Aumann Agreement by Combat

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Abstract

The celebrated Aumann's Agreement Theorem [2] shows that two rational agents with the same priors on an event who make different observations will always converge on the same posteriors after some civilized conversation over tea. Furthermore, they will come to agree even if they do nothing other than simply state their posteriors over and over again.

However, this protocol is widely criticized for being too boring. We therefore introduce a more exciting alternative, which we name *Aumann Agreement by Combat*.

1 Introduction

Informally, Aumann's Agreement Theorem [2] states that two honest Bayesian agents can come to agreement by stating their priors over and over again. (Here, the term *agent* simply means "a person who observes things and has opinions." However, it is more exciting to imagine them as *secret agents*, so we will do so for this paper.)

To take example from Aumann's paper, suppose the agents are investigating the fairness of a particular coin (say, a very rare coin involved in a museum heist). We will call the agents A and B, but of course they are secret agents, so what names these letters stand for is a mystery. The coin is parameterized by a value p, the probability that it will land heads when it is flipped. Each agent begins with the same prior, in this example the uniform prior of p over the interval $[0, 1]$, because the uniform prior is a reasonable prior to assume about a coin that one knows nothing about.

If A flips the coins and sees heads, then her Bayesian calculation will conclude that the probability of the *next* flip being heads will be $2/3$. If B flips and sees tails, they he will think the probability is $1/3$. They are now in disagreement. However, if these agents then meet in a tavern to discuss, then A needs merely to state that her probability is $2/3$, and B will conclude that she must have seen heads in his single flip. With all of that information, she now concludes that the probability of another flip being heads will be $1/2$. Observers will assume that these numbers are all part of a secret code, but A will understand that B has computed the true probability, and she will update her probability to $1/2$ as well. Both agents are now in agreement.

Of course, this scenario assumed that both agents knew that the other would be flipping the coin only once. In more complicated scenarios, the agents may have variable methods of observation, for example, they might flip multiple times, in which case their observations should be weighted more, or they might not. Still, as long as they have the same priors on said methodology, they will be able to come to agreement through a sequence of statements.

Unfortunately, this methodology of merely stating opinions is boring. Furthermore, everybody knows that the goal of arguing is not to reach the truth, but rather to win. Therefore, we have developed a new protocol, *Aumann Agreement by Combat* that alleviates these problems.

2 Aumann Agreement by Combat

Our new protocol works as follows.

1. Agents A and B start with the same priors.
2. A and B each go out and make observations.
3. A and B meet to discuss the probability of some event E. Each agent has some initial estimate of the probability of E.
4. Players alternate in turns, continuing until their estimates match. On each turn, an agent has a choice to either:
   - State their current estimate of E. The other player must use Bayesian reasoning to update their own estimate.
   - Declare COMBAT. The agents will fight it out, and at the end, the loser will adopt the winner's estimate as their own.

The COMBAT phase has no rules. Any player is free to try to win by any means necessary, and they are encouraged to employ all of their wit and resources to the fullest extent. In the base rules, the battle ends when one player loses consciousness, although the players may also agree to a different ending condition, such as getting a flag to one's base, death, or the outcome of a poetry competition.

With this definition in place, we come to our main theorem.

Theorem 1. *After an execution of the Aumann Agreement by Combat protocol, both agents will agree on the correct probability of the event E.*

Proof. Might makes right. □

3 Complexity Analysis

In [1], Aaronson analyzes the complexity of Aumann Agreement, showing that $O(1/\varepsilon^2)$ bits of communication are sufficient to agree within ε.

An advantage of our new protocol is that the analysis is much simpler. The COMBAT phase might take an arbitrary amount of time and resources, depending on the strategies that the agents employ. They might stake the COMBAT phase on the outcome of an intergalactic war, for example. Therefore, there is no complexity bound on time, space, communication, energy, or anything else.

4 Conclusion

We expect that many scholars will welcome this formalization of the game that they already try to play.

References

[1] S. Aaronson. The complexity of agreement. In *Proceedings of the Thirty-seventh Annual ACM Symposium on Theory of Computing*, STOC '05, pages 634–643, New York, NY, USA, 2005. ACM.

[2] R. J. Aumann. Agreeing to disagree. *Ann. Statist.*, 4(6):1236–1239, 11 1976.