程稼夫《力学篇》习题

习题一 1-1 解：$t=\frac{x}{v}=5.56\times10^{17}s=1.76\times10^{10}$年

1-2 解：$t=\frac{x}{v}=6.67\times10^{-24}s$

1-3 解：由题意知 $\Delta T=1\times10^{-4}$ ℃

1-4 解：$\frac{R}{l_{天}}=\theta$ $\frac{R}{l_{1}}=\theta_1$ 解得 $l_{天}=2.7$秒差距$=8.34\times10^{16}m$

1-5 解：$R=\frac{1}{2}\theta\cdot d=1.77\times10^{6}m$

习题二 2-1 解：设列车前端通过0点时为时间 $t=0$，中间时刻为 t_1，列车尾端通过0点为 t_2

由题意有 $u_1t_1+\frac{1}{2}at_1^2=\frac{x}{2}$ ..① $u_1t_2+\frac{1}{2}at_2^2=x$ ..②

$u_2=u_1+at_2$ ..③ $u_{中}=u_1+at_1$ ..④

联立解得 $u_{中}=\sqrt{\frac{1}{2}(u_1^2+u_2^2)}$

2-2 解：$\frac{1}{2}a_1\tau^2=h$ ..① $a_1\tau\cdot\tau'-\frac{1}{2}g\tau'^2=-h$ ..② $a_1=0.8g$ ..③

解得 $\tau'=2\tau$ $\therefore t_{总}=\tau+\tau'=3\tau$

2-3 解：$t_{总}=\frac{v}{a_1}+\frac{v}{a_2}+\frac{s-s_1-s_2}{v}$ 其中 $s_1=\frac{v^2}{2a_1}$ $s_2=\frac{v^2}{2a_2}$

代入解得 $t_{总}=\frac{s}{v}+\frac{v}{2a_1}+\frac{v}{2a_2}$

2-4 解：$\frac{1}{2}a_1t_1^2=l_1$ ..① $\frac{1}{2}a_2t_2^2=l_2$ ..② $a_1t_1=a_2t_2$ ..③ $t_1+t_2=t$ ..④

解得 $a_1=\frac{2(l_1+l_2)^2}{l_1t^2}$ $a_2=\frac{2(l_2+l_1)^2}{l_2t^2}$

2-5 解：$\frac{1}{2}gt^2=h$ ..① $\frac{1}{2}g(t-1)^2=\frac{1}{3}h$ ..② 解得 $h=145.5m$ $t=5.45s$

2-6 解：$\frac{1}{2}gt^2=H$ ..① $\frac{1}{2}g(t+\Delta t)^2=H+l$ ..② 解得 $H=19.42m$

注：本题若认为两球较高一端与楼顶平齐，则解得 $H=21.42m$

2-7 解：$h_1=\frac{1}{2}g\left(\frac{\Delta t_B}{2}\right)^2$ ..① $h_2=\frac{1}{2}g\left(\frac{\Delta t_A}{2}\right)^2$ ..② $\therefore\Delta h=h_2-h_1=\frac{g}{8}[(\Delta t_A)^2-(\Delta t_B)^2]$

2-9 解：$\frac{1}{2}gt_1^2=h$ ..① $gt_1t_2=h\cdot ctg\theta$ ..② $\frac{1}{2}g\sin\theta(t_1+t_2)^2=\frac{h}{\sin\theta}$ ..③

（亦可列等式 $\sqrt{\frac{2h}{g}}+\frac{h\,ctg\theta}{\sqrt{2gh}}=\sqrt{\frac{2h}{g\sin\theta}}$ ）解得 $\sin\theta=\frac{3}{5}$

顺序对换

取小量 有 $g\Delta t\cdot\Delta t'=\frac{4}{3}\cdot\frac{1}{2}g\Delta t^2$ $\therefore\Delta t'=\frac{2}{3}\Delta t$

$\therefore t_{min}=t_1+\frac{2}{3}t_1=\frac{5}{3}t_1$ 其中 $t_1=\sqrt{\frac{2h}{g}}$

$t_{max}=t_1+\frac{4}{3}h/gt_1=\frac{7}{3}t_1$ $\therefore\frac{t_{max}}{t_{min}}=\frac{7}{5}$

2-8 解：由题意知 $L^2=a^2+(\frac{1}{2}gt^2)^2$ 解得 $t=\sqrt{\frac{2}{g}}\cdot(L^2-a^2)^{\frac{1}{4}}$

$\omega_1=\frac{\arccos\frac{a}{L}}{t}=\arccos\frac{a}{L}\cdot\sqrt{\frac{g}{2}}\cdot(L^2-a^2)^{-\frac{1}{4}}$

$\omega_2=\frac{\arccos\frac{a}{L}+2\pi}{t}=\sqrt{\frac{g}{2}}\cdot(L^2-a^2)^{-\frac{1}{4}}\cdot(\arccos\frac{a}{L}+2\pi)$

2-10 解：(1) $t=\frac{R}{v\cdot\cos18^\circ}\approx1.05\frac{R}{v}$ ([illegible]速度分值大小不变)

(2) $\frac{v_0t}{\rho}=\frac{v_0t\ \sin36^\circ}{2R\cos18^\circ}$ 解得 $\rho=\frac{R}{\sin18^\circ}\approx3.24R$

2-11 解：∵ C点速度为0 ∴ $\omega r=\omega_0(R+r)$ 解得 $\omega=\omega_0\frac{R+r}{r}$

2-12 解：(1) ∵ C点速度为0 ∴ $\omega(R_1+R_2)-\omega_1R_1=\omega_2R_2$ 解得 $\omega=\frac{\omega_1R_1+\omega_2R_2}{R_1+R_2}$

(2) $t_1=\frac{2\pi}{\omega}=\frac{2\pi(R_1+R_2)}{\omega_1R_1+\omega_2R_2}$

(3) $t_2=\frac{2\pi}{|\omega-\omega_1|}=\frac{|\omega_2-\omega_1|R_2}{R_1+R_2}$

2-13 解：(1) ∵ C点速度为0 ∴ $\omega(R_1-R_2)=R_1\omega_1-R_2\omega_2$ ∴ $\omega=\frac{R_1\omega_1-R_2\omega_2}{R_1-R_2}$

(2) $t_1=\frac{2\pi}{|\omega|}=\frac{2\pi(R_1-R_2)}{|R_1\omega_1-R_2\omega_2|}$

(3) $t_2=\frac{2\pi}{|\omega-\omega_1|}=\frac{2\pi(R_1-R_2)}{R_2|\omega_1-\omega_2|}$

2-14 解：(1) $\beta=\frac{\Delta\omega}{\Delta t}=\frac{2}{3}\pi\cdot \mathrm{rad/s}$

(2) $\theta=\omega_0t-\frac{1}{2}\beta t^2=\frac{425}{3}\pi=70.83$ 转

(3) $t=\frac{\omega}{\beta}=40s$

2-15 解：(1) 分析竖直方向分运动，有 $v_\perp^2=2gh$ ∴ $v_\perp=\sqrt{2gh}$

(2) $t=\frac{v_\perp}{g}=\sqrt{\frac{2h}{g}}$

(3) $v_{/\!/}=u+\sqrt{v^2-2gh}$ ∴ $x=v_{/\!/}\cdot t=(u+\sqrt{v^2-2gh})\cdot\sqrt{\frac{2h}{g}}$

2-16 解：由题意，有 $x^2+(\frac{1}{2}gt^2)^2=(vt)^2$ 解得 $h=\frac{1}{2}gt^2=5m$ 或 $h=4999.5m$

标答有误

2-17 解：由题意，有 $\frac{v_0^2\sin2\alpha}{g}=x+a$ ‥① $\frac{v_0^2\sin2\beta}{g}=x-b$ ‥②

$\frac{v_0^2\sin2\theta}{g}=x$ ‥③ 联立，解得 $\theta=\frac{1}{2}\arcsin\frac{b\sin2\alpha+a\sin2\beta}{a+b}$

2-18 解：由图可知 $v_1=3v_2$ ∴ $\sqrt{\frac{2H}{g}}+t=3\cdot\sqrt{\frac{2(H-h)}{g}}$ ‥① $\sqrt{2gH}-\sqrt{2g(H-h)}=gt$ ‥②

联立解得 $h=\frac{3}{4}H$

2-19 解：$v_0\cos\alpha_1t=v_0\cos\alpha_2(t+\Delta t)$ ‥①

$v_0\sin\alpha_1t-\frac{1}{2}gt^2=v_0\sin\alpha_2(\Delta t+t)-\frac{1}{2}g(t+\Delta t)^2$ ‥②

联立解得 $\frac{\sin\frac{1}{2}(\alpha_1-\alpha_2)}{\cos\frac{1}{2}(\alpha_1+\alpha_2)}=\frac{g\Delta t}{2v_0}$

2-20 解：$v\cos\alpha(t+\Delta t)=v'\cos\alpha't$ ‥①

$v\sin\alpha(t+\Delta t)-\frac{1}{2}g(t+\Delta t)^2=v'\sin\alpha't-\frac{1}{2}gt^2$ ‥②

联立，解得 $\Delta t=\frac{2vv'\sin(\alpha-\alpha')}{g\cdot(v\cos\alpha+v'\cos\alpha')}$

2-21 解：(1) $v_{p\parallel}=v$ $v_{p\perp}=\frac{1}{2}v_{B\perp}=\frac{\sqrt{3}}{6}v$

∴ $v_p=\sqrt{v_{p\parallel}^2+v_{p\perp}^2}=\sqrt{\frac{13}{12}}v$. $\alpha=\arctan\frac{\sqrt{3}}{6}$

(2) $\alpha=\arctan\frac{\sqrt{3}}{6}<30^\circ$ ∴ P做斜下抛运动

$\frac{1}{2}h=v_1t+\frac{1}{2}gt^2$，其中 $v_1=v_p\sin(30^\circ-\alpha)$

解得 $t=\frac{\sqrt{v^2+16gh}-v}{4g}$

2-22 解：① $\theta=\frac{\pi}{2}-\alpha$ 时，甲球原路返回. $t_1=\frac{2\cdot 2v_0\sin\alpha}{g}$

$t_2=2\sqrt{\frac{2h}{g}}$ 令 $t_1=t_2$ 解得 $\alpha=\arcsin\left(\frac{1}{v_0}\sqrt{\frac{gh}{2}}\right)$ $\left(v_0>\sqrt{\frac{gh}{2}}\right)$

② $\theta=\frac{\pi}{4}$时. 甲球沿 $\alpha'=\frac{\pi}{2}-\alpha$ 的路径返回

$t_1=\frac{2v_0(\sin\alpha+\cos\alpha)}{g}$ $t_2=2\sqrt{\frac{2h}{g}}$ 令$t_1=t_2$ 解得 $\alpha=\arcsin\frac{\sqrt{gh}}{v_0}-\frac{\pi}{4}$

$(\sqrt{gh}<v_0<\sqrt{2gh})$

③ $\theta=\frac{\pi}{4}-\frac{\alpha}{2}$时 甲球碰后做竖直上抛运动，经过二次碰撞再原路返回

$t_1=\left(\frac{2v_0\sin\alpha}{g}+\frac{2v_0}{g}\right)\cdot 2$ $t_2=2\sqrt{\frac{2h}{g}}$ 令$t_1=t_2$

解得 $\alpha=\arcsin\left(\frac{\sqrt{2gh}}{2v_0}-\frac{1}{2}\right)$ $\left(\frac{\sqrt{2gh}}{3}<v_0<\sqrt{2gh}\right)$

注：标准答案不全，均漏掉了第3种情况

2-23 解：$\frac{H}{\cos\theta}=\frac{R}{\sin\alpha}$ ∴ $v_m=\omega R\cdot\cos\theta=\omega H\sin\alpha$

2-24 解：(1) $x=\alpha l\sin\theta$ $y=(1-\alpha)l\cos\theta$ ∴ $\left(\frac{x}{\alpha}\right)^2+\left(\frac{y}{1-\alpha}\right)^2=l^2$

(2) 由相似关系可知 $v_{py}=(\alpha-1)v_B$

又∵ $v_{px}\sin\theta-v_{py}\cos\theta=v_B\cos\theta$ ∴ $v_{px}=\alpha v_B\cot\theta$

2-25 解：$\omega r\sin\alpha=v\cos\alpha$ ∴ $v=\omega r\tan\alpha$

2-26 解：$v_{c\parallel}=v\cos\theta$ $v_{c\perp}=\frac{1}{2}v\sin\theta$ ∴ $v_c=\sqrt{v_{c\parallel}^2+v_{c\perp}^2}=\frac{v}{2}\sqrt{1+3\cos^2\theta}$ ∴ $\frac{v_c}{v_A}=\frac{\sqrt{1+3\cos^2\theta}}{2}$

2-27 解：在O'处看，有 $(V+v)\cos\alpha=v$ 解得 $V=\frac{1-\cos\alpha}{\cos\alpha}\cdot v$

2-28 解：由图可得 $v_P=\frac{v_0/2}{\cos\alpha}=\frac{v_0}{2\cos\alpha}$ $\varphi=\pi-2\alpha$

2-29 解：x方向 $\sum v_2\cos\theta_i\Delta t_i-v_1\sum\Delta t_i=0$ 即 $v_2\cdot\sum\cos\theta\Delta t-v_1t=0$

连线方向 $\sum v_2\Delta t_i-\sum v_1\cos\theta_i\Delta t_i=L$ 即 $v_2t-v_1\sum\cos\theta\Delta t=L$

解得 $t=\frac{Lv_2}{v_2^2-v_1^2}$

2-30 解：画图，易知 $v_a=\sqrt{2}v_0=3.5$ m/s 正东北风向

2-31 解: $u^2+v_1^2-v^2=2uv_1\cos(\beta-\alpha)$ …①

$u^2+v_2^2-v^2=-2uv_2\cos(\beta-\alpha)$ …②

$\frac{x}{v_1}+\frac{x}{v_2}=\frac{2R}{v}$ …③

解得 $x=\frac{2R(v^2-u^2)}{v\sqrt{v^2-u^2\sin^2(\beta-\alpha)}}$

2-32 解: $v_n=\sqrt{\frac{g}{2h}}\cdot R$ $v_\tau=v_0$ ∴ $v_{总}=\sqrt{v_n^2+v_\tau^2}=28.27\ m/s$

$\theta=\frac{\pi}{2}+\arctan\frac{v_\tau}{v_n}=104.34°$

2-33 解: $\frac{V_1-V\sin\theta_0}{V\cos\theta_0}=\tan\theta_1$ …① $\frac{V_2-V\sin\theta_0}{V\cos\theta_0}=\tan\theta_2$ …②

解得 $\frac{V_1}{V_2}=\frac{\tan\theta_1+\tan\theta_0}{\tan\theta_2+\tan\theta_0}$

2-34 解: 当 $\frac{l}{\sqrt{l^2+d^2}}>\frac{v_1}{v_2}$ 即 $l>\frac{dv_1}{\sqrt{v_2^2-v_1^2}}$ 时 此人应由A向B运动

当 $\frac{l}{\sqrt{l^2+d^2}}\le\frac{v_1}{v_2}$. 即 $l\le\frac{dv_1}{\sqrt{v_2^2-v_1^2}}$ 时. 由等效法可知. 人应沿着 $\theta=\arccos\frac{v_1}{v_2}$ 游到岸边. 再上岸行走到B

习题三 3-1 解: $\frac{1}{2}mv^2=mg\cdot R(1-\cos\theta)+\frac{1}{2}mv_0^2$ …① $m\frac{v^2}{R}=mg\cos\theta$ ②

解得 $\theta=\arccos[\frac{1}{3}(\frac{v_0^2}{gR}+2)]$

① 若 $0\le v_0<\sqrt{gR}$ 则 $\theta=\arccos[\frac{1}{3}(\frac{v_0^2}{gR}+2)]$

② 若 $v_0\ge\sqrt{gR}$ 则 $\theta=0$

3-2 解: 由题意知. $\frac{dl}{r\,dt}=\frac{v}{l}$ ∴ $l\,dl=rv\,dt$ ∴ $\frac{1}{2}l^2=rvt$ 即 $l=\sqrt{2rvt}$

3-3 解: (1) 在力的矢量三角形中. 由几何关系. 得 $\frac{\mu mg}{m\omega_0^2R}=\frac{r}{\sqrt{R^2-r^2}}$

整理 解得 $R=\frac{\mu gr}{\sqrt{(\mu g)^2-(\omega_0^2r)^2}}$

(2) 由(1)中结论.知 $\omega_0<\sqrt{\frac{\mu g}{r}}$

3-4 解: (1) 由题意.知 $mg-2T=ma$ …① $T-\mu m_1g=m_1a$ …② $T+\mu m_1g=m_2a$ …③

解得 $\mu_0=\frac{(m_2-m_1)m}{2m_1(m_1+m_2+m)}$ ∴ $\mu\le\mu_0=\frac{(m_2-m_1)m}{2m_1(m_1+m_2+m)}$

(2) $mg-2T=ma_1$ …① $T-\mu m_1g=m_1a_2$ …② $a_1:a_2=1:2$ …③

又∵ $T+\mu m_1g-\mu(m_1+m_2)g\le0$ …④ $a>0$ …⑤

解得 $\frac{2mm_1}{4m_1m_2+mm_2-mm_1}\le\mu<\frac{m}{2m_1}$

3-5 解: $F=(M_1+M_2+M)a$ …① $(M_2g)^2+(M_2a)^2=(M_1a)^2$ …②

解得 $F=392N$

3-6 解：$a<g$ 时 单摆　$a=g$ 时 匀圆.　$a>g$ 时 类抛体后单摆

3-7 解：$F=\mu mg\cos\theta$　其中 $\cos\theta=\frac{v_0}{\sqrt{v_0^2+(\omega r)^2}}$　解得 $F=2.0\ N$

3-8 解：$\sin\alpha\le\frac{h}{L+b}$ ··①　$mg\frac{l}{2}\cos\alpha=ma\frac{l}{2}\sin\alpha$ ··②

解得 $a\ge\frac{g\sqrt{(l+b)^2-h^2}}{h}$

3-9 解：(1) $N\cos\theta=mg$ ··① $N\sin\theta=ma$ ··② $F=(M+m)a$ ··③

解得 $F=(M+m)g\tan\theta$

(2) F较小时、临界情况有 $N\cos\theta+\mu N\sin\theta=mg$ ··① $N\sin\theta-\mu N\cos\theta=ma$ ··②

$F_{min}=(M+m)a$ ··③ 解得 $F_{min}=(M+m)g\cdot\frac{\sin\theta-\mu\cos\theta}{\cos\theta+\mu\sin\theta}$

同理得 $F_{max}=\frac{\sin\theta+\mu\cos\theta}{\cos\theta-\mu\sin\theta}\cdot(M+m)g$　∴ $F_{min}\le F\le F_{max}$

3-10 解：当 $F_0=\mu mg\cos\theta+mg\sin\theta+kx$. 即 $x=\frac{F_0-\mu mg\cos\theta-mg\sin\theta}{k}$ 时 速度极大

$F_0x-(\mu mg\cos\theta+mg\sin\theta)x=\frac{1}{2}kx^2+\frac{1}{2}mv_m^2$

解得 $\boxed{v_m=\frac{F_0-\mu mg\cos\theta-mg\sin\theta}{\sqrt{km}}}$ 标答漏掉此问

当 $F_0x-(\mu mg\cos\theta+mg\sin\theta)x=\frac{1}{2}kx^2$. 即 $x=\frac{2(F_0-\mu mg\cos\theta-mg\sin\theta)}{k}$ 时 物体到达最高点

∴ $W_F=F_0x=\frac{2F_0}{k}(F_0-\mu mg\cos\theta-mg\sin\theta)$

3-11 解：相对速度 $u=2v$　相对加速度 $a=\mu g\cdot\frac{M+m}{M}$

由题意.有. $(2v)^2>2\cdot a\cdot l$　解得 $v>\sqrt{\frac{\mu(M+m)gl}{2M}}$

3-12 解：$Mg\sin\alpha-mg=(M+m)a$ ··①　$T-mg=ma$ ··②

$N=Mg\sin\alpha\cos\alpha-F\cos\alpha$ ··③ 解得 $N=\frac{M\sin\alpha-m}{M+m}\cdot Mg\cos\alpha$

3-13 解：对整体.有 $a=\frac{F}{M_1+M_2+M_3}$ ··①

1° 若 $\mu_1<\tan\theta$

①当a较小时. 在M_3系分析M_1. 有

$M_1g\sin\theta-M_1a\cos\theta-f_1=0$ ··②

$N_1-M_1a\sin\theta-M_1g\cos\theta=0$ ··③

$f_1\le\mu_1N_1$ ··④

综②③④. 解得 $a\ge\frac{\sin\theta-\mu_1\cos\theta}{\cos\theta+\mu_1\sin\theta}g$ ··⑤

②当a较大时、同理.有

$M_1g\sin\theta+f_1-M_1a\cos\theta=0$ ··⑥

联立③④⑥. 解得 $a \le \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta} g$ ⑦

⑤代入① 有 $F \ge \frac{\sin\theta - \mu_1\cos\theta}{\cos\theta + \mu_1\sin\theta}(M_1+M_2+M_3)g$ ⑧

⑦代入① 有 $F \le \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}(M_1+M_2+M_3)g$ ⑨

将 M_1. M_2 看作整体 在 M_3 系中有 $F \le \mu_2(M_1+M_2+M_3)g$ ⑩

讨论⑧⑨⑩ 1) 若 $\mu_2 < \frac{\sin\theta - \mu_1\cos\theta}{\cos\theta + \mu_1\sin\theta}$. 则三物体无法相对静止

2) 若 $\frac{\sin\theta - \mu_1\cos\theta}{\cos\theta + \mu_1\sin\theta} \le \mu_2 \le \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}$

则 $\frac{\sin\theta - \mu_1\cos\theta}{\cos\theta + \mu_1\sin\theta}(M_1+M_2+M_3)g \le F \le \mu_2(M_1+M_2+M_3)g$

3) 若 $\frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta} \le \mu_2$ → 标答此处有误

则 $\frac{\sin\theta - \mu_1\cos\theta}{\cos\theta + \mu_1\sin\theta}(M_1+M_2+M_3)g \le F \le \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}(M_1+M_2+M_3)g$

2° 若 $\mu_1 \ge \tan\theta$ 由于自锁现象. M_1 不会下滑

讨论⑨⑩ 1) 若 $\mu_2 < \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}$ 则 $F \le \mu_2(M_1+M_2+M_3)g$

2) 若 $\mu_2 \ge \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}$ 则 $F \le \frac{\sin\theta + \mu_1\cos\theta}{\cos\theta - \mu_1\sin\theta}(M_1+M_2+M_3)g$

3-14. 解: 在 m 系中. $T + m_1 a_m\cos\alpha_1 - m_1 g\sin\alpha_1 = m_1 a$ ①

$m_2 g\sin\alpha_2 + m_2 a_m\cos\alpha_2 - T = m_2 a$ ②

整体水平方向 $(M+m_1+m_2)a_m - m_1 a\cos\alpha_1 - m_2 a\cos\alpha_2 = 0$ ③

解得 $a_m = \frac{(m_1\cos\alpha_1 + m_2\cos\alpha_2)(m_1\sin\alpha_1 - m_2\sin\alpha_2)}{(m_1+m_2)(M+m_1+m_2) - (m_1\cos\alpha_1 + m_2\cos\alpha_2)^2} g$

$a = \frac{(M+m_1+m_2)(m_1\sin\alpha_1 - m_2\sin\alpha_2)}{(m_1+m_2)(M+m_1+m_2) - (m_1\cos\alpha_1 + m_2\cos\alpha_2)^2} g$

当斜面体静止时. $a_m = 0$ 解得 $m_1\sin\alpha_1 = m_2\sin\alpha_2$

3-15 解: 由题意知 $a_{Bx} = 0$ $a_{By} = a_c\tan\beta$ $a_{B\to c} = \frac{a_c}{\cos\beta}$

又∵绳子一直崩紧. ∴ $a_{A\to c} = a_{B\to c}$

由相对运动可知 $a_{Ax} = a_c - a_{A\to c}\cos\alpha$ $a_{Ay} = -a_{A\to c}\sin\alpha$

$F - f = m_c a_c + m_A a_{Ax} + m_B a_{Bx}$ $N - (m_A+m_B+m_c)g = m_A a_{Ay} + m_B a_{By}$

解得 $\mu = \frac{f}{N} = 0.11$

3-16 解: (1) 由题意有 $N_1\sin\theta - \mu N_1\cos\theta \le \mu N_2$ ① $N_1 = mg\cos\theta$ ②

$N_2 = N_1\cos\theta + \mu N_1\sin\theta + mg$ ③ $\tan\theta > \mu$ ④

解得 $\frac{\sqrt{8\cos^2\theta+1} - (2\cos^2\theta+1)}{\sin 2\theta} \le \mu < \tan\theta$

(2) 代入数据 解得 $\frac{1}{5} < \tan\theta \le \frac{1}{5}(12-\sqrt{69})$ 或 $\tan\theta \ge \frac{1}{5}(12+\sqrt{69})$

即 $\arctan\frac{1}{5} < \theta \le \arctan\frac{12-\sqrt{69}}{5}$ 或 $\arctan\frac{12+\sqrt{69}}{5} \le \theta \le \frac{\pi}{2}$

Mo Tu We Th Fr Sa Su

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3-17 解：由题意知 $a_m=\mu g$ ··① $a_M=\frac{F-\mu mg-\mu(M+m)g}{M}=\frac{F-\mu g(M+2m)}{M}$ ··②

$\frac{1}{2}(a_M-a_m)t^2=\ell$ ··③ 由图像可知 $2\cdot\frac{1}{2}\mu g t^2\le L-\ell$ ··④

