Type-Safe Friends Will Never Hurt You

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1 Introduction

In recent years, the activity commonly known as "friendship" has undergone a precipitous rise in popularity, with recent census numbers suggesting that almost 1 in 3 Americans has friends. As documented in the extensive literature, friendship carries with it a plethora of dangers beginning unpaid lunch debts and culminating in eventual death, even including in some cases situational comedy [2].

Despite extensive efforts to curb this deadly activity, friendship rises across the US and across the world. In an acknowledgement that friendship cannot merely be proscribed by stuffy academics, this stuffy academic applies a formal verification methodology to rule out dangerous friendships while allowing benign friendships to survive.

2 A Taxonomy of Friendships

In order to develop a practical logic for the verification of friendships, one must first categorize the friendships that occur in everyday life. While she is ashamed to admit it, due to the much-maligned American "friendship culture", the author has found herself in a number of so-called friendships. So many, in fact, that one may even call her a "friendxpert". Such a friendxpert, in fact, that she can tell you inventing such words only increases your chances of being accidentally friended. Such are the risks of science.

The most basic ontology consists of Good Friends, Bad Friends, Facebook Friends, and $\gamma-$Friends.

2.1 Good Friends

The best-known and most prized friend is the Good Friend, and a primary goal of friendship research is the identification and preservation of Good Friends. Extensive anecdotal evidence shows the author is a Good Friend. Examples of Good Friend activities include:

- Providing camaraderie at social events in proportion to the quantity of alcohol provided.
- Not interrupting others with questions about homework when I can tell they're trying to focus on research.

- Looking the other way when undesired guests are invited into shared working spaces at 3am right in the middle of working hours.
- Ordering a pizza for the whole office and not complaining when others forget to pay their share
- Providing emotional support for common problems like lack of confidence in one's work, no matter how esoteric and detached from reality.
- Selflessly sharing whiteboard markers even with people who insist on rubbing their greasy monkey paws all over the whiteboard instead of using the eraser like the thumb-opposing human they are.

2.2 Bad Friends

Bad Friends, while nearly as common as Good Friends, are thoroughly despised and are to be avoided at all cost. Despite the numerous benefits of avoiding Bad Friends, prior works have unilaterally failed to do so. The author has much experience with Bad Friends, whose properties include:

- Monopolizing every writing surface within a 10-office radius.
- Using valuable working space as storage for oversized animatronic skulls.
- Treating the office minifridge like their personal minibar.
- Tyranically vetoing all reasonable efforts to acquire matching "Officemates for life" tattoos.

Such a list could fill several PhD theses, and in fact is the subject of theses by such luminaries as [9] and [5]. For brevity we give only the short list above, even though we could say much, much more.

2.3 Facebook Friends

The limited expressive power of strong friendship models led to the development of weak friendship models, starting with the foundational development of partial friendship spaces by [1] and made practical by the introduction of face-oriented social indexing in [12] for which this style of friend is named.

Facebook Friends have numerous useful properties and practical applications:

- Minimal runtime cost, with hundreds of Facebook Friends often costing less than an individual Good Friend
- Enabling work-efficient work-stealing work-avoiding algorithms
- Weak bisimulation with Good Friends
- Constant-time stalking of unrealistic γ-friends (see below)

$$\frac{\Delta \vdash \phi \ \textbf{valid}}{\Delta; \Gamma \vdash \Box\phi \ \textbf{true}} \qquad \frac{\Delta; \cdot \vdash \phi \ \textbf{true}}{\Delta \vdash \Box\phi \ \textbf{valid}} \qquad \frac{\Delta \vdash \phi \ \textbf{true} \quad \Gamma \vdash \psi \ \textbf{true}}{\Delta; \Gamma \vdash \phi \otimes \psi \ \textbf{true}}$$

$$\frac{\Delta \vdash \psi \otimes \psi' \ \textbf{true} \quad \Gamma, u : \psi, w : \psi' \vdash \psi \otimes \psi' \ \textbf{true}}{\Delta; \Gamma \vdash \phi \ \textbf{valid}}$$

$$x : \gamma\text{-Friend} \multimap \ \Diamond !x \quad \Box(\neg \textsc{Give-Up}\ I\ U)$$

$$(\textsc{In-Office}\ \textbf{Evan}) \multimap \neg(\textsc{Sound}\ \textbf{Evan})$$

