# An Analytical and Empirical Survey of Impermanent Loss

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**ABSTRACT** Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

**INDEX TERMS** DAOSYS, Smart DAO, Decentralized Finance, Hyper-diamond proxy, EIP-2535

## I. INTRODUCTION

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## II. ANALYTICAL SURVEY

### A. EXPECTED IMPERMANENT LOSS

Impermanent (divergence) Loss (IL), is derived as:

$$IL = \frac{V_0 - V_H}{V_H} \tag{1}$$

where $V_0$ is the initial value of the portfolio holding,

$$V_0 = 2L\sqrt{P_0 \alpha} \tag{2}$$

where $L$ is the liquidity, $P_0$ is the initial price, and $\alpha$ is the price ratio (i.e., $\frac{p_0}{p_f}$) and $V_H$ is the held portfolio value, and is derived by the following:

$$V_H = 2L\sqrt{P_0(1+\alpha)} \tag{3}$$

Eq. (1) can be derived as shown in XX to provide:

$$IL = \frac{2\sqrt{\alpha}}{1+\sqrt{\alpha}} - 1 \tag{4}$$

However, (1) provides limited insight, as it is represented only in terms of the price ratio, $\alpha$, which provides no insight when considering the stochastic nature of the price behaviour itself. To provide this we assume that price assets move in a Geometric Brownian Motion (GBM) as first done in [5]; see Appendix A. When representing IL stochastically, its expectation can be derived as:

$$E\{IL\} = \frac{e^{-\frac{\sigma^2 t}{8}}}{\cosh(\frac{\mu t}{2})} - 1 \tag{5}$$

see Appendix B for the analytical derivation of (5), and a similar derivation also be found in [6].

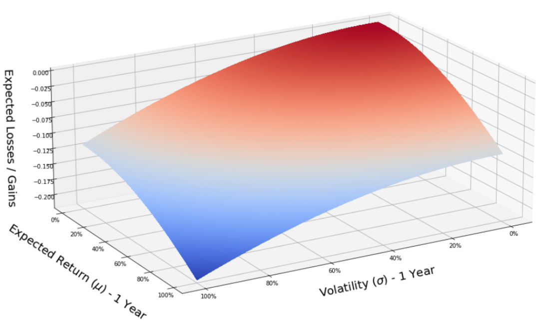


FIGURE 1: Expected impermanent loss

### B. EXPECTED PORTFOLIO VALUES

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$$E\{V_H\} = 2L\sqrt{P_0}(1 + e^{\mu t}) \tag{6}$$

$$E\{V\} = 2L\sqrt{P_0}e^{(\frac{\mu t}{2} - \frac{\sigma^2 t}{8})} \tag{7}$$

### C. EXPECTED RETURNS WITH COMPOUNDING FEES

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$$E\{V\} = 2L\sqrt{P_0}e^{(\frac{\mu t}{2} - \frac{\sigma^2 t}{8})}e^{z_t} \tag{8}$$

$$E\{R\} = \frac{e^{-\frac{\sigma^2 t}{8} + z_t}}{\cosh(\frac{\mu t}{2})} \tag{9}$$

Referring to (8) and (12), the term $e^{z_t}$ represents portfolio growth due to the accumulation of trading fees, where $z_t$ can be defined a myriad of ways. If we take the aggregation of individual deposits $a_i$ over each $t$ in the form of a geometric mean, this can be expressed by the following:

$$(\prod_{i=1}^{n} a_i)^{\frac{1}{n}} = e^{\frac{1}{n}\sum_i^n \ln a_i} \tag{10}$$

For sake of brevity, we assume $\alpha = \frac{1}{n}\sum_i^n \ln a_i$, hence the growth term becomes:

$$F_t = e^{\alpha t} \tag{11}$$

where $\alpha$ represents constant rate of growth, hence assuming GBM, the overall expected returns become:

$$E\{R\} = \frac{e^{-\frac{\sigma^2 t}{8} + \alpha t}}{\cosh(\frac{\mu t}{2})} \tag{12}$$