解得 $\boxed{F\geqslant 2\mu g\left(m+\frac{ML}{L-\ell}\right)}$ 标答此处有误

3-18 解：由题意知 $2\cdot\frac{(M+m)g-Mg}{M+m+M}\cdot h=v^2$ 解得 $g=\frac{(2M+m)v^2}{2mh}$

3-19 解：设人对梯的平向作用力为 F 则 $F-Mg=Ma_1$ ··①

$F-(M+m)g=(M+m)a_2$ ··② $h=\frac{1}{2}a_1t^2$ ··③ $h'=\frac{1}{2}a_2t^2$ ··④ $d=h-h'$ ··⑤

解得 $d=\frac{m}{M+m}\left(h+\frac{1}{2}gt^2\right)$

3-20 解：(1) $2mg-mg=(2m+m)\cdot a$ ··① $v_0=at_0$ ··② $mv_0=3mv_1$ ··③

$t_1=\frac{2v_0}{g}+\frac{2v_1}{a}$ ··④ 解得 $t_1=4s$

(2) 2m第二次落地时 $v_2=\frac{v_1}{3}$ $\therefore t_2=\frac{t_1}{3}$ $t_3=\frac{t_2}{3}=\frac{t_1}{9}$

$\therefore t_总=\sum_{i=2}^{\infty}t_i=\frac{1}{1-\frac{1}{3}}\cdot\frac{4}{3}=2s$

3-21 解：由题意有 $T_2-m_3g=m_3a_3$ ··① $m_1g-T_1=m_1a_1$ ··②

$T_1-m_2g=m_2a_2$ ··③ $T_2=2T_1$ ··④ $a_1-a_3=a_2+a_3$ ··⑤

解得 $a_1=4.2m/s^2$ $a_2=a_3=1.4m/s^2$ $T_1=2.24N$ $T_2=4.48N$

3-22 解：由题意有 $T-m_2g=m_2a_2$ ··① $m_1g-2T=m_1a_1$ ··② $a_2=2a_1$ ··③

解得 $\Delta F=(m_1+m_2)g-3T=26.7N$

3-23 解：$m_Bg'-m_Ag'=(m_A+m_B)a$ ··① $m_Ag=m_Bg'$ ··②

解得 $m_B=5.745kg$

3-24 解：由题意知 $Mg+2T-\rho Vg=Ma$ ··① $T+m_1a-m_1g=m_1a'$ ··②

$m_2g-m_2a-T=m_2a'$ ··③

联立解得 $a=\frac{(m_1+m_2)(M-\rho V)+4m_1m_2}{4m_1m_2+Mm_1+Mm_2}g$

3-25 解：$\tan\theta=\frac{\Delta y}{\Delta x}$ ··① $mg\cdot\tan\theta=m\omega^2x$ ··② 解得 $y=\frac{\omega^2}{2g}x^2$

3-26 解：(1) $\tan\alpha=\frac{mg}{m\frac{v^2}{r}}$ 解得 $v=\sqrt{gr\cot\alpha}$

(2) 同(1)可得 $\omega=\sqrt{\frac{g}{r}\cot\alpha}$

(3) $N\cos\alpha-m\omega^2r-f\sin\alpha=0$ ··① $N\sin\alpha+f\cos\alpha=mg$ ··②

又 $\frac{|f|}{N}\le\mu$ ··③

联立解得 $\sqrt{\frac{\cot\alpha-\mu}{1+\mu\cot\alpha}\cdot\frac{g}{r}}\le\omega\le\sqrt{\frac{\cot\alpha+\mu}{1-\mu\cot\alpha}\cdot\frac{g}{r}}$

3-27 解: $T_1 - T_2' - m_1 g = \frac{m_1 v^2}{a}$ ..① $T_2' - \frac{m_2 v^2}{a} - m_2 g = \frac{m_2 v^2}{b}$ ..②

$T_1 = (m_1+m_2)g$ ..③ $T_2 = m_2 g$ ..④

联立解得 $\frac{\Delta T_1}{\Delta T_2} = 1 + \frac{m_1 b}{m_2(a+b)}$

3-28 解: $MV = (M+2m)v_1$ ..① $\frac{1}{2}MV^2 = \frac{1}{2}Mv_1^2 + 2\times\frac{1}{2}m(v_1^2+v_2^2)$ ..②

$2T = Ma$ ..③ $T + ma = \frac{mv_2^2}{b}$ ..④

解得 $T = \frac{mM^2V^2}{b(M+2m)^2}$

3-29 解: $Mg\cos\theta = mg$ ..① $Mg\sin\theta = m\cdot(\frac{2\pi}{T})^2\cdot l\sin\theta$ ..②

解得 $\cos\theta = \frac{m}{M}$. $T = 2\pi\sqrt{\frac{ml}{Mg}}$ $v = \sqrt{gl\sin\theta\tan\theta}$ $F = mg\tan\theta$

3-30 解: $T_1\cos\alpha = mg + T_2\cos\beta$ ..① $T_1\sin\alpha - T_2\sin\beta = m\omega^2 L\sin\alpha$ ..②

$T_2\cos\beta = mg$ ..③ $T_2\sin\beta = m\omega^2 L(\sin\alpha + \sin\beta)$ ....④

小角度近似解得 $\frac{\alpha}{\beta} = \frac{g}{2g-\omega^2 L}$ $(\beta_{1,2} = \mp\sqrt{2}\alpha)$

∴ $\omega_{1,2} = \sqrt{\frac{(2\mp\sqrt{2})g}{L}}$ $\frac{\alpha}{\beta+\alpha} = 0.414$ 或 $\frac{\alpha}{|\beta|-\alpha} = 2.414$

3-31 解: $v_1\cos[\frac{\pi}{2}-(\alpha+\beta)] = v$ ...① $\frac{m(v_1^2-v^2)}{b} = T_2 - T_1\cos(\alpha+\beta) - mg\sin\beta$ ..②

$m\frac{v_1^2}{a} = T_1 - T_2\cos(\alpha+\beta) - mg\sin\alpha$ ..③ $\frac{a}{\sin\beta} = \frac{b}{\sin\alpha}$ ..④

联立解得 $F = \frac{mg\cos\alpha}{\sin(\alpha+\beta)} + \frac{\cos(\alpha+\beta)}{\sin^4(\alpha+\beta)}\cdot[1+\frac{\sin\beta\cos(\alpha+\beta)}{\sin\alpha}]\cdot\frac{mv^2}{a}$

3-32 解: $v_1^2 = 2\cdot\mu mg\cdot\frac{V_1}{\sqrt{V_0^2+V_1^2}}\cdot\frac{l}{3}$ ..① $V_1^2 = 2\cdot\mu mg\cdot\frac{V_1}{\sqrt{V_2^2+V_1^2}}\cdot\frac{l}{2}$ ..②

解得 $V_2 = \frac{1}{2}\sqrt{9V_0^2+5V_1^2}$

3-33 解: 设杆与墙面夹角为θ时杆与墙分开.

$mg\cdot\frac{l}{2}\cdot(1-\cos\theta) = \frac{1}{2}mv^2$ ..① $mg\cos\theta = \frac{mv^2}{\frac{l}{2}}$ ..②

然后做斜抛运动 $v\cos\theta t + \frac{1}{2}gt^2 = \frac{l}{2}\cdot\cos\theta$ ..③ $v\sin\theta t + \frac{l}{2}\sin\theta = d$ ..④

联立解得 $d = 11.25$ cm

3-34 解: $F_{向} = \sum\frac{M\omega^2 l\Delta l}{d} = \frac{M\omega^2[(d+r)^2-r^2]}{2d} = mg+f$

又 $|f| \le \mu mg$ 解得 $r_{max} = \frac{mg}{M\omega^2} + \frac{\mu g}{\omega^2} - \frac{d}{2}$ $(\omega \le \sqrt{\frac{2(mg+\mu mg)}{Md}})$

$r_{min} = \max\{0, \frac{mg}{M\omega^2} - \frac{\mu g}{\omega^2} - \frac{d}{2}\}$

习题四 4-1 解: 由题意知 $I = -2mv\cdot\vec{j} + \pi mv\cdot\cot\theta\cdot\vec{i} = mv\cdot\sqrt{4+\pi^2\cot^2\theta}$

4-2 解: $\mu mg\Delta t = mv_1$ ..① $-\mu mg\Delta t - \mu(M+m)g\Delta t + I_F = Mv_2$ ..②

解得 $I = (M+2m)v_1 + Mv_2$

4-3 解：$(M+m)g - F_M - F_m = (M+m)a \cdots ①$　$(F_M - Mg)t' = Mat \cdots ②$

$(mg - F_m)t' = mv' - mat \cdots ③$　解得 $v' = \frac{M+m}{m}(t+t')\cdot a$

4-4 解：$mv_0 - mv' = 3ft \cdots ①$　$f\cdot t = (m_1+m_2)v_1 \cdots ②$

$2ft = m_2(V_2 - V_1) \cdots ③$

解得 $V_1 = \frac{m(V_0 - V')}{3(m_1+m_2)}$　$V_2 = \frac{m(2m_1+3m_2)(V_0-V')}{3m_2(m_1+m_2)}$

4-5 解：$F = \frac{\Delta m}{\Delta t}\cdot v + \frac{\Delta m}{\Delta t}\cdot\frac{l}{v}\cdot g\sin\alpha$

$= \mu v + \mu\frac{l}{v}g\sin\alpha \geq 2\mu\sqrt{gl\sin\alpha}$　$\therefore M_{min} = F_{min}R = 2\mu R\sqrt{gl\sin\alpha}$

$v = \sqrt{gl\sin\alpha}$

4-6 解：$m_B a\cos\alpha + m_A a\cdot\cos\beta = f \cdots ①$　$m_B a\sin\alpha - m_A a\sin\beta = N - (m_A+m_B+m_C)g \cdots ②$

联立 解得 $f = 8N$　$N = 103.88N$

4-7 解：$I_2 = m_1V_1 \cdots ①$　$I_1\sin\alpha = m_2V_2' \cdots ②$　$I_1\cos\alpha - I_2 = m_2V_1 \cdots ③$

$I - I_1 = m_3V_3 \cdots ④$　$V_2'\sin\alpha + V_1\cos\alpha = V_3 \cdots ⑤$

联立 解得 $V_A = \frac{m_2 I\cos\alpha}{m_2(m_1+m_2+m_3) + m_1m_3\sin^2\alpha}$　沿AB方向　标答有误

4-8 解：$N = F_t + \frac{x}{l}Mg$　其中 $F_t = \frac{v\Delta t\cdot Mv}{l}\cdot\frac{1}{\Delta t} = \frac{2Mgx}{l}$

$\therefore N = \frac{3x}{l}\cdot Mg$

4-9 解：$F = \frac{x}{l}Mg + \frac{v\Delta t}{l}\cdot Mv\cdot\frac{1}{\Delta t} = \frac{x}{l}Mg + \frac{Mv^2}{l}$

4-10 解：由巴普斯定理，得 $V = 2\pi R\cdot\pi R^2 = 2\pi^2R^3$

4-11 解：易知半圆形质心 $x_1 = \frac{4R}{3\pi}$　设该薄片质心到O点距离为x，有

$6\cdot\frac{1}{2}\pi R^2(x_1 - x) = 6\cdot\pi r^2\cdot(x - \overline{OO_1})$　解得 $x = \frac{4R^3 - 6\pi r^2\cdot\overline{OO_1}}{3\pi(R^2 - 2r^2)}$

4-12 解：利用质心不变原理，得 $(2x)^2 + y^2 = (2l)^2$　$\therefore \frac{x^2}{l^2} + \frac{y^2}{4l^2} = 1$　四分之一椭圆

4-13 解：∵总质心不变 且两质心距离不变 ∴甲虫和环心绕总质心作圆周运动

$R_甲 = \frac{M}{M+m}a$　$R_环 = \frac{m}{M+m}a$

4-14 解：(1)水平方向动量守恒知，船速与m_2的速度同向 ∴ m_2先到

(2) $m_1(u - v) - m_2(u + v) - Mv = 0$　$\therefore v = \frac{(m_1 - m_2)u}{m_1+m_2+M}$

$\therefore t = \frac{\frac{l}{2}}{u+v} = \frac{(m_1+m_2+M)l}{2(2m_1+M)u}$

4-15 解：(1)　$mv_0 = Mv + m(v+u)$　解得 $v = \frac{m(V_0 - u)}{M+m}$　标答有误

(2) $x_C = V_Ct = \frac{mV_0t}{M+m}$　$x_M' = -\frac{m}{M+m}L$　$\therefore x_M = x_C + x_M' = \frac{m(V_0t - L)}{M+m}$

4-16 解：$(M_1+m)V_1 - M_2V_2 = (M_1+M_2+m)V_3$ …①

$(M_1+m)v_1 = M_1v_3 + m(v_1+u)$ …②

解得 $u=9.9\,m/s$

4-17 解：$\frac{1}{2}MV_0^2 = Mg\cdot H_0$ …① $\frac{1}{2}MV_0^2=\frac{1}{2}MV_1^2+MgH$ …②

$MV_1=(M+m)v_2$ …③ $\frac{1}{2}(M+m)V_2^2=(M+m)gH$ …④

解得 $\frac{H}{H_0}=\frac{M^2}{M^2+(M+m)^2}$ 标答有误.

4-18 解：$F\Delta t = \pi R^2\cdot V\Delta t\cdot\rho\cdot V$ … 解得 $F=\frac{\pi D^2V^2\rho}{4}$

由牛顿第三定律，得 $F'=F=\frac{1}{4}\pi D^2V^2\rho$

4-19 解：$(M+m)v_0\cos\theta = Mv'+m(v'-u)$ …①

$\Delta x=\frac{V_0\sin\theta}{g}(v'-V_0\cos\theta)$ …②

联立 解得 $\Delta x=\frac{muv_0\sin\alpha}{g(M+m)}$

4-20 解：$(M+m)v_1 = MV_2$ ∴ $\frac{V_2}{V_1}=\frac{M+m}{M}$

4-21 解：(1) $\frac{1}{2}mv_0^2+mgh=\frac{1}{2}mv_0'^2$ …① $mv_0'=(M+m)v_1$ …②

解得 $v_1=1.33\,m/s$

(2) $a'=\mu g+\frac{\mu mg}{M}$ ∴ $t=\frac{v_0'}{a'}=0.67s$

4-22 解：(1) $(m_1+m)v=m_2(u-v)$ 解得 $v=\frac{m_2u}{m_1+m_2+m}$ ∴ $v_{弟}=\frac{m_1+m}{m_1+m_2+m}u$

(2) $mv_1=m(u-v_1)$ …① $m(u-v_1)=m_2(u-v_1)-mv_1$ …②

解得 $m_1m_2=m(m+m_1)$

习题五. 5-1. 解：$W=\Delta E_{p木}+\Delta E_{p水} = -\frac{1}{2}\rho_0\cdot a^3\cdot g\cdot\frac{3}{2}a+\rho_0\cdot\frac{1}{2}a^3(\frac{7}{4}a-\frac{1}{4}a)g+\rho_0\cdot\frac{1}{2}a^3(\frac{33}{16}a-\frac{3}{4}a)g$

$=\frac{21}{32}\rho_0a^4g$

5-2 解：$W=\frac{1}{2}mv^2+\frac{1}{2}k(x_2^2-x_1^2)=2J$

5-3 解：$W=\mu Mg\cdot\frac{L}{2}\cdot\alpha$

5-4 解：$P=Fv=\frac{\Delta m}{\Delta t}v^2=45W$ 有一半转化成沙子的动能

5-5 解：(1) $t=\frac{v}{a}=\frac{V_0}{g(\sin\alpha+\mu\cos\alpha)}$

(2) $W_f=\frac{\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}\cdot\frac{1}{2}mv_0^2=\frac{\mu mv_0^2}{2(\mu+\tan\alpha)}$

5-6 解：由题意知 $mg-\mu mg\sin\theta=N$ …① $\mu mg+\mu mg\cos\theta-\mu N=ma$ …②

解得 $a=\sqrt{1+\mu^2}\cdot\mu g\cdot\sin(\theta+\varphi)$ 其中 $\cos\varphi=\frac{\mu}{\sqrt{\mu^2+1}}$

又∵ $\frac{1}{2}mv^2 = ma\cdot S$ ∴当 $\theta=\arctan\mu$ 时，合外力做功最大

5-7 解：(1)对整体用动量守恒，有 $Mv_0=(M-m)v$，解得 $v=\frac{M}{M-m}v_0$

(2) $fx=FL$，解得 $x=\frac{M}{M-m}\cdot L$

5-8 解：达到速度 V 时，$t_1=\frac{MV}{P-f}$ $x_1=\frac{MV^2}{2(P-f)}$

功率不变后，$PV(t-t_1)=\frac{1}{2}m(v^2-V^2)+f(x-x_1)$

代入整理得 $v^2=2\frac{PVt-fx}{M}-\frac{PV^2}{P-f}$

5-9 解：作 f-x 图象可知 $h_1:h_2:h_3\cdots=1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})\cdots$

5-10 解：$x_1=\frac{1}{2}\cdot\frac{F}{2M}\cdot t^2$ ..① $V_1=\frac{Ft}{2M}$ ..②

$F(x_1+L)=\frac{1}{2}\times2\times M\cdot(V_1^2+V_2^2)$ ..③ 解得 $V_2=\sqrt{\frac{Fl}{M}}$

∴ $\Delta E=2\times\frac{1}{2}\times V_2^2=Fl$

5-11 解：惯性力 $F=(m'-m)\cdot r\omega^2$ ∴ $W_f=-W_F=-\frac{1}{2}m'(1-\frac{\rho}{\rho_1})\omega^2 l_2(d+\frac{3}{4}l_2)$

代入数据得 $W_f=-5.5\times10^{-7}J$ ∴ $W_{浮}=-W_f=5.5\times10^{-7}J$

同理解得 $W'_{浮}=3.3\times10^{-7}J$

5-12 解：$m\sqrt{2gh}=(m+M_1)v$ ..① $M_1g=kx_0$ ..② $M_2g=kx$ ..③

$\frac{1}{2}(M_1+m)v^2+\frac{1}{2}kx_0^2=(M_1+m)g(x+x_0)+\frac{1}{2}kx^2$ ..④

联立解得 $h_{min}=\frac{g}{2m^2k}(M_1+m)(M_1+M_2)(2m+M_1+M_2)$

∴ $h>h_{min}$

5-13 解：$mgH=\frac{1}{2}mv^2+\frac{1}{2}(2M+m)\cdot v'^2$ ..① $mv=(2M+m)v'$ ..②

$H'=\frac{v^2}{2g}$ ..③ 解得 $H'=\frac{2M+m}{2(M+m)}H$

5-14 解：$Mv=mv_1$ ..① $\frac{1}{2}Mv^2+\frac{1}{2}mv^2=mgH$ ..②

$\frac{1}{2}Mv_1^2=mgH'$ ..③ 解得 $H'=\left(\frac{M}{M+m}\right)^2H$

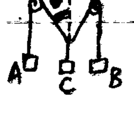
5-15 解：(1) $v_1=\frac{mv_0}{M+m}-\frac{M}{M+m}v_0=1.2m/s$ 其中 $M=8kg$ $m=2kg$

∴ $H=\frac{V_1^2}{2g}=0.072m$

(2) $v_2=\frac{mv_0}{M+m}+\frac{m}{M+m}v_0=0.8m/s$ ∴ $t=\frac{x}{v_2}=0.125s$

(3) $W=\frac{1}{2}\cdot M\cdot V_2^2=2.56J$

5-16 解：(1) $mgh=2\cdot mg\cdot(\sqrt{a^2+h^2}-a)$ 解得 $h=\frac{4}{3}a$



(2) $mga\cdot\cot\theta-2mga\cdot(\frac{1}{\sin\theta}-1)=\frac{1}{2}mv^2+\frac{1}{2}\times2\cdot mv_1^2$ ..① $v\cos\theta=v_1$ ..②

解得 $v^2=\dfrac{2ga(2\sin\theta+\cos\theta-2)}{(2\cos^2\theta+1)\sin\theta}$

利用数值逼近法得，当 $\theta=67.5^\circ$ 时，$v_m=0.6211\sqrt{ga}$

标答认为 $\theta=60^\circ$ 时 v 较大，显然错误，详见《力学方法》

5-17 解：$\frac{1}{2}mv^2=m\cdot\frac{L-a}{L}\cdot g\cdot\left(a+\frac{L-a}{2}+\frac{L-a}{2}\sin\theta\right)$

解得 $v=\left\{\frac{g}{L}\cdot\left[L^2-a^2+(L-a)^2\sin\theta\right]\right\}^{\frac{1}{2}}$ 标答有误

5-18 解：(1) $mv=Mv'$ ‥① $mgR=\frac{1}{2}mv^2+\frac{1}{2}Mv'^2$ ‥②

$N-mg=\dfrac{m(v+v')^2}{R}$ ‥③ 解得 $N=\dfrac{3M+2m}{M}mg$

(2) $W=\frac{1}{2}Mv'^2=\dfrac{m^2gR}{M+m}$

5-19 解：(1) $mv=2m\cdot v'$ ‥① $\frac{1}{2}mv^2+\frac{1}{2}\times 2m\cdot v'^2=mgR$ ‥②

解得 $v'=\sqrt{\dfrac{gR}{3}}$

(2) 对A、C有，$\frac{1}{2}m\cdot(v+v')^2=mgH'$

解得 $H'=\frac{3}{4}R$ $\therefore h'=H'+h-R=h-\frac{1}{4}R$

5-20 解：物体滑入孔内时 $\dfrac{mv_1^2}{R}\le mg\cos\theta$ ‥①

$\frac{1}{2}mv_1^2=mgR(1-\cos\theta)$ ‥②

解得 $\cos\theta\ge\frac{2}{3}$

1° 当 $\cos\theta\ge\frac{2}{3}$ 时，物体滑入孔内，$v_1=\sqrt{2gR(1-\cos\theta)}$

2° 当 $\cos\theta<\frac{2}{3}$ 时 $R\sin\theta=v_2\cos\theta t$ $R(1-\cos\theta)=v_2\sin\theta t-\frac{1}{2}gt^2$

解得 $v_2=\cos\frac{\theta}{2}\cdot\sqrt{\dfrac{gR}{\cos\theta}}$

5-21 解：$mg\sin\theta-ma\cos\theta=\dfrac{mv^2}{r}$ ‥① $mgr(1-\sin\theta)+ma\cos\theta=\frac{1}{2}mv^2$ ‥②

解得 $\theta=65.3^\circ$

5-22 解：(1) $mgR(1-\cos\theta)=\frac{1}{2}mv^2$ ‥① $\dfrac{mv^2}{R}=mg\cos\theta+N$ ‥②

解得 $N=mg(2-3\cos\theta)$

令 $N=0$ 解得 $\theta=\arccos\frac{2}{3}$

(2) $2N\cos\theta=Mg$ ‥③

联立①②③，解得 $\dfrac{M}{m}=2\cos\theta\cdot(2-3\cos\theta)<\dfrac{2}{3}$

5-23 解：$mgh=\frac{1}{2}mv^2$ ‥① $mv=mv'+mu$ ‥② $u-v'=ev$ ‥③

$\therefore h'=\dfrac{u^2}{2g}=0.32\,m$

5-24 解：$mgR\cos\theta = \frac{1}{2}mv^2 \cdots ①$ $N - mg\cos\theta = m\frac{v^2}{R} \cdots ②$

联立 解得 $\theta = 47°$

5-25 解：$mgl(\cos\alpha_0 - \cos\alpha) = \frac{1}{2}mv^2 \cdots ①$

$\frac{mv^2}{R} = mg\cos\alpha \cdots ②$ 解得 $\cos\alpha = \frac{2}{3}\cos\alpha_0$

5-26 解：$\frac{1}{2}k\cdot(2R)^2 - \frac{1}{2}k\cdot(2R\cos\theta)^2 + MgR(1-\cos 2\theta) = \frac{1}{2}Mv^2 \cdots ①$

$k\cdot 2R\cos\theta\cdot\cos\theta + Mg\cos 2\theta - N = \frac{Mv^2}{R} \cdots ②$

解得 $v = 2\sin\theta\cdot\sqrt{gR(1+\frac{kR}{Mg})}$ $N = 2kR\cos^2\theta + Mg\cos 2\theta - 4(Mg+kR)\sin^2\theta$

5-27 解：$2mgR + \frac{1}{2}k(R-d)^2 - \frac{1}{2}k(R+d)^2 = \frac{1}{2}mv^2 \cdots ①$

$k(d+R) - mg = \frac{mv^2}{R} \cdots ②$

解得 $d = \frac{mg}{k} - \frac{R}{5}$

5-28 解：(1) $mg[h - R(1+\cos\alpha)] = \frac{1}{2}mv_0^2 \cdots ①$ $\frac{v_0^2\sin 2\alpha}{g} = 2R\cos\alpha \cdots ③$

$mg\cos\alpha = \frac{mv_0^2}{R} \cdots ③$

解得 $h = R(1 + \cos\alpha + \frac{1}{2\cos\alpha})$

(2) 当 $\cos\alpha = \frac{1}{2\cos\alpha}$. 即 $\alpha = 45°$ 时. $h_{min} = R(\sqrt{2}+1)$

5-29 解：绳子松弛处.有. $mg(L - R - R\cos\theta) = \frac{1}{2}mv_1^2 \cdots ①$

$mg\cos\theta = \frac{mv_1^2}{R} \cdots ②$ $R\sin\theta = v_1\cos\theta t \cdots ③$

$-R\cos\theta = v_1\sin\theta t - \frac{1}{2}gt^2 \cdots ④$ 解得 $R = 2(2-\sqrt{3})L$

