Harder Drive: Hard drives we didn't want or need

Dr. Tom Murphy VII Ph.D.

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1 Introduction

It's currently the longest 2020 ever on record, seemingly with a new annoying or demoralizing twist every week. To me, the most effective distraction is making things, but what to make? And what if it's no better than what was made before? Then perhaps you will experience an annoying or demoralizing twist. So, here's an approach that's so robust I don't even mind giving it away: Make something that nobody would want, or need. The competitive landscape for needless things is relatively uncrowded, for obvious reasons. A good way to think of things that we might not want is to consider common abstractions that have many instantiations (e.g. chess [17], boolean logic [18], integer math [11]), and come up with new ones. Moreover, judge them according to nontraditional criteria. This way, they will "be better"—in at least some sense—than what was made before. This is the secret of SIGBOVIK.

This paper investigates several storage devices ("Harder Drives") that we didn't want, or need.[1] In doing so, we will find inspiration from some vexing current events. Even if only tangentially related, creating structured thoughts on the periphery may help us digest them. It works for me. This is the laxative of SIGBOVIK.

Despite setting out to do an "easy, fun project," of course I managed to make each one much more difficult than I initially anticipated. This is the curse of SIGBOVIK. Bon appétit.

1.1 Chainsaws

At first glance, it seems that the maximum number of chainsaws that a person can wield is two: One per hand. This is known as *dual-wield*. It may be possible to achieve more using "one-man-band" arrangements (shin- and knee-wielded chainsaws, elbow saws, a mouth-held chainsaw activated by blowing into it like a harmonica) but a more natural way to scale is by juggling the saws into the air. Now at second blush it seems that an arbitrary number of saws can be simultaneously equipped, by simply throwing the chainsaws higher into the air. This configuration is known as ∞-wield. However, throwing chainsaws higher and higher eventually reaches physical limits. Once the thrown saws reach the escape velocity of $11.186^{km}/_s$, they will not fall back to Earth, and can hardly be considered brandished by the juggler. Just shy of this, they reach some maximum height before returning. This loop has two problems: First, it is not the longest possible airtime. Second, it can only be used for one chainsaw at a time, as the saws will otherwise interfere with one another along the out-and-back path.

Upon a third look, we need not throw the chainsaws straight upward; instead we can juggle the saws into orbit around Earth. Ignoring air resistance (the chainsaws cut through air like butter) and assuming double precision floating point with discrete time steps of 0.1 seconds will suffice (probably not), my simulation[2] finds the longest orbit that returns the chainsaw to 1.5m above the surface is 335.36 hours.

There are many such orbits to fill with chainsaws, but the limiting factor seems to be the density of chainsaws near the wielder (where the many orbits would interfere). Assuming each orbit is basically parallel here, and consists of chainsaws about 0.2 m wide (they can be efficiently packed in "69 configuration") moving at the maximum velocity of 11,186m/sec, we see 55,930 per second. We could probably fit about three deep and three high on each side of the body, for 18 times that. So we have 1 million chainsaws per second, for 335.36 hours, which is 1.215×10^{12}, a configuration known as tera-wield. This requires expert juggling skills.

Why am I talking about this? Overall, the important lesson here is creativity with dimensional analysis: We can achieve a quantity of *chainsaws* by multiplying some *chainsaws per second* by some *seconds*.

1.2 Juggling with data

Now imagine that instead of chainsaws, we are juggling something more dangerous: Data.

One potential setup would be a powerful directional antenna, which broadcasts a stream of data towards the horizon. For radio waves below 40 MHz, significant reflections off the ionosphere occur, bouncing the waves back to Earth.

[1]Source code can be found at `sourceforge.net/p/tom7misc/svn/HEAD/tree/trunk/pingu/`.

[2]Someone equipped with Johann Sebastian Kepler's laws could just solve this exactly.

They may also reflect off the ground, and again off the ionosphere, and in principle make their way fully around the planet. The antenna is paired with a receiver in the same location, which accepts the signal and retransmits it, "juggling" it back into circulation.

Since a full trip around the Earth is 40,000 km (and significantly more in this setup due to reflections) and radio waves take time to propagate, this orbit takes at least 150ms to complete. As a result, we can have 0.15 sec $\times$ 40,000,000 bits/sec = 750 kilobytes of data outstanding in the steady state. When data we are interested reaches the receiver we can "read" it, and of course we can choose to retransmit modified data to perform a "write," This is similar to the rotation of a hard drive, with the fixed read/write head waiting for the "orbiting" platter.

This author does not have sufficient skill to construct such a system, which would probably not work in practice anyway. The reflections probably do not make it all the way around the Earth, and the noise from other radio waves would offer significant interference.

A superior arrangement would place repeater towers along the great circle. This would clearly work but would require an investment in real estate throughout the world. But the main reason not to do this is that 750kb is a trivial amount of storage; a similar magnitude is found incidentally in disposable consumer devices (Section 4).

Of course, if we are retransmitting the signal anyway, we do not need to send it in the same direction. It is much simpler to send it directly back, like an echo.

1.3 ICMP Echo

Sorry to remind you about the Internet, which seems to be making the whole world collectively dumber and meaner, but this section concerns a hard drive made from the transmission delays of the network itself.

Back when the internet was a collaborative and basically nice place, the Internet Control Message Protocol (ICMP) was proposed as a way for network nodes to help each other out. This protocol allows sending messages like "hey this address is down!" or other tips. Since it is easy to forge ICMP messages, most off these have potential for abuse (like telling you the site you're talking to is down) and are no longer commonly honored. (As another indicator of the era, these internet standards are known as "Requests for Comment", with ICMP described in RFC 792 [21]. On the modern internet we still have requests for Comment, but they are almost universally accompanied by requests for Like and Subscribe.)

However, many hosts will still respond to an ECHO packet with ECHO REPLY. This is typically used to "ping" a host: The source sends ECHO to the destination with some identifying information and an embedded timestamp; the destination sends ECHO REPLY with that same data back to the source, and the source can calculate the round-trip time.

Since there are hosts throughout the world that will already reply to ECHO messages, this could be a perfect setup for juggling data! The data field of the ECHO can store the bytes of interest. When we receive an ECHO REPLY we will "read" or "write" that data if needed, and then immediately broadcast another ECHO. Since we do not retain the data otherwise, it will be stored "inside" the internet itself: Inside the buffers of routers but also as moving photons inside fiber optics, flowing charge in ethernet cables, and so on.

In principle we should be able to saturate our internet connection with outgoing ECHO and incoming ECHO REPLY; even on a consumer plan (these days on the order of 1 gigabit/sec) we may be able to store significant amounts of data. If we ping a host on the Earth's antipode, the round trip time will be at least 150ms (speed of light and circumference are limits here as well). $1Gb/sec \times 0.15sec$ = 833 Megabytes.

In practice this proves to be much more difficult. Alas, even the apparently harmless ECHO has been regularly abused for denial-of-service attacks, such as the "Ping of Death" [27] and "Smurf attack" [28]. Thus, hosts almost always have hard limits that we have to work within. We will face the following difficulties that cause us to fall far short of the ideal above:

1. Hosts limit the size of an ECHO packet they will respond to. 512 bytes of payload is a typical limit for a fairly permissive host,[3] but many will reject payloads more than a few dozen bytes. The IP header (20 bytes) and ICMP header (8 bytes) thus contribute significant overhead.

2. Hosts have global limits on the rate of incoming and outgoing ICMP messages.

3. Consumer internet connections have built-in throttling of ICMP messages, perhaps to limit the impact of Denial-of-Service attacks originating from their networks.

4. Pings are "best effort" and readily dropped by congested routers without retries (this can even be a desirable property for measuring network congestion).

1.4 Pinging the internet

While developing code that can process many thousands of pings per second and investigating these limitations, I figured I might as well ping the entire internet.

Here by internet I mean "IPv4 address space." I don't care about IPv6 which has *way too many addresses* (plus like, call me when you are at least version 7, right?). There are only 2^{32} IPv4 addresses, which is no longer that big of a number. I wrote a fairly simple program pingy.exe which pings all of the hosts of the form *.*.c.* for some c $\in \{0, \ldots, 255\}$ in a random order. For each one it saves

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[3] Allegedly, "all hosts are required to be able to reassemble datagrams of size up to 576 bytes,"[22] but I guess most do not care about this or consider it better than dropping all pings. There are not many legitimate uses for a payload of this size, anyway.

the number of milliseconds of round-trip time (or records special sentinel values for "timeout" or "wrong data returned") in a single byte. This results in 256 files, which assembled are 4.2 Gigabytes.

This turned out to be much more logistically challenging than I expected. Naïvely I should be able to send millions of pings per second, but the packets are dropped somewhere if I exceed about 1000 pings/sec. Even at 1000 pings/sec (a trivial amount of bandwidth) this behavior seems to wreak havoc on my home network; other devices sharing the connection get extremely bad performance, a no-no for the Work-from-Home video call lifestyle of the pandemic. This could be because my internet provider throttles ICMP; it could also be that some hardware or software in the path is not able to handle thousands of different IP addresses each second (e.g. there may be fixed-size NAT tables). I tried using a VPN, but this had a much lower success rate; the VPN egress point probably throttles ICMP to prevent DDoS attacks, and it's possible that many internet gateways also simply block ICMP from known VPN endpoints since they are obvious choices for people up to no good. Anyway, what I thought might take a few hours or a weekend ended up taking months. Eventually I rented time on several machines in different data centers to parallelize the process; this also produced a higher ping response rate than my home network, so I redid all of the already-completed sections for uniformity. The results make a nice poster, though, and are in Figure 1.

9.18% of addresses responded successfully within 4 seconds. Only 4,529 hosts (0.000105%) replied with the wrong data.

2   Harder Drive: Pingu

At last, I yearn to build a virtual hard drive using the ideas above. It will be called `pingu`.[4] To do that, I first had to figure out how to make a hard drive. This is no big deal.

2.0.1   nbdkit

In UNIX, storage systems are abstracted as "block devices." Like all things in UNIX, it is conceptually "just a file," but then gets complicated with all sorts of concessions for efficiency. Fortunately, efficiency is a non-goal for this project. We could implement these drives as kernel modules that implement the basic operations of a block device. This would be a bad choice because of the number of userspace facilities we want to use, and also because I would have to do a *lot* of rebooting as my myriad bugs panicked the kernel.

`nbdkit` (for "no big deal" kit) is a library for creating and mounting Network Block Devices in userspace. Network Block Device (for "NBD") is a protocol for communicating with a quote-unquote block device (for example, a physical

hard drive, or a virtual drive like a file containing a DVD-ROM image, or a block of memory, or the variety of weird drives considered in this paper) over a network. It's also straightforward to use with a local quote-unquote network (i.e. UNIX domain socket).

In order to create a device, you implement functions like `pread` (read some bytes from a region in the device and copy them into the caller's buffer) and `pwrite` (same in reverse). There are also many optional operations (e.g. "fast zero" a region) for efficiency, plus hints for `nbdkit` or the kernel to know how to optimize data layout on the drive. For example I was charmed to see a flag `is_rotational` that describes whether the drive is based on spinning platters, which presumably is used as a hint that sequential reads/writes are more efficient. The block device is compiled as a shared object that can be loaded into `nbdkit`'s server, then attached (as root) as a block device like `/dev/nbd0`. At this point, the device can be formatted with some filesystem.

2.0.2   Implementation

The smallest drive that can be formatted and mounted on a normal Linux machine is 51,200 bytes, using the FAT12 filesystem common on DOS floppy disks in the 1980s [26].[5] So the device consists of one hundred 512-byte blocks. Each block will be stored inside multiple outstanding pings (for redundancy) with a 512-byte payload.

Despite pinging the whole internet in Section 1.4, we use a fixed set of IP addresses here. The reason for this is that we want a set of IP addresses that are stable, reliable, and have high latency. They must respond to pings with a 512-byte payload. We would also prefer these to have uncorrelated failures, because if all of the outstanding pings fail for a block, then the data is permanently lost. So we want them to be geographically diverse, for example. I also prefer to use major commercial sites that can clearly bear the load, as opposed to e.g. someone's cell phone (who may even have metered bandwidth). These are actually hard to identify from the full data set, particularly the last criterion, but it is not hard to find candidates by hand. I produced the list of $\sim$75 hosts by searching for "most popular websites in Madagascar" (etc.) and manually pinging them to make sure the criteria are met, particularly the latency. Many sites worldwide use content networks (or are simply hosted in the United States) and so they are much faster than the speed of light would suggest. I found that database-backed sites (like e-commerce pages) were less likely to be on content networks than e.g. news sites, which makes sense.

Implementing this block device is tricky: We can only process a read or write at the moment a ping returns from the network.

Blocks.   Each block contains a sequence and version counter, as well as the set of outstanding pings (send time

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[4]Named for the classic stop-motion penguin of the same name. Also as in "i ping u 2 store data thx 4 ur help"

