The Pacman Effect and the Disc-Sphere Duality

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Abstract

In this paper, we study the Pacman effect, where the Pacman disappears from the leftmost part of the screen when reaching it, and re-appears from the rightmost part of the screen. We apply the Pacman effect to unify the theory of flat and spherical Earth, proposing the Disc-Sphere Duality. We conclude that a spherical Earth is basically the same as a flat Earth under the Pacman effect, bring peace to believers of both theories.

1 Introduction

The aim of this paper is to bring peace! For years very smart people have been fighting over the shape of our beautiful, albeit shitty, planet. Two equally reasonable positions have mainly emerged: the Earth is a disc VS the Earth is an oblate spheroid (that we approximate with a sphere, from now on). This two very different bidimensional manifolds have both equal rights to claim themselves the proper geometrical model for Earth. Thus they started fighting. With what army? Well, Flat Earth Societies are fighting in the blue corner alongside the multiple-millennia champion Disc. In the red corner we see the young arrogant opponent, the Sphere, supported by its crew: Science and everyone else.

They have been fighting round after round for centuries, with equal credibility, and the match seemed impossible to settle. There was this dark moment in history where the evil newcomer was unfairly holding our beloved champion against the ropes. All seemed lost. How can planes fly in circumferences around the globe if the globe is not a globe? That was a tough one, also a shitty move in the authors' opinion. Then Pacman, the fairest referee of all, came to save the day.

The main contribution of this work is none. However, we would like to remind everyone that the Pacman effect is a thing. Airplanes can fly to the extreme boundary of Earth, but they are bound to reappear on the diametrically opposite side of the disc, just like Pacman in the famous Namco video game. Now when forced to apply the rigour of mathematics (reluctantly: reasoning is for nerds! Nerds suck!), we see that the Pacman effect can be modelled as the identification of the boundary of the disc to one single point.

Counterintuitively this Pacman effect sheds new light on the Truth. The disc with such identified border is nothing but a surface diffeomorph to the sphere. As Einstein once said:

> "It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two

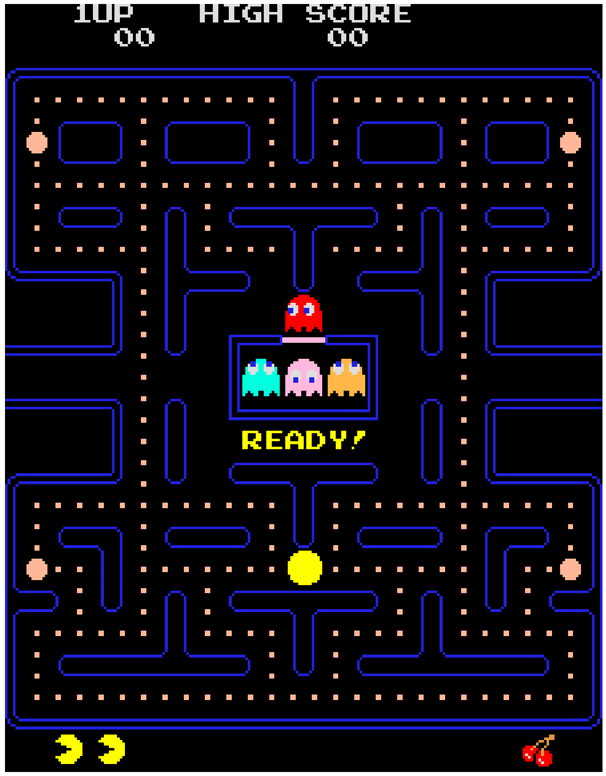


Figure 1: A Screenshot of Pacman. When reaching the leftmost part of the screen, Pacman wraps to the rightmost part of the screen.

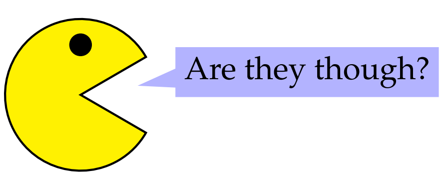
> contradictory pictures of reality; separately neither of them fully explains the phenomena of ~~light~~ Earth, but together they do."

