

Universidade de São Paulo - São Carlos

Consulta

Instituto de Ciências Matemáticas e de Computação

MFP Unicamp

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Contest (1)

template.cpp17 lines

```
#include <bits/stdc++.h>
using namespace std;

// No decorrer do notebook, podem aparecer alguns dos seguintes
// comandos, para diminuir a quantidade de caracteres e
// facilitar copiar o codigo (nao eh necessario copiar isso,
// apenas entender caso apareca algum assim la na frente)
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
    ios::sync_with_stdio(false); cin.tie(nullptr);
    int x;
    cin >> x;
    cout << x << "\n";
}
```

template.py8 lines

```
# ler string
x = input()
# ler int
x = int(input()) # le int
# ler array de string na mesma linha separado por espaco
l = input().split(' ')
# ler array de int na mesma linha
l = list(map(int, input().split()))
```

terminal.txt10 lines

```
# compilar com flags de warning e sanitizers
g++ -Wall -std=c++17 -fsanitize=address,undefined -g -gdb3 \
    sol.cpp -o sol;

# testar o input no arquivo 1.in
./sol < 1.in

# comparar a sua saida com a do arquivo 1.out
./sol < 1.in > my_out
diff -w 1.out my_out

troubleshoot.txt52 lines
```

6

Pre-submit:

Write a few simple test cases if sample is not enough.

Are time limits close? If so, generate max cases.

Is the memory usage fine?

Could anything overflow?

Make sure to submit the right file.

9

Wrong answer:

Print your solution! Print debug output, as well.

Are you clearing all data structures between test cases?

Can your algorithm handle the whole range of input?

Read the full problem statement again.

Do you handle all corner cases correctly?

Have you understood the problem correctly?

Any uninitialized variables?

Any overflows?

Confusing N and M, i and j, etc.?

Are you sure your algorithm works?

What special cases have you not thought of?

Are you sure the STL functions you use work as you think?

Add some assertions, maybe resubmit.

Create some testcases to run your algorithm on.

Go through the algorithm for a simple case.

Go through this list again.

Explain your algorithm to a teammate.

Ask the teammate to look at your code.

Go for a small walk, e.g. to the toilet.

Is your output format correct? (including whitespace)

Rewrite your solution from the start or let a teammate do it.

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Runtime error:

Have you tested all corner cases locally?

Any uninitialized variables?

Are you reading or writing outside the range of any vector?

Any assertions that might fail?

Any possible division by 0? (mod 0 for example)

Any possible infinite recursion?

Invalidated pointers or iterators?

Are you using too much memory?

Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:

Do you have any possible infinite loops?

What is the complexity of your algorithm?

Are you copying a lot of unnecessary data? (References)

How big is the input and output? (consider scanf)

Avoid vector, map. (use arrays/unordered_map)

What do your teammates think about your algorithm?

Memory limit exceeded:

What is the max amount of memory your algorithm should need?

Are you clearing all data structures between test cases?

STL (2)

Possui algoritmos e containers que podem ser utilizados.

2.1 Estruturas de dados

2.1.1 vector<T>

Um vetor dinâmico, pode ter seu tamanho alterado durante a execucao e tudo mais

vector<int> v; // criar um vetor q armazena int

v.push_back(1); // insere no final. O(1)

v.pop_back(); // remove o cara do final. O(1)

v.insert(v.begin(), 1); // insere no comeco. O(n) – CUIDADO COM TLE!!

v.size(); // retorna a qtd de elementos no vetor

vector<long long> x(5, 1); // cria um vetor de 5 elementos

valendo 1 cada

for (int i = 0; i < (int) x.size(); i++) cout << x[i]; //

imprime 11111

for (int i : x) cout << i; // imprime 11111. Esse for e tipo um

"for each"

2.1.2 string

Um vector de caracteres. Bem melhor do que manipular string em C

string x = "aba";

string y = "xxx";

string s = x + y; // concatena, s = abaxx

cin >> s; // le do input

cout << s; // imprime na tela

x[1] = 'c'; // x = "aca" agora

2.1.3 pair<T1, T2>

Um par de elementos quaisquer. Ja possui o comparador de "<" implementado, facilitando a comparacao (considera o primeiro e desempata pelo segundo)

pair<int, double> p = {1, 3.5};

cout << p.first << " " << p.second; // 1 3.5

p.first = 10;

cout << p.first << " " << p.second; // 10 3.5

// pode ter pair de qqr coisa basicamente

pair<vector<int>, string> p2;

pair<pair<pair<int, int>, int>, int> p3; // tem q pegar p3.

first.first.first

2.1.4 stack<T>

Uma estrutura de dados de pilha (FILO).

stack<int> st;

st.push(1);

st.push(2);

int old = st.top(); // old = 2

st.pop(); // tira da pilha

int cur = st.top(); // cur = 1

2.1.5 queue<T>

Uma estrutura de dados de fila (FIFO).

queue<int> qu;

qu.push(1);

qu.push(2);

```
int old = qu.front(); // old = 1
qu.pop(); // tira da pilha
int cur = qu.front(); // cur = 2
```

2.1.6 deque<T>

Double ended queue: uma fila dupla. Pode inserir e tirar do comeco ou do final.

```
deque<int> dq;
dq.push_back(3); // dq = {3}
dq.push_front(1); // dq = {1, 3}
dq.push_back(2); // dq = {1, 3, 2}
dq.back(); // retorna 2
dq.front(); // retorna 1
dp.pop_back(); // dq = {1, 3}
```

2.1.7 priority_queue<T>

Uma *max heap*. Faz operacoes de insercao e retornar o topo em O(logn).

```
priority_queue<int> pq;
pq.push(50); // pq = {50}
pq.push(10); // pq = {10, 50}
pq.top(); // retorna 50
pq.pop(); // pq = {10}
```

2.1.8 set<T>

Um conjunto matematico ordenado. Nao armazena valores repetidos. Utiliza uma arvore balanceada. Operacoes em O(logn).

```
set<int> st;
st.insert(10); // st = {10}
st.insert(1); // st = {1, 10}
st.insert(10); // st = {1, 10}
st.erase(10); // st = {1}
cout << *st.begin() << "\n"; // imprime 1
```

```
for (int x : st) cout << x << " "; // imprime todos os caras do set
```

