Erdös-Bacon-Sabbath Numbers
Reductio ad Absurdum

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Abstract

A small Erdös number – the "collaborative distance" of authorship between oneself and Paul Erdös – has long been a source of pride for mathematicians, computer scientists and other geeks. Utilizing similar collaborative distance metrics, a small Bacon number (the degree of separation from Kevin Bacon) has been a source of pride for actors, while a small Sabbath number (the degree of separation from Black Sabbath) has been a source of pride for musicians. Previous research in Erdös-Bacon number minimization has reduced the Erdös number of a number of computer scientists to two, which is believed optimal, although the reduction of the Bacon number to four was clearly suboptimal. We extend and improve on this previous work to provide a Erdös-Bacon-Sabbath number minimization that is believed to be close to optimal in all axes.

Derivative Introduction [2]

Paul Erdös co-authored nearly 1500 papers (until his death in 1996), working with nearly 500 collaborators achieving the status of the most prolific mathematician in modern times [4]. Mathematicians thus humorously defined Erdös numbers. A person's Erdös number is the distance between that person and Paul Erdös in the academic paper collaboration graph [3]. Succinctly, Paul Erdös is the unique person with Erdös number zero; all of Erdös' immediate co-authors have Erdös number one; in general, if you publish an academic paper with a collaborator who has Erdös number N and none of your other co-authors has Erdös number less than N, your Erdös number is $N + 1$.

A similar Bacon number [16] has been proposed for actor Kevin Bacon, except using collaborations in film instead of collaborations in academic papers. Likewise, a similar Sabbath number [12] has been proposed to connect to the members of

the musical group Black Sabbath, using collaborations in musical performances.

Erdös-Bacon-Sabbath numbers were subsequently defined [5] to be the sum of each person's Erdös, Bacon, and Sabbath numbers.

There is a long tradition of posthumous publication [7], and authors claiming to have collaborated with Erdös have brought his total number of known publications to 1525, his collaborator count to 511, and the Erdös number of the chutzpah-bearing mathematician to one. The latest publications co-written with Paul Erdös appeared more than ten years after his death. With additional rumored works in progress, Erdös's publication list is expected to grow. In fact, Paul Erdös himself has published a solo work 15 years after his death [8].

In this paper, we describe and demonstrate a technique called Erdös-Bacon-Sabbath Number Minimization.

Rules of the Games

The rules of Erdös number calculation are clear: author a paper in a peer-reviewed publication, either with someone connected by co-authorship with Paul Erdös, or with Paul Erdös himself (the latter being unlikely unless pre-demise work is used in a posthumous publication, or if you are better a communicating with the dead than Edgar Cayce). Simply putting Paul Erdös' name on your paper does not count (and changing your name to Paul Erdös for purposes of publication is *definitely* cheating).

The rules of Bacon number calculation [16] are also clear: act in or be otherwise credited in a film with someone connected by film credit with Kevin Bacon[1], or with Kevin Bacon himself. We tried contacting Kevin Bacon, but his agent refused to put us in touch. We contemplated inserting a fair-use clip of Kevin Bacon from an unrelated movie, but knew that would be cheating (but we *did* think of it).

Finally, the rules of Sabbath number calculation [12] are also clear: connections between a musician and a band or solo artist can only be made if they actually performed or recorded together. However, "session musicians" are valid connections, so musicians who perform live or record with an artist, but are not strictly committed to that band are valid. A new recording [11] can serve as adequate proof, but singing along to a Black Sabbath record is cheating. We also contemplated contacting Ozzy Osbourne, but he postponed the 2019 tour that would have taken him through the Pittsburgh area, and then cancelled the tour altogether.

The theoretically achievable absolute minimum Erdös-Bacon-Sabbath number is two: if Kevin Bacon (who has a Bacon Number of zero) were to become a member of Black Sabbath (thus receiving a Sabbath number of zero), and was to publish a paper with a person with an Erdös number of one (since Paul Erdös is dead, and ineligible as a co-author). The practically achievable minimum Erdös-Bacon-Sabbath number is four (by authoring a paper with a someone with an Erdös number of one, appear in a movie with Kevin Bacon, and perform with a member of Black Sabbath), but the realistically achievable minimum is somewhat higher than that.

[1] The Internet Movie Database http://www.imdb.com is typically used for verification of film/video/YouTube credits.

Computation of Erdös, Bacon, and Sabbath Numbers

In [1], Maria Klawe co-authored with Paul Erdös and has an Erdös number of one, thus guaranteeing that all authors on this paper have an Erdös number no greater than two.

