Week 6: Visualizing the Bayesian Workflow

27/02/23

Introduction

This lab will be looking at trying to replicate some of the visualizations in the lecture notes, involving prior and posterior predictive checks, and LOO model comparisons.

The dataset is a 0.1% of all births in the US in 2017. I've pulled out a few different variables, but as in the lecture, we'll just focus on birth weight and gestational age.

The data

Read it in, along with all our packages.

```
# A tibble: 6 x 8
  mager mracehisp meduc   bmi sex   combgest  dbwt ilive
  <dbl>     <dbl> <dbl> <dbl> <chr>    <dbl> <dbl> <chr>
1    16         2     2  23   M           39  3.18 Y
2    25         7     2  43.6 M           40  4.14 Y
3    27         2     3  19.5 F           41  3.18 Y
4    26         1     3  21.5 F           36  3.40 Y
5    28         7     2  40.6 F           34  2.71 Y
6    31         7     3  29.3 M           35  3.52 Y
```

Brief overview of variables:

- `mager` mum's age
- `mracehisp` mum's race/ethnicity see here for codes: https://data.nber.org/natality/2017/natl2017.pdf page 15
- `meduc` mum's education see here for codes: https://data.nber.org/natality/2017/natl2017.pdf page 16

- `bmi` mum's bmi
- `sex` baby's sex
- `combgest` gestational age in weeks
- `dbwt` birth weight in kg
- `ilive` alive at time of report y/n/ unsure

I'm going to rename some variables, remove any observations with missing gestational age or birth weight, restrict just to babies that were alive, and make a preterm variable.

Question 1

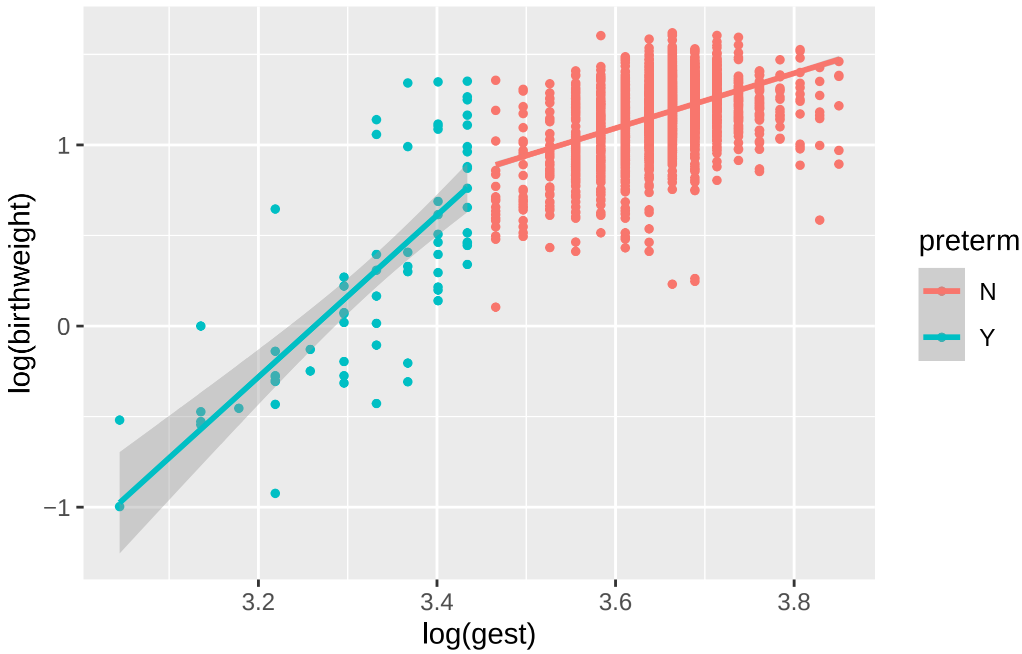
Use plots or tables to show three interesting observations about the data. Remember:

- Explain what your graph/ tables show
- Choose a graph type that's appropriate to the data type
- If you use `geom_smooth`, please also plot the underlying data

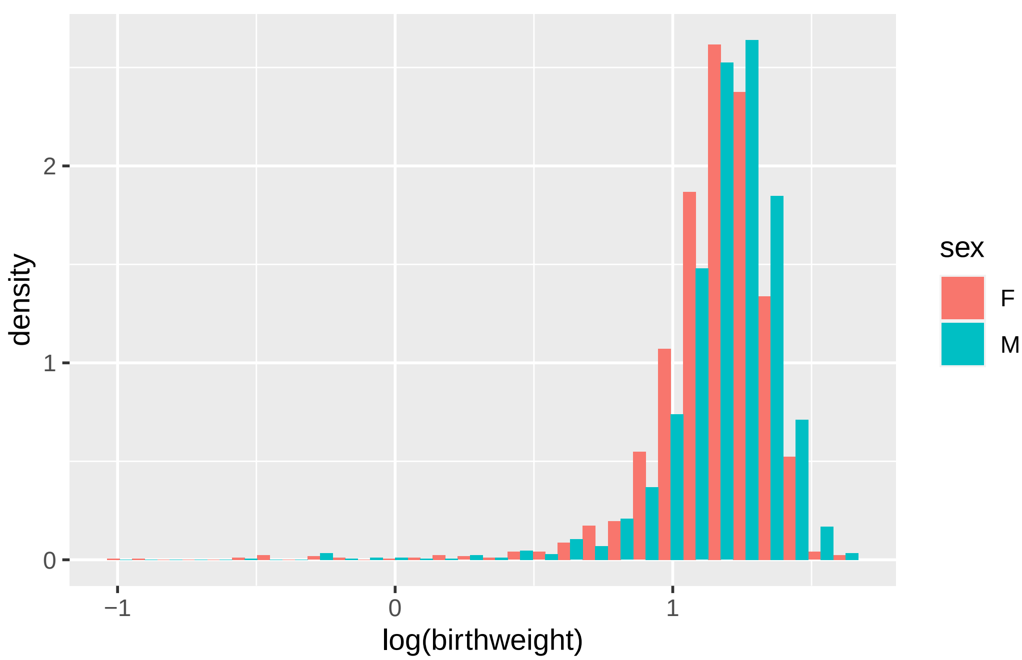
Feel free to replicate one of the scatter plots in the lectures as one of the interesting observations, as those form the basis of our models.

