# Interpretation and Compilation of Languages

Master Programme in Computer Science

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Lecture 3

based on lectures by Jean-Christophe Filliâtre and Léon Gondelman previous editions by João Costa Seco, Luís Caires, and Bernardo Toninho

Heróisdomar, nobrepovo, Naçãovalente, imortal, Levantaihojedenovo Oesplendorde Portugal! Entreas brumas da memória, Ó Pátria, sente-seavoz Dosteus egrégios avós, Quehá-deguiar-teàvitória!

Heróis do mar, nobre povo, Nação valente, imortal, Levantai hoje de novo O esplendor de Portugal! Entre as brumas da memória, Ó Pátria, sente-se a voz Dos teus egrégios avós, Que há-de guiar-te à vitória!

Asarmaseosbarõesassinalados Queda Ocidental praia Lusitana, Pormares nuncadantes na vegados Passarama inda além da Taprobana, Emperigos eguerras esforçados Maisdo que prometia a força humana Eentregente remota edificaram Novo Reino, que tanto sublimaram;

As armas e os barões assinalados Que da Ocidental praia Lusitana, Por mares nunca dantes navegados Passaram ainda além da Taprobana, Em perigos e guerras esforçados Mais do que prometia a força humana E entre gente remota edificaram Novo Reino, que tanto sublimaram;

# Lexical Analysis

Lexical analysis goal is to split the input into "words".

As with natural languages, this splitting phase eases the work of the next step, the syntactic analysis.

The resulting words are called tokens.

# Today: Lexical Analysis

- 1. Regular expressions
- 2. Finite automata
- 3. Lexical analyzer
  Principles
  Construction
- 4. Lexing tools ocamllex iflex
- 5. Lexers in practice
  - ITA Demo: using ocamllex for fun and profit

source = sequence of characters

lexical analysis

sequence of tokens



abstract syntax

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Blanks (spaces, newlines, tabs, etc.) play a role in lexical analysis; they can be used to separate two tokens.

For instance, funx is understood as a single token (identifier funx) and fun x is understood as two tokens (keyword fun and identifier x).

Yet several blanks are useless (as in x + 1) and simply ignored.

Blanks do not appear in the returned sequence of tokens.

Lexical conventions differ according to the languages, and some blanks may be significant.

#### Examples:

- tabs for make
- newlines and indentation in Python or Haskell (indentation defines the structure of blocks)

#### Comments act as blanks

```
fun(* go! *)x -> x + (* adding one *) 1
```

Here the comment (\* go! \*) is a significant blank (splits two tokens) and the comment (\* adding one \*) is a useless blank.

Note: comments are sometimes interpreted by other tools (javadoc, ocamldoc, etc.), which handle them differently in their own lexical analysis

```
val length: 'a list -> int
  (** Return the length (number of elements) of ...
```

#### Which tools?

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To implement lexical analysis, we are going to use

- regular expressions to describe tokens
- finite automata to recognize them

We exploit the ability to automatically construct a deterministic finite automaton recognizing the language described by a regular expression.

# Regular Expressions

#### Let A be some alphabet

$$r ::= \emptyset$$
 empty language  $\mid \epsilon \mid$  empty word  $\mid a \mid$  character  $a \in A$   $\mid rr \mid$  concatenation  $\mid r \mid r \mid$  Alternation  $\mid r \star \mid$  Kleene star

Conventions: in forthcoming examples, star has strongest priority, then concatenation, then alternation e.g.  $r_0 r_1 | r_2 r_{3} \star$  means  $(r_0 r_1) | (r_2 (r_3) \star)$ 

The language defined by the regular expression r is the set of words L(r) defined as follows:

$$L(\emptyset) = \emptyset$$

$$L(\epsilon) = \{\epsilon\}$$

$$L(a) = \{a\}$$

$$L(r_1 r_2) = \{w_1 w_2 \mid w_1 \in L(r_1) \land w_2 \in L(r_2)\}$$

$$L(r_1 \mid r_2) = L(r_1) \cup L(r_2)$$

$$L(r \star) = \bigcup_{n \ge 0} L(r^n) \quad \text{where } r^0 = \epsilon, \ r^{n+1} = r r^n$$