[5]Try: `mkfs.vfat -F 12 -v -a -n "PINGU"`
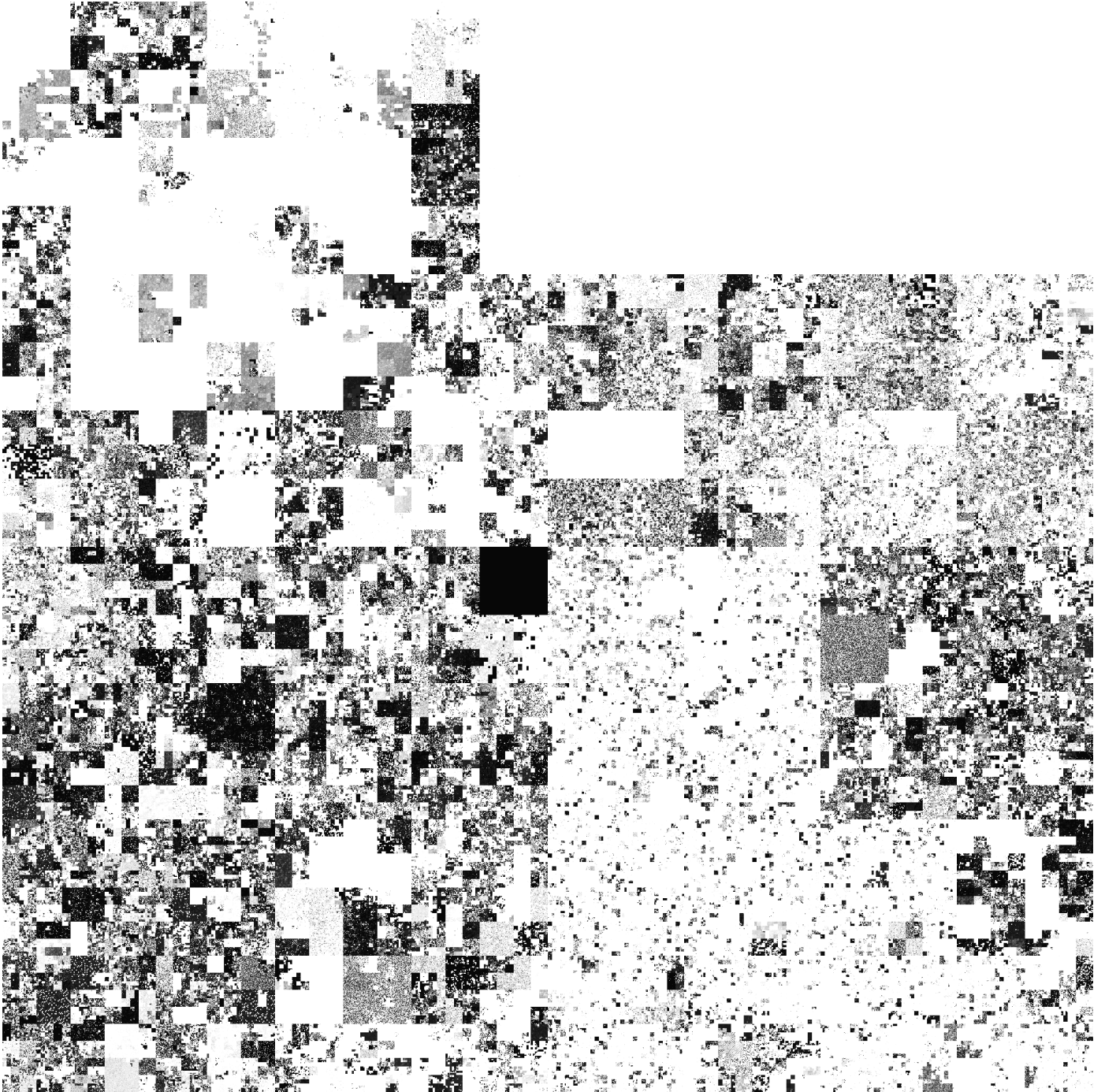
Figure 1: The results of pinging all 2^{32} IPv4 addresses in early 2022. The IP addresses are plotted along a 16-level Hilbert curve [9]. The full image is 65536 × 65536 pixels (and 4.2 Gigabits), which alas cannot be fit within the preposterously limited SIGBOVIK page and PDF size guidelines. This image is 2048 × 2048, so each pixel represents 32 × 32 hosts, with the level of grey giving the response rate (white = no response). There are several obvious patterns, which can be cross-referenced with Figure 2 to see the first octet of the IP address. Some interesting regions: There are almost no responses in the top-right region, which is `224.*` to `255.*`; these are the former "Class D" and "Class E" segments which are *multicast* and *reserved* respectively. There is a dark block at the center that almost always receives responses, which is from the `127.*` "loopback" addresses. This all makes sense. It is interesting to see how active regions allocate their space; some have a variety of distinctive patterns and others seem uniformly random (Figure 2). This way of plotting the address space is basically canonical, so it is a bit disappointing not to find any graphical messages. Like, how cool would it be to embed a micro QR code in some 16x16 subnet that says `IPv6sux`? It would be a little bit cool, is how cool.

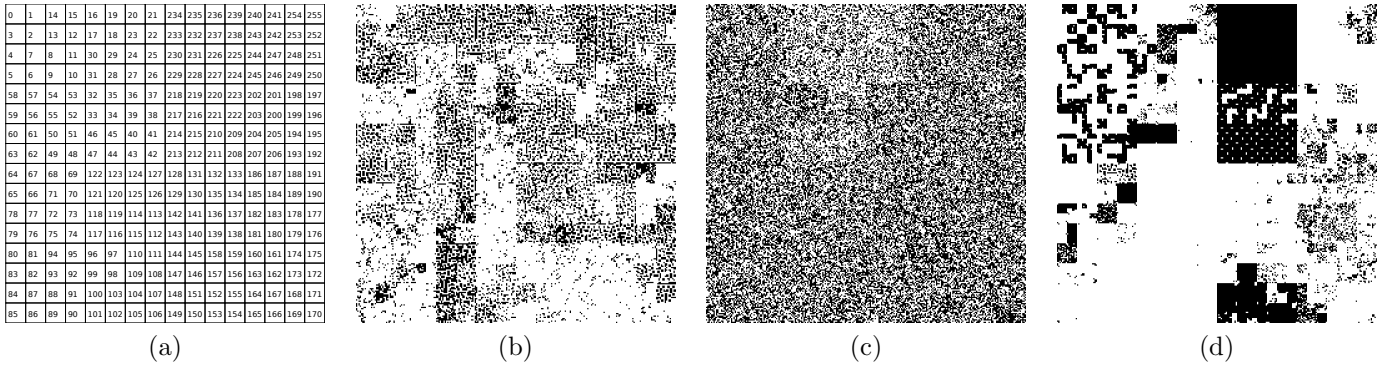Figure 2: (a) An *internet legend*: The location of top-level octets in the Hilbert curve used to create Figure 1. Then, several 1:1 zoomed regions, showing how much the textures vary: (b) The subnet `213.6.*.*` (Palestine Telecommunications Company) shows some curious patterns of clumps or lines surrounded by whitespace, almost like a map of ancient city ruins. (c) The subnet `5.138.*.*` (Rostelecom Macroregional Branch South) is almost uniformly random, although it does look like it might be hiding a faint spooky skull at the top. (d) The subnet `45.195.*.*` (CloudInnovation) clearly has distinct subregions, which makes sense as it is an IP address management company. In that sense it Fractally resembles larger portions of the Internet. We also see some missing regions in the shape of Tetris pieces (Section 3).

and host IP, so that we can detect timeouts and update host stats). There is also a queue of pending reads and writes. The contents of the block is not stored.

Reading. A call to read a block inserts itself in a queue and then waits on a condition variable; it will not return until we receive a ping that belongs to that block.

Writing. A call to write is accompanied by some data (the caller has allocated it). These are also enqueued and wait synchronously until we receive a ping from the host and can process it. In the general case we cannot process a write without receiving the ping, because the write may only be to a portion of the block (and so we need to know the data outside that region). When we process the write we update the version counter so that we don't later use the data from any other outstanding redundant pings.

Hosts. With each host (IP address) we also keep track of its recent latency and reliability (exponentially-weighted moving averages) as well as a token bucket to prevent exceeding a prescribed number of pings per second to that host.

Processing. A single thread calls `select` to see if the socket is ready. For juggling we need to simultaneously be ready for both *reading* and *writing*. We then read a ping, and use its sequence number to route it to the correct block. The block validates the ping (if it has the wrong version it's just discarded, for example) and uses it to fulfill any outstanding reads (copying into their buffers and notifying the condition variable so those calls can return). We then process any writes to compute the updated data, and juggle the data back onto the network by sending pings until we are at the target redundancy (there will be at least one, since we just received one of the outstanding pings). For each outgoing ping we prefer a host with high latency, high reliability, and which has not recently been used. We also avoid using the same host more than once for a block, because if we lose all the outstanding pings, the data are
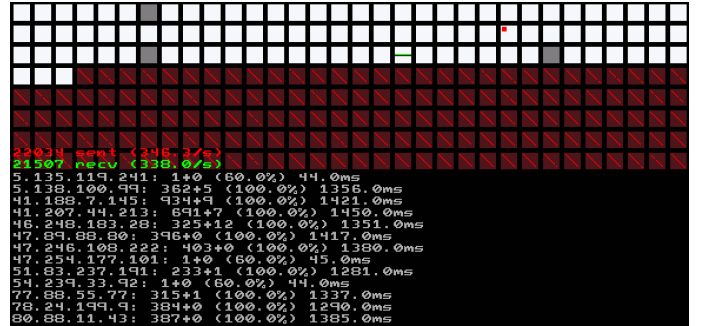
Figure 3: Visualization of the `pingu` drive (truncated). The squares at the top are the data blocks; where white indicates a healthy block with a full complement of outstanding pings, and darker colors less so. The crossed-out blocks have not yet been written (and so store no data). The red dot indicates a block with an outstanding write, and the green bar the current block for the round-robin initialization. At the bottom, some of the hosts and their recent statistics.

forever lost.

Initialization. The loop just described is driven by the receipt of pings, so we also need to kick off the process by sending initial pings for each block. After each call to select we initialize a single block if it has not yet been, and has at least one outstanding write.

Visualization. The block device runs in userspace, but not in a way that supports a UI. To view the device while it's being used, I send text status updates to `nbdkit`'s debugging interface, and then pipe these to an SDL-based visualization (Figure 3). It shows the status of each block and statistics on each host, as well as read/write activity. It is fun to watch the process of formatting it for FAT12 and reading/writing files.

2.0.3 Results

Next we want to evaluate this Harder Drive according to various criteria. Our goal is not to create a drive that is "good" according to normal criteria like speed, but it is still interesting to benchmark it.

For each benchmark in this paper we will store a single file on the drive. The choice of file typically doesn't matter for the benchmark (which will compute "bytes per second" etc.) but it very much matters for the aesthetics of the project. In each case we'll choose a file that establishes a kind of "improper hierarchy" [16]. For `pingu`, we'll store RFC 792 [21], a 29,186-byte text file that describes ICMP, including the `ECHO` and `ECHO REPLY` messages with which we've constructed the drive.

Before benchmarking, we format the drive with a FAT12 filesystem and mount it (`-noatime`, etc.). We then `sync` and flush the kernel cache.[6] Flushing cache is very important, as these tiny drives easily fit entirely within the cache and appear to be very fast if you don't do this. Then, the benchmark writes the entire file to the test drive, syncs and clears cache again, and reads it back, comparing to make sure the correct bytes were written. We repeat this process over and over, for at least one minute (though some drives will only complete a single pass, taking much longer than a minute). The sync/flush between writing and reading is attributed to the write time, because this is when the writes are actually taking place.

Qualitative. This is a good Harder Drive. It solves a problem we don't have, which is to unreliably store a small amount of data in an even smaller amount of memory.[7] It treats latency as a desirable quantity, contrary to the usual preference. Implementing the drive was much more difficult than expected from back-of-the-envelope calculations.

Cost. The cost consists of an up-front cost (a computer and network interface; even a $35 Raspberry Pi should work fine) and an incremental cost per byte stored. This drive is unusual in that storage is derived from network bandwidth,

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[6] `echo 3 > /proc/sys/vm/drop_caches`

[7] I considered whether it would be possible to create a drive that used *no* memory for each additional block, and how I might even define/measure that. This lead me to a brief experiment with `compu`, which never graduated to a proper Harder Drive. This drive compiles the drive's contents as code, namely a large switch statement that is of the form `case ADDRESS: return DATA;` for each address in storage. The joke is of course that if you don't count the "code" towards memory, you can sneak memory into the code. Of course, to write to the drive, we need to rewrite the code and recompile it (this is done dynamically by forking `g++` and then loading the recompiled symbol with `dlopen` (which obviously uses memory)). I was hoping that the compiler would be able to do some clever optimizations on the switch statement, which might have led to some interesting developments. But the only ones I observed were the cases where the entire contents are the same byte, or where each address contains the low byte of the address as data. Otherwise it always just compiled as a table lookup, which is pretty uninspiring. For completeness, this drive was annoyingly fast in benchmarks (of course using its own source code as the benchmark test file): 6,119 bytes/sec writing; 10 Megabytes/sec reading.

which is measured in bytes per second. My home network is "1 Gigabit/sec" and $80/month ($3.04 \times 10^{-5}$ / second). Storing 51,200 bytes renders the connection otherwise unusable, so we assume this is close to the maximum storage. This is a cost of 0.156 cents per byte per month, which is 5.94×10^{-8} cents, or 59.4 nanodollars, per byte per second.

Longevity. Longevity is poor. The one-minute benchmark succeeds with 100% accuracy, but data will readily be lost if the drive is left running for several minutes. We can increase longevity by using hosts with higher latency, although this reduces speed.[8] Since the data are stored externally using untrusted hosts around the world, it is easy for adversaries to tamper with it by sending us back the wrong data. This could be mitigated with checksums or error correcting codes [23], although we want to avoid anything that might resemble "storing" the data locally (this is cheating). On the other hand, since we do not store data locally, this drive could be considered "non-volatile" in the sense that if we completely lose power and reboot, we can still recover the data as the pings are received from the network. Such a reboot would need to happen in less than about 100 milliseconds, though.

Speed. The drive is slow but tolerable. In the benchmark wrote and read the test file 15 times in one minute, and achieved 15,286 bytes/sec writing and 13,239 bytes/sec reading. Reads and writes are basically the same operation so this gives a small indication of the variance (high) as well. We can get better I/O performance by using hosts with lower latency, but this increases the (local) cost and decreases longevity.

Power. Power consumption is low. The up-front power cost for a computer and network connection is small (Raspberry Pi 3 is about 3 Watts). The data are actually stored externally, and if we were the only use of the Internet, a very significant amount of power would be consumed in transmission lines. In the benchmarked configuration, each 512-byte block has 4 outstanding pings, for which we assume a mean cable length of 1/4 Earth circumference (10 Mm). A copper connector like Cat6 UTP has nominal DC resistance of 84 Ω/km, so the loop resistance would be 840 MΩ, which is actually rather high. The math to figure out the power per byte eludes me (not to mention that undersea cables are usually fiber optics), but it is not trivial. A typical undersea data cable's excitation power is on the order of tens of kilowatts, with repeaters every 100 km or so. Fortunately, the total bandwidth of such cables is extremely high, with these 512-byte packets representing a minuscule fraction. Rather than try to multiply a big estimate by a small estimate, it is better to work from a known quantity: Assuming that the cost of the consumer internet connection also covers the marginal cost of the power in

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[8] One easy approach is to deliberately incur additional overhead. For example by connecting to a VPN in Nigeria, I ensure a round-trip to Africa before the pings even make their way to the open Internet, which increases latency significantly. This reduces throughput to 2,948 bytes/sec., however.

these backbones, at $0.15c/kwH$, this seems to be at most 5.8 μWatts per byte-second.

Is rotational?   One of our criteria will be whether the drive should set the `is_rotational` flag for `nbdkit` (Section 2.0.1). The inspiration for this drive (orbital chainsaws or radio towers around the world) would be rotational, but this drive is not. Although the initialization happens round-robin, due to the stochastic timing of the outstanding pings, the drive soon thereafter processes the blocks in a random order.

Harm to society.   The drive is definitely harmful to my home network; whether that can be considered a positive or negative to society is left as an exercise for the reader. At the benchmark scale of 51,200 bytes, the effect on the broader internet is trivial, and I took care to not overwhelm any particular host. However, at scale this drive would be harmful to the shared infrastructure, and carries the moral hazard of "freeloading" off the willingness of hosts to reply to `ECHO` messages.

3   Tetris, the Soviet Mind Game

Sorry to remind you about Vladimir Putin's illegal invasion of Ukraine, but this section concerns a hard drive made from the best Russian (actually, Soviet) video game, Tetris [20].