Answer

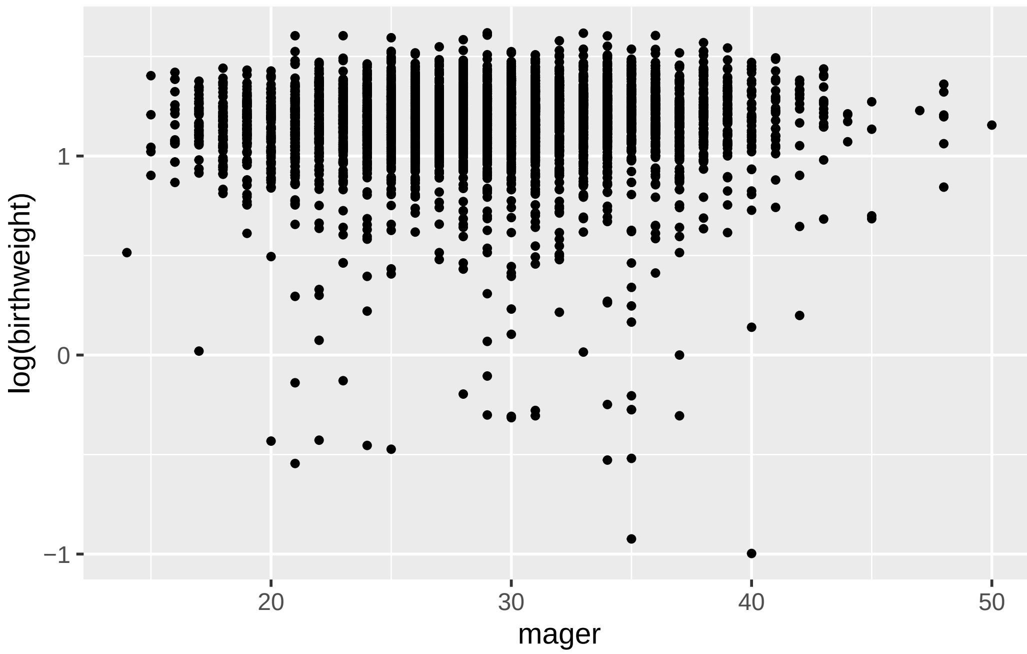
The following plots is the scatter plots in the lecture replicated. It indicates linear relation between (log)birthweight and log(gest). It also indicates that 'preterm' would be a significant covariate.



The following histogram was created to check if ‘sex’ has something to with ‘birthweight’. We can see from this plot that male are distributed slightly larger than female. Based on this observation, we’ll choose ‘sex’ as the additional covariate in Question 8.



We also created the following plot to see if mother’s age has something to do with birthweight. However, the plot does not indicate strong evidence that the covariate matters.



The model

As in lecture, we will look at two candidate models

Model 1 has log birth weight as a function of log gestational age

$$\log(y_i) \sim N(\beta_1 + \beta_2 \log(x_i), \sigma^2)$$

Model 2 has an interaction term between gestation and prematurity

$$\log(y_i) \sim N(\beta_1 + \beta_2 \log(x_i) + \beta_2 z_i + \beta_3 \log(x_i) z_i, \sigma^2)$$

- y_i is weight in kg
- x_i is gestational age in weeks, CENTERED AND STANDARDIZED
- z_i is preterm (0 or 1, if gestational age is less than 32 weeks)

Prior predictive checks

Let's put some weakly informative priors on all parameters i.e. for the βs

$$\beta \sim N(0, 1)$$

and for σ

$$\sigma \sim N^+(0, 1)$$

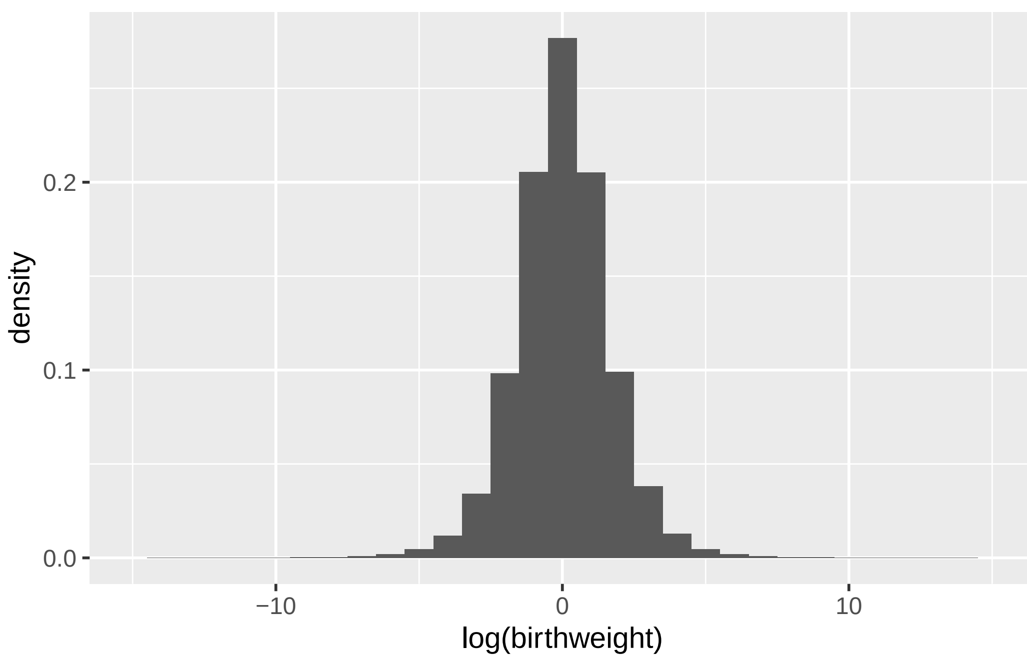
where the plus means positive values only i.e. Half Normal.