#### With alphabet $\{a,b\}$

• words with exactly three letters

words ending with a

$$(a|b)*a$$

words alternating a and b

$$(b|\epsilon)(ab)\star(a|\epsilon)$$

indeed, we have  $\epsilon = \epsilon(ab)^0 \epsilon$ ,  $a = \epsilon(ab)^0 a$ ,  $b = b(ab)^0 \epsilon$  and

	aa		ab		bb		ba	
			ab	$= \epsilon (ab)^1 \epsilon$			ba	$= b(ab)^0a$
	aba	$= \epsilon (ab)^1 a$	abab	$= \epsilon (ab)^2 \epsilon$		$=b(ab)^1\epsilon$	baba	$= b(ab)^1 a$
ab(a	ab) <sup>n</sup> a	$=\epsilon(ab)^{n+1}a$	a(ba) <sup>n</sup> b	$=\epsilon(ab)^{n+1}\epsilon$	b(ab) <sup>n</sup> ab	$=b(ab)^{n+1}\epsilon$	b(ab) <sup>n</sup> a	$=b(ab)^{n+1}a$

# Integer Literals

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Decimal integer literals, possibly with leading zeros

 $\big(0|1|2|3|4|5|6|7|8|9\big)\big(0|1|2|3|4|5|6|7|8|9\big)\star$ 

### **Identifiers**

Identifiers composed of letters, digits and underscore, starting with a letter

$$(a|b|...|z|A|B|...|Z)(a|b|...|z|A|B|...|Z|_0|1|...|9)$$

### Floating Point Literals

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$$dd\star (.d\star | (\epsilon|.d\star)(e|E)(\epsilon|+|-)dd\star)$$

with 
$$d = 0|1|...|9$$

Comments such as (\* ... \*), (\*\* ... \*) but not nested (that is, not (\* ... (\* ... \*) ... \*)), can be described with the following regular expression

$$( * (* \star r_1 | r_2) \star * * \star)$$

where  $r_1 =$  all characters but \* and ) and  $r_2 =$  all characters but \*

#### **Nested Comments**

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Regular expressions are not expressive enough to describe nested comments (we say that the language of balanced parentheses is not regular).

We will explain later how to get around this problem.

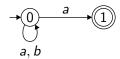
# Finite Automata

#### Definition

A finite automaton over some A is a tuple (Q, T, I, F) where

- Q is a finite set of states
- $T \subseteq Q \times A \times Q$  is a set of transitions
- $I \subseteq Q$  is a set of initial states
- $F \subseteq Q$  is a set of final states

Example: 
$$Q = \{0,1\}$$
,  $T = \{(0,a,0),(0,b,0),(0,a,1)\}$ ,  $I = \{0\}$ ,  $F = \{1\}$ 



A word  $a_1 a_2 ... a_n \in A^*$  is recognized by the automaton (Q, T, I, F) if and only if

$$s_0 \stackrel{a_1}{\rightarrow} s_1 \stackrel{a_2}{\rightarrow} s_2 \cdots s_{n-1} \stackrel{a_n}{\rightarrow} s_n$$

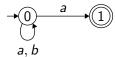
with  $s_0 \in I$ ,  $(s_{i-1}, a_i, s_i) \in T$  for all i, and  $s_n \in F$ .

The language defined by an automaton is the set of words it recognizes.