$\therefore d = L - R = (2\sqrt{3}-3)L$

5-30 解：$mgh = \frac{1}{2}mv_1^2 \cdots ①$ $mv_1 = Mv_2 - mv_1' \cdots ②$

$\frac{1}{2}Mv_2^2 = Mgh_0 + \frac{1}{2}Mv_3^2 \cdots ③$ $Mg\cdot\frac{h_0 - R}{R} = \frac{Mv_3^2}{R} \cdots ④$

解得 $h = 2(3h_0 - R)$ $\frac{m}{M} = \frac{1}{3}$

5-31 解：(1) $\frac{1}{2}Mv_0^2 = Mg\cdot 2l\sin\alpha + \frac{1}{2}Mv_1^2 \cdots ①$ $\frac{Mv_1^2}{l} = Mg\sin\alpha \cdots ②$

解得 $v_0 = \sqrt{5gl\sin\alpha}$

(2) 由(1)可知. $v_1 = \sqrt{gl\sin\alpha}$ $a = \frac{v_1^2}{l} = g\sin\alpha$

(3) $\frac{1}{2}mv_0^2 - mgl\sin\alpha(1-\cos\theta) = \frac{1}{2}mv^2 \cdots ③$

$\frac{mv^2}{l} = T - mg\sin\alpha\cos\theta \cdots ④$ 解得 $T = 3mg\sin\alpha(1+\cos\theta)$

(4) 若换杆,则要求 $v_1 = 0$ $\therefore v_0 = \sqrt{4gl\sin\alpha}$

5-32 解：$MV=(m+M)V'$ ‥① $\frac{1}{2}\mu(V-V')^2=\frac{1}{2}\mu V_1^2+\mu g\cdot 2l$

$\frac{\mu V_1^2}{l}=\mu g$ ‥③ 解得 $V=\frac{M+m}{m}\sqrt{5gl}$

5-33 解：(1) $mgl(1-\cos\theta_0)=\frac{1}{2}mv^2$ ‥① $mgl(1-\cos\theta_0)+W_1=\frac{1}{2}m(v_0-v)^2-\frac{1}{2}mv_0^2$ ‥②

解得 $W_1=-mv_0\cdot\sqrt{2gl(1-\cos\theta_0)}$

(2) $mgl(1-\cos\theta_0)=\frac{1}{2}mv^2$ ‥① $mgl(1-\cos\theta_0)+W_2=\frac{1}{2}m(v_0+v)^2-\frac{1}{2}mv_0^2$ ‥②

解得 $W_2=mv_0\cdot\sqrt{2gl(1-\cos\theta_0)}$

5-34 解：(1) $(mg\cos\alpha-ma\sin\alpha)\cdot\sin\alpha=Ma$ ‥①

$mg\sin\alpha+ma\cos\alpha=ma'$ ‥② $t_1=\sqrt{\frac{2l}{a'}}$ ‥③

解得 $t_1=\sqrt{\frac{(4M+3m)l}{\sqrt{3}(M+m)g}}$

(2) $MV_1=mV_2$ ‥④ $\frac{1}{2}MV_1^2+\frac{1}{2}mV_2^2=mgh$ ‥⑤

解得 $V'=V_1+V_2=\sqrt{\frac{\sqrt{3}(M+m)gl}{M}}$

(3) $t_2=\frac{l}{V'}=\sqrt{\frac{Ml}{\sqrt{3}(M+m)g}}$

$\therefore t=2\cdot(2t_1+t_2)=2(2\sqrt{4M+3m}+\sqrt{M})\cdot\sqrt{\frac{l}{\sqrt{3}(M+m)g}}$

5-35 解：(1) $\mu mg\cdot L=\frac{1}{2}\frac{Mm}{M+m}\cdot V_0^2$ 解得 $\mu=\frac{MV_0^2}{2(M+m)gL}$

(2) $\mu g(1+\frac{m}{M})t=V_0$ ‥① $\frac{1}{2}\cdot\mu g\cdot\frac{m}{M}\cdot t^2=x$ ‥②

解得 $x=\frac{m}{M+m}L$

5-36 解：$mv_0=mv+MV_1$ ‥① $\frac{V_0^2-V_1^2}{2(L+l)}=\frac{V_1^2}{2L}$ ‥②

解得 $L=\frac{ml(V_0-V)^2}{M(V_0^2-V^2)-m(V_0-V)^2}$

5-37 解：$\frac{1}{2}mV_0^2-\frac{1}{2}\cdot\frac{m^2}{2m}\cdot V_0^2-\mu mgl=\frac{1}{2}\cdot 3m(\frac{V_0}{3})^2$

解得 $V_0=2\sqrt{3\mu gl}$

5-38 解：(1) $MV_0=(M+2m)V_x$ ‥①

$\frac{1}{2}MV_0^2=\frac{1}{2}MV_x^2+\frac{1}{2}\cdot 2m\cdot(V_x^2+V_y^2)$ ‥②

解得 $V_M=V_x=\frac{MV_0}{M+2m}$ $V_m=\sqrt{V_x^2+V_y^2}=\frac{V_0}{2m+M}\cdot\sqrt{2M(M+m)}$

(2) $\frac{mV_x^2}{a}=T+\frac{2T}{M}\cdot m$ 解得 $T=\frac{mM^2V_0^2}{(M+2m)^2a}$ 标答有误

(3) $\frac{Mx+2m(x-a)}{M+2m}=\frac{MV_0}{M+2m}\cdot T$ 整理得 $(M+2m)x=MV_0T+2ma$

5-39 解：$mV=mV_1+m_HV_H$ ‥① $\frac{1}{2}mV^2=\frac{1}{2}mV_1^2+\frac{1}{2}m_HV_H^2$ ‥②

$mV=mV_2+m_NV_N$ ‥③ $\frac{1}{2}mV^2=\frac{1}{2}mV_2^2+\frac{1}{2}m_NV_N^2$ ‥④

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解得 $m=1.16$ 原子质量单位

5-40 解: $\gamma m_B V_A - m_B V_B = m_B v > 0$ ① $\frac{1}{2}\gamma m_B V_A^2 = \frac{1}{2} m_B V_B^2$ ··②

$\frac{1}{2}\frac{m_B \cdot \gamma m_B}{m_B+\gamma m_B}(V_A+V_B)^2 \geq \frac{1}{2}\frac{m_B\cdot\gamma m_B}{m_B+\gamma m_B}\cdot v^2$ ··③

解得 $1<\gamma\leq(\sqrt{2}+1)^2$

5-41 解: $m v_1 = m_1 v_1' + m_2 v_2'$ ··① $v_1 = v_2' - v_1'$ ··②

解得 $v_2' = \frac{2m_1 v_1}{m_1+m_2}$. 同理 $v_3' = \frac{2m_2}{m_2+m_3}\cdot\frac{2m_1 v_1}{m_1+m_2}$

易得 $v_3' = \frac{4m_1 v}{m_1+m_3+m_2+\frac{m_1m_3}{m_2}} \leq \frac{4m_1 v}{m_1+m_3+\sqrt{m_1m_3}}$ 当 $m_2=\sqrt{m_1m_3}$ 时,取等号

5-42 解: 对非弹性碰撞. 有. $E_k' = E_k\cdot\frac{M}{M+m}$

1与2碰前 $E_{k0} = mgd\sin\theta = A$ 碰后 $E_{k1} = \frac{m}{m+m}E_{k0} = \frac{A}{2}$

$E_{k1}' = E_{k1} + 2mgd - 2\mu mgd\cos\theta = \frac{5}{2}A - 2B$

$E_{k2} = \frac{2}{3}E_{k1}' = \frac{5}{3}A - \frac{4}{3}B$ $E_{k2}' = E_{k2} + 3(A-B) = \frac{14}{3}A - \frac{13}{3}B > 0$

$E_{k3} = \frac{3}{4}E_{k2}' = \frac{7}{2}A - \frac{13}{4}B$ $E_{k3}' = E_{k3} + 4(A-B) \leq 0$

解得 $\frac{10\sqrt{3}}{29}\leq\mu<\frac{14\sqrt{3}}{39}$

5-43 解: $MV_0 = (M+nm)V_m$ ··① $\frac{1}{2}(M+nm)V_m^2 = \frac{1}{2}kl_n^2$ ·② $\frac{1}{2}mV_0^2 = \frac{1}{2}kl_0^2$ ·③

解得 $l_n = l_0\cdot\sqrt{\frac{M}{M+nm}}$

5-44 解: (1) 由能量守恒. 易得 $x_1=0.5m$ $x_2=0$ $x_3=0.42m$ $x_4=0$ $x_5=0.37m$

$x_6=0$ $\Delta t_1=0.05\pi$ s $\Delta t_2=0$ $\Delta t_3=0.06\pi$ s $\Delta t_4=0$ $\Delta t_5=0.067\pi$ s $\Delta t_6=0$

(2) $\frac{1}{2}mV_0^2 - \frac{1}{2}\cdot\frac{m[(n-1)m+M]}{M+nm}v_0^2 = \frac{1}{2}kx^2$ 解得 $n=17$

5-45 解: (1) 在C系中看. $a_1=1.5\mu g$ $V_0^2-2aL\geq0$ 解得 $V_0\geq\sqrt{3\mu gL}$

(2) 在C系中看. A与B碰后交换速度. $V_0^2-2a_1\cdot2L\geq0$ 解得 $V_0\geq\sqrt{6\mu gL}$

(3) C系中 A与B速度相同 ∴ 不能碰上

(4) C系中看. $a_2=3\mu g$ $V_0^2-2a_1\cdot2L-2a_2L\geq0$. 解得 $V_0\geq\sqrt{12\mu gL}$

(5) C系中看 $a_3=2\mu g$ $V_0^2-2a_1\cdot2L-2a_2L-2a_3L\geq0$ 解得 $V_0\geq\sqrt{16\mu gL}$

5-46 解: $\frac{1}{2}mV_0^2 = \frac{1}{2}mV^2 + \frac{1}{2}mu^2$ ··① $mV_0 = mV\cos60^\circ + mu\cos\theta$ ··②

$mV\sin60^\circ = mu\sin\theta$ ··③

解得 $v=\frac{1}{2}V_0=1.50m/s$ $u=\frac{\sqrt{3}}{2}V_0=2.60m/s$ $\theta=30^\circ$

5-47 解: $\frac{1}{2}m_1v_0^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$ ··① $(m_1v_0)^2 + (m_2V_2)^2 - (m_1V_1)^2 = 2m_1V_0\cdot m_2V_2\cdot\cos\theta_2$

解得 $v_1 = \frac{v_0}{m_1+m_2}\cdot\sqrt{m_1^2+m_2^2-2m_1m_2\cos 2\theta_2}$

$v_2 = \frac{2m_1v_0\cos\theta_2}{m_1+m_2}$

5-48 (1) $T_1+T_2+T_3=Q$ ‥ ①

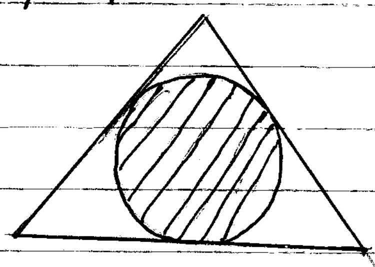
$p_3^2 = p_2^2+p_1^2+2p_1p_2\cos\theta$ ‥ ②　　$T=\frac{p^2}{2m}$ ‥ ③

解得 $(T_3-T_1-T_2)^2 \le 4T_1T_2$

又∵ $T_3=\frac{Q}{3}+\rho\cos\varphi$　　$T_2=\frac{Q}{3}-\rho\cos(\varphi+60^\circ)$

$T_1=\frac{Q}{3}-\rho\cos(\varphi-60^\circ)$

代入解得 $\rho\le\frac{Q}{3}$　如图



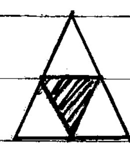
(2) $T=c\cdot p$

得 $\left|\frac{T_3^2-T_1^2-T_2^2}{2T_1T_2}\right|\le 1$

解得 $T_3\le(T_1+T_2)$　即 $2T_3\le T_1+T_2+T_3=Q$　∴ $T_3\le\frac{Q}{2}$

同理 $T_1\le\frac{Q}{2}$　$T_2\le\frac{Q}{2}$

如图



5-49 解：$2mgR\left(\frac{\sqrt{6}}{3}-\sin\theta\right)=\frac{1}{2}mv_1^2+\frac{3}{2}mv_2^2$ ‥ ①　　$v_2=v_1\tan\theta$ ‥ ②

$\frac{m(v_1^2+v_2^2)}{2R}=mg\sin\theta$ ‥ ③　$\frac{1}{2}mv_1^2+2mgR\sin\theta=\frac{1}{2}mv^2$ ‥ ④

解得 $v=1.621377951\sqrt{gR}$

注：此处需解三次方程，若滑板上方小球质量改为3m，则整体代换可解得标答。

习题六. 6-1 解：$\frac{G\cdot\lambda_1\cdot 2\pi R\cdot m}{R^2}=\frac{G\lambda_2\cdot 2\pi r m}{r^2}$　解得 $\frac{\lambda_1}{\lambda_2}=\frac{R}{r}$

6-2 解：$\frac{Gm_1m_2}{r^2}=m_1\cdot\left(\frac{2\pi}{T}\right)^2\cdot\frac{m_2}{M}\cdot r$　解得 $r=\left(\frac{GM}{4\pi^2}\cdot T^2\right)^{\frac{1}{3}}$

6-3 解：$\frac{E_{k2}}{E_{k1}}=\frac{\frac{1}{2}m(u+v)^2}{\frac{1}{2}m(u-v)^2}=\frac{(\sqrt{gR_e}+\frac{2\pi R_e}{T})^2}{(\sqrt{gR_e}-\frac{2\pi R_e}{T})^2}=1.27$

6-4 解：(1) $\frac{G\cdot\frac{4}{3}\pi R^3\cdot\rho m}{R^2}=m\cdot\left(\frac{2\pi}{T}\right)^2\cdot R$　解得 $T=\sqrt{\frac{3\pi}{\rho G}}$　仅与密度有关

∴月球的平均密度与地球相同

(2) $T_{min}=\sqrt{\frac{3\pi}{\rho G}}=1.19\times10^{-3}$ s

6-5 解：$\frac{GMm}{(ct)^2}=m\left(\frac{2\pi}{T}\right)^2\cdot(ct)$ ‥ ①　$M=\frac{gR_e^2}{G}$ ‥ ②

解得 $\frac{M}{m}=3\times10^5$

6-6 解：相同。忽略地球上各点到太阳距离之差，太阳对物体的引力与物体的惯性离心力相抵消。

6-7 解：(1) $\frac{GM_em}{R^2}=m\frac{v^2}{R}=mg$ $\therefore T=\frac{2\pi R}{v}=2\pi\sqrt{\frac{R}{g}}=84\ \text{min}$

(2) $ma=-G\cdot\frac{\frac{4}{3}\pi\rho x^3\cdot m}{x^2}=-\frac{4}{3}\pi\rho G\cdot mx$ $\therefore T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{R}{g}}$

(3) $T=2\pi\sqrt{\frac{R}{g}}$

(4) $f=-G\frac{Mm}{R^2}\cdot\frac{x}{R}$ $\therefore \omega=\sqrt{\frac{g}{R}}$ $\therefore T=2\pi\sqrt{\frac{R}{g}}$

6-8 解：$\frac{1}{2}mv^2-\frac{GMm}{R}=-\frac{GMm}{2R}$ …① $\frac{1}{2}mv_1^2-\frac{GMm}{a}=\frac{1}{2}mv_2^2-\frac{GMm}{b}$ …②

$v_1a=v_2b$ …③ 解得 $a=3.0\times10^8\ \text{km}$

6-9 解：$\frac{GM_sm}{r^2}=m(\frac{2\pi}{T_0})^2\cdot r$ …① $\frac{GM_sM}{R^2}=M_s(\frac{2\pi}{T})^2R$ …②

解得 $M=1.533\times10^{11}M_s$

6-10 解：$g=G\cdot\frac{\frac{4}{3}\pi R_e^3\cdot\rho_e}{R_e^2}$ …① $\Delta g=G\cdot\frac{\frac{4}{3}\pi R^3\cdot 2\rho_e}{h^2}$ …②

$\Delta T=T\cdot\frac{\Delta g}{2g}$ …③ 解得 $\frac{\Delta T}{T}=1\times10^{-6}$

6-11 解：$R\sin\alpha\cdot v_0=(R+h)v$ …① $\frac{1}{2}mv_0^2-\frac{GMm}{R}=\frac{1}{2}mv^2-\frac{GMm}{R+h}$ …②

解得 $h=R\cos\alpha$

6-12 解：$\frac{1}{2}mc^2-G\frac{Mm}{r_0}<0$ …① $m\frac{v^2}{r}=G\frac{Mm}{r^2}$ …②

解得 $r<5.3\times10^5\ \text{km}$

6-13 解：(1)由题意，有 $\frac{GM^2}{L^2}=M(\frac{2\pi}{T_{计算}})^2\cdot\frac{L}{2}$ 解得 $T_{计算}=\sqrt{\frac{2\pi^2L^3}{GM}}$

(2) 设该暗物质的密度为ρ，则有

$\frac{GM'M}{(\frac{L}{2})^2}+\frac{GM^2}{L^2}=M(\frac{2\pi}{T_{观测}})^2\cdot\frac{L}{2}$ 且 $M'=\rho\cdot\frac{4}{3}\pi(\frac{L}{2})^3$，$T_{观测}=\frac{T_{计算}}{\sqrt{N}}$

解得 $\rho=\frac{3M}{2\pi L^3}(N-1)$

6-14 解：$\frac{GM_em}{R_{月地}^2}=m(\frac{2\pi}{T})^2\cdot R_{月地}$ 解得 $R_{月地}=\sqrt[3]{\frac{GM_eT^2}{4\pi^2}}$ 又$\because gR^2=GM_e$

代入数据，得

距今(10^8年)	0.29	1	1.8	3.2	4.7
月地距离(10^8m)	3.72	3.32	2.91	2.58	1.83

作图（图像略）由图可知，月亮大致以 $v=0.62$ m/年的速度远离我们

6-15 解：(1) $\frac{Gm^2}{L^2}=m\cdot(\frac{2\pi}{T})^2\cdot\frac{L}{2}$ 解得 $T=\pi L\sqrt{\frac{2L}{Gm}}$ $v=\frac{2\pi\cdot\frac{L}{2}}{T}=\sqrt{\frac{Gm}{2L}}$

(2) 设两星最近距离为2a 则有

$\frac{v_0}{2}\cdot\frac{L}{2}=v_m\cdot a$ ..① $2\times\frac{1}{2}m(\frac{v_0}{2})^2-\frac{Gm^2}{L}=2\times\frac{1}{2}\times mv_m^2-\frac{Gm^2}{2a}$ ..②

解得 $a=\frac{L}{14}$ $\therefore a'=(a+\frac{L}{2})\times\frac{1}{2}=\frac{2}{7}L$

$\frac{T'}{T}=(\frac{a'}{\frac{L}{2}})^{\frac{3}{2}}=(\frac{4}{7})^{\frac{3}{2}}$

6-16 解：$v_{min}\cdot a=v_{max}\cdot b$ ..① $\frac{1}{2}mv_{max}^2-\frac{1}{2}mv_{min}^2=Mg(a-b)$ ..②

解得 $\frac{1}{2}mv_{max}^2=\frac{Mga^2}{a+b}$, $\frac{1}{2}mv_{min}^2=\frac{Mgb^2}{a+b}$

6-17 解：$mva=mv_{\perp 1}\cdot r_1$ ..① $mv\cdot 3a=mv_{\perp 2}\cdot(4a-r_1)$ ..②

$2\times\frac{1}{2}mv^2=\frac{1}{2}m(v_{//}^2+v_{\perp 1}^2)+\frac{1}{2}m(v_{//}^2+v_{\perp 2}^2)$ ..③

令 $v_{//}=0$. 得 $r=1.63a$

6-18 解：整体动能定理. 有 $Fs+mgl(1-\sin\theta)=\frac{1}{2}\times 2m(v_{c\perp}^2+v_{c//}^2)+\frac{1}{2}\cdot\frac{1}{2}ml^2w^2$ ..①

对质心. 有 $Ft=2mv_{//}$ ..② $\frac{1}{2}\frac{F}{2m}\cdot t^2=s-\frac{l}{2}\cos\theta$ ..③

对B. 有 $F-T\cos\theta=ma$ ..④

在B系看A. 有. $T+mg\sin\theta-ma\cos\theta=m\omega^2 l$ ..⑤

B的合速度水平. 有. $v_\perp=\frac{1}{2}wl\cos\theta$ ..⑥

又. $N=mg-T\sin\theta$ ..⑦

联立①-⑦. 解得 $N=mg-\frac{mg(4\sin\theta-6\sin^2\theta+\sin^4\theta)+F(3\cos\theta\sin\theta+\cos^3\theta\sin\theta)}{(1+\cos^2\theta)^2}$

标答似有误

习题七 7-1 解：以A为转轴. 有. $M_1gL\sin\theta=M_2g(R-L\sin\theta)$ 解得 $\sin\theta=\frac{M_2R}{(M_1+M_2)L}$

7-2 解：以O点为转轴. $\frac{1}{2}mgl=Fl$ 解得 $F=\frac{1}{2}mg$. 方向如图.

7-3 解：$2N\cos\beta=mg$ ..① $\frac{mg}{\sin(\beta-\alpha)}=\frac{N}{\sin\alpha}$ ..② 解得 $\tan\beta=3\tan\alpha$

7-4 解：如图所示. $x_1=\frac{\sqrt{3}}{2}a-\frac{1}{3}\cdot\frac{\sqrt{3}}{2}a=\frac{\sqrt{3}}{3}a$ $x_2=\frac{\sqrt{3}}{6}a$ .. $x_n=\frac{\sqrt{3}}{3n}a$

7-5 解：(1) $2N\cos\theta=Q$ ..① $T=N\sin\theta$ ..② $\sin\theta=\frac{r}{R+r}$ ..③

解得 $T=\frac{Qr}{2\sqrt{R^2+2Rr}}$

(2) $2N'=2P+Q$ 解得 $N'=P+\frac{Q}{2}$

(3) $N=\frac{Q}{2\cos\theta}=\frac{Q(R+r)}{2\sqrt{R^2+2Rr}}$

7-6 解：$2N=2Q$ ..① $N-\frac{Q}{2}=\mu N\cdot\tan\alpha$ ..②

$\cos\alpha=\frac{b}{4r}$ ..③

解得 $b=2\sqrt{2}\cdot r$

7-7 解: $P\cdot\frac{a}{2}\cot\alpha = N\cdot r\cdot\cot\alpha$ ..① $2f\cos\alpha+2N\sin\alpha=Q$ ..②

$\because |f|\le\mu N$ $\therefore P_{min}=\frac{Qr}{a(\sin\alpha+\mu\cos\alpha)}$ $P_{max}=\frac{Qr}{a(\sin\alpha-\mu\cos\alpha)}$

1° 当 $\tan\alpha>\mu$ 则 $\frac{Qr}{a(\sin\alpha+\mu\cos\alpha)}\le P\le\frac{Qr}{a(\sin\alpha-\mu\cos\alpha)}$

2° 当 $\tan\alpha\le\mu$ 则 $P\ge\frac{Qr}{a(\sin\alpha+\mu\cos\alpha)}$

7-8 解: 开始木条与A不动. B向左移动 (x表示相对滑动的木棒与棒重心距离)

临界时. 有. $N_A\cdot\frac{L}{2}=N_B\cdot x$ ..① $\mu_0\cdot N_A=\mu N_B$ ..②

解得 $x=\frac{L}{4}=0.16m$ 然后木条与B一起向左运动. 同理类推得

① B向左. 至 $x_1=0.16m$ 处 ② 一起向左. 至 $x_2=0.08m$ 处 ③ B向左. 至 $x_3=0.04m$ 处

④ 一起向左. 至 $x_4=0.02m$ 处 ⑤ B向左. 至 $x_5=0.01m$ 处 ⑥ 一起向左. 至 $x_6=0.01m$ 处.

此时 A与B两木棒接触. 系统停止运动

7-9 解: 由图. 可知. $3G\cdot(r+2r\cos30^\circ)+2G\cdot\frac{l}{2}\cos30^\circ+Nr\cot30^\circ=Tl\sin30^\circ$ ..①

$N=3G\tan30^\circ$ ..② 联立解得 $T=\frac{r}{l}\cdot(6+8\sqrt{3})G$

7-10 解: 由摩擦角可知 $\theta=15\sim30^\circ$ 时 木棍在木板上滑动. 在墙上无滑动滚动

$\theta=30^\circ-60^\circ$ 时 木棍在木板上无滑动滚动. 在墙上滑动

由图可知. $h_1=r\cdot\cot(\frac{15^\circ}{2})$ $h_2=r\cdot\cot(\frac{30^\circ}{2})$ $h_3=r\cot(\frac{60^\circ}{2})$

$\therefore \varphi_1=\frac{h_1-h_2}{r}$ $\varphi_2=\frac{h_2-h_3}{r}$ 本身转动 $\varphi_3=60^\circ-30^\circ=30^\circ$

$\therefore \varphi=\varphi_1+(\varphi_2)+\varphi_3=135^\circ$. 顺时针转动

7-11 解: $N_左 l\cdot\sin\alpha_1=P_1\cdot\frac{l}{2}\cos\alpha_1$ ..① $N_右 l\cdot\sin\alpha_2=P_2\cdot\frac{l}{2}\cos\alpha_2$ ..②

又 $\because N_左=N_右$ ..③ 解得 $\frac{P_1}{\tan\alpha_1}=\frac{P_2}{\tan\alpha_2}$

7-12 解: $N_A=T_B\cos30^\circ$ ..① $T_A=T_B\cos60^\circ$ ..② $N_B=Mg$ ..③

以CE为轴 $\frac{1}{2}Mgl\cos60^\circ\cdot\cos30^\circ+N_A l\sin60^\circ=N_B l\cos60^\circ\cdot\cos30^\circ$ ..④

解得 $T_A=11.5N$ $T_B=23.1N$ $N_A=20N$ $N_B=80N$

7-13 解: $T\sin\varphi+N=mg$ ..① $T\cos\varphi=\mu N$ ..②

$\frac{1}{2}mgl\cos\alpha=Nl\cos\alpha+\mu Nl\sin\alpha$ ..③ 解得 $\varphi=\arctan(2+\frac{1}{\mu})$

7-14 解: $N=\frac{mg}{\sin\frac{\varphi}{2}}$ ..① $mgl\sin\frac{\varphi}{2}=\frac{mg}{\sin\frac{\varphi}{2}}\cdot\frac{r}{\tan\frac{\varphi}{2}}$ ..②

整理得 $l\cdot\sin^3\frac{\varphi}{2}-r\cos\frac{\varphi}{2}=0$

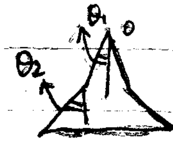
7-15 解: (1) 由图可知. $\theta=\theta-\alpha=\frac{\pi}{2}-2\alpha$ $\overline{AO}=2l\cdot\sin\alpha$

(2) 作图 可知 $\alpha-2\varphi \le \theta \le \alpha+2\varphi$

7-16 解：如图.