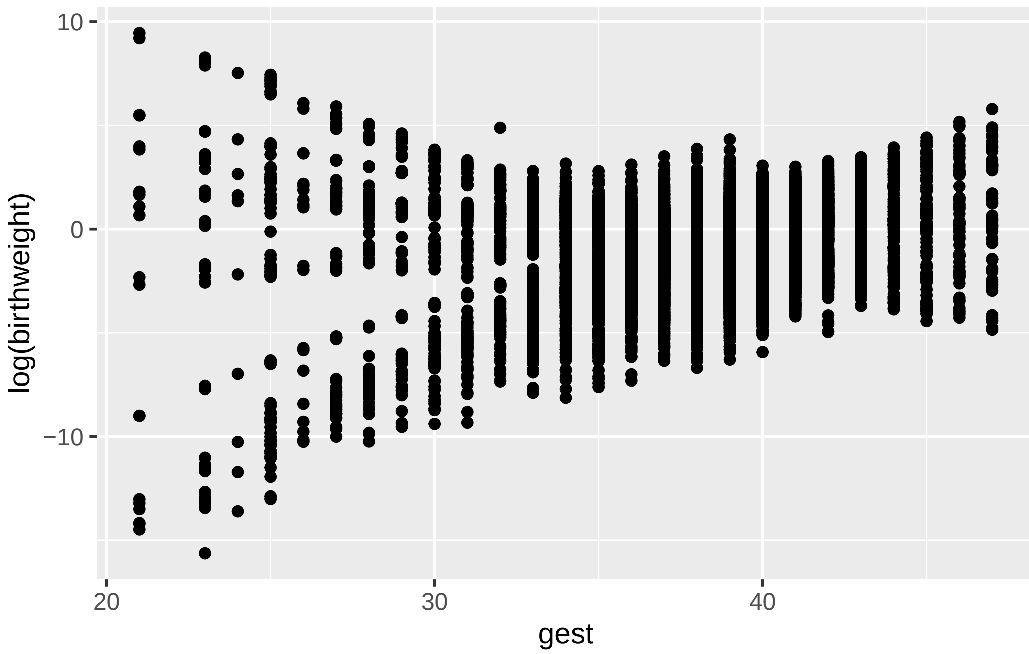
Let's check to see what the resulting distribution of birth weights look like given Model 1 and the priors specified above, assuming we had no data on birth weight (but observations of gestational age).

Question 2

For Model 1, simulate values of βs and σ based on the priors above. Do 1000 simulations. Use these values to simulate (log) birth weights from the likelihood specified in Model 1, based on the set of observed gestational weights. **Remember the gestational weights should be centered and standardized**.

- Plot the resulting distribution of simulated (log) birth weights.
- Plot ten simulations of (log) birthweights against gestational age.





Run the model

Now we're going to run Model 1 in Stan. The stan code is in the `code/models` folder.

First, get our data into right form for input into stan.

Now fit the model

```
SAMPLING FOR MODEL 'simple_weight' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 0.000343 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 3.43 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 1: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 1: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 1: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 1: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 1: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 1: Iteration: 251 / 500 [ 50%]  (Sampling)
```

```
Chain 1: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 1: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 1: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 1: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 1: Iteration: 500 / 500 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 0.628292 seconds (Warm-up)
Chain 1:                0.444071 seconds (Sampling)
Chain 1:                1.07236 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'simple_weight' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 0.000158 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 1.58 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 2: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 2: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 2: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 2: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 2: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 2: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 2: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 2: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 2: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 2: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 2: Iteration: 500 / 500 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 0.508863 seconds (Warm-up)
Chain 2:                0.511455 seconds (Sampling)
Chain 2:                1.02032 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'simple_weight' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 0.000177 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 1.77 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
```

```
Chain 3: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 3: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 3: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 3: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 3: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 3: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 3: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 3: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 3: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 3: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 3: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 3: Iteration: 500 / 500 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 0.53542 seconds (Warm-up)
Chain 3:                0.492603 seconds (Sampling)
Chain 3:                1.02802 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'simple_weight' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 0.000219 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 2.19 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 4: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 4: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 4: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 4: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 4: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 4: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 4: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 4: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 4: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 4: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 4: Iteration: 500 / 500 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 0.559021 seconds (Warm-up)
Chain 4:                0.516387 seconds (Sampling)
Chain 4:                1.07541 seconds (Total)
Chain 4:
```

```
             mean      se_mean          sd      2.5%       25%       50%
beta[1] 1.1626250 7.634607e-05 0.002583881 1.1575321 1.1609497 1.1626383
beta[2] 0.1436183 8.105504e-05 0.002791943 0.1380281 0.1417563 0.1436199
sigma   0.1689127 1.051837e-04 0.001979909 0.1650908 0.1676042 0.1688619
              75%     97.5%     n_eff      Rhat
beta[1] 1.1643919 1.1677313 1145.4383 0.9970543
beta[2] 0.1455075 0.1489575 1186.4598 0.9984953
sigma   0.1701148 0.1728405  354.3181 1.0046933
```

Question 3

Based on model 1, give an estimate of the expected birthweight of a baby who was born at a gestational age of 37 weeks.

Answer

```r
exp(1.1626250 + 0.1436183*(log(37)-m)/s)
```

```
[1] 2.93654
```

Question 4

Write a stan model to run Model 2, and run it.

Answer

Now fit the model

```
SAMPLING FOR MODEL 'weight2' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 0.001179 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 11.79 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 1: Iteration:  50 / 500 [ 10%]  (Warmup)
```

```
Chain 1: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 1: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 1: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 1: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 1: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 1: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 1: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 1: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 1: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 1: Iteration: 500 / 500 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 2.68849 seconds (Warm-up)
Chain 1:                2.21765 seconds (Sampling)
Chain 1:                4.90613 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'weight2' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 0.000422 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 4.22 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 2: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 2: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 2: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 2: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 2: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 2: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 2: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 2: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 2: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 2: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 2: Iteration: 500 / 500 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 2.78167 seconds (Warm-up)
Chain 2:                2.86418 seconds (Sampling)
Chain 2:                5.64584 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'weight2' NOW (CHAIN 3).
Chain 3:
```

```
Chain 3: Gradient evaluation took 0.000506 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 5.06 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 3: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 3: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 3: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 3: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 3: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 3: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 3: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 3: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 3: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 3: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 3: Iteration: 500 / 500 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 2.97391 seconds (Warm-up)
Chain 3:                2.45044 seconds (Sampling)
Chain 3:                5.42435 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'weight2' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 0.000463 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 4.63 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 4: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 4: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 4: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 4: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 4: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 4: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 4: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 4: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 4: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 4: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 4: Iteration: 500 / 500 [100%]  (Sampling)
Chain 4:
```

```
Chain 4:  Elapsed Time: 2.99115 seconds (Warm-up)
Chain 4:                1.93982 seconds (Sampling)
Chain 4:                4.93097 seconds (Total)
Chain 4:
```

