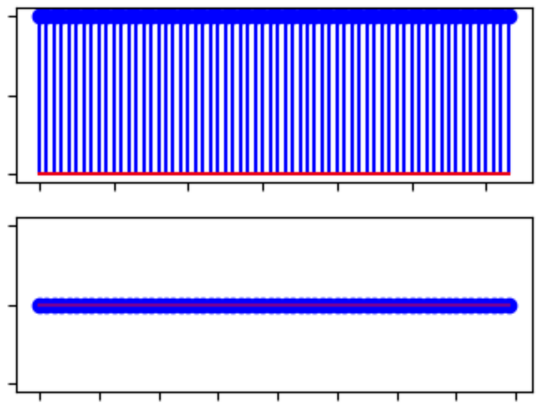
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j(\frac{2\pi}{N})kn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j(\frac{2\pi}{N})kn}$$

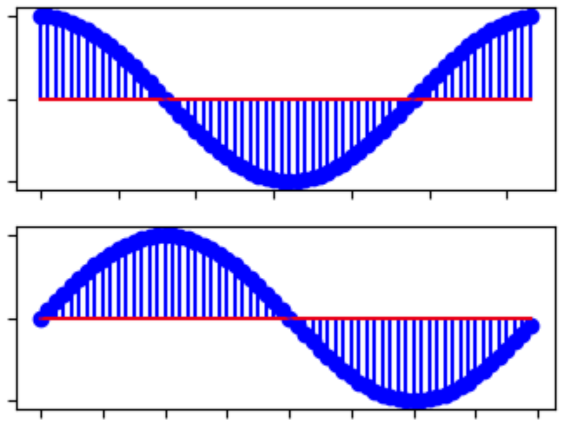
$x[n]$



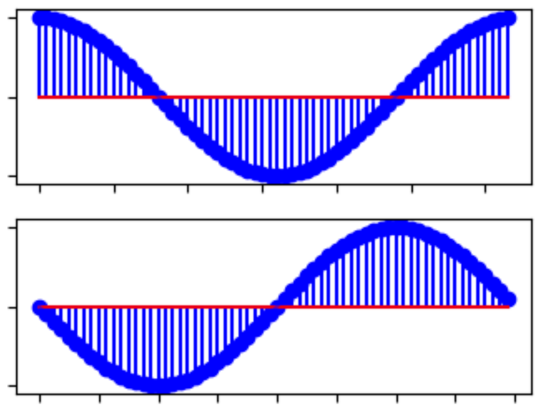
$= \frac{1}{N}\Big(X[0]$



e^{j0} $+\ X[1]$



$e^{j\frac{2\pi}{N}n}$ $+ \ldots +\ X[N-1]$

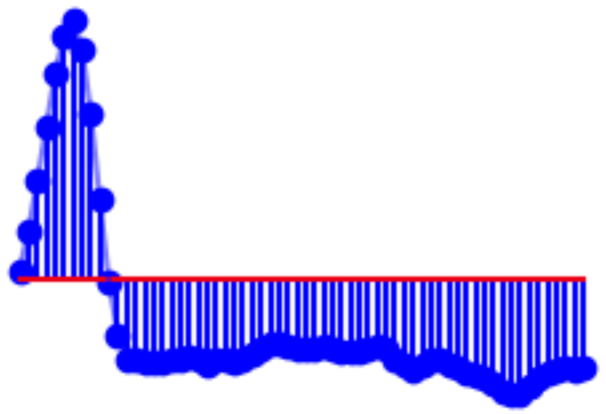


$e^{j\frac{2\pi}{N}(N-1)n}$ $\Big)$

Discrete Fourier Transform (DFT)

Time domain

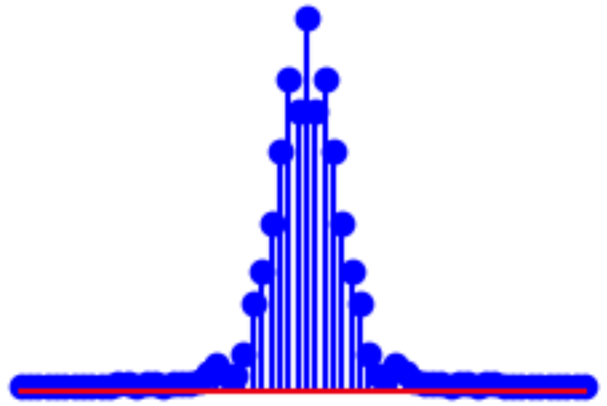
$x[n]$



$$\underset{x[n]}{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}} \overset{DFT}{\longleftrightarrow} \underset{X[k]}{\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}}$$

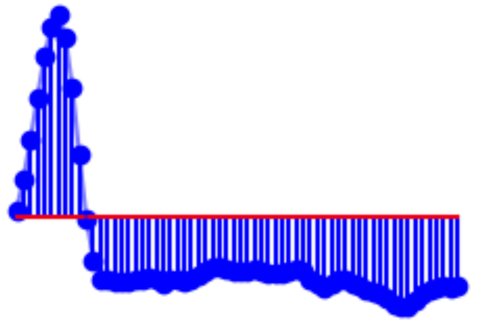
Frequency domain

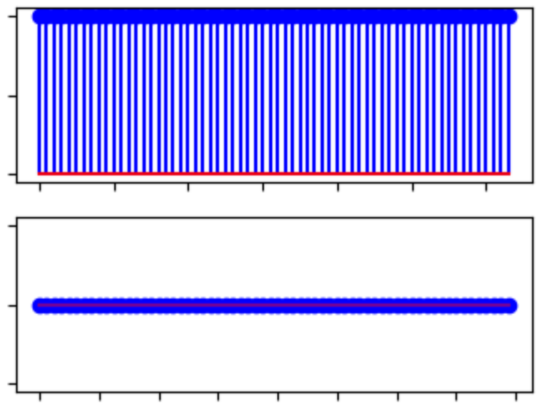
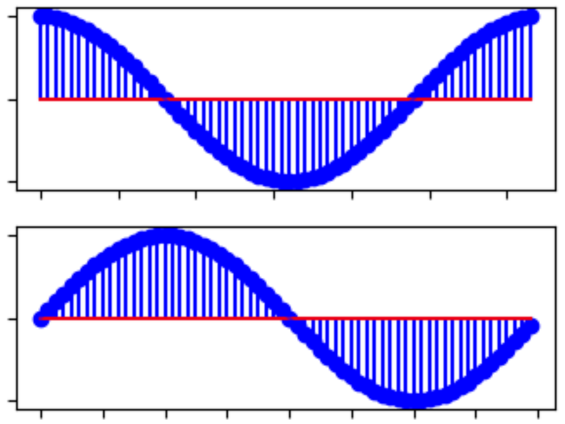
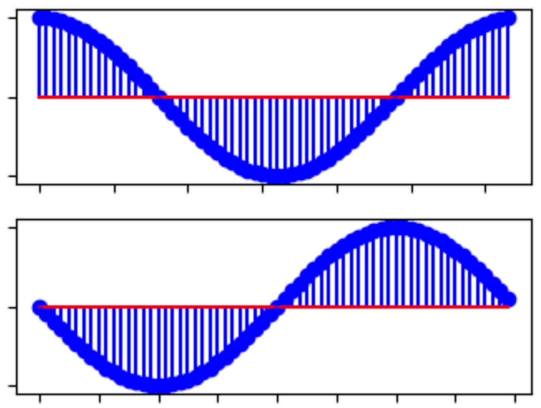
$|X[k]|$



$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j(\frac{2\pi}{N})kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]e^{-j(\frac{2\pi}{N})kn}$$

$x[n]$

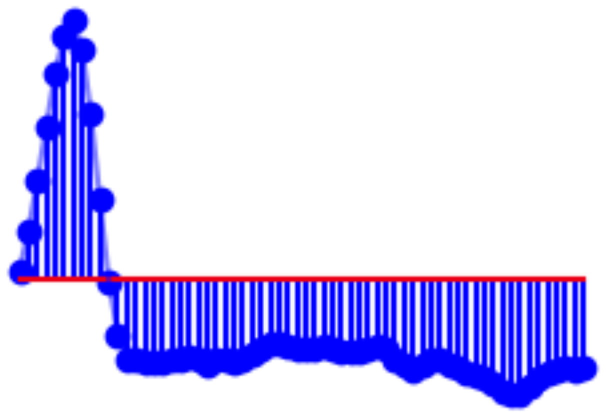


$= \frac{1}{N}\Big(X[0]$  $+ X[1]$  $+ \ldots + X[N-1]$  $\Big)$

Discrete Fourier Transform (DFT)

Time domain

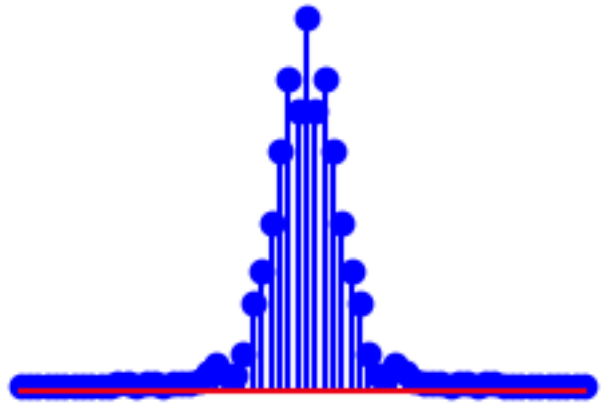
$x[n]$



$$\begin{array}{ccc} x[n] & & X[k] \\ \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} & \overset{DFT}{\longleftrightarrow} & \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} \end{array}$$

