

An Analytical and Empirical Survey of Impermanent Loss

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• **ABSTRACT** Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

• **INDEX TERMS** DAOSYS, Smart DAO, Decentralized Finance, Hyper-diamond proxy, EIP-2535

I. INTRODUCTION

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II. ANALYTICAL SURVEY

A. EXPECTED IMPERMANENT LOSS

Impermanent (divergence) Loss (IL), is derived as:

$$IL = \frac{V_0 - V_H}{V_H} \quad (1)$$

where V_0 is the initial value of the portfolio holding,

$$V_0 = 2L\sqrt{P_0\alpha} \quad (2)$$

where L is the liquidity, P_0 is the initial price, and α is the price ratio (i.e., $\frac{P_0}{P_f}$) and V_H is the held portfolio value, and is derived by the following:

$$V_H = 2L\sqrt{P_0(1+\alpha)} \quad (3)$$

Eq. (1) can be derived as shown in XX to provide:

$$IL = \frac{2\sqrt{\alpha}}{1 + \sqrt{\alpha}} - 1 \quad (4)$$

However, (1) provides limited insight, as it is represented only in terms of the price ratio, α , which provides no insight when considering the stochastic nature of the price behaviour itself. To provide this we assume that price assets move in a Geometric Brownian Motion (GBM) as first done in [5]; see Appendix A. When representing IL stochastically, its expectation can be derived as:

$$E\{IL\} = \frac{e^{-\frac{\sigma^2 t}{8}}}{\cosh(\frac{\mu t}{2})} - 1 \quad (5)$$

see Appendix B for the analytical derivation of (5), and a similar derivation also be found in [6].

B. EXPECTED PORTFOLIO VALUES

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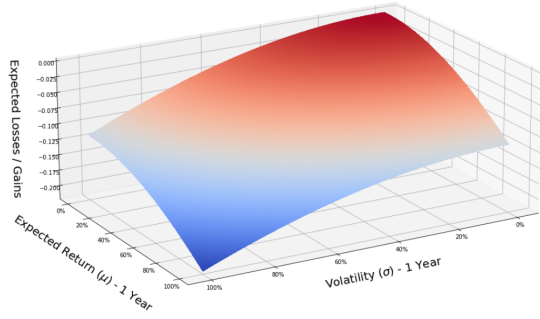


FIGURE 1: Expected impermanent loss for first year (ie, $t = 1$)

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$$E\{V_H\} = 2L\sqrt{P_0}(1 + e^{\mu t}) \quad (6)$$

$$E\{V\} = 2L\sqrt{P_0}e^{\left(\frac{\mu t}{2} - \frac{\sigma^2 t}{8}\right)} \quad (7)$$

C. EXPECTED RETURNS WITH COMPOUNDING FEES

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$$E\{V\} = 2L\sqrt{P_0}e^{\left(\frac{\mu t}{2} - \frac{\sigma^2 t}{8}\right)}e^{z_t} \quad (8)$$

$$E\{R\} = \frac{e^{-\frac{\sigma^2 t}{8} + z_t}}{\cosh\left(\frac{\mu t}{2}\right)} \quad (9)$$

Referring to (8) and (12), the term e^{z_t} represents portfolio growth due to the accumulation of trading fees, where z_t can be defined a myriad of ways. If we take the aggregation of individual deposits a_i over each t in the form of a geometric mean, this can be expressed by the following:

$$\left(\prod_{i=1}^n a_i\right)^{\frac{1}{n}} = e^{\frac{1}{n} \sum_{i=1}^n \ln a_i} \quad (10)$$

For sake of brevity, we assume $\alpha = \frac{1}{n} \sum_{i=1}^n \ln a_i$, hence the growth term becomes:

$$F_t = e^{\alpha t} \quad (11)$$

where α represents constant rate of growth, hence assuming GBM, the overall expected returns become:

$$E\{R\} = \frac{e^{-\frac{\sigma^2 t}{8} + \alpha t}}{\cosh\left(\frac{\mu t}{2}\right)} \quad (12)$$