Question 5

For reference I have uploaded some model 2 results. Check your results are similar.

```
             mean      se_mean          sd       2.5%        25%       50%
beta[1] 1.1697241 1.385590e-04 0.002742186 1.16453578 1.16767109 1.1699278
beta[2] 0.5563133 5.835253e-03 0.058054991 0.43745504 0.51708255 0.5561553
beta[3] 0.1020960 1.481816e-04 0.003669476 0.09459462 0.09997153 0.1020339
beta[4] 0.1967671 1.129799e-03 0.012458398 0.17164533 0.18817091 0.1974114
sigma   0.1610727 9.950037e-05 0.001782004 0.15784213 0.15978020 0.1610734
              75%     97.5%     n_eff      Rhat
beta[1] 1.1716235 1.1750167 391.67359 1.0115970
beta[2] 0.5990427 0.6554967  98.98279 1.0088166
beta[3] 0.1044230 0.1093843 613.22428 0.9978156
beta[4] 0.2064079 0.2182454 121.59685 1.0056875
sigma   0.1623019 0.1646189 320.75100 1.0104805
```

Answer

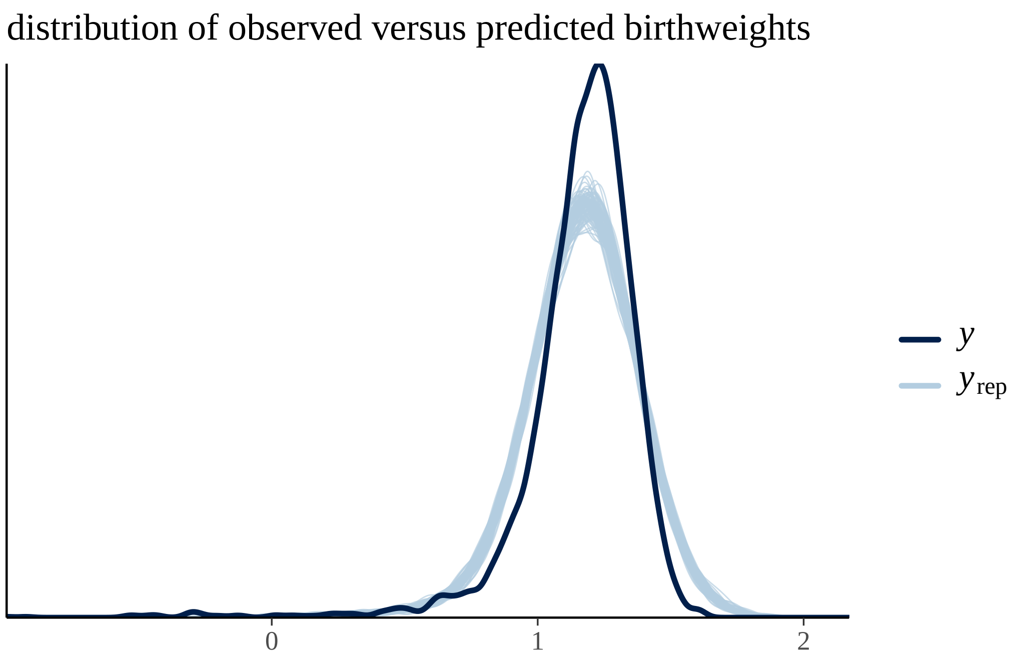
My results are similar to the one above except that 'beta[2]' and 'beta[3]' are flipped.

```
             mean      se_mean          sd       2.5%        25%       50%
beta[1] 1.1695474 7.775215e-05 0.002730748 1.16432439 1.16774539 1.1694545
beta[2] 0.1020646 1.333365e-04 0.003540319 0.09526538 0.09965927 0.1020458
beta[3] 0.5634542 4.555121e-03 0.066073353 0.42870273 0.52058743 0.5622696
beta[4] 0.1983429 9.280153e-04 0.013709344 0.17026957 0.18956716 0.1986937
sigma   0.1611931 7.237822e-05 0.001813104 0.15776604 0.16004710 0.1610828
              75%     97.5%     n_eff     Rhat
beta[1] 1.1714191 1.1746801 1233.4982 1.000121
beta[2] 0.1044228 0.1088863  704.9956 1.008594
beta[3] 0.6072238 0.6902591  210.4035 1.031391
beta[4] 0.2076363 0.2246584  218.2343 1.027843
sigma   0.1624286 0.1648346  627.5230 1.000475
```

PPCs

Now we've run two candidate models let's do some posterior predictive checks. The `bayesplot` package has a lot of inbuilt graphing functions to do this. For example, let's plot the distribution of our data (y) against 100 different datasets drawn from the posterior predictive distribution:

