CHAPTER 2

Number Systems

(Solutions to Practice Set)

Review Questions

1. A number system shows how a number can be represented using distinct symbols.
2. In a positional number system, the position of a symbol determines the value it represents. In a nonpositional number system each symbol has a value but the position of a symbol normally has no relation to its value; the value of each symbol is fixed.
3. The base (or radix) is the total number of symbols used in a positional number system.
4. The decimal system is a positional number system that uses ten symbols to represent a number. The word decimal is derived from the Latin root *decem* (ten) or *decimalis* (related to ten). In the decimal system, the base is 10.
5. The binary system is a positional number system that uses two symbols (0 and 1) to represent a number. The word binary is derived from the Latin root *bini* (two by two) or *binarius* (related to two). In the binary system, the base is 2.
6. The octal system is a positional number system that uses eight symbols to represent a number. The word octal is derived from the Latin root *octo* (eight) or *octalis* (related to eight). In the octal system, the base is 8.
7. The hexadecimal system is a positional number system with sixteen symbols. The word hexadecimal is derived from the Greek root *hex* (six) and the Latin root *decem* (ten). To be consistent with decimal and binary, it should have been called *sexadecimal*, from Latin roots *sex* and *decem*. In the hexadecimal system, the base is 16.
8. Conversion is easy because there is a direct relationship between the two systems (see the answer to question 9).
9. Four bits in binary is one hexadecimal digit.
10. Three bits in binary is one octal digit.

Multiple-Choice Questions

11. c	12. a	13. b	14. d	15. a	16. b
17. b	18. a	19. c	20. d	21. b	22. a

Exercises

23.

Place values	64	32	16	8	4	2	1	1/2	1/4	1/8	
$(01101)_2$ =		+	+ 0	+ 8	+ 4	+ 0	+ 1	+	+	+	= 13
$(1011000)_2$ =	64	+ 0	+ 16	+ 8	+ 0	+ 0	+ 0	+	+	+	= 88
$(011110.01)_2$ =		+ 0	+ 16	+ 8	+ 4	+ 2	+ 0	+ 0	+ 1/4	+	= 30.25
$(111111.111)_2$ =		+ 32	+ 16	+ 8	+ 4	+ 2	+ 1	+ 1/2	+ 1/4	+ 1/8	= 63.875

24.

Place values	256	16	1	1/16	1/256	
$(AB2)_{16}$ =	10×256	+ 11×16	+ 2×1	+	+	= 2738
$(123)_{16}$ =	1×256	+ 2×16	+ 3×1	+	+	= 291
$(ABB)_{16}$ =	10×256	+ 11×16	+ 11×1	+	+	= 2747
$(35E.E1)_{16}$ =	3×256	+ 5×16	+ 14×1	+ $14 \times 1/16$	+ $1 \times 1/256$	≈ 862.879

25.

Place values	512	64	8	1	1/8	1/64	
$(237)_8$ =		+ 2×64	+ 3×8	+ 7×1	+	+	= 159
$(2731)_8$ =	2×512	+ 7×64	+ 3×8	+ 1×1	+	+	= 1497
$(617)_8$ =		+ 6×64	+ 1×8	+ 7×1	+ $7 \times 1/8$	+	= 399.875
$(21.11)_8$ =		+	+ 2×8	+ 1×1	+ $1 \times 1/8$	+ $1 \times 1/64$	≈ 17.141

26.

a. 1234 = $(100\ 1101\ 0011)_2$ as shown below:

0	←	1	←	2	←	4	←	9	←	19	←	38	←	77	←	154	←	308	←	617	←	1234
		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓
		1		0		0		1		1		0		1		0		0		1		0

b. 88 = $(1011000)_2$ as shown below:

