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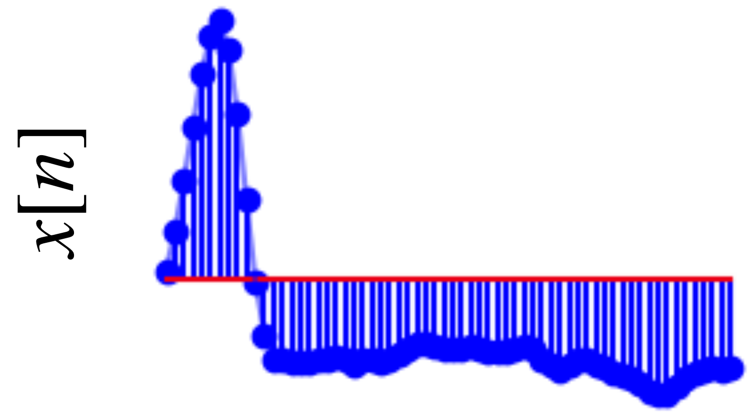
Some Properties of the DFT

Itthi Chatnuntawech

Discrete Fourier Transform (DFT)

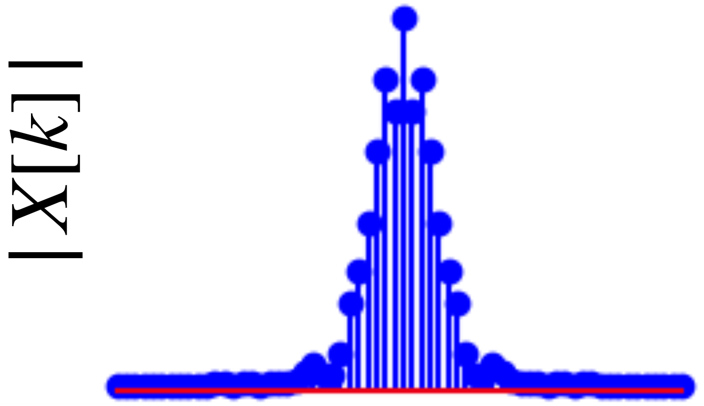
Time domain



$$\underbrace{\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}}_{} \quad \overset{DFT}{\longleftrightarrow} \quad \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

$x[n]$ $\quad$ $X[k]$

Frequency domain



scipy.fft.fft

```
scipy.fft.fft(x, n=None, axis=-1, norm=None, overwrite_x=False, workers=None, *, plan=None)    [source]
```

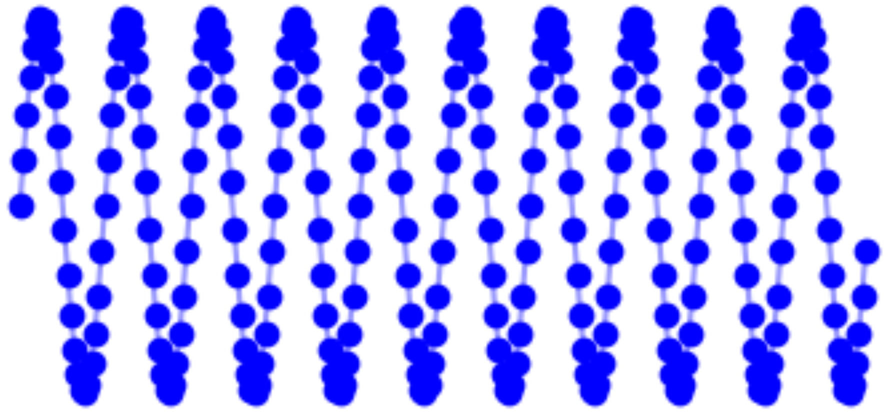
Compute the 1-D discrete Fourier Transform.

This function computes the 1-D *n*-point discrete Fourier Transform (DFT) with the efficient Fast Fourier Transform (FFT) algorithm [1].

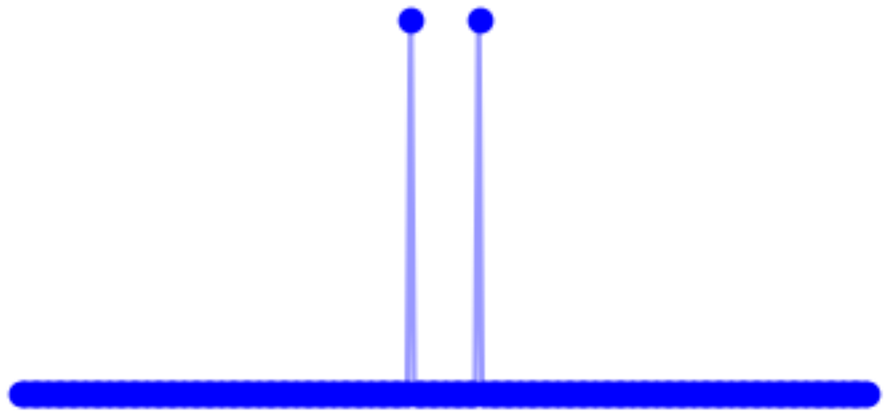
Fast Fourier Transform (FFT) - An efficient algorithm that computes the DFT of a signal