2.1.9 multiset<T>

Um conjunto matematico ordenado. **Armazena** valores repetidos. Utiliza uma arvore balanceada. Operacoes em O(logn).

```
multiset<int> ms;
ms.insert(10); // ms = {10}
ms.insert(10); // ms = {10, 10}
ms.insert(1); // ms = {1, 10, 10}
ms.insert(1); // ms = {1, 1, 10, 10}
ms.erase(10); // remove TODOS os 10. ms = {1, 1}
ms.erase(ms.begin()); // remove so o cara do iterador que mandar. ms = {1}
cout << *ms.begin() << "\n"; // imprime 1
```

```
for (int x : ms) cout << x << " "; // imprime todos os caras do multiset
```

2.1.10 map<K, V>

Um dicionario "chave, valor". Pra cada chave, tem um valor associado. Caso vc acesse uma chave nao inicializada, e criado uma instância com valor (construtor vazio padrao).

```
map<string, int> mp;
```

template template terminal troubleshoot

```
mp["aba"] = 10;
mp["axxx"] = 11;
cout << mp["aba"]; // imprime 10
cout << mp["aaaaaaa"]; // imprime 0, inteiro padrao
mp.erase("aba"); // remove a chave "aba" e o valor q tava nela
```

```
for (auto p : mp) { // percorre todos os pair<key, value> do map
    cout << p.first << ": " p.second << " ";
} // nesse caso, imprimira "axxx: 11 aaaaaaa: 0 "
```

2.1.11 unordered

As estruturas unordered_set<T> e unordered_map<K, V> sao como os set e map, mas utilizam Hash para fazer as operacoes em O(1). Nao garantem ordem, e podem ser inclusive mais lentos que o set e map (podem ser criados casos de teste que "quebram" o hash, fazendo com que tenha muito conflito).

2.2 Iteradores

Um iterador e "como um ponteiro". Sao meio complicados de entender de primeira, mas sao muito utilizados pelos algoritmos da STL. Exemplos:

```
vector<int> v = {1, 2, 3};
vector<int>::iterator it; // tipo do iterador
// se fizer *it, pega o valor do cara daquele endereco
```

```
// exemplo
vector<int>::iterator ini = v.begin();
// E um iterador que aponta pro primeiro cara. No caso, *ini = 1
```

```
auto ini = v.begin(); // tambem podemos utilizar a tipagem automatica "auto"
// pra nao precisar ficar escrevendo muita coisa
```

```
v.end(); // -> um elemento depois do ultimo cara. *v.end() daria erro
v.rbegin(); // iterador reverso (comeca do final pro comeco).
// no caso, v.rbegin() e o ultimo cara (*v.rbegin() = 3)
v.rend(); // iter. reverso, um cara antes do primeiro cara (*v.rend() da erro)
```

2.3 Algoritmos

2.3.1 sort(begin, end, <comparador>)

```
vector<int> v = {4, 2, 1};
sort(v.begin(), v.end()); // ordena em forma crescente
sort(v.begin(), v.end(), [](int a, int b) {
    return a > b;
}); // utiliza uma funcao lambda para ordenar de forma decrescente
```

```
// vc tambem pode fazer com uma funcao sem ser lambda
bool cmp(int a, int b) { return a > b; }
...
sort(v.begin(), v.end(), cmp);
```

- **lower_bound:** retorna um iterador para o primeiro elemento maior ou igual ao valor enviado.
- **upper_bound:** retorna um iterador para o primeiro elemento maior que o valor enviado.

```
vector<int> v = {1, 2, 3, 3, 6}; // o vetor deve estar ordenado
```

```
auto it1 = lower_bound(v.begin(), v.end(), 1); // it pro indice 0
auto it2 = upper_bound(v.begin(), v.end(), 1); // it pro indice 1
auto it3 = lower_bound(v.begin(), v.end(), 3); // it pro indice 2
auto it4 = upper_bound(v.begin(), v.end(), 3); // it pro indice 4
auto it5 = lower_bound(v.begin(), v.end(), 6); // it pro indice 4
auto it6 = upper_bound(v.begin(), v.end(), 6); // it do v.end()
auto it7 = lower_bound(v.begin(), v.end(), 7); // it do v.end()
auto it8 = lower_bound(v.begin(), v.end(), -1); // it pro indice 0
```

2.3.3 next_permutation(begin, end)

Gera permutacoes, muito util para problemas em que precisa testar algo para todas as permutacoes em O(n!)

```
vector<int> v = {3, 4, 1};
sort(v.begin(), v.end()); // ordena para ficar na "permutacao inicial"
do {
    for (int i : v) cout << i << " ";
    cout << "\n";
} while(next_permutation(v.begin(), v.end())); // imprime todas as 6 permutacoes do vetor v
```

2.3.4 min(val1, val2), max(val1, val2)

Retorna o valor minimo para os valores val1 e val2. Eles devem ser do mesmo tipo, e este tipo deve ter o operador "i" definido.

```
int a = 4, b = 10;
long long x = 1321, y = 923;
```

```
min(a, b); // 4
max(a, b); // 10
min(x, y); // 932
min(a, x); // ERRO, nao sao do mesmo tipo
min( (long long) a, x); // 4, cast antes para long long
min(1LL * a, x); // 4, multiplicacao de long long com INT gera long long
```

Mathematics (3)

3.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e & x &= \frac{ed - bf}{ad - bc} \\ cx + dy &= f & y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i 'th column replaced by b .

3.2 Recurrences

If $a_n = c_1a_{n-1} + \dots + c_ka_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k + c_1x^{k-1} + \dots + c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1n + d_2)r^n$.