$E_p = -\left[2mg\cdot\frac{l}{2}\cos\theta_1 + 2mg\left(l\cos\theta_1+\frac{l}{2}\cos\theta_2\right) + mgl(\cos\theta_1+\cos\theta_2)\right]$

$= -2mgl(2\cos\theta_1+\cos\theta_2)$



又∵ $\sin\theta_1+\sin\theta_2=\frac{1}{2}$

令E_p取极值 得 $\theta_1=9.87^\circ$ $\theta_2=19.18^\circ$

上方顶角19.8°. ~~侧~~下方顶角70.8° 侧方顶角189.3°. 棱形成

~~标答有误~~

7-17 解：$N_{AH}=T+\mu N$ ..① $N=\frac{1}{2}P$ ..②

$(N_{AH}+\mu N)\frac{1}{2}l\cos\alpha=(N+\frac{P}{2})\cdot\frac{l}{2}\sin\alpha$ ..③

解得 $2\alpha=76^\circ50'$

7-18 解：(1) $N=T\cos\theta+mg$ ..① $F=T\sin\theta+f$ ..②

$F(L-h)=fL$ ..③ 解得 $F=383N$

(2) 解①②③得 $F=\frac{mgL\tan\theta}{(L-h)\tan/\mu-h}$

当 $F\le0$. 即 $h\ge0.66L$ 时 杆无论如何不会滑倒

∵ $h=0.8L\ge0.66L$ ∴ 杆不滑倒

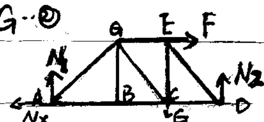
7-19 解：$\frac{1}{2}mgl\sin\alpha=(F_x\cos\alpha+F_y\sin\alpha)l$ ..①

$F_x=\mu N$ ..② $F_y+mg=N$ ..③

$(F_x+\mu N)\frac{l}{2}\cdot\sin\beta=(N+F_y)\cdot\frac{l}{2}\cos\beta$ ..④

联立 解得 $\mu=\frac{2}{\cot\alpha+3\cot\beta}$

7-20 解：$N_1 2l+Fl'=N_2\cdot l$ ..① $N_1+N_2=G$ ..②



解得 $N_1=300N$ $N_2=900N$

对整体 有 $N_x=F=400N$ 取压力为正 则 $F_{AG}=500N$ $F_{AB}=-800N$

$F_{DE}=1500N$ $F_{CD}=-1200N$ $F_{EG}=800N$

$F_{CE}=-900N$ $F_{CG}=-500N$ $F_{BC}=-800N$

$F_{BG}=0$

7-21 解：$\mu\ge\tan\theta$ ..① $x=\frac{p_1h+pl}{p+p_1}$ ..②

$\frac{(2l-x)\sin\varphi}{\sin(\frac{\pi}{2}-\varphi-\theta)} = \frac{x}{\sin\theta}$ ··③ 解得 $\mu \geq \frac{p_1h+pl}{2l(p+p_1)}\cdot\cot\varphi$

7-22 解: $\rho_0\cdot Sxg\cdot(l-\frac{x}{2})\sin\theta = \rho Slg\cdot\frac{l}{2}\sin\theta$

解得 $x = l\cdot(1-\sqrt{1-\frac{\rho}{\rho_0}})$

7-23 解: (1) 易知 $\theta=45^\circ$ ∴ $K\cdot 2\pi(\sqrt{2}-1)\frac{R}{2}\Delta\theta = \frac{\sqrt{2}R\cdot\Delta\theta}{2}\cdot\frac{Mg}{2\pi\cdot\sqrt{2}\frac{R}{2}}$

解得 $K = \frac{(\sqrt{2}+1)Mg}{2\pi^2 R}$

(2) 设所在处圆半径为r. 则有

$K\cdot 2\pi(r-\frac{R}{2})\Delta\theta = \frac{r\Delta\theta}{2\pi r}\cdot Mg\frac{r}{\sqrt{R^2-r^2}}$ 该方程无解

∴物体不可能静止在球上 ∴最后停留在桌面上 长度为 πR

7-24 解: $0.9\rho_0\cdot Vg = \rho_0 V_排 g - F$ ① $\Delta h = \left|\frac{\frac{9}{10}\rho_0 V - \rho_0 V_排}{\rho_0 S}\right|$ ··②

解得 $\Delta h = 0.1m$

7-25 解: $2\pi R\cdot(R-2h) = \frac{4}{5}\pi R^2$ ··① $\rho_0\cdot\frac{\pi h^2}{3}(3R-h) = \rho'\cdot\frac{4}{3}\pi R^3$ ··②

解得 $\rho' = \frac{27}{32}\rho_0 = 844\,kg/m^3$

7-26 解: $a^2(p_0+\rho_0 gh) = 7a^3\rho_0 g$ 解得 $h=2m$

∴上表面距水面2m时立方体脱离 不能

7-27 解: (1). $F=\rho g\cdot\pi R^3$ ··① $G=\rho_1\cdot\frac{2}{3}\pi R^3\cdot g$ ··② $m=\rho_0\cdot\frac{1}{3}\pi R^3$

$F = G+N$ ··③ 又 $N = mg+\rho g\cdot(h-R)\cdot\pi R^2$ ··④

解得 $h = R(1+\frac{\rho_1}{3\rho}-\frac{\rho_0}{3\rho})$

(2) $N' = mg = \frac{1}{3}\pi R^3\cdot\rho_0 g$ ··⑤

$N' + [\frac{2\pi R^3}{3} - \frac{2}{3}\pi R^3 + \pi R^2\cdot h_1 + \frac{1}{3}\pi h_1^3)\rho_1 g = \pi R^2\cdot\rho h_1 g$ ··⑥

解得 $h_1 = \sqrt[3]{\frac{\rho_0}{\rho_1}}\cdot R$

7-28 解: $T\cos\theta = Mg$ ·① $T\sin\theta = N$ ··② $Mgl\cos\theta = T\cdot\frac{s}{\cos\theta}$ ··③

解得 $\theta = \arccos(\frac{s}{l})^{\frac{1}{3}}$

7-29 解: $T\sin\alpha = N$ ··① $T\cos\alpha = mg$ ··② $N\cdot 2a\cos\alpha = mg\frac{\sqrt{2}}{2}a\cdot\sin(\alpha+\frac{\pi}{4})$ ··③

解得 $T = \frac{\sqrt{10}}{3}mg$

习题八 8-1 解: (1) $\frac{1}{n} = 2\pi\sqrt{\frac{m_2}{k}}$ ··① $\frac{1}{n'} = 2\pi\sqrt{\frac{m_1}{K}}$ ··②

解得 $n' = n\cdot\sqrt{\frac{m_2}{m_1}}$

(2) $\frac{1}{n''} = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1+m_2)}}$ 解得 $n'' = n\cdot\sqrt{\frac{m_1+m_2}{m_1}}$

8-2 解：$t_1=\frac{L}{2v}$ $t_2=\pi\sqrt{\frac{m}{2k}}$

$\therefore t=2t_1+t_2=\frac{L}{v}+\pi\sqrt{\frac{m}{2k}}$

8-3 解：$F=-(k\Delta x+\rho\cdot\pi r^2\Delta x\cdot g)=-(k+\rho\pi r^2 g)\Delta x$

∴物体作简谐运动 $T=2\pi\sqrt{\frac{m}{k+\rho\pi r^2 g}}$

8-4 解：$2k\Delta x-\Delta T=m_1\cdot 2a$ ··① $2\Delta T=m_2 a$ ··②

解得 $4k\cdot\Delta x=(4m_1+m_2)a$ $\therefore T=2\pi\sqrt{\frac{4m_1+m_2}{4k}}$

8-5 解：$Mg+2T-kx=Ma$ ··① $m_1g-T=m_1a_1$ ··②

$m_2g-T=m_2\cdot(2a-a_1)$ ··③

整理得 $(M+\frac{4m_1m_2}{m_1+m_2})a+kx=[(M+m_1+m_2)-\frac{(m_1-m_2)^2}{m_1+m_2}]g$

$\therefore T=2\pi\sqrt{\frac{M(m_1+m_2)+4m_1m_2}{k(m_1+m_2)}}$

8-6 解：$mg=kd$ $\therefore T=2\pi\sqrt{\frac{d}{g}}$

又∵ $\frac{1}{2}k(c+d)^2=mg(c+d)+\frac{1}{2}mv_0^2$

解得 $v_0=\sqrt{\frac{g}{d}(c^2-d^2)}$

$\therefore t=\frac{\pi}{2}\sqrt{\frac{d}{g}}+(\arcsin\frac{d}{c})\sqrt{\frac{d}{g}}+\sqrt{\frac{c^2-d^2}{dg}}$

8-7 解：$F_1=k_1\Delta x_1$ ··① $F_2=k_2\Delta x_2$ ··② $F_1a=F_2\cdot l$ ··③

$\Delta x=\Delta x_1+\frac{a}{L}\cdot\Delta x_2$ ··④ 解得 $F=\frac{k_1k_2l^2}{k_1a^2+k_2l^2}\Delta x$

$\therefore T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m(k_1a^2+k_2l^2)}{k_1k_2l^2}}$

8-8 解：$mg\sin\theta=\mu mg\cos\theta+kx$ ··①

$mg\sin\theta\cdot x-\mu mg\cos\theta\cdot x-\frac{1}{2}kx^2=\frac{1}{2}mv^2$ ··②

解得 $v_m=0.24\,m/s$

8-9 解：(1) $T=2\pi\sqrt{\frac{\sqrt{3}a}{2g}}$

(2) $T=2\pi\cdot\sqrt{\frac{\sqrt{3}a}{2g\cos\varphi}}$ 找等效摆长、等效重力加速度

8-10 解：该本题可看成 x、y 两方向简谐运动的叠加。

(1) $t=\frac{1}{4}T=\frac{\pi}{2}\sqrt{\frac{m}{k}}$

(2) $\frac{1}{2}mv_0^2+\frac{1}{2}kl^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2$ ·①

$v_0 l=vx$ ··②

解得 $v=\sqrt{\frac{k}{m}}\cdot l$ $x=\sqrt{\frac{m}{k}}\cdot v_0$

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8-11 解：$E = mgl\cdot(1-\cos\theta) + \frac{1}{2}\mu\cdot l^2\cdot(\frac{\Delta\theta}{\Delta t})^2$

$= \frac{1}{2}mgl\cdot\theta^2 + \frac{1}{2}\frac{mM}{M+m}\cdot l^2\cdot(\frac{\Delta\theta}{\Delta t})^2$

$\therefore T = 2\pi\sqrt{\frac{Ml}{(M+m)g}}$

8-12 解：物体回到原位置. $t = nT = n\cdot 2\pi\sqrt{\frac{m}{k}}$

8-13 解：$\frac{1}{2}MV_0^2 + \frac{1}{2}kl^2 = \frac{1}{2}kA^2 + \mu mg(l+A)$ …①

$\frac{1}{2}MV_0^2 = 2\mu Mg(l+A)$ …②

解得 $V_0 = \sqrt{8\mu g(l+\frac{\mu mg}{k})}$

8-14 解：$\frac{1}{2}kx_0^2 - \frac{1}{2}kx_1^2 = \mu mg(x_0+x_1)$. 解得 $x_0 = x_1 + \frac{2\mu mg}{k}$

同理 $x_1 = x_2 + \frac{2\mu mg}{k}$ …… $x_{n-1} = \frac{2\mu mg}{k}$

$\therefore x_0 = \frac{2n\cdot\mu mg}{k}$ $\therefore \mu = \frac{kx_0}{2nmg}$

8-15 解：$\rho Vg = \rho_0\cdot V\cdot[1-(\frac{h}{H})^3]g$ 解得 $H = \frac{h}{\sqrt[3]{1-\frac{\rho}{\rho_0}}}$

$F = -\frac{3h^2}{H^3}\rho Vg\cdot\Delta h$ $\therefore T = 2\pi\sqrt{\frac{\rho Vg}{\frac{3h^2}{H^3}\rho Vg}} = 2\pi\sqrt{\frac{\rho_0 h}{3(\rho_0-\rho)g}}$ 未给有误

8-16 解：(1) $\frac{1}{2}g\cdot(\frac{T}{4})^2 = \frac{\sqrt{2gh}}{\omega}$. 其中 $T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{2k}}$

解得 $k = \frac{\pi^4\cdot mg}{256h}$

(2) $V_A^2 = \frac{\sqrt{2gh}}{\omega}\cdot 2g$ 解得 $V_A = \frac{4}{\pi}\sqrt{2gh}$

8-17 解：取盒中点为坐标原点. 向右为正方向. 设平衡位置为 $-x_0$

有. $\frac{mF}{M+m} = 2kx_0$. 解得 $x_0 = \frac{mF}{2k(M+m)}$

设物体相对平衡位置移动 Δx

$F = -2k\cdot(1+\frac{m}{M})\cdot\Delta x$ $\therefore \omega^2 = \frac{2k(M+m)}{Mm}$

$\therefore x = -\frac{mF}{2k(M+m)} + \frac{mF}{2k(M+m)}\cos\sqrt{\frac{2k(M+m)}{Mm}}t$

8-18 解：$N_1(l+\Delta l) = N_2(l-\Delta l)$ …① $N_1+N_2 = P$ ②

解得 $N_1 = \frac{P}{2}(1-\frac{\Delta l}{l})$ $N_2 = \frac{P}{2}(1+\frac{\Delta l}{l})$

$\therefore F = \mu\cdot\Delta N = -\frac{\mu\cdot P}{l}\cdot\Delta l$ $\therefore T = 2\pi\sqrt{\frac{l}{\mu g}}$

8-19 解：(1) 等效成长为 l_0 的杆.

$mgx(\cos\theta' - \cos\theta) + mgL(\cos\theta' - \cos\theta) = \frac{1}{2}m\omega^2(x^2+L^2)$ …①

$mgl_0(\cos\theta' - \cos\theta) = \frac{1}{2}m\omega^2 l_0^2$ …②

解得 $l_0 = \frac{x^2+L^2}{x+L}$ $\therefore T = 2\pi\sqrt{\frac{l_0}{g}} = 2\pi\sqrt{\frac{x^2+L^2}{(x+L)g}}$

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(2) $\frac{l^2+x^2}{x+l}=\frac{l^2+2lx+x^2}{x+l}-\frac{2xl+2l^2}{x+l}+\frac{2l^2}{x+l}$

$=x+l-2l+\frac{2l^2}{x+l}\geq 2\sqrt{2l^2}-2l$

当且仅当 $x=(\sqrt{2}-1)l$ 时等号成立

$\therefore$ 当 $x=(\sqrt{2}-1)l$ 时，系统周期最小

8-20. 解：(1) $2ka=\mu mg$ $\therefore a=\frac{\mu mg}{2k}$

若 $|x_n|\leq a$，一旦静止就不会再运动

$x_1=-(x_0-2a)$ $x_2=-(x_1-2a)=x_0-4a$ $\cdots$ $x_n=(-1)^n(x_0-2na)$

令 $-a\leq x_n\leq a$ 且 $|x_{n-1}|>a$ 解得 $\frac{x_0-a}{2a}\leq N<\frac{x_0+a}{2a}$

$\therefore N=\begin{cases}\frac{x_0-a}{2a} & \frac{x_0+a}{2a}\text{ 为整数}\\ \left[\frac{x_0+a}{2a}\right] & \frac{x_0+a}{2a}\text{ 不为整数}\end{cases}$

(2) $t=N\cdot\frac{T}{2}=N\pi\sqrt{\frac{m}{k}}$

(3) $x_N=(-1)^N\cdot(x_0-2Na)$

(4) $W=\frac{1}{2}\cdot 2k\cdot(x_0^2-x_N^2)=4kNa(x_0-Na)$

8-21 解：$mg+k_2l_2=k_1l_1$，解得 $m=0.028\ kg$

1° 最高点处上绳未恢复原长，有

$\frac{1}{2}k(l_1+l_3)^2=mgh+\frac{1}{2}k(l_1+l_3-h)^2+\frac{1}{2}k_2(h-l_3+l_2)^2$

解得 $h=0.150m$

$\omega_1=\sqrt{\frac{k}{m}}=\frac{20\sqrt{35}}{7}rad/s$ $l_0=\frac{mg}{k_1}=0.035m$

$A=\overline{GD}=l_1+l_3-l_0=0.125m$

$\therefore t_1=\frac{1}{\omega_1}\arccos\frac{\overline{GD_2}}{A}=0.055s$ $\omega_2=\sqrt{\frac{k_1+k_2}{m}}=\frac{60\sqrt{14}}{7}rad/s$

$A'=\overline{HC}=h-l_3=0.070m$ $t_2=\frac{1}{\omega_2}\arcsin\frac{\overline{B_2C}}{A'}=0.017s$

$t_3=\frac{\pi}{2\omega_2}=0.059s$

$t_总=2(t_1+t_2+t_3)=0.26s$

2° 最高点处上绳恢复原长，有

$\frac{1}{2}k_1(l_1+l_3)^2=mgh+\frac{1}{2}k_2(h-l_3+l_2)^2$

无合理解，$\therefore$ 舍去

8-22 解：$p_0=\frac{4\alpha}{r_0}$ ① $p=\frac{V_0}{V}\cdot p_0=\frac{r_0^3}{r^3}\cdot p_0$ ② $r=r_0+x$ ③

$\therefore F_{合} = 4\pi r^2 \cdot p - 4\pi r^2 \cdot \frac{4\alpha}{r} = -32\pi\alpha \cdot x$

$\therefore T = 2\pi\sqrt{\frac{m}{32\pi\alpha}} = \sqrt{\frac{\pi m}{8\alpha}}$

8-23 解：(1) $\frac{1}{2}kL_0^2 = \frac{1}{2}(M+m)v^2 \cdots ①$ $2\pi\sqrt{\frac{m}{k}} = \frac{2L}{v} \cdots ②$

解得 $L_0 = 2.01\ cm$

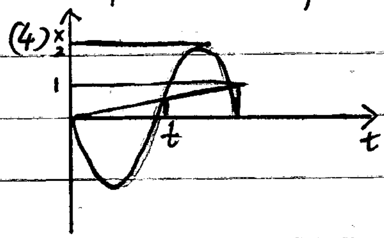
(2) $\frac{1}{2}mv^2 = \frac{1}{2}kL_1^2 \cdots ③$ 联立①③.得 $L_1 = 1.08\ cm$

(3) $mv - Mv = mv_1 + Mv_2 \cdots ④$

$\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \cdots ⑤$

解得 $v_1 = -\frac{13}{7}v$ $\therefore L_2 = \frac{13}{7}L_1 = 2cm$

(4)



习题九 9-1 解：(1) $\lambda = \frac{v}{f} = 8cm$ (2) $y = 2\cdot\cos(20\pi t - \frac{\pi}{2})$

(3) $y = 2\cos(20\pi t - \frac{\pi}{4}x - \frac{\pi}{2})$ (4) $\varphi = -\frac{3}{2}\pi$

9-2 解：(1) 由题意可知.此波沿正x方向传播. ~~标答有误~~

(2) $\lambda = \frac{2\pi}{\Delta\varphi}\cdot\Delta x = 24\ cm$ $v = \lambda f = 48\ cm/s$

9-3 解：$p\cdot 4\pi R^2\cdot\Delta R = P\cdot\Delta t \cdots ①$ $c\cdot\Delta t = \Delta R \cdots ②$

解得 $R = 34.5\ km$

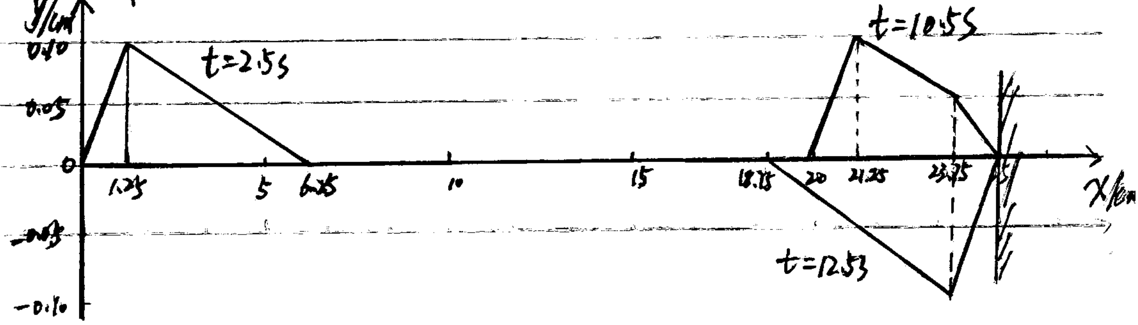
9-4 解：(1) $\Delta\nu = \nu_0\cdot(\frac{1}{1-\frac{u}{v}} - \frac{1}{1+\frac{u}{v}}) = 73.9\ Hz$

(2) $\nu' = \nu_0\cdot\frac{1+\frac{u}{v}}{1-\frac{u}{v}} = 568.3\ Hz$

9-5 解：$\nu' = \nu_0\cdot\frac{1+\frac{u}{v}}{1-\frac{u}{v}} = 41\ kHz$

9-6 解：$\nu_0\cdot\frac{1+\frac{u}{v}}{1-\frac{u}{v}} - \nu_0 = \Delta\nu$ 解得 $v = 6\ m/s$

9-7 解：如图所示



程稼夫《电磁学篇》习题

习题一 1-1 解：$F=\dfrac{k(0.01\cdot\frac{m}{M}\cdot N_A\cdot e)^2}{R^2}=1.71\times10^{15}N$

1-2 解：$T\Delta\theta=\dfrac{k\cdot q_0\cdot\frac{q}{2\pi r}\cdot r\Delta\theta}{r^2}$．解得 $T=\dfrac{q_0q}{8\pi^2\varepsilon_0r^2}$

1-3 解：(1) $\dfrac{k(\frac{Q_{min}}{2})^2}{r^2}=\dfrac{GMm}{r^2}$ 解得 $Q_{min}=2\sqrt{\dfrac{GMm}{k}}=1.14\times10^{14}C$

(2) $\dfrac{kq_1q_2}{r^2}=\dfrac{GMm}{r^2}$ ‥① $\dfrac{q_1}{M}=\dfrac{q_2}{m}$ ‥② $Q=q_1+q_2$ ‥③

解得 $Q=5.21\times10^{14}C$

1-4 解：Q移动 Δx．$F=-\dfrac{9\sqrt{3}kQq}{a^3}\cdot\Delta x$

$\therefore f=\dfrac{1}{T}=\dfrac{1}{2\pi}\cdot\sqrt{\dfrac{9\sqrt{3}kQq}{ma^3}}=2.096\times10^{13}\ Hz$ 标答令 $\pi\approx3$ 有误差

1-5 解：$\dfrac{kq_1q_2}{r_{12}^2}=\dfrac{kq_2q_3}{r_{23}^2}=\dfrac{kq_1q_3}{r_{13}^2}$ 解得 $q_3=\dfrac{q_1q_2}{(\sqrt{q_1}+\sqrt{q_2})^2}$ $\vec{r_3}=\dfrac{\vec{r_1}\sqrt{q_2}+\vec{r_2}\sqrt{q_1}}{\sqrt{q_2}+\sqrt{q_1}}$

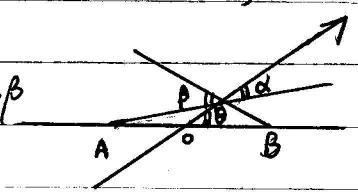
1-6 解：(1) $\dfrac{ke^2}{(2a)^2}=\dfrac{2ke^2}{a^2+b^2}\cdot\dfrac{a}{\sqrt{a^2+b^2}}$ ‥① $\dfrac{ke^2}{(2b)^2}=\dfrac{2ke^2\cdot b}{(a^2+b^2)\sqrt{a^2+b^2}}$ ‥②

易知 ①② 无法同时成立 ∴ 不能静止

(2) $\dfrac{-ke^2}{(2a)^2}+\dfrac{2ke^2}{a^2+b^2}\cdot\dfrac{a}{\sqrt{a^2+b^2}}=m\omega^2a$ ‥③

联立①③．解得 $\omega=e\cdot\sqrt{\dfrac{k}{m}\cdot\left[\dfrac{2}{(a^2+b^2)^{\frac{3}{2}}}-\dfrac{1}{4a^3}\right]}$