Frequency domain

$|X[k]|$



$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j(\frac{2\pi}{N})kn} \qquad X[k] = \sum_{n=0}^{N-1} x[n]e^{-j(\frac{2\pi}{N})kn}$$

scipy.fft.fft

```
scipy.fft.fft(x, n=None, axis=-1, norm=None, overwrite_x=False,
workers=None, *, plan=None)
```

[source]

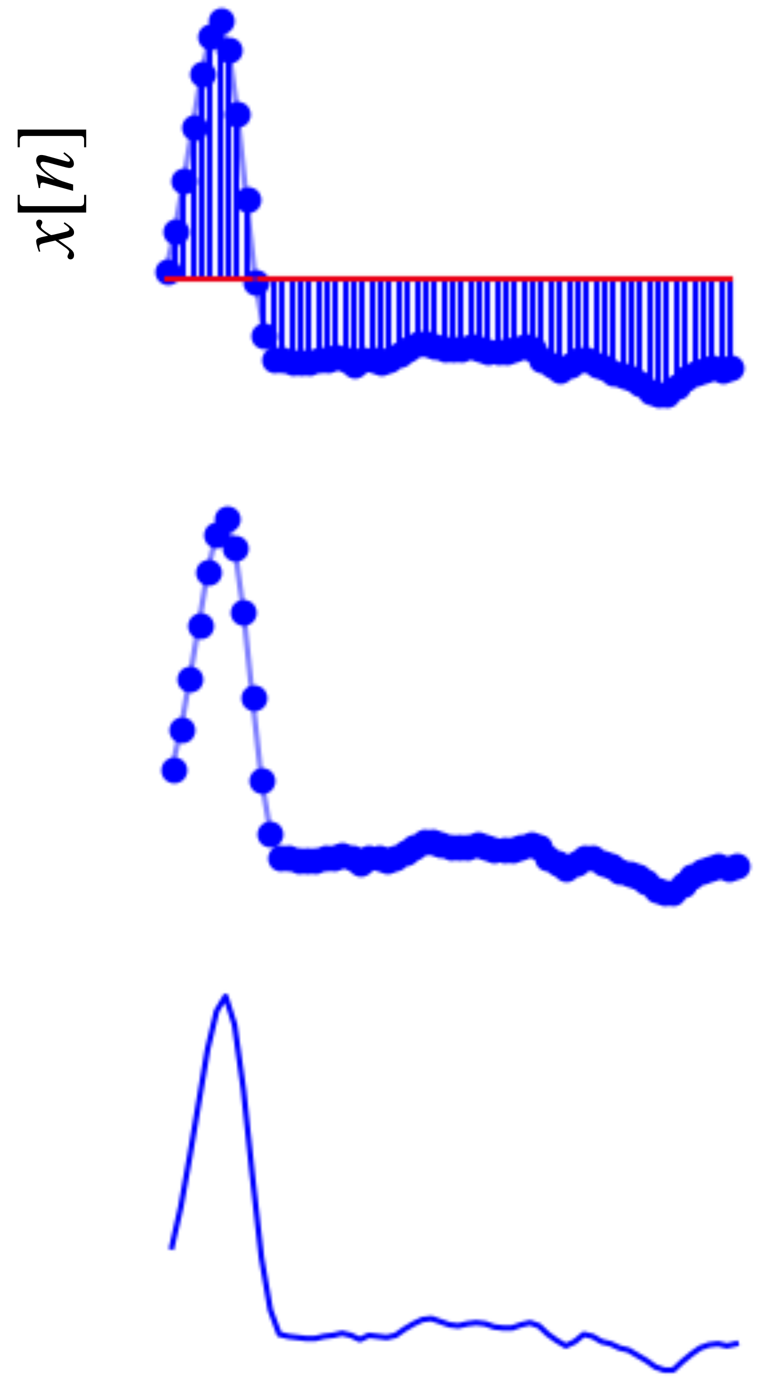
Compute the 1-D discrete Fourier Transform.

This function computes the 1-D *n*-point discrete Fourier Transform (DFT) with the efficient Fast Fourier Transform (FFT) algorithm [1].

Fast Fourier Transform (FFT) - An efficient algorithm that computes the DFT of a signal

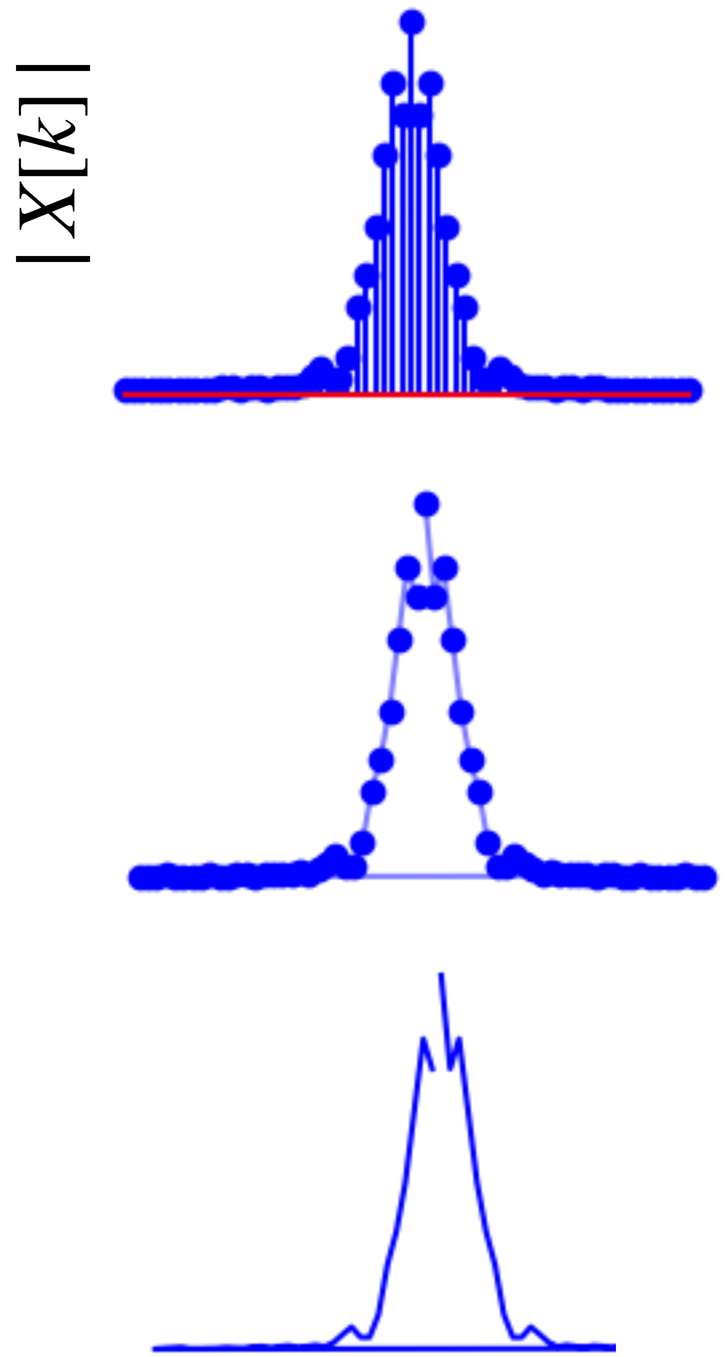
Discrete Fourier Transform (DFT)

Time domain



$$\underbrace{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}}_{}^{x[n]} \overset{DFT}{\longleftrightarrow} \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}^{X[k]}$$