3.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\begin{aligned} \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2} \end{aligned}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

3.4 Geometry

3.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b + c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

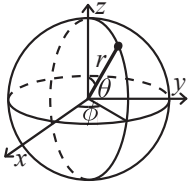
3.4.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

3.4.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

3.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

3.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n + 1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

3.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

3.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

3.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \dots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu = np, \sigma^2 = np(1 - p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $Fs(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1 - p)^{k-1}, \, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $Po(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \, \sigma^2 = \lambda$$

3.8.2 Continuous distributions
Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $U(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a + b}{2}, \, \sigma^2 = \frac{(b - a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $Exp(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

3.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let X_1, X_2, \dots be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi \mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i . π_j / π_i is the expected number of visits in state j between two visits in state i .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets \mathbf{A} and \mathbf{G} , such that all states in \mathbf{A} are absorbing ($p_{ii} = 1$), and all states in \mathbf{G} leads to an absorbing state in \mathbf{A} . The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j , is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i , is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

Algorithms (4)

DivideAndConquer.h	
Description: Divide and Conquer pseudocode	
Time: $\mathcal{O}(n \log n)$	f26c5b, 16 lines
<pre>// Estrutura basica (pseudocodigo) int solve(vector<int> a) { int n = a.size(); if (n == 1) return solucao(a); // divide vector<int> vl = a[0..n/2]; vector<int> vr = a[n/2..n]; // pega a resposta pra cada metade solucao_left = solve(vl); solucao_right = solve(vr); // "conquista" return merge(solucao_left, solucao_right); }</pre>	
BinarySearch.h	
Description: Binary search on the answer	
Time: $\mathcal{O}(\log(hi - lo))$	d65547, 44 lines

bool check(int k) { // int ou double // check se pode realizar a operacao com K }	
<pre>// dada uma funcao que retorna 0 0 0 0 0 1 1 1 1.. // retorna a posicao que ocorre o primeiro 1 int binary_search1(int n) { // trocar o LO e HI por valores limite int lo = 0, hi = n - 1, mi; while(lo < hi) { mi = (lo + hi) / 2; if (check(mi)) hi = mi; else lo = mi + 1; } return lo; }</pre>	
<pre>// dada uma funcao que retorna 1 1 1 1 1 0 0 0 0... // retorna a posicao que ocorre o ultimo 1 int binary_search2(int n) { // trocar o LO e HI por valores limite int lo = 0, hi = n - 1, mi; while(lo < hi) { mi = (lo + hi + 1) / 2; if (check(mi)) lo = mi; else hi = mi - 1; } return lo; }</pre>	
<pre>// codigo para double double binary_search() { const double EPS = 1e-9; // limiar de erro aceitavel double lo = 0, hi = 1e9, mi; while(hi - lo > EPS) { mi = (lo + hi) / 2; // nao esqueca de trocar tipo do check para double if (check(mi)) lo = mi; else hi = mi; } return lo; }</pre>	
TernarySearch.h	
Description: Ternary search on a function. Get the minimum or maximum of a parabola, for example.	
Time: $\mathcal{O}(\log(hi - lo))$	9bda2b, 40 lines
<pre>const double EPS = 1e-9; // function f() to be defined. Can be int as well double f(double x) { ... } double maximum(double lo, double hi) { while (hi - lo > EPS) { double m1 = lo + (hi - lo) / 3.0; double m2 = lo + 2.0 * (hi - lo) / 3.0; if (f(m1) < f(m2)) lo = m1; else hi = m2; } return (lo + hi) / 2; } double minimum(double lo, double hi) { while (hi - lo > EPS) { double m1 = lo + (hi - lo) / 3.0; double m2 = lo + 2.0 * (hi - lo) / 3.0; if (f(m1) < f(m2)) hi = m2; else lo = m1; } }</pre>	

```
        return (lo + hi) / 2.0;
    }
    // F(x) has to be strictly incr. or decr. where x is not max
    int maximum(int lo, int hi) {
        while (lo < hi) {
            int mi = (lo + hi) / 2;
            if (f(mi) <= f(mi + 1)) lo = mi + 1;
            else hi = mi;
        }
        return lo;
    }
    // F(x) has to be strictly incr. or decr. where x is not min
    int minimum(int lo, int hi) {
        while (lo < hi) {
            int mi = (lo + hi) / 2;
            if (f(mi) <= f(mi + 1)) hi = mi;
            else lo = mi + 1;
        }
        return lo;
    }
}
```

Data structures (5)

UnionFind.h
Description: Disjoint-set data structure.
Time: $\mathcal{O}(\alpha(N))$

```
struct UF {
    vi e;
    UF(int n) : e(n, -1) {}
    bool sameSet(int a, int b) { return find(a) == find(b); }
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x]); }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        e[a] += e[b]; e[b] = a;
        return true;
    }
};
```

FenwickTree.h
Description: Computes partial sums $a[0] + a[1] + \dots + a[\text{pos} - 1]$, and updates single elements $a[i]$, taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

```
struct FT {
    vector<ll> s;
    FT(int n) : s(n) {}
    void update(int pos, ll dif) { // a[pos] += dif
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
    }
    ll query(int pos) { // sum of values in [0, pos)
        ll res = 0;
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];
        return res;
    }
    int lower_bound(ll sum) { // min pos st sum of [0, pos] >= sum
        // Returns n if no sum is >= sum, or -1 if empty sum is.
        if (sum <= 0) return -1;
        int pos = 0;
        for (int pw = 1 << 25; pw; pw >= 1) {
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
                pos += pw, sum -= s[pos-1];
        }
        return pos;
    }
};
```

```
    }
};
```

Numerical (6)

Determinant.h
Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}
```

SolveLinear.h
Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2m)$

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i,0,n) {
        double v, bv = 0;
        rep(r,i,n) rep(c,i,m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j,i,n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j,0,n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j,i+1,n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k,i+1,m) A[j][k] -= fac*A[i][k];
        }
        rank++;
    }

    x.assign(m, 0);
    for (int i = rank; i--;) {
        b[i] /= A[i][i];
        x[col[i]] = b[i];
        rep(j,0,i) b[j] -= A[j][i] * b[i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h
Description: Invert matrix A . Returns rank; result is stored in A unless singular ($\text{rank} < n$). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of $A \pmod p$, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) {
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
        }
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    }

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    }

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

FastFourierTransform.h
Description: $\text{fft}(a)$ computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k . N must be a power of 2. Useful for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n , reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16} ; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double)
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    }
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
        }
    }
```

```
        a[i + j + k] = a[i + j] - z;
        a[i + j] += z;
    }
}

vd conv(const vd& a, const vd& b) {
    if (a.empty() || b.empty()) return {};
    vd res(sz(a) + sz(b) - 1);
    int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
    vector<C> in(n), out(n);
    copy(all(a), begin(in));
    rep(i,0,sz(b)) in[i].imag(b[i]);
    fft(in);
    for (C& x : in) x *= x;
    rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
    fft(out);
    rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
    return res;
}
```

