$$F_{01}^{spring} = k_{01} \left(y_1 - \ell_{01} \right)$$

$$F_{12}^{spring} = k_{12} \left(y_2 - y_1 - \ell_{12} \right)$$

$$F_1^{ext} = m_1 g$$

Force balance for m_1

$$\vec{F}_{01}^{spring} + \vec{F}_{12}^{spring} + \vec{F}_1^{ext} = 0$$

$$\vec{F}_{01}^{spring} + \vec{F}_{12}^{spring} + \vec{F}_1^{ext}$$
$$= -F_{01}^{spring}\hat{y} + F_{12}^{spring}\hat{y} + F_1^{ext}\hat{y}$$
$$= -k_{01} \left(y_1 - \ell_{01} \right) \hat{y} + k_{12} \left(y_2 - y_1 - \ell_{12} \right) \hat{y} + m_1 g \hat{y}$$
$$= - \left(k_{01} + k_{12} \right) y_1 + k_{12} y_2 + k_{01} \ell_{01} - k_{12} \ell_{12} + m_1 g$$
$$= 0$$

$$\Rightarrow \left(k_{01} + k_{12} \right) y_1 - k_{12} y_2 = m_1 g + k_{01} \ell_{01} - k_{12} \ell_{12}$$

Force balance for m_2

$$\vec{F}_{12}^{spring} + \vec{F}_{23}^{spring} + \vec{F}_2^{ext} = 0$$

$$F_{12}^{spring} = k_{12} \left(y_2 - y_1 - \ell_{12} \right)$$

$$F_{23}^{spring} = k_{23} \left(y_3 - y_2 - \ell_{23} \right)$$

$$\vec{F}_2^{ext} = m_2 g$$

$$-F_{12}^{spring}\hat{y} + F_{23}^{spring}\hat{y} + F_2^{ext}\hat{y} = - k_{12} \left(y_2 - y_1 - \ell_{12} \right) \hat{y} + k_{23} \left(y_3 - y_2 - \ell_{23} \right) \hat{y} + m_2 g \hat{y}$$
$$= k_{12} y_1 - \left(k_{12} + k_{23} \right) y_2 + k_{23} y_3 + k_{12} \ell_{12} - k_{23} \ell_{23} + m_2 g$$
$$= 0$$

$$\Rightarrow -k_{12} y_1 + \left(k_{12} + k_{23} \right) y_2 - k_{23} y_3 = m_2 g + k_{12} \ell_{12} - k_{23} \ell_{23}$$

Force balance for m_3

$$\vec{F}_{23}^{spring} + \vec{F}_{34}^{spring} + \vec{F}_3^{ext} = 0$$

$$F_{23}^{spring} = k_{23} \left(y_3 - y_2 - \ell_{23} \right)$$
$$F_{34}^{spring} = k_{34} \left(y_4 - y_3 - \ell_{34} \right)$$
$$\vec{F}_3^{ext} = m_3 g$$

$$-F_{23}^{spring} \hat{y} + F_{34}^{spring} \hat{y} + F_3^{ext} \hat{y} = - k_{23} \left(y_3 - y_2 - \ell_{23} \right) \hat{y} + k_{34} \left(y_4 - y_3 - \ell_{34} \right) \hat{y} + m_3 g \hat{y}$$
$$= k_{23} y_2 - \left(k_{23} + k_{34} \right) y_3 + k_{34} y_4 + k_{23} \ell_{23} - k_{34} \ell_{34} + m_3 g$$
$$= 0$$

$$\Rightarrow -k_{23} y_2 + \left(k_{23} + k_{34} \right) y_3 - k_{34} y_4 = m_3 g + k_{23} \ell_{23} - k_{34} \ell_{34}$$

Force balance for m_4

$$\vec{F}_{34}^{spring} + \vec{F}_4^{ext} = 0$$

$$F_{34}^{spring} = k_{34} \left(y_4 - y_3 - \ell_{34} \right)$$
$$\vec{F}_4^{ext} = m_4 g$$

$$-F_{34}^{spring} \hat{y} + F_4^{ext} \hat{y} = - k_{34} \left(y_4 - y_3 - \ell_{34} \right) \hat{y} + m_4 g \hat{y}$$
$$= k_{34} y_3 - k_{34} y_4 + k_{34} \ell_{34} + m_4 g$$
$$= 0$$

$$\Rightarrow -k_{34} y_3 + k_{34} y_4 = m_4 g + k_{34} \ell_{34}$$

system of linear equations:

$$\left(k_{01} + k_{12} \right) y_1 - k_{12} y_2 = m_1 g + k_{01} \ell_{01} - k_{12} \ell_{12}$$
$$-k_{12} y_1 + \left(k_{12} + k_{23} \right) y_2 - k_{23} y_3 = m_2 g + k_{12} \ell_{12} - k_{23} \ell_{23}$$
$$-k_{23} y_2 + \left(k_{23} + k_{34} \right) y_3 - k_{34} y_4 = m_3 g + k_{23} \ell_{23} - k_{34} \ell_{34}$$
$$-k_{34} y_3 + k_{34} y_4 = m_4 g + k_{34} \ell_{34}$$

$$A \vec{y} = \vec{b}$$

$$A = \begin{pmatrix} k_{01} + k_{12} & -k_{12} & 0 & 0 \\ -k_{12} & k_{12} + k_{23} & -k_{23} & 0 \\ 0 & -k_{23} & k_{23} + k_{34} & -k_{34} \\ 0 & 0 & -k_{34} & k_{34} \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} m_1 g + k_{01}\ell_{01} - k_{12}\ell_{12} \\ m_2 g + k_{12}\ell_{12} - k_{23}\ell_{23} \\ m_3 g + k_{23}\ell_{23} - k_{34}\ell_{34} \\ m_4 g + k_{34}\ell_{34} \end{pmatrix}$$

=====