Computational model of lambda calculus

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1 General Definitions

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Definition 1.1. An enumeration operator (or e-operator) ψ^A is a r.e. set. For any $A \subseteq \mathbb{N}$

$$x \in \psi^A \iff \exists u \ (finite \ D_u \subseteq A)((x, u) \in \psi)$$
 (1)

Definition 1.2. If A is a r.e. set then ψ_A is the enumeration operator defined by it, namely

$$x \in \psi_A^B \iff \exists u \ (D_u is finite)((x, u) \in A \land D_u \subseteq B)$$
 (2)

Definition 1.3. If θ is an enumeration operator then G_{θ} is a well-defined r.e. set defining it, namely

$$(x,u) \in G_{\theta} \iff x \in \theta^{D_u}$$
 (3)

Lemma 1.1. If ψ is an enumeration operator, then $\psi_{G_{\psi}} = \psi$

Definition 1.4. Let η be an assignment of r.e. sets to the variables of lambda calculus. With every λ -term E we inductively associate a r.e. set $[\![E]\!]_{\eta}$:

- 1. $[\![x]\!]_{\eta} = \eta(x)$
- 2. $[E_1E_2]_{\eta} = \psi_{[E_1]_{\eta}}([E_2]_{\eta})$
- 3. $[\![\lambda x.E]\!]_{\eta} = G_{\lambda X.[\![E]\!]_{\eta[x:=X]}}$

Where $\lambda X.[E]_{\eta[x:=X]}$ is a function

$$A \in W \mapsto \llbracket E \rrbracket_{\eta[x:=A]} \tag{4}$$

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References

- $[1]\,$ S. B. Cooper, $Computability\ Theory.\ 2003.$
- $[2]\,$ P. Odifreddi, Classical recursion theory. 1989.