1-7 解：(1) 如图



$F_{合}=\dfrac{kQq}{AP^2}\cdot\cos\alpha-\dfrac{kQq}{BP^2}\cos\beta$

又 $\because AP^2=r^2+s^2+2rs\cos\theta$

$BP^2=r^2+s^2-2rs\cos\theta$

$\cos\alpha=\dfrac{s+r\cos\theta}{(r^2+s^2+2rs\cos\theta)^{\frac{1}{2}}}$ $\cos\beta=\dfrac{r\cos\theta-s}{(r^2+s^2-2rs\cos\theta)^{\frac{1}{2}}}$

小量处理后整理得 $F_{合}=\dfrac{2kqQ}{r^3}(1-3\cos^2\theta)\cdot s$

固保略．$q>0$ 时．$\cos^2\theta>\frac{1}{3}$ 为稳定平衡 $q<0$ 时 $\cos^2\theta<\frac{1}{3}$ 为稳定平衡．

(2) $T=2\pi\sqrt{\dfrac{mr^3}{2kQq\,|3\cos^2\theta-1|}}$

1-8 解：$F_{合}=\dfrac{kQq}{(r+\Delta r)^2}-\dfrac{kQq}{(r-\Delta r)^2}+\dfrac{2kQq}{r^2+\Delta r^2}\cdot\dfrac{\Delta r}{\sqrt{r^2+\Delta r^2}}$

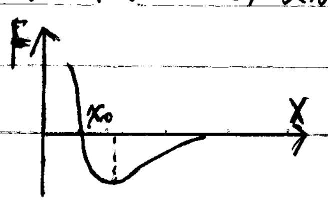
$=-\dfrac{2kQq}{r^3}\Delta r$ 其中 $r=\dfrac{\sqrt{2}}{2}a$

$\therefore T=2\pi\sqrt{\dfrac{\sqrt{2}\varepsilon_0\pi ma^3}{Qq}}$

1-9 解：(1) 易知 q应在Q右边．$\dfrac{kQq}{x^2}=\dfrac{k\cdot2Qq}{(x+l)^2}$ 解得 $x=(\sqrt{2}+1)l$

根据微小扰动后受力 易知该平衡为稳定平衡

(2)



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1-10 解：$|\vec{r}-\vec{r_0}| = 10\,m$ $\therefore E = \frac{kq}{r^2} = 4500\,N\cdot C^{-1}$ 方向 $0.6\vec{i} - 0.8\vec{j}$

1-11 解：将线段等效成圆弧，易知 $E_0 = 0$

1-12 解：(1) 利用对称性可知 $E = \frac{k\cdot q'}{r^2}\cdot\frac{\vec{r}}{r}$ 其中 $q' = \frac{q}{2l}\cdot\frac{2lr}{\sqrt{r^2+l^2}}$

所得 $E = \frac{1}{4\pi\varepsilon_0 r^2}\cdot\frac{q\vec{r}}{\sqrt{r^2+l^2}}$

(2) $E = \int_{r+l}^{r-l}\frac{k\cdot dx\cdot\frac{q}{2l}}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2-l^2}\cdot\frac{\vec{r}}{r}$

(3) $\lambda = \frac{q}{2l}$ $E = \frac{k\cdot\lambda\cdot x}{r^2-l^2}$ 其中 $x = 2\cdot\sqrt{r^2-x^2}\cdot\sin\theta$

又 $\because \tan 2\theta = \frac{2l}{\sqrt{r^2-l^2}}$

联立解得 $E = \frac{q}{4\pi\varepsilon_0 l}\sqrt{\frac{\sqrt{r^2+3l^2}-\sqrt{r^2-l^2}}{2(r^2-l^2)\sqrt{r^2+3l^2}}}$ $\theta = \arctan\frac{\sqrt{r^2+3l^2}-\sqrt{r^2-l^2}}{2l}$

1-13 解：n区内部 $E(x) = \frac{\rho_n(-x)}{2\varepsilon_0} + \frac{\rho_n(x_n+x)}{2\varepsilon_0} + \frac{\rho_p\cdot x_p}{2\varepsilon_0} = \frac{N_D\cdot e}{\varepsilon_0}(x_n+x)$

p区内部 $E(x) = \frac{\rho_p\cdot(x_p-x)}{2\varepsilon_0} + \frac{\rho_n\cdot x_n}{2\varepsilon_0} - \frac{\rho_p x}{2\varepsilon_0} = \frac{N_A e}{\varepsilon_0}(x_p-x)$

外部 $E(x) = \frac{\rho_p x_p}{2\varepsilon_0} - \frac{\rho_n x_n}{2\varepsilon_0} = 0$

1-14 解：(1) $\vec{E} = \frac{k\cdot\rho\cdot\frac{4}{3}\pi r^3}{r^2}\cdot\frac{\vec{r}}{r} = \frac{\rho}{3\varepsilon_0}\cdot\vec{r}$

(2) $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi r_1^3\rho}{r_1^2}\cdot\frac{\vec{r_1}}{r_1} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi r_2^3\rho}{r_2^2}\cdot\frac{\vec{r_2}}{r_2} = \frac{\rho}{3\varepsilon_0}\cdot(\vec{r_1}-\vec{r_2}) = \frac{\rho\cdot\vec{a}}{3\varepsilon_0}$

1-15 解：$F = \frac{\sigma}{2\varepsilon_0}\cdot\sigma\cdot\pi R^2 = 500\,N$

1-16 解：$E(x) = \frac{kq}{R^2+x^2}\cdot\frac{x}{\sqrt{R^2+x^2}} = \frac{kqx}{(R^2+x^2)^{\frac{3}{2}}}$

$E'(x) = kq\cdot\frac{(R^2+x^2)^{\frac{3}{2}} - x^2\cdot 3\cdot(R^2+x^2)^{\frac{1}{2}}}{(R^2+x^2)^3} = \frac{R^2-2x^2}{(R^2+x^2)^{\frac{5}{2}}}$

$\therefore$ 当 $E'(x)=0$，即 $x = \frac{R}{\sqrt{2}}$ 时 $E(x)_{max} = \frac{2\sqrt{3}}{9}\cdot\frac{kq}{R^2} = \frac{\sqrt{3}\cdot q}{18\pi\varepsilon_0 R^2}$

1-17 解：$\frac{kq^2}{a^2}\cdot\frac{\sqrt{2}}{2}\cdot 2 + \frac{kq^2}{(\sqrt{2}a)^2} - \frac{kQq}{(\frac{\sqrt{2}}{2}a)^2} > 0$ 解得 $|Q| < \frac{2\sqrt{2}+1}{4}q$

1-18 解：$W = 12\cdot\frac{kq^2}{a} + 12\cdot\frac{kq^2}{\sqrt{2}a} + 4\cdot\frac{kq^2}{\sqrt{3}a} + 8\,\frac{kQq}{\frac{\sqrt{3}}{2}a} = \frac{kq}{a}(12q + 6\sqrt{2}q + \frac{4\sqrt{3}}{3}a + \frac{16\sqrt{3}}{3}Q)$

1-19 解：$U_1' = \frac{3}{4}U_1$ $U_2' = U_2 - \frac{U_2 - \frac{U_1}{4}}{3} = \frac{2}{3}U_2 + \frac{U_1}{12}$

1-20 解：易知 $\varphi_0 = \frac{\varphi_1+\varphi_2+\varphi_3+\varphi_4+\varphi_5+\varphi_6}{6}$

1-21 解：$U_1 = \frac{kq}{R} + \frac{kq}{\frac{R}{2}} + \frac{kq}{\frac{R}{4}} + \frac{kq}{\frac{3R}{8}} = \frac{29}{3}\frac{kq}{R}$ $U_4 = \frac{kq}{R} + \frac{kq}{\frac{R}{2}} + \frac{kq}{\frac{R}{4}} + \frac{kq}{\frac{R}{8}} = \frac{15kq}{R}$

$\therefore U_{41} = U_4 - U_1 = \frac{16kq}{3R}$

1-22 解：易知 $U = U_1 + U_2 = \begin{cases} 2\pi k(R_1\sigma_1 + R_2\sigma_2) & r \le R_2 \\ 2\pi k(R_1\sigma_1 + \frac{R_2^2\sigma_2}{r}) & R_2 < r \le R_1 \end{cases}$

1-23 解：$E_1\cdot\frac{1}{3}d = E_2\cdot\frac{2}{3}d$ ‥① $E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$ ‥②

解得 $E_1 = \frac{2\sigma}{3\varepsilon_0}$ $E_2 = \frac{\sigma}{3\varepsilon_0}$

1-24 解：$U_P + U_Q = \frac{2kq}{R}$ ∴ $U_Q = \frac{2kq}{R} - U_P$

1-25 解：水平、竖直 t 相同，Δv 相同 ∴ h 相同

∴ $U_{MN} = \frac{v_0^2}{2g}\cdot\frac{U}{d}$

1-26 解：(1) 易知 $U_E = U_A$ ∴ $W=0$

(2) $U_{TE} = -U_{PE}$ ∴ $W' = -W$

1-27 解：将速度分解，加速度合成 ∴ $v_{min} = v_0\cos\theta = v_0\frac{qE}{\sqrt{(qE)^2+(mg)^2}}$

1-28 解：$\frac{kq_1}{a}+\frac{kq_2}{b}=0$ 解得 $q_2 = -\frac{b}{a}q_1$

$$U=\begin{cases}0 & r\le a\\ kq_1\left(\frac{1}{r}-\frac{1}{a}\right) & a<r<b\\ \frac{kq_1}{r}\left(1-\frac{b}{a}\right) & r\ge b\end{cases}$$

1-29 解：$t=0.3\,s$ 时，由矢量图可知 $v'=\sqrt{\frac{3}{2}}v_0$

∴ $E_0 = \frac{2}{3}E = 300\,eV$

1-30 解：1与2接触后，$Q_2=Q_1'=\frac{Q}{2}$

1与3接触后 $Q_3=Q_1''=\frac{Q}{4}$

1与4接触后 $\frac{kQ_1}{a}+\frac{kQ_2}{r}+\frac{kQ_3}{\sqrt{2}r}+\frac{kQ_4}{r}=\frac{kQ_1}{r}+\frac{kQ_2}{\sqrt{2}r}+\frac{kQ_3}{r}+\frac{kQ_4}{a}$

可整理后，解得 $Q_1=\frac{1-a(\sqrt{2}-1)/\sqrt{2}r}{8}\cdot Q$

$Q_4=\frac{1+a(\sqrt{2}-1)/\sqrt{2}r}{8}\cdot Q$

1与地接触后 $\frac{kq_1}{a}+\frac{kQ_2}{r}+\frac{kQ_3}{\sqrt{2}r}+\frac{kQ_4}{r}=0$

解得 $q_1=-\frac{aQ}{r}\left(\frac{5+\sqrt{2}}{8}\right)$ ∴ $\Delta Q=|q_1|+Q_1=\frac{Q}{8}\left[1+\frac{a}{r}\left(4+\frac{3\sqrt{2}}{2}\right)\right]$

1-31 解：(1) $F_电=\frac{\varepsilon_0 S U^2}{2d_1^2}=k(d_0-d_1)$ 解得 $k=\frac{\varepsilon_0SU^2}{2(d_0-d_1)d_1^2}$

(2) $F_合=-k\Delta x-\frac{\varepsilon_0SU^2}{2d_1^2}+\frac{\varepsilon_0SU^2}{2(d_1-\Delta x)^2}=-\frac{\varepsilon_0SU^2(3d_1-2d_0)}{2d_1^3\cdot(d_0-d_1)}\cdot\Delta x$

又∵ $mg=kx_0$ 解得 $f=\frac{1}{2\pi}\sqrt{\frac{(3d_0-2d_1)g}{d_1x_0}}$

1-32 解：$\sqrt{\frac{2h}{g}}=n\cdot\frac{2\pi R}{v_0}$ 解得 $v_0=n\pi R\cdot\sqrt{\frac{2g}{h}}$ $n\in N_+$

$E=\frac{U_R}{d}$ ‥① $QE=\frac{mv_0^2}{R}$ ‥② $C_R\cdot U_R=C_x\cdot U_x$ ‥③ $U_R+U_x=U$ ‥④

$C_R=\frac{RL}{2kd}$ ‥⑤ 解得 $C_x=\frac{n^2\pi^2R^2Lmg}{k(hQU-2n^2\pi^2Rdmg)}$ $n\in N_+$

1-33 解：$\frac{1}{2}mv_0^2-\frac{kq_1e}{r_1}=\frac{1}{2}mv_1^2-\frac{kq_1e}{r_2}$ ‥① $v_0r_1=v_1r_2$ ‥②

解得 $v_0=\sqrt{\frac{2kq_1er_2}{mr_1(r_2+r_1)}}$

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1-34 解：由题意知 $F_{电}=q\cdot\frac{nq}{Cd}$ $\therefore a'=\frac{F_{电}}{m}-g=\frac{nq^2}{mCd}-g$

又 $\because 2gh=2a'\cdot d$ 解得 $n=\frac{mgC\cdot(d+h)}{q^2}$

$\therefore N=\left[\frac{mgC\cdot(d+h)}{q^2}+1\right]$

1-35 解：(1) 由题意知 $\frac{kq}{\sqrt{a^2+r^2}}+\frac{kQ_1}{R}=0$ 解得 $Q_1=-\frac{R}{\sqrt{a^2+r^2}}\cdot q$

(2) $\varphi_0=\frac{kq}{\sqrt{a^2+r^2}}$

(3) $\frac{kQ_1'}{R}+\frac{kq}{\sqrt{a^2+r^2}}=V_0$ $\therefore Q_1'=\frac{RV_0}{k}-\frac{Rq}{\sqrt{a^2+r^2}}$

(4) $\Delta F=\Delta Q_1\cdot\frac{kq}{r^2+a^2}\cdot\frac{a}{\sqrt{r^2+a^2}}=\frac{RaqV_0}{(r^2+a^2)^{\frac{3}{2}}}$ 方向水平向右

(5) 令 $V_0=\varphi_0$ 得 $\Delta F'=\frac{kq^2\cdot aR}{(r^2+a^2)^2}$ 方向水平向右

1-36 解：$E_k=\frac{1}{2}m_H V_0^2$ …① $m_H V_0 l=m_H\cdot v'\cdot R$ …② $\frac{1}{2}m_H V_0^2=\frac{1}{2}m_H v'^2+eU$ …③

解得 $l=\frac{\sqrt{2}}{2}R$ 改成电子同理可得 $l=\frac{\sqrt{6}}{2}R$

1-37 解：(1) $\frac{k}{R}\cdot q=\frac{mv_0^2}{R}$ 解得 $v_0=\sqrt{\frac{kq}{m}}$

(2) $-q\cdot\frac{k}{r}=-\frac{mv_\theta^2}{r}+F$ …① 又 $v^2=v_\theta^2+v_r^2$ …②

$\frac{1}{2}mv^2+(-q)\cdot U(r)=\frac{1}{2}mv_0^2+(-q)\cdot U(R)$

化简得 $F=-\frac{2mv_0^2}{R^2}\cdot\Delta r$

$\therefore T=2\pi\sqrt{\frac{R^2}{2v_0^2}}=\frac{\sqrt{2}\pi R}{v_0}$ $\theta=\frac{v_0}{R}\cdot\frac{T}{2}=\frac{\pi}{\sqrt{2}}=127^\circ$

1-38 解：(1) e 从 2d 到 d. $a_1=\frac{ebd}{m}$ $\therefore t_1=\sqrt{\frac{2d}{a_1}}=\sqrt{\frac{2m}{eb}}$

$v_1=a_1t_1=\sqrt{\frac{2eb}{m}}\cdot d$ 从 d 到 0. $F=-ebx=-kx$

$A=\sqrt{x_0^2+\frac{v_1^2}{\omega^2}}=\sqrt{3}d$ $\therefore t_2=T_0\cdot\frac{\arcsin\frac{1}{\sqrt{3}}}{2\pi}=0.615\sqrt{\frac{m}{eb}}$

$\therefore T=4(t_1+t_2)=8.12\sqrt{\frac{m}{eb}}$

(2) $\Delta y=\frac{v_0T}{2}=4.06 v_0\sqrt{\frac{m}{eb}}$

1-39 解：$\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\cdot v_1^2=\frac{k\cdot Q^2}{r}$ 解得 $r=\frac{Q^2}{v_1^2}\cdot\frac{2k(m_1+m_2)}{m_1m_2}$

代入数据得 $d=0.2m$ （(1).(2)问答案相同）

1-40 解：(1) $F(r)=U'(r)=k\cdot\frac{4a_s}{3r^2}$ 由题意有 $\frac{mv^2}{\frac{r_0}{2}}=F(r_0)$ …①

$2\cdot mv\cdot(\frac{r_0}{2})=\frac{h}{2\pi}$ …② 解得 $r_0=\frac{3h^2}{8\pi^2 m a_s k}=1.4\times10^{-17}m$

(2) 易知 $v=\frac{4\pi a_s k}{3h}$ $\therefore T=\frac{2\pi\cdot\frac{r_0}{2}}{v}=\frac{9h^3}{32\pi^3 m a_s^2 k^2}=1.83\times10^{-24}s$

$\therefore \frac{\tau}{T}=0.22$ $\therefore$ 束缚态不存在

1-41 解：$\sin\alpha=\frac{R}{L}$ …① $mg=\frac{kQ\cdot x}{(R^2+x^2)^{\frac{3}{2}}}\cdot\tan\alpha$ …② $R=x\tan\alpha$ …③

联立,解得 $L=\sqrt[3]{\frac{kRQ^2}{mg}}=7.2\ cm$

1-42 解:(1) 对B,有 $mg=qE\cdot\tan30^\circ$ ∴ $E=\frac{\sqrt{3}mg}{q}$

(2) 对整体,有 $2mg\cdot\tan\beta=qE$ 解得 $\beta=\arctan(\frac{\sqrt{3}}{2})$

(3) $T=\sqrt{(2mg)^2+(qE)^2}=\sqrt{7}mg$

1-43 解:(1) $\left|\frac{k\cdot8Q\cdot x}{48r^2}\right|=\frac{k\cdot(8Q+x)^2}{4r^2}$ 解得 $x=-6Q$ 或 $-10.67Q$

(2) 最后 $Q_A=4Q$ $Q_B=Q_C=-Q$

∴对B. $F_x=\frac{k\cdot Q_A\cdot Q_B}{(2a)^2}\cdot\cos60^\circ+\frac{kQ_BQ_C}{a^2}=\frac{KQ^2}{2a^2}$

$F_y=-\frac{kQ_AQ_B}{(2a)^2}\cdot\cos30^\circ=\frac{\sqrt{3}}{2}\cdot\frac{kQ^2}{a^2}$

∴ $F_合=\frac{kQ^2}{a^2}$. 方向与AB连线成6°角. 与BC连线成120°角

1-44 解: $\frac{Q-q}{q}=\frac{Q}{q_m}$ 解得 $q_m=\frac{Qq}{Q-q}$

1-45 解: ∵小球带负电 ∴有无穷远的电场线指向小球 ∴ Q_A. $Q_B<Q$

用a图减去b图. 左边为一个不带电导体, 右边为一个大导体, 右端负电

不接地(注意) ∴左端带负电 ∴ $Q_1<Q_2$ 即 $Q_1<Q_2<Q$

1-46 解: K将A与外壳相连 $\varphi_A=\varphi_0$ ∴指针闭合

移去K. 用手摸A后. 指针重新张开. 此时A球带负电

1-47 解: $k\cdot x=\frac{q^2}{4\pi\varepsilon_0(2l)^2}$ 解得 $q=2l\cdot\sqrt{4\pi\varepsilon_0\cdot kx}$

1-48 解: $W=\frac{1}{2}\cdot\frac{k\cdot q^2}{2l}=\frac{kq^2}{4l}$

1-49 解:(1) 由电像法. 写出镜像电荷 $F=\frac{kq^2}{l^2}\sqrt{2}-\frac{kq^2}{(\sqrt{2}l)^2}=\frac{(2\sqrt{2}-1)q^2}{8\pi\varepsilon_0l^2}$

(2) $E=2\times\frac{kq^2}{(\frac{l}{2})^2}-\frac{2kq^2}{(\frac{\sqrt{5}}{2}l)^2}\cdot\frac{1}{\sqrt{5}}=2(1-\frac{\sqrt{5}}{25})\frac{q}{\pi\varepsilon_0l^2}$

1-50 解:(1) $\frac{k\cdot q^2}{(2d)^2}=2\cdot\frac{k\cdot\frac{r}{d}q^2}{d^2}$ 解得 $r=\frac{d}{8}$

(2) $\frac{kQ}{R}=V$ $F=\frac{kQq}{d^2}=\frac{qV}{8d}$

注. 可能有人认为 $r=\frac{d}{8}$ 与 $r\ll d$ 矛盾. 可以用精确解计算. 得

$r=0.1249087439\cdot d$ 与前者仅差万分之一

1-51 解: 由静电屏蔽. 可知. $F_{q_1}=F_{q_2}=0$

$F_A=F_q=\frac{k(Q'+q_1+q_2)q}{r^2}-k\cdot\frac{\frac{R}{r}\cdot q^2}{(r-\frac{R^2}{r})^2}$

$=\frac{kq}{r^2}(q_1+q_2+\frac{R}{r}q)-\frac{kRrq^2}{(r^2-R^2)^2}$

1-52 解: ∵做匀速圆周运动 ∴圆周为等势面

由像电荷知识 易知 $r'=\frac{R^2}{r}$ $q'=-\frac{R}{r}\cdot q$

1-53 解: $f=-\frac{kqq'}{(d-x)^2}+\frac{kq(Q+q')}{d^2}$ 其中 $q'=\frac{R}{d}q$ $x=\frac{R^2}{d}$

令 $f<0$. 解得 $Q<q\cdot\frac{R^3}{d}\cdot\frac{2d^2-R^2}{(d^2-R^2)^2}$

1-54 解 $f=-\frac{kqq'}{(d-x)^2}+\frac{kq(q+q')}{d^2}$ 其中 $q'=\frac{R}{d}\cdot q$ $x=\frac{R^2}{d}$

令 $f=0$. 化简得 $d^5-2R^2d^3-2R^3d^2+R^4d+R^5=0$

$\therefore (d^5-Rd^4+Rd^4-R^2d^3-R^2d^3-R^3d^2)+(-R^3d^2+R^4d+R^5)=0$

$\therefore d^2(d^3+Rd^2-R^2d^2-R^2d-R^2d-R^3)-R^3(d^2-Rd-R^2)=0$

$\therefore d^2(d+R)(d^2-Rd-R^2)-R^3(d^2-Rd-R^2)=0$

$\therefore (d^3+d^2R-R^3)(d^2-Rd-R^2)=0$

又 $\because d\geq R$. 解得 $d=\frac{\sqrt{5}+1}{2}R$

1-55 解: (1) 易知. $U_0=\frac{kq}{r}-\frac{kq}{R_1}+\frac{kq}{R_2}$

(2) $U_{\text{球壳}}=\frac{kq}{R_2}-\frac{kq}{r_0}+\frac{kq}{r_0}=\frac{kq}{R_2}$

1-56 解: $\because \frac{kq_1q_3}{l_1^2}=\frac{kq_2q_3}{l_2^2}$ ..① 又 $\because$ A_1对A_3的力与A_2对A_3的力相等

$\therefore$ A_1与A_2受到的合力相等 $\therefore m_1\omega^2 l_1=m_2\omega^2 l_2$ ..②

①② 联立. 得 $\frac{\mu_1}{\mu_2}=\frac{l_1^3}{l_2^3}$

1-57 解: $F=-Q(E_1-E_2)=-kQq\cdot\left\{\frac{L-\Delta X}{[R^2+(L-\Delta X)^2]^{\frac{3}{2}}}-\frac{L+\Delta X}{[R^2+(L+\Delta X)^2]^{\frac{3}{2}}}\right\}$

$=-2kQq(R^2+L^2)^{-\frac{5}{2}}\cdot(2L^2-R^2)\cdot\Delta X$

作稳定小振动的条件为 $k>0$. 即 $L>\frac{R}{\sqrt{2}}$

$T=2\pi\sqrt{\frac{m(R^2+L^2)^{\frac{5}{2}}}{2kQq\cdot(2L^2-R^2)}}$

1-58 解: $\because l\gg a$ $\therefore$ 那大接地金属板无用途

$C=4\pi\varepsilon_0 a$

1-59 解: (1) $C=\left(\frac{1}{4\pi\varepsilon_0 a}+\frac{1}{4\pi\varepsilon_0 b}\right)^{-1}=4\pi\varepsilon_0\frac{ab}{a+b}$ 看作串联

(2) $C=4\pi\varepsilon_0(a+b)$ 看作并联

1-60 解: (1) $\frac{kQ}{R_2}+\frac{kq}{R_1}=0$ 解得 $q=-\frac{R_1}{R_2}\cdot Q$

$\varphi_{外}=\frac{kq}{R_2}+\frac{kQ}{R_2}=\frac{kQ(R_2-R_1)}{R_2^2}$

(2) $C=4\pi\varepsilon_0 R_2+\frac{4\pi\varepsilon_0 R_1R_2}{R_2-R_1}=\frac{4\pi\varepsilon_0 R_2^2}{R_2-R_1}$

1-61 解: 当 $U_1=6kV$时. $U_2=3kV$ $\therefore U_{总}=9kV$

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当 $U_2'=4kV$ 时，$U_1'=8kV>6kV$ ∴不成立

∴最大可承受 9kV 电压

1-62 解：(1) 画等效电路图. $C_{12}=\frac{1}{\frac{1}{2C_0}+\frac{1}{C_0}}=\frac{2}{3}C_0=\frac{2}{3}\frac{\varepsilon_0 S}{d}$