Linearity

20 Hz sine wave

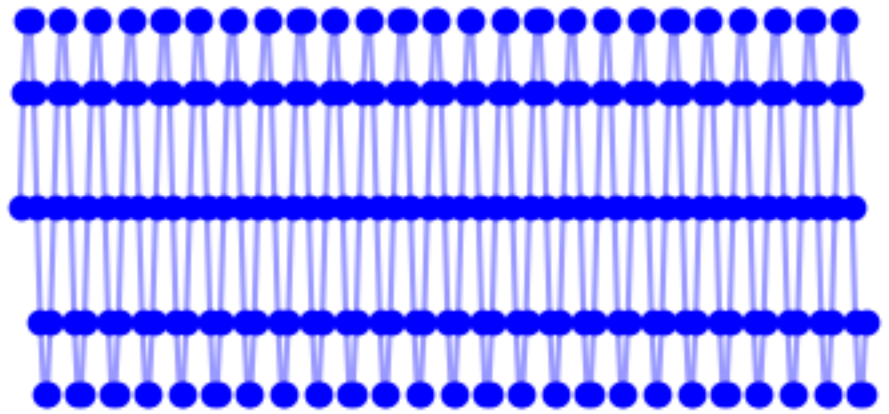


$$x_1[n] \overset{DFT}{\longleftrightarrow} X_1[k]$$

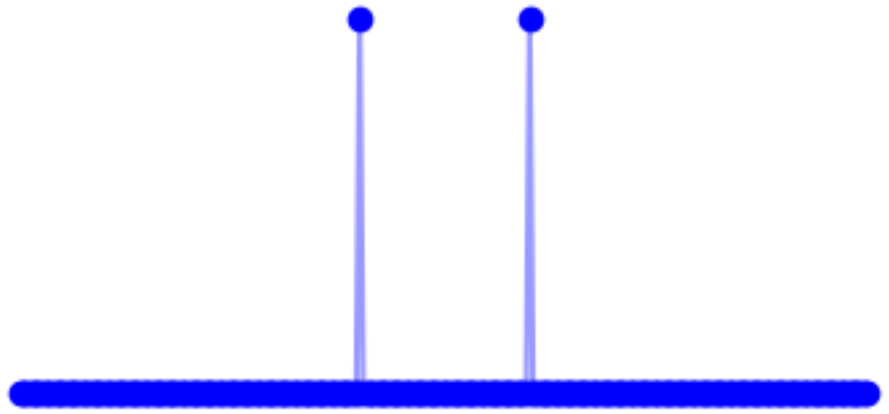


± 20Hz

50 Hz sine wave

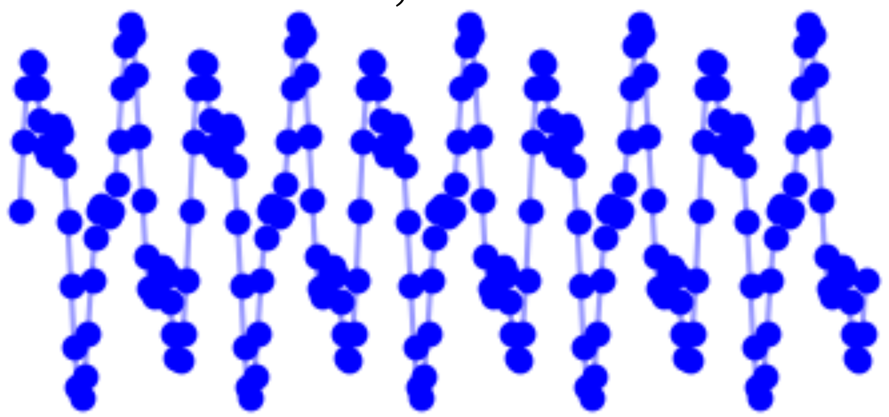


$$x_2[n] \overset{DFT}{\longleftrightarrow} X_2[k]$$

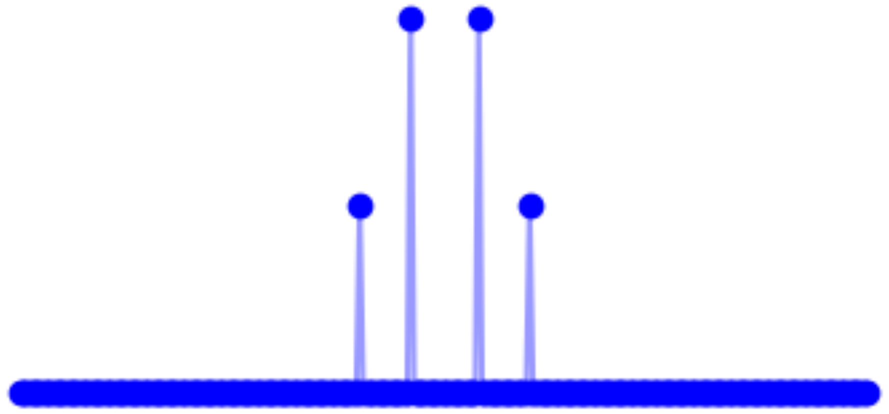


± 50Hz

$a = 1,\ b = 0.5$



$$ax_1[n] + bx_2[n] \overset{DFT}{\longleftrightarrow} aX_1[k] + bX_2[k]$$



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The Convolution Theorem

$$x_1[n] \quad \overset{DFT}{\longleftrightarrow} \quad X_1[k]$$

$$x_2[n] \quad \overset{DFT}{\longleftrightarrow} \quad X_2[k]$$

circular convolution

$$x_1[n] \circledN x_2[n] \quad \overset{DFT}{\longleftrightarrow} \quad X_1[k]X_2[k]$$

Convolution in the time domain corresponds to multiplication in the frequency domain

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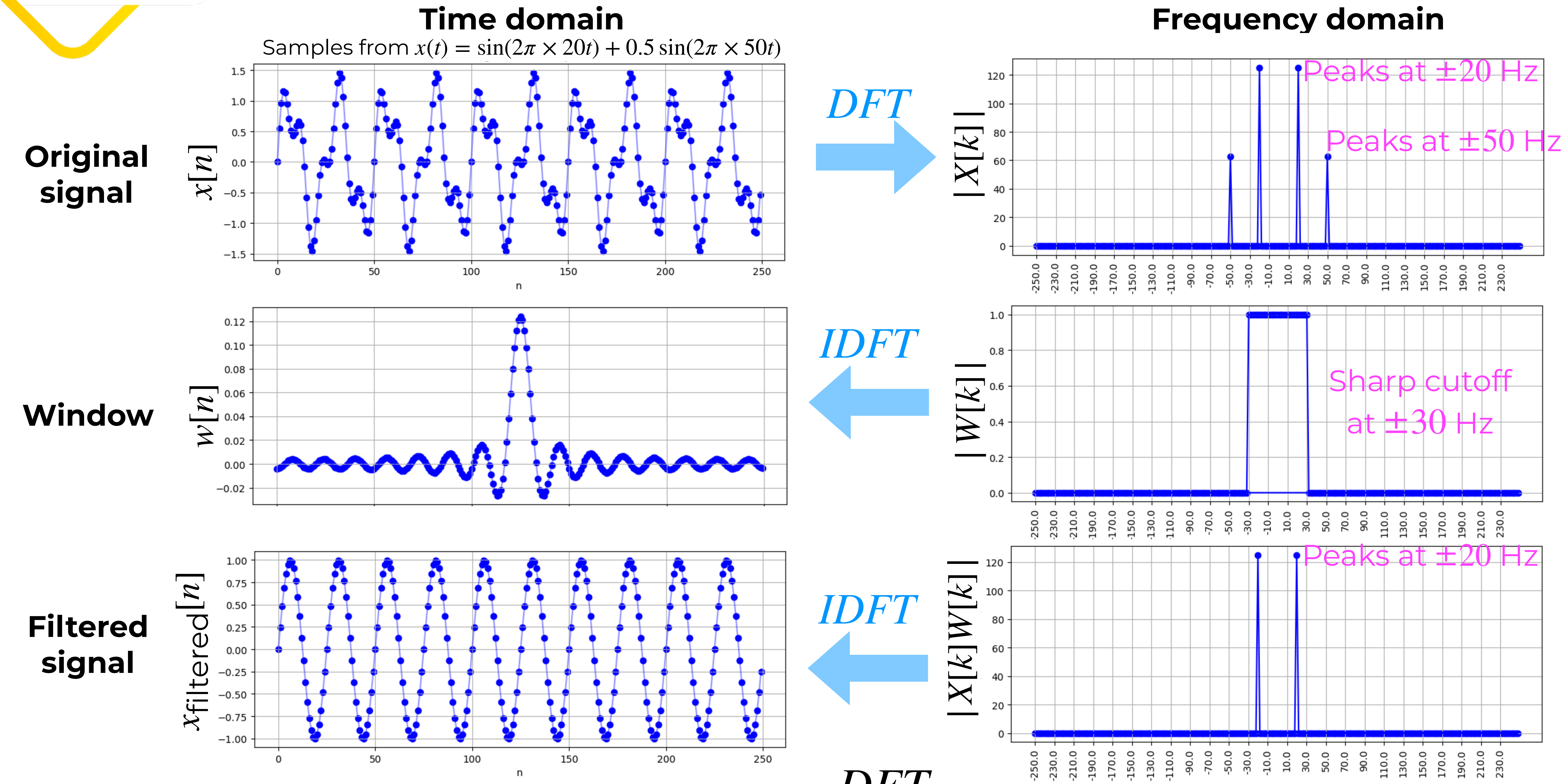
A Simple Filtering