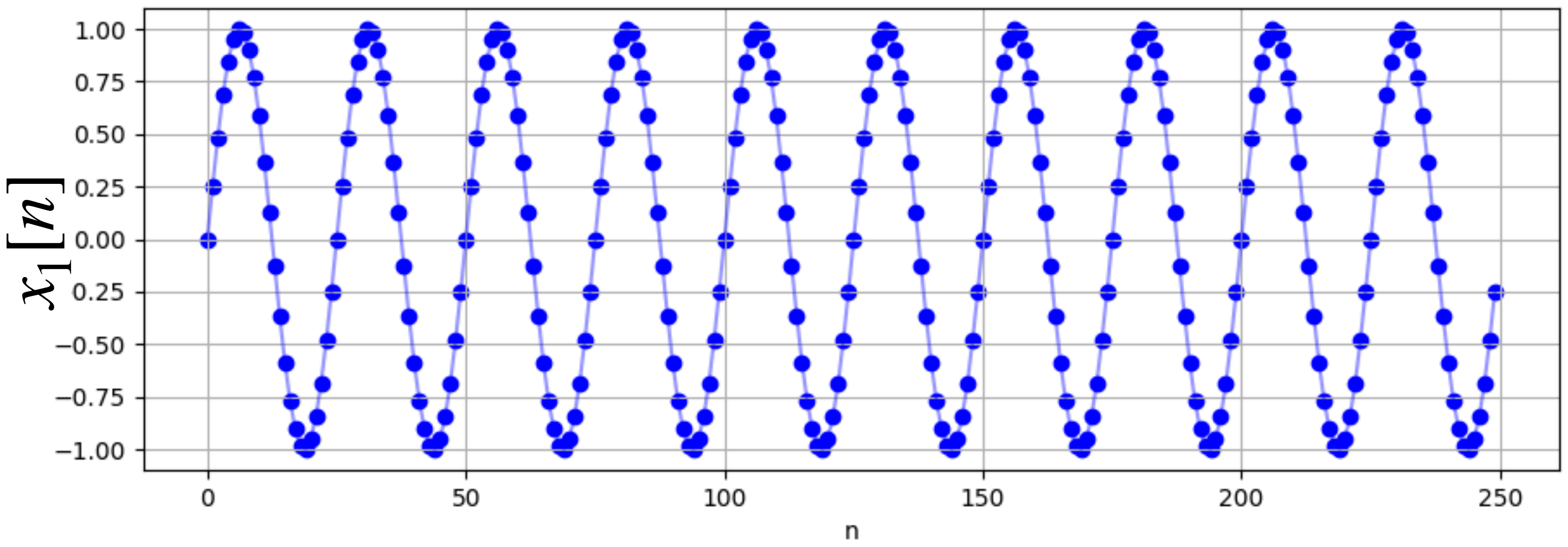
Frequency domain



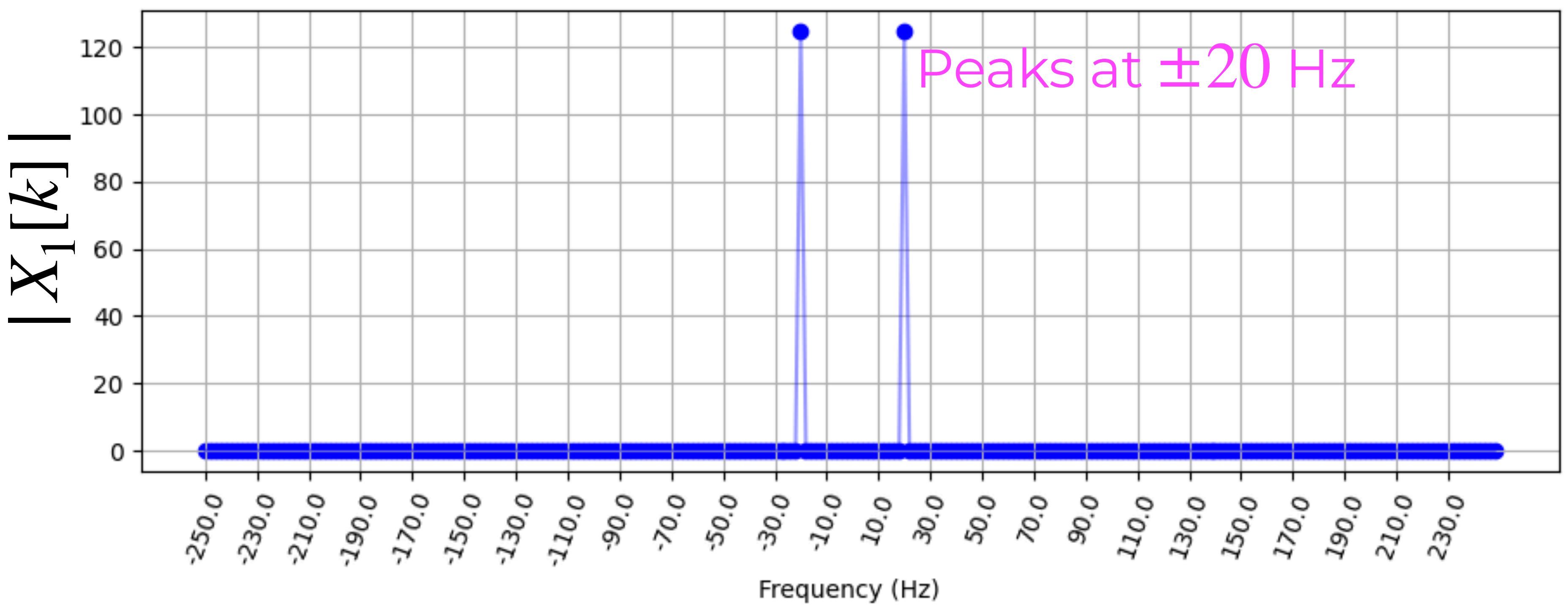
Discrete Fourier Transform (DFT)

Time domain

Samples from $x(t) = \sin(2\pi \times 20t)$



Frequency domain

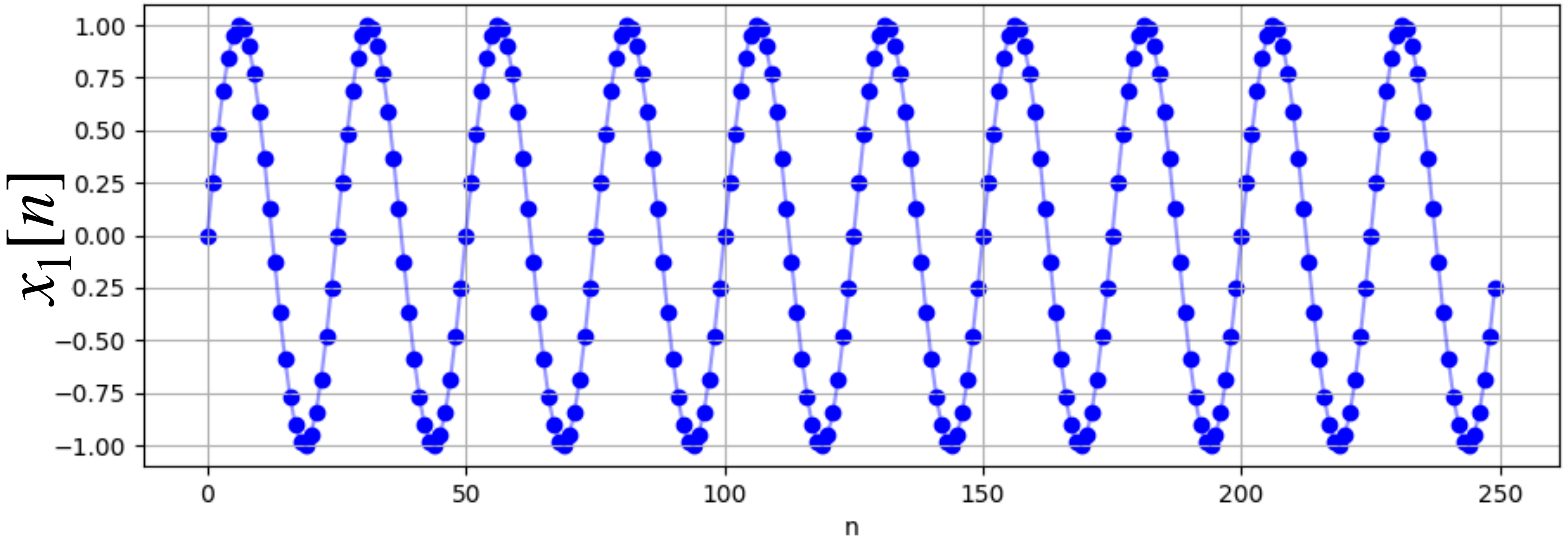


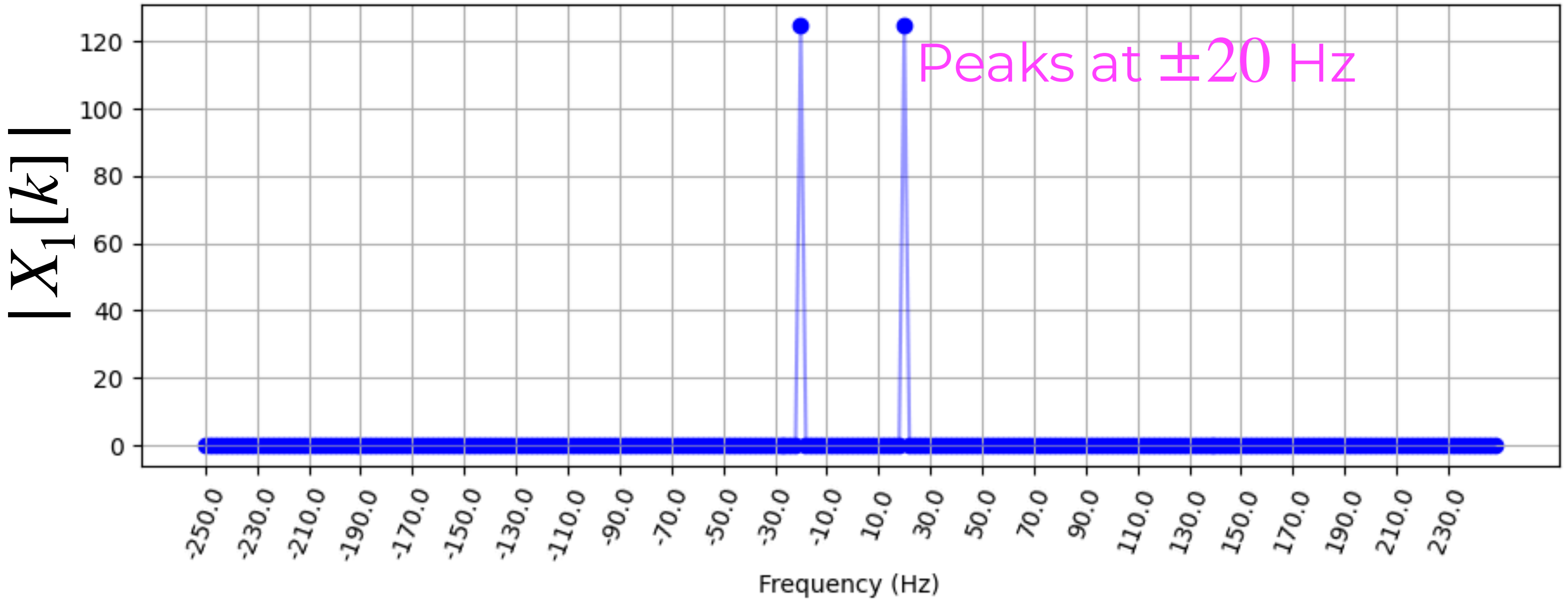
Discrete Fourier Transform (DFT)

