Stochastic Properties of EIP-1559 Basefees

A Preprint

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Abstract

EIP-1559 is a new proposed pricing mechanism for the Ethereum protocol developed to mitigate short-term volatility in demand for transactions. To properly understand this as a stochastic process, it is necessary to develop the mathematical foundations to understand under what conditions the base fee gas price outcomes behave as a stationary process, and when it does not. We believe understanding these mathematical fundamentals is critically needed to properly engineering a well-designed system.

Keywords EIP 1559 · Base Fees · Stochastic Processes · Stationarity

1 Introduction

The current Ethereum pricing mechanism employs the price auction model where high-value use cases are prioritized over lower ones. The problem with this approach is that as Ethereum has grown more popular, users have difficulty estimating optimal gas fees. In 2018, EIP-1559 was proposed by Ethereum Founder, Vitalik Buterin, as a major change to Ethereum's transaction fee mechanism, and introduces two different types of fees, which is the base fee and inclusion fee [1, 4]. The idea is the base fee is relatively stable while the inclusion fee serves as an additional tip offered to compensate miners. This mechanism allows for the base fee to be adjusted after every block in accordance to demand. Hence, when demand for gas is higher than the target gas price, the block size is adjusted upwards, and when demand for gas is lower than the target gas, the block size is adjusted downwards. Blocks are allowed to grow as large as double the target block size.

In this study, we investigate the stochastic properties of EIP-1559 base fees. As the main motivation behind EIP-1559 is to address the volatility and uncertainty associated with gas fees, we look at the mathematical conditions that this mechanism must have to achieve stationarity. We open up this discussion in Section 2 where we go over the EIP 1559 pricing mechanism. Using these simulated gas demands, we analyze the stationary properties in Section 3. In Section 4 we simulate the gas demand. Finally, we close off with our conclusions in Section 6.

2 EIP 1559 Pricing Mechanism

The basic premise of the new EIP 1559 proposal begins with setting the base fee which increases when the network capacity exceeds the target per-block gas usage, and decreases when the capacity is below the target. The calculation of the updated basefee, b_k at the k^{th} block, is as follows:

$$b_k = b_{k-1} f_k \tag{1}$$

$$f_k = 1 + \frac{\delta_k}{c} \tag{2}$$

where $\delta_k = demand_k - Target\ Gas\ Fee$ and $c = Target\ /\ Base\ Fee\ Max\ Change$; for the purpose of this study, we applied setup from Table 1, and assumed δ_k to be a random stationary process. However before we do that, we run the *abm1559* simulator [3] to show that this is the case, under the assumption that new users are Poisson distributed.

Constant	Value
Target Gas Fee	12,500,000
Base Fee Max Change	50
Initial Basefee	10,000,000,000

Table 1: EIP 1559 constants

3 Analyze Stationarity

Definition: A time series $\{x_t\}$ is said to be strictly stationary if random vectors $(x_{t_1}, ..., x_{t_n})^T$ and $(x_{t_1+\tau}, ..., x_{t_n+\tau})^T$ have the same joint distribution for all $\{t \in \mathbb{N}^0\}$ and $n, \tau \in \mathbb{N}^+$. This is defined as:

$$(x_{t_1}, ..., x_{t_n})^T \stackrel{d}{=} (x_{t_1+\tau}, ..., x_{t_n+\tau})^T, \tag{3}$$

where $\stackrel{d}{=}$ means equivalence in distribution.

3.1 Augmented Dickey Fuller Test

Given the following higher-order autoregressive processes:

$$\Delta y_t = \alpha y_{t-1} + \theta_1 \Delta y_{t-1} + ... + \theta_p \Delta y_{t-p} + \epsilon_t, \tag{4}$$

the Augmented Dickey Fuller (ADF) test checks the existence of a unit root $\alpha = 1$ (ie, H0: $\alpha = 1$) where the null hypothesis is non-stationary. The unit root in (5) is characteristic of a time series that makes it non-stationary.

4 Gas Demand Simulations

To simulate gas demand δ_k in (2) we use the *abm1559* simulator [3] where the following assumptions were made: (a) users are Poisson distributed; and (b) every TX consumes 21000 Gas. For our analysis, we are interested in simulating a year's worth of gas costs, which is infeasible as running the full Ethereum chain simulator consumes a lot of overhead.

Hence, we used the simulator to generate a sample, which was analysed using Exploratory Data Analysis (EDA) techniques, as shown in Figure 2. As we can see the gas demand is behaving as a normal random sample.

We also applied the ADF test to the simulated gas demands, shown in top image of Figure 1, and found it to be statistically significant (ie, p-val = 2.33e-29). Hence, rejecting HO, which indicate simulated gas demands to be stationary.

5 Gas Fee Simulations

In Section 4 we simulated gas demand δ_k in (2) and found them to be stationary and normally distributed with μ = 1.36e7, and variance σ^2 = 5.51e5. To simulate gas prices using EIP 1559 we used these parameter estimates to model a year's worth of gas demands and fed it through the system described in (1) and (2). A series of these simulations can be seen in Figure 3.