(2) 同上，得 $C_{12}=C_0+\frac{1}{2}C_0=\frac{3\varepsilon_0 S}{2d}$

1-63 解：$E_{12}=\frac{U}{d}$　$E_{13}=E_{24}=\frac{U}{2d}$

易知 $\sigma_1=\frac{C_{总}U}{S}=\frac{3\varepsilon_0 U}{2d}$　$\sigma_2=-\frac{3\varepsilon_0 U}{2d}$

$\sigma_3=-\frac{C\cdot\frac{1}{2}U}{S}=-\frac{\varepsilon_0 U}{2d}$　$\sigma_4=\frac{\varepsilon_0 U}{2d}$

1-64 解：(1) 易知 $C_{ab}=C$ 且 $C_k=C$

又由电容串并联得 $U_k=\frac{1}{3}\cdot U_{k-1}$

递推得 $U_k=(\frac{1}{3})^{k-1}\cdot U$

$W_k=\frac{1}{2}C_kU_k^2=\frac{1}{2}\cdot(\frac{1}{3})^{2k-2}\cdot CU^2$

(2) 由题意，有 $Q=\frac{1}{3}U\cdot 3C=CU$ ‥①

$q_1-q_3=0$ ‥②　$q_2-q_1=-\frac{1}{3}Q$ ‥③

$\frac{q_1}{3C}+\frac{q_2}{2C}+\frac{q_3}{3C}=0$ ‥④

解得 $q_1=\frac{1}{7}Q$　$q_2=-\frac{4}{21}Q$　$q_3=\frac{1}{7}Q$

$\therefore W=\frac{1}{2}\cdot\frac{q_1^2}{3C}+\frac{1}{2}\cdot\frac{q_2^2}{2C}+\frac{1}{2}\cdot\frac{q_3^2}{3C}=\frac{1}{63}\cdot CU^2$　标答有误

1-65 解：(1) 设板1右端带电Q ∴ $Q_{n左}=-(q_2+q_3+\cdots+q_{n-1}+Q)$

$Q_{n右}=q_2+q_3+\cdots+q_n+Q$

又∵板1与板100接地 ∴ $\sum_{i=2}^{100}\frac{Q_{i左}}{C}=0$

解得 $Q=-1682\cdot\frac{1}{3}\cdot q_1$

$\therefore \Delta q_1=1683\frac{1}{3}q_1$　$\Delta q_{100}=3366\cdot\frac{2}{3}q_1$

(2) $U_{x(x+1)}=\frac{Q_{(x+1)左}}{C}$ 令 $U_{(x-1)x}\geq 0$ 且 $U_{x(x+1)}\leq 0$

解得 $57.5\leq x\leq 58.5$

∴ $x=58$，即第58块板上电势最高

$U_{58}=\frac{1}{C}\sum_{i=2}^{58}Q_{i左}=63441\cdot\frac{4\pi k d q_1}{L^2}$

1-66 解：当Q恒定时 $U=\frac{Q}{C}=\frac{Q}{\varepsilon_0\varepsilon S}\cdot d\propto d$　$\therefore\frac{\Delta U}{U}=\frac{\Delta d}{d}$

受力分析有 $pS=2k\cdot(\sqrt{L^2+\Delta d^2}-L)\cdot\frac{\Delta d}{\sqrt{L^2+\Delta d^2}}$

化简后得 $pS=\frac{k\cdot d^3}{L^2}$ $\therefore p=\frac{kd^3}{L^2S}\cdot\left(\frac{\Delta U}{U}\right)^3$

1-67 解：由题意.有 $U\cdot 4\pi\varepsilon_0 R=\frac{\frac{4}{3}\pi R^3}{\pi(\frac{a}{2})^2\cdot h}\cdot\frac{\varepsilon\varepsilon_0\cdot\pi ah}{d}\cdot E$

解得 $U=\frac{4\varepsilon R^2E}{3ad}=457.1\,kV$ 原题中心应是指内直径

1-68 解：由题意. ε_1与ε_3串再并上 ε_2与ε_3的串联

$\therefore C=\left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}+\left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}$ 其中 $C_i=\frac{\varepsilon_i\varepsilon_0\cdot a^2}{d}$

代入解得 $C=\frac{\varepsilon_0\varepsilon_3}{d}a^2\left(\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_3}+\frac{\varepsilon_2}{\varepsilon_2+\varepsilon_3}\right)$

令 $C'=\frac{\varepsilon_0\varepsilon_x a^2}{d}=C$ 得 $\varepsilon_x=\frac{\varepsilon_3\varepsilon_1}{\varepsilon_1+\varepsilon_3}+\frac{\varepsilon_2\varepsilon_3}{\varepsilon_2+\varepsilon_3}$

若为导体面.则 $C'=\left[(C_1+C_2)^{-1}+(2C_3)^{-1}\right]^{-1}$

$=\frac{\varepsilon_0a^2}{d}\cdot\frac{2(\varepsilon_1+\varepsilon_2)\varepsilon_3}{\varepsilon_1+\varepsilon_2+2\varepsilon_3}$

1-69 解：由题意.知 $\frac{F}{\rho Vg}=\frac{F}{\varepsilon_r(\rho-\rho_0)Vg}$ 解得 $\rho=\frac{\varepsilon_r}{\varepsilon_r-1}\rho_0=1.6\,g\cdot cm^{-3}$

1-70 解：(1) $E_{01}=\frac{U_0}{d}=\frac{Q_0}{\varepsilon_0 S}$ $E_{02}=\frac{U}{d}=\frac{Q_0}{\varepsilon_r\varepsilon_0 S}$

$E_a=E_b=\frac{U}{d}=\frac{1}{d}\cdot\frac{Q_0}{C}=\frac{2}{\varepsilon_r+1}\cdot\frac{Q_0}{\varepsilon_0 S}$

(2) $Q_2'=-Q_0\cdot\left(1-\frac{1}{\varepsilon_r}\right)=-\frac{\varepsilon_r-1}{\varepsilon_r}Q_0$

(3) $Q_a=C_aU=\frac{1}{2}\varepsilon_r\cdot C_0\cdot E_a\cdot d=\frac{\varepsilon_r}{\varepsilon_r+1}Q_0$

$Q_b=C_b\cdot U=\frac{1}{2}C_0\cdot E_b\cdot d=\frac{1}{\varepsilon_r+1}\cdot Q_0$

(4) $\frac{Q_a+Q_a'}{C_0/2}=\frac{Q_b}{C_0/2}$ 解得 $Q_a'=-\frac{\varepsilon_r-1}{\varepsilon_r+1}\cdot Q_0$

1-71 解：法一：受力分析法. 对极化电介质.故有 $\rho gSh=\sigma'\cdot S\cdot E$

其中 $\sigma'=\frac{\varepsilon_r-1}{\varepsilon_r}\cdot\sigma$ $E=\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0\varepsilon_r}$

解得 $h=\frac{(\varepsilon_r^2-1)\sigma^2}{2\varepsilon_0\varepsilon_r^2\rho g}$

法二：能量密度+虚功原理 $\sigma'=\frac{\varepsilon_r-1}{\varepsilon_r}\cdot\sigma$ $E_2=\frac{\sigma}{\varepsilon_0\varepsilon_r}$ $E_1=\frac{\sigma}{\varepsilon_0}$

$\therefore W_1=\frac{1}{2}\varepsilon_0E_1^2Sd_1=\frac{\sigma^2Sd_1}{2\varepsilon_0}$ $W_2=\frac{1}{2}\varepsilon_0E_2^2S(d-d_1)=\frac{\sigma^2S(d-d_1)}{2\varepsilon_0\varepsilon_r^2}$

$\rho gSh\cdot\Delta d_1=\Delta W_1+\Delta W_2=\frac{\sigma^2\cdot(\varepsilon_r^2-1)S}{2\varepsilon_0\varepsilon_r^2}\cdot\Delta d_1$

解得 $h=\frac{(\varepsilon_r^2-1)\sigma^2}{2\varepsilon_0\varepsilon_r^2\rho g}$

法三：形成两电容器 上方为 σ. d_1 下方为 $(\sigma-\sigma')$. $(d-d_1)$

$\therefore W_1=\frac{\sigma^2Sd_1}{2\varepsilon_0}$ $W_2=\frac{(\sigma-\sigma')^2S\cdot(d-d_1)}{2\varepsilon_0}=\frac{\sigma^2S(d-d_1)}{2\varepsilon_0\varepsilon_r^2}$

以下同解法二

法四：能量求导 令下极板为电势零点

$U_1 = E_2(d-d_1) = \frac{\sigma(d-d_1)}{\varepsilon_0\varepsilon_r}$ $\quad U_2 = U_1 + E_1 d_1 = \frac{\sigma(d-d_1)}{\varepsilon_0\varepsilon_r} + \frac{\sigma d_1}{\varepsilon_0}$

$\therefore W_{总} = \frac{1}{2}\cdot[U_1\cdot(-\sigma_1)\cdot S + U_2\sigma\cdot S] = \frac{\sigma^2 S}{2\varepsilon_0\varepsilon_r^2}[(\varepsilon_r^2-1)d_1+d]$

$\therefore F = \frac{\Delta W_{总}}{\Delta d_1} = \frac{\sigma^2 S\cdot(\varepsilon_r^2-1)}{2\varepsilon_0\varepsilon_r^2} = \rho g h S$

解得 $h = \frac{\sigma^2(\varepsilon_r^2-1)}{2\varepsilon_0\varepsilon_r^2\rho g}$

1-72 解：法一：能量+受力 取 $x\to 0$

$C_1 = \frac{\varepsilon_0 S}{x}$ $\quad C_2 = \frac{\varepsilon_r\varepsilon_0 S}{d-x}$ $\quad \therefore C = \frac{C_1C_2}{C_1+C_2} = \frac{\varepsilon_0\varepsilon_r S}{d+(\varepsilon_r-1)x}$

又 $\because Q = C\cdot U = \sigma\cdot S$

$\therefore \sigma = \frac{\varepsilon_0\varepsilon_r U}{d+(\varepsilon_r-1)x}$ $\quad \sigma' = \frac{\varepsilon_r-1}{\varepsilon_r}\sigma = \frac{\varepsilon_0\cdot(\varepsilon_r-1)U}{d+(\varepsilon_r-1)x}$

$W_1 = \frac{\sigma^2 S^2}{2\varepsilon_0 S}\cdot x = \frac{\sigma^2 S}{2\varepsilon_0}\cdot x$ $\quad W_2 = \frac{(\sigma-\sigma')^2 S^2}{2\varepsilon_0 S}(d-x) = \frac{\sigma^2 S(d-x)}{2\varepsilon_0\varepsilon_r^2}$

$\therefore W = W_1 + W_2 = \frac{\varepsilon_0 S U^2[(\varepsilon_r^2-1)x+d]}{2[d+(\varepsilon_r-1)x]^2}$

$\therefore F_{电} = \frac{\partial W}{\partial x} = \frac{\varepsilon_0 S U^2}{2}\cdot\frac{(\varepsilon_r^2-1)[d+(\varepsilon_r-1)x]^2 - [(\varepsilon_r^2-1)x+d]\cdot 2(\varepsilon_r-1)[d+(\varepsilon_r-1)x]}{[d+(\varepsilon_r-1)x]^4}$

令 $x=0$ 得 $F_{电} = \frac{\varepsilon_0 U^2 S\cdot(\varepsilon_r-1)^2}{2d^2}$

上.下极板给介质极化电荷的力 $F' = \sigma'\cdot S\cdot\frac{\sigma}{\varepsilon_0} = \frac{\varepsilon_0\varepsilon_r(\varepsilon_r-1)SU^2}{d^2}$

$\therefore F_{总} = F' - F_{电} = \frac{\varepsilon_0 U^2 S\cdot(\varepsilon_r^2-1)}{2d^2}$

$\therefore \Delta p = \frac{F_{总}}{S} = \frac{\varepsilon_0 U^2(\varepsilon_r^2-1)}{2d^2} = 7.26\ kPa$

法二：虚功原理

$\Delta\sigma\cdot S\cdot U + F_{电}\cdot\Delta x + \Delta p\cdot S\cdot\Delta x = 0$

解得 $\Delta p = \frac{\varepsilon_0 U^2(\varepsilon_r^2-1)}{2d^2} = 7.26\ kPa$

法三：电势能 令下极板为势能零点

$W_{总} = \frac{1}{2}\cdot U\cdot\sigma S + \frac{1}{2}U_1\cdot(-\sigma')S$ $\quad$ 其中 $U_1 = U - \frac{\sigma S}{C_1}$

$= \frac{\varepsilon_0 S U^2[(\varepsilon_r^2-1)x+d]}{2[d+(\varepsilon_r-1)x]^2}$ 以下同解法一

法四：受力分析法

$F = \sigma'\cdot S\cdot E$ $\quad E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma'}{2\varepsilon_0}$ $\quad \Delta p = \frac{F}{S}$

解得 $\Delta p = \frac{\varepsilon_0 U^2(\varepsilon_r^2-1)}{2d^2} = 7.26\ kPa$ 标答有误

1-73 解：(1) $C_1 = \left(\frac{d-t}{\varepsilon_0 S} + \frac{t}{\varepsilon_0\varepsilon_r S}\right)^{-1} = \frac{\varepsilon_0\varepsilon_r S}{\varepsilon_r d-(\varepsilon_r-1)t}$ $\quad C_2 = \frac{\varepsilon_0 S}{d}$

$\therefore W = (C_1U)^2\cdot\left(\frac{1}{2C_2} - \frac{1}{2C_1}\right) = \frac{\varepsilon_0\varepsilon_r(\varepsilon_r-1)StU^2}{2[\varepsilon_r d-(\varepsilon_r-1)t]^2}$

(2) $\Delta C\cdot U^2 + W_1 = \frac{1}{2}\Delta C\cdot U^2$ $\quad \therefore W_1 = \frac{1}{2}(C_1-C_2)U^2$

$= \frac{\varepsilon_0 S\cdot(\varepsilon_r-1)tU^2}{2[\varepsilon_r d-(\varepsilon_r-1)t]d}$

(3) $C_1' = \frac{\varepsilon_0 S}{d-t}$ $C_2' = \frac{\varepsilon_0 S}{d}$

$\therefore W' = \frac{(C_1'V)^2}{2}\left(\frac{1}{C_2'} - \frac{1}{C_1'}\right) = \frac{\varepsilon_0 S t V^2}{2(d-t^2)}$

同理 $W_1' = \frac{1}{2}(C_1 - C_2)\cdot V^2 = \frac{\varepsilon_0 S t V^2}{2(d-t)\cdot d}$

1-74 解：由题意知. $C_1 = \frac{\varepsilon_0 b(a-x)}{d}$ $C_2 = \frac{\varepsilon_0 \varepsilon_r b x}{\varepsilon_r(d-t)+t}$

$\therefore C = C_1 + C_2 = \varepsilon_0\left[\frac{b(a-x)}{d} + \frac{\varepsilon_r b x}{\varepsilon_r(d-t)+t}\right] = \varepsilon_0\left[\frac{b(a-x)}{d} + \frac{bx}{d-t'}\right]$

$= \frac{\varepsilon_0\cdot[s\cdot(d-t') + bxt']}{(d-t')\cdot d}$ 其中 $S = ab$、$t' = \frac{\varepsilon_r - 1}{\varepsilon_r} t$

$\therefore W = \frac{Q^2}{2C}$ $F = -\frac{\partial W}{\partial x} = \frac{Q^2 b t'(d-t')d}{2\varepsilon_0[S(d-t') + bxt']^2}$

1-75 解：$\Delta W = W_{电源} + A - W_R$

易知 $W_{电源} = -C\varepsilon^2$ $\Delta W = -\frac{1}{2}C\varepsilon^2$

$\therefore W_R = A - \frac{1}{2}C\varepsilon^2$

1-76 解：$U_1 = 300\,V$ $U_2 = \frac{C_1 U_1}{C_2} = 300\,V$

$W = \frac{(C_1U_1)^2}{2}\cdot\left(\frac{1}{C_2} - \frac{1}{C_1}\right) = 4.05\times10^{-5}\,J$ 标答有误

1-77 解：$\frac{kQ_0}{R} = U_0$ ‥① $r = \frac{R}{\sqrt[3]{2}}$ ‥② $W = \frac{kQ^2}{2R}$ ‥③

$W_1 - W_2 = 2E_k$ ‥④ 解得 $E_k = \frac{1}{8}(2-\sqrt[3]{2})Q_0U_0$

1-78 解：(1) $W = \frac{k\cdot(Ne)^2}{2R} = 1.06\times10^{-10}\,J = 662.4\,MeV$

(2) $Q = \frac{k\cdot(Ne)^2}{2R} - \frac{k\cdot(\frac{N}{2}e)^2\cdot 2}{2r}$ 其中 $r = \frac{R}{\sqrt[3]{2}}$

代入数据得 $Q = 3.92\times10^{-11}\,J = 245.1\,MeV$

(3) $Q' = N_A\cdot Q\cdot\frac{m}{M} = 1.0\times10^{14}\,J = 6.28\times10^{26}\,MeV$

1-79 解：$\frac{kQ^2}{2R} = \Delta E$. 解得 $R = 1.57\times10^{-16}\,m$

1-80 解：(1) $\Delta W = \frac{kQ^2}{2}\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{kQ^2(R_2-R_1)}{2R_1R_2}$

(2) $W' = \left(\frac{kq}{2R} + \frac{kq_0}{R}\right)q = \frac{k(q+2q_0)q}{2R}$

$\therefore \Delta W' = \frac{(q+2q_0)q}{8\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

1-81 解：(1) $W_1 = \frac{kq_1^2}{2R_1}$ $W_2 = \frac{kq_2^2}{2R_2}$

(2) $W = W_1 + W_2 + \frac{kq_1q_2}{R_2} = \frac{q_2^2}{8\pi\varepsilon_0 R_2} + \frac{q_1^2}{8\pi\varepsilon_0 R_1} + \frac{q_1q_2}{4\pi\varepsilon_0 R_2}$

1-82 解：$W_总 = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_3} = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right)$

1-83 解：设原电容器静电能为 E_1 和 E_2. 其中 $E_1 = \frac{Q^2}{2\varepsilon_0 S}\cdot d$ $E_2 = \frac{Q^2}{2\varepsilon_0 S}\cdot d$

1° 异号极板相对时 $E_0 = \frac{Q^2}{4\varepsilon_0 S}\cdot d$ $\therefore \frac{2E_0}{E_1+E_2} = \frac{1}{3}$

$\therefore$ 静电能变为原先的 $\frac{1}{3}$

2° 同号极板相对时

$E_0'=\frac{Q^2}{4\varepsilon_0 S}\cdot d$ $E_0''=\frac{4Q^2}{2\varepsilon_0 S}\cdot d$

$\therefore \frac{2E_0'+E_0''}{E_1+E_2}=\frac{5}{3}$ $\therefore$ 静电能变为原先的 $\frac{5}{3}$

习题二 2-1 解：$R=2\cdot\int_a^\infty \rho\cdot\frac{dx}{4\pi x^2}=\frac{\rho}{2\pi a}$

2-2 解：$2\left(\frac{U}{R+r}\right)^2\cdot r=\left(\frac{U}{r_1+\frac{R}{2}}\right)^2\cdot r_1$

解得 $r_1=45\Omega$ 或 $r_1'=5\Omega$

2-3 解：由题意 $P_0=\frac{U^2}{R_1+R_2+R_3}$ $P_1=\frac{U^2}{R_3}$ $P_2=\frac{U^2}{R_1}$

$\therefore P_4=U^2\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$

$=P_1+P_2+\frac{P_0P_1P_2}{P_1P_2-P_0P_1-P_0P_2}$

2-4 解：(1) $\because I=I_1$ $P=P_1$ $\therefore R=R_1$ 即 $\frac{U}{I}=1000$

作直线 与伏安图像交点 (3mA, 3V)

$\therefore Q_t=I^2R=9\times10^{-3}$ J/s

则 $t=t_0+\frac{Q_t}{\alpha}=40°C$

(2) $R_2=0$ 有 $U=7-1000I$

作该直线 与伏安图像交于 (3.7mA, 3.3V)

$\therefore R'=\frac{U}{I}=8.9\times10^2\Omega$

2-5 解：(1) $P_2=I^2\cdot R_2$ $\therefore I$ 最大 即 $R_1=0$ 时 P_2 最大

(2) 将电源内阻看作 R_1+r 易知

当 $R_2=R_1+r$ 时 P_2 最大

2-6 解：(1) $P_1=\left(\frac{E}{R+r}\right)^2\cdot R=\frac{E^2}{4r}$ $P_2'=\left(\frac{E}{r+\frac{R}{2}}\cdot\frac{1}{2}\right)^2\cdot R=\frac{E^2}{9r}$

$\therefore P_1:P_2=9:4$

(2) $P_0=E\cdot\frac{E}{R+r}=\frac{E^2}{2r}$ $P_0'=E\cdot\frac{E}{r+\frac{R}{2}}=\frac{2E^2}{3r}$

$\therefore P_0:P_0'=3:4$

2-7 解：$P=\left(\frac{U}{\frac{R}{3}+r}\right)^2\cdot\frac{R}{3}$ $P'=\left(\frac{U}{\frac{R}{2}+r}\right)^2\cdot\frac{R}{2}$

解得 $P:P'=242:243$

2-8 解：$Pt=cm\cdot\Delta T$ 得 $t=42s$

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2-9 解：$Pt_1 = k\cdot(T_1 - T_0)\cdot(t_1 + t_2) \cdots$ ①

$Pt_1' = k(T_1' - T_0)(t_1' + t_2') \cdots$ ②

$Pt = k(T_1'' - T_0)\cdot t \cdots$ ③

解得 $T_1' = 180°C \quad T_1'' = 420°C$

2-10 解：$\frac{P_2'}{P_1'} = \frac{I_2^2R_2}{I_1^2R_1} = \frac{I_2^2(1+\alpha t_2)}{I_1^2(1+\alpha t_1)} = \frac{t_2 - t_0}{t_1 - t_0}$

解得 $P_2 = \sqrt{\frac{(t_2-t_0)(1+\alpha t_1)}{(t_1-t_0)(1+\alpha t_2)}}\cdot P_1 = 1.4kW$

2-11 解：$(P_0 - P_{损})\Delta t_1 = (0.81P_0 - P_{损})\cdot\Delta t_2$

解得 $P_{损} = 0.62P_0$

$\therefore U_{min} = \sqrt{0.62}\,U_0 = 0.8U_0$

2-12 解：$P_m = k_1\cdot S(T^4 - T_{外界}^4) = k_1\cdot(T^4 - T_{外界}^4)\cdot 2\pi r\cdot l$

又∵ $P_m = I^2\cdot\rho\frac{l}{\pi r^2}$

联立得 $I^2 = \frac{k_1(T^4 - T_{外界}^4)\cdot 2\pi^2}{\rho}\cdot r^3 \propto r^3$

∴ I 与 l 无关

2-13 解：设破损部分产生的电阻为r，则 $R' = \frac{9}{10}R + \left(\frac{10}{R} + \frac{1}{r}\right)^{-1}$

解得 $r = 0.5\Omega$

2-14 解：(1) 由图可知，作 $U = 110V$ 直线，与A交于 $(110V, 0.335A)$

与B交于 $(110, 0.22A)$ $\therefore P_A = 37W \quad P_B = 24W$

(2) 作其中一条关于 $U = 110V$ 的对称找交点，由图像

解得 $P_A = 18W \quad P_B = 39W$

2-15 解：$t_1 = \frac{Q}{I} = \frac{CU}{I} = 10^9 s \quad \Delta U = 1.5V$

2-16 解：对灯泡 $I_0 = \frac{U_0}{R_0} = 2.25A$. $P_0 = I_0U_0 = 10.125A$

又∵ $\eta = \frac{P_0}{UI}$ 解得 $I = 2.81A$

$R_2 = \frac{U - U_0}{I} = 0.53\Omega \quad R_1 = \frac{U_0}{I - I_0} = 8\Omega$

$\therefore R = R_1 + R_2 = 8.53\Omega$

当I最小，即 $I = I_0$ 时，η 最大

$R' = \frac{U - U_0}{I_0} = 0.67\Omega \quad \eta_m = \frac{U_0}{U} = 0.75$

2-17 解：由题意，有 $\frac{R}{90+R}\cdot\frac{\frac{180(90+R)}{270+R}}{90 + \frac{180(90+R)}{270+R}} = \frac{\frac{90R}{90+R}}{180 + \frac{90R}{90+R}}$ 解得 $R = 30\Omega$

$\therefore U = 54 \times \frac{72}{72+90} \times \frac{30}{90+30} = 6V$

2-18 解 (1) $I = \frac{\Sigma \varepsilon_i}{\Sigma r_i} = \alpha$

(2) $U_{AB} = \sum_{k}^{k} \varepsilon_i - I r_i = 0$

2-19 解 $\frac{2\varepsilon}{R+R_1+R_2} \cdot R_2 = \varepsilon$. 解得 $R = R_2 - R_1$

2-20 解：设通过 R_1 和 R_2 的电流为 I_1 和 I_2（I_1 向左. I_2 向右）

有. $I_1 \cdot R_1 = (I_2 - I_1) r_1 - \varepsilon_1$ ‥①

$I_1 R_1 + I_2 R_2 + I_2 r_2 + \varepsilon_2 = 0$ ‥②

解得 $I_1 = -1A \quad I_2 = -0.5A$

2-21 解：将 A.B 左端等效为一个电压源.