Daniel V. Klein has a Bacon number of 2, having appeared in [9] with Steve Guttenberg, who appeared in [10] with Kevin Bacon. Mike Ancas also has a Bacon number of 2, having appeared as an extra in [17] with Tom Hanks, who appeared in [18] with Kevin Bacon. All authors on this paper also appear in the documentary about the writing of this paper [11], and thus have a Bacon number no greater than three.

Additionally, Mike Ancas also has a Sabbath number of 1, having been a member of the Bloomsburg PA High School rock band The Rubber Band. In 1971 The Rubber Band performed as one of several warm-up acts for Ronnie James Dio (who in 1979 became the lead singer for Black Sabbath). The authors of this paper have recorded a special musical piece, composed specifically for this paper, and captured in [11]. Their Sabbath number is therefore no greater than two.[2]

The combined Erdös-Bacon-Sabbath number for all the authors of this paper is therefore no greater than seven, surpassing the rarified company of the only three other people hitherto known to have an Erdös-Bacon-Sabbath number of eight: Stephen Hawking, Ray Kurzweil, and Daniel Levitin, a professor of psychology and behavioral neuroscience at McGill University [6]. They now are tied with the previous record-low Erdös-Bacon-Sabbath number (held by Lawrence Krauss [13]) with an Erdös-Bacon-Sabbath number of seven!

Not to brag or nothin', but Maria Klawe (E=1, B=3, S=2) and Daniel V. Klein (E=2, B=2, S=2) have now beaten that record with an Erdös-Bacon-Sabbath number of six. Finally, with a near-optimal EBS number minimization, Mike Ancas (E=2, B=2, S=1) has algorithmically achieved an incredible Erdös-Bacon-Sabbath number of five!

Sheet Music & Documentary

You've read the paper, now read the music (in the appendix)! Lastly, you get to watch the documentary at [11].

Acknowledgements

Thanks, Mom! Thanks, Dad! Thanks to spouse(s) and/or partner(s), significant other(s), groupies, and random tinder dates!

References

[1] Paul Erdös, Frank Harary, and Maria Klawe. "Residually complete graphs." In *Proc. Sympos. Combinatorial Mathematics and Optimal Design*, pages 117–123, Colorado State University, Fort Collins, Colorado, 1978

[2] Charles Garrod, Maria Klawe, Iain Matthews, *et. magnum al.* "An Algorithm for Erdos-Bacon Number Minimization." In *A Record of The Proceedings of SIGBOVIK 2010*, pages 3-4, Carnegie Mellon University, Pittsburgh, Pennsylvania, 2010

[2] Early research on low Sabbath Numbers included DK Fackler, who has a Sabbath number of 2. DK performed with Roger Daltrey [14] in 1994, who in turn performed in Wembley Stadium with "The Who" at *Live Aid* (1985), where Black Sabbath also performed [15].

[3] Caspar Goffman. “What is your Erdös number?” The American Mathematical Monthly, 76(7):791, 1969

[4] Jerry Grossman, Patrick Ion, and Rodrigo De Castro. Erdös number project, 2010. http://www.oakland.edu/enp/.

[5] Richard Sear. “Erdos-Bacon-Sabbath numbers.” Times Higher Education, 2016. http://blogs.surrey.ac.uk/physics/2012/09/15/erdos-bacon-sabbath-numbers/comment-page-1/

[6] Scott Seckel. “Rarified air: Do you have an Erdos-Bacon-Sabbath number?” University of Arizona, 2016. https://asunow.asu.edu/20160126-creativity-lawrence-krauss-erdos-bacon-sabbath-score

[7] Myles McWeeney. “Voices from beyond the grave - authors whose legacies live on through posthumous publishing” The Independent (Ireland), 2017. https://www.independent.ie/entertainment/books/voices-from-beyond-the-grave-authors-whose-legacies-live-on-through-posthumous-publishing-35804952.html

[8] Paul Erdös. “Some Problems On The Distribution Of Prime Numbers.” In: Ricci G. *(eds)* Teoria dei numeri. C.I.M.E. Summer Schools, vol 5. Springer, Berlin, Heidelberg https://doi.org/10.1007/978-3-642-10892-1_3