### Theorem (Kleene, 1951)

Regular expressions and finite automata define the same languages.

$$(a|b) \star a$$



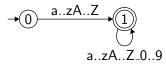
# Integer Literals

#### Regular expression

$$(0|1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)$$
\*

#### Regular expression

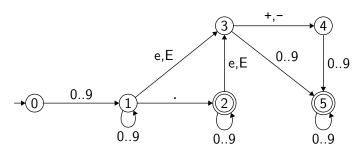
$$(a|b|...|z|A|B|...|Z)(a|b|...|z|A|B|...|Z|\_|0|1|...|9)$$



#### Regular expression

$$dd \star (.d \star | (\epsilon | .d \star)(e|E)(\epsilon | + | -)dd \star)$$

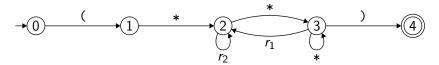
where 
$$d = 0|1|...|9$$



#### Regular expression

$$\boxed{( * (* \star r_1 | r_2) \star * * \star )}$$

where  $r_1 =$  all characters but \* and ) and  $r_2 =$  all characters but \*



# Lexical Analyzer

# Lexical Analyzer

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A lexical analyzer is a finite automaton for the "union" of all regular expressions describing the tokens.

However, it differs from the mere analysis of a single word by an automaton, since

- we must split the input into a sequence of words
- there are possible ambiguities
- we have to build tokens (final states contain actions)

# **Ambiguities**

The word funx is recognized by the regular expression for identifiers, but contains a prefix recognized by another regular expression (keyword fun)

⇒ we choose to match the longest token

The word **fun** is recognized by the regular expression for the keyword **fun** but also by that of identifiers

 $\Rightarrow$  we order regular expressions using priorities

With the three regular expressions

a lexical analyzer will fail on input

abc

(ab is recognized, as longest, then failure on c)

Yet the word abc belongs to the language (a|ab|bc)\*

Tokens are output one by one, on demand (from the syntax analyzer).

The lexical analyzer memorizes the position where the analysis will resume.



When a new token is required, we start from the initial state of the automaton, from position current\_pos.

As long as a transition exists, we follow it, while memorizing any token that was recognized (any final state that was reached).



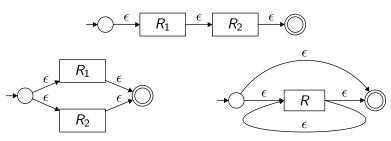
When there is no transition anymore, there are two cases:

- if a token was recognized, we return it and current\_pos←last
- otherwise, we signal a lexical error

# **Building the Automaton**

### Thompson Algorithm

We can build a finite automaton corresponding to a regular expression by means of an intermediate non-deterministic automaton.



We can then convert it into a deterministic automaton, even minimize it

but...

# Direct Construction (Berry & Sethi, 1986)

... we can also build directly a deterministic automaton.

Starting idea: we can match characters from a recognized word and those from and those appearing in a regular expression.

Example: the word aabaab is recognized by  $(a|b) \star a(a|b)$ , as follows

a 
$$(a|b) * a(a|b)$$
  
a  $(a|b) * a(a|b)$   
b  $(a|b) * a(a|b)$   
a  $(a|b) * a(a|b)$   
a  $(a|b) * a(a|b)$   
b  $(a|b) * a(a|b)$ 

Let us distinguish the different characters of the regular expression:

$$(a_1|b_1) \star a_2(a_3|b_2)$$

We build an automaton where the states are sets of characters.

The state s recognizes suffixes of L(r) for which the first character belongs to s.

Example: the state  $\{a_1, a_2, b_1\}$  recognizes words where the first is either an a matching  $a_1$  or  $a_2$ , or a b matching  $b_1$ .

To build transitions from  $s_1$  to  $s_2$ , one needs to compute the characters that can occur after another in a recognized word (follow).