$$x_{\text{filtered}}[n] = x[n] \, Ⓝ \, w[n] \quad \xleftrightarrow{DFT} \quad X_1[k]X_2[k]$$

The Modulation or Windowing Theorem

$$x_1[n] \overset{DFT}{\longleftrightarrow} X_1[k]$$

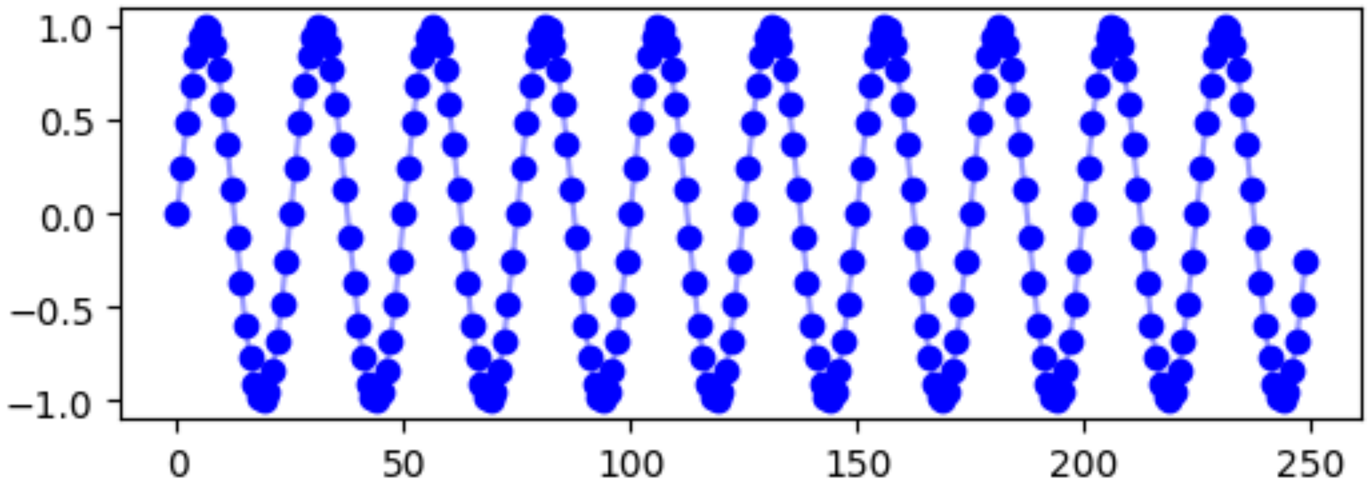
$$x_2[n] \overset{DFT}{\longleftrightarrow} X_2[k]$$

circular
convolution

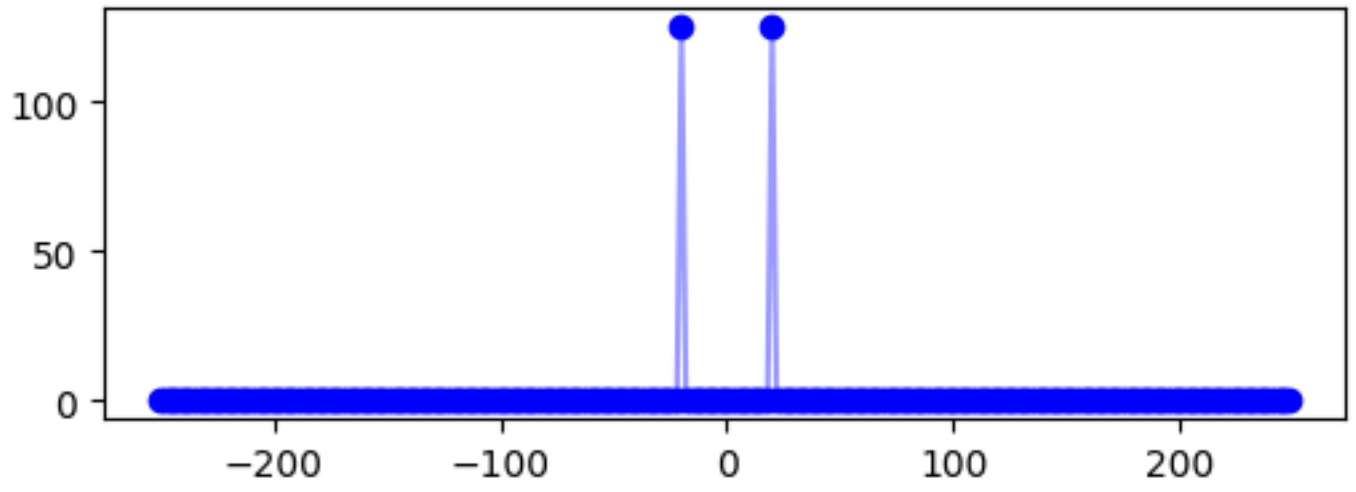
$$x_1[n]x_2[n] \overset{DFT}{\longleftrightarrow} \frac{1}{N} X_1[k] \, \textcircled{\scriptsize N} \, X_2[k]$$

Multiplication in the time domain corresponds to circular convolution in the frequency domain