Figure 1: Selected Rules and Axioms

Furthermore, due to their extensive knowledge of Facebook Walls, the Facebook Friends are being considered for important applications in national security [3].

2.4 γ-Friends

The most dangerous friend is the γ-friend. The γ-friend is a generalization of the classical concepts of boyfriend and girlfriend. The γ-friend is an incredibly general concept which can encode esoteric genders including: dragon, code, space, squid, dog, cat, demon, angel, mermaid, lycanthrope, celestial, vampire, pumpkin and even *ceiling fan* [6].

The sheer generality of this relationship makes it of great interests to academics, yet analysis of γ-Friends has long been considered the holy grail of Friendship Theory, being the subject of a plurality of publications in the BMG, EMI, Universal and Warner conferences, including a majority of Grammy winners. Despite such an exhaustive literature, little inroads have been made, as outlined in the retrospective “What is Love?” [4].

The author must concede she is no expert in this branch of Friendship Theory, though she has heard extensive rumors that such friendships exist.

3 Modal Linear Friendship Logical Framework

In the present work we focus on the analysis of Good Friends and Bad Friends. The examples given in the previous section have a common theme of reasoning about situations involving change, possibility and certainty, but are otherwise quite varied. For this reason our logic extends previous dual-intuitionistic linear logics with modal operators $\Box\phi$ and $\Diamond\phi$ which say a formula ϕ holds in all or in some future world, respectively. To ensure sufficient generality we take a logical framework approach, where new propositions and axioms can be introduced as needed.

4 Case Study: Office Relations

A common friendship scenario is that of the graduate school office mates. The many hours spent in the office each day make the office mate bond one of great importance, but due to the randomness inherent in the process, the risk of a Bad Friend is alarmingly high. In this section we use MLFLF to verify the safety of one student and the unsafety of their office mate.

In the first scenario we verify a common safety property: Paying debts that are owed. We assume the following axioms:

$$\mathit{A1}\text{: } \Box(\mathsf{Owes}\ P\ N \multimap \mathsf{Pays}\ P\ N) \multimap \mathsf{Safe}\ P$$

$$\mathit{A2}\text{: } \Diamond\mathsf{Spins} \multimap \Diamond P \multimap \Box P$$

$$\mathit{A3}\text{: } \Box\mathsf{Spins} \multimap \Box P \multimap \Diamond P$$

$$\mathit{A4} : \Diamond\mathsf{Paid\ Rose\ 10}$$

$$\mathit{A5}\text{: } \Box\mathsf{Owes}\ P\ N \multimap N = 10$$

$$\mathit{A6}\text{: } \Box\mathsf{Spins}$$

$$\mathit{A7}\text{: } \mathsf{Paid\ Rose\ 10} \multimap \mathsf{Pays\ Rose\ 10}$$

For our second scenario we prove a common unsafety property: Theft of valuables, under the axioms

$$\mathsf{A1} : (\mathsf{Friend\text{-}Of\ A\ B} \otimes \mathsf{Owns\ A\ X} \otimes \mathsf{Takes\ B\ X}) \multimap \mathsf{Unsafe\ B}$$

$$\mathsf{A2} : (\mathsf{Owns\ A\ X} \otimes \mathsf{On\ Y\ X}) \multimap \mathsf{Owns\ A\ Y}$$

$$\mathsf{A3} : \mathsf{Friend\text{-}Of\ Rose\ B} \multimap \mathsf{Takes\ B\ X}$$