Example: still using 
$$r = (a_1|b_1) \star a_2(a_3|b_2)$$
, we have

$$follow(a_1, r) = \{a_1, a_2, b_1\}$$

To compute *follow*, we need to compute the first (resp. last) possible characters in a recognized word (*first*, resp. *last*)

Example : still using  $r = (a_1|b_1) \star a_2(a_3|b_2)$ , we have

$$first(r) = \{a_1, a_2, b_1\}$$
  
 $last(r) = \{a_3, b_2\}$ 

To compute *first* and *last*, we need a last notion : does the empty word belongs to the recognized language ? (*null*)

```
null(\emptyset) = 	ext{false}
null(\epsilon) = 	ext{true}
null(a) = 	ext{false}
null(r_1r_2) = null(r_1) \wedge null(r_2)
null(r_1|r_2) = null(r_1) \vee null(r_2)
null(r_*) = 	ext{true}
```

We can thus define first...

$$first(\emptyset) = \emptyset$$
  
 $first(\epsilon) = \emptyset$   
 $first(a) = \{a\}$   
 $first(r_1 r_2) = first(r_1) \cup first(r_2)$  if  $null(r_1)$   
 $= first(r_1)$  otherwise  
 $first(r_1 | r_2) = first(r_1) \cup first(r_2)$   
 $first(r \star) = first(r)$ 

#### ...and last

$$last(\emptyset) = \emptyset$$
  
 $last(\epsilon) = \emptyset$   
 $last(a) = \{a\}$   
 $last(r_1 r_2) = last(r_1) \cup last(r_2)$  if  $null(r_2)$   
 $= last(r_2)$  otherwise  
 $last(r_1 | r_2) = last(r_1) \cup last(r_2)$   
 $last(r \star) = last(r)$ 

#### And finally we can define follow

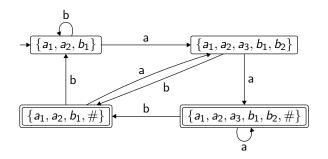
```
 follow(c,\emptyset) = \emptyset 
 follow(c,\epsilon) = \emptyset 
 follow(c,a) = \emptyset 
 follow(c,r_1r_2) = follow(c,r_1) \cup follow(c,r_2) \cup first(r_2) \text{ if } c \in last(r_1) 
 = follow(c,r_1) \cup follow(c,r_2) \text{ otherwise} 
 follow(c,r_1|r_2) = follow(c,r_1) \cup follow(c,r_2) 
 follow(c,r_*) = follow(c,r) \cup first(r) \text{ if } c \in last(r) 
 = follow(c,r) \text{ otherwise}
```

We have everything we need to build the automaton for regular expression s.

We begin by adding a character # (*End of File*) to the end of r. The algorithm is as follows:

- 1. the initial state is the set first(r#)
- 2. while there is a state s for which one needs to compute transitions for each character c of the alphabet let s' be the state  $\bigcup_{c_i \in s} follow(c_i, r\#)$  add the transition  $s \xrightarrow{c} s'$
- 3. The accepting states are those that contain #

For  $(a|b) \star a(a|b)$  we get



Reference : Gérard Berry & Ravi Sethi, From regular expressions to deterministic automata, 1986

# Lexing Tools

In practice, we have tools to build lexical analyzers from a description with regular expressions and actions.

This is the lex family: lex, flex, jflex, ocamllex, etc.

We illustrate jflex (for JAVA) and ocamllex (for OCAML).

## Minimal Example

To illustrate these tools, let us write a lexical analyzer for a language of arithmetic expressions with

- integer literals
- parentheses
- subtraction

# jflex

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#### A jflex file has suffix .flex and the following structure

```
... preamble ...
%{
... some Java code
%}
%%
<YYINITIAL> {
  regular expression { action }
  ...
  regular expression { action }
}
```

where each action is Java code (returning a token most of the time).

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We set up a file Lexer.flex for our language

```
import static sym.*; /* imports the tokens */
%%
                     /* our class will be Lexer */
%class Lexer
%unicode
                     /* we use unicode characters */
%cup
                     /* syntax analysis using cup */
%line
                     /* activate line numbers */
%column
                     /* and column numbers
%yylexthrow Exception /* we can raise Exception */
%{
  /* no need for a Java preamble here */
```

```
WhiteSpace
               = [ \t\r\n] +
                                 /* shortcuts */
                = [:digit:]+
Integer
%%
<YYINITIAL> {
 11 _ 11
     { return new Symbol(MINUS, yyline, yycolumn); }
 "(" { return new Symbol(LPAR, yyline, yycolumn); }
 ")" { return new Symbol(RPAR, yyline, yycolumn); }
 {Integer}
       { return new Symbol(INT, yyline, yycolumn,
                         Integer.parseInt(yytext())); }
 {WhiteSpace}
       { /* ignore */ }
      { throw new Exception (String.format (
          "Line %d, column %d: illegal character: '%s'\n",
         yyline, yycolumn, yytext())); }
```