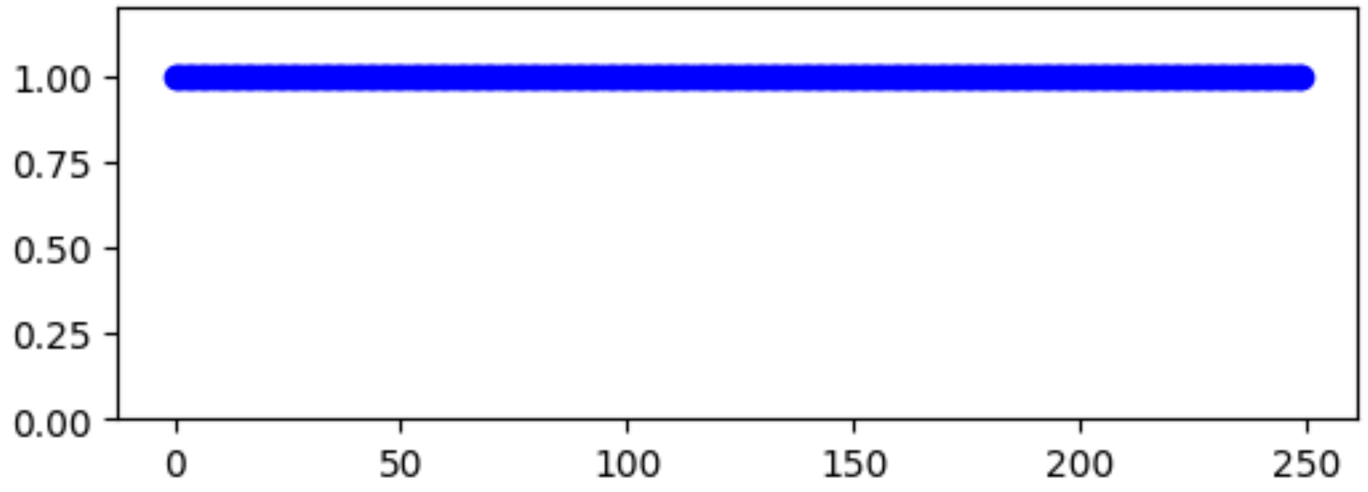
$x[n]$



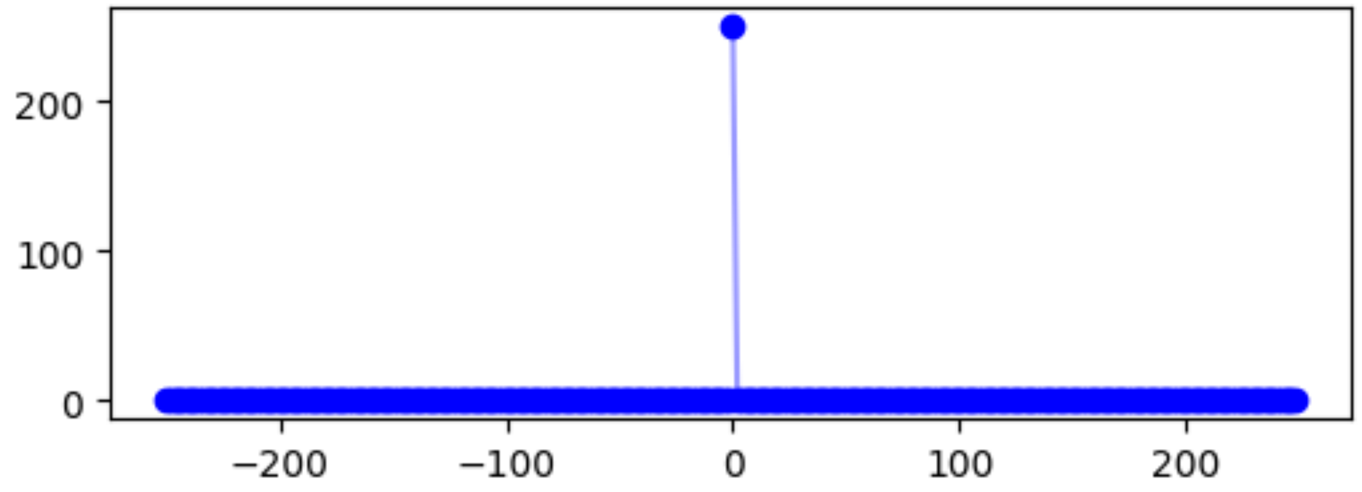
$X[k]$



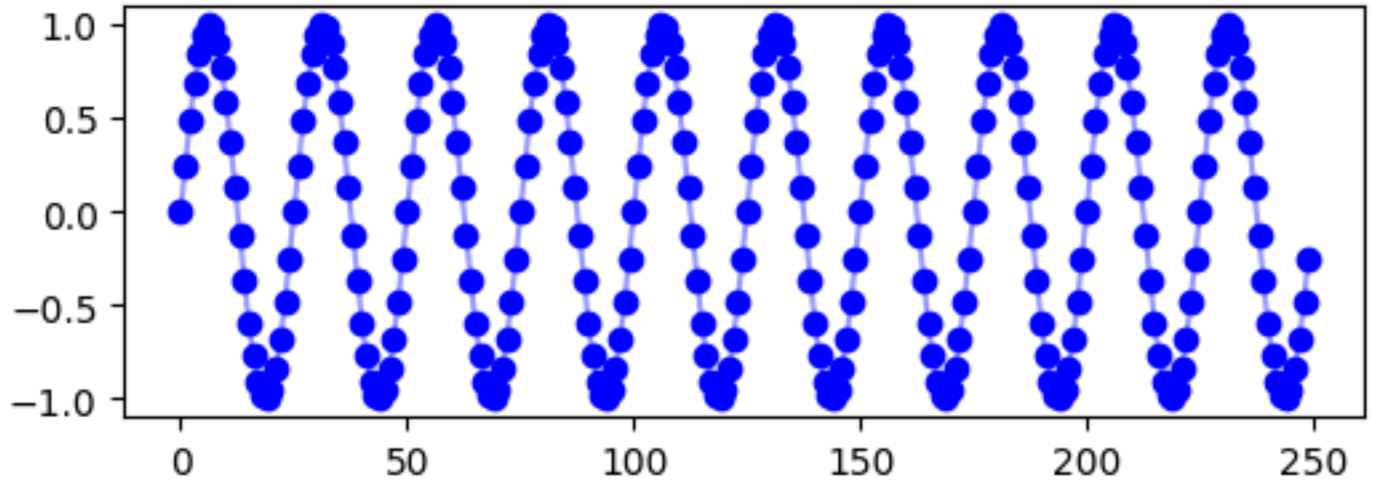
$w[n]$



$W[k]$

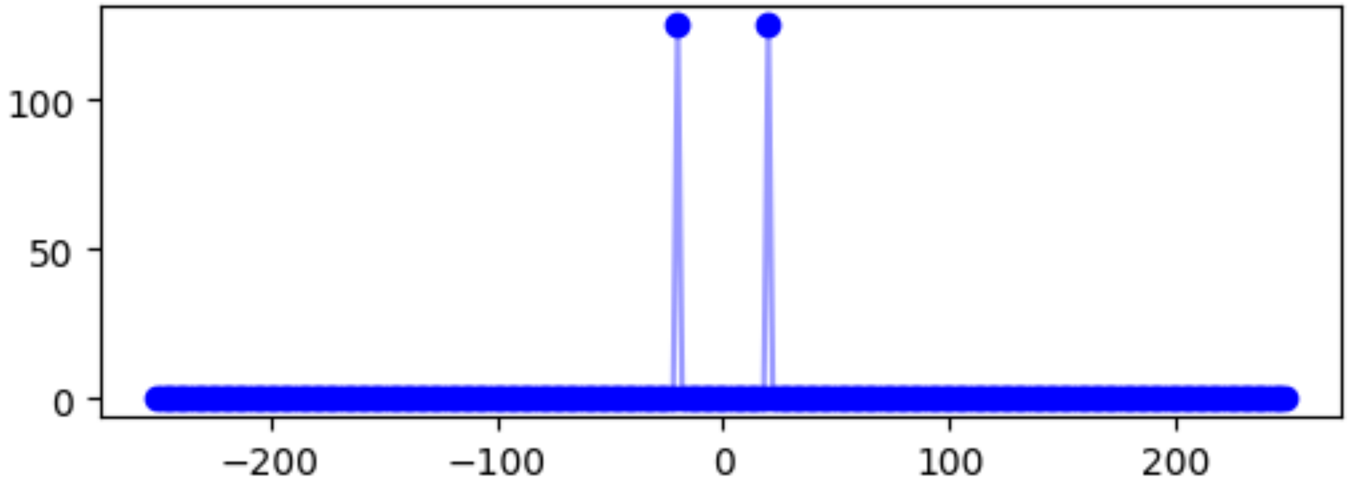


$x[n]w[n]$