Number theory (7)

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set MOD to some number first and then you can use the structure.

```
2b6487, 22 lines

const int MOD = 1e9 + 7;
struct mint {
    int v;
    mint() : v{} {}
    mint(int v) : v(v) { go(); }
    inline void go() { if (abs(v) >= MOD) v %= MOD; if (v < 0) v
        += MOD; }
    inline static mint fpow(mint x, long long y) { // x ^ y
        mint r = 1;
        for (; y > 0; y /= 2, x *= x) if (y%2 != 0) r *= x;
        return r;
    }
    mint inv() const { return fpow(*this, MOD - 2); }
    mint operator+(mint const& m) const { return v + m.v >= MOD ?
        v + m.v - MOD : v + m.v; }
    mint operator-(mint const& m) const { return v - m.v < 0 ? v
        - m.v + MOD : v - m.v; }
    mint operator*(mint const& m) const { return (long long) v *
        m.v % MOD; }
    mint operator/(mint const& m) const { return (*this) * m.inv
        (); }
    mint operator^(long long m) const { return fpow(*this, m); }
    mint& operator+=(mint const& m) { return (*this) = (*this) +
        m; }
    mint& operator-=(mint const& m) { return (*this) = (*this) -
        m; }
    mint& operator*=(mint const& m) { return (*this) = (*this) *
        m; }
    mint& operator/=(mint const& m) { return (*this) = (*this) /
        m; }
};
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \bmod c$.

```
"ModMuLL.h"
60dcd1, 12 lines

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
```

```
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

```
"ModMuLL.h", "MillerRabin.h"
a33cf6, 18 lines

ull pollard(ull n) {
    auto f = [n](ull x) { return modmul(x, x, n) + 1; };
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}

vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
}
```

7.1 Divisibility

euclid.h

Description: Finds two integers x and y , such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

```
33ba8f, 5 lines

ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

7.2 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

7.3 Estimates

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (8)

8.1 Permutations

8.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL.MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$

```
044568, 6 lines

int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

8.1.2 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n + 1$ leaves (0 or 2 children).
- ordered trees with $n + 1$ vertices.
- ways a convex polygon with $n + 2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (9)

9.1 Fundamentals

9.1.1 Como representar

Em C++, podemos representar uma lista de adjacencias como um vector de vectors, ou um array de vectors:

```
int n, m; cin >> n >> m; // n=vertices, m=arestas
vector<vector<int>>> edges(n); // edges[i] = {adjacentes a i}

for (int i = 0; i < m; i++) {
    int u, v; cin >> u >> v;
    edges[u].push_back(v); // u->v
    edges[v].push_back(u); // v->u (para grafos nao direcionados)
}

// tb poderia fazer um dos arrays estaticos:
```

```
const int MAXN = 1e5 + 5; // falando que o maximo de N eh 10^5
vector<int> edges[MAXN];
```

DFS.h

Description: Busca em profundidade, com implementacao recursiva.
Time: $\mathcal{O}(N + M)$

e4de52, 11 lines

```
const int MAXN = 1e5 + 5;

vector<int> edges[MAXN];
bool vis[MAXN];

void dfs(int u) {
    vis[u] = true;
    for (int v : edges[u]) if (!vis[v]) {
        dfs(v);
    }
}
```

BFS.h

Description: Busca em largura, com implementacao com fila.
Time: $\mathcal{O}(N + M)$

e4992f, 18 lines

```
const int MAXN = 1e5 + 5;

vector<int> edges[MAXN];
int dist[MAXN];

void bfs(int s) { // vertice para iniciar a busca
    dist[s] = 1;
    queue<int> qu;
    qu.emplace(s);
    while(!qu.empty()) {
        int u = qu.front();
        qu.pop();
        for (int v : edges[u]) if (!dist[u]) {
            dist[v] = dist[u] + 1;
            qu.emplace(v);
        }
    }
}
```

Dijkstra.h

Description: Caminho mínimo a partir de um vértice para todos os outros em um grafo ponderado.
Time: $\mathcal{O}((N + M) \log n)$

12544c, 22 lines

```
const int MAXN = 1e5 + 5;

vector<pair<int, int>> edges[MAXN]; // edges[u] = {pair(v, w)}
ll dist[MAXN];

void dijkstra(int s) { // vertice para iniciar a busca
    memset(dist, 0x3f, sizeof dist); // inicializa todo mundo com INF

    dist[s] = 0;
    priority_queue<pair<ll, int>> pq;
    pq.emplace(0, 0);
    while(!pq.empty()) {
        auto [d, u] = pq.top();
        pq.pop();
        if (-d > dist[u]) continue;

        for (auto [v, w] : edges[u]) if (dist[v] > dist[u] + w) {
            dist[v] = dist[u] + w;
            pq.emplace(-dist[v], v);
        }
    }
}
```

```
}

FloydWarshall.h
Description: Calcula a distsancia minima entre todos os pares de vertices em um grafo direcionado e ponderado (suporta tambem arestas negativas). A matriz de input m possui o valor das arestas i, j em m[i][j], e m[i][j] = inf se i e j nao sao adjacentes. Como output, m[i][j] possui a distancia minima entre i e j, e inf se nao ha caminho.
Time:  $\mathcal{O}(N^3)$ 

d7f3b5, 9 lines

ll m[MAXN][MAXN];

void floydWarshal() {
    for (int k = 0; k < n; k++)
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                if (m[i][j] > m[i][k] + m[k][j])
                    m[i][j] = m[i][k] + m[k][j];
}
```

TopoSort.h

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V| + |E|)$

66a137, 14 lines

```
vi topoSort(const vector<vi>& gr) {
    vi indeg(sz(gr)), ret;
    for (auto& li : gr) for (int x : li) indeg[x]++;
    queue<int> q; // use priority_queue for lexic. largest ans.
    rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
    while (!q.empty()) {
        int i = q.front(); // top() for priority queue
        ret.push_back(i);
        q.pop();
        for (int x : gr[i])
            if (--indeg[x] == 0) q.push(x);
    }
    return ret;
}
```