0	←	1	←	2	←	5	←	11	←	22	←	44	←	88
		↓		↓		↓		↓		↓		↓		↓
		1		0		1		1		0		0		0

c. 124.02 = $(111\ 1110.00000101)_2$ as shown below:

$$\begin{array}{ccccccccccccccc} 0 & \leftarrow & 1 & \leftarrow & 3 & \leftarrow & 7 & \leftarrow & 15 & \leftarrow & 31 & \leftarrow & 62 & \leftarrow & 124 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & & \mathbf{0} & & \mathbf{0} \end{array}$$

$$\begin{array}{cccccccccccccccccc} & .02 & \rightarrow & .04 & \rightarrow & .08 & \rightarrow & .16 & \rightarrow & .32 & \rightarrow & .64 & \rightarrow & .28 & \rightarrow & .56 & \rightarrow & .12 \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ \bullet & \mathbf{0} & & \mathbf{0} & & \mathbf{0} & & \mathbf{0} & & \mathbf{0} & & \mathbf{1} & & \mathbf{0} & & \mathbf{1} & & \end{array}$$

d. 14.56 = $(1110.100011)_2$ as shown below:

$$\begin{array}{ccccccccccccccccccccc} 0 & \leftarrow & 1 & \leftarrow & 3 & \leftarrow & 7 & \leftarrow & 14 & & .56 & \rightarrow & .12 & \rightarrow & .24 & \rightarrow & .48 & \rightarrow & .96 & \rightarrow & .92 & \rightarrow & .84 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ & & \mathbf{1} & & \mathbf{1} & & \mathbf{1} & & \mathbf{0} & \bullet & \mathbf{1} & & \mathbf{0} & & \mathbf{0} & & \mathbf{0} & & \mathbf{1} & & \mathbf{1} & & \end{array}$$

27.

a. 1156 = $(2204)_8$ as shown below:

$$\begin{array}{ccccccccc} 0 & \leftarrow & 2 & \leftarrow & 18 & \leftarrow & 144 & \leftarrow & 1156 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ & & 2 & & 2 & & 0 & & 4 \end{array}$$

b. 99 = $(134)_8$ as shown below:

$$\begin{array}{ccccccc} 0 & \leftarrow & 1 & \leftarrow & 12 & \leftarrow & 99 \\ & & \downarrow & & \downarrow & & \downarrow \\ & & 1 & & 4 & & 3 \end{array}$$

c. 11.4 = $(13.3146)_8$ as shown below:

$$\begin{array}{cccccccccccccc} 0 & \leftarrow & 1 & \leftarrow & 11 & & .4 & \rightarrow & .2 & \rightarrow & .6 & \rightarrow & .8 & \rightarrow & 4 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ & & 1 & & 3 & \bullet & 3 & & 1 & & 4 & & 6 & & \end{array}$$

d. 72.8 = $(110.6314)_8$ as shown below:

$$\begin{array}{ccccccccccccccccc} 0 & \leftarrow & 1 & \leftarrow & 9 & \leftarrow & 72 & & .8 & \rightarrow & .4 & \rightarrow & .2 & \rightarrow & .6 & \rightarrow & .8 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ & & 1 & & 1 & & 0 & \bullet & 6 & & 3 & & 1 & & 4 & & \end{array}$$

28.

a. 576 = $(237)_{16}$ as shown below:

$$\begin{array}{ccccccc} 0 & \leftarrow & 2 & \leftarrow & 35 & \leftarrow & 567 \\ & & \downarrow & & \downarrow & & \downarrow \\ & & 2 & & 3 & & 7 \end{array}$$

b. 1411 = $(583)_{16}$ as shown below:

$$\begin{array}{ccccccc} 0 & \leftarrow & 5 & \leftarrow & 88 & \leftarrow & 1411 \\ & & \downarrow & & \downarrow & & \downarrow \\ & & 5 & & 8 & & 3 \end{array}$$

c. 12.13 = $(C.2147AE)_{16}$ as shown below:

$$\begin{array}{cccccccccccccc} 0 & \leftarrow & 12 & & .13 & \rightarrow & .08 & \rightarrow & .48 & \rightarrow & .68 & \rightarrow & .88 & \rightarrow & .08 \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\ & & C & \bullet & 2 & & 1 & & 7 & & A & & E & & \end{array}$$

d. 16.5 = $(10.8)_{16}$ as shown below:

$$\begin{array}{ccccccccc} 0 & \leftarrow & 1 & \leftarrow & 16 & & .5 & \rightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 1 & & 0 & \bullet & 8 & & \end{array}$$

29.

$(514)_8$	=		101	001	100			=	1	0100	1100			=	**$(14C)_{16}$**
$(411)_8$	=		100	001	001			=	1	0000	1001			=	**$(109)_{16}$**
$(13.7)_8$	=			001	111	•	111	=		00	1011	•	1110	=	**$(B.E)_{16}$**
$(1256)_8$	=	001	010	101	110			=	0010	0101	1110			=	**$(25E)_{16}$**

30.