Time domain

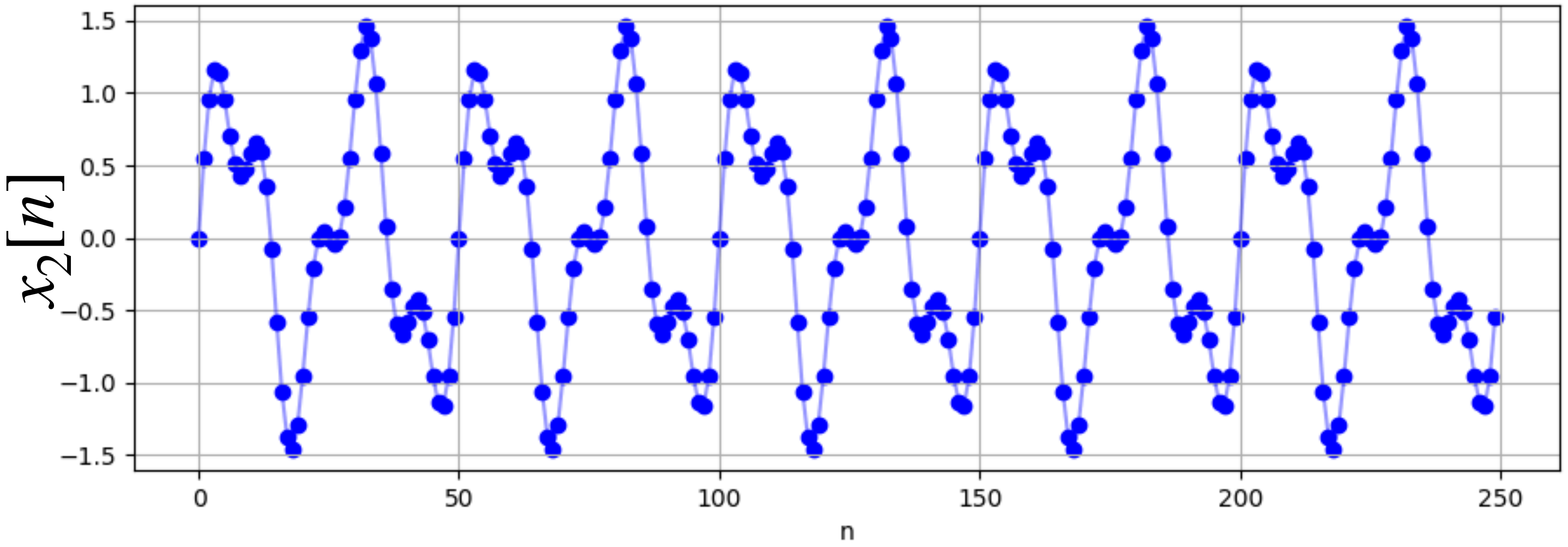
Frequency domain

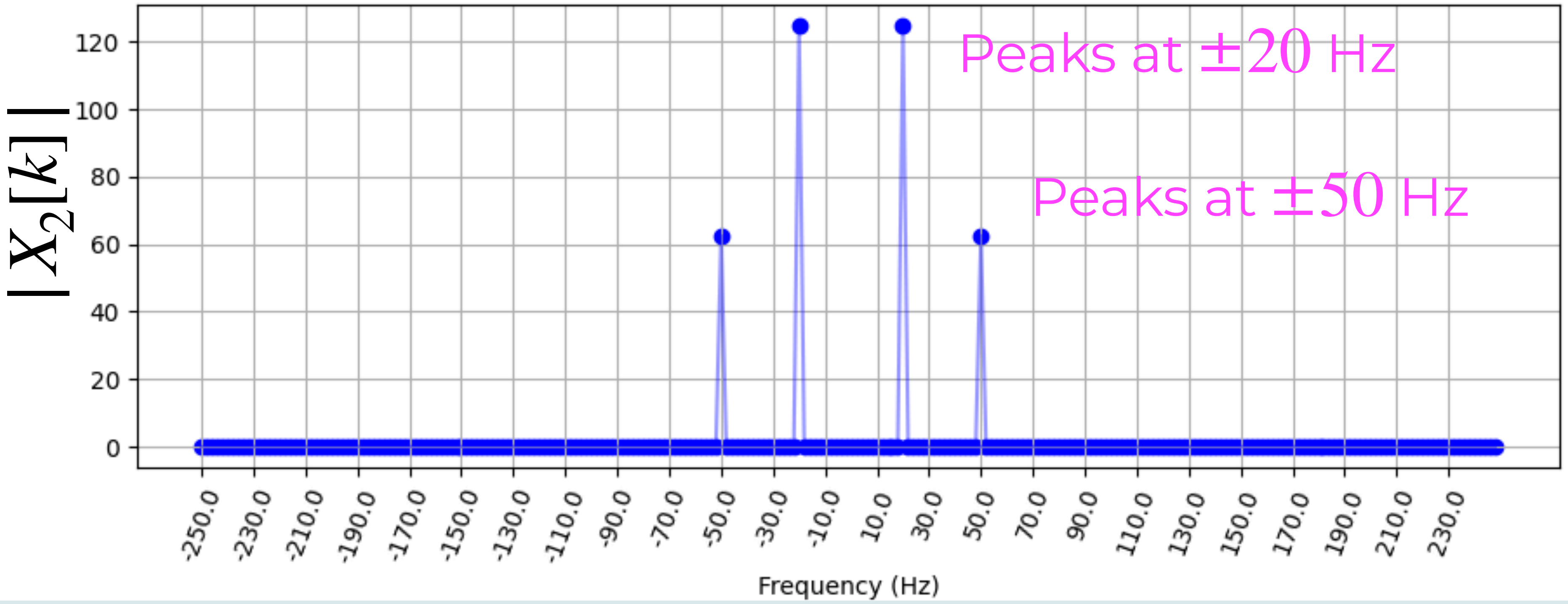
Samples from $x(t) = \sin(2\pi \times 20t)$





