

# Erdős-Bacon-Sabbath Numbers

## *Reductio ad Absurdum*

Maria Klawe  
Harvey Mudd College  
Claremont CA 91711

Daniel V. Klein  
Google  
Pittsburgh PA 15206

D K Fackler  
University of Edgestow  
Thulcandra

Mary Shaw  
Carnegie Mellon University  
Pittsburgh PA 15203

Michael Ancas  
Crazy Pants Productions  
McDonald PA 15057

Augie Fackler  
Google  
Pittsburgh PA 15206

Sarah R. Allen  
Google  
Pittsburgh PA 15206

Harry Q. Bovik  
Carnegie Mellon University  
Doha Qatar

Salim Virji  
Google  
New York NY 10011

## Abstract

A small Erdős number – the “collaborative distance” of authorship between oneself and Paul Erdős – has long been a source of pride for mathematicians, computer scientists and other geeks. Utilizing similar collaborative distance metrics, a small Bacon number (the degree of separation from Kevin Bacon) has been a source of pride for actors, while a small Sabbath number (the degree of separation from Black Sabbath) has been a source of pride for musicians. Previous research in Erdős-Bacon number minimization has reduced the Erdős number of a number of computer scientists to two, which is believed optimal, although the reduction of the Bacon number to four was clearly suboptimal. We extend and improve on this previous work to provide a Erdős-Bacon-Sabbath number minimization that is believed to be close to optimal in all axes.

## Derivative Introduction [2]

Paul Erdős co-authored nearly 1500 papers (until his death in 1996), working with nearly 500 collaborators achieving the status of the most prolific mathematician in modern times [4]. Mathematicians thus humorously defined Erdős numbers. A person’s Erdős number is the distance between that person and Paul Erdős in the academic paper collaboration graph [3]. Succinctly, Paul Erdős is the unique person with Erdős number zero; all of Erdős’ immediate co-authors have Erdős number one; in general, if you publish an academic paper with a collaborator who has Erdős number  $N$  and none of your other co-authors has Erdős number less than  $N$ , your Erdős number is  $N + 1$ .

A similar Bacon number [16] has been proposed for actor Kevin Bacon, except using collaborations in film instead of collaborations in academic papers. Likewise, a similar Sabbath number [12] has been proposed to connect to the members of

the musical group Black Sabbath, using collaborations in musical performances.

Erdős-Bacon-Sabbath numbers were subsequently defined [5] to be the sum of each person's Erdős, Bacon, and Sabbath numbers.

There is a long tradition of posthumous publication [7], and authors claiming to have collaborated with Erdős have brought his total number of known publications to 1525, his collaborator count to 511, and the Erdős number of the chutzpah-bearing mathematician to one. The latest publications co-written with Paul Erdős appeared more than ten years after his death. With additional rumored works in progress, Erdős's publication list is expected to grow. In fact, Paul Erdős himself has published a solo work 15 years after his death [8].

In this paper, we describe and demonstrate a technique called Erdős-Bacon-Sabbath Number Minimization.

## Rules of the Games

The rules of Erdős number calculation are clear: author a paper in a peer-reviewed publication, either with someone connected by co-authorship with Paul Erdős, or with Paul Erdős himself (the latter being unlikely unless pre-demise work is used in a posthumous publication, or if you are better at communicating with the dead than Edgar Cayce). Simply putting Paul Erdős' name on your paper does not count (and changing your name to Paul Erdős for purposes of publication is *definitely* cheating).

The rules of Bacon number calculation [16] are also clear: act in or be otherwise credited in a film with someone connected by film credit with Kevin

Bacon<sup>1</sup>, or with Kevin Bacon himself. We tried contacting Kevin Bacon, but his agent refused to put us in touch. We contemplated inserting a fair-use clip of Kevin Bacon from an unrelated movie, but knew that would be cheating (but we *did* think of it).

Finally, the rules of Sabbath number calculation [12] are also clear: connections between a musician and a band or solo artist can only be made if they actually performed or recorded together. However, "session musicians" are valid connections, so musicians who perform live or record with an artist, but are not strictly committed to that band are valid. A new recording [11] can serve as adequate proof, but singing along to a Black Sabbath record is cheating. We also contemplated contacting Ozzy Osbourne, but he postponed the 2019 tour that would have taken him through the Pittsburgh area, and then cancelled the tour altogether.