9.2 Network flow

MinCostMaxFlow.h

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}(E^2)$

fe85cc, 81 lines

```
#include <bits/stdc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
    int N;
    vector<vi> ed, red;
    vector<VL> cap, flow, cost;
    vi seen;
    VL dist, pi;
    vector<pii> par;

    MCMF(int N) :
        N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap),
        seen(N), dist(N), pi(N), par(N) {}

    void addEdge(int from, int to, ll cap, ll cost) {
        this->cap[from][to] = cap;
```

```
        this->cost[from][to] = cost;
        ed[from].push_back(to);
        red[to].push_back(from);
    }

    void path(int s) {
        fill(all(seen), 0);
        fill(all(dist), INF);
        dist[s] = 0; ll di;

        __gnu_pbds::priority_queue<pair<ll, int>> q;
        vector<decltype(q)::point_iterator> its(N);
        q.push({0, s});

        auto relax = [&](int i, ll cap, ll cost, int dir) {
            ll val = di - pi[i] + cost;
            if (cap && val < dist[i]) {
                dist[i] = val;
                par[i] = {s, dir};
                if (its[i] == q.end()) its[i] = q.push({-dist[i], i});
                else q.modify(its[i], {-dist[i], i});
            }
        };

        while (!q.empty()) {
            s = q.top().second; q.pop();
            seen[s] = 1; di = dist[s] + pi[s];
            for (int i : ed[s]) if (!seen[i])
                relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
            for (int i : red[s]) if (!seen[i])
                relax(i, flow[i][s], -cost[i][s], 0);
        }
        rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
    }

    pair<ll, ll> maxflow(int s, int t) {
        ll totflow = 0, totcost = 0;
        while (path(s), seen[t]) {
            ll fl = INF;
            for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
                fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p]);
            totflow += fl;
            for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
                if (r) flow[p][x] += fl;
                else flow[x][p] -= fl;
        }
        rep(i,0,N) rep(j,0,N) totcost += cost[i][j] * flow[i][j];
        return {totflow, totcost};
    }
}
```

```
// If some costs can be negative, call this before maxflow:
void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
        rep(i,0,N) if (pi[i] != INF)
            for (int to : ed[i]) if (cap[i][to])
                if ((v = pi[i] + cost[i][to]) < pi[to])
                    pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
}

};
```

Dinic.h

Description: Flow algorithm with complexity $\mathcal{O}(VE \log U)$ where $U = \max |cap|$. $\mathcal{O}(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $\mathcal{O}(\sqrt{V}E)$ for bipartite matching.

d7f0f1, 42 lines

```
struct Dinic {
```



```
struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need flows
};
vi lvl, ptr, q;
vector<vector<Edge>> adj;
Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].push_back({b, sz(adj[b]), c, c});
    adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap});
}
ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {
        Edge& e = adj[v][i];
        if (lvl[e.to] == lvl[v] + 1)
            if (ll p = dfs(e.to, t, min(f, e.c))) {
                e.c -= p, adj[e.to][e.rev].c += p;
                return p;
            }
    }
    return 0;
}
ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    rep(L, 0, 31) do { // 'int L=30' maybe faster for random data
        lvl = ptr = vi(sz(q));
        int qi = 0, qe = lvl[s] = 1;
        while (qi < qe && !lvl[t]) {
            int v = q[qi++];
            for (Edge e : adj[v])
                if (!lvl[e.to] && e.c >> (30 - L))
                    q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
        }
        while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
}
bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

MinCut.h
Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s , only traversing edges with positive residual capacity.

9.3 DFS algorithms

SCC.h
Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.
Time: $\mathcal{O}(E + V)$

76b5c9, 24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
    int low = val[j] = ++Time, x; z.push_back(j);
    for (auto e : g[j]) if (comp[e] < 0)
        low = min(low, val[e] ?: dfs(e, g, f));

    if (low == val[j]) {
        do {
            x = z.back(); z.pop_back();
```

```
        comp[x] = ncomps;
        cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
}
return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
    int n = sz(g);
    val.assign(n, 0); comp.assign(n, -1);
    Time = ncomps = 0;
    rep(i, 0, n) if (comp[i] < 0) dfs(i, g, f);
}
```

2sat.h
Description: Calculates a valid assignment to boolean variables a, b, c, \dots to a 2-SAT problem, so that an expression of the type $(a|||b)&\&(a|||c)&\&(d|||!b)&\&\dots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ($\sim x$).
Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3);` // Var 0 is true or var 3 is false
`ts.setValue(2);` // Var 2 is true
`ts.atMostOne({0, ~1, 2});` // ≤ 1 of vars 0, ~1 and 2 are true
`ts.solve();` // Returns true iff it is solvable
`ts.values[0..N-1]` holds the assigned values to the vars
Time: $\mathcal{O}(N + E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 56 lines

```
struct TwoSat {
    int N;
    vector<vi> gr;
    vi values; // 0 = false, 1 = true

    TwoSat(int n = 0) : N(n), gr(2*n) {}

    int addVar() { // (optional)
        gr.emplace_back();
        gr.emplace_back();
        return N++;
    }

    void either(int f, int j) {
        f = max(2*f, -1-2*f);
        j = max(2*j, -1-2*j);
        gr[f].push_back(j^1);
        gr[j].push_back(f^1);
    }

    void setValue(int x) { either(x, x); }

    void atMostOne(const vi& li) { // (optional)
        if (sz(li) <= 1) return;
        int cur = ~li[0];
        rep(i, 2, sz(li)) {
            int next = addVar();
            either(cur, ~li[i]);
            either(cur, next);
            either(~li[i], next);
            cur = ~next;
        }
        either(cur, ~li[1]);
    }
}
```

```
vi val, comp, z; int time = 0;
int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
        low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
```

```
        x = z.back(); z.pop_back();
        comp[x] = low;
        if (values[x>>1] == -1)
            values[x>>1] = x&1;
        while (x != i);
        return val[i] = low;
    }

    bool solve() {
        values.assign(N, -1);
        val.assign(2*N, 0); comp = val;
        rep(i, 0, 2*N) if (!comp[i]) dfs(i);
        rep(i, 0, N) if (comp[2*i] == comp[2*i+1]) return 0;
        return 1;
    }
};
```