$\varepsilon' = \frac{\varepsilon_0 \cdot R_3}{R_2+R_3}$. $r = \frac{R_2 R_3}{R_2+R_3}$

$\therefore I = \frac{\varepsilon' + \varepsilon}{r+R} = \frac{\varepsilon_0 R_3 + \varepsilon(R_2+R_3)}{RR_2 + RR_3 + R_2R_3}$

2-22 解：设电流分别为 $I_1. I_2. I_3$（以流向 O 为正）

$I_1 R_1 + I_3 R_3 = 5$ ‥① $\quad I_1 R_1 + I_2 R_2 = 4$ ‥② $\quad I_1 + I_2 + I_3 = 0$ ‥③

解得 $I_1 = 0.2A$

2-23 解：易知 $R_{13} = \frac{3}{4}\Omega \quad R_{24} = \frac{4}{3}\Omega$

$\therefore U_{13} = 9V \quad U_{24} = 16V \quad \therefore I_1 = 9A \quad I_3 = 8A$

$\therefore I_{CD} = 1A$. 从C向D

2-24 解：设干路电流为I. 支路电流为 $I_1. I_2$ 有

$\varepsilon_1 = I_1 r_1 + IR$ ·① $\quad \varepsilon_2 = I_2 r_2 + IR$ ‥② $\quad I = I_1 + I_2$ ‥③

又 $\because I = \frac{\varepsilon}{R+r}$ ‥④ 解得 $\varepsilon = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} \quad r = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}$

n个时. $\varepsilon = \sum_{i=1}^{n} \frac{\varepsilon_i}{r_i} \Big/ \sum_{i=1}^{n} \frac{1}{r_i} \quad r = \left(\sum_{i=1}^{n} \frac{1}{r_i}\right)^{-1}$

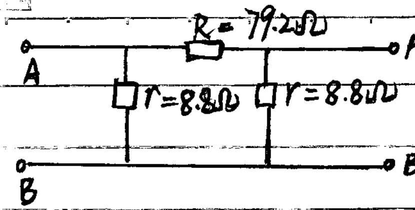
2-25 解：$P_{电池} = \frac{\varepsilon^2}{4R} = \frac{5}{4}W \quad \therefore n_0 = \frac{P}{P_{电池}} = 120$ 个

设 x组 每组 y个电池串联. 有. $\frac{yR}{x} = \frac{U^2}{P}$ ‥①

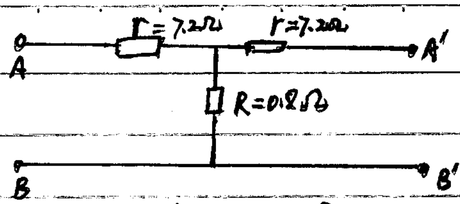
$xy = 120$ ‥② $\quad$ 联立解得 $\quad x = 6 \quad y = 20$

$\therefore$ 最少需120个电池. 分 6组. 每组 20个电池串联 $\quad$ 标答有误

2-26 解：电路图. 如图所示.



或

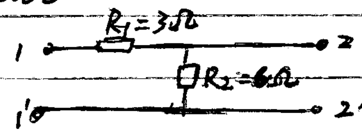


2-27 解：由题意，有 $\frac{U_0}{R_1+R_3}\cdot R_3=U\cdots①$ $\frac{U_0}{R_1+\frac{R_2R_3}{R_2+R_3}}\cdot\frac{R_3}{R_2+R_3}=I\cdots②$

$\frac{U_0}{R_2+R_3}\cdot R_3=U\cdots③$

解得 $R_1=R_2=10\Omega$ $R_3=20\Omega$

2-28 解：(i) 如图所示



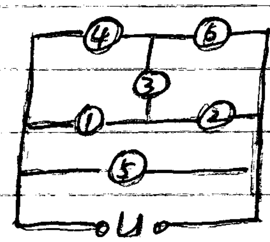
(ii) $I_2=\frac{R_2}{R_2+R_L}\cdot\frac{E}{r+R_1+\frac{R_2R_L}{R_2+R_L}}=0.5A$

$\therefore P_L=I_2^2\cdot R_L=1.5W$

2-29 解：取电源负极为电势零点，$\therefore U_d=U_b=0$ $U_a=10V$ $U_c=5V$

$\therefore I_2=1A$ $I_3=0.5A$ $\therefore I=I_2+I_3=1.5A$

2-30 解：等效电路图如图



可知 $U_3=0$

若 $U_5=10V$ 则 $U_1=U_2=U_4=U_6=5V$

若 U_1、U_2、U_4、U_6 有一个为10V，则其余为10V，

$U_5=20V$

2-31 解：易知 $U_2=7V$ 由节点电流方程可知 $I_1+I_3=I_2$

$\therefore U_1+U_3=U_2$ 即 $U_3=U_2-U_1=5V$

2-32 解：$I_{U_1}=I_2-I_1=0.25mA$ $\therefore R_V=\frac{U_1}{I_{U_1}}=1\ kV$

$R_A=\frac{U_1}{I_1}=333\Omega$ $E=U_1+I_2(R_V+R_A)=1.58V$

2-33 解：设表头满偏电流为 I_0 (mA) 内阻为 r (Ω) 有

$\frac{I_0}{1-I_0}=\frac{R_1+R_2+R_3}{r}\cdots①$ $\frac{I_0}{10-I_0}=\frac{R_2+R_3}{r+R_1}\cdots②$ $\frac{I_0}{100-I_0}=\frac{R_3}{r+R_1+R_2}\cdots③$

整理得 $I_0(r+160)=160$

$\therefore$ 应将B表头串联40Ω的电阻使用

2-34 解：易判断 $R>20k\Omega$

由题意，有 $\frac{E}{\frac{50R_g}{50+R_g}+R}\cdot\frac{50}{50+R_g}=3.9I_0\cdots①$

$$\frac{E}{\frac{100R_g}{100+R_g}+R}\cdot\frac{100}{100+R_g}=1.2I_0\cdots②$$

$$\frac{E}{\frac{10R_g}{10+R_g}+220000}\cdot\frac{10}{10+R_g}=7.8I_0\cdots③$$

小主处理后. 解得 $R=120\,k\Omega$

2-35 解: $\frac{U_{AB}}{U_{BC}}=\frac{200}{\frac{100r}{100+r}}$ 解得 $r=100\,k\Omega$

b图 $\frac{U_{BC}'}{U_{AB}'}=\frac{12-x}{x}=\frac{100}{\frac{200\times100}{200+100}}=\frac{3}{2}$

解得 $x=4.8V$

2-36 解: $\frac{1}{r}=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}$ 解得 $r=1\Omega$

测 AB. AC. BC 间电阻.

若外围电阻不准 (如AB) 则 $r_{CA}=r_{BC}\neq r_{AB}$ 且 $r_{AB}\neq 1\Omega$

若内侧电阻不准确 (如AD) 则 $r_{AC}=r_{AB}\neq r_{BC}$ 且 $r_{BC}=1\Omega$

2-37 解: 设 $AC=x$ 在A端导线和屏蔽金属包皮间测 i_A

在B端测得 i_B

则有 $E=x\cdot R_0\cdot i_A=(l-x)R_0\cdot i_B$

解得 $x=\frac{l i_B}{i_A+i_B}$

2-38 解: $(50+50-x)\cdot 6=r+6x$ 解得 $x=20\,km$

2-39 解: $220\times\frac{R}{xR_0+R}=40\cdots①$ $300\times\frac{R}{(l-x)R_0+R}=40\cdots②$

解得 $x=\frac{9}{22}l=20.5\,km$ 标答有误

2-40 解: (1) 共有 $2^3=8$ 种可能情况. 如下表

V_1	0	1	0	0	1	1	0	1
V_2	0	0	1	0	1	0	1	1
V_3	0	0	0	1	0	1	1	1
V_B	0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{7}{12}$

∴有8种可能的电压值

(2) 若 $V_i=1V$ 则 $U_i=\frac{1}{3}V$ ∴ $U_{i\to B}=\frac{1}{3}\cdot\left(\frac{1}{2}\right)^{n-i}$

∴ $U_B=\sum U_{i\to B}=\frac{2}{3}\left[1-\left(\frac{1}{2}\right)^n\right]$

∴ $n\to\infty$ 时 $U_B=\frac{2}{3}V$

2-41 解: 如图所示.

易知 $i_2=6A$ $i_3=-1A$

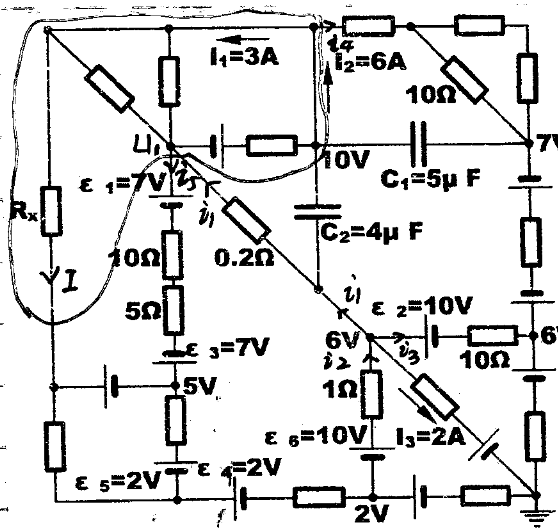
∴ $i_1=5A$ ∴ $U_1=5V$

又∵ $i_4=3A$ $i_5=0$

对图示区域，流入=流出

有 $i_1=i_4+i_5+I$

解得 $I=2A$，方向如图



图复15 － 6

2-42 解: $C=\frac{\varepsilon_0\varepsilon_r S}{d}$

∴ $I=\frac{U}{R}=\frac{US}{\rho d}=\frac{UC}{\rho\varepsilon_0\varepsilon_r}$

$=1.5\mu A$

2-43 解: (1) ∵ $I_1=I_2$ 又 $I_1=\frac{U_1}{R_1}=\frac{E_1S}{\rho_1}$ $I_2=\frac{E_2S}{\rho_2}$

∴ $\frac{E_1}{\rho_1}=\frac{E_2}{\rho_2}$ ··① 又∵ $E_1d_1+E_2d_2=V$ ··②

解得 $E_1=\frac{\rho_1V}{\rho_1d_1+\rho_2d_2}$ $E_2=\frac{\rho_2V}{\rho_1d_1+\rho_2d_2}$

(2) $I=\frac{E_1S}{\rho_1}=\frac{SV}{\rho_1d_1+\rho_2d_2}$

(4) $\sigma=\frac{C_2V_2-C_1V_1}{S}=\varepsilon_0V\frac{\varepsilon_{r2}\rho_2-\varepsilon_{r1}\rho_1}{\rho_1d_1+\rho_2d_2}$

(3) $\sigma'=\frac{Q_2'-Q_1'}{S}$ 又∵ $Q_1'=\frac{Q_1}{\varepsilon_{r1}}$ $Q_2'=\frac{Q_2}{\varepsilon_{r2}}$

解得 $\sigma'=\frac{\varepsilon_0V(\rho_2-\rho_1)}{\rho_1d_1+\rho_2d_2}$

2-44 解: 短接桥上电阻，$R_{AB}>100\Omega$

短接其他电阻，200Ω时 $R_{AB}<100\Omega$

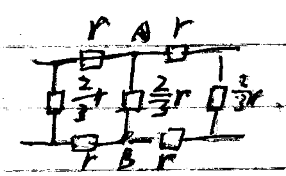
其它均大于100Ω ∴应短接200Ω的电阻

2-45 解: $\frac{1}{R_{34}}=\frac{1}{r_1+r_2}+\frac{1}{r_3+r_4}$ ··① $\frac{1}{R_{12}}=\frac{1}{r_1+r_3}+\frac{1}{r_2+r_4}$ ··②

又∵ $R_{34}=R_{12}$ 解得 $r_1=r_4$ 或 $r_2=r_3$

2-46 解: 由题意知，$\frac{1}{R_{AB}}=\frac{1}{lr_0}+\frac{1}{\frac{\frac{R_{AB}}{2}\cdot lr_0}{\frac{R_{AB}}{2}+lr_0}+lr_0}$ 解得 $R_{AB}=\frac{1}{3}(\sqrt{7}-1)lr_0$

2-47 解: 对称后，$R_{AB}=2\cdot\left(\frac{1}{R}+\frac{1}{R+\frac{R}{3}}\right)^{-1}=\frac{8}{7}R$



2-48 解: 利用等势点，将网络变为如图所示

设无穷二端网络电阻为 R_x

则 $\frac{R_x\cdot\frac{2}{3}r}{R_x+\frac{2}{3}r}+2r=R_x$ 解得 $R_x=\frac{3+\sqrt{21}}{3}r$

$\therefore R_{AB}=\left(\frac{2}{R_x}+\frac{1}{\frac{1}{3}r}\right)^{-1}=\frac{2\sqrt{21}}{21}r$

2·49 解：将C点拆开，利用简单串并联，解得 $R_{AB}=2.4\Omega$

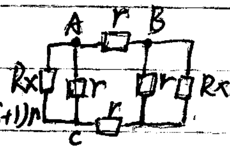
2-50 解：设连接在 P_m 和 P_n 上

要使题中条件满足，需 $U_{AB}=U_{CD}$

即 $\frac{R_m}{r_m}=\frac{R_n}{r_n}$

$\therefore \frac{R_1}{r_1}=\frac{R_2}{r_2}=\cdots=\frac{R_{200}}{r_{200}}$　　$\Delta U=0$

2-51 解：作等效电路图



$\because R_x=2r+\frac{rR_x}{r+R_x}$　　解得 $R_x=(\sqrt{3}+1)r$

$\therefore R_{AC}=\left(\frac{1}{R_x}+\frac{1}{r}\right)^{-1}=(\sqrt{3}-1)r$

$\therefore R_{AB}=\left(\frac{1}{r}+\frac{1}{2R_{AC}+r}\right)^{-1}=\frac{6-\sqrt{3}}{6}\cdot r$

2-52 解：(1) $n=0$ 时，$R_{AC}=\frac{2}{3}r$　　$n=1$ 时，$R'_{AC}=\frac{10}{9}\cdot\frac{1}{2}r=\frac{5}{9}r$

将 R'_{AC} 等效成 r_{AC} 形式，可知 $\frac{2}{3}r_x=\frac{5}{9}r$　$\therefore r_x=\frac{5}{6}r$

递推，得 $R_{(3)}=\left(\frac{5}{6}\right)^3\cdot R_{AC}=\frac{125}{324}r$

(2) 由(1)可知 $R_{(n)}=\left(\frac{5}{6}\right)^n\cdot\frac{2}{3}r$

(3) $R_n=\left(\frac{5}{6}\right)^n\cdot\frac{2}{3}r$　又$\because L=\left(\frac{1}{2}\right)^n\cdot L_0$

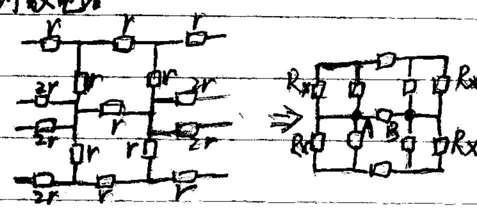
$\therefore R_n=k\cdot L^s$.　$s=\frac{\ln\frac{5}{6}}{\ln\frac{1}{2}}=0.263$

2-53 解：a图，$C_1.C_2.C_3$ 并联　$\therefore C_a=C_1+C_2+C_3$

b图，将中间电容拆掉，$C_b=\frac{1}{2}\cdot 2C_0=C_0$

2-54 解：将电容全部看做电阻，利用倒数电容

将中间部分拆开，如图



$R_x=r+2r+\frac{rR_x}{r+R_x}$

解得 $R_x=\frac{3+\sqrt{21}}{2}r$

$\therefore R_{AB}=\frac{rR}{R+r}$，其中 $R=\frac{1}{2}\left(r+\frac{2rR_x}{r+R_x}\right)=\frac{1}{2}(\sqrt{21}-2)r$

$\therefore R_{AB}=\left(1-\frac{2\sqrt{21}}{21}\right)r$

$\therefore C_{AB}=\frac{21+2\sqrt{21}}{17}\cdot C$

2-55 解：$C_{AB}=C_1'+C_2'+C_3'$　　其中 $C_1'=\left[\left(\frac{C_2}{2}+C_1\right)^{-1}+\frac{1}{C_1}+\frac{1}{C_1}\right]^{-1}$

$C_2'=\frac{C_2}{2}$　　$C_3'=\frac{C_3}{2}$　　解得 $C_{AB}=2.9\mu F$

2-56 解: $R_{并} = \left(\frac{1}{R} + \frac{n-2}{2R}\right)^{-1} = \frac{2R}{n}$ ∴ $I = \frac{\varepsilon}{r+R_{并}} = \frac{n\varepsilon}{nr+2R}$

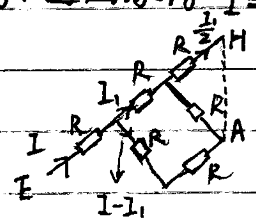
∴ $I' = I\cdot\frac{\frac{2R}{n-2}}{\frac{2R}{n-2}+R} = \frac{2}{n}I = \frac{2\varepsilon}{nr+2R}$ 标答有误

2-57 解: 易知 $E=3V$ 当 $U_R=3V$ 时, 由图像得 $I=0.55A$

∴ $U = E + I\cdot R' = 6.3V$

2-58 解: (1) 读图得 $I=25mA$

EG等效电阻如图.



$R_{EG} = R + \frac{(R+\frac{R}{2})\cdot 2R}{R+\frac{R}{2}+2R} = \frac{13}{7}R$

$U_D + I(R'+R_{EG}+r) = E$. 解得 $R=30\Omega$

如图 $(I-I_1)\cdot 2R = I_1R + \frac{I_1}{2}R$ 解得 $I_1 = \frac{4}{7}I = 7.15mA$

∴ $U_{EA} = IR + I_1R + \frac{I_1}{2}R = 0.695V$

(2) 易得 $R_{BD} = \frac{5}{7}R = 21.4\Omega$

$U_D' + I_D'(R_{BD}+R'+r) = E$. 由图可得 $I_D = 40.5mA$

∴ $U_{BD} = I_D\cdot R_{BD} = 0.867V$ ∴ $U_{EG} = \frac{3}{5}U_{BD} = 0.52V$

2-59 解: 图线I. $U = 6.2 - 23.85I$

图线II $U = 3 - 5I$ 交点 $(0.17A, 2.15V)$

$P_1 = U_1I_1 = (6.2-23.85I)\cdot I$ 易知 $I_1=0.13A$ 时 $(P_1)_m = 0.403W$

$P_2 = U_2\cdot I_2 = (3-5I)\cdot I$ 易知 $I_2 = 0.3A$ 时 $(P_2)_m = 0.45W$

∴ $P_m = 0.45W$ $R = \frac{U_2}{I_2} = 5\Omega$

2-60 解: 由题意有. $\varepsilon = 2U_D + (I_D + \frac{2U_D}{R_2})\cdot R_1$

整理得 $U_D = 1.5 - 0.25\times10^3\cdot I_D$ 作该直线 找交点

由图像得. 交点为 $(1V, 2mA)$ ∴ $I_D = 2mA$ $P_R = \frac{(\varepsilon-2U_D)^2}{R_1} = 16mW$

2-61 解: (1) 断开时. $U_a = 4V$ $U_b = 2V$

(2) 接通时 $U_a = U_b = 4V$

(3) 断开时 电量改变为0. 闭合后 $\Delta Q = -CU_1 + CU_2 = 6\mu C$

2-62 解: 闭合前. $U_1 = U_2 = \frac{\varepsilon}{6}$ ∴ $Q_总 = 0$

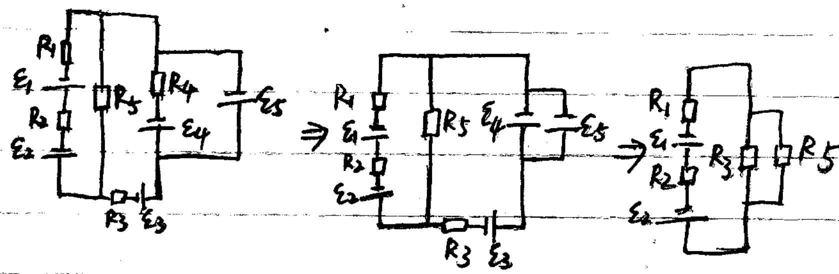
闭合后 $U_1' = \frac{\varepsilon}{2}$ $U_2' = \frac{5}{6}\varepsilon$ ∴ $Q_总' = C(U_1+U_2) = \frac{4}{3}CU$

∴ $\Delta Q = Q_总' - Q_总 = \frac{4}{3}CU$ 标答有误

2-63 解：由题意，得 $\frac{1}{2}C\cdot(2\varepsilon)^2=\frac{1}{2}C\varepsilon^2+C\varepsilon^2+2Q_2$

解得 $Q_2=\frac{1}{4}C\varepsilon^2$

2-64 解：a与e等势 ∴将a与e连接，电路图变换如下



$\therefore I_5=\frac{\varepsilon_1+\varepsilon_2}{R_1+R_2+\frac{R_3}{2}}\cdot\frac{1}{2}=0.35A$

习题三 3-1 解：$2\rho Sag\cdot\frac{a}{2}\sin\theta+\rho Sag\cdot a\sin\theta=BIa\cdot a\cos\theta$

解得 $B=9.35\times10^{-3}T$ 标答有误，将15°当45°算

3-2 解：$\Delta f_1=\frac{m}{2\pi R}\cdot\Delta l\cdot\omega^2R$ ··① $\Delta f_2=B\cdot\frac{\omega}{2\pi}\cdot Q\cdot\Delta l$ ··②

又∵ $(\frac{\Delta f_1}{\Delta l}+\frac{\Delta f_2}{\Delta l})\cdot2R=2T$ ··③

解得 $T=\frac{R\omega}{2\pi}(m\omega+BQ)$

3-3 解：由题意知，$eU=\frac{1}{2}mv^2$ ··① $v\cdot\frac{2\pi m}{eB}\cdot n=L$ ··②

解得 $B=\frac{2\pi n}{L}\cdot\sqrt{\frac{2mU}{e}}$

3-4 解：∵ $r=\frac{mv_1}{qB}\le\frac{R}{2}$ ∴ $\sin\theta\le\frac{qBR}{2mv_0}$

即 $\sin\theta_m=\frac{qBR}{2mv_0}$

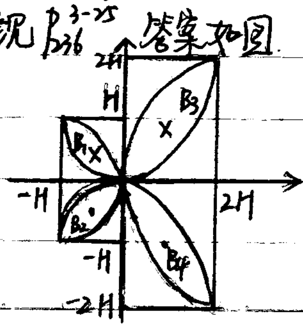
$\eta=\frac{2\pi R^2(1-\cos\theta_m)}{4\pi R^2}=\frac{1}{2}\left[1-\sqrt{1-(\frac{qBR}{2mv_0})^2}\right]$

3-5 解：(1) $r=\frac{b}{2\sin\alpha}$ ··① $\frac{1}{2}mv^2=eU$ ··② $evB=\frac{mv^2}{r}$ ··③

解得 $B=3.7\times10^{-3}T$

(2) $k\cdot\frac{2\pi m}{qB}\cdot v\cos\alpha=b$ 解得 $B=6.7k\times10^{-3}T$ α取60°

3-6 解：参见 P_{236}^{3-25} 答案如图

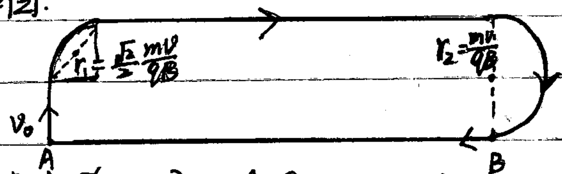


$B_1=B_2=\frac{mev}{eH}$

$B_3=B_4=\frac{mev}{2eH}$

答案不唯一

3-7 解：如图.