The theoretically achievable absolute minimum Erdős-Bacon-Sabbath number is two: if Kevin Bacon (who has a Bacon Number of zero) were to become a member of Black Sabbath (thus receiving a Sabbath number of zero), and was to publish a paper with a person with an Erdős number of one (since Paul Erdős is dead, and ineligible as a co-author). The practically achievable minimum Erdős-Bacon-Sabbath number is four (by authoring a paper with a someone with an Erdős number of one, appear in a movie with Kevin Bacon, and perform with a member of Black Sabbath), but the realistically achievable minimum is somewhat higher than that.

---

<sup>1</sup> The Internet Movie Database <http://www.imdb.com> is typically used for verification of film/video/YouTube credits.

# Computation of Erdős, Bacon, and Sabbath Numbers

In [1], Maria Klawe co-authored with Paul Erdős and has an Erdős number of one, thus guaranteeing that all authors on this paper have an Erdős number no greater than two.

Daniel V. Klein has a Bacon number of 2, having appeared in [9] with Steve Guttenberg, who appeared in [10] with Kevin Bacon. Mike Ancas also has a Bacon number of 2, having appeared as an extra in [17] with Tom Hanks, who appeared in [18] with Kevin Bacon. All authors on this paper also appear in the documentary about the writing of this paper [11], and thus have a Bacon number no greater than three.

Finally, DK Fackler has a Sabbath number of 2, having performed with Roger Daltrey [14] in 1994, who in turn performed in Wembley Stadium with The Who at Live Aid in 1985, where Black Sabbath also performed[15]. The authors of this paper have recorded a special musical piece, composed specifically for this paper, and captured in [11]. Their Sabbath number is therefore no greater than three.

The combined Erdős-Bacon-Sabbath number for all the authors of this paper is therefore no greater than eight, which puts them in the rarified company of the only three other people hitherto known to have an Erdős-Bacon-Sabbath number of eight: Stephen Hawking, Ray Kurzweil, and Daniel Levitin, a professor of psychology and behavioral neuroscience at McGill University [6].

However, four principal authors of this paper have beaten this value and tied the record-low (held by Lawrence Krauss [13]) with an Erdős-Bacon-Sabbath number of seven! Maria Klawe (E=1, B=3, S=3), Daniel V. Klein and Mike

Ancas (E=2, B=2, S=3) and DK Fackler (E=2, B=3, S=2) now join that rarified company.

## Sheet Music & Documentary

You've read the paper, now read the music (in the appendix)! Lastly, you get to watch the documentary at [11].

## Acknowledgements

Thanks, Mom! Thanks, Dad! Thanks to spouse(s) and/or partner(s), significant other(s), groupies, and random tinder dates!

## References

- [1] Paul Erdős, Frank Harary, and Maria Klawe. "Residually complete graphs." In *Proc. Sympos. Combinatorial Mathematics and Optimal Design*, pages 117–123, Colorado State University, Fort Collins, Colorado, 1978
- [2] Charles Garrod, Maria Klawe, Iain Matthews, *et. magnum al.* "An Algorithm for Erdos-Bacon Number Minimization." In *A Record of The Proceedings of SIGBOVIK 2010*, pages 3-4, Carnegie Mellon University, Pittsburgh, Pennsylvania, 2010
- [3] Caspar Goffman. "What is your Erdős number?" *The American Mathematical Monthly*, 76(7):791, 1969
- [4] Jerry Grossman, Patrick Ion, and Rodrigo De Castro. Erdős number project, 2010.  
<http://www.oakland.edu/enp/>.
- [5] Richard Sear. "Erdos-Bacon-Sabbath numbers." *Times Higher Education*, 2016.  
<http://blogs.surrey.ac.uk/physics/2012/09/15/erdos-bacon-sabbath-numbers/comment-page-1/>
- [6] Scott Seckel. "Rarified air: Do you have an Erdos-Bacon-Sabbath number?" *University of Arizona*, 2016.

<https://asunow.asu.edu/20160126-creativity-lawrence-krauss-erdos-bacon-sabbath-score>

[7] Myles McWeeney. “Voices from beyond the grave - authors whose legacies live on through posthumous publishing” The Independent (Ireland), 2017.