$$\mathsf{A4} : \mathsf{On\ Paper\ Table}$$

$$\mathsf{A5} : \mathsf{Owns\ Rose\ Table}$$

$$\mathsf{A6} : \mathsf{Friend\text{-}Of\ Rose\ Evan}$$

$$\dfrac{\text{A1} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{A4}}{\Diamond\text{Paid Rose}} \qquad \dfrac{\dfrac{\text{A6}}{\Box\text{Spins}} \qquad \dfrac{\text{A2}}{\Box\text{Spins}, \Diamond\text{Paid Rose} \vdash \Box\text{Paid Rose}}}{\Diamond\text{Paid Rose} \vdash \Box\text{Paid Rose}}}{\Box\text{Paid Rose}}}{\text{Paid Rose}} \qquad \dfrac{\dfrac{\text{A5}}{\text{Paid Rose}, \text{Owes Rose N} \vdash \text{N} = 10} \qquad \dfrac{\dfrac{\text{A7}}{\text{Paid Rose} \vdash \text{Pays Rose } 10}}{\text{Paid Rose}, \text{Owes Rose } 10 \vdash \text{Pays Rose } 10}}{\text{Paid Rose}, \text{Owes Rose N} \vdash \text{Pays Rose N}}}{\text{Owes P N} \vdash \text{Pays P N}}}{\text{Owes P N} \multimap \text{Pays P N}}}{\Box(\text{Owes P N} \multimap \text{Pays P N})}}{\text{Safe Rose}}$$

Proof 1: A Safe Friend

$$\dfrac{\text{A1} \qquad \dfrac{\dfrac{\text{A6}}{\text{Friend-Of Rose Evan}} \qquad \dfrac{\text{A5} \quad \text{A4} \quad \text{A2}}{\text{Owns Rose Paper}} \qquad \dfrac{\text{A3} \quad \text{A6}}{\text{Takes Evan Paper}}}{\text{Friend-Of Rose Evan} \otimes \text{Owns Rose Paper} \otimes \text{Takes Evan Paper}}}{\text{Unsafe Evan}}$$

Proof 2: A Unsafe Friend

5 Formalization

We have developed an extensive formalization of the MLFLF framework in the linear logic proof assistant Cwelf, available upon request. We show the soundness of all inference rules and validity of all axioms presented in this paper, along with many more, as well as all of our examples. Further case studies for friendship logic formalized in Cwelf will be presented in follow-up paper.

Building on the successes of MLFLF, we show that Cwelf itself is sound from within Cwelf, following in the footsteps of [8].

6 Future Directions

While this work has developed an effective framework for the verification of Good and Bad friends, it can say little about Facebook Friends and even less about γ-Friends. We wish to address these limitations in future works. Because many properties of Facebook Friends seem to involve evaluation cost, we wish to develop a cost semantics for friendship. In order to tackle the complexity of γ-Friends, we wish to employ automation in the form of linear logical frameworks.

> Before you can love another, you must love your Celf.
> — Chris Martens

Furthermore, while it is extremely useful to verify whether friends are safe, this provides us little recourse in the all-to-common case where a friend is, in fact, not safe. Therefore we wish to build upon our experience in friend verification to perform *friendship synthesis*: the automatic generation of provably-safe friends from a formal specification.

7 Related Work

To our knowledge, the only other attempt at formal verification of friendship properties is the set-theoretic approach taken in "You've Got a Friend in Me" [7]. However, this approach has never been formalized and does not express state as easily as our linear-logical approach does.

The complexity of friendship has been studied more extensively than formal verification. In particular, it has been shown that friendship can be detected in $O(n^2)$ time with high probability in some important special cases [11]. Hardness results have been show for a number of key friendship problems: Most famously it was shown that optimizing the size of social networks is NP-hard by a reduction from the max-clique problem [10].

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