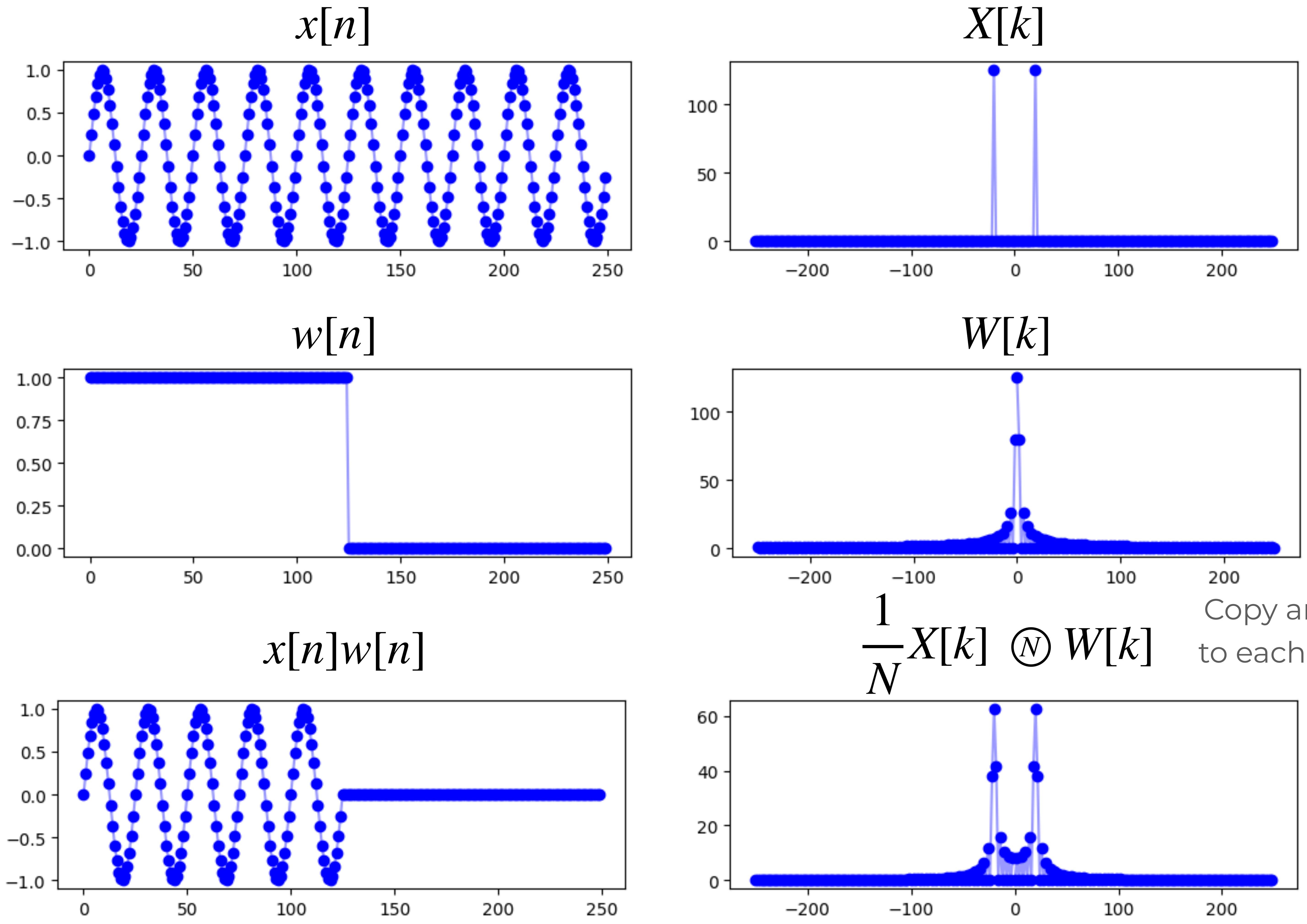


$$\frac{1}{N} X[k] \; \textcircled{N} \; W[k]$$

Copy and paste $W[k]$ to each non-zero $X[k]$



Multiplication in the time domain corresponds to circular convolution in the frequency domain