$(51A)_{16}$	=	0101	0001	1010			=	010	100	011	010				=	**$(2432)_8$**
$(4E1)_{16}$	=	0100	1110	0001			=	010	011	100	001				=	**$(2341)_8$**
$(BB.C)_{16}$	=		1011	1011	•	1100	=		010	111	011	•	110		=	**$(273.6)_8$**
$(ABC.D)_{16}$	=	1010	1011	1100	•	1101	=	101	010	111	100	•	110	100	=	**$(5274.64)_8$**

31.

$(01101)_2$	=		001	101			=	**$(15)_8$**
$(1011000)_2$	=	001	011	000			=	**$(130)_8$**
$(011110.01)_2$	=		011	110	•	010	=	**$(36.2)_8$**
$(111111.111)_2$	=		111	111	•	111	=	**$(77.7)_8$**

32.

$(01101)_2$	=		1101			=	$\mathbf{(0D)_{16}}$
$(1011000)_2$	=	0101	1000			=	$\mathbf{(58)_{16}}$
$(011110.01)_2$	=	0001	1110	•	0100	=	$\mathbf{(1E.4)_{16}}$
$(111111.111)_2$	=	0011	1111	•	1110	=	$\mathbf{(3F.E)_{16}}$

33.

121	=	0	+	64	+	32	+	16	+	8	+	0	+	0	+	1	=	$\mathbf{(01111001)_2}$
78	=	0	+	64	+	0	+	0	+	8	+	4	+	2	+	0	=	$\mathbf{(01001110)_2}$
255	=	128	+	64	+	32	+	16	+	8	+	4	+	2	+	1	=	$\mathbf{(11111111)_2}$
214	=	128	+	64	+	0	+	16	+	0	+	4	+	2	+	0	=	$\mathbf{(11010110)_2}$

34.

3 5/8	=	0	+	0	+	2	+	1	+	1/2	+	0	+	1/8	+	0	+	0	+	0	+	0	=	$\mathbf{(0011.1010000)_2}$
12 3/32	=	8	+	4	+	0	+	0	+	0	+	0	+	0	+	1/16	+	1/32	+	0	+	0	=	$\mathbf{(1100.0001100)_2}$
4 13/64	=	0	+	4	+	0	+	0	+	0	+	0	+	1/8	+	1/16	+	0	+	1/64	+	0	=	$\mathbf{(0100.0011010)_2}$
12 5/8	=	8	+	4	+	0	+	0	+	0	+	0	+	0	+	0	+	1/32	+	0	+	1/128	=	$\mathbf{(1100.0000101)_2}$

35.

a. binary: $2^6 - 1 = 63$

b. decimal: $10^6 - 1 = 999{,}999$

c. hexadecimal: $16^6 - 1 = 16{,}777{,}215$

d. octal: $8^6 - 1 = 262{,}143$

36.

a. $\lceil 5 \times (\log 10) / (\log 2) \rceil = \lceil 16.6 \rceil = 17$

b. $\lceil 4 \times (\log 10) / (\log 8) \rceil = \lceil 4.4 \rceil = 5$

c. $\lceil 7 \times (\log 10) / (\log 16) \rceil = \lceil 5.8 \rceil = 6$

37.

a. $\lceil 5 \times (\log 2) / (\log 10) \rceil = \lceil 16.6 \rceil = 2$

b. $\lceil 3 \times (\log 8) / (\log 10) \rceil = \lceil 16.6 \rceil = 3$

c. $\lceil 3 \times (\log 16) / (\log 10) \rceil = \lceil 16.6 \rceil = 4$

38.