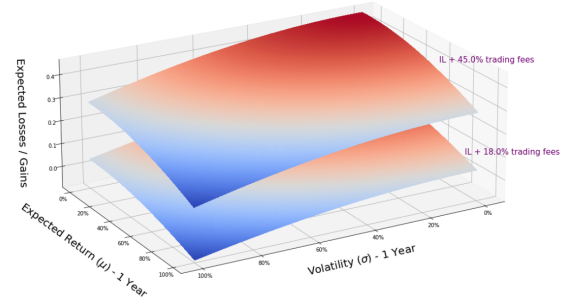


FIGURE 2: Expected impermanent loss with compounding trading fees for first year (ie, $t = 1$)

III. DEFI PYTHON SIMULATOR

To realize proper tokenomics design, it is highly inefficient to invest resources into development without first conducting a proper simulations of the design to test specifications for various outcomes. This is what every DeFi project in the crypto space is not doing. This is why we are introducing an open source python package to simulate various sandboxed DeFi components of DAOSYS so that project engineers, managers and designers can pre-plan outcomes prior to investing valuable resources into development.

With this tool, DAOSYS designers can utilize the plug and play components of our simulator to build tokenomics mockups for business planning purposes so that teams can come together and get collective consensus alongside potential users and investors. Not only is this tool applicable to DAOSYS, it can be used as a general purpose tool to simulate DEX activity for anyone wishing to setup their own liquidity pool (LP) and wish to stress test their ideas prior to development. This can be used as a powerful design tool to explore the limitations of an DeFi project idea prior to committing valuable resources on development costs (ie, Devs and Project Managers). Table 1 highlights the main components of the simulator, so that potential users of the system can get a higher level understanding of the DeFi simulator package.

We have a working beta version of the simulator which is available through Syscoin's Github repository. This simulator is being built alongside DAOSYS, and we are using this to understand the ROI and design considerations for DAOSYS's first usecase (ie, Masternode Yield Farming). Since the simulator is still in the beta stage, the setup is currently not

Simulator Component	Description
Agents	Entities that engage with the system, and are subcategorized into tokens and users
Events	Agnostic events that take place within the system (eg. mint, deposit, withdraw, swap, and rebase)
ModelQueue	Queue of univariate events that are modelled aprior that can be fed into the system as events
Actions	Event actions that are fed into the system performed by agents; they can either be single stand-alone independent event actions or chained together with dependency
ActionChains	Actions that have dependencies on other actions as inputs
ActionBatch	Batches of actions placed together into a repeatable sequence; there is only one assigned time delta per the pass of each batch, and there is no limit as to the number of batches that can be created
Liquidity Pools	Pool of two token agents managed by constant product trading
Orchestrator	Manages agents and actions working within the system
Event Queue	Queue of storable actions
Event Executor	Final step which executes queue of action events

TABLE 1: Descriptions of DeFi python simulator components; refer to Fig. 3 to see how components interact with one another.

realized to its full intention. However, a downloadable demo of this tool is available from Syscoin’s Github repository for the community to begin using. For a mini python tutorial on how to use this tool, please refer to [10] and refer to [11] for a series of example Jupyter notebooks.

The purpose of this tool is primarily for the design aspect of DAOSYS, and is still in its early stages of development. The next stages involve feeding the output of various mathematical models into the simulator framework and to use this tool to simulate other DeFi usecases. This is so that we can test a roster of edge cases to build more robust systems. The final goal is to bring this system to a level of maturity so that it can be utilized in parallel with the contracts in real time. Hence, exposing DeFi to a scientific way of design, testing, and implementation.

