On the Intractability of Multiclass Restroom Queues with Perfect Stall Etiquette

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ABSTRACT

We extend prior work on queueing-theoretic bathroom humor. Our results aren't as good, but the system model is funnier.

ACH Reference format:
Sarah Allen and Ziv Scully. 2018. On the Intractability of Multiclass Restroom Queues with Perfect Stall Etiquette. In *Proceedings of SIGBOVIK 2018, Pittsburgh, PA, USA, March 29, 2018 (SIGBOVIK '18),* 2 pages.

1 INTRODUCTION

The complex social dynamics of *homo sapiens* results in intricate etiquette protocols for many activities, including restroom usage. Such protocols are a significant mathematical obstacle for queueing theorists who wish to rigorously analyze the performance metrics of restrooms. A recent breakthrough by Gardner and Scully [2] presented the first theoretical analysis of a restroom queue that accounted for restroom etiquette. The work addresses the so-called M/M/3/C2UPN, which handles the case of urinals in a men's restroom under the usual rule: no two adjacent urinals may be simultaneously occupied.

By now, the careful reader will have noticed that Gardner and Scully [2] consider only one of the two traditional genders served by multioccupancy restrooms and only one of its two customer classes [3]. Men, as notoriously simple creatures, employ a urinal protocol that admits exact analysis in almost alarming generality. In contrast, in this work we show that the stall protocol employed in women's restrooms results in a Markov chain whose behavior is *impossible to exactly analyze* using known techniques, except under very specific conditions. The intractability arises from the convoluted interaction between customers of both classes, which has not been considered in prior work.

2 SYSTEM MODEL

We consider a women's restroom with k servers, namely stalls. Customers arrive with a Poisson process of rate λ. Each customer is independently Class 1, with probability p_1, or Class 2, with probability $p_2 = 1 - p_1$. Class 1 customers have an exponential service time distribution of rate μ_1, and similarly for Class 2 with rate μ_2. However, the story for Class 2 customers, described in detail below, is complicated due to the following restriction.

> A Class 2 customer can only be served while it is the *only customer of any class* in the system.

That is, in a women's restroom, *no one can hear you poop*. This protocol ensures that every customer can plausibly maintain the facade that they are not a Class 2 client [4].

The protocol for Class 2 customers is defined formally in Algorithm 2.1. We now describe the intuition behind the protocol. A Class 2 customer, instead of beginning service immediately upon entering a server, initially *stalls*, or blocks, until all k servers are empty or contain other Class 2 customers. We conjecture this behavior is the namesake of the colloquial term for servers. Once only Class 2 customers remain at servers, they begin a *sitoff*, during which each customer may leave, dethroning themselves as a contender to be the lone Class 2 customer to receive service. This occurs at stochastic rate ν_2, and the leaving customer has to find a new place to number-2. Until the queue is empty, the Class 2 customers at the servers alternate between stalls and sitoffs, depending on whether there is a Class 1 customer in service.

Algorithm 2.1 Protocol for Class 2 Customers

Begin as Inactive.
- If there is a Class 1 customer at another server, *stall*, namely do nothing.
- If only Class 2 customers occupy other servers, begin a *sitoff*, abandoning at rate ν_2.
  - If Number-2-ing, instead do not abandon.
  - If one of the other customers is Number-2-ing, instead abandon at increased rate $\nu_2 + \xi_2$.
- If there are no other customers in the system, permanently transition from Inactive to Number-2-ing.
  - While Number-2-ing with no other customers in the system, complete service at rate μ_2.

If the system is stable, eventually a single Class 2 customer will occupy the system, at which point they finally begin service. They become the sole *number-2-ing* customer. As other customers arrive, they occupy servers as normal, with other Class 2 customers experiencing stalls and sitoffs as normal. The number-2-ing customer stalls during sitoffs between the other Class 2 customers. A suspicious air about the number-2-ing customer gives sitoff participants a chance to sniff them out. When a sitoff participant discovers a number-2-ing customer, they know they will not win the sitoff, so they abandon the system. This discovery happens at rate ξ_2, so the abandonment rate of sitoff participants in the presence of a number-2-er to $\nu_2 + \xi_2$.

3 INTRACTABILITY OF EXACT ANALYSIS

We can describe the system as a Markov chain whose states are 4-tuples of natural numbers (n, s_1, s_2, g):
- n is the number of customers in the queue,
- s_1 is the number of Class 1 customers at a server,
- s_2 is the number of Class 2 customers at a server, and
- g_2 is the number of Class 2 customers number-2-ing.