Tetris is an inventory-management survival-horror game with 19 principal characters, each with its own story arc; they are: ▙▄▟▛▜▞▞▌▐▛▐▌▙▟▛▟▐▌▛▐▞▌▐▛

Like all living things, these characters are made up of four individual pixels, or "blocks." By being confused about the fact that words can have multiple meanings, we can have an idea: Make a block device from these blocks, using their presence or absence in the playfield to store data. A Tetris board is 10 columns wide and 20 rows high. Even if we could use every one to store a bit, 200 bits is far too few to create a filesystem. Therefore we'll use an array (or if you will, a *Beowulf cluster*) of Tetris games to create the block device.

We will store a bit pattern in a Tetris game by playing a series of moves to create a specific pattern in the playfield. We can then read the data directly from that pattern. If we need to write a new pattern, we reset the game and begin again.

Each "line" of the playfield has 10 positions, each of which could have a block in it or not, so we could consider storing 10 bits. However, due to the rules of Tetris, if all of the cells are filled, then the line is cleared. This would make it impossible to store the pattern `1111111111`. It will also be impossible to store the pattern `0000000000`, because an empty line cannot support any pieces above it, so empty lines can only appear in some completely-empty prefix of the playfield. Additionally, observe that a Tetris board always has an even number of cells filled. We can only add 4 blocks by dropping a piece, or remove 10 blocks by clearing a line, which can only yield even numbers. So it will also benefit us to have one free cell per line for parity.

```
uint16_t NextRNG(uint16_t state) {
  uint16_t carry = ((state >> 9) ^ (state >> 1)) & 1;
  return (state >> 1) | (carry << 15);
}
```

Figure 4:   The simplified code for NES Tetris's pseudorandom number generator, which resides at address `0xAB47` in the Tetris code. This is a 16-bit two-tap LFSR: The `carry` is the exclusive-or of the second and tenth least significant bits. We right-shift off the least significant bit and use the carry as the new most-significant bit.

A good choice is to use 8 cells per line for data, encoding a single byte, which is a nice round number.

We can't use the full height of the playfield, since we need some room in which to maneuver pieces. 8 is a convenient choice here as well (although more is possible). Each Tetris game will thus store 64 bits: Eight lines, each with eight bits. We'll use the venerable NES Tetris (Nintendo, 1989), which is also 8 bits.

3.1   Playing Tetris

Now the problem is: Given a blank board, what sequence of moves do we make in order to produce the target pattern?

This is not easy. To begin with, Tetris normally gives the player pieces at random. As anyone who plays Tetris knows, it can be very disruptive to your strategy when you don't get the piece you need for some time. The first step will be to reverse engineer the pseudorandom piece drop logic so that we can influence the sequence of pieces that are dropped.

3.1.1   Random pieces

The core of the piece drop logic is a 16-bit linear feedback shift register [8].[9] Equivalent C code is given in Figure 4. This 16-bit state is updated on every frame (and sometimes more; see below), and has period 32767.[10]

The pseudorandom state is extended with two additional bytes: One giving the last dropped piece (a piece is "dropped" into a queue so this is actually the "next piece" to the player) and the count of pieces dropped (mod 256). When the player places a piece, the routine at address `0x9907` uses the LFSR state and these two bytes to drop a new piece (and update the state):

```
RNGState NextPiece(RNGState s) {
  constexpr std::array<uint8_t, 8> PIECES = {
    0x02, 0x07, 0x08, 0x0A, 0x0B, 0x0E, 0x12,
    /* not used */ 0x02,
```

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[9]Fittingly, Golomb was a pioneer in both shift registers *and* polyominoes, the latter which influenced Tetris itself!

[10]It is possible to create a 16-bit LFSR with a period of 65535, but this one is simply deficient. This is one of several small problems with the code. I hope to one day release a "hot fix" ROM that fixes this and other bugs and inefficiencies.

```
    };

    s.drop_count++;

    uint8_t a = (s.lfsr_hi + s.drop_count) & 7;

    if (a == 7 || PIECES[a] == s.last_drop) {
        // re-roll if out of bounds, or repeat
        s = NextRNG(s);
        // mod 7 forces in-bounds, but allows repeats
        a = ((s.lfsr_hi & 7) + s.last_drop) % 7;
    }

    s.last_drop = PIECES[a];
    return s;
}
```

It uses three bits of the RNG state to pick a random
piece (there are 7 different shapes, and the game always
drops a shape in the same orientation). If it rolls an 8,
or if the selected piece is the same as the last one, then it
re-rolls: Another update of the LFSR, and then a different
weird procedure to pick the piece index. Here the result
is mod 7, so it is always in bounds. The code only re-rolls
once, so it is possible to drop the same piece twice in a row,
just less unlikely.

Ideally we would be able to select a sequence of pieces
that we want, and then force Tetris to give us those pieces.
Since the LFSR update runs every frame, we can use a
different number of frames while placing a piece, and get
a different LFSR state at the point NextPiece is called.
The LFSR is "pretty good," so we can easily cause the first
roll to be whatever value we want by just waiting. In the
worst possible case we need to pause 98 additional frames
before seeing all 8 rolls (Figure 5), which is 1.6 seconds at
the NES frame rate.

However, this does not work for re-rolls, which is the only
way to get the same piece twice in a row. Even though we
use the same "pretty good" LFSR to get the second pseu-
dorandom number, two successive calls are highly corre-
lated. There are only two possible new values for the high
byte of the LFSR (s.lfsr_hi): (s.lfsr_hi >> 1) and
128 + (s.lfsr_hi >> 1). Worse, since these are congru-
ent modulo 8, we really just have (s.lfsr_hi >> 1).

As a result, even if you have complete control over the
LFSR state (but not the previous piece nor drop count),
there are a limited number of outcomes possible from the
reroll. We can just inspect all the possible combinations
of previous piece and drop count to see that with some
there are at most 4 possible rerolls, and as few as 2. For
example, if the previous piece is ⌐ and 253 pieces have
been dropped so far, then only ⌐, ⌐, and ⌐ can result.
So here it is possible to get two ⌐ pieces in a row. But if
the last piece was ▬, and 3 pieces have been dropped so
far, then only ⌐ and ⌐ are possible from a re-roll. Since
▬ cannot result from the first roll and is not possible for
the re-roll in this state, it is impossible to get two ▬ pieces
in a row on the 3rd and 4th drop. All pieces other than ■
periodically have this problem.[11] Even when it is possible,

----

[11]This is another example of a deficiency in the code that could eas-

it may require a long drought to get the LFSR in a rare
working state.

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
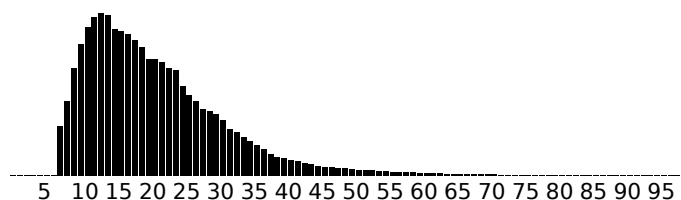
Figure 5: The maximum possible "drought" for the NES
Tetris LFSR. This is the number of iterations before we see
all possible rolls (all eight possible values for the low three
bits), histogrammed over all meaningful start states (LFSR
state and drop count). The minimal value is 8 (obviously,
by pigeonhole) and rarely it can be as high as 99, but most
of the time we have seen all rolls by a few dozen iterations.

By manipulating the RNG we will be able to achieve any
sequence of pieces, except if that sequence contains repeats.
There is a similar consideration for the first two pieces of
the game, which are generated on the same frame (cur-
rent and next piece) and so only certain combinations are
possible. To simplify this issue away, we begin by always
playing the same starting sequence: ■0 ⌐3 ⌐6 ⌐2 ⌐7.
This means to drop a ■ in column 0 (the leftmost column),
⌐ in column 3 (the left edge of the piece goes in column
3), etc. This sequence clears two lines, leaving the board
empty, with the only constraint now being that we cannot
start with a ⌐ piece. In fact we will always start with a ⌐
piece in the leftmost column to make use of the modular
plans described in the next section.

3.1.2 Planning

With a smart algorithm, it would be possible to plan moves
within these constraints to generate moves for any byte
pattern that we want to create. However, this is not an
easy feat. Creating very sparse patterns (few 1 bits) is
pretty challenging because pieces can only be placed on
top of existing blocks (Figure 6).

Instead, the approach I took was to build a modular solu-
tion for each byte. We always place the board in a standard
configuration (Figure 7) where a ⌐ piece is in column 0. A
portion of the board below it contains the encoded bytes
(one per row) and parity (first two columns). We do not
depend on the contents of this portion at all, so the first
moves must hang off of the ⌐ piece. For each byte, we need
to come up with a series of moves that takes a starting con-
figuration like this, encodes the byte (and its parity) in the
bottom row, and then recreates the starting configuration
with the ⌐ piece moved up one line. These plans are also

----

ily be fixed. If we simply remove the instruction at 0x9925, AND #$07
(so that the re-roll just uses (s.lfsr_hi + s.last_drop) % 7) then
all configurations can now produce all pieces upon re-roll. The mod-
ulus is computed with a loop, so this is not a strict efficiency im-
provement, but efficient alternatives with this property exist, like
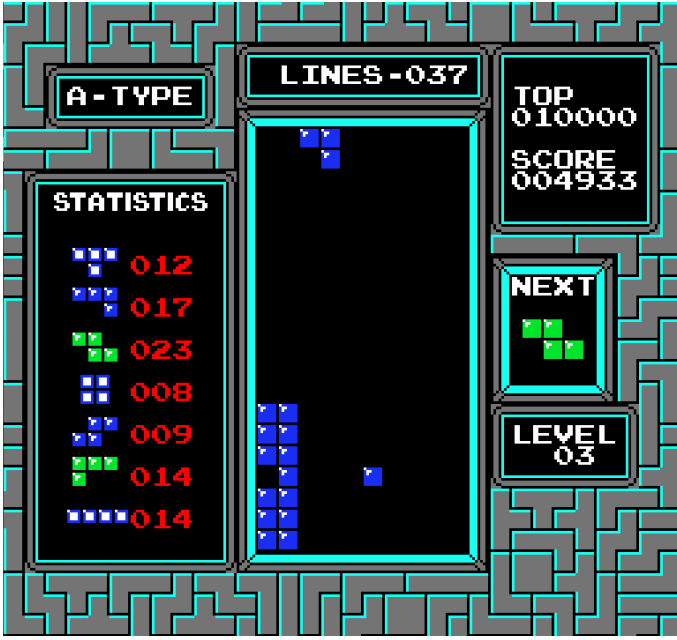((s.rng2 & 15) ^ s.last_drop) % 7.

Figure 6: A screenshot of NES Tetris after playing for 2 minutes and 46 seconds, encoding the sequence 0, 0, 0, 0, 64, 0, 0, 0. Floating blocks like this are very difficult to make. Try creating a position like this, even by selecting your favorite piece at each step!

nontrivial to construct, but we can take our time to solve each byte offline. Then, because they are compositional, we can assemble them to create any sequence of bytes at runtime.[12]

An example sequence is illustrated in Figure 8. They are not easy to construct by hand, but it is not too hard to find them with computer search. I have a two-phase heuristic search: First, a heuristic that measures how close we are to placing the correct bit pattern in the bottom row (with penalties for covering bits that we still need to set, or for growing the pile too high). Second, measuring how close we are to reaching the standard start position (by clearing any leftover stuff).[13] This finds solutions for all bytes easily; to find really good short sequences we just run it for hundreds of hours. This search uses my own implementation of Tetris, which can be simplified because e.g. we only drop pieces straight down. It is orders of magnitude faster than emulating the NES Tetris ROM.

With this setup, the best solutions I know of for each byte (as of publication) are as follows:

- 00: ⌐1 ⌐0 ⌐3 ⌐5 ⌐3 ⌐8 ⌐5 ⌐5 ⌐8 ⌐0 ⌐5 ⌐2 ⌐0
- 01: ⌐1 ⌐3 ⌐5 ⌐2 ⌐7 ⌐0 ⌐3 ⌐7 ⌐4 ⌐0 ⌐6 ⌐8 ⌐0
- 02: ⌐1 ⌐0 ⌐3 ⌐2 ⌐0 ⌐5 ⌐7 ⌐6 ⌐8 ⌐4 ⌐5 ⌐7 ⌐0
- 03: ⌐1 ⌐0 ⌐3 ⌐3 ⌐5 ⌐7 ⌐5 ⌐7 ⌐1 ⌐7 ⌐8 ⌐0 ⌐2 ⌐0 ⌐4 ⌐6

---

[12]As one final complication, when we concatenate two of these sequences we still need to avoid repeat pieces. We do this by only allowing a sequence to end with the ⌐ or ⌐ pieces, and not allowing either of those to start sequences.

[13]Details here are in encode.cc. My first attempt worked pretty well, so I didn't fiddle with it much; no doubt it can be improved!
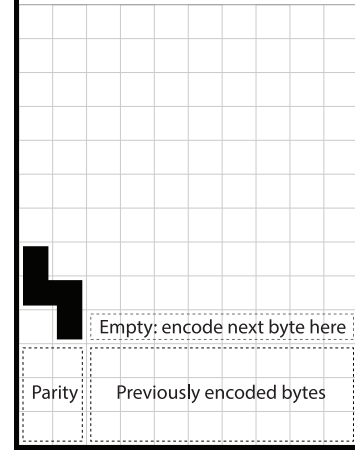
Figure 7: The start state as we encode each byte. The ⌐ piece in the left column is known to be present, but we do not know the state of any blocks below it. We will only build off of this block so that the solution works regardless of what is below, and at any height (as long as there is enough headroom in the playfield). The parity columns will be 0b01 or 0b11 as appropriate, except in the case of the byte 0xFF we use 0b00 so that we don't create and clear a line. A specific example solution is given in Figure 8.

(a)   (b)
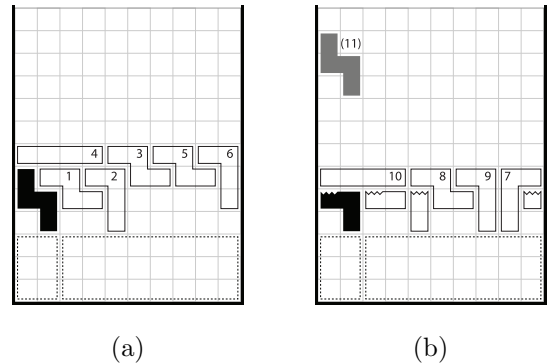
Figure 8: Encoding 0x26, which is 0b00100110. This is one of the easiest bytes. In (a), drops 1–6 build off of the starting ⌐, conveniently dropping a 1 bit in column 4 along the way. These moves clear the two top lines, leaving some partial blocks. Note that we use ⌐ and ⌐ multiple times, but never consecutively. In (b) we use some of the floating pieces to fill the remaining 1 bits, clean up by making two more lines, and then drop a ⌐ onto known support in order to put ourselves back in standard position, one line higher. Many sequences create the "⌐" shape through means other than placing it literally.
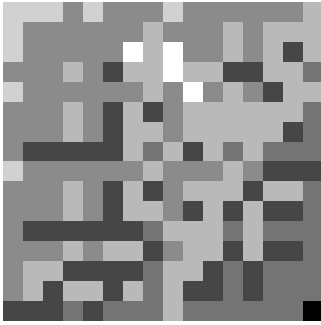
Figure 9: The number of Tetris pieces placed to create each byte, with light being the fewest (11) and dark being the most (22). The first row is (0x00, 0x01, ...). We tend to need fewer moves to create bytes with fewer 1 bits, and there are some symmetries and patterns visible. On the other hand, I was somewhat disappointed not to see the familiar Triforce pattern appear here, as it has so many times before [13; 14; 15]. It may be that these solutions are not yet optimal (but they are probably close). Alternatively, it could be that this is the *exception that proves the Hyrule*.