EulerWalk.h
Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with `src` at both start and end, or empty list if no cycle/path exists. To get edge indices back, add `.second` to `s` and `ret`.
Time: $\mathcal{O}(V + E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
    int n = sz(gr);
    vi D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++; // to allow Euler paths, not just cycles
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        if (it == end){ ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e]) {
            D[x]--, D[y]++;
            eu[e] = 1; s.push_back(y);
        }
    }
    for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
    return {ret.rbegin(), ret.rend()};
}
```

BinaryLifting.h
Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P){
    int on = 1, d = 1;
    while(on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]];
    return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
    rep(i, 0, sz(tbl))
        if (steps&(1<<i)) nod = tbl[i][nod];
    return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
    a = jmp(tbl, a, depth[a] - depth[b]);
    if (a == b) return a;
    for (int i = sz(tbl); i--;) {
        int c = tbl[i][a], d = tbl[i][b];
        if (c != d) a = c, b = d;
    }
}
```

```
    return tbl[0][a];
}
```

Geometry (10)

10.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

<pre>template <class T> int sgn(T x) { return (x > 0) - (x < 0); } template<class T> struct Point { typedef Point P; T x, y; explicit Point(T x=0, T y=0) : x(x), y(y) {} bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); } bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); } P operator+(P p) const { return P(x+p.x, y+p.y); } P operator-(P p) const { return P(x-p.x, y-p.y); } P operator*(T d) const { return P(x*d, y*d); } P operator/(T d) const { return P(x/d, y/d); } T dot(P p) const { return x*p.x + y*p.y; } T cross(P p) const { return x*p.y - y*p.x; } T cross(P a, P b) const { return (a-*this).cross(b-*this); } T dist2() const { return x*x + y*y; } double dist() const { return sqrt((double)dist2()); } // angle to x-axis in interval [-pi, pi] double angle() const { return atan2(y, x); } P unit() const { return *this/dist(); } // makes dist()==1 P perp() const { return P(-y, x); } // rotates +90 degrees P normal() const { return perp().unit(); } // returns point rotated 'a' radians ccw around the origin P rotate(double a) const { return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } friend ostream& operator<<(ostream& os, P p) { return os << "(" << p.x << ", " << p.y << ")"; } };</pre>	47ec0a, 28 lines
---	------------------

lineDistance.h

Description: Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

<pre>"Point.h" template<class P> double lineDist(const P& a, const P& b, const P& p) { return (double) (b-a).cross(p-a) / (b-a).dist(); }</pre>	f6bf6b, 4 lines
---	-----------------

SegmentDistance.h

Description: Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

<pre>"Point.h" typedef Point<double> P; double segDist(P& s, P& e, P& p) { if (s==e) return (p-s).dist(); auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s))); return ((p-s)*d-(e-s)*t).dist()/d;</pre>	5c88f4, 6 lines
--	-----------------

```
}
```

SegmentIntersection.h

Description: If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;

<pre>"Point.h", "OnSegment.h" template<class P> vector<P> segInter(P a, P b, P c, P d) { auto oa = c.cross(d, a), ob = c.cross(d, b), oc = a.cross(b, c), od = a.cross(b, d); // Checks if intersection is single non-endpoint point. if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0) return {(a * ob - b * oa) / (ob - oa)}; set<P> s; if (onSegment(c, d, a)) s.insert(a); if (onSegment(c, d, b)) s.insert(b); if (onSegment(a, b, c)) s.insert(c); if (onSegment(a, b, d)) s.insert(d); return {all(s)}; }</pre>	9d57f2, 13 lines
--	------------------

lineIntersection.h

Description: If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;

<pre>"Point.h" template<class P> pair<int, P> lineInter(P s1, P e1, P s2, P e2) { auto d = (e1 - s1).cross(e2 - s2); if (d == 0) // if parallel return {-(s1.cross(e1, s2) == 0), P(0, 0)}; auto p = s2.cross(e1, e2), q = s2.cross(e2, s1); return {1, (s1 * p + e1 * q) / d}; }</pre>	a01f81, 8 lines
---	-----------------

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: bool left = sideOf(p1,p2,q)==1;

<pre>"Point.h" template<class P> int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }</pre>	3af81c, 9 lines
---	-----------------

<pre>template<class P> int sideOf(const P& s, const P& e, const P& p, double eps) { auto a = (e-s).cross(p-s); double l = (e-s).dist()*eps;</pre>	
---	--

```
    return (a > 1) - (a < -1);
}
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p) <= epsilon) instead when using Point<double>.

<pre>"Point.h" template<class P> bool onSegment(P s, P e, P p) { return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0; }</pre>	c597e8, 3 lines
---	-----------------

```
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description: Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

<pre>"Point.h" typedef Point<double> P; P linearTransformation(const P& p0, const P& p1, const P& q0, const P& q1, const P& r) { P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq)); return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2(); }</pre>	03a306, 6 lines
---	-----------------

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

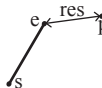
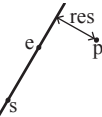
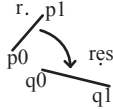
Usage: vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