The states are divided into those in the *repeating portion*, which have $n \geq 1$, and those in the *initial portion*, which have $n = 0$. States in the repeating portion obey the constraint $s_1 + s_2 + g_2 = k$, and those in the initial portion obey $s_1 + s_2 + g_2 \leq k$. All states obey $g_2 \leq \min\{s_2, 1\}$.

Transitions out of states in the repeating portion of the Markov chain are as follows:

- $(n, s_1, s_2, g) \to (n+1, s_1, s_2, g)$ at rate λ, due to an arrival;
- $(n, s_1, s_2, g) \to (n-1, s_1, s_2, g)$ at rate $p_1 s_1 \mu_1$, due to a Class 1 completion and the next customer being Class 1;
- $(n, s_1, s_2, g) \to (n-1, s_1-1, s_2+1, g)$ at rate $p_2 s_1 \mu_1$, due to a Class 1 completion and the next customer being Class 2;
- $(n, 0, s_2, g) \to (n-1, 1, s_2-1, g)$ at rate $p_1 s_2(\nu_2 + g\xi_2)$, due to a Class 2 abandonment and the next customer being Class 1; and
- $(n, 0, s_2, g) \to (n-1, 0, s_2, g)$ at rate $p_2 s_2(\nu_2 + g\xi_2)$, due to a Class 2 abandonment and the next customer being Class 2.

Transitions out of states in the initial portion are routine to state and thus omitted. The highlight is the transition $(0, 0, 0, 1) \to (0, 0, 0, 0)$ at rate μ_2, due to a Class 2 customer's completion.

The analysis of Gardner and Scully [2] took advantage of the *recursive renewal-reward* (RRR) technique [1]. The RRR technique applies, roughly speaking, when the states in the repeating portion can be partitioned into *layers* such that transitions between layers form a directed acyclic graph. Our system's Markov chain has one layer for each triple (s_1, s_2, g). The layers form two connected components, one for $g = 0$ and another for $g = 1$. Unfortunately, both components have cyclic transitions between layers: $(1, k-1, 0) \leftrightarrow (0, k, 0)$ and $(1, k-2, 1) \leftrightarrow (0, k-1, 1)$.

We have seen that *RRR cannot exactly solve this Markov chain.* Similar issues occur when attempting matrix analytic methods and other techniques. A glimmer of hope comes from Gardner and Scully [2], who found that RRR applies to a 5-urinal system, which also had cyclic transitions between layers of its Markov chain. This is because for each transition from layer A to layer B in that Markov chain, the *total rate of transitions towards the initial portion never increases* going from layer A to layer B, ensuring the existence of some matrix's square root or something[1]. This yields the following conclusion.

THEOREM 3.1. *We can only analyze the present system if*

$$3\nu_2 = 2(\nu_2 + \xi_2) = \mu_1.$$

And that is just a ridiculously specific assumption, even for a SIGBOVIK paper.

4 SUGGESTED PROTOCOLS

Here we present some alternative strategies for rendering the above system analyzable and suggest that customers of women's restrooms implement them so that we can rigorously demonstrate the suboptimality of the system.

Get Your Shit Together. Class 2 customers stall while Class 1 customers remain in the system. When a potential standoff is reached, all Class 2 customers simultaneously initiate service. Note: this technique has been observed in practice, as long as no two customers occupy the common area at the same time, thus assuring plausible anonymity (unless, of course, the bathroom is in a computer science department, where number of clients who use women's restrooms is regrettably low).

Shit or Get Off the Pot. If the customer encounters a situation in which they would like to run a Class 2 job, but cannot doo-doo due to other customers in the system, they must immediately call process *Not Giving a Shit* or process *Full of Shit*, both of which are defined below.

Not Giving a Shit. The customer decides that their re-poo-tation is worth tarnishing for the purposes of optimality and brazenly uses the available resource, regardless of the state of other servers.

Full of Shit. The customer refrains from using the Class 2 services provided by the public restroom and blocks until they can use a guaranteed private resource at home[2].

Shit Yourself. Not recommended.

[1] Lossy personal communication from past Ziv to present Ziv.

[2] In the first named author's experience, this approach yields poor results when one lives in a dormitory hall with a common restroom.

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