- tokens are freely implemented;
   here, we use the class Symbol that comes with cup<sup>1</sup>
- MINUS, LPAR, RPAR and INT are integers (token kinds) built by the tool cup and imported from sym.java
- variables yyline and yycolumn are updated automatically
- yytext() returns the string that was recognized by the regular expression

We compile file Lexer.flex with jflex jflex Lexer.flex

We get pure Java code in Lexer.java, with

a constructor

Lexer(java.io.Reader)

a method

Symbol next\_token()

### jflex Regular Expressions – Summary

```
any character
                   the character 'a'
а
                   the string "foobar" (in particular \epsilon = "")
"foobar"
                   set of characters (e.g. [a-zA-Z])
[characters]
[^characters]
                   set complement (e.g. [^"])
                   predefined set of characters (e.g. [:digit:])
[:ident:]
{ident}
                   named regular expression
r_1 \mid r_2
                   alternation
                   concatenation
r1 r2
r *
                   star
                   one or more repetitions of r \stackrel{\text{def}}{=} r r \star
r +
                   zero or one occurrence of r \stackrel{\text{def}}{=} \epsilon | r
r ?
(r)
                   grouping
```

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## ocamllex

An ocamllex file has suffix .mll and the following structure

```
{
    ... some OCaml code ...
}
rule ident = parse
| regular expression { action }
| regular expression { action }
| ...
{
    ... some OCaml code ...
}
```

where each action is some OCaml code.

```
let white_space = [' ' '\t' '\n']+
let integer = ['0'-'9']+
rule next_token = parse
  | white_space
     { next_token lexbuf }
  | integer as s
     { INT (int_of_string s) }
  | '-'
    { MINUS }
  1 '('
    { LPAR }
  | ')'
    { RPAR }
  l eof
     { EOF }
  l as c
      { failwith ("illegal character" ^ String.make 1 c) }
```

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we assume the following type for the tokens

```
type token =
| INT of int
| MINUS
| LPAR
| RPAR
| EOF
```

(will be built by the syntax analyzer)

- contrary to jflex
  - we explicitly call next\_token to ignore blanks
  - we do not handle lines and columns explicitly

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We compile lexer.mll with ocamllex

```
% ocamllex lexer.mll
```

that generates OCaml lexer.ml file that defines a function for each analyzer f1, ..., fn:

```
val f1 : Lexing.lexbuf -> type1
val f2 : Lexing.lexbuf -> type2
...
```

.

val fn : Lexing.lexbuf -> typen

and

```
val next_token : Lexing.lexbuf -> token
```

### The Type Lexing.lexbuf

The type Lexing.lexbuf is the one of the data structure for the state of the lexical analyzer.

The module Lexing of the standard library gives different ways to construct a value of this type such as

```
val from_channel : in_channel -> lexbuf
```

```
val from_string : string -> lexbuf
```

### ocamllex Regular Expressions – Summary

```
any character
'na,
                   the character 'a'
                   the string "foobar" (in particular \epsilon = "")
"foobar"
                   set of characters (e.g. ['a'-'z' 'A'-'Z'])
[characters]
                   set complement (e.g. [^ '"'])
[^characters]
ident
                   named regular expression
                   alternation
r_1 \mid r_2
                   concatenation
r_1 r_2
                   star
r *
                   one or more repetitions of r \stackrel{\text{def}}{=} r r \star
                   zero or one occurrence of r \stackrel{\text{def}}{=} \epsilon | r
r ?
(r)
                   grouping
                   end of input
eof
```