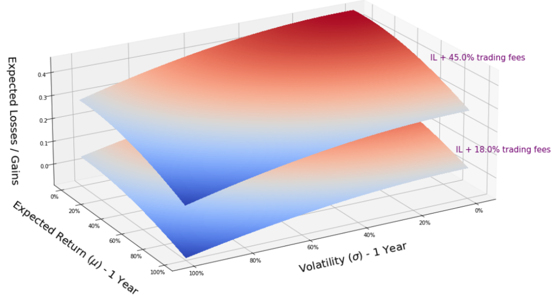


FIGURE 2: Expected impermanent loss with trading fees

## III. DEFI PYTHON SIMULATOR

To realize proper tokenomics design, it is highly inefficient to invest resources into development without first conducting a proper simulations of the design to test specifications for various outcomes. This is what every DeFi project in the crypto space is not doing. This is why we are introducing an open source python package to simulate various sandboxed DeFi components of DAOSYS so that project engineers, managers and designers can pre-plan outcomes prior to investing valuable resources into development.

With this tool, DAOSYS designers can utilize the plug and play components of our simulator to build tokenomics mockups for business planning purposes so that teams can come together and get collective consensus alongside potential users and investors. Not only is this tool applicable to DAOSYS, it can be used as a general purpose tool to simulate DEX activity for anyone wishing to setup their own liquidity pool (LP) and wish to stress test their ideas prior to development. This can be used as a powerful design tool to explore the limitations of an DeFi project idea prior to committing valuable resources on development costs (ie, Devs and Project Managers). Table 1 highlights the main components of the simulator, so that potential users of the system can get a higher level understanding of the DeFi simulator package.

We have a working beta version of the simulator which is available through Syscoin's Github repository. This simulator is being built alongside DAOSYS, and we are using this to understand the ROI and design considerations for DAOSYS's first usecase (ie, Masternode Yield Farming). Since the simulator is still in the beta stage, the setup is currently not realized to its full intention. However, a downloadable demo of this tool is available from Syscoin's Github repository for the community to begin using. For a mini python tutorial on how to use this tool, please refer to [10] and refer to [11] for a series of example Jupyter notebooks.

| Simulator Component | Description |
|---|---|
| Agents | Entities that engage with the system, and are subcategorized into tokens and users |
| Events | Agnostic events that take place within the system (eg, mint, deposit, withdraw, swap, and rebase) |
| ModelQueue | Queue of univariate events that are modelled aprior that can be fed into the system as events |
| Actions | Event actions that are fed into the system performed by agents; they can either be single stand-alone independent event actions or chained together with dependency |
| ActionChains | Actions that have dependencies on other actions as inputs |
| ActionBatch | Batches of actions placed together into a repeatable sequence; there is only one assigned time delta per the pass of each batch, and there is no limit as to the number of batches that can be created |
| Liquidity Pools | Pool of two token agents managed by constant product trading |
| Orchestrator | Manages agents and actions working within the system |
| Event Queue | Queue of storable actions |
| Event Executor | Final step which executes queue of action events |

TABLE 1: Descriptions of DeFi python simulator components; refer to Fig. 3 to see how components interact with one another.

