1) P(no student has to answer twice)

$$= \underset{1st}{\frac{15}{15}} \times \underset{2nd}{\frac{14}{15}} \times \underset{3rd...}{\frac{13}{15}} \times \frac{12}{15} \times \frac{11}{15} \times \frac{10}{15} \times \frac{9}{15} \times \frac{8}{15}$$

$$= \frac{\frac{15!}{7!}}{15^8} = \frac{259459200}{2562890625} \approx 0.101$$

2) $\underset{odd}{5} \cdot \underset{odd}{4} \cdot \underset{\text{7 choices}}{7} \cdot \underset{\text{6 choices}}{6} \cdot \underset{even}{5} = 4200$

odd: 1, 3, 5, 7, 9

odd: ~~1~~, 3, 5, 7, 9

7 choices: already picked 2 odds + 1 even to fit condition

6 choices: already chose 1 from the remaining

even: 0, 2, 4, 6, 8

5 choices 4 choices: already chose a number from odds

$$\frac{4200}{100000} = 0.042$$

3) event A: at least 2 dice show 4 or above
event B: all 3 dice show the same value

$P(A) \cdot P(B) = P(A \cap B)$

$P(A) = \underbrace{P(\text{just 2 out of 3 dice show} \geq 4)}_{A_1} + \underbrace{P(\text{all 3 dice show} \geq 4)}_{A_2}$

binomial distribution: $P(k) = \binom{n}{k} p^k (1-p)^{n-k}$

$P(A_1) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8}$

$P(A_2) = \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8}$

$P(A) = P(A_1) + P(A_2) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

$P(B) = \underset{\text{1st dice}}{\frac{6}{6}} \times \underset{\text{2nd dice}}{\frac{1}{6}} \times \underset{\text{3rd dice}}{\frac{1}{6}} = \frac{1}{36}$

$P(A \cap B) = \frac{3}{6 \times 6 \times 6} = \frac{1}{72}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$

independent

4) geometric distribution

$$P(\text{flush}) = \binom{4}{1} \cdot \binom{13}{5} = 4 \cdot 1287 = 5148 \text{ hands}$$

choose 1 suit out of 4

choose 5 cards out of 13 (of chosen suit)

$$P(\text{all possible hands}) = \binom{52}{5} = 2598960 \text{ total hands}$$

$$p = \frac{5148}{2598960} \approx 0.002$$

googled E[X] of geometric distribution derivation

$$E[X] = \sum_{k} k \cdot p(k) = \frac{1}{p} = \frac{1}{0.002} \approx 505 \text{ hands}$$

5) E: team wins 4/5 games

F = superstar played

F^c = superstar doesn't play

$$P(F) = \frac{3}{4} = 0.75$$

$$P(F^c) = 1 - P(F) = 0.25$$

P(superstar played given team won 4 out of 5 games)

$$= P(F|E) = \frac{P(E|F) \cdot P(F)}{P(E|F) \cdot P(F) + P(E|F^c) \cdot P(F^c)} = \frac{(0.36)(0.75)}{(0.36)(0.75) + (0.16)(0.25)}$$

$$= \frac{0.27}{0.27 + 0.04} = 0.87$$

binomial distribution: $\binom{n}{k} p^k (1-p)^{n-k}$

$$P(E|F) = \binom{5}{4} (0.70)^4 (0.3)^1 = 0.36$$

$$P(E|F^c) = \binom{5}{4} (0.5)^4 (0.5)^1 = 0.16$$