Samples from $x(t) = \sin(2\pi \times 20t) + 0.5\sin(2\pi \times 50t)$



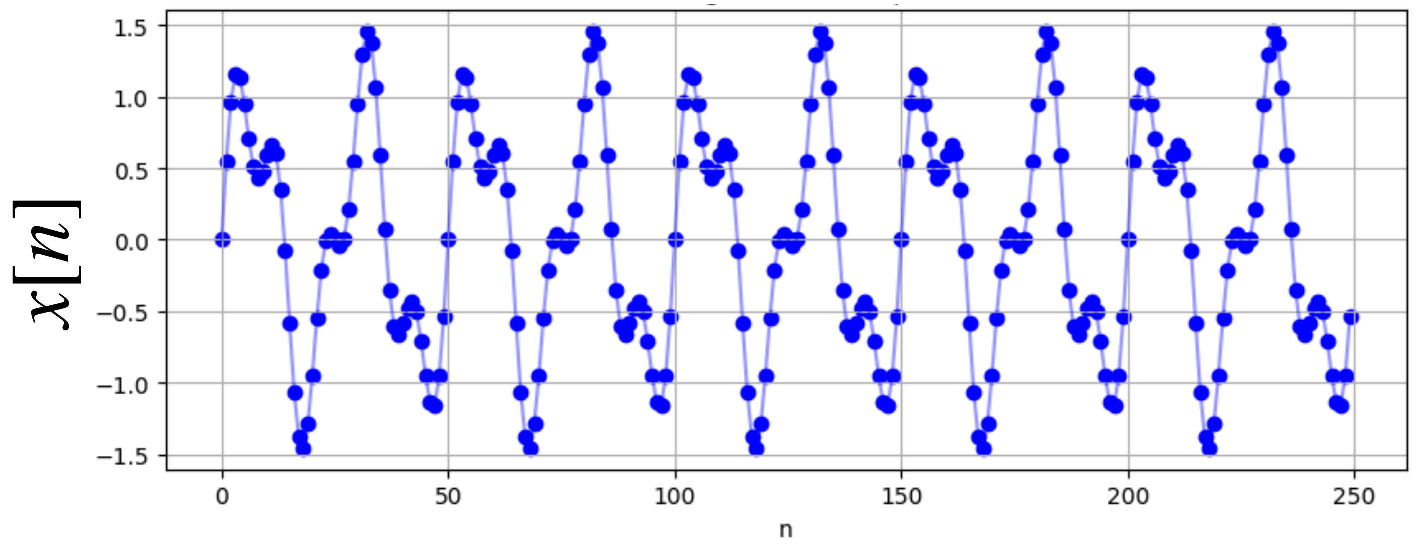


A Simple Filtering

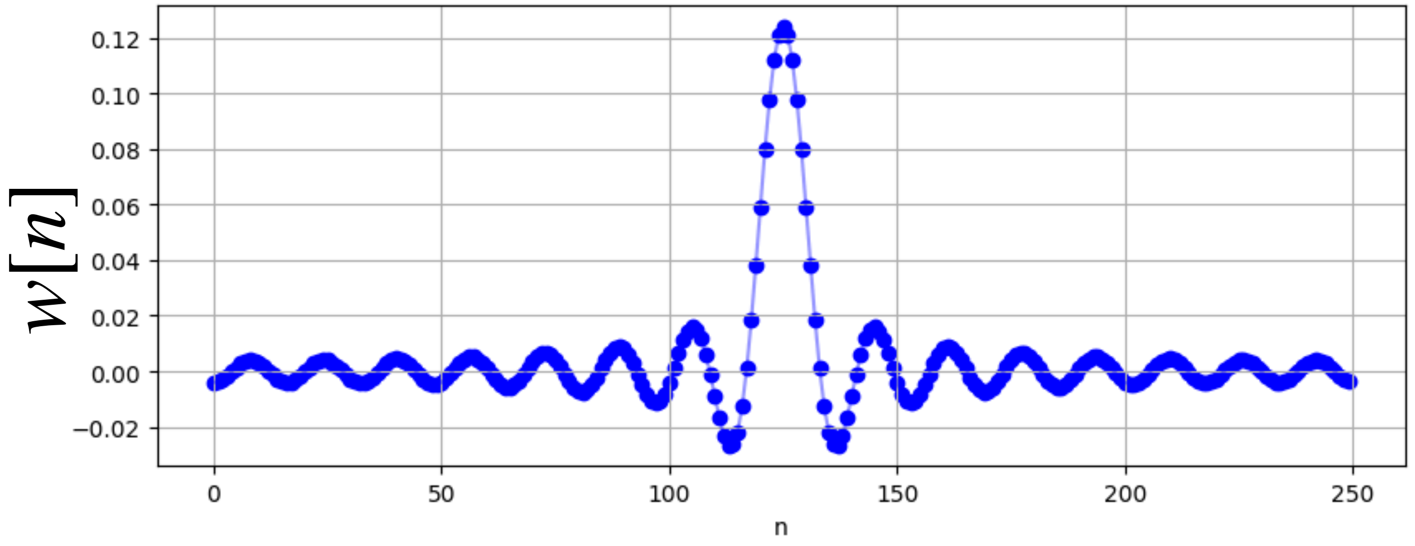
Time domain

Samples from $x(t) = \sin(2\pi \times 20t) + 0.5\sin(2\pi \times 50t)$

Original signal



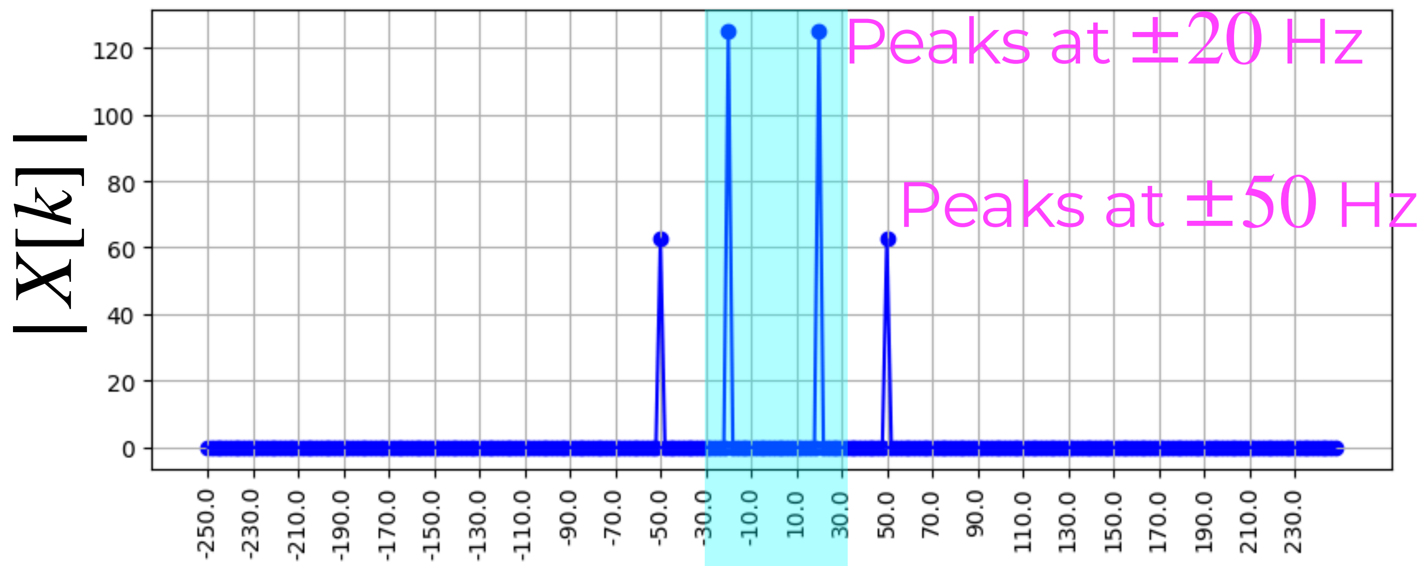