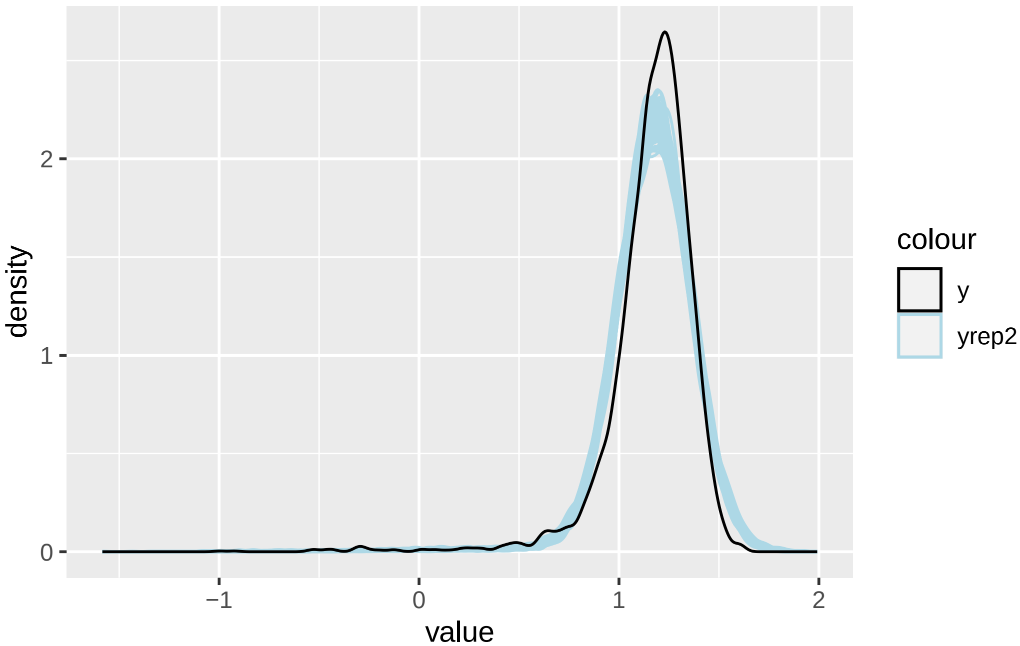
```
[1] 1000 3842
```



Question 6

Make a similar plot to the one above but for model 2, and **not** using the bayes plot in built function (i.e. do it yourself just with `geom_density`)

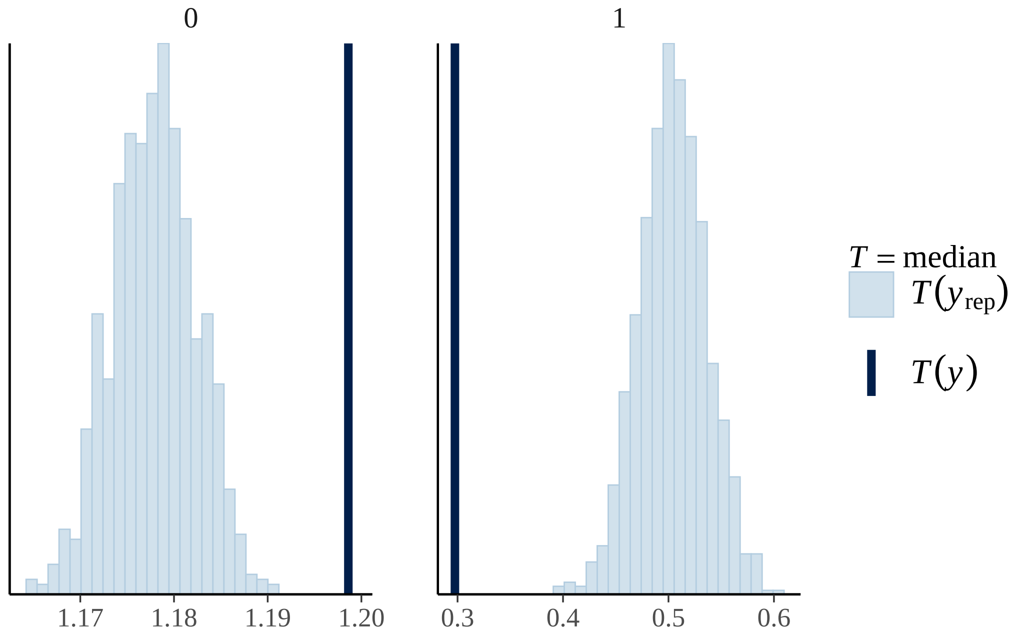
Answer



Test statistics

We can also look at some summary statistics in the PPD versus the data, again either using `bayesplot` – the function of interest is `ppc_stat` or `ppc_stat_grouped` – or just doing it ourselves using ggplot.

