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Hopfield neural network based on ant system

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Abstract: Hopfield neural network is a single layer feedforward neural network. Hopfield network requires some control parameters to be carefully selected, else the network is apt to converge to local minimum. An ant system is a nature inspired meta heuristic algorithm. It has been applied to several combinatorial optimization problems such as Traveling Salesman Problem, Scheduling Problems, etc. This paper will show an ant system may be used in tuning the network control parameters by a group of cooperated ants. The major advantage of this network is to adjust the network parameters automatically, avoiding a blind search for the set of control parameters. This network was tested on two TSP problems, 5 cities and 10 cities. The results have shown an obvious improvement.

Key words: hopfield network; ant system; TSP; combinatorial optimization problem

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Traveling Salesman Problem (TSP) is a class of combinatorial optimization problems. It has a variety of important applications, mostly in the area of routing and scheduling problems. The TSP is considered to be one of the standard problems in the operation research/management science literature.

Recent advances in the study of artificial neural networks have resulted, among others, in the algorithm of Hopfield who shows the TSP may be solved by neuron-like computational networks. This work has attracted the attention of many researchers to the field of neural networks. Unfortunately, the deterministic nature of Hopfield network, which minimizes a global energy function, is heavily dependent on initial values of parameters and variables. With this nature, the original network is apt to converge to local minimum, never to escape from this stationary point. Thus, it is very important to use appropriate ranges of randomized initial states and of some multipliers. Furthermore when the TSP is simulated with neural networks, the inter-relationships among the variables are too complex to be refined decisively.

In recent years, Ant system has been developed to solve several combinatorial optimization problems such as Traveling Salesman Problem, Scheduling Problems etc. Ant system is a nature inspired meta heuristic algorithm. Ant system has some properties: positive feedback, distributed computation, and use of a constructive greedy heuristic. Positive feedback accounts for rapid discovery of good solutions, distributed computation avoids premature convergence, and the greedy heuristic helps find acceptable solutions in the early stages of the search process.

TSP problem and Hopfield Neural network will be briefly presented in Section 1. In Section 2, Hopfield neural network based on ant system is introduced. Section 3 presents the simulation results and, finally, Section 4 summarizes conclusions.

1 TSP and Hopfield Neural Network

Traveling Salesman Problem is a problem that is computationally expensive to solve. The premise of TSP is typically taken to be a traveling salesman needs to visit a list of cities. Each city only connects to certain other cities, and there is a weight associated with that connection. The weight can be a time, how much fuel is consumed, a combination of the two or some other arbitrary condition. The goal is to find a path that allows the salesman to visit each city exactly once with the lowest cost possible. Often the problem requires that he end in the same city he began in.

In the TSP, we are given n cities and a distance matrix $[d_{ij}]$ where d_{ij} is the distance between city i and city j. TSP is a permutation problem, i. e. a feasible solution to TSP is a permutation of the given n cities.

Hopfield Networks have been widely used to solve a variety of combinatorial optimization problems. It can be proved that a Hopfield Network with discrete output neurons operating in an appropriate manner is a stable system in the Lyapunov sense. Hopfield Neural net is

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usually described by energy function in which the lowest energy state corresponds to the optimum solution.

In the case of n cities problem, make a $n \times n$ neuron matrix. For example, illustrate a 5×5 matrix in Fig. 1. In this network each neuron is connected to all other neurons (fired neuron symbol = "1" and not fired = symbol "0"). In this matrix, a row is the number of the city and a column is the number of the visit. If a city was visited, the corresponding neuron in this matrix is fired as shown in Fig. 2.

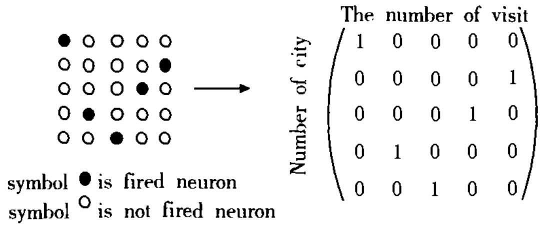

Fig. 1 5 × 5 matrix

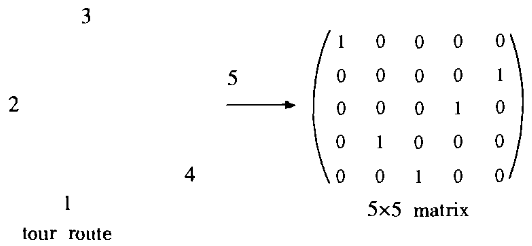

Fig. 2 A tour route and matrix

Let V_{Ai} denote the state of the neuron in row A, column i. Hopfield proceeds further to developuuline an energy function:

$$E_{TSP} = E_1 + E_2 + E_3, \tag{1}$$

$$E_1 = C_1 \sum_A \sum_i \sum_{j \neq i} V_{Ai} V_{Aj} + C_2 \sum_i \sum_A \sum_{B \neq A} V_{Ai} V_{Bi}, \tag{2}$$

$$E_2 = C_3 \left(\sum_A \sum_i V_{Ai} - n \right)^2, \tag{3}$$

$$E_3 = C_4 d_{AB} V_{Ai} (V_{B,i+1} + V_{B,i-1}) . \tag{4}$$

The subscripts on V's are to be treated modulo n to take care of connecting the last city in the tour to the first one. The parameters C is are to be chosen.

Minimizing the term E_1 forces the network to a state with no more than one neuron turned on in any row or column—a requirement for the solution to be a permutation. The term E_2 is minimized when there are exactly n neurons turned on. This is useful to make sure that neurons do not converge to the trivial "all-zero" state. However, this term can be replaced by a global-bias term feeding all the neurons to produce the same effect. The term E_3 takes care of the actual distance minimization.