- 04: ↳1 ⊤3 ↳5 ⌐7 ↲2 ⌐8 ⌐0 ↳4 ⌐6 ⎯3 ⌐0 ⌐7 ↳0
- 05: ↳1 ⌐0 ↳3 ⎯5 ⌐8 ⊤5 ⌐3 ⌐6 ⊤2 ⌐8 ⌐0 ⌐8 ⊓6 ⌐3 ↳0
- 06: ↳1 ⊤3 ↳5 ↳2 ⌐7 ⌐8 ⊓0 ↳4 ⌐6 ⊤2 ⌐8 ⌐0 ↳5 ⎯1 ↳0 ⎯6
- 07: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⊤7 ↳5 ⌐8 ⌐7 ⌐4 ⌐6 ⌐0 ⊓8 ⌐3 ↳0 ↳0
- 08: ↳1 ⊓0 ↳3 ⊓2 ⌐5 ⊓4 ↳6 ⌐0 ⌐4 ⌐8 ⌐7 ⌐4 ↳0
- 09: ↳1 ↳0 ↳3 ⌐5 ↳6 ⌐8 ↳3 ↳4 ⊓2 ⌐6 ⌐8 ⌐0 ⌐8 ⎯0 ↳0 ⎯4
- 0a: ↳1 ⌐0 ↳3 ⌐5 ↳6 ⌐4 ⌐3 ⌐8 ⌐0 ⌐6 ⌐0 ⊤2 ⌐5 ⌐8 ↳0
- 0b: ↳1 ↳0 ↳3 ⊓3 ⌐5 ↳6 ↳5 ⌐1 ⌐8 ⌐8 ⌐6 ⌐8 ⊓0 ⎯5 ⌐2 ↳0
- 0c: ↳1 ⌐0 ↳3 ⊓3 ↳5 ↳8 ↳3 ⌐8 ⌐0 ⌐4 ⌐2 ⌐0 ⊤7 ↳5 ↳0
- 0d: ↳1 ↳0 ↳3 ⌐5 ↳6 ⌐8 ↳2 ↳4 ⌐8 ⌐6 ⌐0 ⌐8 ↳1 ↳0 ⌐5 ⎯3
- 0e: ↳1 ⌐0 ↳3 ⌐5 ↳6 ⌐8 ↳3 ⌐8 ⌐6 ⌐2 ⌐0 ⎯4 ⌐8 ⎯0 ⌐5 ↳0
- 0f: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ⌐6 ⌐0 ⌐8 ⎯1 ⌐5 ↳0
- 10: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ⌐8 ⌐6 ⌐5 ⎯6 ↳0 ⌐2 ↳0
- 11: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ⌐8 ⊓1 ⌐6 ⌐5 ⌐8 ⌐0 ⌐0 ⎯3 ⌐6 ⎯2
- 12: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ⊓5 ⌐7 ⌐7 ⌐2 ⌐0 ↳4 ↳6 ⎯0 ⌐7 ↳0
- 13: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ⌐7 ⌐5 ⌐8 ⌐0 ↳4 ↳6 ⌐0 ↳7 ⌐3 ↳0
- 14: ↳1 ⌐0 ↳3 ⎯5 ↳2 ⌐8 ⊤5 ⌐8 ↳6 ↳3 ⌐0 ⌐5 ⌐7 ⎯3 ⌐0 ↳0
- 15: ↳1 ↳0 ↳3 ⎯5 ⌐8 ⊤5 ⌐1 ⊓3 ↳8 ↳6 ⌐5 ⌐8 ↳0 ⎯1 ⌐5 ↳0
- 16: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ⌐7 ↳8 ⌐7 ⎯3 ↳6 ⌐0 ↳0
- 17: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ⌐7 ⌐5 ⌐8 ⌐0 ⌐2 ⌐5 ⌐7 ↳0
- 18: ↳1 ↳0 ↳3 ↳5 ↳6 ⎯3 ↳2 ⌐8 ↳6 ⌐0 ⌐7 ⌐4 ⌐7 ⎯3 ⊓0 ↳0
- 19: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳1 ⌐8 ↳2 ⌐0 ↳6 ⌐5 ⌐8 ⎯4 ↳0
- 1a: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ⌐8 ↳4 ⌐3 ⌐7 ⌐0 ⎯5 ⌐0 ↳0
- 1b: ↳1 ⌐0 ↳3 ⎯5 ⌐7 ↳5 ⊓3 ↳6 ⌐8 ⌐0 ⎯1 ⌐5 ⌐7 ↳0
- 1c: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ↳8 ↳4 ↳5 ⌐0 ⌐7 ⌐0 ⌐3 ↳0
- 1d: ↳1 ↳0 ↳3 ⎯5 ⌐7 ⊓3 ↳5 ↳6 ⌐8 ↳5 ↳0 ⌐7 ↳0
- 1e: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ⌐8 ↳0 ↳4 ⎯6 ⌐1 ↳0
- 1f: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ⊓4 ↳2 ↳6 ⌐8 ⌐0 ⌐4 ⌐7 ↳0
- 20: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐3 ↳5 ⌐7 ⌐2 ⊓0 ⌐7 ↳0
- 21: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳8 ⎯0 ↳6 ⌐0 ⎯3 ⌐8 ⎯0 ↳0 ⎯4
- 22: ↳1 ⌐0 ↳3 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳7 ↳0 ⌐1 ⌐7 ↳5 ⌐3 ⎯0 ↳0
- 23: ↳1 ⊓3 ↳4 ⊓1 ↳6 ⌐8 ↳0 ⌐8 ↳4 ↳6 ⌐2 ↳8 ↳4 ⎯5 ⌐0 ↳0
- 24: ↳1 ⊓3 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳8 ↳6 ↳0 ⎯4 ⌐1 ⌐3 ⌐7 ↳0
- 25: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ↳4 ↳6 ↳8 ↳2 ⌐8 ⌐3 ↳6 ⌐0 ↳0
- 26: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⎯0 ↳0
- 27: ↳1 ⊓3 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐7 ⌐0 ⎯3 ↳0
- 28: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐3 ↳7 ↳6 ⌐0 ↳0
- 29: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳0 ⌐5 ↳8 ⊓3 ⌐3 ⌐8 ⌐0 ↳6 ↳0
- 2a: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐2 ⌐7 ↳6 ↳0 ↳4 ⌐6 ↳1 ⌐7 ⎯3 ↳0
- 2b: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳8 ⌐1 ⌐5 ⌐4 ⌐7 ↳0 ↳0
- 2c: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳4 ↳7 ↳5 ↳1 ⊓2 ⌐7 ⌐4 ↳0 ↳0
- 2d: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ⌐7 ⌐5 ↳0 ⌐1 ↳5 ⎯7 ↳0
- 2e: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ↳4 ↳6 ⌐8 ↳2 ⌐5 ⌐2 ⌐7 ↳0 ⎯1 ↳5 ⌐7 ↳0
- 2f: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐4 ⎯1 ⌐8 ↳6 ⌐7 ↳0
- 30: ↳1 ⊓3 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ⌐8 ↳4 ↳7 ↳5 ↳0 ⎯6 ↳1 ⌐3 ↳0
- 31: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐1 ↳4 ⌐7 ↳5 ↳7 ⌐8 ⎯0 ⌐4 ⎯0 ↳0
- 32: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ⊓4 ↳2 ⌐7 ↳5 ⌐0 ⌐7 ⎯3 ⌐0 ↳0
- 33: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⎯0 ⌐8 ↳4 ↳6 ⌐7 ⌐0 ⌐3 ↳0
- 34: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ↳4 ↳6 ⌐1 ↳8 ↳1 ⎯6 ⌐3 ⎯6 ↳0
- 35: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐2 ↳4 ↳6 ⌐8 ↳0 ⌐4 ⌐2 ⌐7 ⌐8 ↳5 ⊓3 ⊓0 ↳0
- 36: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳6 ⌐8 ↳0 ↳1 ⎯3 ⎯6 ↳0
- 37: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐0 ⌐8 ↳8 ↳4 ⊓2 ↳6 ⌐4 ⌐7 ↳0
- 38: ↳1 ↳0 ⊓3 ↳4 ↳1 ↳6 ⌐8 ⌐7 ↳6 ↳4 ↳0
- 39: ↳1 ⌐0 ↳3 ⎯3 ↳8 ⊓3 ⌐5 ↳5 ↳7 ↳0 ↳8 ⌐4 ⎯1 ↳0
- 3a: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ↳0 ↳6 ⌐7 ⌐4 ⎯1 ↳0
- 3b: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ⌐7 ⊓6 ⌐8 ⌐0 ⎯6 ⌐1 ⊓5 ⊓3 ⊓8 ⌐0 ⌐5 ↳0
- 3c: ↳1 ⊓3 ↳4 ⎯1 ↳6 ⌐8 ↳0 ⌐7 ↳6 ↳4 ⌐1 ↳5 ⌐3 ↳7 ↳0 ⌐7 ⊓5 ⌐1 ↳0
- 3d: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ⎯1 ↳5 ↳0 ↳4 ↳5 ⌐7 ↳0
- 3e: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐8 ↳4 ⎯0 ↳6 ↳4 ⎯0 ↳0 ⎯6
- 3f: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ↳0 ⌐7 ↳6 ⌐4 ⌐2 ⎯4 ⌐0 ⊓8 ↳0
- 40: ↳1 ↳0 ↳3 ⎯5 ↳8 ↳5 ↳6 ⌐3 ⌐8 ⊓2 ⎯5 ⊓0 ↳0
- 41: ↳1 ↳0 ↳3 ⎯5 ↳8 ⊤5 ⌐3 ↳6 ⌐1 ↳7 ⌐8 ↳0 ⎯1 ↳5 ↳0 ⎯2
- 42: ↳1 ↳0 ↳3 ⊓5 ⌐7 ↳4 ⌐8 ⎯0 ↳6 ⌐8 ⌐2 ↳0 ↳4 ⎯0 ⎯6
- 43: ↳1 ⊓3 ↳5 ⎯1 ⌐7 ↳0 ⌐8 ↳3 ⌐5 ⎯4 ↳0 ⌐8 ⌐4 ⌐7 ⌐1 ↳0
- 44: ↳1 ⌐0 ↳3 ⎯5 ↳8 ⌐5 ⊓3 ↳6 ↳4 ⌐8 ↳0 ⎯1 ↳5 ⎯0 ↳0
- 45: ↳1 ⌐0 ↳3 ⎯5 ↳8 ⊓3 ⌐5 ⊓0 ⌐7 ⌐4 ↳7 ↳8 ⊓3 ⌐5 ⎯0 ↳0
- 46: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ⌐7 ⌐1 ↳0 ↳4 ⌐3 ⎯6 ↳0 ⎯6
- 47: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ⌐6 ⌐8 ↳0 ⌐1 ↳0 ⎯4
- 48: ↳1 ⌐0 ↳3 ↳5 ↳3 ↳6 ⌐8 ⌐7 ⊓8 ⌐0 ⊓2 ⌐5 ⌐0 ⌐3 ↳6 ↳0
- 49: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⌐3 ↳6 ⌐8 ⌐1 ↳0
- 4a: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ⊓8 ↳6 ⊓2 ⌐8 ⊤4 ⊓0 ⎯0 ↳0 ⎯4
- 4b: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⌐3 ↳6 ⌐8 ⌐1 ⌐0 ⌐7 ↳0 ⎯3
- 4c: ↳1 ⌐0 ↳3 ↳5 ↳6 ↳3 ↳3 ⌐8 ⌐0 ⎯6 ↳8 ⌐6 ↳3 ⎯5 ⌐0 ↳0
- 4d: ↳1 ⌐3 ↳5 ⌐7 ⊓0 ⌐8 ↳3 ⌐2 ⎯5 ⌐4 ⌐8 ⊓6 ⌐0 ⌐1 ↳6 ⊓8 ⎯3 ⌐0 ↳0
- 4e: ↳1 ⌐0 ↳3 ↳5 ↳3 ⌐8 ⊓8 ⊓4 ↳6 ↳0 ⌐1 ↳0 ⎯6
- 4f: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⌐3 ↳6 ⌐8 ⌐1 ⌐7 ⎯0 ↳0 ⎯4
- 50: ↳1 ⌐0 ↳3 ↳5 ↳6 ↳3 ↳8 ⌐1 ⌐0 ⌐3 ⌐7 ⎯0 ⌐4 ↳0
- 51: ↳1 ⌐3 ↳5 ⎯1 ⌐0 ↳6 ⌐8 ↳3 ↳6 ⌐8 ↳4 ↳0 ⌐8 ⌐1 ↳0 ⎯4
- 52: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ⌐0 ⌐7 ⌐5 ↳8 ↳4 ⌐3 ⌐0 ⌐7 ↳5 ↳0
- 53: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ⌐7 ⌐5 ↳8 ↳0 ↳4 ⌐1 ⎯6 ↳0
- 54: ↳1 ⌐0 ↳3 ⎯5 ↳8 ⌐5 ⌐3 ↳6 ↳8 ⌐1 ⌐5 ⎯1 ⌐0 ↳0 ⌐7 ⎯3
- 55: ↳1 ⌐0 ↳3 ⎯5 ↳8 ⌐5 ⌐3 ↳6 ↳8 ↳4 ↳0 ⌐8 ⌐1 ↳6 ⎯1 ⌐7 ⌐3 ⌐0 ↳0
- 56: ↳1 ⌐0 ↳3 ⊓3 ↳5 ↳6 ⌐8 ↳8 ↳6 ↳0 ↳4 ⌐1 ↳0 ⎯6
- 57: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⎯4 ↳8 ↳7 ↳0 ⌐1 ⎯6 ⌐3 ⎯6 ↳0 ⎯2 ↳0
- 58: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ⌐2 ⎯0 ↳4 ↳7 ⌐8 ⌐4 ⊓0 ↳0
- 59: ↳1 ⌐0 ↳3 ⎯5 ⊓3 ⌐7 ↳5 ↳6 ⌐8 ↳0 ↳4 ⌐1 ⎯6 ↳0
- 5a: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ⎯5 ↳8 ↳0 ↳6 ⊓8 ↳4 ⌐1 ↳0
- 5b: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳8 ↳6 ⌐1 ↳4 ⌐7 ⎯0 ↳0
- 5c: ↳1 ↳0 ↳3 ⎯5 ↳8 ↳5 ↳3 ↳5 ↳8 ↳0 ↳7 ⌐4 ⌐1 ↳0
- 5d: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ↳3 ↳6 ⌐8 ⌐1 ⎯0 ↳5 ⌐7 ↳0
- 5e: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ⌐8 ⌐1 ⌐1 ⎯0 ↳0
- 5f: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ⌐8 ↳0 ⌐7 ↳6 ⌐4 ⌐1 ⊓8 ⎯0 ↳0 ⎯4
- 60: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐4 ⎯0 ⌐3 ↳5 ⌐7 ⎯0 ↳8 ⎯0 ↳0 ⎯4
- 61: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ⌐7 ⎯0 ↳5 ⊓2 ↳6 ⌐8 ⎯2 ⊓0 ↳0
- 62: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐3 ⊓1 ↳5 ⊓8 ⎯2 ⌐0 ↳0
- 63: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳2 ⌐8 ↳4 ↳6 ⌐0 ⌐3 ⌐7 ↳0
- 64: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐3 ⊓1 ↳5 ⌐0 ⎯2 ⌐7 ↳0
- 65: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ↳2 ↳8 ↳6 ⌐8 ↳0 ⎯4 ⌐3 ⎯0 ↳0 ↳5 ⎯6
- 66: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐1 ⎯6 ↳0 ↳0 ⎯2
- 67: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐1 ↳0 ⌐7 ⎯3 ↳0
- 68: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐1 ⌐7 ↳6 ⊓8 ↳4 ⌐1 ↳6 ⎯3 ⌐0 ↳0
- 69: ↳1 ⌐0 ↳3 ⎯5 ⊓3 ⌐8 ⌐5 ↳7 ↳5 ↳0 ⌐8 ⌐1 ⌐4 ↳0
- 6a: ↳1 ⌐0 ↳3 ⊓3 ↳5 ↳7 ↳5 ⌐8 ↳0 ↳6 ⌐7 ⌐4 ⌐1 ↳0
- 6b: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳8 ⌐0 ↳5 ⊓2 ⌐7 ⌐4 ↳0
- 6c: ↳1 ↳0 ↳3 ⎯5 ↳8 ⌐5 ↳5 ↳8 ↳0 ⌐7 ⌐4 ⌐1 ↳0
- 6d: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ⌐7 ⊓0 ⊓2 ⌐5 ⌐4 ⌐7 ↳0
- 6e: ↳1 ⌐0 ↳3 ⊓3 ↳5 ⌐7 ⌐7 ↳6 ⌐1 ↳8 ↳4 ↳0 ↳6 ⌐4 ⌐3 ⎯7 ⎯0 ↳0 ⎯5
- 6f: ↳1 ⌐0 ↳3 ⊓3 ↳5 ↳6 ↳5 ↳8 ↳8 ↳6 ↳0 ⌐1 ↳8 ⌐4 ⌐6 ⎯0 ↳0
- 70: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳0 ⌐5 ↳5 ⊓2 ⌐5 ⊓8 ⊓2 ⌐1 ↳0
- 71: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⊓4 ⌐4 ↳6 ↳0 ⌐3 ⌐7 ⌐4 ⎯0 ⌐8 ⎯0 ↳0 ⎯4
- 72: ↳1 ⊓3 ↳4 ↳0 ⌐3 ↳6 ⌐8 ↳8 ⌐1 ↳0 ↳6 ↳4 ↳8 ↳5 ↳7 ⌐2 ⌐0 ↳0 ⎯3
- 73: ↳1 ⊓3 ⎯0 ↳4 ⎯6 ↳6 ⌐8 ⎯0 ↳4 ↳5 ↳7 ⌐2 ↳8 ↳5 ⌐0 ⌐3 ⎯6 ⌐0 ↳0
- 74: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⎯0 ↳4 ⌐7 ↳5 ↳7 ↳3 ↳0 ⎯5 ⌐2 ⎯4 ↳8 ⌐0 ↳0
- 75: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳4 ⎯0 ↳5 ↳7 ⌐1 ⌐3 ↳8 ⌐2 ↳5 ⌐7 ↳0 ↳0
- 76: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐1 ⌐0 ⌐3 ⎯6 ↳0
- 77: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ↳3 ⌐0 ↳6 ↳8 ↳5 ⊓0 ⌐2 ↳0
- 78: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐1 ⎯2 ↳0 ⎯6 ↳0
- 79: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐2 ⌐7 ↳6 ⌐0 ↳5 ↳3 ↳6 ⌐8 ↳4 ⌐2 ⎯6 ⊓0 ↳0
- 7a: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ⊓3 ⎯6 ⌐8 ⌐7 ⌐4 ⌐7 ↳0
- 7b: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ⎯0 ↳6 ↳4 ⌐1 ↳8 ↳0 ⎯2 ↳6 ↳0
- 7c: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳8 ↳4 ↳7 ⌐4 ⌐2 ↳0
- 7d: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳4 ↳6 ⌐0 ↳3 ↳6 ↳8 ↳0 ⌐5 ⎯1 ↳0
- 7e: ↳1 ↳0 ⊓3 ↳4 ⌐1 ↳6 ⌐8 ⌐7 ↳4 ↳6 ↳6 ↳0 ↳3 ↳8 ⌐1 ↳5 ↳0
- 7f: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⊓8 ↳0 ↳6 ⌐4 ⌐2 ↳8 ⎯0 ↳0 ⎯4
- 80: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ↳5 ⌐7 ⎯0 ↳0
- 81: ⊓1 ↳2 ⊓0 ↳4 ⎯2 ↳6 ⌐8 ↳7 ↳6 ↳4 ⎯6 ↳0 ⌐8 ⎯1 ↳0
- 82: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ↳5 ↳8 ↳0 ⌐1 ⎯3 ⌐7 ↳0
- 83: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⎯2 ↳8 ↳2 ⊓0 ↳4 ↳8 ↳6 ⌐7 ⎯0 ↳0 ⎯2
- 84: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ↳8 ↳6 ⌐1 ↳8 ↳0 ⊓2 ⌐6 ⌐4 ↳0
- 85: ↳1 ↳0 ↳3 ⎯5 ↳8 ⊓3 ⌐5 ⊓5 ⌐7 ↳0 ↳7 ↳1 ↳8 ↳0 ↳2 ⎯4
- 86: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⎯2 ↳2 ⌐8 ↳4 ↳6 ⎯6 ⊓0 ⎯2 ↳0
- 87: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ↳8 ↳0 ⌐7 ⌐2 ⌐7 ⌐4 ↳0
- 88: ↳1 ⌐0 ↳3 ↳5 ↳6 ↳3 ↳8 ↳4 ↳6 ↳1 ↳0 ↳5 ⌐7 ⌐7 ↳0
- 89: ↳1 ⌐0 ↳3 ↳5 ↳6 ⌐8 ↳3 ⌐7 ↳1 ↳4 ↳7 ⌐8 ⊓0 ↳2 ⊓5 ↳0
- 8a: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ↳1 ↳2 ↳5 ↳8 ↳0 ⌐7 ⎯3 ↳0
- 8b: ↳1 ⌐0 ↳3 ⎯5 ⌐7 ↳5 ⊓3 ↳6 ⌐8 ↳2 ↳4 ⊓0 ⌐7 ↳0
- 8c: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⌐3 ↳6 ↳8 ⌐1 ↳6 ⎯2 ⊓0 ⎯2 ⌐7 ↳0
- 8d: ⊓1 ↳2 ↳0 ↳4 ⎯6 ↳6 ⌐8 ↳4 ⌐0 ⌐7 ↳7 ↳3 ↳5 ↳0 ↳8 ⎯4 ⌐1 ↳0
- 8e: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⎯2 ↳8 ↳2 ↳8 ↳4 ↳6 ↳0 ⌐6 ⌐7 ⌐4 ↳0 ⌐2 ↳0
- 8f: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ↳8 ↳5 ↳0 ⌐1 ↳7 ⌐4 ↳6 ↳8 ⌐2 ↳0 ↳0
- 90: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ⌐4 ⌐7 ↳0 ⌐1 ⎯3 ⌐7 ↳0
- 91: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ↳5 ↳7 ↳8 ↳2 ⌐1 ↳0
- 92: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ⌐8 ⎯0 ↳5 ⌐4 ⎯0 ⌐7 ↳0
- 93: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳7 ↳5 ↳8 ↳2 ↳4 ↳0 ⎯6 ↳0
- 94: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ⎯4 ⊓0 ↳7 ⎯0 ⎯4 ↳0
- 95: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ⌐7 ⌐4 ↳2 ↳0 ↳7 ↳8 ⌐3 ⊓5 ⌐0 ↳0
- 96: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ↳8 ↳7 ↳2 ↳4 ↳0 ⎯6 ↳0
- 97: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ↳8 ⊓5 ↳0 ↳2 ↳7 ↳8 ⌐3 ⎯0 ⊓5 ↳0
- 98: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐7 ↳6 ⌐2 ↳5 ⎯7 ⊓0 ⊓2 ↳0 ⎯5
- 99: ↳1 ↳0 ↳3 ⎯5 ⌐7 ⊓3 ↳5 ↳6 ⌐8 ↳0 ↳4 ⌐1 ↳0 ⎯6
- 9a: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳7 ↳5 ↳8 ↳4 ↳0 ↳6 ⌐1 ↳0
- 9b: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳8 ↳6 ⌐1 ↳4 ⌐7 ⎯0 ↳0
- 9c: ⊓1 ↳2 ⊓0 ↳4 ⎯6 ↳6 ⌐8 ↳2 ↳4 ⌐7 ↳5 ↳0 ↳6 ⌐3 ↳8 ⎯4 ⌐2 ⊓0 ↳0
- 9d: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⌐3 ↳6 ↳8 ⌐1 ⌐7 ⌐4 ⎯0 ↳0
- 9e: ↳1 ↳0 ↳3 ↳5 ↳6 ⌐8 ↳3 ↳6 ⌐8 ⌐1 ↳4 ⎯0 ↳0 ⎯6
- 9f: ↳1 ↳0 ↳3 ⎯5 ↳8 ⌐3 ↳8 ↳6 ⊓1 ↳3 ↳0 ⌐7 ↳5 ⎯1 ↳0 ⎯6
- a0: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ⌐7 ⊓1 ↳0 ⌐7 ↳5 ⎯6 ⌐3 ⎯0 ↳0
- a1: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ⌐7 ⌐1 ⌐6 ↳0 ↳8 ⎯2 ⌐7 ⎯1 ↳0
- a2: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⌐1 ↳4 ⌐8 ↳5 ↳6 ⌐1 ↳8 ↳7 ⌐0 ↳0
- a3: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐3 ⎯0 ⌐0 ⌐7 ↳0
- a4: ↳1 ⊓3 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳6 ⎯0 ↳4 ⌐3 ⎯0 ⎯3 ⌐5 ↳0
- a5: ↳1 ⊓3 ↳4 ↳0 ↳6 ⌐8 ⊓2 ↳4 ↳6 ↳8 ↳0 ⊓1 ↳4 ↳7 ↳8 ⌐5 ⊓0 ⊓3 ↳0
- a6: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳6 ⌐1 ⎯2 ↳0 ⎯6 ↳0
- a7: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⎯0 ⌐0 ⎯3 ⌐7 ↳0
- a8: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳6 ↳8 ↳1 ⎯6 ⌐3 ⎯6 ↳0
- a9: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ⌐7 ↳6 ↳3 ⌐5 ⌐1 ↳7 ↳8 ↳0 ⌐1 ↳5 ⌐3 ↳0
- aa: ↳1 ⌐0 ↳3 ⎯5 ↳8 ⊓3 ⌐5 ↳5 ↳8 ↳0 ⌐7 ↳1 ⌐4 ↳0
- ab: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐1 ↳4 ↳8 ↳6 ⌐7 ⌐1 ↳3 ↳0 ↳6 ⌐1 ⌐7 ⌐4 ↳0
- ac: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳4 ↳6 ↳8 ⌐1 ↳0 ⎯6 ⎯2 ↳0
- ad: ↳1 ↳0 ⊓3 ↳4 ⌐1 ↳6 ⌐8 ⌐7 ↳4 ↳5 ↳7 ⎯1 ↳0 ⎯8 ⌐3 ⎯6 ⌐0 ↳0
- ae: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ⌐7 ↳6 ⊓2 ↳4 ↳6 ⎯3 ↳8 ⌐2 ⊓0 ⎯4 ⌐7 ↳0
- af: ↳1 ↳0 ↳3 ⊓3 ↳5 ⌐7 ↳5 ↳8 ↳0 ↳5 ⌐2 ⌐7 ↳4 ↳6 ⌐8 ⎯0 ↳0
- b0: ↳1 ⊓3 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳7 ↳5 ⎯0 ⎯6 ⌐3 ⎯0 ↳0
- b1: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ⌐7 ↳4 ↳6 ⌐3 ⊓1 ↳6 ↳8 ↳0 ⊓4 ⌐1 ↳0 ⎯6
- b2: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳4 ↳8 ⌐2 ↳7 ↳5 ⌐7 ↳5 ⌐7 ↳8 ⌐1 ↳0 ↳4 ↳0 ⌐2 ⎯6
- b3: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⎯0 ⌐8 ⎯4 ↳4 ↳6 ⌐1 ↳8 ⎯4 ⌐7 ↳0 ↳1 ↳0 ⎯3
- b4: ↳1 ⊓3 ↳4 ⎯0 ↳4 ⎯0 ↳6 ⌐8 ↳4 ⌐7 ↳4 ↳6 ⌐1 ↳0 ↳5 ⌐8 ⌐3 ⎯0 ↳0
- b5: ↳1 ⊓3 ↳4 ↳1 ↳6 ⌐8 ↳0 ↳4 ⌐7 ⌐2 ↳5 ↳7 ⊓5 ↳8 ↳3 ↳2 ⌐7 ⊓0 ↳0
- b6: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⎯0 ↳4 ↳8 ↳5 ↳1 ↳6 ↳3 ↳5 ⊓8 ↳0 ⌐1 ↳0
- b7: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳8 ↳4 ↳6 ⌐1 ↳0 ↳8 ⎯3 ⌐6 ⎯3 ⊓0 ↳0
- b8: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ⌐7 ↳6 ↳4 ⌐1 ↳0 ⎯2 ↳0 ⎯6
- b9: ↳1 ↳0 ↳3 ⎯5 ↳8 ↳5 ⊓5 ⊓0 ↳7 ↳8 ⌐4 ⌐2 ↳0
- ba: ↳1 ⌐0 ↳3 ↳5 ↳6 ↳5 ⊓3 ↳8 ↳8 ⊓0 ⌐7 ⌐4 ⌐2 ↳0
- bb: ↳1 ⊓3 ↳4 ⎯0 ↳6 ⌐8 ↳7 ↳4 ↳6 ↳3 ↳0 ⊓1 ⌐6 ↳8 ⎯0 ↳0 ⎯4
- bc: ↳1 ↳0 ↳3 ⎯5 ⌐7 ↳5 ⊓3 ↳6 ↳8 ⊓0 ⌐7 ⌐4 ⌐2 ↳0