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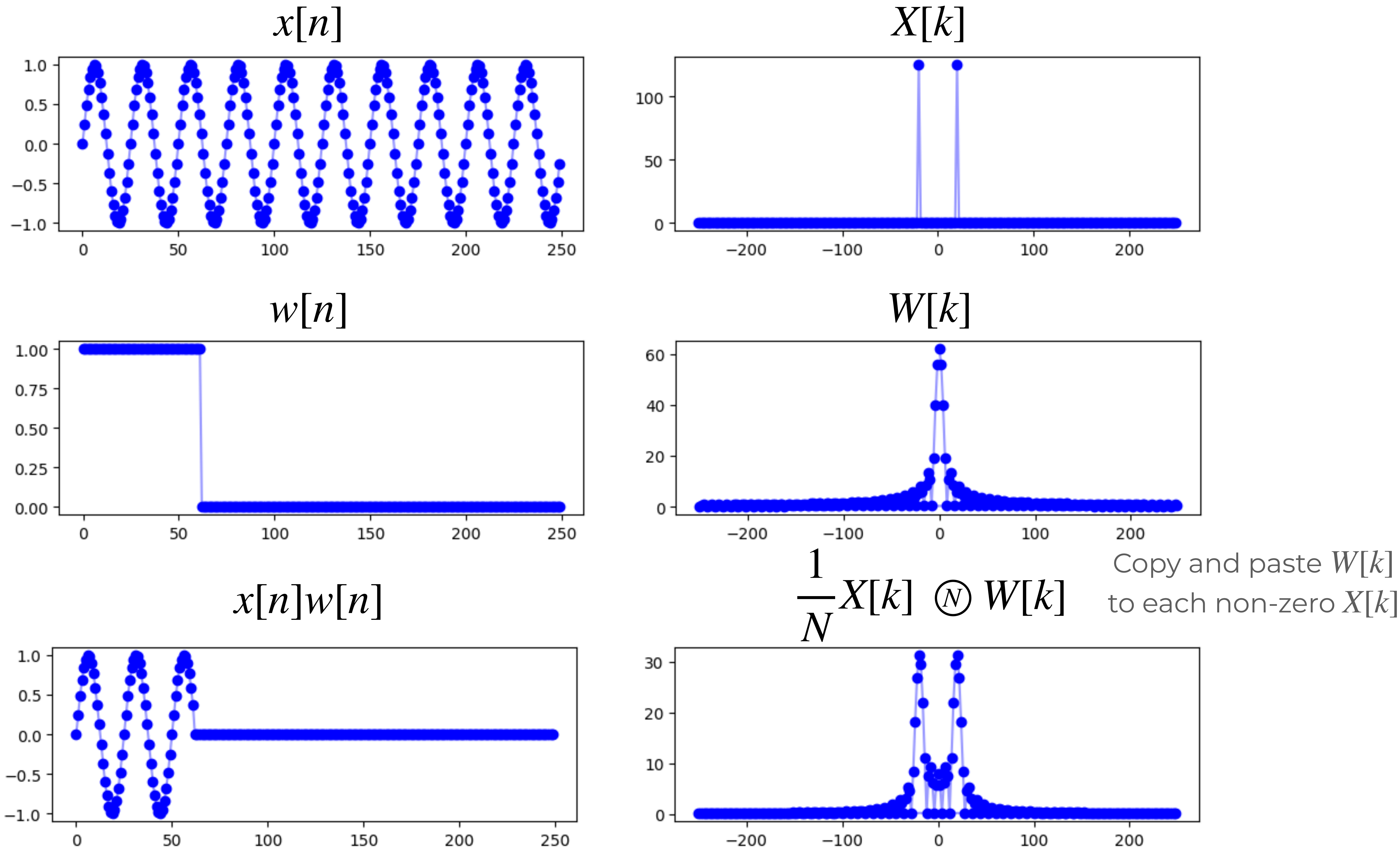
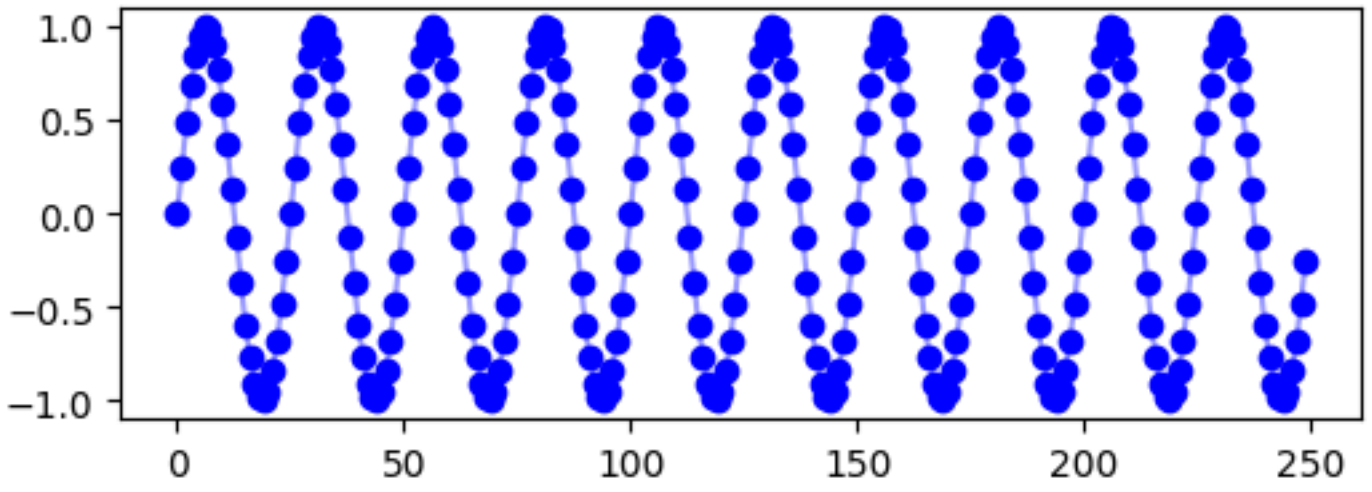
Multiplication in the time domain corresponds to circular convolution in the frequency domain

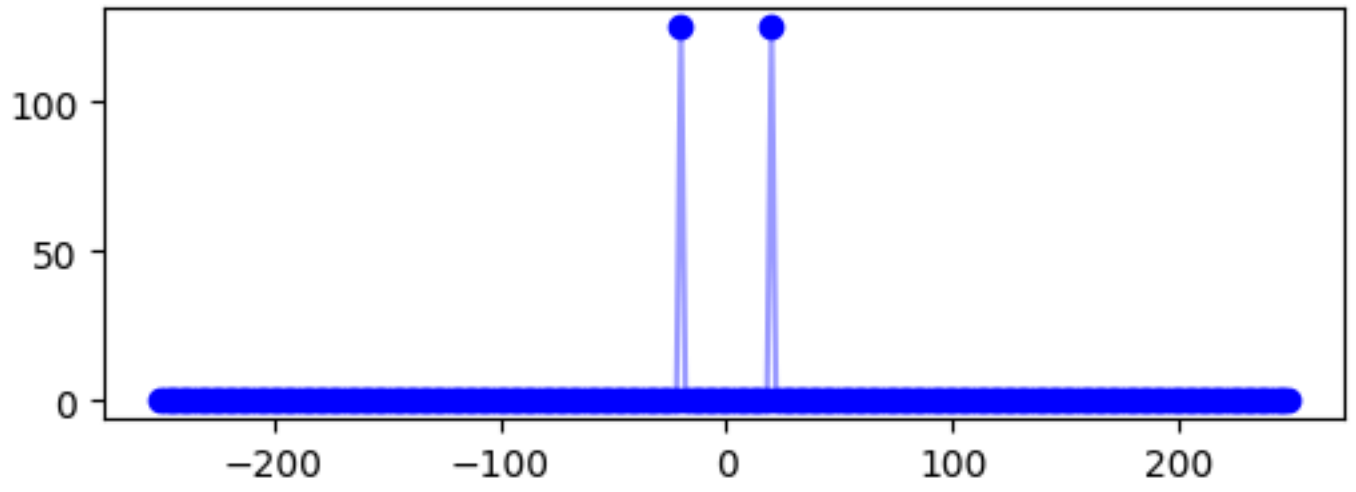


Multiplication in the time domain corresponds to circular convolution in the frequency domain