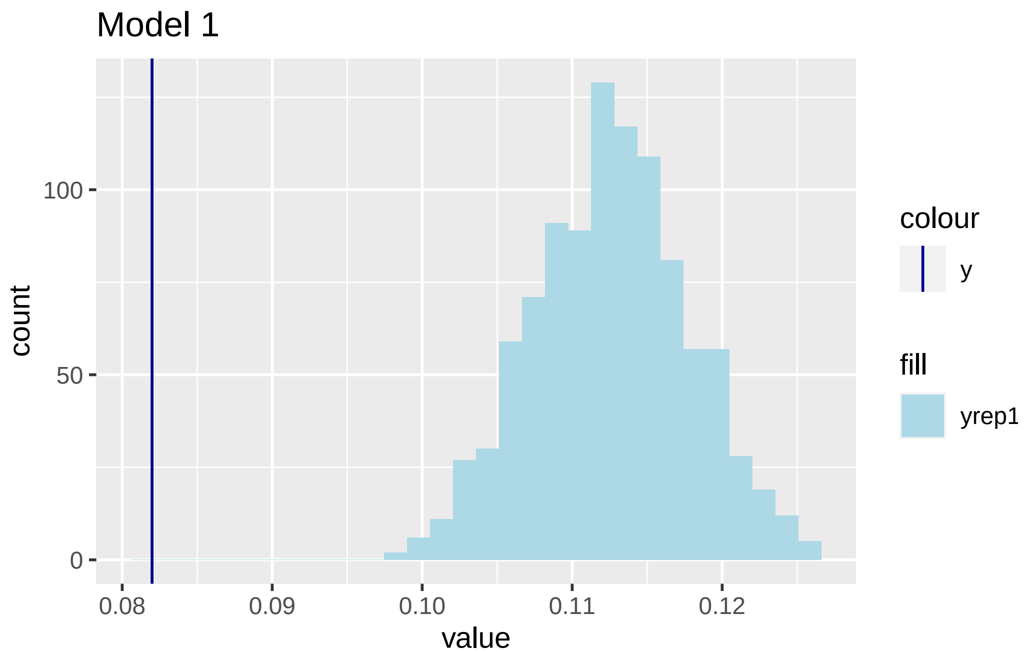
E.g. medians by prematurity for Model 1

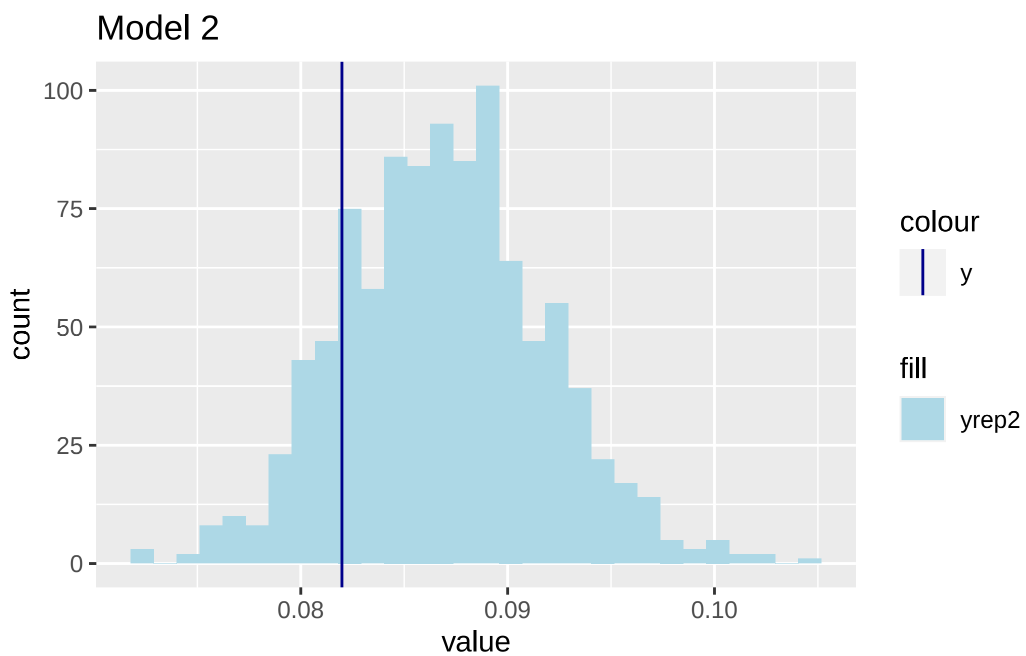


Question 7

Use a test statistic of the proportion of births under 2.5kg. Calculate the test statistic for the data, and the posterior predictive samples for both models, and plot the comparison (one plot per model).

Answer


Model 1
count
100
50
0
0.08
0.09
0.10
0.11
0.12
value
colour
y
fill
yrep1


Model 2
count
100
75
50
25
0
0.08
0.09
0.10
value
colour
y
fill
yrep2

LOO

Finally let's calculate the LOO elpd for each model and compare. The first step of this is to get the point-wise log likelihood estimates from each model:

And then we can use these in the `loo` function to get estimates for the elpd. Note the `save_psis = TRUE` argument saves the calculation for each simulated draw, which is needed for the LOO-PIT calculation below.

Look at the output:

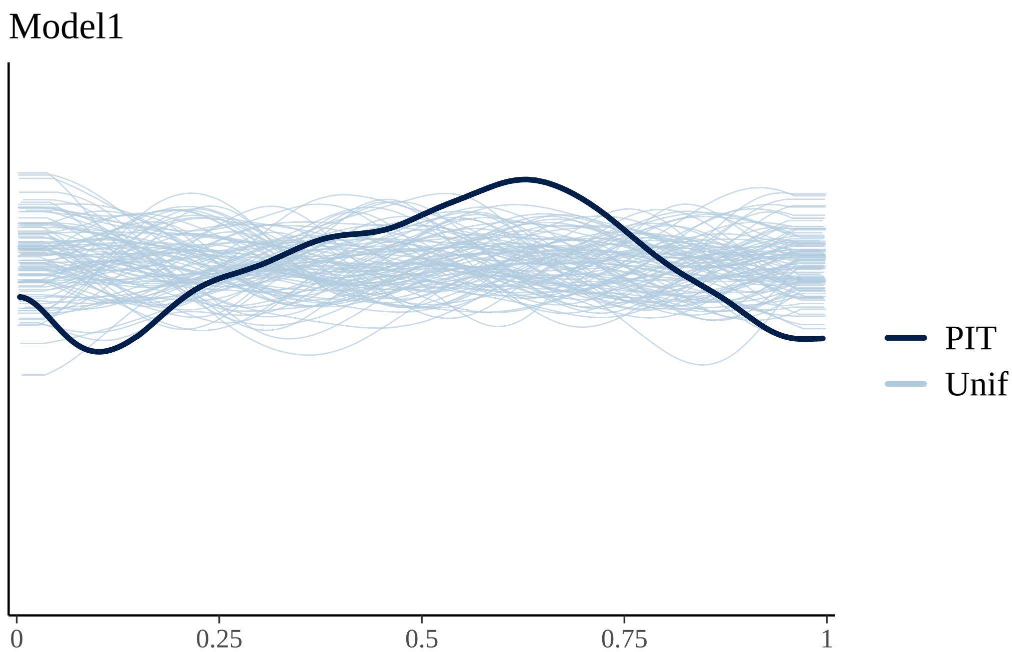
```
Computed from 1000 by 3842 log-likelihood matrix

         Estimate    SE
elpd_loo   1377.2  72.6
p_loo         9.6   1.5
looic     -2754.5 145.2
------
Monte Carlo SE of elpd_loo is 0.1.

All Pareto k estimates are good (k < 0.5).
See help('pareto-k-diagnostic') for details.
```