- bd: ↳1 ↱3 ↳4 ⅃1 ↳6 ⌐8 ⌐0 ↲7 ⌐6 ↱4 ⌐2 ↴6 ⌐0 ⌐8 ↴2 ↲5 ↳0
- be: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ⌐8 ↳4 ⌐6 ↴1 ⌐0 ↲5 ↲5 ⌐3 ↴0 ⌐8 ↳0
- bf: ↳1 ↳0 ↱3 ⌐3 ↳5 ↴7 ⌐7 ⌐6 ⌐8 ⌐5 ↳1 ↲7 ↴2 ↲0 ↳3 ⌐6 ↳0
- c0: ↳1 ↳0 ↱3 ⅃5 ⌐8 ↳5 ⌐6 ⌐3 ⌐7 ⌐0 ⌐8 ↳1 ↳3 ⅃0 ↲5 ↳0
- c1: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ↳5 ↳7 ↴1 ⌐8 ⌐3 ↲0 ↳0
- c2: ⌐1 ↳2 ↲0 ↳4 ↳3 ↳6 ⌐8 ⌐8 ⌐5 ⌐0 ⌐7 ⌐2 ↴7 ↲4 ⅃1 ↳0
- c3: ↳1 ⌐0 ↱3 ⌐3 ↳5 ⌐5 ⌐7 ↴7 ⌐8 ⌐0 ⌐7 ↳1 ⌐3 ↳0
- c4: ⌐1 ↳2 ↲0 ↳4 ↳3 ↳6 ⌐8 ⌐0 ↳5 ↴7 ⌐7 ⌐2 ↴7 ↲4 ⅃1 ↳0
- c5: ↳1 ⌐0 ↱3 ⌐3 ↳5 ↴7 ⌐5 ⌐8 ⌐7 ⌐0 ↴7 ↳1 ⌐3 ↳0
- c6: ↳1 ⌐0 ↱3 ⌐3 ↳5 ⌐7 ⌐5 ↲7 ⌐8 ⌐0 ⌐7 ↳1 ⌐3 ↳0
- c7: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ⌐8 ↴5 ⌐0 ⌐7 ↳2 ⌐8 ⌐0 ↳3 ↴5 ↳0
- c8: ↳1 ↳0 ↱3 ⅃5 ↳6 ↳3 ⌐8 ↴4 ⌐8 ⌐7 ↳1 ↳2 ⌐0 ⌐6 ↳3 ↳0
- c9: ↳1 ⌐0 ↱3 ⅃5 ⌐3 ⌐8 ↴5 ⌐5 ↳7 ⌐0 ⌐8 ↳1 ⌐3 ↳0
- ca: ↳1 ⌐0 ↱3 ⌐3 ↳5 ⌐7 ↳5 ⌐8 ⌐0 ⌐6 ↴7 ↳1 ↳0 ⌐3
- cb: ↳1 ↳0 ↱3 ⌐3 ↳5 ⌐7 ⌐5 ↴7 ⌐8 ↴1 ⌐7 ↴2 ⌐6 ⌐0 ⅃4 ⌐8 ⌐0 ↳3 ↳0
- cc: ↳1 ↳0 ↱3 ⅃5 ⌐3 ↳5 ⌐5 ↲8 ⌐7 ⌐0 ⌐6 ↴7 ↳1 ↳0 ⌐3
- cd: ↳1 ↳0 ↱3 ⅃5 ↴7 ⅃5 ↳3 ↳6 ⌐0 ⌐8 ⌐3 ↴6 ⌐2 ↴4 ⌐0 ⅃2 ↴7 ⌐6 ↳0
- ce: ⌐1 ↳2 ↲0 ↳4 ⌐6 ↳6 ⌐8 ⌐0 ↳2 ↴4 ↲7 ⌐6 ↳3 ↳6 ↴1 ⌐8 ⌐0 ↳0 ⅃4
- cf: ↳1 ↳0 ↱3 ⅃5 ⌐4 ↴1 ↳6 ⌐8 ⌐8 ⌐6 ↳1 ↴7 ⌐3 ⌐0 ⅃2 ↳0 ⌐6
- d0: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ⌐5 ↲7 ⌐7 ↲7 ⅃2 ⌐0 ↳0
- d1: ↳1 ↳0 ↱3 ⌐3 ↳5 ⌐5 ↴7 ↲7 ⌐8 ⌐0 ↴7 ↳1 ⌐3 ↳0 ⌐6
- d2: ↳1 ⌐0 ↱3 ⌐3 ↳5 ⌐7 ⌐6 ⌐8 ⌐8 ⌐5 ⌐0 ↳1 ↳0 ⅃3
- d3: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ↳5 ⌐7 ↴2 ⌐8 ↴1 ⌐0 ↳3 ↳5 ⌐6
- d4: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ↳5 ↲7 ↲2 ↲0 ⌐7 ↲2 ↴7 ↲3 ↲0 ↳0
- d5: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↲2 ↳2 ↳4 ↲7 ⌐0 ⌐5 ⌐7 ↴2 ⌐8 ↲0 ↳0 ⅃4
- d6: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ⌐8 ↳5 ↲2 ↲7 ⌐0 ↴7 ⌐3 ↲0 ↳0 ⅃3
- d7: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ↳5 ⌐7 ⌐0 ⌐8 ↴2 ↴5 ⅃1 ↳0
- d8: ↳1 ⌐0 ↱3 ⅃5 ↳6 ⌐8 ↳3 ↲7 ⌐0 ↲7 ↴4 ↳1 ⌐3 ↳0
- d9: ↳1 ↴3 ↳5 ⅃0 ↲7 ↱3 ↴7 ⅃5 ↳0 ↳6 ⌐8 ↴1 ⌐3 ↳0
- da: ↳1 ↳0 ↱3 ⌐3 ↳5 ↴7 ⌐7 ↲5 ⌐8 ↴1 ⌐0 ↱4 ⌐3 ↳5 ⌐8 ⌐7 ⅃3 ↴0 ↳0
- db: ⌐1 ↳2 ⌐0 ↳4 ⅃2 ↳6 ⌐8 ↲7 ⌐6 ↳2 ↳4 ↴6 ⌐0 ⌐8 ↴1 ⅃4 ↳0
- dc: ⌐1 ↳2 ⌐0 ↳4 ⌐6 ↳6 ⌐8 ↳2 ↳4 ↲7 ↳5 ↴2 ⌐6 ⌐8 ↲0 ⌐0 ↲5 ⌐3 ↲0 ↳0
- dd: ↳1 ↳0 ↱3 ⅃5 ↳6 ⌐3 ⌐8 ↲5 ↲7 ⌐7 ⌐0 ⌐8 ↳1 ↳3 ⌐0 ↳0 ⅃4
- de: ↳1 ↳0 ↱3 ⌐3 ↳5 ⌐7 ⌐5 ⌐8 ⌐0 ↲7 ⌐6 ↳1 ↴3 ⌐0 ⌐8 ⅃4 ↳0
- df: ↳1 ↳0 ↱3 ⌐3 ↳5 ⌐7 ⌐5 ↴7 ⌐0 ↴7 ↳1 ↴3 ↲6 ⌐8 ⅃1 ↳5 ↳0
- e0: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ↲7 ↳1 ↲2 ↳5 ⌐0 ⅃1 ⌐8 ⌐6 ↳0
- e1: ↳1 ↱3 ⅃0 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ⌐8 ↲5 ↲2 ⌐0 ↳0
- e2: ↳1 ↱3 ↳4 ↳1 ↳6 ⌐8 ⌐0 ⌐8 ↳4 ↳6 ⌐3 ↴1 ⌐8 ↲5 ⌐0 ↳3 ⅃1 ⌐6 ↳0
- e3: ↳1 ↱3 ↳4 ↲1 ⌐0 ↳6 ⌐8 ⌐8 ↳4 ↳6 ↴1 ↴7 ⌐7 ⌐3 ↳0
- e4: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ↳0 ⅃8 ↲2 ⌐0 ⌐0 ↳5 ↳0
- e5: ↳1 ↱3 ↳4 ↳0 ↳6 ⌐8 ↲2 ↳4 ⌐6 ⌐0 ⌐8 ↳3 ⌐6 ⌐8 ↲2 ⌐6 ⌐4 ⌐0 ↳0
- e6: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ↳6 ↳1 ⌐3 ⌐6 ⌐0 ↳0
- e7: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ⌐8 ↳4 ⌐6 ↳1 ⌐0 ⌐3 ⅃8 ↲6 ⌐3 ↴0 ↳0
- e8: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↲7 ↳4 ↳5 ↳1 ⌐0 ↲2 ⌐7 ↳0
- e9: ↳1 ↱3 ↳4 ↳0 ↳6 ⌐8 ↴1 ↲7 ⌐6 ⌐6 ↳4 ⅃2 ⌐5 ↳0 ⌐8 ↳0 ↳0 ↳2 ⌐4
- ea: ↳1 ↳0 ↱3 ↳4 ↲2 ↳6 ⌐8 ↲7 ⌐0 ↲5 ⌐6 ↲2 ⌐7 ↴4 ⌐8 ⅃2 ↳0 ↳0 ⅃4
- eb: ↳1 ↳0 ↱3 ⌐3 ↳5 ↴7 ⌐7 ⌐6 ⌐8 ⌐5 ⌐0 ↳2 ↴5 ⌐8 ↲2 ↳0
- ec: ↳1 ↳0 ↱3 ⅃5 ↳6 ⌐8 ↳3 ↲7 ⌐2 ↴5 ↲7 ⌐0 ⌐4 ↴1 ⌐0 ↴7 ⌐3 ↳0 ⅃4
- ed: ↳1 ↳0 ↱3 ⅃5 ↳6 ⌐8 ↳3 ↲7 ↴1 ↲5 ⌐0 ↴7 ↳4 ↴4 ↲1 ↳0 ⌐6
- ee: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ⌐8 ↳4 ⌐6 ↳1 ⌐0 ⌐2 ↲6 ⌐4 ⅃0 ⌐8 ↳0
- ef: ↳1 ↳0 ↱3 ⅃5 ↴7 ⌐5 ⌐8 ↲7 ⅃0 ⌐6 ⌐4 ↳1 ↴1 ↳0 ⌐8 ⅃4
- f0: ↳1 ↱3 ⅃0 ↳4 ⅃0 ↳6 ⌐8 ↲7 ⌐6 ↱4 ⌐0 ↲2 ↲1 ↳4 ↳6 ⌐8 ⌐0 ↳0 ⅃5
- f1: ↳1 ↱3 ⅃0 ↳4 ⅃2 ↳6 ⌐8 ⌐6 ⌐0 ↱4 ↲7 ⅃5 ⌐3 ↴1 ⌐7 ⌐8 ⅃3 ↲0 ↳0
- f2: ↳1 ↱3 ↳4 ↳0 ↳6 ⌐8 ↲2 ⌐8 ↳4 ↴4 ↲0 ⌐7 ⌐8 ↴4 ↳1 ↲0 ⌐6 ↲3 ↳0
- f3: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳3 ⌐8 ↳4 ⌐7 ⅃0 ↳5 ↳1 ⌐8 ⌐0 ↳2 ⅃4 ↳0
- f4: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ⅃0 ⌐8 ↲5 ↳1 ⌐3 ⌐6 ⌐0 ⌐0 ↴4 ↴7 ⌐0
- f5: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ⌐8 ⅃0 ↳5 ⌐0 ↲2 ⌐8 ⅃4 ↴1 ↳0
- f6: ↳1 ↱3 ⅃0 ↳4 ⅃0 ↳6 ⌐8 ⌐8 ↳4 ↳6 ↴4 ⌐3 ↴1 ⌐0 ⌐6 ↳0 ⌐6
- f7: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ⌐8 ↳4 ⅃0 ⌐6 ↴4 ↴7 ↳1 ⌐0 ⌐6 ⌐3 ↳0
- f8: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↲7 ↳4 ⌐6 ↳1 ⌐3 ⌐6 ⌐0 ↳0
- f9: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↲7 ⌐6 ↳4 ↳1 ⌐3 ⌐0 ↲5 ⌐8 ↴0 ⌐4 ↳0
- fa: ↳1 ↳0 ↱3 ⅃5 ↳6 ⌐8 ↳3 ↲7 ↲5 ↴1 ↴6 ⌐0 ↲3 ↳1 ⌐3 ↴7 ↳0
- fb: ↳1 ↳0 ↳4 ⅃0 ↳6 ⌐8 ↲7 ↳4 ⌐6 ↲2 ↴6 ↴8 ⌐0 ↲1 ⅃5 ↳0
- fc: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐8 ⌐6 ⌐3 ↴1 ⌐0 ↲5 ↴4 ⅃0 ↴7 ↳0
- fd: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ⅃8 ⌐0 ⌐2 ↴4 ⌐8 ⌐6 ↲2 ⌐0 ↳0
- fe: ↳1 ↱3 ↳4 ⅃0 ↳6 ⌐8 ↳4 ⌐6 ↴0 ⌐2 ↳4 ⌐6 ⌐0 ⌐6 ↳0 ⅃2
- ff: ↳1 ↱3 ↳4 ⅃0 ↳6 ↳4 ⌐8 ⌐6 ↳1 ↲2 ⌐0 ↲5 ⌐8 ↴0 ↲4 ⌐3 ↳6 ⌐8 ⅃0 ↳0 ⅃4

