

Computational model of lambda calculus

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1 General Definitions

Definition 1.1. An enumeration operator (or e-operator) Ψ^A is a r.e. set. For any $A \subseteq \mathbb{N}$

$$x \in \Psi^A \iff \exists u (\text{finite } D_u \subseteq A)((x, u) \in \Psi) \quad (1)$$

Definition 1.2. If A is a r.e. set then Ψ_A is the enumeration operator defined by it, namely

$$x \in \Psi_A^B \iff \exists u (D_u \text{ is finite})((x, u) \in A \wedge D_u \subseteq B) \quad (2)$$

Definition 1.3. If θ is an enumeration operator then G_θ is a well-defined r.e. set defining it, namely

$$(x, u) \in G_\theta \iff x \in \theta^{D_u} \quad (3)$$

Lemma 1.1. If Ψ is an enumeration operator, then $\Psi_{G_\Psi} = \Psi$

Proof. TO BE DONE @anton □

Definition 1.4. Let η be an assignment of r.e. sets to the variables of lambda calculus. With every λ -term E we inductively associate a r.e. set $\llbracket E \rrbracket_\eta$:

1. $\llbracket x \rrbracket_\eta = \eta(x)$
2. $\llbracket E_1 E_2 \rrbracket_\eta = \Psi_{\llbracket E_1 \rrbracket_\eta}(\llbracket E_2 \rrbracket_\eta)$
3. $\llbracket \lambda x. E \rrbracket_\eta = G_{\lambda X. \llbracket E \rrbracket_{\eta[x:=X]}}$

Where $\lambda X. \llbracket E \rrbracket_{\eta[x:=X]}$ is a function

$$A \in W \mapsto \llbracket E \rrbracket_{\eta[x:=A]} \quad (4)$$

Lemma 1.2. For any environment η and term t , $\llbracket t \rrbracket_\eta$ is an c.e. set.

Proof. By structural induction on the definition of $\llbracket t \rrbracket_\eta$.

1. $\llbracket x \rrbracket_\eta = \eta(x)$ by definition
2. To show that $\Psi_{\llbracket E_1 \rrbracket_\eta}(\llbracket E_2 \rrbracket_\eta)$ is a c.e. set we prove that Ψ_A^B is an enumeration operator which follows from

$$n \in \Psi_A^B \iff \exists u (D_u \subseteq B < n, u > \in A) \quad (5)$$

3. To be done. □

Lemma 1.3. For the following theorem we will need one lemma ahead for better readability

Proof. TO BE DONE @anton □

Theorem 1.4. *If $E_1 \xrightarrow{\beta} E_2$ then $\llbracket E_1 \rrbracket_\eta = \llbracket E_2 \rrbracket_\eta$ for any η .*

Proof. We will prove one step of the β reduction and then by induction the rest will follow

We have that $(\lambda x E_1)E_2 = E_1[x \mapsto E_2]$ and we will prove that $\llbracket (\lambda x E_1)E_2 \rrbracket_\eta = \llbracket E_1[x \mapsto E_2] \rrbracket_\eta$

$$\begin{aligned}
\llbracket (\lambda x E_1)E_2 \rrbracket_\eta &\stackrel{def.1.4.2}{=} \Psi_{\llbracket \lambda x E_1 \rrbracket_\eta}(\llbracket E_2 \rrbracket_\eta) \\
&\stackrel{def.1.4.3}{=} \Psi_{G_{\Lambda X} \llbracket E_1 \rrbracket_{\eta[x:=X]}}(\llbracket E_2 \rrbracket_\eta) \\
&\stackrel{Lemma1.1}{=} \Lambda X \llbracket E_1 \rrbracket_{\eta[x:=X]}(\llbracket E_2 \rrbracket_\eta) \\
&\stackrel{def\Lambda}{=} \llbracket E_1 \rrbracket_{\eta[x:=\llbracket E_2 \rrbracket_\eta]} \\
&\stackrel{Lemma1.3}{=} \llbracket E_1[x \mapsto E_2] \rrbracket_\eta
\end{aligned}$$

□

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References

- [1] S. B. Cooper, *Computability Theory*. 2003.
- [2] P. Odifreddi, *Classical recursion theory*. 1989.