a. $0.1875 = 0.125 + 0.0625 = (1/8) + (1/16) = (3/16)$

b. $0.640625 = 0.5 + 0.125 + 0.015625 = (1/2) + (1/8) + (1/64) = (41/64)$

c. $0.40625 = 0.25 + 0.125 + 0.03125 = (1/4) + (1/8) + (1/32) = (13/32)$

d. $0.375 = 0.25 + 0.125 = (1/4) + 1/8) = 3/8$

39. Using the result of previous exercise, we can find the equivalent as:

a. $7.1875 = (111)_2 + (0.001)_2 + (0.0001)_2 = (111.0011)_2$

b. $12.540625 = (1100)_2 + (0.1)_2 + (0.001)_2 + (0.000001)_2 = (1100.101001)_2$

c. $11.40625 = (1011)_2 + (0.01)_2 + (0.001)_2 + (0.00001)_2 = (1011.01101)_2$

d. $0.375 = (0.01)_2 + (0.001)_2 = (0.011)_2$

40.

a. $10^{10} - 1 = 999$

b. $22^1 - 1 = 4095$

c. $8^8 - 1 = 16{,}777{,}215$

d. $16^7 - 1 = 268{,}435{,}455$

41.

a. $\lceil \log_2 1000 \rceil = \lceil \log 1000 / \log 2 \rceil = \lceil 9.97 \rceil = 10$

b. $\lceil \log_2 100{,}000 \rceil = \lceil \log 100{,}000 / \log 2 \rceil = \lceil 16.6 \rceil = 17$

c. $\lceil \log_2 64 \rceil = \lceil \log_2 2^6 \rceil = \lceil 6 \times \log_2 2 \rceil = \lceil 6 \rceil = 6$

d. $\lceil \log_2 256 \rceil = \lceil \log_2 2^8 \rceil = \lceil 8 \times \log_2 2 \rceil = \lceil 8 \rceil = 8$

42.

a. 14

b. 8

c. 13

d. 4

43.

a.	17×256^3	+	234×256^2	+	34×256^1	+	14×256^0	=	300,556,814
b.	14×256^3	+	56×256^2	+	234×256^1	+	56×256^0	=	238,611,000
c.	110×256^3	+	14×256^2	+	56×256^1	+	78×256^0	=	1,864,425,678
d.	24×256^3	+	56×256^2	+	13×256^1	+	11×256^0	=	406,326,539

44.

a.	00010001	11101010	00100010	00001110
b.	00001110	00111000	11101010	00111000
c.	01101110	00001110	00111000	01001110
d.	00011000	00111000	00001101	00001011

45.

a. 15

b. 27

c. This is not a valid Roman Numeral (V cannot come before L)

d. 1157

46.

a. XVII

b. XXXVIII

c. LXXXII
d. CMXCIX

47.

a. Not valid because I cannot come before M
b. Not valid because I cannot come before C
c. Not valid because V cannot come before C
d. Not valid because 5 is written as V not VX

48. First, we convert the four numbers to base 20 as shown below:

0	←	12	
		↓	
		12	

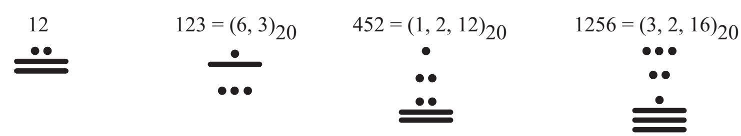
0	←	6	←	123
		↓		↓
		6		**3**

0	←	1	←	22	←	452
		↓		↓		↓
		1		**2**		**12**

0	←	3	←	62	←	1256
		↓		↓		↓
		3		2		**16**

The equivalent Myan numerals are shown in Figure S2.48.

Figure S2.48 *Exercise 48*



49.

a. First, we convert the three numbers to base 60 as shown below:

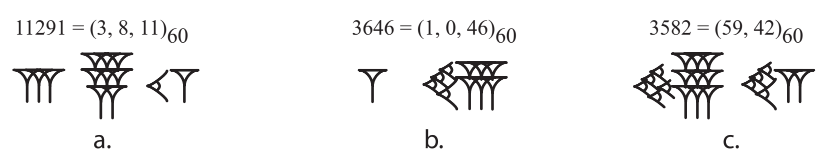
0	←	3	←	188	←	11291
		↓		↓		↓
		3		**8**		**11**

0	←	1	←	60	←	3646
		↓		↓		↓
		1		**0**		**46**

0	←	59	←	3582
		↓		↓
		59		**42**

The equivalent Babylonian numerals are shown in Figure S2.49

Figure S2.49 *Exercise 49*



b. In Babylonian numerals, they used extra space when a zero was needed in the middle of the number. When a zero was need at left, they did not use anything; They probably recognized it from the context.