An earlier version of this table did not fit neatly into the column. Rather than fiddle with LATEX layout, I instead expended significant CPU time to further optimize the problematic rows until they would be short enough to fit! This is the *true spirit of typography*.

3.1.3 Executing

Now that we have a solution for each byte, we want to play the NES game to put the desired pattern in the playfield. We use a NES emulator (my version of FCEUX [1] which I've heavily modified for e.g. thread-safety) which allows us to save and restore states, inspect RAM, and execute frames much faster than real time.

We always input a fixed sequence of button presses to the emulator to begin a game with the right starting pieces. We then start executing the plan, which consists of the fixed starting sequence (puts us in an empty board with ↳ in the leftmost column) and then the concatenation of the plans for the bytes we want to encode. All that is left is to place pieces according to this plan, while ensuring that we get the desired sequence of "random" piece drops.

At the start of each piece, we save the emulator state, and

then navigate the piece into the correct column. Holding down on the D-pad causes the piece to drop as fast as possible. When it lands, if we got the correct piece by chance, we just continue. Otherwise, we inspect the state of the random number generator (right *before* the piece dropped), and then simulate it using our reverse engineered version. We tabulate the piece that would drop if we were to take one additional frame to get here, then two, then three, and so on up to some fixed horizon.[14] We can then restore the saved state and drop the piece more slowly (not pressing down on the controller), delaying for the correct number of frames to get the piece that we want. We can usually get this exactly right on the first try, but if not, we try again with slightly longer or shorter delays until successful. It is also possible to pause the game [12] if the delay needs to be longer than the natural descent of the piece, which can happen when the game speeds up at later levels and the pile is high.