Window

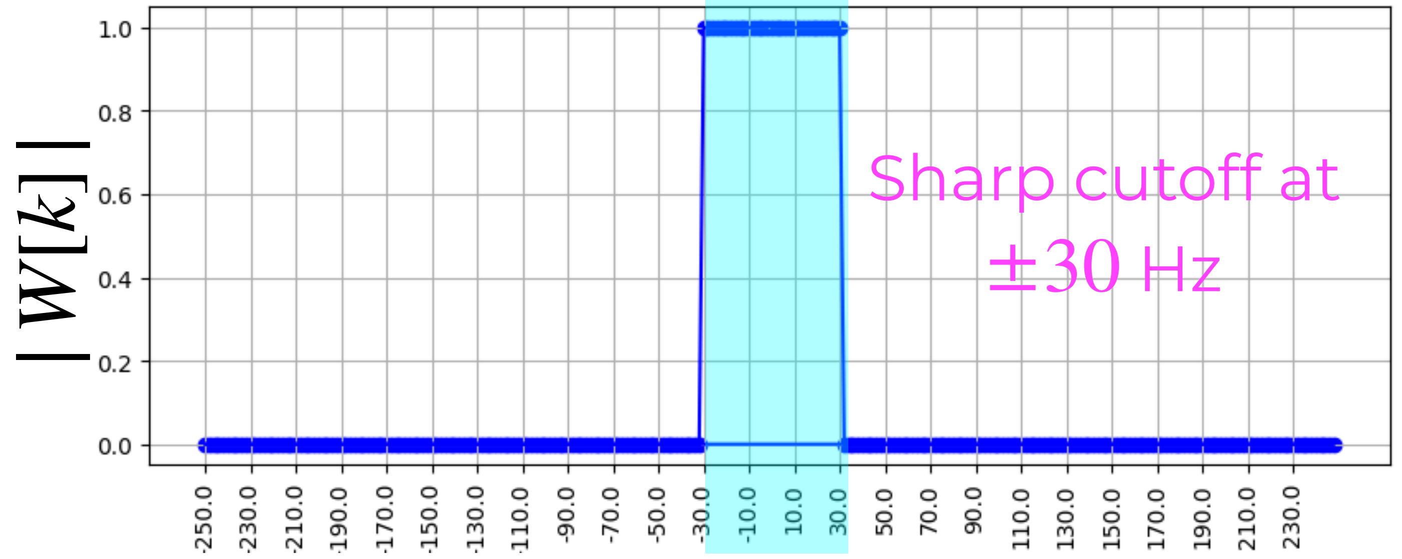


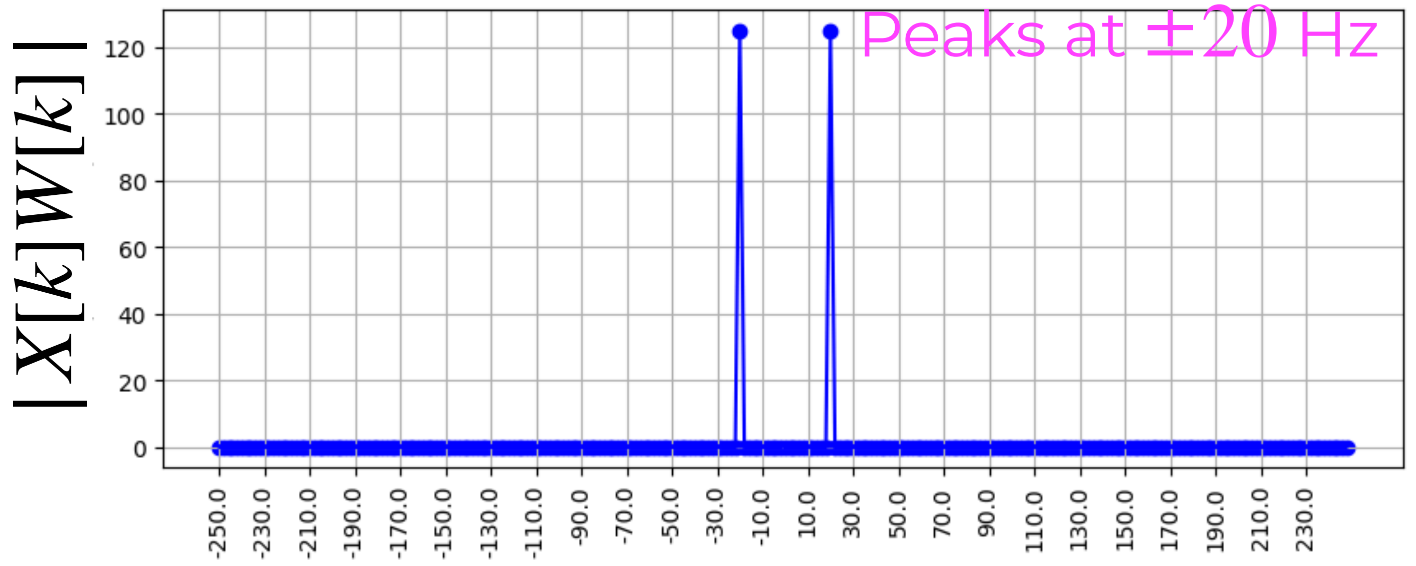
DFT

IDFT

Frequency domain







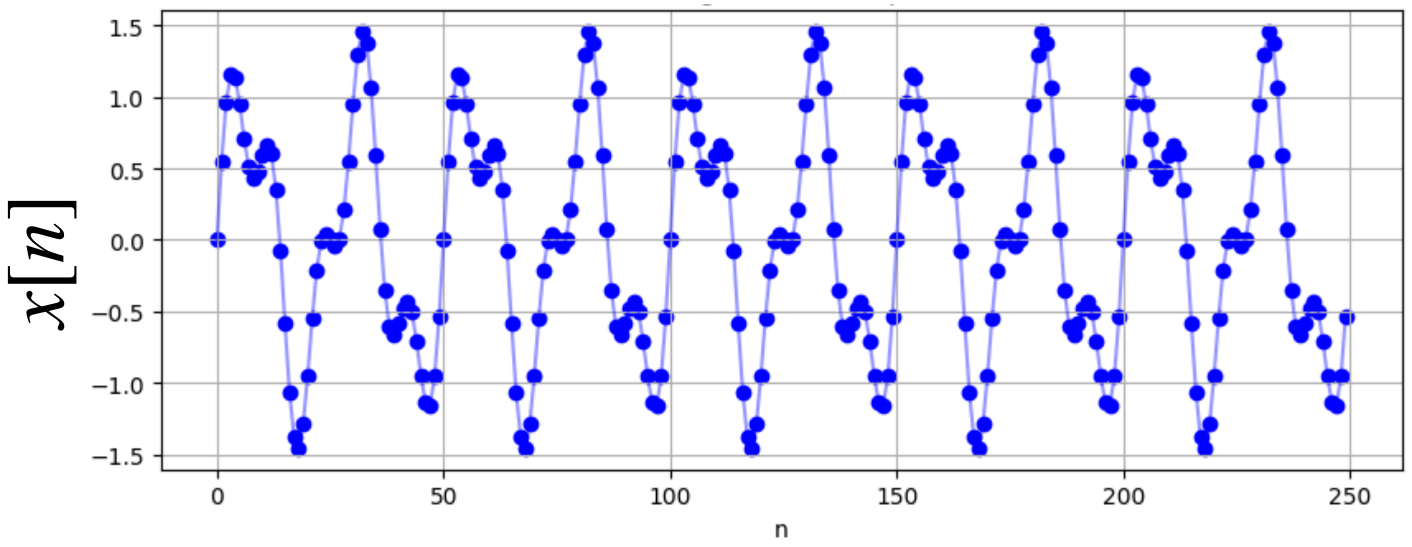
A Simple Filtering

Time domain

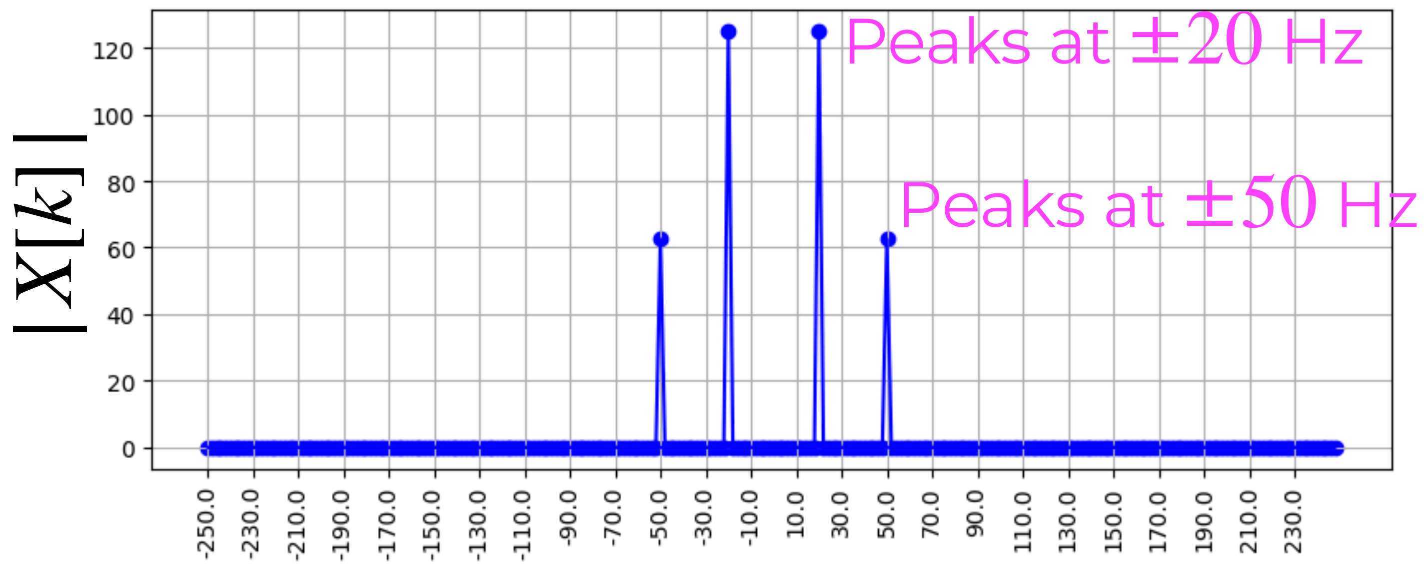
Samples from $x(t) = \sin(2\pi \times 20t) + 0.5\sin(2\pi \times 50t)$