IV. NUMERICAL SIMULATIONS

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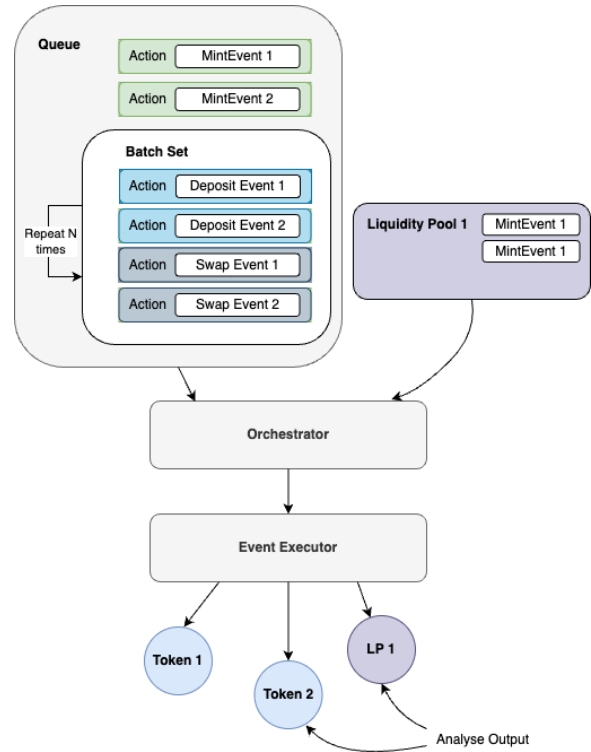


FIGURE 3: System components to DeFi Python simulator for DAOSYS

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V. SUMMARY

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APPENDIX A GEOMETRIC BROWNIAN MOTION

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \quad (13)$$

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APPENDIX B EXPECTED IMPERMANENT LOSS DERIVATION

Under the assumption of GBM, equations (2) and (3) can be represented as:

$$V_0 = 2L\sqrt{P_0}e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t}, \quad (14)$$

and

$$V_H = 2L\sqrt{P_0}(1 + e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t}). \quad (15)$$

Using (14) and $E\{e^{aX}\} = e^{\mu a + \frac{a^2 \sigma^2}{2}}$, we first can calculate $E\{V_0\}$:

$$\begin{aligned} E\{V_0\} &= E\{2L\sqrt{P_0}e^{(\frac{\mu}{2} - \frac{\sigma^2}{4})t}e^{\frac{\sigma W_t}{2}}\} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2} - \frac{\sigma^2}{4})t}E\{e^{\frac{\sigma W_t}{2}}\} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2} - \frac{\sigma^2}{4})t}e^{0 + \frac{\sigma^2 t}{8}} \\ &= 2L\sqrt{P_0}e^{(\frac{\mu}{2} - \frac{\sigma^2}{8})t} \end{aligned}$$

Likewise, using (15) and the same identity, we calculate $E\{V_H\}$:

$$\begin{aligned} E\{V_H\} &= E\{L\sqrt{P_0}(1 + e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t})\} \\ &= L\sqrt{P_0}(1 + E\{e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t}\}) \\ &= L\sqrt{P_0}(1 + e^{(\mu - \frac{\sigma^2}{2})t}E\{e^{\sigma W_t}\}) \\ &= L\sqrt{P_0}(1 + e^{(\mu - \frac{\sigma^2}{2})t}e^{0 + \frac{\sigma^2 t}{2}}) \\ &= L\sqrt{P_0}(1 + e^{\mu t}) \end{aligned}$$

Therefore using the the identity $\cosh(x) = \frac{1+e^{2x}}{2e^x}$, and our definition of IL outlined in (1) we have:

$$\begin{aligned} E\{IL\} &= \frac{E\{V_0\} - E\{V_H\}}{E\{V_H\}} \\ &= \frac{2L\sqrt{P_0}e^{(\frac{\mu}{2} - \frac{\sigma^2}{8})t}}{L\sqrt{P_0}(1 + e^{\mu t})} \\ &= \frac{2e^{(\frac{\mu}{2} - \frac{\sigma^2}{8})t}}{1 + e^{\mu t}} - 1 \\ &= \frac{e^{-\frac{\sigma^2 t}{8}}}{\cosh(\frac{\mu t}{2})} - 1 \end{aligned}$$