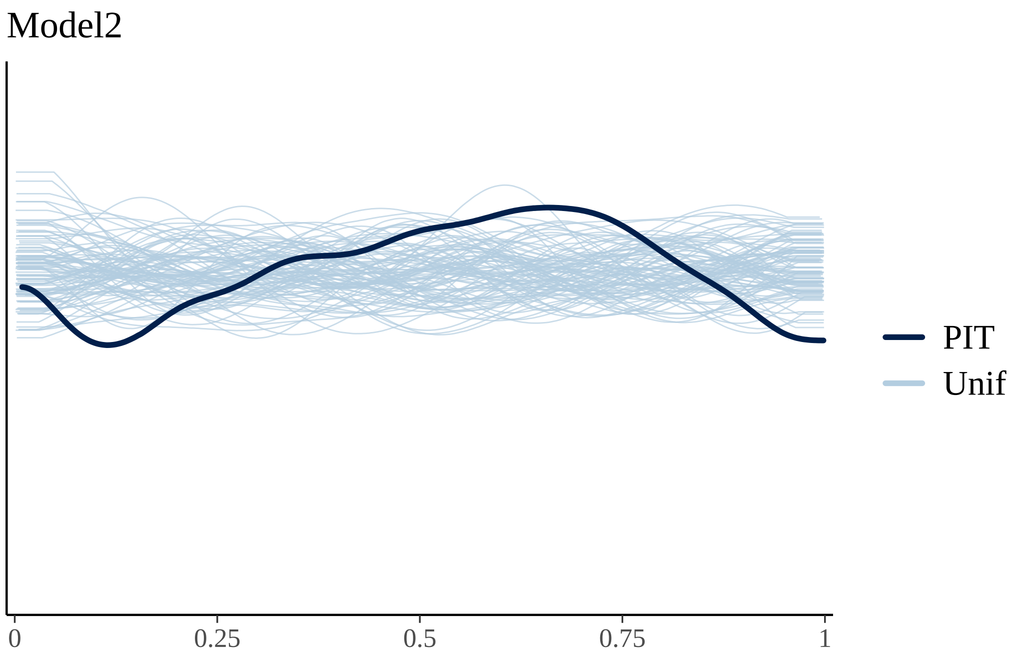
```
Computed from 1000 by 3842 log-likelihood matrix

         Estimate    SE
elpd_loo   1552.3  69.9
p_loo        15.7   2.4
looic     -3104.7 139.7
------
Monte Carlo SE of elpd_loo is 0.2.

Pareto k diagnostic values:
                         Count Pct.    Min. n_eff
(-Inf, 0.5]   (good)     3841  100.0%  220
 (0.5, 0.7]   (ok)          1    0.0%  462
   (0.7, 1]   (bad)         0    0.0%  <NA>
   (1, Inf)   (very bad)    0    0.0%  <NA>

All Pareto k estimates are ok (k < 0.7).
See help('pareto-k-diagnostic') for details.
```

Comparing the two models tells us Model 2 is better:

```
       elpd_diff se_diff
model2    0.0       0.0
model1 -175.1      36.5
```

We can also compare the LOO-PIT of each of the models to standard uniforms. The both do pretty well.





Bonus question (not required)

Create your own PIT histogram "from scratch" for Model 2.

Question 8

Based on the original dataset, choose one (or more) additional covariates to add to the linear regression model. Run the model in Stan, and compare with Model 2 above on at least 2 posterior predictive checks.

Answer

We chose 'sex' as a covariate and build a model as follows:

$$\log(y_i) \sim N(\beta_1 + \beta_2 \log(x_i) + \beta_3 z_i + \beta_4 \log(x_i) z_i + \beta_5 w_i, \sigma^2),$$

where w_i is sex (0 or 1, if male).

Now fit the model

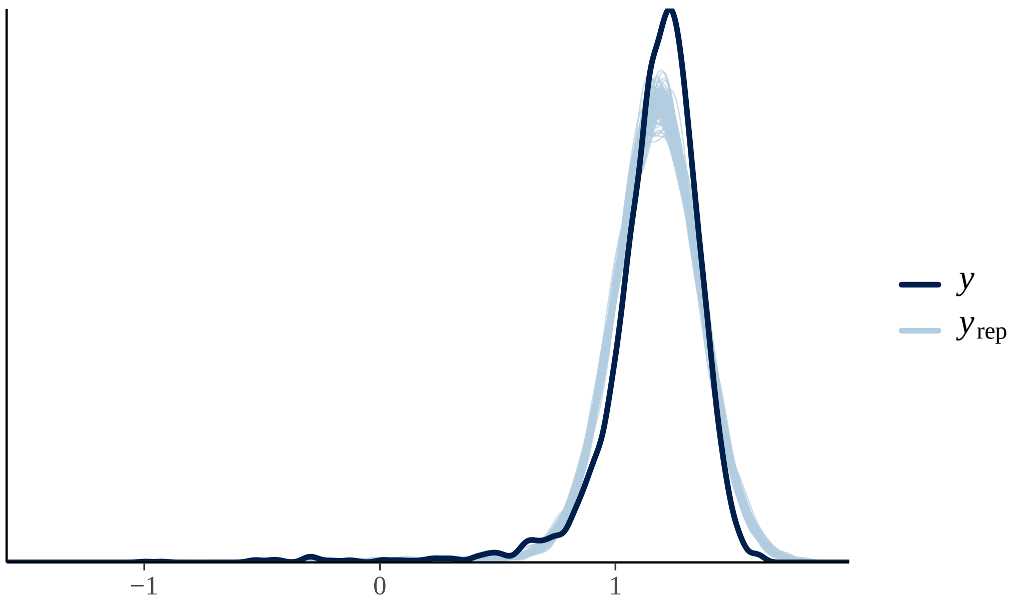
```
SAMPLING FOR MODEL 'weight3' NOW (CHAIN 1).
Chain 1:
Chain 1: Gradient evaluation took 0.001482 seconds
Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 14.82 seconds.
Chain 1: Adjust your expectations accordingly!
Chain 1:
Chain 1:
Chain 1: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 1: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 1: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 1: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 1: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 1: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 1: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 1: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 1: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 1: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 1: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 1: Iteration: 500 / 500 [100%]  (Sampling)
Chain 1:
Chain 1:  Elapsed Time: 3.54391 seconds (Warm-up)
Chain 1:                3.80446 seconds (Sampling)
Chain 1:                7.34838 seconds (Total)
Chain 1:

SAMPLING FOR MODEL 'weight3' NOW (CHAIN 2).
Chain 2:
Chain 2: Gradient evaluation took 0.000623 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 6.23 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 2: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 2: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 2: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 2: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 2: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 2: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 2: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 2: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 2: Iteration: 400 / 500 [ 80%]  (Sampling)
```

```
Chain 2: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 2: Iteration: 500 / 500 [100%]  (Sampling)
Chain 2:
Chain 2:  Elapsed Time: 3.82262 seconds (Warm-up)
Chain 2:                3.14318 seconds (Sampling)
Chain 2:                6.96579 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'weight3' NOW (CHAIN 3).
Chain 3:
Chain 3: Gradient evaluation took 0.00064 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 6.4 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 3: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 3: Iteration: 100 / 500 [ 20%]  (Warmup)
Chain 3: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 3: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 3: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 3: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 3: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 3: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 3: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 3: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 3: Iteration: 500 / 500 [100%]  (Sampling)
Chain 3:
Chain 3:  Elapsed Time: 4.32154 seconds (Warm-up)
Chain 3:                2.83039 seconds (Sampling)
Chain 3:                7.15193 seconds (Total)
Chain 3:

SAMPLING FOR MODEL 'weight3' NOW (CHAIN 4).
Chain 4:
Chain 4: Gradient evaluation took 0.000755 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 7.55 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:   1 / 500 [  0%]  (Warmup)
Chain 4: Iteration:  50 / 500 [ 10%]  (Warmup)
Chain 4: Iteration: 100 / 500 [ 20%]  (Warmup)
```