Frequency domain

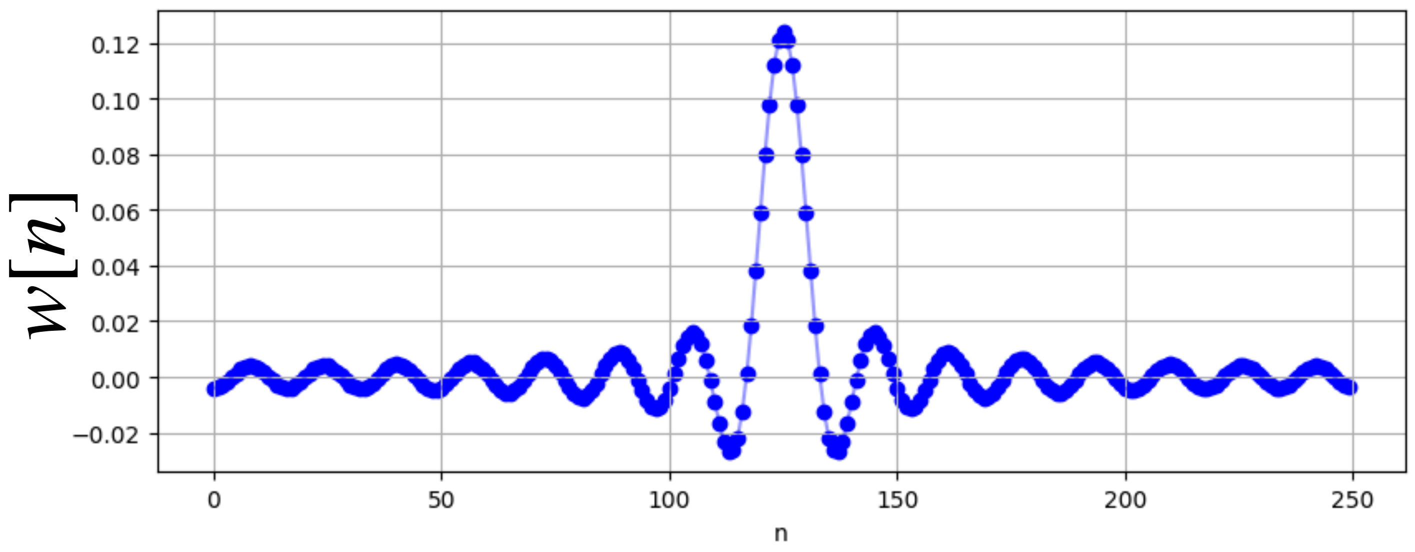
Original signal



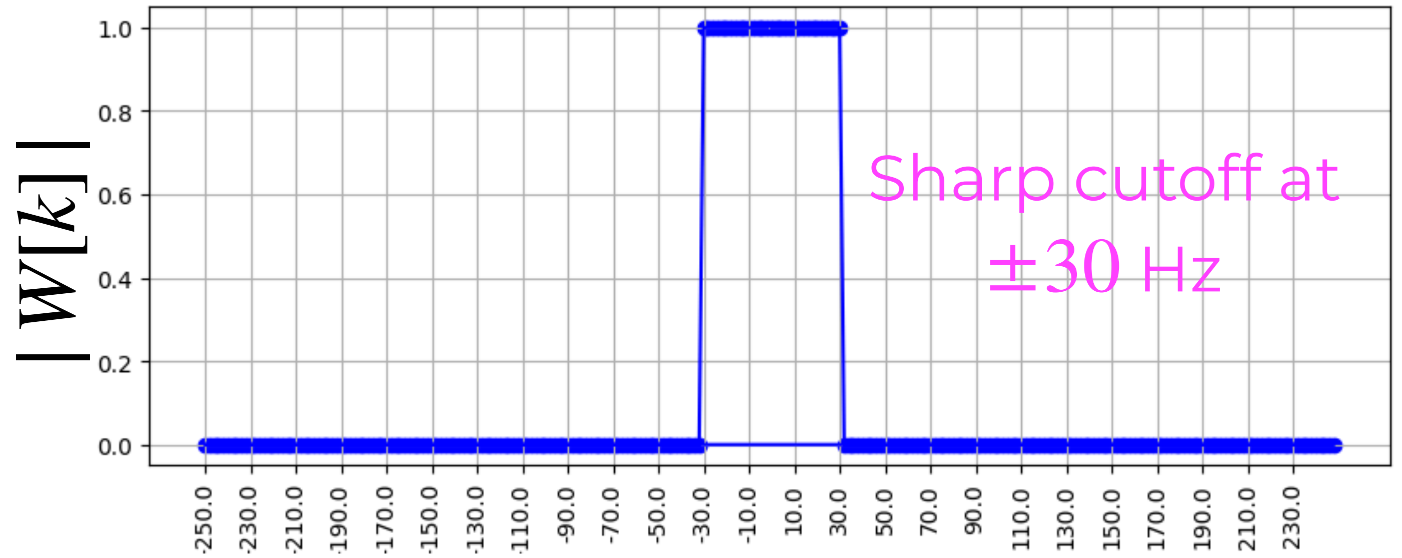
DFT



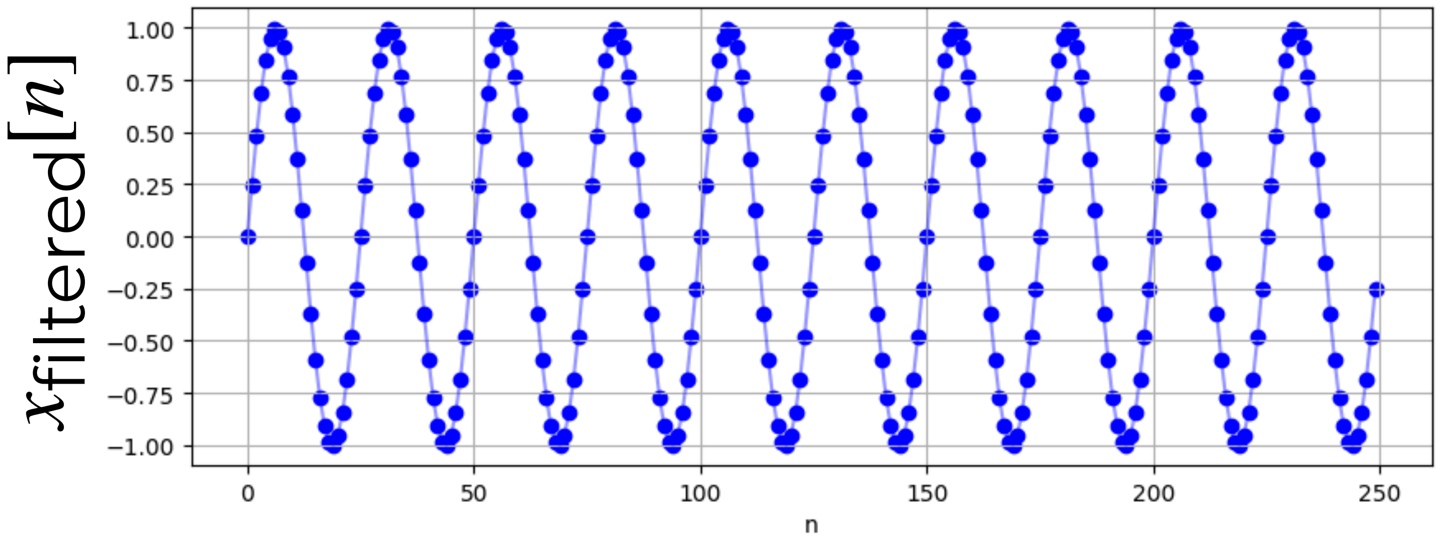
Window



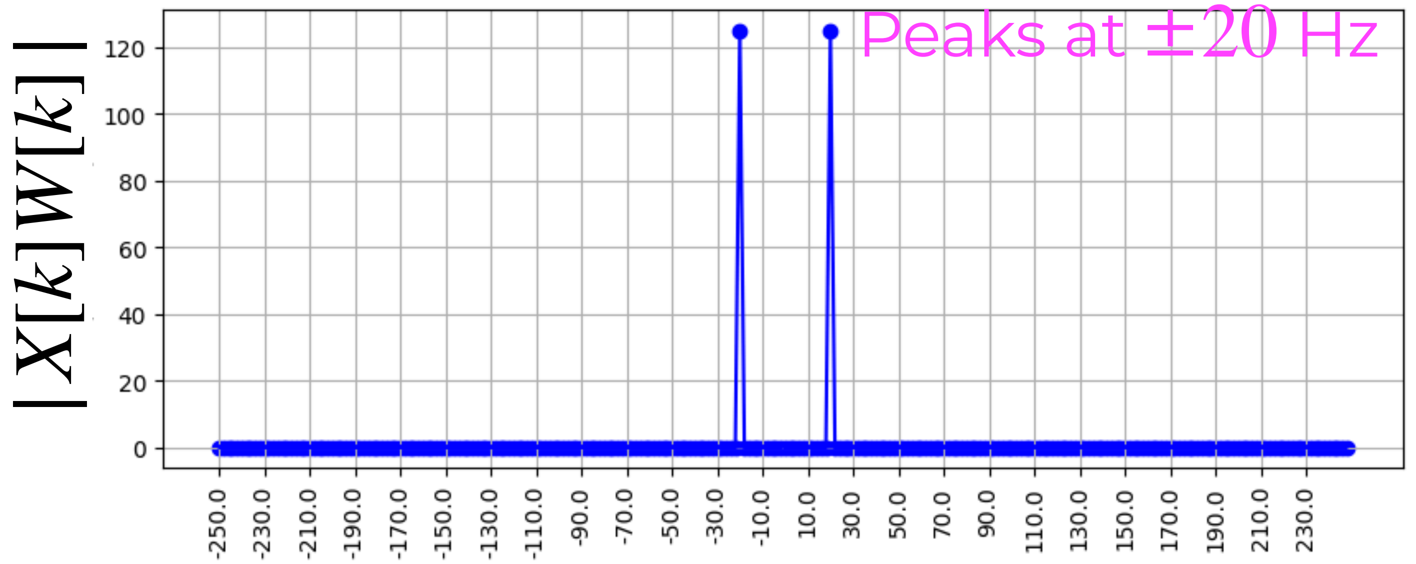
IDFT



Filtered signal

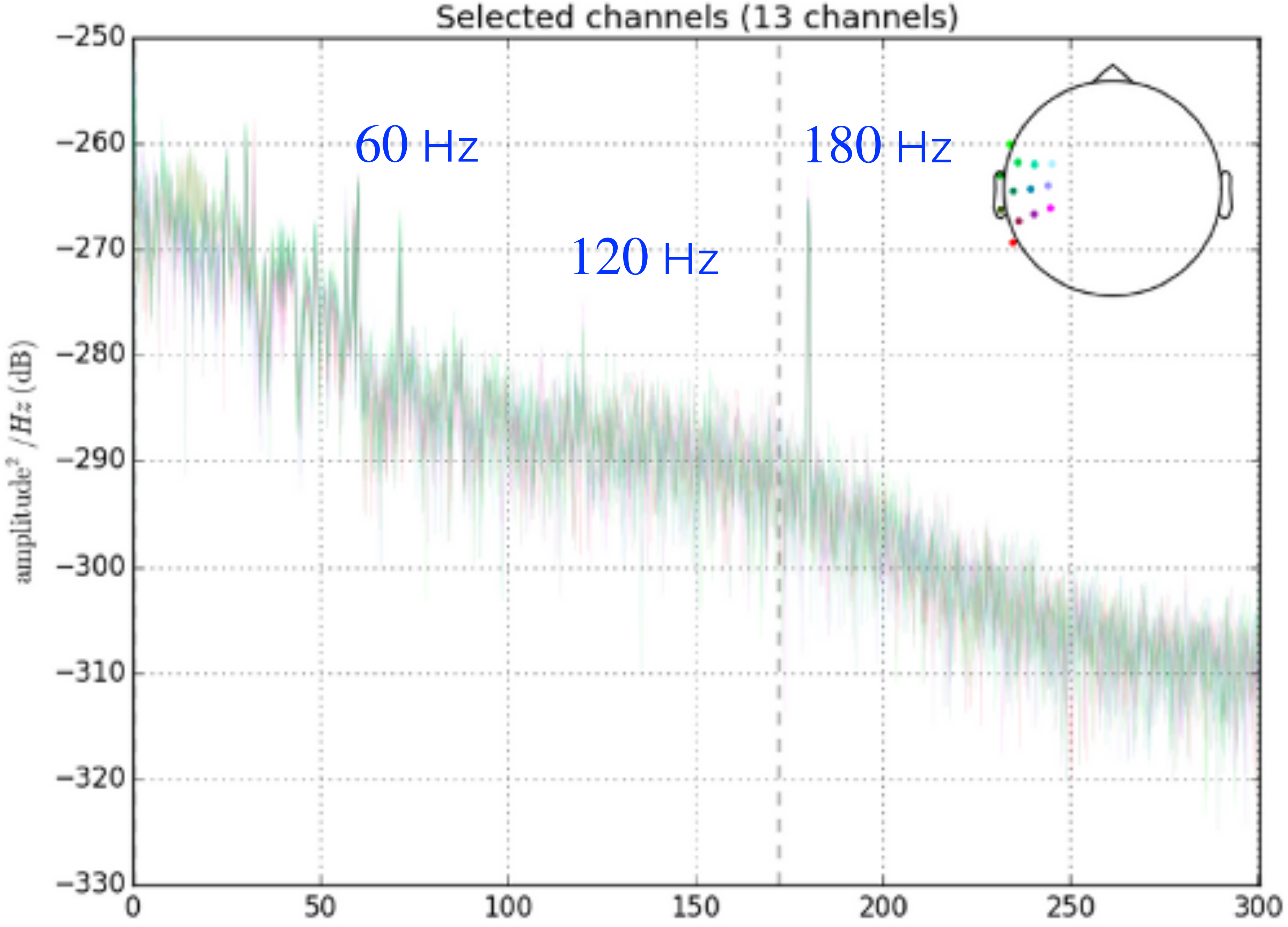


IDFT



EEG Preprocessing

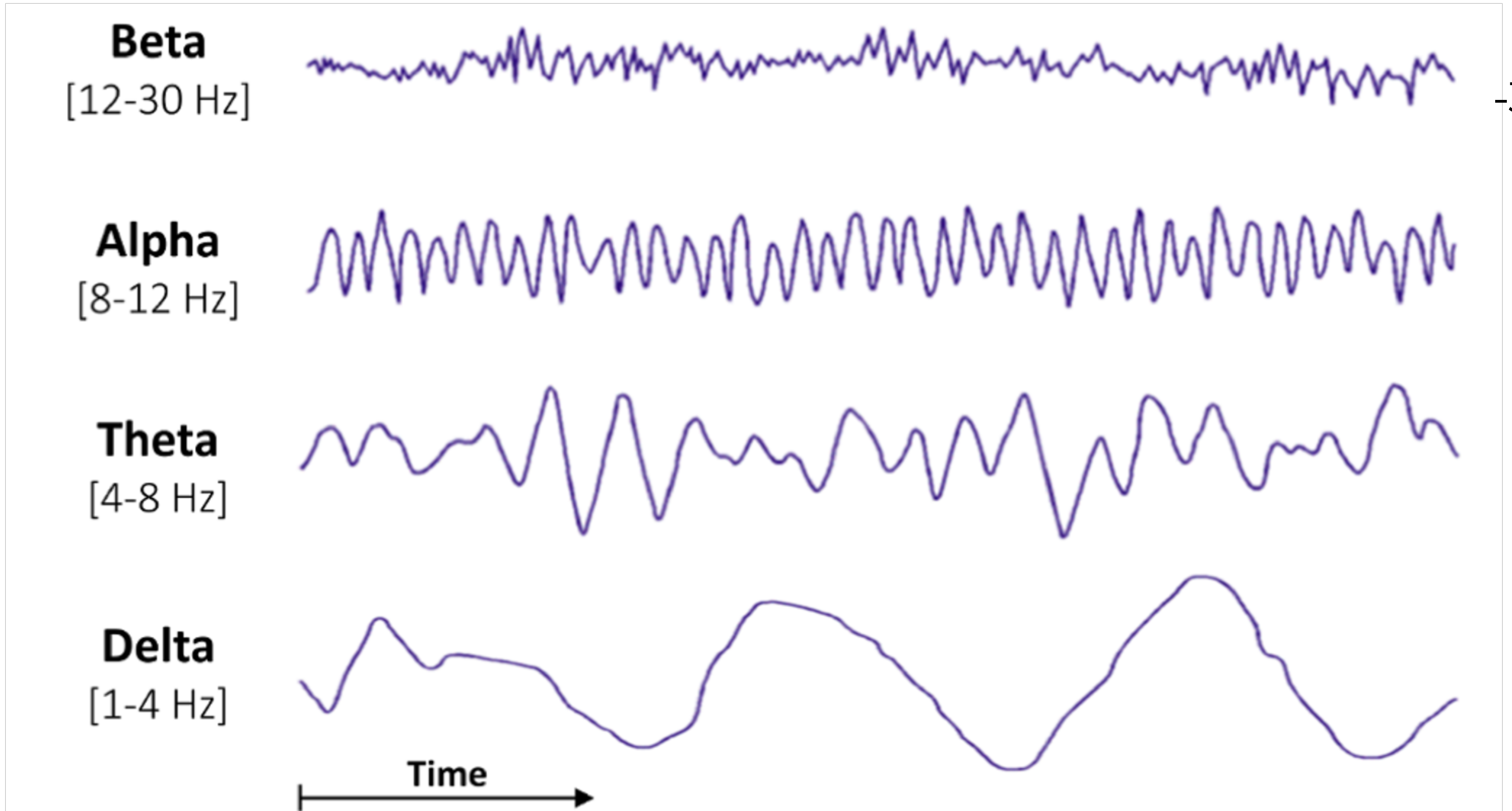
AC power line noise

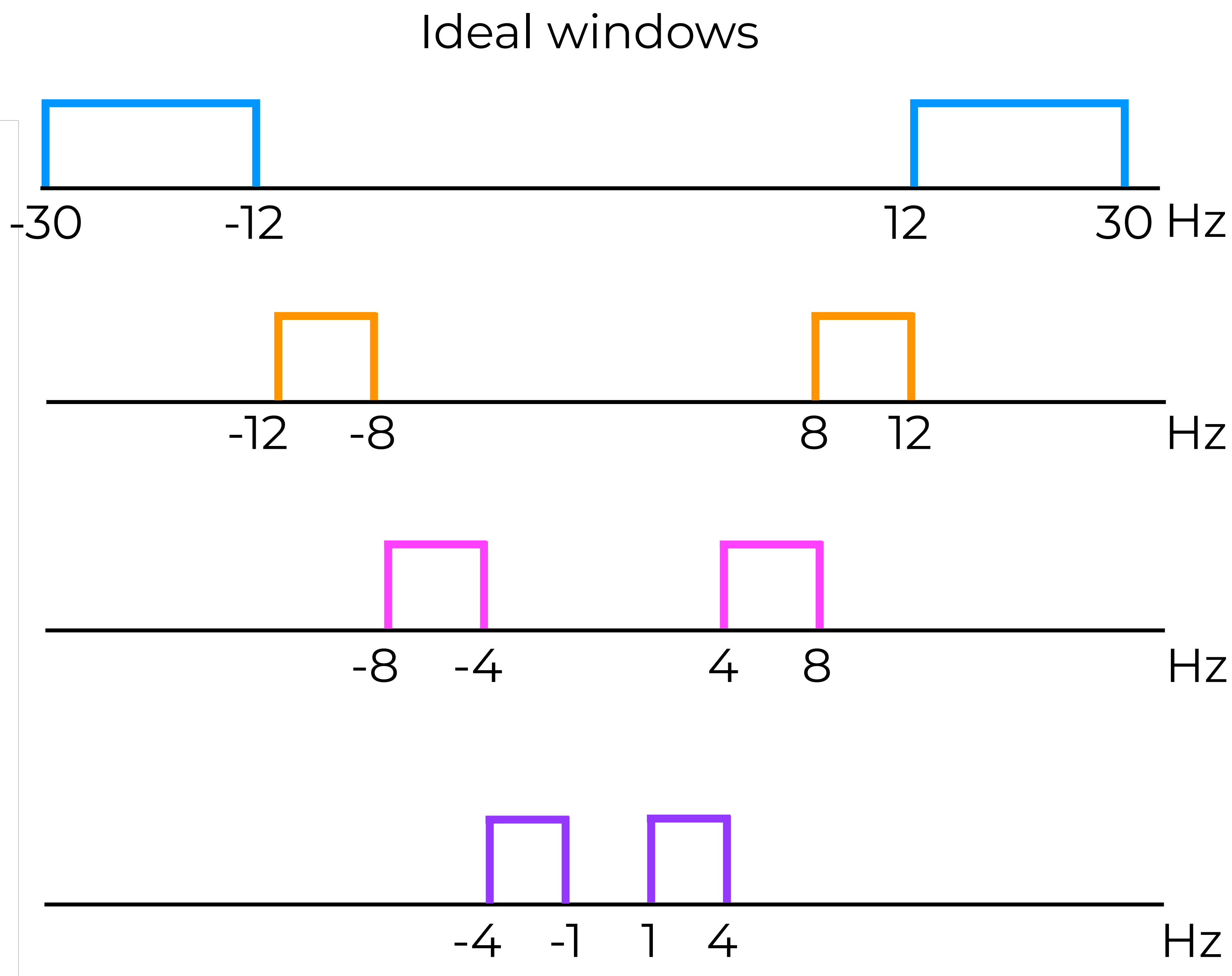


MNE's filtering and resampling data (v0.15)

EEG Preprocessing

