Pittsburgh is a Grid

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Abstract

Anyone who's been to Pittsburgh knows its streets are laid out in a weird way. Most of it looks like a grid locally, but then streets run into each other at angles downtown, streets intersect themselves, they do weird things to avoid going up hills, or sometimes they go straight up a hill. In this paper, we show that much of this confusion stems from the fact that Pittsburgh is a triangle and not a rectangle. We show this constructively, by exhibiting a mapping from Pittsburgh into the unit square. The images of many of Pittsburgh's streets under this mapping form a nice grid. Sort of.

1 Introduction

Historically, many great cities, and also some not-so-great cities, have been established near large bodies of water. Presumably, these locations were convenient for the transportation of goods and people to and from the city. I don't know; I'm a computer scientist, not a historian. Furthermore, through a combination of sheer luck and the fact that a large enough curve locally resembles a straight line, the waterfront of many such cities forms a flat border for at least part of the city. Examples include midtown Manhattan in New York, the Back Bay in Boston, Center City Philadelphia, all of Chicago, and all of those interchangeable cities where much higher-paid people with my training live. This sort of situation makes things very easy for city planners: put half the streets parallel to the water and the other half perpendicular. This leaves you with a nice convenient grid of streets so people can figure out where they're going, builds in nice water views on many of your streets, and gives the general impression that you know what you're doing and put a lot of thought into this.

Enter Pittsburgh. Pittsburgh is not bordered by a body of water. For a bunch of complicated reasons involving George Washington, the French, the British and probably Andrew Carnegie at some point, Pittsburgh is bordered by three bodies of water[1], but its designers still wanted it to feel like New York, and can you blame them?[Aurand(2006)] Now, having multiple waterfronts isn't necessarily a problem. If they're parallel to each other, you still get the nice street grid and visitors to your lovely city have the opportunity to enjoy both the more polluted waterway and the even more polluted waterway. In Pittsburgh, however, the rivers which border our city are not even parallel. In fact, at one Point, two of them intersect to form a Pointed shape which we will, of course, call The Acute Angle. This left the planners with a difficult choice of which river to use as the basis for the grid of downtown Pittsburgh. As it happens, they chose "both". Furthermore, Pittsburgh has many hills which would disrupt an orderly grid in any case.

In this paper, we construct an embedding of Pittsburgh into a flat, rectangular space. The images of the streets under this embedding form an orderly grid. This shows that Pittsburgh is just as beautiful and well-organized as other cities; it simply exists in a higher-dimensional, more complex metric space. This is probably related in some way to the technique of kernel methods, but I don't really understand those and so I didn't investigate that connection.

[1] Depending on how you count.

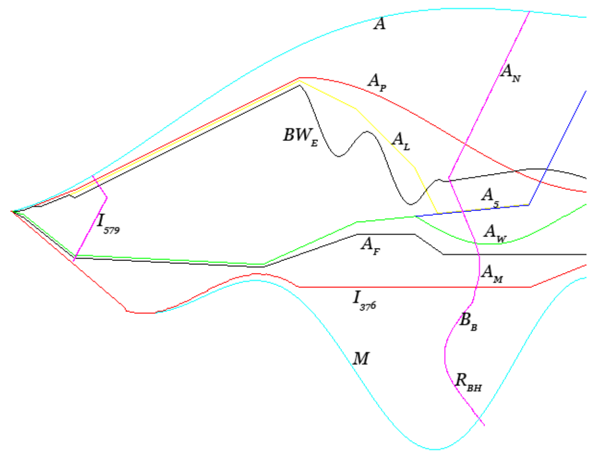

Figure 1: A model of the space $\mathcal{P}$ with some representative subsets.