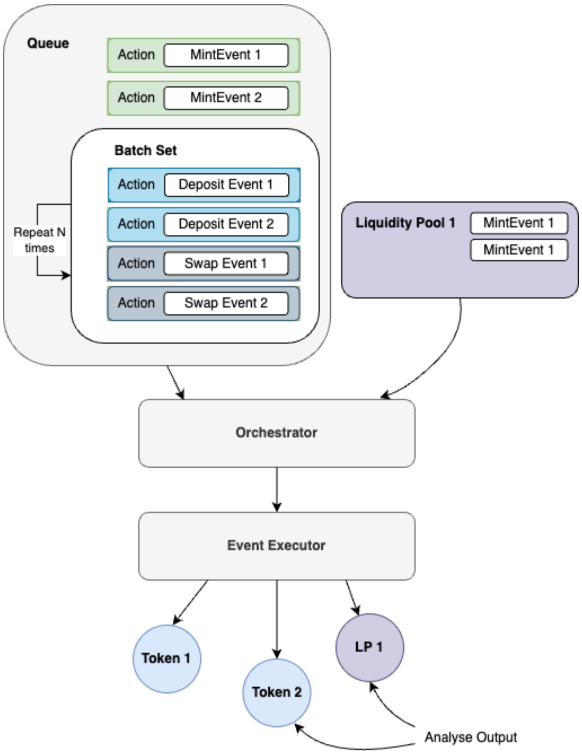


FIGURE 3: System components to DeFi Python simulator for DAOSYS

The purpose of this tool is primarily for the design aspect of DAOSYS, and is still in its early stages of development. The next stages involve feeding the output of various mathematical models into the simulator framework and to use this tool to simulate other DeFi usecases. This is so that we can test a roster of edge cases to build more robust systems. The final goal is to bring this system to a level of maturity so that it can be utilized in parallel with the contracts in real time. Hence, exposing DeFi to a scientific way of design, testing, and implementation.

## IV. NUMERICAL SIMULATIONS

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## V. SUMMARY

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## APPENDIX A GEOMETRIC BROWNIAN MOTION

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \tag{13}$$

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## APPENDIX B EXPECTED IMPERMANENT LOSS DERIVATION

Under the assumption of GBM, equations (2) and (3) can be represented as:

$$V_0 = 2L\sqrt{P_0 e^{(\mu-\frac{\sigma^2}{2})t+\sigma W_t}}, \tag{14}$$

and

$$V_H = 2L\sqrt{P_0}(1+e^{(\mu-\frac{\sigma^2}{2})t+\sigma W_t}). \tag{15}$$

Using (14) and $E\{e^{aX}\} = e^{\mu a+\frac{a^2\sigma^2}{2}}$, we first can calculate $E\{V_0\}$:

$$\begin{aligned} E\{V_0\} &= E\{2L\sqrt{P_0}e^{(\frac{\mu}{2}-\frac{\sigma^2}{4})t}e^{\frac{\sigma W_t}{2}}\} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2}-\frac{\sigma^2}{4})t}E\{e^{\frac{\sigma W_t}{2}}\} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2}-\frac{\sigma^2}{4})t}e^{0+\frac{\sigma^2 t}{8}} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2}-\frac{\sigma^2}{8})} \end{aligned}$$

Likewise, using (15) and the same identity, we calculate $E\{V_H\}$:

$$\begin{aligned} E\{V_H\} &= E\{L\sqrt{P_0}(1+e^{(\mu-\frac{\sigma^2}{2})t+\sigma W_t})\} \\ &= L\sqrt{P_0}(1+E\{e^{(\mu-\frac{\sigma^2}{2})t+\sigma W_t}\}) \\ &= L\sqrt{P_0}(1+e^{(\mu-\frac{\sigma^2}{2})t}E\{e^{\sigma W_t}\}) \\ &= L\sqrt{P_0}(1+e^{(\mu-\frac{\sigma^2}{2})t}e^{0+\frac{\sigma^2}{2}t} \\ &= L\sqrt{P_0}(1+e^{\mu t}) \end{aligned}$$

Therefore using the the identity $\cosh(x) = \frac{1+e^{2x}}{2e^x}$, and our definition of IL outlined in (1) we have:

$$\begin{aligned} E\{IL\} &= \frac{E\{V_0\}-E\{V_H\}}{E\{V_H\}} \\ &= \frac{2L\sqrt{P_0}e^{(\frac{\mu}{2}-\frac{\sigma^2}{8})}}{L\sqrt{P_0}(1+e^{\mu t})} \\ &= \frac{2e^{(\frac{\mu}{2}-\frac{\sigma^2}{8})}}{1+e^{\mu t}} - 1 \\ &= \frac{e^{-\frac{\sigma^2 t}{8}}}{cosh(\frac{\mu t}{2})} - 1 \end{aligned}$$

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