[9] Ravi Godse. “Help Me Help You”, 2009. https://www.imdb.com/title/tt1417297/

[10] Barry Levinson. “Diner”, 1982. https://www.imdb.com/title/tt0083833/

[11] Michael Ancas, Daniel Klein. “Creating a Tiny Erdös-Bacon-Sabbath Number for Fun (and No Profit) (2020)” https://www.imdb.com/title/tt11833110/ https://youtu.be/ssbObQeaH4

[12] Bill M. “The Black Sabbath Game”, 2003 http://web.archive.org/web/20031121110828/http://www.imjustabill.com/blacksabbathgame/index.html

[13] Scott Seckel. “This guy? He knows people” Arizona State University, 2016. https://sese.asu.edu/about/news/article/1645

[14] Roger Daltrey. Live in Chicago, 1994. http://www.thewholive.net/concert/index.php?id=1423

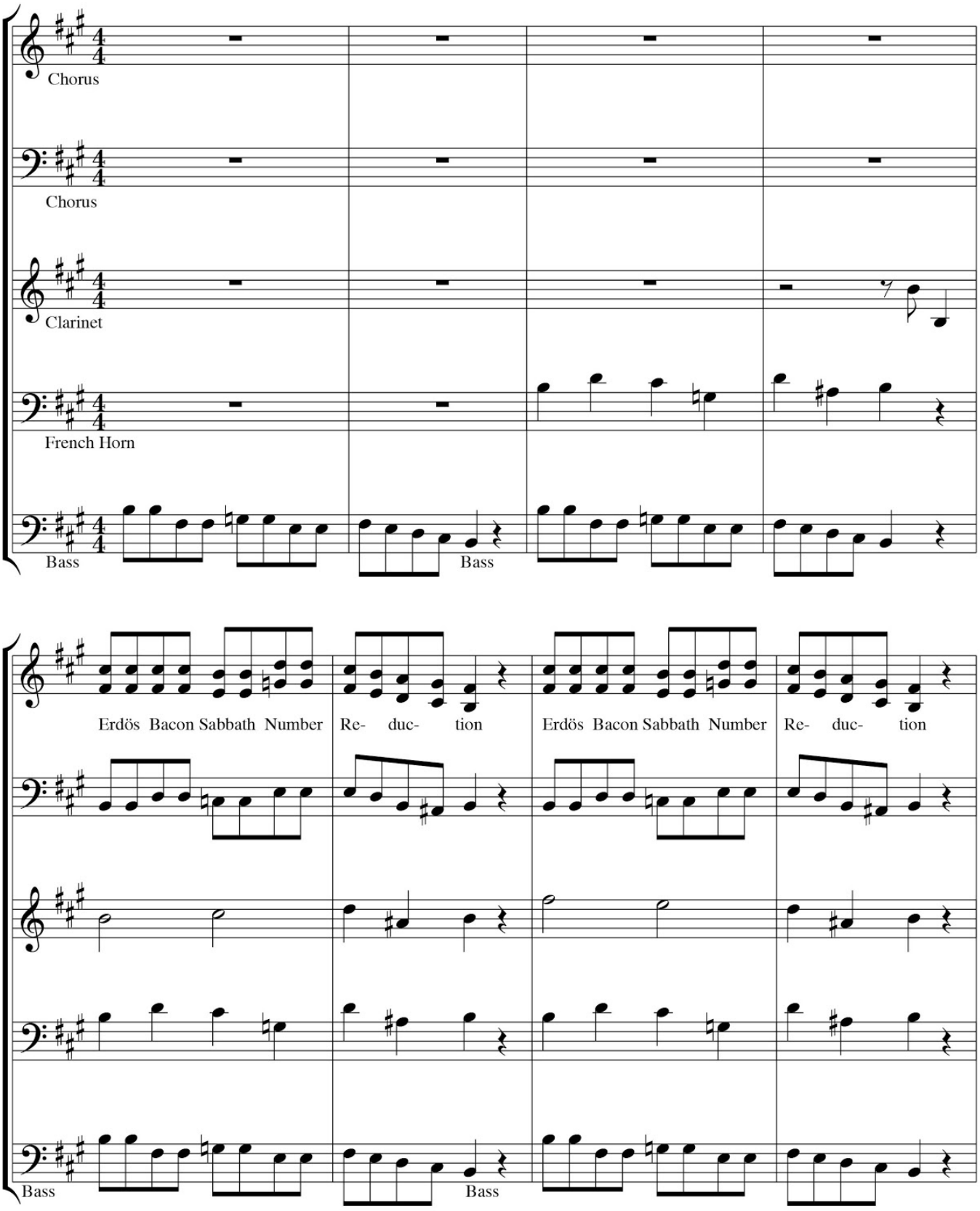
[15] Black Sabbath at “Live Aid”, 1985. https://ultimateclassicrock.com/black-sabbath-live-aid/