3.2 Harder Drive: Tetru

We can now build a hard drive with Tetris. It's called `tetru`, following the convention from Section 2.[15] The setup is straightforward: The block size is 8 bytes, and each block consists of a NES Tetris emulator. When we first write a block, we allocate the emulator, load the Tetris ROM, and use the procedure above to supply inputs to the game. The driver is multithreaded so 16 concurrent Tetris games can be in progress, although the block size is so small that we need several serial passes to write one "normal" 512-byte sector. To read a block, we just inspect the 200 bytes of memory at `0x400`. The byte `0xEF` is an empty cell (`0`) and any other byte is some part of a tetromino (`1`).

If we re-write a cell, we reset the emulator and start again. It might be possible to clear the playfield by playing the game (for example it would be straightforward to precompute a plan that clears any bit pattern on a single line, parity issues notwithstanding). But as the game gets faster and faster we may be unable to drop pieces in the correct locations, so it is safer to reset. This is something the player can do anyway, so there is no loss of authenticity.

Finally, as an optimization, we also cache the input sequence that we compute for the 8 bytes the first time we do it; if we write that same pattern again then we can just replay the inputs rather than search for them a second time. Both the FAT12 directory structure and benchmark file have many repeated patterns (especially 8 × `0x00`), giving us a good cache hit rate of 46% during benchmarking.

Of course, there would be other ways to make this faster, too. For example, just the pointer to the emulator object

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[14] The longest possible drought before we see the desired piece (if we missed it on the first attempt) is 98 frames (Figure 5). But it is helpful to build in significant redundancy, since there are some unusual situations where we cannot drop exactly on a desired frame, and must overshoot.

[15] Also because Tetris is Russian and `.ru` is the TLD for Russia. Classic retcon which I just nailed.
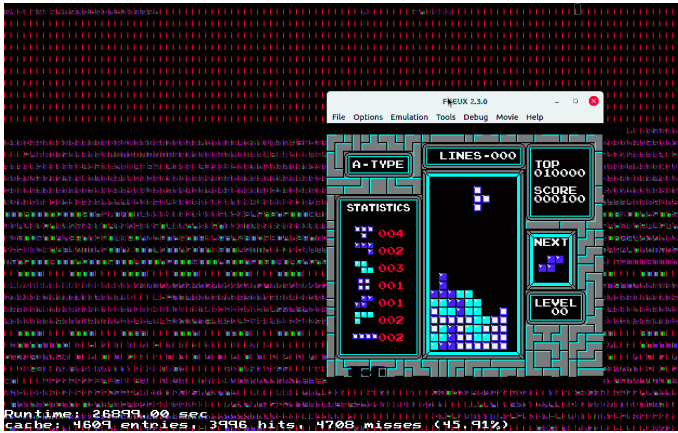
Figure 10: Playing a game of Tetris in the FCEUX emulator, in which we loaded the `tetris.nes` ROM from the `tetru` drive. The backdrop is a truncated portion of the visualization showing the contents of the 8,640 Tetris boards being emulated. The upper portion is the FAT-12 header and directory entries (it is mostly `0x00`) and the lower portion is the ROM data. I only have 100 points right now but as you can see I am gearing up to complete some sweet, high-scoring Tetrises.

for each block is 64 bits, or 8 bytes itself, suggesting a form of "content-addressed storage."

3.2.1 Results

In order to benchmark we need some file to write to the filesystem. A beautiful choice is the `tetris.nes` ROM file, which is 49,168 bytes. Though the minimal filesystem for FAT12 requires a device with 51,200 bytes, there is significant overhead from the filesystem header, directory entry, and so on. So to store this ROM we create a device with 69,120 bytes, which is 8,640 NES emulators. It is straightforward to scale to thousands of emulators, with the biggest challenge being to fit them all on the screen for some kind of visualization.

We benchmark as before, but then of course it is important for aesthetic reasons to load the ROM that we stored inside the drive to play a game of Tetris. Figure 10 shows this in practice.

Qualitative. This is a good Harder Drive. It solves a problem we don't have, which is that typical hard drive "blocks" are not made of actual "blocks," but Tetris players will recognize that the data on the drive is indeed made from blocks. It is very satisfying to watch the thousands of Tetris games drop pieces to encode the data.

Cost. Simulated on a computer, the up-front cost of storing data is fairly low. A basic 16-core desktop computer is about $1,800 in 2022. The software NES emulator uses 1,652,372 bytes of RAM for Tetris on a 64-bit machine, which is 650 instances per gigabyte. So we can store about 5200 bytes in about $4.37 worth of RAM,[16] which is 0.084 cents per byte. This could easily be improved; the emulators could be stored much more efficiently, because they are all emulating the same ROM. If we built this with actual Nintendo hardware, we would need one NES Console and one Tetris cartridge (or bootleg) per eight bytes. A used NES runs about $150 and a Tetris cartridge about $10. This is $20 per byte, which 24 thousand times more expensive.

Longevity. The stored data lasts indefinitely, as long as the computer (or Beowulf cluster of NES consoles) remains powered.

Speed. Writing the 49,168-byte test file `tetris.nes` takes 3 hours, 18 minutes and 52 seconds, for a data rate of 2.57 bytes/sec. Due to its caching nature, writing to the hard drive gets faster as it stores more data. Reading is much faster at 61,430 bytes/sec.

Power. On a modern computer, a gigabyte of RAM uses about 375 mW of power [10], so the marginal cost is 72 μW per byte. The NES console uses about 10 Watts, which would give us about 1.25 W per byte.

Is rotational? This drive `is_rotational`, because the Tetris pieces are rotated to place them in the correct orientation.

Harm to society. There is no harm to society for the software emulation. If built on real NES consoles, hoarding thousands of these machines and cartridges would be considered antisocial, as they are historic items that are in limited supply, and many people still enjoy collecting and using them for their intended purpose.

4 Cue the coronavirus

Speaking of using things for their intended purpose: Sorry to remind you about the worldwide pandemic still killing thousands of people every day, but this section concerns a hard drive made from COVID-19 tests.

SARS-CoV-2 is an RNA coronavirus first isolated in January 2020 [29]. Since it is highly contagious and can cause severe illness (especially in the immune-naïve), testing is an important part of the worldwide response. There are two widely available approaches to testing: Lateral flow antigen tests and PCR. Lateral flow tests detect a target molecule (e.g. the SARS-CoV-2 spike protein) by binding a tagged complementary molecule (antibody) to it as the sample flows along a capillary bed. This is awesome. The tests are fast and cheap. Polymerase Chain Reaction (PCR) [24] tests work by amplifying a target sequence of DNA exponentially. It heats and cools the sample in the presence of a heat-stable DNA polymerase (typically *Taq* polymerase, which was isolated from the thermophilic *Thermus aquaticus* bacterium) and a bubble-bath of nucleotides that can be used to create more DNA. Each thermal cycle first unzips a double-stranded molecule into two pieces, and then

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[16]In 2022, a Corsair 16GB DDR4 DIMM is only $70.

reassembles each one, doubling the target. Properly, the tests are reverse transcription PCR [4], because first we need to turn the viral RNA into DNA. PCR tests are more sensitive (they can detect a *single* molecule) and specific (they detect a particular genetic sequence). This is also awesome.

Cue is a bougie COVID-19 test that launched in 2021. The test consists of a reusable reader ($200) and a single-use cartridge ($65 *each*!). Notwithstanding the eye-popping expense, the system is pretty good. My employer provides these tests for free (!), so I started collecting the used cartridges over the course of several months, and soliciting them from my friends as well.

The cartridge itself (Figure 12) is fairly ingenious and deserves to be disassembled.[17] When you stick the nose swab into the cartridge, it of course is delivering the snotty sample, but the insertion force also mechanically actuates a number of plastic thorns which pierce foil seals on reagent ampules, allowing them to start their thing. The assay is described as "nucleotide amplification," which would suggest something like RT-PCR, but since the cartridge does not significantly change temperature during a test, it is probably not literally PCR. LAMP [19] is a similar technique which is *isothermal* and seems like a credible choice. Again it would be preceded by a reverse transcription step to turn RNA into DNA. These things are all awesome and deserve to be learned about; for example did you know that the most frequently used reverse transcriptases (which turns RNA into DNA to begin amplification) were isolated from RNA viruses (which those sneaky bastards use to turn their own RNA into DNA so that it can be transcribed by the host)? So now we're using virus machinery to detect and fight other viruses? Hell yeah we are!

If you pull off all the chemistry pieces from the cartridge's endoskeleton, you'll also find a tiny 8-pin microchip onboard. And if you have a microscope you can read that it says ST 24C04WP, which is a serial EEPROM [25]. An EEPROM is a programmable ROM (so it is "read-only" but also writable?). It is probably used to store the cartridge's serial number, expiration, what kind of test it is, and maybe calibration data. This stuff would be pretty small, so it's no surprise that the chip can only store 512 bytes. A dump of one of these ROMs is in Figure 11.

The EEPROM is an I^{2}C device, so I could use pre-existing code to send commands to it. Reading the EEPROM is normal difficulty. I found writing to be more like "hurt me plenty" difficulty: When you write a line, the EEPROM drops off the bus temporarily (it may need to do this because it is internally stepping up voltage for the erase operation). You then have to either wait "enough time" or poll it to see when it's ready to write the next line. What I did is to repeatedly try to read back the same line we just wrote (this also allows us to check that the

```
00 00 b6 98 d0 19 5a 45 3b ef a1 d1 41 37 e6 ec | .....ZE;...A7..
49 91 6f 3b ab 7d 59 32 8f d4 e1 a6 33 bf 66 4b | I.o;.}Y2....3.fK
d6 fb 6a 9d f6 d4 89 72 a4 3d 8a 5b 62 10 4b 07 | ..j....r.=.[b.K.
d4 c3 15 52 01 d9 20 c1 97 87 4d c2 34 df 2a af | ...R......M.4.*.
cc 05 01 00 00 00 2e 00 0a 19 08 c6 ad d6 40 10 | ..............@.
13 20 a3 82 01 30 80 d6 89 99 06 3a 06 32 30 39 | . ...0.....:.209
34 35 48 12 11 9a 01 0e 08 02 15 00 80 88 c5 20 | 45H............
01 2d 00 00 c8 c3 ff ff ff ff ff ff ff ff ff ff | .-..............
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 | ................
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 | ................
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 | ................
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 | ................
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00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 | ................
```

Figure 11: ROM dump from a Cue COVID-19 test's on-board 512-byte EEPROM. After soldering tiny wires onto the tiny pins and writing a driver for it, I had hoped to see a secret message congratulating me on my steady hands and the beginning of an Alternate Reality Game whose prize was the inheritance of an eccentric billionaire (but who's got time for that?). Alas there is nothing that can be easily deciphered on here other than perhaps 20945H. Note how much of the EEPROM is unused, but I'm glad they sprung for 512 bytes, or else this project would not have been so *feasible*.

data were successfully written). However, sometimes the chip would come online *during* the read command, producing unpredictable results. Basically you have to be tolerant of errors in some situations, but not *too* tolerant, or else you don't detect real failures. It gives me some sympathy for terrible dedicated EEPROM programmers I have used [15].

4.1 Harder Drive: Cu

Having committed to the naming scheme where I replace some of the last letters of the thing with the letter u, it seems the best name for this drive is Cu. For one thing, this is the chemical symbol for copper, and the drive uses copper to function.

With the ability to read and write a single Cue cartridge, the remainder is just a matter of straightforward engineering and tedious manual labor. Of course, you want to do all of this on a manufactured printed circuit board (Figure 13). The job here is to make it possible to individually address a single EEPROM to read and write its data. Though I^{2}C does support multiple devices on the same bus, these chips all have the same I^{2}C address and so they would all try to reply to the same commands. The ST 24C04WP EEPROM does have "chip enable" pins that would allow it to share a bus with others, by selectively enabling only the chip of interest. Unfortunately, these pins are not connected to any of the exposed connectors on the cartridge. Instead, I use a bus switch (which is basically this same "chip enable"

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[17]I recommend only disassembling a "negative" test, in case that is not obvious. The typical reagents used in RT-LAMP are not particularly dangerous; for example *Bst* polymerase is "not hazardous" according to OSHA, although it may be an "eye irritant" [2].
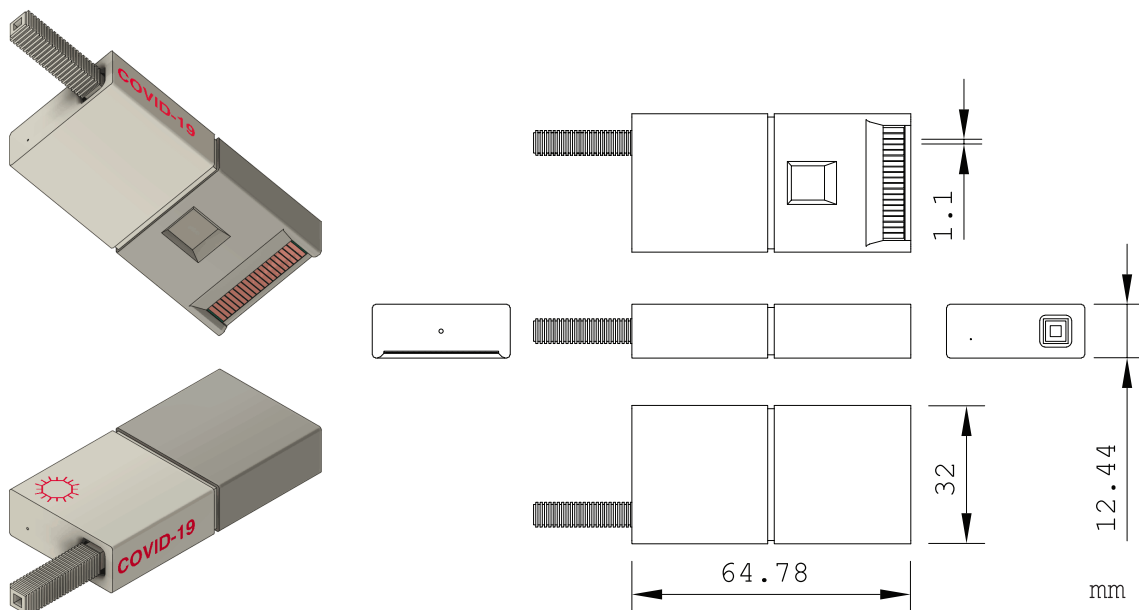
Figure 12: Mechanical drawing of the Cue COVID-19 test cartridge. The protruding stick is the nasal swab, which is permanently captured during use with zip-tie–like ratcheting. The card edge connector is the low tolerance piece here, whose small size (0.05 inch pitch with 1.1mm fingers) requires special consideration for mounting and soldering. (In its normal usage, this connector mates with some spring-loaded pins when the cartridge is inserted in the Cue reader.)

circuitry) to connect each EEPROM to the I^2C bus. Each SN74CBTLV3125 is a quad bus switch, so I can switch the two I^2C lines (SDA, SCL) for two Cue cartridges with each chip. Then, we can select one of 8 cartridges (a single daughter board) with a demultiplexor, which takes 3 address bits and sets exactly one of its 8 output lines to 0. For decorative purposes, I associate a colored LED with each cartridge; this LED foolishly ends up accounting for most of the components on the board and most of the assembly time, since I also have to build logical NOT gates (demultiplexor outputs logical 0). Finally, the cartridges themselves are very tricky to incorporate. They have very small plated connectors that normally mate with spring-loaded pins in the reader, but that component is not readily available (and would probably be expensive). Instead, I mount it at 90° through a hole in the PCB, where the PCB has its own matching edge connector made with castellated holes. I also 3D printed a plastic jig that could hold the cartridge at the right angle during soldering. With generous acid flux and a steady hand, soldering these worked quite well. Only 4 pins need to be connected (3v3, GND, SDA, SCL) but I also soldered some distal pins, since these joints are the only mechanical connections for the cartridges.

