

$$\begin{aligned}
& \sum_{i=1}^n \frac{1}{n} \eta_y^2 \frac{1}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \sum_{j=0}^{k-1} \tilde{h}_{i,y}^{(t,j)} \right\|^2 \\
& \leq \sum_{i=1}^n \frac{1}{n} \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \sum_{j=0}^{k-1} \frac{1}{c_t} \nabla_y f_i(x_i^{(t,j)}, y_i^{(t,j)}) + \nabla_y g_i(x_i^{(t,j)}, y_i^{(t,j)}) - \frac{1}{\gamma} (y_i^{(t,j)} - \theta_i^{(t,j)}) \right\|^2 \\
& \leq 4 \sum_{i=1}^n \frac{1}{n} \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \sum_{j=0}^{k-1} \nabla_y g_i(x_i^{(t,j)}, y_i^{(t,j)}) - \frac{1}{\gamma} (y_i^{(t,j)} - \theta_i^{(t,j)}) - (\nabla_y g_i(x^{(t)}, y^{(t)}) - \frac{1}{\gamma} (y^{(t)} - \theta^{(t)})) \right\|^2 \\
& \quad + 2 \sum_{i=1}^n \frac{1}{n} \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \sum_{j=0}^{k-1} \frac{1}{c_t} \nabla_y f_i(x_i^{(t,j)}, y_i^{(t,j)}) \right\|^2 + 4 \sum_{i=1}^n \frac{1}{n} \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \nabla_y g_i(x^{(t)}, y^{(t)}) - \frac{1}{\gamma} (y^{(t)} - \theta^{(t)}) \right\|^2 \\
& \leq 2\tau \eta_y^2 \frac{L_f^2}{c_t^2} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_\theta^{(t)} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_y^{(t)} + 8 \sum_{i=1}^n \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \frac{1}{n} \sum_{i=1}^n \nabla_y g_i(x^{(t)}, y^{(t)}) + \frac{1}{\gamma} (y^{(t)} - \theta^{(t)}) \right\|^2 \\
& \quad + 8 \sum_{i=1}^n \frac{1}{n} \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \nabla_y g_i(x^{(t)}, y^{(t)}) - \frac{1}{\gamma} (y^{(t)} - \theta^{(t)}) - \frac{1}{n} \sum_{i=1}^n \nabla_y g_i(x^{(t)}, y^{(t)}) + \frac{1}{\gamma} (y^{(t)} - \theta^{(t)}) \right\|^2 \\
& \leq 2\tau \eta_y^2 \frac{L_f^2}{c_t^2} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_\theta^{(t)} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_y^{(t)} + 8\eta_y^2 \tau \Delta^2 + 16 \sum_{i=1}^n \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \frac{1}{n} \sum_{i=1}^n \nabla_y \frac{1}{c_t} f_i(x^{(t)}, y^{(t)}) \right\|^2 \\
& \quad + 16 \sum_{i=1}^n \frac{\eta_y^2}{\tau} \sum_{k=0}^{\tau-1} \mathbb{E} \left\| \frac{1}{n} \sum_{i=1}^n \nabla_y \frac{1}{c_t} f_i(x^{(t)}, y^{(t)}) + \nabla_y g_i(x^{(t)}, y^{(t)}) + \frac{1}{\gamma} (y^{(t)} - \theta^{(t)}) \right\|^2, \\
& \leq 18\tau \eta_y^2 \frac{L_f^2}{c_t^2} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_\theta^{(t)} + \frac{12}{\gamma^2} \tau \eta_y^2 \Delta_y^{(t)} + 8\eta_y^2 \tau \Delta^2 + 16 \frac{\eta_y^2}{\lambda_y^2} \tau \left\| \mathbb{E}[y^{(t+1)} - y^{(t)}] \right\|^2
\end{aligned}$$