<https://www.independent.ie/entertainment/books/voices-from-beyond-the-grave-authors-whose-legacies-live-on-through-posthumous-publishing-35804952.html>

[8] Paul Erdős. “Some Problems On The Distribution Of Prime Numbers.” In: Ricci G. (eds) Teoria dei numeri. C.I.M.E. Summer Schools, vol 5. Springer, Berlin, Heidelberg [https://doi.org/10.1007/978-3-642-10892-1\\_3](https://doi.org/10.1007/978-3-642-10892-1_3)

[9] Ravi Godse. “Help Me Help You”, 2009. <https://www.imdb.com/title/tt1417297/>

[10] Barry Levinson. “Diner”, 1982. <https://www.imdb.com/title/tt0083833/>

[11] Michael Ancas, Daniel Klein. “Creating a Tiny Erdős-Bacon-Sabbath Number for Fun (and No Profit) (2020)” <https://www.imdb.com/title/tt11833110/>

[12] Bill M. “The Black Sabbath Game”, 2003 <http://web.archive.org/web/20031121110828/http://www.imjustabill.com/blacksabbathgame/index.html>

[13] Scott Seckel. “This guy? He knows people” Arizona State University, 2016. <https://sese.asu.edu/about/news/article/1645>

[14] Roger Daltrey. Live in Chicago, 1994. <http://www.thewholive.net/concert/index.php?id=1423>

[15] Black Sabbath at “Live Aid”, 1985. <https://ultimateclassicrock.com/black-sabbath-live-aid/>

[16] Isabel Teotonio, “Google adds Six Degrees of Kevin Bacon to search engine”, The Star (Toronto, Canada), 2012. [https://www.thestar.com/entertainment/2012/09/13/google\\_adds\\_six\\_degrees\\_of\\_kevin\\_bacon\\_to\\_search\\_engine.html](https://www.thestar.com/entertainment/2012/09/13/google_adds_six_degrees_of_kevin_bacon_to_search_engine.html)

[17] Marielle Heller, “A Beautiful Day in the Neighborhood”, 2019 <https://www.imdb.com/title/tt3224458/>

[18] Ron Howard, “Apollo 13”, 1995 <https://www.imdb.com/title/tt0112384>

Appendix: “Erdős Bacon Sabbath Number Reduction” in A Maj  
Daniel V. Klein, op 2+2+3

The first system of the musical score consists of five staves. The top staff is for the Chorus in treble clef, and the second staff is for the Chorus in bass clef. Both are in A major (three sharps) and 4/4 time. The third staff is for the Clarinet in treble clef, and the fourth staff is for the French Horn in bass clef. The bottom staff is for the Bass in bass clef. The music is mostly rests, with some notes appearing in the Clarinet and French Horn parts in the final measures.

Chorus

Chorus

Clarinet

French Horn

Bass

The second system of the musical score consists of five staves. The top staff is for the Chorus in treble clef, and the second staff is for the Chorus in bass clef. Both are in A major (three sharps) and 4/4 time. The third staff is for the Clarinet in treble clef, and the fourth staff is for the French Horn in bass clef. The bottom staff is for the Bass in bass clef. The music features a vocal melody in the Chorus parts, with lyrics "Erdős Bacon Sabbath Number Re-duc-tion" repeated. The Clarinet and French Horn parts provide harmonic support, and the Bass part provides a steady rhythm.

Erdős Bacon Sabbath Number Re-duc-tion

Erdős Bacon Sabbath Number Re-duc-tion

Bass

Erdős Bacon Sabbath Number Re-duc-tion Re-duc-tion!

Bass

This musical score is for a piece titled 'Erdős Bacon Sabbath Number'. It is written in 4/4 time with a key signature of three sharps (F#, C#, G#). The score consists of five staves. The top staff is a treble clef with a melody of eighth and sixteenth notes. The second staff is a bass clef with a similar melodic line. The third staff is a treble clef and is mostly empty, with a few notes in the final measure. The fourth staff is a bass clef with a steady eighth-note accompaniment. The fifth staff is a bass clef with a similar eighth-note accompaniment. The lyrics 'Erdős Bacon Sabbath Number' are under the first two measures, and 'Re-duc-tion' is under the next two. 'Re-duc-tion!' is under the final measure.

This block contains the continuation of the musical score from the first block. It consists of five staves. The top two staves (treble and bass clef) are empty. The third staff (treble clef) contains a melodic line of eighth and sixteenth notes. The fourth staff (bass clef) contains a steady eighth-note accompaniment. The fifth staff (bass clef) contains a similar eighth-note accompaniment.