<pre>struct Angle { int x, y; int t; Angle(int x, int y, int t=0) : x(x), y(y), t(t) {} Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; } int half() const { assert(x y); return y < 0 (y == 0 && x < 0); } Angle t90() const { return {-y, x, t + (half() && x >= 0)}; } Angle t180() const { return {-x, -y, t + half()}; } Angle t360() const { return {x, y, t + 1}; } };</pre>	0f0602, 35 lines
---	------------------

```
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()}; }
    Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
        make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
```

```
bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
        make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
```

```
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180() ?
        make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
    Angle r(a.x + b.x, a.y + b.y, a.t);
    if (a.t180() < r) r.t--;
    return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
    int tu = b.t - a.t; a.t = b.t;
    return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```



10.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	84d6d3, 11 lines
<pre>typedef Point<double> P; bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) { if (a == b) { assert(r1 != r2); return false; } P vec = b - a; double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2, p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2; if (sum*sum < d2 dif*dif > d2) return false; P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2); *out = {mid + per, mid - per}; return true; }</pre>	

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	b0153d, 13 lines
<pre>template<class P> vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) { P d = c2 - c1; double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr; if (d2 == 0 h2 < 0) return {}; vector<pair<P, P>> out; for (double sign : {-1, 1}) { P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2; out.push_back({c1 + v * r1, c2 + v * r2}); } if (h2 == 0) out.pop_back(); return out; }</pre>	

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../content/geometry/Point.h"	a1ee63, 19 lines
<pre>typedef Point<double> P; #define arg(p, q) atan2(p.cross(q), p.dot(q)) double circlePoly(P c, double r, vector<P> ps) { auto tri = [&](P p, P q) { auto r2 = r * r / 2; P d = q - p; auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2(); auto det = a * a - b; if (det <= 0) return arg(p, q) * r2; auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det)); if (t < 0 1 <= s) return arg(p, q) * r2; P u = p + d * s, v = p + d * t; return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2; }; auto sum = 0.0; rep(i,0,sz(ps)) sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c); return sum; }</pre>	

circumcircle.h

Description:

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h"	1caa3a, 9 lines
<pre>typedef Point<double> P; double ccRadius(const P& A, const P& B, const P& C) { return (B-A).dist()*(C-B).dist()*(A-C).dist()/ abs((B-A).cross(C-A))/2; } P ccCenter(const P& A, const P& B, const P& C) { P b = C-A, c = B-A; return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2; }</pre>	

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h"	09dd0a, 17 lines
<pre>pair<P, double> mec(vector<P> ps) { shuffle(all(ps), mt19937(time(0))); P o = ps[0]; double r = 0, EPS = 1 + 1e-8; rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) { o = ps[i], r = 0; rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) { o = (ps[i] + ps[j]) / 2; r = (o - ps[i]).dist(); rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) { o = ccCenter(ps[i], ps[j], ps[k]); r = (o - ps[i]).dist(); } } } return {o, r}; }</pre>	

10.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);

"Point.h", "OnSegment.h", "SegmentDistance.h"	2bf504, 11 lines
<pre>template<class P> bool inPolygon(vector<P> &p, P a, bool strict = true) { int cnt = 0, n = sz(p); rep(i,0,n) { P q = p[(i + 1) % n]; if (onSegment(p[i], q, a)) return !strict; //or: if (segDist(p[i], q, a) <= eps) return !strict; cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0; } return cnt; }</pre>	

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"	f12300, 6 lines
<pre>template<class T> T polygonArea2(vector<Point<T>>& v) {</pre>	

T a = v.back().cross(v[0]); rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]); return a; }
--

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h"	9706dc, 9 lines
<pre>typedef Point<double> P; P polygonCenter(const vector<P>& v) { P res(0, 0); double A = 0; for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) { res = res + (v[i] + v[j]) * v[j].cross(v[i]); A += v[j].cross(v[i]); } return res / A / 3; }</pre>	

PolygonCut.h

Description: Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h"	f2b7d4, 13 lines
<pre>typedef Point<double> P; vector<P> polygonCut(const vector<P>& poly, P s, P e) { vector<P> res; rep(i,0,sz(poly)) { P cur = poly[i], prev = i ? poly[i-1] : poly.back(); bool side = s.cross(e, cur) < 0; if (side != (s.cross(e, prev) < 0)) res.push_back(lineInter(s, e, cur, prev).second); if (side) res.push_back(cur); } return res; }</pre>	

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

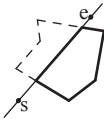
Time: $\mathcal{O}(n \log n)$

"Point.h"	310954, 13 lines
<pre>typedef Point<ll> P; vector<P> convexHull(vector<P> pts) { if (sz(pts) <= 1) return pts; sort(all(pts)); vector<P> h(sz(pts)+1); int s = 0, t = 0; for (int it = 2; it--; s = --t, reverse(all(pts))) for (P p : pts) { while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--; h[t++] = p; } return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])}; }</pre>	

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

"Point.h"	c571b8, 12 lines
<pre>typedef Point<ll> P;</pre>	



```
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (; j = (j + 1) % n) {
            res = max(res, {{S[i] - S[j]}.dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"	71446b, 14 lines
--------------------------------------	------------------

typedef Point<ll> P;

```
bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet(-1, -1)$ if no collision, $\bullet(i, -1)$ if touching the corner i , $\bullet(i, i)$ if along side $(i, i + 1)$, $\bullet(i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h"	7cf45b, 39 lines
-----------	------------------

```
#define cmp(i,j) sgn(dir.perp().cross(poly[i]%n-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}
```

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
```

```
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    }
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]};
        }
    return res;
}
```

10.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

"Point.h"	ac41a6, 17 lines
-----------	------------------

typedef Point<ll> P;

```
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(lo - p).dist2(), {lo, p}});
        S.insert(p);
    }
    return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h"	bac5b0, 63 lines
-----------	------------------

typedef long long T;

typedef Point<T> P;

const T INF = numeric_limits<T>::max();

```
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
```

```
struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if width >= height (not ideal...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
```

```
            // divide by taking half the array for each child (not
            // best performance with many duplicates in the middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()});
        }
    }
};

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }

        Node *f = node->first, *s = node->second;
        T bfirst = f->distance(p), bsec = s->distance(p);
        if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

        // search closest side first, other side if needed
        auto best = search(f, p);
        if (bsec < best.first)
            best = min(best, search(s, p));
        return best;
    }

    // find nearest point to a point, and its squared distance
    // (requires an arbitrary operator< for Point)
    pair<T, P> nearest(const P& p) {
        return search(root, p);
    }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise.