2 The Definition of Pittsburgh

We begin by formally defining Pittsburgh. In this paper, we will be working with the set $\mathcal{P}$, defined as follows

$$\mathcal{P} = \{(x, y) \mid 0 < x \leq 1 \wedge \mathcal{M}(x) \leq y \leq \mathcal{A}(x)\}$$

where

$$\mathcal{A}(x) = 2\left((1 - \cos\frac{9x}{2})/(4x + \frac{1}{200}) + \frac{x}{2}\right)$$

$$\mathcal{M}(x) = \begin{cases} -\frac{17x}{20} & x < \frac{1}{5} \\ -10(2x\sin(11x) + \sqrt{10x}) & x \geq \frac{1}{5} \end{cases}$$

Note that this model only accounts for the area of Pittsburgh between the Allegheny and Monongahela Rivers. This seems reasonable. After all, many cities have central areas that consist of grids as well as outlying areas that don't. In addition, we assume throughout this paper that Pittsburgh is planar. This is not strictly true, but close enough.

We also identify a number of subsets of $\mathcal{P}$, which we notate A_5, A_P, A_L, I_{579}, BW_E, A_W, A_F, I_{376}, A_N, A_M, B_B and R_{BH}. A representation of this model of $\mathcal{P}$ is shown in Figure 1.

3 A 2D Embedding

We begin by constructing an embedding ignoring elevation and pretending that Pittsburgh is flat. The embedding is a function $E : \mathcal{P} \to [0, 1] \times [0, 1]$ First, let d be the standard Euclidean distance metric on $\mathcal{P}$:

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let $m_{\mathcal{A}}(x, y)$ be the point on $\mathcal{A}$ closest to (x, y):

$$m_{\mathcal{A}}(x, y) = \arg\min_{0<t\leq 1} d((x, y), \mathcal{A}(t))$$

and similar for $\mathcal{M}$:

$$m_{\mathcal{M}}(x, y) = \arg\min_{0<t\leq 1} d((x, y), \mathcal{M}(t))$$

For convenience, we let $d_{\mathcal{A}}$ and $d_{\mathcal{M}}$ be the respective distances:

$$d_{\mathcal{A}}(x,y) = d((x,y), m_{\mathcal{A}}(x,y)) \qquad d_{\mathcal{M}}(x,y) = d((x,y), m_{\mathcal{M}}(x,y))$$

We can now define E.

$$E(x,y) = \left(\frac{m_{\mathcal{A}}(x,y) + m_{\mathcal{M}}(x,y)}{2}, \frac{d_{\mathcal{M}}(x,y)}{d_{\mathcal{A}}(x,y) + d_{\mathcal{M}}(x,y)} \right)$$

The function E maps $\mathcal{P}$ into the unit square by normalizing the distance between the rivers to 1, and it ensures that streets parallel and perpendicular to the closest point on the closest river will form a grid in the image of E.

4 A 3D embedding

The above embedding accounts for the unusual shape of Pittsburgh, but not its hills. We account for this with the observation that it's hard to go up hills and thus our distance metric should include vertical distance:

$$d((x_1,y_1),(x_2,y_2)) = \int_0^1 \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + \frac{dH'}{dt}} dt$$

where

$$H'(t) = H(x_1 + (x_2-x_1)t, y_1 + (y_2-y_1)t)$$

The function $H(x,y)$ is an extension to our model of Pittsburgh:

$$H_D(x,y) = 400e^{-\left(\frac{(x-\frac{1}{3})^2}{2(\frac{1}{6})^2} + \frac{(y-\frac{1}{15})^2}{2(\frac{1}{6})^2}\right)} \qquad H_{Sq}(x,y) = 450e^{-\left(\frac{(x-\frac{3}{4})^2}{2(\frac{1}{4})^2} + \frac{(y+\frac{1}{15})^2}{2(\frac{1}{4})^2}\right)}$$

$$H(x,y) = L(700 + H_D(x,y) + H_{Sq}(x,y))/25000$$

The factor of 25000 is to provide appropriate scaling between the entirely arbitrary units ("feet") used in the elevation calculations above and the much more sensible units ("Pittsburgh-widths") in which x and y are expressed. The constant $L = 100$ is the "laziness" factor which approximates how difficult walking on hills is: the constant indicates that 1 vertical foot is approximately equal to 100 horizontal feet. Is this constant reasonable? Sure, because it makes the results look nice.