2 Applying Ant System to the Hopfield Neural Network

For parameters C_{1-4}, take all possible values to compose a set B_{C_i} ($i = 1 \sim 4$). Assume that there are M ants in the constructed algorithm. These ants start from the ant nest to search for food, every ant choosing randomly an element from every set B_{C_i} ($i = 1 \sim 4$) in turn according to the amount of pheromones corresponding to every element in the set and roulette algorithm. When an ant finishes choosing the elements in four sets, it arrives at a food source, and tunes pheromone elements according to rules for adjusting the amount of pheromone.

Assume that the set B_{C_i} have N_i ($i = 1 \sim 4$) elements, $P(B_{C_{ij}})$ ($j = 1 \sim N_i$) denoting its jth elements, whose corresponding amount of pheromones is denoted $pher\ P(B_{C_{ij}})$. Assume Inc denotes the increment of the pheromone amount corresponding to the element which an ant chooses during its going from ant nest to food source.

The steps that ants use to search are the optimal parameters for a Hopfield network:

Step 1: Initializing the amount of pheromone $pher\ P(B_{Cij})$ of each element $P(B_{Cij})$ of all the sets B_{Cij}, and the M ants start from the nest.

Step 2: Ants choose elements in each set in turn according to path choosing rule:

Path choosing rule: for set B_{C_i}, an ant chooses randomly its jth element according to the probability.

$$P^k(B_{Cij}) = \frac{| pherP(B_{Cij}) |}{\sum_{j=1}^{N_i} | pherP(B_{Cij}) |}. \tag{5}$$

Step3: repeat Step 2, all ants arrive at the food source.

Step4: For an ant, after it finishes its searching parameters, it adjusts the amount of pheromones corresponding to the chosen elements according to the following rule.

Rule for adjusting the amount of pheromone: After ants finish their one cycle, every one of them adjusts the amount of pheromone according to equation (6)

$$pherP(B_{Cji})(t + 4) = \rho \cdot pherP(B_{Cij})(t) + Inc(B_{Cij}), \tag{6}$$

where ρ is a coefficient such that $(1 - \rho)$ represents the evaporation of trail between time t and $t + 4$,

$$Inc(B_{Cij}) = \sum_{k=1}^{m} Inc^k(B_{Cij}), \tag{7}$$

where $Inc^k(B_{Cij})$ is the substance (pheromone in real ants) quantity laid on parameter $P(B_{C_{ij}})$ by the kth ant between time t and $t + 4$; it is given by

$Inc^k(B_{ij}) =$

$$\begin{cases} \dfrac{Q}{E_{TSP}^k} & \text{if } k\text{th ant uses parameter } P(B_{ij}), \\ & \text{in its tour (between time } t \text{ and } t+4), \\ 0 & \text{otherwise}, \end{cases} \tag{8}$$

where Q is a positive constant, it is used to adjust the speed of adjusting the amount of pheromone.

Step 5: The above steps continue, until all ants converge on the same path or a generation number is lager than the given number.

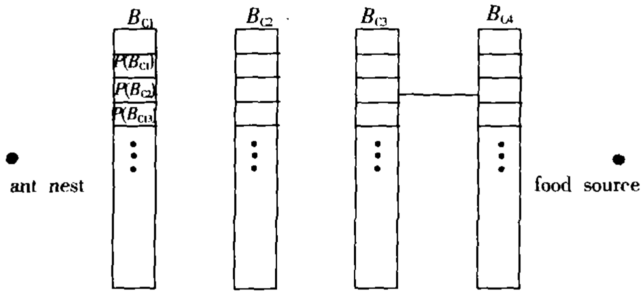

Fig. 3 Selecting paremeters

3 Simulation Results

The above method has been tested on two TSP problems, 5 cities and 10 cities. Coordinates in these problems are shown in Tabs. 1 and 2. The results obtained after 50 tests are shown in Tab. 3. The best tour results are shown in Fig. 4. They are compared with the results using standard Hopfield neural network. In the 5 cities problem, the solution from our algorithm is the same as a Standard Hopfield network. In the 10 cities problem, this method can produce 48 (96%) valid solutions, and the best tour has a length of 3.46, the same as the optimal tour. While using a standard Hopfield neural network, only 37 valid solutions could be obtained in 50 experiments, and the best solution has a length of 3.63, 5% longer than the optimal tour.

Tab. 1 Coordinates in 5 cities

x	y
0	0.400
0.757	0.286
1.135	0.022
1.249	0.551
0.568	0.589

Tab. 2 Coordinates in 10 cities

x	y
0	0.300
0.418	0.237
0.627	0.091
0.732	0.300
1.045	0.614
0.962	0.948
0.836	1.136
0.627	0.865
0.690	0.384
0.314	0.405

Tab 3 Simulation Results

	HP with ant system	Standard HP	HP with ant system	Standard HP
Cities	5	5	10	10
Ant Size	10		28	
Feasible Solutions	50	50	48	37
Best Length	3.05	3.05	3.46	3.63
Optimum Length	3.05	3.05	3.46	3.46

Fig. 4 Best tour results

4 Conclusion

Hopfield neural network based on ant system is presented in this paper. Hopfield neural network has been widely used to solve optimization problems. However, the control parameters for this work have to be carefully selected, or else the network is apt to converge to local minimum. In this paper, these parameters could be obtained by a group of cooperating working ants. This network has been demonstrated in TSP problem of 5 cities and 10 cities. The results have shown a significant improvement using the algorithm.

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