We have discovered (maybe, "invented"?) what we call the "disc-sphere duality".

2 Flat and Spherical Earth are Very Different Views of the World

Try and play Ultimate with a football, or football with a flying disc. You silly. Readers are encouraged to use their imagination, or alternatively refer to Fig. 2.

3 Pacman Says: "Are they though?"



We show that the closed disc with the boundary identified to a point (with the quotient topology induced by the topology of the plane) and the sphere (with its natural Euclidean topology) are homeomorphic topological manifolds. In fact, the homeomorphism can be used to transfer the differential structure of the sphere to the disc with the boundary identified to a point.

(a) A Football

(b) Ultimate Frisbee, with a Disc

Figure 2: Flat and spherical objects are different.

Our reference is [2], but the background provided by any textbook in basic topology will suffice.

We set the following notation:

- $\mathbb{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ is the (open) disc;
- $\overline{\mathbb{D}} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ is the closed disc, with the topology induced by $\mathbb{R}^2$;
- $\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ is the sphere;

Let $\sim$ be the equivalence relation on $\overline{\mathbb{D}}$ which identifies the boundary to a point; write $[\partial]$ for the equivalence class of the boundary. We have $\overline{\mathbb{D}}/\sim = \mathbb{D} \cup [\partial]$.

The fact that $\overline{\mathbb{D}}/\sim$ is homeomorphic to a sphere is an immediate consequence of the fact that they can both be identified with the 1-point identification of the disc. We give an explicit homeomorphism, using the stereographic projection of the sphere.

Recall the stereographic projection

$$\begin{aligned} \Sigma : \mathbb{S}^2 \setminus \{(0,0,1)\} &\to \mathbb{R}^2 \\ (x, y, z) &\mapsto \left(\frac{x}{1-z}, \frac{y}{1-z} \right). \end{aligned}$$

Then Σ homeomorphically maps $\mathbb{S}^2 \setminus \{(0,0,1)\}$ onto $\mathbb{R}^2$. This homeomorphism extends to the one points compactifications setting $\Sigma((0,0,1)) = \infty$, where ∞ is the point at infinity of $\mathbb{R}^2$.

The plane $\mathbb{R}^2$ is homeomorphic to the unit disc via a two dimensional tangent map and the homeomorphism extends to the one point compactifications:

$$\tan : \mathbb{R}^2 \cup \{\infty\} \to \overline{\mathbb{D}}$$
$$\begin{cases} (x, y) & \mapsto & \frac{(x,y)}{x^2+y^2} \cdot \frac{2}{\pi} \arctan(x^2 + y^2) & \text{if } (x, y) \in \mathbb{R}^2, (x, y) \neq (0, 0) \\ (0, 0) & \mapsto & (0, 0) \\ \infty & \mapsto & [\partial]. \end{cases}$$

You wanted it? You've got it.

4 No. They aren't.

No. They aren't. You silly.

5 Conclusion and Future Work

Our work clearly shows that, thanks to the Pacman effect, the two main mathematical models from literature, applied to describe the planet Earth, are in fact the same. The first critique that comes to mind to the presented results can be synthetically expressed as follows.

> In order to be able to model the Earth as flat, you need to introduce ad hoc technicalities, like the identification of the border to a point (a.k.a. Pacman effect), that de facto allow you to use a spherical model, while calling it "flat".

First of all the guy above likes Latin a lot and we don't trust people that try to trick us with different languages. Secondly we finds that who writes such critics is a quite biased and narrow-minded person, not willing to accept point of views and opinions different from their owns.

The authors of this paper instead will not discriminate any model on basis of shape, race, colour, sex, language, religion, political or other opinion, national or social origin, property, birth or other status such as disability, age, marital and family status, sexual orientation and gender identity, health status, place of residence, economic and social situation.

As for related and future work, finding out whether Australia factually exists and, if so, where it is located, goes beyond the aim of this paper. Indeed, we have discovered a truly marvellous proof of this, which this margin is too narrow to contain.

References

[1] Wikipedia

[2] Munkres, James R, *Topology: A First Course*, Prentice Hall International, 1974.