Brain rhythm frequency bands associated with cognitive processes



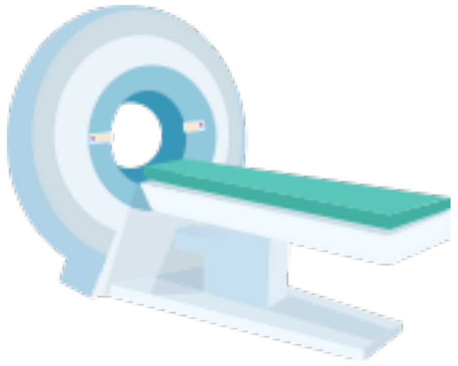


Vallat R. Compute the average bandpower of an EEG signal (2018)

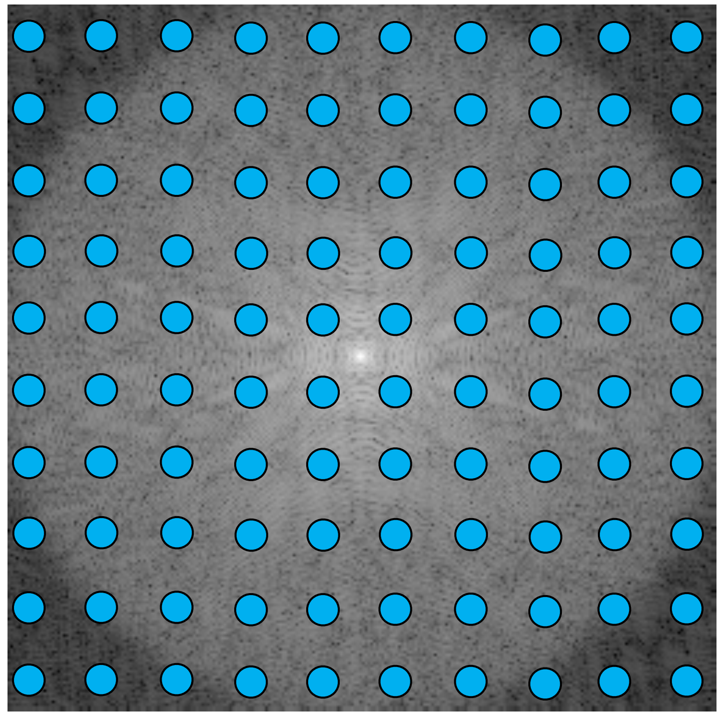
MRI Acquisition and Reconstruction

- The acquired data are the DFT samples of the object being imaged
- If the sampling rate is high enough, the image can be reconstructed by applying the inverse DFT to the k-space data

MRI scanner



Acquired data
(k-space)



2D-IDFT

Reconstructed data
(image-space)