The motherboard has its own demultiplexor to select the daughter board, as well as an ad hoc pair of "group selector" pins wired directly to GPIO. Together it supports 7-bit addresses, for up to 128 cartridges, which is 64 kilobytes. I did not collect enough used tests to fill the address space, but I did connect 72 of them, which is enough to do something interesting at least! Except . . .

4.2   Failure!

I blew it! Literally! On the evening of the SIGBOVIK deadline, in an attempt to be simultaneously *expedient* and *careful*, I soldered the Cu motherboard while it was plugged in, and fully toasted it and the connected Raspberry Pi. My best guess is that the soldering iron had a very different idea of "ground" than the device under test. It made an upsetting pop noise, an upsetting burn smell, an upsetting spark and smoke sight, an upsettingly warm touch,[18] and it made it impractical to fix before the paper is due. You can at least see what the tabletop device looks like in Figure 14. It should be possible to replace the Pi and motherboard, so perhaps the by the accompanying video or talk I will be able to finish the task and get some benchmark numbers.

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[18]I did not attempt to taste it; the board is not RoHS-compliant due to the copious lead used. Despite the panoply of upsetting sensations, the obviousness of the failure was a blessing that saved me time trying to debug! Even if I plug the Pi in on its own, it makes a pathetic, obviously unhealthy whine. It is so dead.
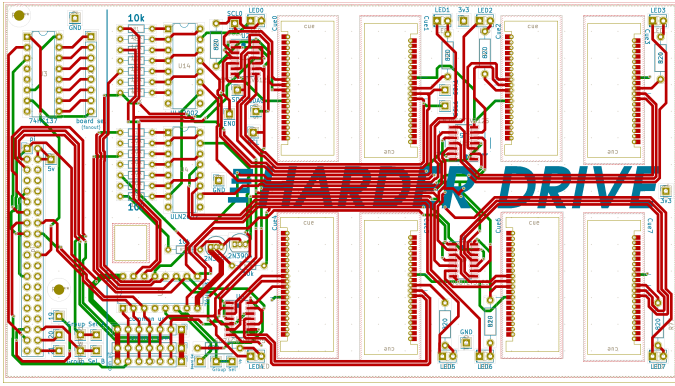
Figure 13: The two-layer printed circuit board for the Cue drive. Because the boards must be ordered in quantity, one board contains the layout for both the motherboard (used once) and daughter boards (used many times). On the far left is the motherboard, for example the header for interfacing with the Raspberry Pi and the 3:8 demultiplexor for selecting a daughter board. These parts are only populated once. The remainder is the daughter board: Eight cutouts for cue cartridges mounted at 90° with castellated edges; an LED and support logic for each; the surface-mount bus switch ICs; another 3:8 demultiplexor for selecting the cartridge on this board. The design can accommodate 16 daughter boards, each with 8 cartridges, for a total of 64 kilobytes.

4.2.1 Results

We need a file to store on the drive to tie the knot and to run the benchmark. A beautiful choice here is the genetic sequence of SARS-CoV-2 (ancestral) [30] from GenBank [5]. This is a 77343 base-pair sequence; it would be too large to store in ASCII.[19] Each nucleotide is only two bits of information, though, so we pack four of these into each byte for only 19336 bytes.

Qualitative. This is a decent Harder Drive. It solves a problem we don't have, which is what to do with all those COVID-19 tests that we'd otherwise just throw away? It took significant effort to create, although most of the difficulty was from problems (e.g. how to solder these tiny pins) that are not interesting from a computer science perspective. EEPROMs are fundamentally data-storage devices, so this usage of them cannot be considered clever, but arraying dozens of them to create a modest-sized non-volatile memory that could be easily replaced with a single 30-cent NAND Flash IC is at least "very silly." Mucho demerits because I broke it during the final assembly.

Cost. The cost is significant. The up-front cost is a Raspberry Pi 3 (nominally $35), accessories, and a demultiplexor IC ($0.60), plus scrap plywood for mounting. Then, per board, we have the following bill of materials:
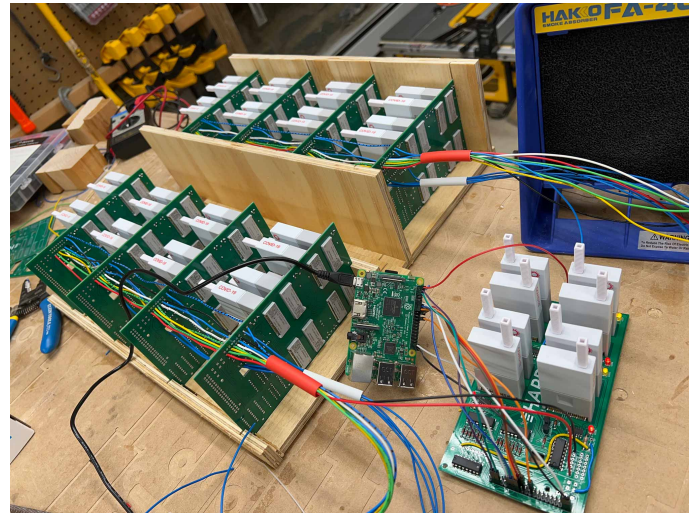
Figure 14: Assembled `Cu` drive with 72 Cue cartridges. Imagine trying to explain to someone that this homemade thing that has "COVID-19" written on it 72 times, and has got all sorts of colored wires everywhere, is not some instrument of bioterror. In fact it does not even drive hard: In my rush to meet the preposterously strict SIGBOVIK deadline, I fried the Raspberry Pi *and* motherboard (perhaps you can see that multiple LEDs are lit on the motherboard, which clearly violates invariants). Fortunately due to its modular design, it can likely be fixed with a few more hours of manual labor.

Qty.	Part no.	Description	Price ea.	Total
8	Cue L2900006	Used COVID-19 Test	$65.00	$520.00
1	custom	2-layer PCB 162x92mm	$5.766	$5.766
2	497-2340-5-ND	Transistor array IC	$0.5768	$1.154
4	SN74CBTLV3125	Bus switch IC	$0.6636	$2.654
2	2N3904	NPN BJT transistor	$0.09	$0.18
20	jellybean	$10k\Omega$ resistor	$0.0155	$0.31
8	jellybean	845Ω resistor	$0.02428	$0.194
8	jellybean	3mm LED 2v 20mA	$0.01499	$0.12
1	CD74HC137E	3:8 demultiplexor IC	$0.6048	$0.605

$$\$530.98$$

This does not include consumables like solder and hook-up wire, nor the considerable time to assemble each board (about 1 hour with practice).

We need 13 boards to store a full FAT12 filesystem, for a total cost of $6,936.04. The marginal price per byte is 12.96 cents.

Longevity. This is the only drive considered where data are retained when powered down. The M24C04-W EEPROM is rated for 200 years of data retention, and 4 million write cycles [25]. At the current pace, this is likely to outlast the human race.

Speed. Unknown as of publication! As described in Section 4.2, the motherboard was damaged on the eve of the deadline and no benchmark was conducted. Reading the EEPROM is fast, but writing a block takes a few hundred milliseconds. The speed is expected to be high (for the drives considered here).

Power. The up-front power cost is low; we need to power the Raspberry Pi and various chips on the boards. Only one of the decorative LEDs is lit at a time, using about

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[19]Plus, I'm a biology noob and I may just be missing something, but GenBank uses "T" in this sequence even though it should be U (uracil) in RNA, which seems very non-canonical?

1 mW. The total is about 3 Watts. The marginal power cost is excellent: On the Cue cartridge, only the EEPROM is powered. During standby it uses no more than 3 μA at 3.3 V, which is 9.9 μW for 512 bytes, or 19.3 nW per byte.

Is rotational? This drive is not rotational; it provides us SSD-like random access to the Cue cartridges, and the EEPROMs on board allow random access to each line of data.

Harm to society. Arguably, the drive is beneficial to society. First, it is built mostly from trash. Second, coronavirus testing prevents death or other hardship by informing infected people that they may be contagious; at a minimum it is good for the spirit by facilitating lower-anxiety gatherings. Finally, since the tests contain captured body fluids, this adds an all-too-rare "human element" to computing.

5 Other hard drives we really didn't need

Here are some things I hate: (1) The name of TDAmeritrade's stock trading app, which is "thinkorswim." This is of course a play on the idiom to "sink or swim," meaning metaphorically to toss someone into deep water to survive by their own efforts, or else drown. The analogy is certainly apt for an app that lets consumers trade derivatives, but the obvious problem here is that if it is "think or swim," then we are now asking the subject to survive by their own efforts (swim) or else "think"? Huh? Or is it that they must think carefully about their trades, or else they will survive? Wha?   (2) Poison ivy. This plant has no purpose other than to make you itch. It doesn't even get anything out of that trick, since I wasn't going to eat it anyway. Nevertheless it spreads.   (3) Cryptocurrency. I have no objection to the use of cryptography in finance, but there aren't enough vomiting emojis in Unicode to appropriately react to the current hype. Cryptocurrency significantly harms the planet while taking advantage of many people's technical and financial illiteracy.[20]

Nonetheless, the common prefix between "block device" and "blockchain" is hard to avoid noticing, and a head-to-head comparison may be instructive. So I put on incognito mode, a VPN, an N-95 mask and six condoms in order to research some numbers for this section.

Bitcoin is "append-only" by design, so it does not have the same abstraction as other Harder Drives. For comparison sake, we consider a usage where the head of the blockchain contains the full data; a write is accomplished by mining a new block and a read is accomplished by reading from the current head. For Bitcoin, the block size is 1 Mb, and the network automatically adjusts to mine a single block every ten minutes. I did *not* actually implement this drive, both because of the gag reflex and because I do not have that kind of money!

Qualitative. Despite hating it, I must admit that Bitcoin meets the criteria for a Harder Drive pretty well. It solves a problem that we don't have, by imagining a world where we cannot agree on a small set of trustworthy parties, a majority of which must be acting in good faith. Its approach is elegant in the small but for its obvious fatal flaws, and comically absurd if taken to its logical extreme. It is impressively inefficient, and grows less efficient over time. In short, it would make a solid SIGBOVIK paper. The only problem is that people are actually using it in seriousness, and the social problems that result from the value it has attained.

Cost. The cost is extremely high. The reward for mining one Bitcoin is currently 6.25 BTC, plus an average of about 0.97 BTC in transaction fees, which totals $342,000 in March 2022. This gives us an approximate upper bound on the cost to mine (by assuming the marginal cost is profitable) a block, which is 34.2 cents per byte. This does not include the up-front cost of hardware and facilities, which is of course monumental.[21]

Longevity. The data has excellent longevity, in fact, it is impossible to erase previous data once written. Of course, "forks" of the chain can make it unclear what version of the data is correct, or if > 50% of the untrusted miners

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[20]Note to cryptocurrency apologists: This short section does not have room for a full criticism, nor would such a thing be "fun" enough for SIGBOVIK. Briefly, there are five principal problems. (1) Proof of Work is incredibly wasteful (see the stats below; this is just *one* of the cryptocurrencies). Of course I am aware of "Proof of Stake." I remain very skeptical that miners with large capital investments in (otherwise useless) ASICs will be willing to salvage them, but I would gladly celebrate this and by all means, please do make this happen. (2) I believe that regulation of finance is *good*, both formal regulation with law and self-regulatory organizations like FINRA, as well as informal practices like rolling back erroneous transactions or returning stolen funds, which are regulated indirectly by the desire to maintain valuable public reputations. Unregulated markets have many problems (insider trading, etc.) and avoiding regulation mostly seems to be useful for tax evasion and other crimes. (3) In attempt to avoid "decentralization", control is nonetheless effectively centralized in the hands of a small number of actors anyway (large-capacity miners and exchanges). However, these actors are set up as adversarial, or at best as some kind of wild-West "disruptors." I'd trust these skeezy guys way less than I trust banks, and rightly so: They rou-

tinely front-run transactions, just as one example. (4) The space is riddled with Ponzi schemes and scams, as exemplified (but certainly not limited to) NFTs. This is plainly immoral. (5) Cryptocurrency aficionados are insufferable, presumably because they feel like they need to convince you to get in on their tokens so that they grow in value (which presumably they hope to then sell to get real money). I get an enormous amount of cryptocurrency spam. The only words I've muted on Twitter other than five green Unicode squares are cryptocurrency terms, and this has improved the experience greatly. If you are a cryptocurrency aficionado reading this that feels like you need to "educate" me about how I am misinformed (despite what I can clearly see in front of me!), case in point. That said, I will happily discuss interesting ideas with informed computer scientists over beer.

[21]Nor the surrounding apparatus like "bitcoin ATMs" and "crypto exchanges" (the kind of stuff that apologists are talking about when they tell you that "regular money uses a lot of electricity too!"), although it is probably not fair to count these as part of the cost when used as a pure data storage mechanism.

disagree, they can change the data at will.

Speed.   The network is slow, although not the slowest considered here. It takes an expected ten minutes to write 1 Mb of data. This is a data rate of 1,747 bytes/sec, approximately the speed of a 14.4 kbaud modem.

Power.   The power usage is incredibly high. Mining Bitcoins uses about 0.31% of the entire world's energy production, 15.74 GW [6]. Remember that this does not solve any interesting computational problems or accomplish anything useful; the only purpose is to create an expensive waste of power in order to avoid trusting a bank or set of banks. To store one megabyte on an ongoing basis, this is 15.74 kW per byte.

Is rotational?   It's not even rotational!

Harm to society.   The harm to society is significant. Aside from the catastrophic waste of resources, the primary use case is speculation (at best morally neutral, but probably tends to harm small investors). As a slow, expensive, non-atomic yet irrevocable payment mechanism, they are best suited for extortive transactions like Ransomware.

6   Conclusion

In this paper, we decided that sometimes it's more fun to do things the hard way, and then did so. Using several different techniques and some needless digressions, we created block devices that could support small filesystems, which then could host a fitting file. Each filesystem was bad when considered as a regular hard drive, but good when considered as a Harder Drive. We also compared these drives to the most popular cryptocurrency. The idea was to make the point that cryptocurrency is so egregiously bad that it resembles a "SIGBOVIK joke gone wrong" more than something one would make on purpose. This part may not have been as fun.

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