We've forgotten how to do integrals and that one looks particularly scary. Fortunately, we have computers to numerically aproximate these things for us. Figure 2 shows the image of the curves from Figure 1 under E. Figure 3 shows the same results, but with some minor noise cleaned up.

5 Conclusion

In this paper, we have shown that all that's required to turn Pittsburgh's complex mass of streets and avenues into the orderly grid enjoyed by many other cities is a 3D model of the city, line integrals and a surprising amount of compute power. There are many aspects of Pittsburgh that this model fails to consider. We have already acknowledged that Pittsburgh is non-Euclidean and therefore any attempt to project it onto a plane is necessarily an approximation. We also don't consider the topological structure induced by Pittsburgh's many bridges. For an exploration of this complex topic, see [Hanneman and Blum(2015)].

References

[Aurand(2006)] M. Aurand. *The spectator and the topographical city.* University of Pittsburgh Press, Pittsburgh, PA, 2006.

[Hanneman and Blum(2015)] G. Hanneman and B. Blum. A constrvctive solvtion to the Königs-Pittsbvrgh bridge problem. *The ninth annual intercalary robot dance party in celebration of workshop on symposium about 2^6th birthdays: in particular, that of Harry Q. Bovik (SIGBOVIK)*, 2015.

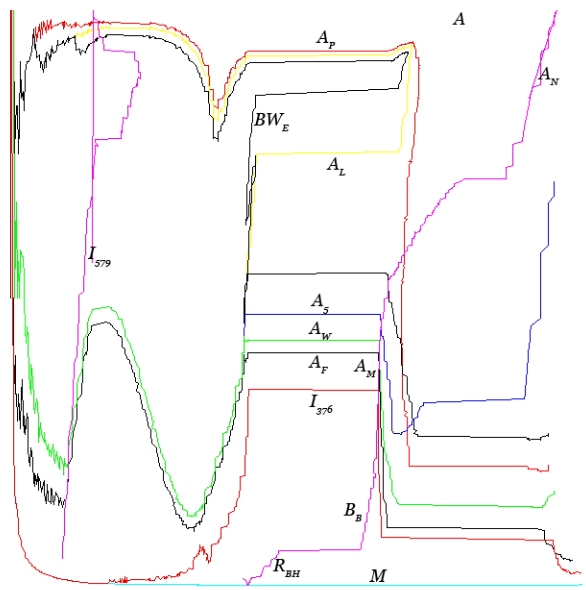

Figure 2: The image of $\mathcal{P}$ under E.

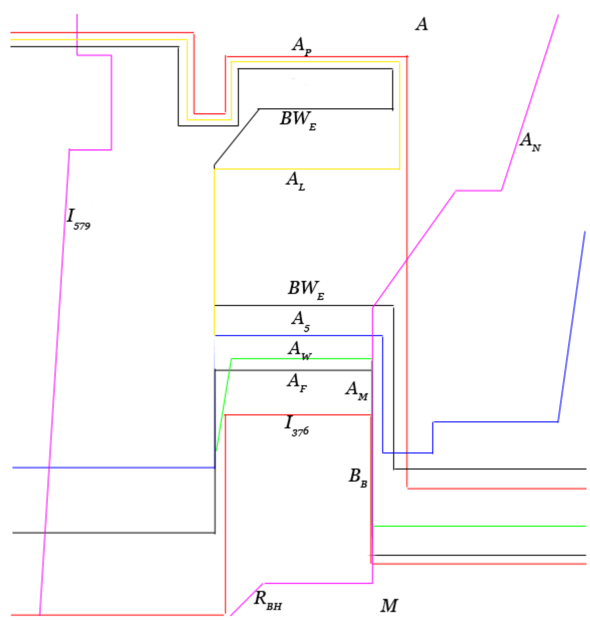

Figure 3: The image of $\mathcal{P}$ under E, with some minor smoothing operations.