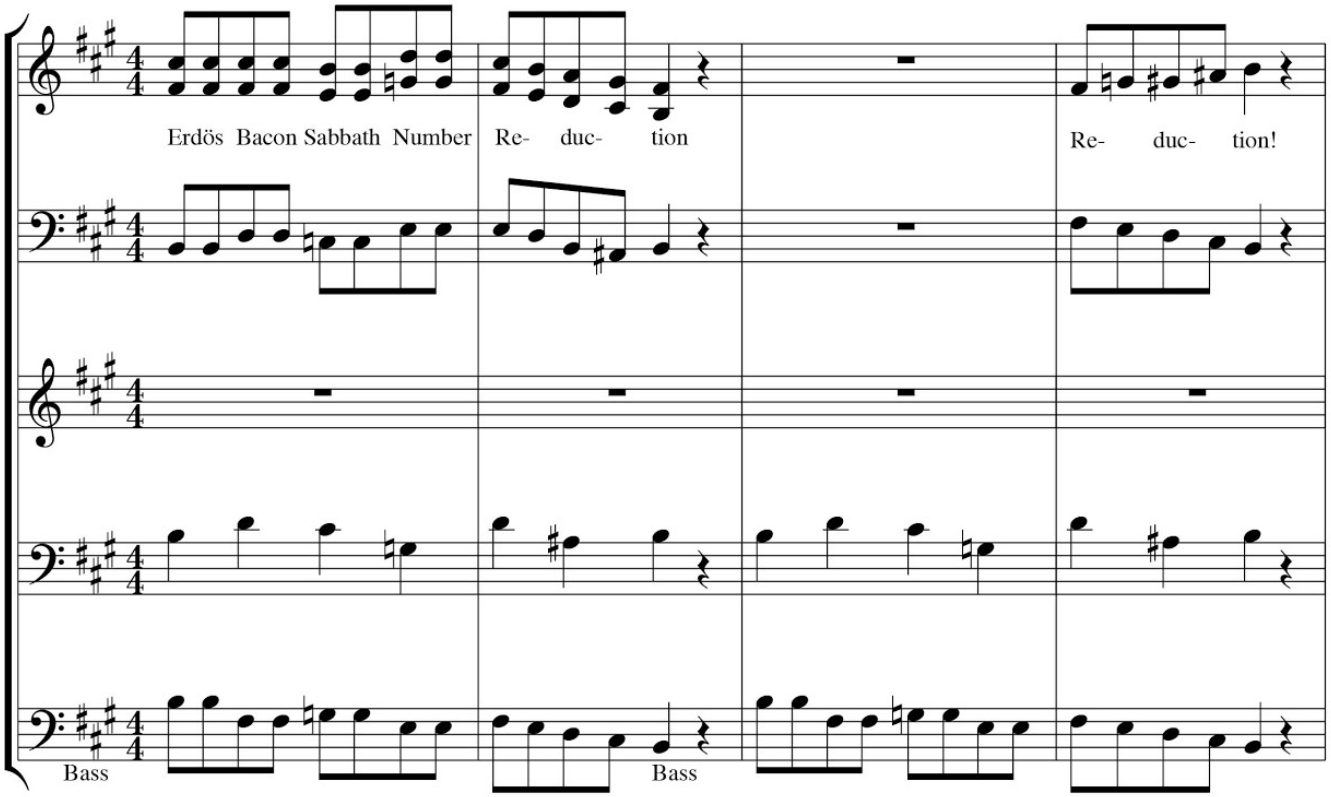
[16] Isabel Teotonio, “Google adds Six Degrees of Kevin Bacon to search engine”, The Star (Toronto, Canada), 2012. https://www.thestar.com/entertainment/2012/09/13/google_adds_six_degrees_of_kevin_bacon_to_search_engine.html

[17] Marielle Heller, “A Beautiful Day in the Neighborhood”, 2019 https://www.imdb.com/title/tt3224458/

[18] Ron Howard, “Apollo 13”, 1995 https://www.imdb.com/title/tt0112384

Appendix: “Erdös Bacon Sabbath Number Reduction” in A Maj
Daniel V. Klein, op 2+2+3




Erdös Bacon Sabbath Number
Re- duc- tion
Re- duc- tion!
Bass
Bass