Time: $\mathcal{O}(n \log n)$

"Point.h"	eefdf5, 88 lines
-----------	------------------

typedef Point<ll> P;

typedef struct Quad* Q;

typedef __int128_t ll1; // (can be ll if coords are < 2e4)

P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

```
struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    ll1 p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
```

```
    return r;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}
```

10.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y), z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
```

```
vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
vector<F> FS;
auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
        q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
};
rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
        F f = FS[j];
        if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
            E(a,b).rem(f.c);
            E(a,c).rem(f.b);
            E(b,c).rem(f.a);
            swap(FS[j-], FS.back());
            FS.pop_back();
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        }
        for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
            A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
        return FS;
    };
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius *radius* between the points with azimuthal angles (longitude) *f1* (ϕ_1) and *f2* (ϕ_2) from *x* axis and zenith angles (latitude) *t1* (θ_1) and *t2* (θ_2) from *z* axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. *dx***radius* is then the difference between the two points in the *x* direction and *d***radius* is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

Strings (11)

KMP.h

Description: *pi*[*x*] computes the length of the longest prefix of *s* that ends at *x*, other than *s*[0...*x*] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

d4375c, 16 lines

```
vi pi(const string& s) {
    vi p(sz(s));
    rep(i,1,sz(s)) {
        int g = p[i-1];
```

```
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
}
return p;
}

vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
    rep(i,sz(p)-sz(s),sz(p))
        if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
    return res;
}
```

Zfunc.h
Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

```
vi Z(const string& S) {
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i,l,sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
    return z;
}
```

Manacher.h
Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi,2> p = {vi(n+1), vi(n)};
    rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    }
    return p;
}
```

MinRotation.h
Description: Finds the lexicographically smallest rotation of a string.
Usage: rotate(v.begin(), v.begin()+minRotation(v), v.end());
Time: $\mathcal{O}(N)$

```
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
        if (s[a+k] > s[b+k]) {a = b; break;}
    }
    return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. sa[i] is the starting index of the suffix which is *i*'th in the sorted suffix array. The returned vector is of size $n + 1$, and sa[0] = n. The lcp array contains longest common prefixes for neighbouring strings in the suffix array: lcp[i] = lcp(sa[i], sa[i-1]), lcp[0] = 0. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

```
struct SuffixArray {
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++;
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        }
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i+]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    }
};
```

SuffixTree.h
Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r] into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r] substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

```
struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;

    void ukkadd(int i, int c) { suff:
        if (r[v]<=q) {
            if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
                p[m++]=v; v=s[v]; q=r[v]; goto suff; }
            v=t[v][c]; q=l[v];
        }
        if (q==-1 || c==toi(a[q])) q++; else {
            l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
            p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
            l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
            v=s[p[m]]; q=l[m];
            while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
            if (q==r[m]) s[m]=v; else s[m]=m+2;
            q=r[v]-(q-r[m]); m+=2; goto suff;
        }
    }

    SuffixTree(string a) : a(a) {
        fill(r,r+N,sz(a));
        memset(s, 0, sizeof s);
        memset(t, -1, sizeof t);
        fill(t[1],t[1]+ALPHA,0);
        s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
        rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
    }
}
```

```
    }

    // example: find longest common substring (uses ALPHA = 28)
    pii best;
    int lcs(int node, int i1, int i2, int olen) {
        if (l[node] <= i1 && i1 < r[node]) return 1;
        if (l[node] <= i2 && i2 < r[node]) return 2;
        int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
        rep(c,0,ALPHA) if (t[node][c] != -1)
            mask |= lcs(t[node][c], i1, i2, len);
        if (mask == 3)
            best = max(best, {len, r[node] - len});
        return mask;
    }

    static pii LCS(string s, string t) {
        SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
        st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
        return st.best;
    }
};
```

Hashing.h
Description: Self-explanatory methods for string hashing.

2d2a67, 44 lines

// Arithmetic mod $2^{64}-1$. $2x$ slower than mod 2^{64} and more
// code, but works on evil test data (e.g. Thue–Morse, where
// ABBA... and BAAB... of length 2^{10} hash the same mod 2^{64}).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
 ull x; H(ull x=0) : x(x) {}
 H operator+(H o) { return x + o.x + (x + o.x < x); }
 H operator-(H o) { return *this + ~o.x; }
 H operator*(H o) { auto m = (__uint128_t)x * o.x;
 return H((ull)m) + (ull)(m >> 64); }
 ull get() const { return x + !~x; }
 bool operator==(H o) const { return get() == o.get(); }
 bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (1l)1e11+3; // (order ~ $3e9$; random also ok)

```
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
};

vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i,0,length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    rep(i,length,sz(str)) {
        ret.push_back(h = h * C + str[i] - pw * str[i-length]);
    }
    return ret;
}
```

```
H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

AhoCorasick.h

Description: AhoCorasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

f35677, 66 lines

```
struct AhoCorasick {
    enum {alpha = 26, first = 'A'}; // change this!
    struct Node {
        // (nmatches is optional)
        int back, next[alpha], start = -1, end = -1, nmatches = 0;
        Node(int v) { memset(next, v, sizeof(next)); }
    };
    vector<Node> N;
    vi backp;
    void insert(string& s, int j) {
        assert(!s.empty());
        int n = 0;
        for (char c : s) {
            int& m = N[n].next[c - first];
            if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
            else n = m;
        }
        if (N[n].end == -1) N[n].start = j;
        backp.push_back(N[n].end);
        N[n].end = j;
        N[n].nmatches++;
    }
    AhoCorasick(vector<string>& pat) : N(1, -1) {
        rep(i, 0, sz(pat)) insert(pat[i], i);
        N[0].back = sz(N);
        N.emplace_back(0);

        queue<int> q;
        for (q.push(0); !q.empty(); q.pop()) {
            int n = q.front(), prev = N[n].back;
            rep(i, 0, alpha) {
                int &ed = N[n].next[i], y = N[prev].next[i];
                if (ed == -1) ed = y;
                else {
                    N[ed].back = y;
                    (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
                        = N[y].end;
                    N[ed].nmatches += N[y].nmatches;
                    q.push(ed);
                }
            }
        }
    }
    vi find(string word) {
        int n = 0;
        vi res; // ll count = 0;
        for (char c : word) {
            n = N[n].next[c - first];
            res.push_back(N[n].end);
            // count += N[n].nmatches;
        }
        return res;
    }
    vector<vi> findAll(vector<string>& pat, string word) {
        vi r = find(word);
        vector<vi> res(sz(word));
        rep(i, 0, sz(word)) {
```

```
int ind = r[i];
while (ind != -1) {
    res[i - sz(pat[ind]) + 1].push_back(ind);
    ind = backp[ind];
}
}
return res;
}
};
```