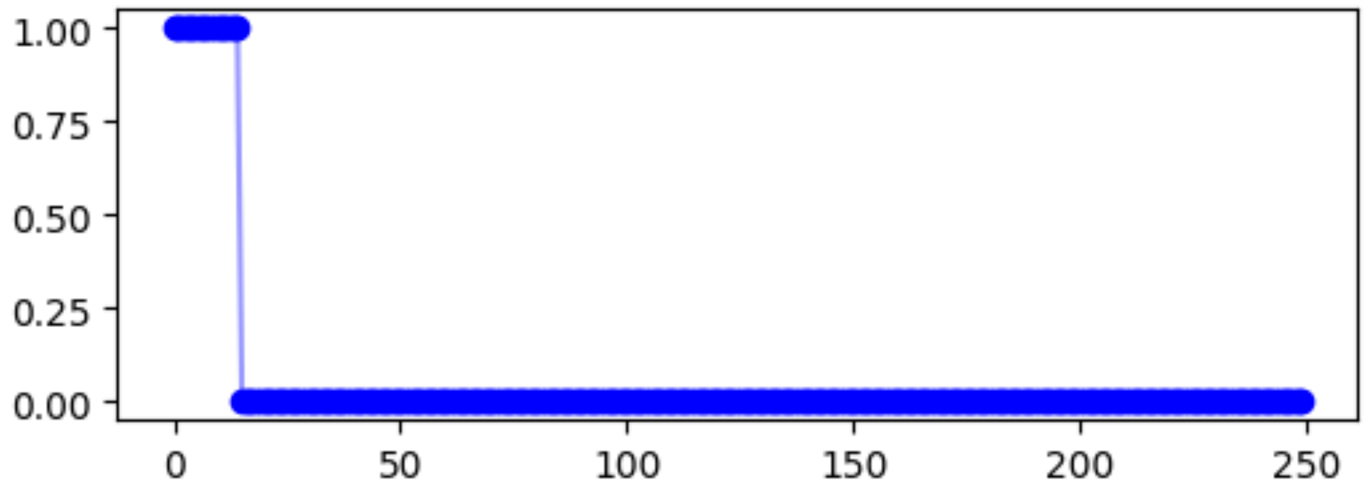
$x[n]$



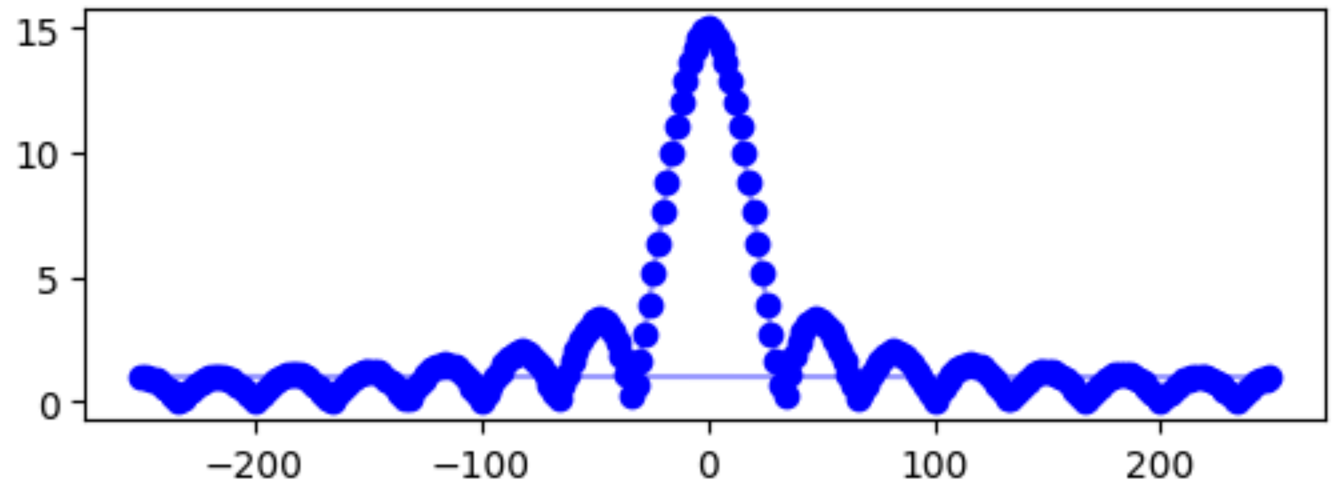
$X[k]$



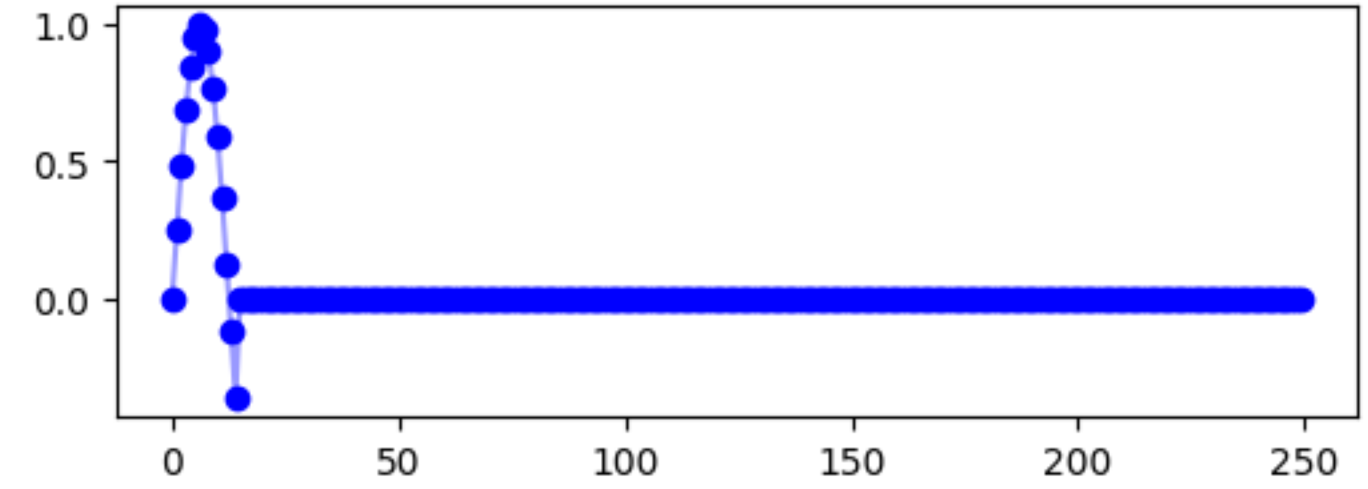
$w[n]$



$W[k]$

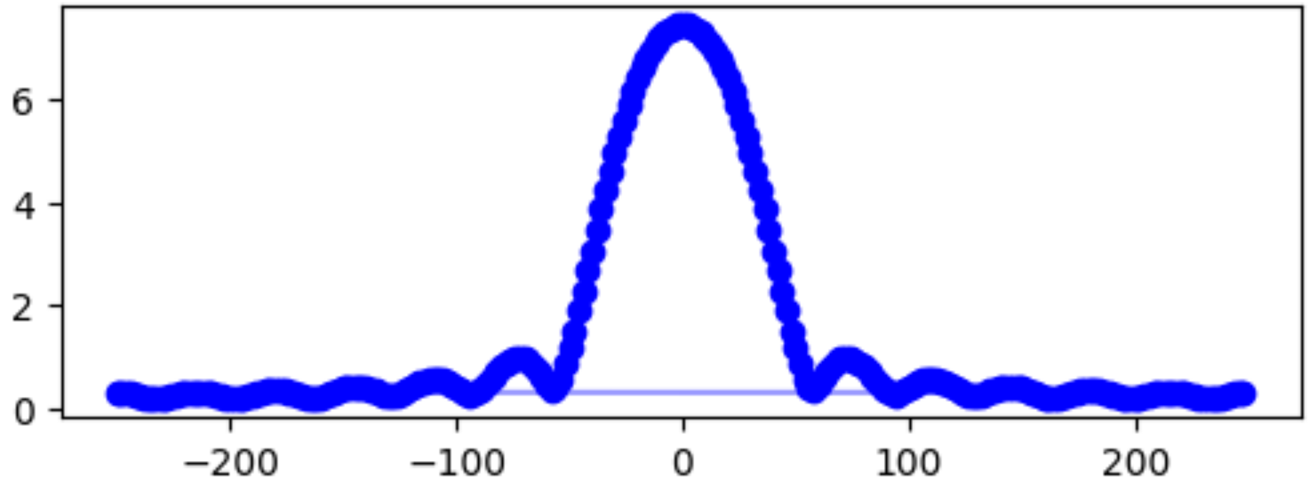


$x[n]w[n]$



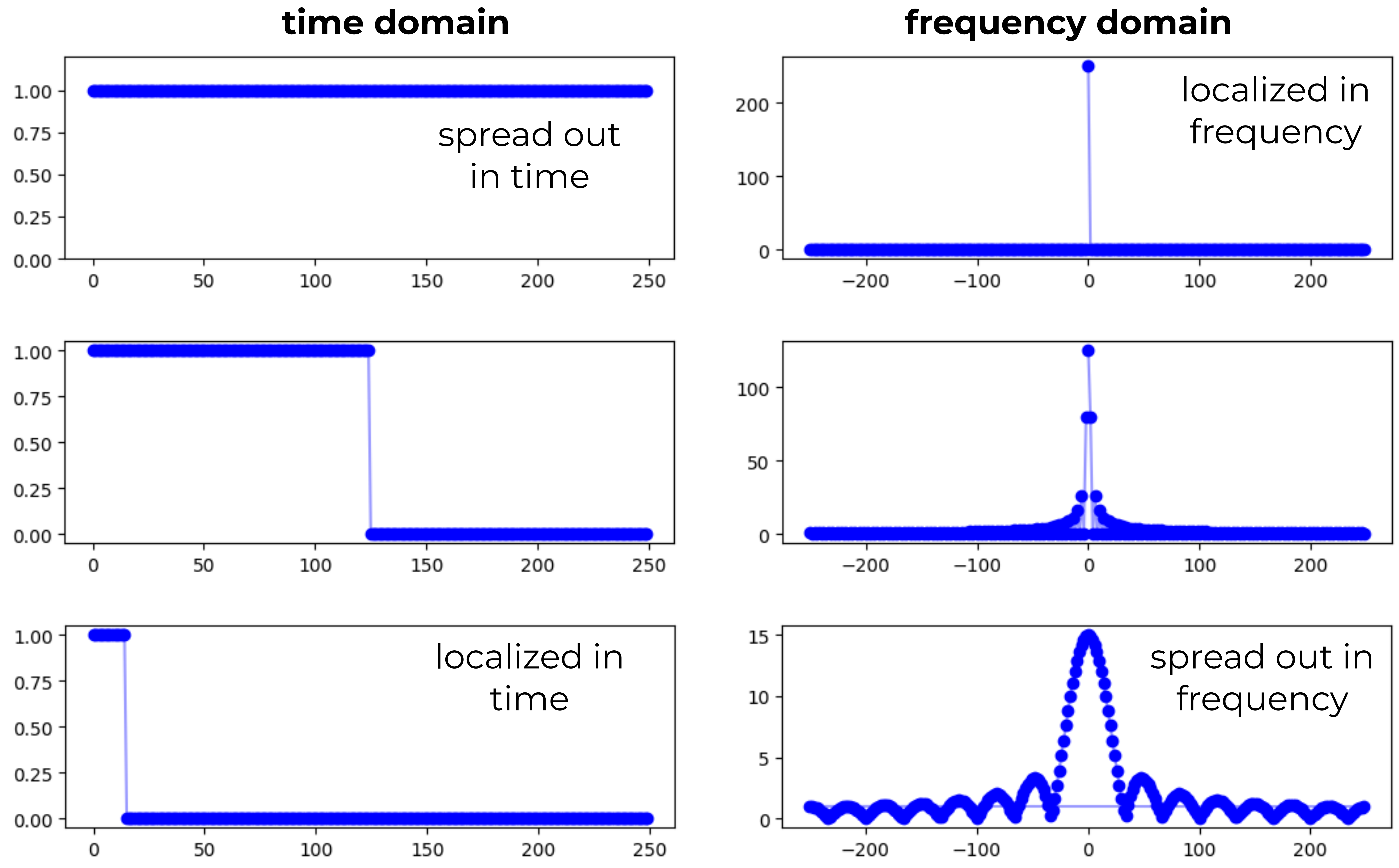
$\frac{1}{N} X[k] \; \textcircled{N} \; W[k]$

Copy and paste $W[k]$ to each non-zero $X[k]$



Multiplication in the time domain corresponds to circular convolution in the frequency domain

The Inverse Relationship Between Time and Frequency



The Inverse Relationship Between Time and Frequency