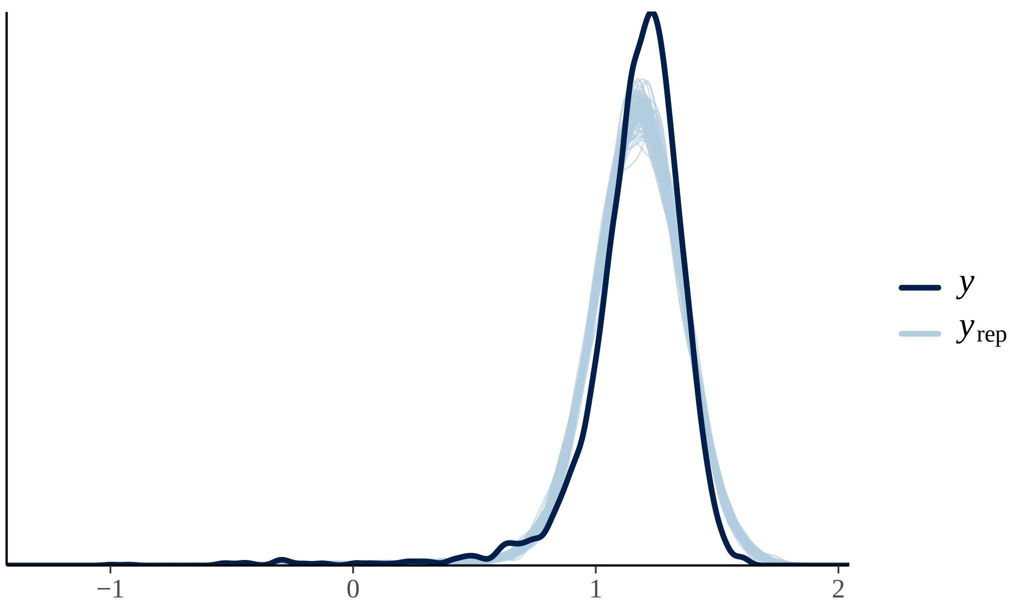
```
Chain 4: Iteration: 150 / 500 [ 30%]  (Warmup)
Chain 4: Iteration: 200 / 500 [ 40%]  (Warmup)
Chain 4: Iteration: 250 / 500 [ 50%]  (Warmup)
Chain 4: Iteration: 251 / 500 [ 50%]  (Sampling)
Chain 4: Iteration: 300 / 500 [ 60%]  (Sampling)
Chain 4: Iteration: 350 / 500 [ 70%]  (Sampling)
Chain 4: Iteration: 400 / 500 [ 80%]  (Sampling)
Chain 4: Iteration: 450 / 500 [ 90%]  (Sampling)
Chain 4: Iteration: 500 / 500 [100%]  (Sampling)
Chain 4:
Chain 4:  Elapsed Time: 3.91606 seconds (Warm-up)
Chain 4:                3.5004 seconds (Sampling)
Chain 4:                7.41646 seconds (Total)
Chain 4:

              mean      se_mean          sd       2.5%         25%         50%
beta[1] 1.14844358 0.0001291497 0.003692184 1.14087258 1.14614805 1.14849077
beta[2] 0.10244761 0.0001087468 0.003497191 0.09601436 0.09993039 0.10226833
beta[3] 0.55433633 0.0032287618 0.065870173 0.42585550 0.51003456 0.55391326
beta[4] 0.19599569 0.0006725433 0.013724689 0.16955548 0.18636853 0.19611854
beta[5] 0.04224789 0.0001877414 0.005139763 0.03203777 0.03879210 0.04226883
sigma   0.15990533 0.0000813047 0.001858884 0.15652342 0.15864097 0.15975673
               75%      97.5%     n_eff      Rhat
beta[1] 1.15099808 1.15561356  817.2963 0.9992439
beta[2] 0.10491610 0.10930858 1034.2030 0.9994427
beta[3] 0.59698866 0.68957203  416.2034 1.0093294
beta[4] 0.20501423 0.22307944  416.4516 1.0059028
beta[5] 0.04575562 0.05199171  749.4905 0.9985962
sigma   0.16111271 0.16378550  522.7251 0.9974357
```

We now compare distribution of predicted birthweights with model 2 and model 3. From the following plots, we cannot see evident difference between them.

Model 2: distribution of observed versus predicted birthweights



Model 3: distribution of observed versus predicted birthweights



Next, we use leave one out method and compare ELPD_loo with model 2.

```
Computed from 1000 by 3842 log-likelihood matrix

         Estimate    SE
```

```
elpd_loo   1552.3  69.9
p_loo        15.7   2.4
looic     -3104.7 139.7
------
Monte Carlo SE of elpd_loo is 0.2.

Pareto k diagnostic values:
                         Count Pct.    Min. n_eff
(-Inf, 0.5]   (good)     3841  100.0%  220
 (0.5, 0.7]   (ok)          1    0.0%  462
   (0.7, 1]   (bad)         0    0.0%  <NA>
   (1, Inf)   (very bad)    0    0.0%  <NA>

All Pareto k estimates are ok (k < 0.7).
See help('pareto-k-diagnostic') for details.


Computed from 1000 by 3842 log-likelihood matrix

         Estimate    SE
elpd_loo   1584.2  70.4
p_loo        17.0   2.5
looic     -3168.5 140.7
------
Monte Carlo SE of elpd_loo is 0.2.

All Pareto k estimates are good (k < 0.5).
See help('pareto-k-diagnostic') for details.
```

Comparing these two, model 3 has slightly larger elpd_loo and thus we conclude model 3 is better. However, the difference is small. So, it may also be reasoable to choose model 2 for simplicity.