3-8 解：由题意，有 $v^2=2gl\cdot(\cos\theta-\cos\alpha)$ ··① $mg\cos\theta+m\frac{v^2}{l}\ge qvB$ ··②

整理得 $B\le\frac{3m\sqrt{g}\cos\theta-2m\sqrt{g}\cos\alpha}{q\sqrt{2l(\cos\theta-\cos\alpha)}}=\frac{m\sqrt{g\cos\alpha}}{q\sqrt{l}}\cdot\frac{3x-2}{\sqrt{2x-2}}$ 其中 $x=\frac{\cos\theta}{\cos\alpha}$

设 $f(x)=\frac{3x-2}{\sqrt{2x-2}}$ 求导得 $x=\frac{4}{3}$ 时，$f(x)_m=\sqrt{6}$

∴ $B\le\frac{m}{q}\cdot\sqrt{\frac{6g\cos\alpha}{l}}$

3-9 解：(1) 易知粒子在YOZ平面作匀圆，X正方向匀减速直线运动，速度减到0

以后沿X负方向作匀加速直线运动

(2) $x=\frac{2\pi m}{qB}\cdot v_0\cos60^\circ-\frac{1}{2}\cdot\frac{qE}{m}\cdot\left(\frac{2\pi m}{qB}\right)^2=1.98\times10^{-5}m$

(3) $x'=v_0\cos60^\circ\cdot t-\frac{1}{2}\cdot\frac{qE}{m}\cdot t^2=0$

空间范围为一圆柱. $r=5.5\times10^{-8}m$. $l=5\times10^{-4}m$

∴ $V=4.75\times10^{-18}\ m^3$

3-10 解：$qE=qv\cdot B\cos\varphi$ ··① $h=v\cdot\sin\varphi\cdot\frac{2\pi m}{qB}$ ··②

解得 $h=\frac{2\pi mE}{eB^2}\tan\varphi=6.1\ cm$

3-11 解：易知 $v=\frac{E}{B}$ ∴ $F=\frac{\Delta mv}{\Delta t}=\frac{I\Delta t\cdot mv}{e\Delta t}=\frac{ImE}{eB}=2\times10^{-5}N$ 速度选择器

3-12 解：本题为XOZ平面匀圆与Y轴正向匀加速直线运动合成

(1) $y_n=\frac{1}{2}\cdot\frac{qE}{m}\cdot\left(n\cdot\frac{2\pi m}{qB}\right)^2=\frac{2n^2\pi^2mE}{qB^2}$

(2) $v_y=\frac{qE}{m}\cdot n\frac{2\pi m}{qB}=\frac{2\pi nE}{B}$ ∴ $\alpha=\arctan\frac{v_0B}{2\pi nE}$

3-13 解：(1) X轴正反向分别加 $v_0=\frac{E}{B}$ ∴ 水平方向的匀直与xoy平面匀圆叠加

易解得 $x(t)=\frac{mE}{qB^2}(\omega t-\sin\omega t)$ $y(t)=\frac{mE}{qB^2}\cdot(1-\cos\omega t)$

其中 $\omega=\frac{qB}{m}$ [illegible]

(2) $S=2\pi\cdot\frac{mE}{qB^2}=\frac{2\pi mE}{qB^2}$ (3) $\overline{v_x}=v_0=\frac{E}{B}$

3-14 解：$\frac{kQ^2}{r^2}=qv\cdot\frac{a}{r^2}\cdot\tan\theta$ ··① $qv\cdot\frac{a}{r^2}\cdot\frac{1}{\cos\theta}=m\frac{v^2}{R}$ ··② $r=\frac{R}{\sin\theta}$ ··③

解得 $R=\frac{a^2}{km}\cdot\frac{\sin^3\theta}{\cos\theta}$

3-15 解：$qvB-\frac{kQq}{x^2}\cdot\sin\theta=\frac{mv^2}{R}$ ··① $\frac{kQq}{x^2}\cos\theta=mg$ ··② $\tan\theta=\frac{R}{y}$ ··③

$x^2=R^2+y^2$ ··④ $v=\omega R$ ··⑤ 解得 $y=\frac{mg}{\omega(qB-m\omega)}$

3-16 解：令 $v_0=\frac{U}{Bd}$. 向上的抵消重力. 向下的作圆周运动

∴ $\frac{2mv_0}{qB_{min}}=d$ ∴ $B_{min}=\frac{1}{ed}\cdot\sqrt{2meU}$

3-17 解：(1) $v_y=v_0\sin\varphi$ $v_x=v_0-v_0\cos\varphi$

∴ $v^2=v_x^2+v_y^2=2v_0^2(1-\cos\varphi)$

又∵ $v_0=\frac{E}{B}$, $y=\frac{mE}{qB^2}(1-\cos\varphi)$

代入解得 $v=\sqrt{\frac{2qyE}{m}}$ 题目中 g 应为 q

(2) $v_{ym}=2v_0=\frac{2E}{B}$

3-18 解：由题意,有, $qv_0B_0=mg$ ①

$\frac{2\cdot mv_0}{qB_0}=h$ ② 解得 $B_0=\frac{m}{q}\sqrt{\frac{2g}{h}}$

3-19 解：易知脱离时 $qv_1B=mg$ ① 令 $v_2=\frac{E}{B}$ ②

易知 $v_m=\sqrt{v_1^2+v_2^2}+v_2=7.4\,m/s$

3-20 解：$\Delta p=\frac{F}{S}=\frac{BI\cdot b}{ab}=\frac{BI}{a}=0.5\,kPa$

3-21 解：由左手定则易知,板终M板带正电 ∴电流应由P→Q

$U=Ed=Bvd$ $r=2\cdot\rho\frac{c}{ab}$ $I=\frac{U}{R+r}$

解得 $I=\frac{Bvd}{R+\frac{2\rho c}{ab}}$

3-22 解：$v_{min}=v_0\cos\theta=\frac{v_0qE}{\sqrt{(mg)^2+(qE)^2}}$

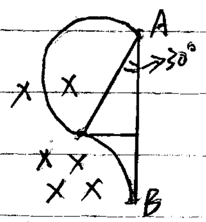
题意不明 3-23 解：$E_\perp=vB$ ∴ $E_{//}=\frac{v}{\mu}=\eta vB$ 解得 $\mu=\frac{1}{\eta B}=3.2\times10^{-3}\,m^2/(V\cdot s)$

3-24 解：∵ $\frac{2\pi m}{qB}=\frac{1}{f}$. 解得 $B=\frac{2\pi mf}{e}=1.6\,T$

又∵ $R=\frac{mv}{qB}$ ∴ $E_{km}=\frac{1}{2}m\left(\frac{qBR}{m}\right)^2=2.6\times10^{-12}\,J$

3-25 解：(1) $\frac{2\cdot mv}{eB}=l$. 即 $v_{max}=\frac{eBl}{2m}$

(2) 区域如图. $l_{总}=(1+\sqrt{3})l$



3-26 解：(1) $\frac{1}{2}mv_0^2=Ue$ ① $\frac{mv_0}{eB_2}=R$

未给数据 解得 $B_2=\frac{1}{R}\sqrt{\frac{2mU}{e}}$

(2) 因题意,有, $\frac{2\pi m}{qB_1}\cdot4=\frac{2\pi m}{qB_2}$ ∴ $B_1=4B_2=\frac{4}{R}\sqrt{\frac{2mU}{e}}$

3-27 解：$\sin\theta_m=\sqrt{\frac{B_0}{B_m}}=\frac{1}{2}$ ∴ $\theta_m=30°$

∴ $k=\frac{2\pi R^2(1-\cos\theta)\times2}{4\pi R^2}=1-\frac{\sqrt{3}}{2}$

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习题四. 4-1 解：(1) 易知 $I=\frac{Blv}{R+r}$ $\therefore P_R=\frac{B^2l^2v^2R}{(R+r)^2}$

$\therefore r$ 最小，即 $\theta=\frac{\pi}{2}$ 时 P_R 最大

(2) $P_{MN}=B^2l^2v^2\cdot\frac{1}{R^2+\frac{r^2}{R}+2r}$

1° 若 $l\le 1m$ 则 $\theta=\arcsin l$ 时 P_{MN} 最大

2° 若 $l>1m$ 则 $\theta=\frac{\pi}{2}$ 时 P_{MN} 最大

4-2 解：将 oab 平面在原平面内投影

此转动可看成 C点的平动与杆绕C点转动的叠加

又∵转动时 U_{ab} 无贡献 $\therefore U_{ab}=-B\cdot\frac{\sqrt{2}}{2}l_0\cdot\omega l_0=-\frac{\sqrt{2}}{2}B\omega l_0^2$

注意：此题注意读题！教书图(b)中所标示的转动方向ω有误，实际应为杆在oab平面绕O作圆周运动。v_c 与 oc 垂直且在 oab平面内。

4-3 解：(1) 当OP经过YOZ平面的瞬间，两端电势相等。

(2) 当OP处于YOZ平面右侧时，$U_P>U_O$。

(3) $(U_{PO})_m=B\cdot l\cos\theta\cdot\bar{v}=\frac{1}{2}\omega l^2B\sin\theta\cos\theta$

4-4 解：$E=\frac{\Delta\Phi}{\Delta t}=\frac{\mu_0\cdot\Delta I}{2R}\cdot\frac{\pi R^2}{\Delta t}=\rho\cdot\frac{2\pi R}{\pi\cdot(\frac{d}{2})^2}$ 答案中间约3.

解得 $\rho_{max}=7.82\times10^{-29}\,\Omega\cdot m$ 即 $\rho<7.82\times10^{-29}\,\Omega\cdot m$

4-5 解：易知 $E_m=Bv_0\cdot\Delta l$

4-6 解：$BIa\frac{a}{2}=mg\cdot\frac{a}{2}\cdot\sin\omega t$ ·· ① $\mathcal{E}-\frac{1}{2}Ba^2\omega=IR$ ·· ②

解得 $\mathcal{E}=\frac{1}{2}Ba^2\omega+\frac{mgR}{aB}\cdot\sin\omega t$

4-7 解：(1) 易知 $\mathcal{E}=\frac{1}{2}Ba^2\omega$

(2) 当 $R_外=R_内$ 即 $R=\frac{r}{4}$ 时，$P_m=\frac{\mathcal{E}^2}{4r_内}=\frac{B^2a^4\omega^2}{4r^2}$

4-8 解：(1) $I=\frac{\frac{1}{2}Bl^2\omega_1-\frac{1}{2}Bl^2\omega_2}{\frac{r}{4}+\frac{r}{4}}=\frac{Bl^2(\omega_1-\omega_2)}{r}$

$BIl\cdot\frac{l}{2}=Fl$ 解得 $\omega_2=\omega_1-\frac{2Fr}{B^2l^3}$

(2) $P=\frac{1}{2}Bl^2\omega_1\cdot I=Fl\omega_1$

4-9 解：$A=\frac{v}{\omega}$ ·· ① $E=Blv$ ·· ② $F=BIl$ ·· ③

联立解得 $F=0.16N$

4-10 解：(1) $|BIL-mg\sin\alpha|\le\mu mg\cos\alpha$ 解得 $1.5\Omega\le R\le4.7\Omega$

(2) $\frac{\mathcal{E}-Blv}{R+r}\cdot Bl=mg\sin\alpha+\mu mg\cos\alpha$ 解得 $v=15\,m/s$

4-11 解：(1) I 变大 $\therefore V_1=\varepsilon-Ir$ 变小． $V_2=IR$ 变大

(2) $mg\sin\theta-Bl\cos\theta\dfrac{E+Blv\cos\theta}{R+r}=ma$

解得 $v=6.25\,m/s$ 代入解得 $U_1=\varepsilon-Ir=6.75V$

标答有误，注意B方向 图中画错了，该题

4-12 解：$D\cdot 4L\cdot S\cdot g=\dfrac{B^2L^2v}{\rho\cdot\frac{4L}{S}}$ 解得 $h=\dfrac{128gD^2\rho^2}{B^4}$

4-13 解：(1) $\varepsilon=B\cdot b\cdot v=2.58\times10^{-3}V$ 标答单位有误

(2) $\sigma=\varepsilon_0\cdot B\cdot v=9.15\times10^{-15}\ C\cdot m^{-2}$

4-14 解：$P_m=\dfrac{\varepsilon^2}{4r}=\dfrac{(BLv)^2}{4\cdot\rho\frac{l}{S}}=\dfrac{B^2v^2lS}{4\rho}=10^{-3}W$

4-15 解：$\sigma=\varepsilon_r\cdot\varepsilon_0\cdot Bv=4.0\times10^{-13}\ C\cdot m^{-2}$

4-16 解：$\vec{M}=|\vec{m}\times\vec{B}|=NBI\cdot 2RL\cdot\sin\theta=mgR\sin\theta$

解得 $I=\dfrac{mg}{2NBL}=2.15A$

4-17 解：$U=Bdv$． $U'=Bd(v+\Delta v)$

$mg\frac{1}{2}(v+v+\Delta v)\Delta t=\frac{1}{2}m[(v+\Delta v)^2-v^2]+\frac{1}{2}C(U'^2-U^2)$

解得 $a=\dfrac{\Delta v}{\Delta t}=\dfrac{mg}{m+CB^2d^2}=(1-\frac{1}{1000})\cdot g$

又：$m=\rho\cdot\pi R^2\cdot d$ $C=\dfrac{\varepsilon\pi R^2}{d}$ 解得 $B=10^6T$

4-18 解：$Q=\frac{1}{2}\mu u^2=\dfrac{m_1m_2}{2(m_1+m_2)}\cdot v_0^2$

4-19 解：$\dfrac{\Delta\Phi}{\Delta t}=E=\dfrac{\Delta B}{\Delta t}\cdot\pi R^2$ 又 $\because I=\dfrac{E}{\rho\cdot\frac{2\pi R}{\pi r^2}}$ $M=D\cdot\pi r^2\cdot 2\pi R$

解得 $I=\dfrac{M}{4\rho\pi D}\cdot\dfrac{\Delta B}{\Delta t}$

4-20 解：$E=\dfrac{\pi D^2}{4}\cdot b$ $\therefore I=\dfrac{E}{3R}$ $\therefore U_{MN}=I\cdot\dfrac{R}{2}-\dfrac{E}{4}=-\dfrac{1}{48}\pi D^2\cdot b$

4-21 解：(1) 设 acb 上的感应电动势为 E_1，其他导线上为 E_2．

有 $E_1+E_2=\frac{1}{4}\pi D^2\cdot k$

$\therefore I=\dfrac{E_1+E_2}{r+\frac{r}{3}}=\dfrac{3\pi D^2k}{16r}$． $I_G=\dfrac{I}{3}=\dfrac{\pi D^2k}{16r}$

(2) adb 绕 ab 转动无影响 $\therefore I_G'=I_G=\dfrac{I}{3}=\dfrac{\pi D^2k}{16r}$

(3) $I'=\dfrac{E_1+E_2}{\frac{r}{2}+\frac{r}{2}}=\dfrac{\pi D^2k}{4r}$ $\therefore I_G''=\frac{1}{2}I=\dfrac{\pi D^2k}{8r}$

4-22 解：$E=\dfrac{\Delta\Phi}{\Delta t}=\dfrac{\Delta(B_0\cdot\sin\omega t\cdot S\cdot\cos\omega t)}{\Delta t}=BS\omega\cdot\cos2\omega t$

$\therefore f=\dfrac{2\omega}{2\pi}=\dfrac{\omega}{\pi}$

4-23 解：(1) 设 O 与 MN 距离为 d 导杆向滑动的角速率同为 ω

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$\therefore \frac{d\,\omega B q}{2\cos\theta_0}\cos\theta_0 = m\cdot\omega^2\cdot d\tan\theta_0$ 解得 $\omega = \frac{qB}{2m\tan\theta_0}$

(2) $N_y = qE_{感}\cdot\sin\theta - q\cdot v B\cdot\cos\theta$ ..① $qE_{感} = \frac{kx}{\cos\theta}$ ..②

$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\cdot R^2$ ..③ $\cos\theta = \frac{\sqrt{R^2-x^2}}{R}$ ..④

联立解得 $N_y = \frac{q\omega B_0}{2R}(3x^2-2R^2)$

4-24 解：$\mathcal{E}_{ac} = k\cdot S$ ..① $S = \frac{\sqrt{3}}{4}R^2 + \frac{1}{2}\cdot\frac{\pi}{6}\cdot R^2$ ..②

解得 $\mathcal{E}_{ac} = \frac{kR^2}{4}(\sqrt{3}+\frac{\pi}{3})$

4-25 解：$I_1 + I_2 = I_1' + I_2'$ ..① $2I_1 r_1 + I_1' r_1 = k\pi a^2$ ..②

$2I_2 r_2 + 2I_2' r_2 = \frac{3\sqrt{3}}{4}ka^2$ ..③ $I_1 r_1 - I_2 r_2 = (\frac{\pi}{3}-\frac{\sqrt{3}}{4})ka^2$ ..④

解得 $U_{AB} = I_1 r_1 - \mathcal{E}_1 = -\frac{\sqrt{3}}{32}ka^2$

4-26 解：$P = \frac{(k\pi a^2)^2}{2\pi a r_0} = \frac{\pi k^2 a^3}{2r_0}$

4-27 解：(1) $I = \frac{\Delta\Phi}{\Delta t\cdot R} = \frac{\Delta[(B_0-kt)(x_0+vt)\cdot l]}{R\Delta t} = \frac{B_0 l v - k x_0 l - 2kvtl}{R}$

(2) $F = (B_0-kt)Il = \frac{(B_0-kt)(B_0v-kx_0-2kvt)l^2}{R}$

4-28 解：$W = BI_1 l\cdot l + BI_2 l^2$

其中 $I_1 = \frac{Blv}{R_1}$ $I_2 = \frac{Blv}{R_2}$ 又∵ $R_1 = 5r$ $R_2 = \frac{10}{3}R$

代入解得 $W = \frac{B^2 l^3 v}{2r}$

4-29 解：$\frac{2NB\cdot\pi\cdot(\frac{d}{2})^2}{R} = I\Delta t = \Delta q$ 解得 $B = 1.3\times10^{-4}\,T$

4-30 解：$\frac{Q_1^2}{2C} = Q_1 U$ 解得 $Q_1 = 2CU$

$\frac{1}{2}C(U_n^2 - U_{n-1}^2) = C\cdot(U_n + U_{n-1})\cdot U$ 解得 $U_n = U_{n-1} + 2U$

联立得 $U_n = 2nU$ $\therefore Q_n = 2n\cdot CU$

4-31 解：易知 $\mathcal{E}_L = I(R_2+R_3) = \frac{R_2+R_3}{R_1+R_2+r}\cdot\mathcal{E}$

4-32 解：$\Phi = \frac{LI}{N} = 10^{-7}\,Wb$

4-33 解：$\frac{\mathcal{E}}{R_1+r} = I_2$ ..① $\frac{\mathcal{E}}{R_1+R_2+r} = I_1$ ..② 解得 $\mathcal{E} = \frac{I_1 I_2}{I_2-I_1}R_2$

$r = \frac{I_1}{I_2-I_1}R_2 - R_1$

4-34 解：(1) 易知 $R_V = \frac{U_g}{I_g} = 4\times10^5\,\Omega$

合上K $U_m = \frac{\mathcal{E}}{R_V+R}\cdot R_V = 12V$ $I_m = \frac{\mathcal{E}}{R} = 12A$

(2) 电流表由12A减少到0. 电压表由0变到4.8×10^6V再减到0

(3) 电压表与朕易损坏. 断开K前先取下电压表

4-35 解：$\frac{N^2}{(N_1+N_2)^2}\times 8=\left(\frac{N}{N_2}\right)^2\times 3.5$ 解得 $\frac{N_1}{N_2}=\frac{4\sqrt{7}}{7}-1=0.51$

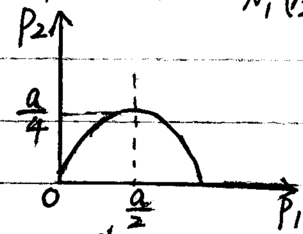
4-36 解：由题意 有 $I_1^2 r_1+I_2U_2=I_1U_1$ ① $U_2=I_2(r_2+R)$ ②

$\frac{I_2}{I_1}=\frac{N_1}{N_2}$ ③ $P_1=U_1I_1$ ④ $P_2=I_2^2R_2$ ⑤

整理得 $P_1=P_2+P_1^2\cdot\frac{N_1^2(r_2+r_1)}{N_2^2\cdot U_1^2}$

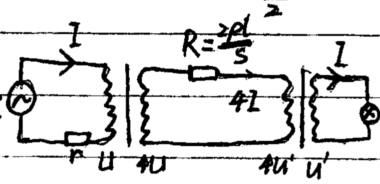
或 $P_2=-\frac{1}{a}\left(P_1-\frac{a}{2}\right)^2+\frac{a}{4}$ 其中 $a=\frac{N_2^2U_1^2}{N_1^2(r_2+r_1)}$

图象如下



标答有误

4-37 解：(1) 如图.



易知 $I=\frac{N\cdot P}{U}=10A$

$\therefore 4U=4U'+4I\cdot\frac{2\rho l}{S}$ 解得 $U=222.5V$

$\therefore \varepsilon=U+Ir=232.5V$

$P_{出}=I\varepsilon-I^2r=2.225kW$

(2) $R_{总}=r+\frac{2\rho l}{S}=5\Omega$

$\therefore N=\frac{\Delta U}{R_{总}I}=\frac{\Delta U\cdot U}{R_{总}\cdot P}=22$个 ∴不能使全部正常发光

4-38 解：(1) 易知 $I=\frac{\varepsilon}{2r}$ $P_{max}=\frac{\varepsilon^2}{4r}$

(2) $U_1=\varepsilon-Ir=\frac{\varepsilon}{2}$ $U_2=\sqrt{RP_m}=\frac{\varepsilon}{2}\cdot\sqrt{\frac{R}{r}}$

(3) $\frac{N_1}{N_2}=\frac{U_1}{U_2}=\sqrt{\frac{r}{R}}$

习题五 5-1 解：对整流后的直流电 $\bar{I}=\frac{1}{2}\cdot\frac{2}{\pi}\cdot\sqrt{2}I_0=\frac{\sqrt{2}}{\pi}I_0$

$\therefore t'=\frac{I_0}{\bar{I}}t_0=\frac{\pi}{\sqrt{2}}t_0$

5-2 解：(1) 易知 $i=\frac{N\cdot B\omega\cdot S}{R+r}\cos(\omega t+\theta)$

(2) $P(t)=i^2\cdot R=\frac{N^2\omega^2B^2S^2R}{(R+r)^2}\cos^2(\omega t+\theta)$

(3) $\vec{M}=N\cdot B\cdot i\cdot S_{有}=\frac{N^2B^2\omega S^2}{R+r}\cos^2(\omega t+\theta)$

5-3 解：(1) $U_{ad}=\sqrt{(U_{ab}-U_{cd})^2+U_{bc}^2}=18.0V$

(2) $i=\frac{U_{bc}}{R}=0.15A$

$\therefore\ i\cdot \omega L = U_{ab}$ 解得 $L=0.21H$

$i\cdot\frac{1}{\omega C}=U_{cd}$ 解得 $C=23.9\mu F$

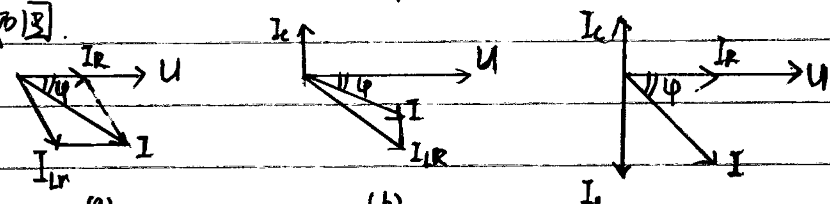
5-4 解：易知 $i=\frac{P}{U_1}=1.33A$

$\therefore R=\frac{U-U_灯}{i}=131\Omega$

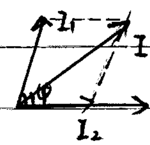
$i\cdot\omega L=\sqrt{U^2-U_灯^2}$ 解得 $L=0.514H$

$i\cdot\frac{1}{\omega C}=\sqrt{U^2-U_灯^2}$ 解得 $C=19.7\mu F$

5-5 解：如图.



5-6 解：如图 $I^2=I_1^2+I_2^2+2I_1I_2\cos\varphi$ ..①



$I_2\cdot R=I_1\cdot\frac{r}{\cos\varphi}$ ..②

$P=I_1^2\cdot r$ ..③

联立解得 $P=\frac{1}{2}(I^2-I_1^2-I_2^2)\cdot R=2.5W$

5-7 解：(1) $X_总=\frac{(R+j\omega L)(-j\frac{1}{\omega C})}{R+j(\omega L-\frac{1}{\omega C})}=\frac{\frac{RL}{C}-\frac{RL}{C}+\frac{R}{\omega^2C^2}}{R^2+(\omega L-\frac{1}{\omega C})^2}+\frac{\frac{1}{\omega^2C^2}-\frac{\omega L^2}{C}-\frac{R^2}{\omega C}}{R^2+(\omega L-\frac{1}{\omega C})^2}\cdot j$

$\therefore$ 虚部为零 即 $\omega_0=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}=3\times10^4\ rad\cdot s^{-1}$

(2) $I=\frac{U}{X_总'}=\frac{URC}{L}=3mA$ $\quad I_L=\frac{U}{X_L'}=0.98A$

$I_c=I_L=0.98A$ 补漏求 I_c

习题六 6-1 解：易知 $f=\frac{1}{2\pi\sqrt{LC}}=39.8Hz$. $\frac{1}{2}CU_m^2=\frac{1}{2}LI^2$ 解得 $U_m=5000V$

6-2 解：$f=\frac{c}{\lambda}=\frac{1}{2\pi\sqrt{LC}}$ 易得 $C_{max}=390\ pF$ $\quad C_{min}=43\ pF$

6-3 解：f最大对应 L.C 均为最小 $\therefore L_{min}=2.27\times10^{-4}H$

$L_{max}=2.52\times10^{-4}H$

6-4 解：(1) $L\frac{di}{dt}+\frac{q}{C}=0$ 易解得 $i(t)=U_m\sqrt{\frac{C}{L}}\cdot\sin\frac{1}{\sqrt{LC}}t$

(2) $\frac{1}{2}C\cdot\varepsilon^2=\frac{1}{2}\cdot\frac{1}{2}CU_m^2$ 解得 $\varepsilon=\frac{\sqrt{2}}{2}U_m$

6-5 解：$f'=\eta f_0$ $\quad\therefore C=\frac{1}{\eta^2}\cdot C_0$ $\quad\therefore W'=\frac{Q^2}{2C'}=\eta^2W_0$

$\therefore \Delta W=W'-W_0=(\eta^2-1)W_0$

6-6 解：(1) 易知 $T_固=2\pi\sqrt{L(C_1+C_2)}=7.02\times10^{-4}s$

(2) $\because \frac{1}{2}(C_1+C_2)U^2 = \frac{1}{2}L I_m^2$. 解得 $I_m = \sqrt{\frac{C_1+C_2}{L}} \cdot U = 8.05\,A$

6-7 解：$L\frac{\Delta i}{\Delta t} + \frac{q_{左}}{C} - \frac{q_{右}}{C} = 0$ $\quad q_{左} + q_{右} = CU_0$

易得 $U_{左} = U_0(1+\cos\omega t)$ $\quad U_{右} = U_0(1-\cos\omega t)$

其中 $\omega = \sqrt{\frac{2}{LC}}$

6-8 解：$\frac{W_{磁}}{W_{电}} = \frac{\frac{1}{2}LI^2}{\frac{1}{2}C\cdot(RI)^2} = \frac{L}{CR^2} = 5$

6-9 解：$\Delta\Phi = L\cdot\Delta I$. 两边积分得 $I_0 = \frac{\Phi}{L}$

$\therefore i(t) = I_0 \cdot \cos\frac{1}{\sqrt{LC}} \cdot t$