Making Explicit a Constant Appearing in an Impossible Equation of Carl and Moroz

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Abstract

Recently, Carl and Moroz [1] constructed a Diophantine (integer polynomial) equation for which it is impossible to decide whether the equation has any (integer) solutions. However, their presentation is not as explicit as it could be, instead involving implicitly defined polynomials spread out over many pages. In particular, they give a two page description of a crucial constant without bothering to explicitly compute it. We resolve this issue using a bit of Python, SageMath, and 878 MB of disk space.

1 Introduction

The Matiyasevich–Robinson–Davis–Putnam (MRDP) theorem [4] states that Diophantine equations (polynomial equations with integer coefficients) are "Turing complete", in the sense that for any recursively enumerably set of integers S, there is a polynomial f with integer coefficents such that $S = \{n \mid \exists x_1, \ldots, x_k \in \mathbb{Z}.\ f(n, x_1, \ldots, x_k) = 0\}$. This famously implies that is undecidable to determine whether a Diophantine equation has any (integer) solutions, giving a negative answer to Hilbert's Tenth Problem.

Another consequence is that it is in principle possible to take your favorite axiomatization of mathematics, such as ZFC set theory, and construct a Diophantine equation such that, within that axiomatization, one can neither prove nor disprove whether the equation has any solutions. Recently, Carl and Moroz [1] did exactly this for Gödel-Bernays set theory, which has the same set of provable statements about sets as ZFC [5]. That is, they describe how to write down an "impossible" Diophantine equation such that, within "mathematics as we know it", one cannot prove or disprove whether it has a solution. However, their presentation is not as explicit as it could be—the equations are spread out over many pages, and some of them are not written explicitly as polynomials, instead involving polynomial functions of other polynomials.

It would be aesthetically pleasing to write down their impossible equation in an explicit, easily interpreted form. Such forms are known for a separate consequence of the MRDP theorem: just as one can construct universal Turing machines, one can construct *universal Diophantine equations*, which can be made to emulate any Turing machine by fixing some variable values. For example, there is a computable encoding of Turing machines as triples of integers (z, u, y), and Turing machine inputs as integers x, such that the Turing machine corresponding to (z, u, y) accepts input x iff the following system of equations has a solution [3]:

$$\begin{gathered}
elg^2 + \alpha = (b - xy)q^2, \quad q = b^{5^{60}}, \quad \lambda + q^4 = 1 + \lambda b^5, \quad \theta + 2z = b^5, \quad l = u + t\theta, \\
e = y + m\theta, \quad n = q^{16}, \quad r = [g + eq^3 + lq^5 + (2(e - z\lambda)(1 + xb^5 + g)^4 \\
+ \lambda b^5 + \lambda b^5 q^4)q^4]\,[n^2 - n] + [q^3 - bl + l + \theta\lambda q^3 + (b^5 - 2)q^5]\,[n^2 - 1], \\
p = 2ws^2r^2n^2, \quad p^2k^2 - k^2 + 1 = \tau^2, \quad 4(c - ksn^2)^2 + \eta = k^2, \\
k = r + 1 + hp - h, \quad a = (wn^2 + 1)rsn^2, \quad c = 2r + 1 + \phi, \\
d = bw + ca - 2c + 4a\gamma - 5\gamma, \quad d^2 = (a^2 - 1)c^2 + 1, \quad f^2 = (a^2 - 1)i^2c^4 + 1, \\
(d + of)^2 = ((a + f^2(d^2 - a))^2 - 1)(2r + 1 + jc)^2 + 1.
\end{gathered}$$

A related fun result is the statement that any provable mathematical statement has an alternative proof consisting of 100 additions and multiplications of integers [3, Theorem 5].

Our work is a first step towards putting Carl and Moroz's equation into an aesthetically pleasing form. Specifically, we compute a constant which is defined implicitly in their work using two pages of equations, many of which are not true polynomials but instead involve polynomial functions of other polynomials.

2 The Impossible Equation and its Constant

Briefly, Carl and Moroz construct their equation as follows. Let $\mathcal{L}$ denote the language of first-order logic with a single binary predicate symbol. First, they construct a polynomial $f(t, \vec{x})$ which is a Diophantine version of a computer program that checks whether $\vec{x}$ encodes a proof of t in $\mathcal{L}$. Thus for any statement P of $\mathcal{L}$, letting t_P be their integer encoding of P, P is provable in $\mathcal{L}$ iff there exist integers $\vec{x} \in \mathbb{Z}^{14558112}$ such that $f(t, \vec{x}) = 0$.

Next, they explain how to write down the integer encoding of the statement "the axioms of Gödel-Bernays set theory imply a contradiction".[1] Specifically, the authors encode the statement

$$A_1 \Rightarrow (A_2 \Rightarrow (\cdots (A_{15} \Rightarrow (\forall x. x \in x)) \cdots)), \tag{1}$$

where $A_1, \ldots, A_{15}$ are the axioms of the theory, and $\in$ is the binary predicate symbol of $\mathcal{L}$. The axioms are meant to model that $\in$ denotes ordinary set inclusion, and in particular, they imply that $\forall x. x \in x$ is false, so this statement is provable iff Gödel-Bernays set theory is inconsistent. Thus by Gödel's second incompleteness theorem [2], the statement can be neither proved nor disproved within Gödel-Bernays set theory, hence within all of ordinary mathematics.[2]

In the author's notation, the integer encoding of (1) is denoted $f_{15}(\vec{a}, 3)$, where f_{15} is defined in equation (5) on page 49, and $\vec{a}$ is the vector of the values $a_1, \ldots, a_{15}$ defined in Section 8.

Our computation computes this $f_{15}(\vec{a}, 3)$.

3 Computation

Our program[3] is a straightforward Python transcription of the equations defining $f_{15}(\vec{a}, 3)$ in [1]. One exception is that we re-order the axioms in (1) by the magnitude of their integer encodings. This helps to keep the magnitude of the overall result small, since the magnitude of $B \Rightarrow C$ is approximately the square of the sum of the magnitudes of B and C. With the original axiom ordering, the program ran out of memory before finishing despite having 16 GB of RAM.

While the program could be run in Python, we actually ran it using SageMath [6]. This is about 100× faster due to SageMath's use of the GMP library for integer arithmetic. It takes 11 minutes to run on the author's CMU-issued laptop.

The resulting value is 877,757,576 decimal digits long.

4 Result

We now include as many digits of the result as the editor will allow. We hope the reader finds its aesthetics more pleasing than the original presentation in [1], perhaps even on-par with the universal Diophantine equation reproduced above. The full result is available on a flashdrive by request from Gates 5005.

[1] They use Gödel-Bernays set theory instead of the better-known ZFC set theory because Gödel-Bernays has only finitely many axioms, whereas some of ZFC's "axioms" are actually axiom schemas which describe infinite sets of first-order statements.

[2] Unless the axioms are inconsistent, in which case we have bigger problems to worry about!

[3] `http://mattweidner.com/diophantine_impossible/diophantine_impossible.sage`

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