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#### Identifiers

#### Integer constants

#### Floating number constants

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```
let letter = ['a'-'z', 'A'-'Z']
let digit = ['0'-'9']
let decimals = '.' digit*
let exponent = ['e' 'E'] ['+' '-']? digit+
rule token = parse
  | letter (letter | digit | '_')*
                                             { ... }
                                            { ... }
  | digit+
  | digit+ (decimals | decimals? exponent) { ... }
```

When using the keyword parse, it is a rule of the longest recognized token that is applied.

And for two recognized tokens of equal length, it's the first rule that applies

For the shortest, one can use the keyword shortest instead of parse

```
rule scan = shortest
  | regexp1 { action1 }
  | regexp2 { action2 }
```

One can give a name to a recognized token or a sub-token using as construct and then use it inside the action

Inside of an action, it is possible to call the lexical analyzer recursively, or call other mutually recursively defined rules

The buffer of the lexical analyzer must be passed as argument; it is contained inside a variable called lexbuf

For example; it's easy to treat white spaces:

Note how we handle properly an error of non-terminated comment

We can also handle nested comments using a lexical analyzer.

First try: using a counter. . . rule token = parse | "(\*" { comment 1 lexbuf; token lexbuf } and comment level = parse | "\*)" { if level > 1 then comment (level - 1) lexbuf } | "(\*" { comment (level + 1) lexbuf } { comment level lexbuf } | eof { failwith "non terminated comment" }

... or even without a counter!

Note: by doing so we actually go beyond regular expressions!

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Four rules to keep in mind when writing a lexical analyzer:

- 1. handle white spaces
- 2. the rules with highest priory to define first (e.g. language keywords before identifiers)
- 3. signal lexical errors (e.g. illegal characters, non terminated comments, etc.)
- 4. handle the end of the input (eof)

- Regular expressions are the basis of lexical analysis
- The job is automatized with tools such as jflex and ocamllex
- jflex/ocamllex are more expressive than regular expressions indeed, actions can call the lexical analyzer recursively ⇒ allows us to recognize nested comments for instance

# It's Demo Time!

### ocamllex Applications

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The use of ocamllex is not even limited to *classical* lexical analysis.

To analyze a text (string, file, stream) based on regular expressions, ocamllex is the tool of choice.

Nice example: write filters, *i.e.*, programs that translate a language into another, following local and simple modifications.

Example 1: counting the occurrences of a word in a text.

```
let word = Sys.argv.(1)
rule scan c = parse
  | ['a'-'z' 'A'-'Z']+ as w
     { scan (if word = w then c+1 else c) lexbuf }
    { scan c lexbuf }
  l eof
   { c }
  let c = open_in Sys.argv.(2)
  let n = scan 0 (Lexing.from_channel c)
  let () = Printf.printf "%d occurrence(s)\n" n
```

#### Example 2: remove blanks from a text.

```
that's right, I used ocamllex to build the first two slides;)
{
 let buffer = Buffer.create 1024
let blank = ['\t' '\r' '\n' ' ']
rule token = parse
  | blank+ { token lexbuf }
   _ as c { Buffer.add_char buffer c; token lexbuf }
   eof { () }
  let () =
    let fin = Sys.argv.(1) in
    let cin = open_in fin in
    token (Lexing.from channel cin);
    let cout = open_out (fin ^ "_out") in
    Buffer.output buffer cout buffer
```

### Example 3: while2html

Example 3: a small translator from  $W_{\rm HILE}$  language into HTML to prettify the online source code.

#### Goal

- usage: while2html file.wl, that outputs file.wl.html
- keywords in green, comments in red
- numbering lines

### Bibliography

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 Compilers, Principles, Techniques, and Tools, Alfred Aho and Monica S. Lam and Ravi Sethi and Jeffrey D. Ullman, Second Edition[Chapter 3], Addison-Wesley (2006) ("the dragon book").