REFERENCES

- [1] Pintail, *Uniswap: A Good Deal for Liquidity Providers?*, Jan. 2019. Accessed on: Dec 2022. [Online]. Available: <https://pintail.medium.com/uniswap-a-good-deal-for-liquidity-providers-104c0b6816f2>
- [2] Pintail, *Understanding Uniswap Returns*, Feb. 2019. Accessed on: Dec 2022. [Online]. Available: <https://pintail.medium.com/understanding-uniswap-returns-cc593f3499ef>
- [3] P. Erins, *How to calculate Impermanent Loss: full derivation*, Jun. 2021. Accessed on: Dec 2022. [Online]. Available: <https://medium.com/auditless/how-to-calculate-impermanent-loss-full-derivation-803e8b2497b7>
- [4] A. Aigner and G. Dhaliwal, *UNISWAP: Impermanent Loss and Risk Profile of a Liquidity Provider*, Jun. 2021. Accessed on: Dec 2022. [Online]. Available: <https://arxiv.org/pdf/2106.14404.pdf>
- [5] G. Lambert, *Calculating the Expected Value of the Impermanent Loss in Uniswap*, Oct. 2021. Accessed on: Dec 2022. [Online]. Available: <https://lambert-guillaume.medium.com/an-analysis-of-the-expected-value-of-the-impermanent-loss-in-uniswap-bfbfbefbed2>
- [6] Danr, *Expected Impermanent Loss in Uniswap V2 & V3*, Mar. 2022. Accessed on: Dec 2022. [Online]. Available: <https://medium.com/gammaswap-labs/expected-impermanent-loss-in-uniswap-v2-v3-7fb8103bd81>
- [7] Danr, *Total Returns and Impermanent Loss in Uniswap V2*, Dec. 2021. Accessed on: Dec 2022. [Online]. Available: <https://medium.com/gammaswap-labs/total-returns-and-impermanent-loss-in-uniswap-v2-9f3d5b6ebc89>
- [8] Uniswap Blog, *A short history of Uniswap*, Sept. 2019. Accessed on: Sept 2022. [Online]. Available: <https://uniswap.org/blog/uniswap-history>
- [9] I.C. Moore, *Mastermode Yield Farming as a DAOSYS Usecase: Part 1*, Medium Article, Aug. 2022. Accessed on: Sept 2022. [Online]. Available: <https://icmoore.medium.com/mastermode-yield-farming-as-a-daosys-usecase-part-1-8485ab7ba721>
- [10] I.C. Moore, *Mastermode Yield Farming as a DAOSYS Usecase: Part 2*, Medium Article, Aug. 2022. Accessed on: Sept 2022. [Online]. Available: <https://icmoore.medium.com/mastermode-yield-farming-as-a-daosys-usecase-part-2-8906d0a11c27>
- [11] I.C. Moore, *Simulation: Mastermode Yield Farming*, Syscoin Github repos, Accessed on: Apr 2021. [Online]. Available: <https://github.com/syscoin/daosys/tree/main/notebooks/simulation>
- [12] Syscoin News, *Introducing Rollux, Syscoin's Rollup Suite Ready to Take Market by Storm*, Syscoin, Jun. 2022. Accessed on: Sept 2022. [Online]. Available: <https://syscoin.org/news/introducing-rollux-syscoins-rollup-suite-ready-to-take-market-by-storm>
- [13] J. Sidhu, *A Design For An Efficient Coordinated Financial Computing Platform*, Feb 2021. Accessed on: Sep 2021. [Online]. Available: <https://jsidhu.medium.com/a-design-for-an-efficient-coordinated-financial-computing-platform-ab27e8a825a0>
- [14] J. Sidhu and I.C. Moore, *Syscoin 4: A Peer-to-Peer Electronic Cash System Built For Decentralized Web 3.0 Business Applications*, Syscoin Platform, Dec. 2021. Accessed on: Sept 2022. [Online]. Available: https://syscoin.org/syscoin4_whitepaper.pdf

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