We tested gas demands for stationarity. In Table 3 we fail to reject to H0 for all ten tests indicating insufficient evidence to conclude that the effect of stationarity exists in the gas fee simulations.

5.1 Problem Statement

For sake of comparison, let's look at the stationarity for a standard first-order auto-regressive process (ie, AR(1)) defined as:

$$x_t = \mu + \alpha(x_{t-1} - \mu) + \epsilon_{t-1} \qquad \epsilon_t \sim IID\ N(0, \sigma^2) \tag{5}$$

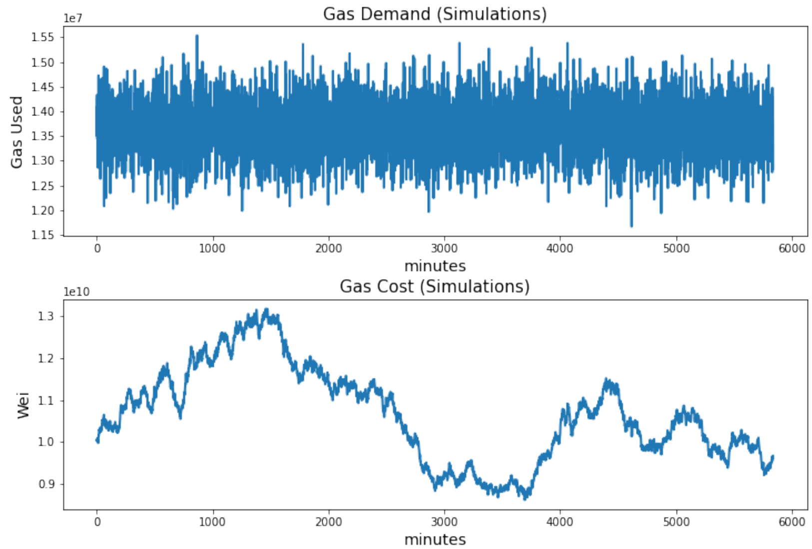

Figure 1: Simulations of; (top) gas demands using random samples from a normal distribution; and (bottom) gas costs using (1). As we can see, the gas cost simulations (bottom) are behaving like a random walk process

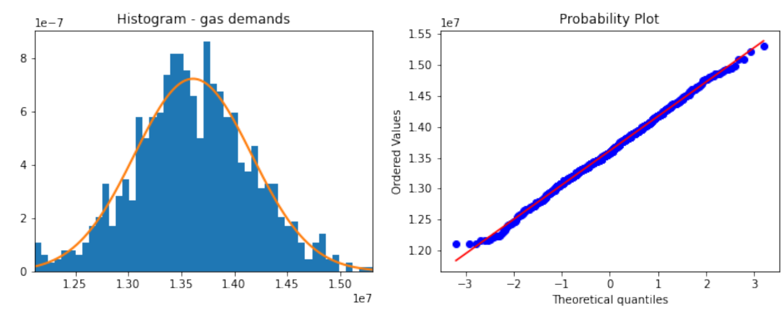

Figure 2: Testing normality of gas demands (top image of Fig. 1) using; (left) histogram with normal distribution fit of demands; and (right) Normal Q-Q plot indicates sample and theoretical quantiles match

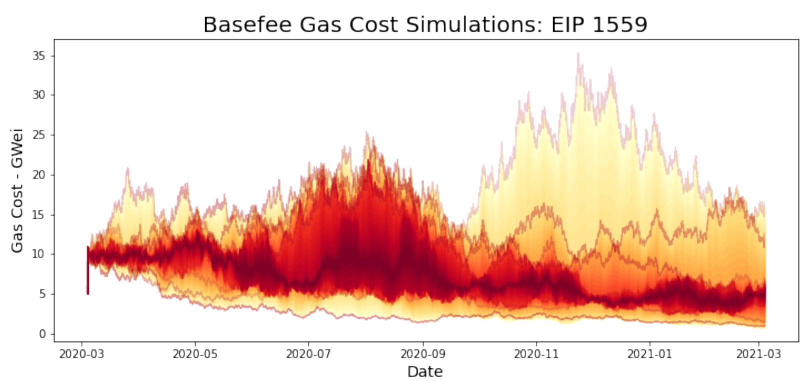

Figure 3: Gas cost simulations using EIP 1559; we can visual see the non-stationary behaviour in price over time, which is indicative of a random walk.

Simulation	ADF Statistic	p-value	Outcome
1	-1.506	0.531	not stationary
2	-1.260	0.647	not stationary
3	-2.170	0.217	not stationary
4	-2.432	0.133	not stationary
5	-1.844	0.359	not stationary
6	-1.323	0.618	not stationary
7	-1.645	0.459	not stationary
8	-1.662	0.451	not stationary
9	-1.124	0.705	not stationary
10	-1.930	0.318	not stationary

Table 2: ADF test on basefee simulations from Figure 3 to test null hypothesis (H0) that a unit root is present; when unit root is present, then sample is considered to be non-stationary. As we can see, we fail to reject H0 for all ten tests indicating insufficient evidence to conclude that the effect of stationarity exists.

The above process is stationary when $|\alpha|$<1, hence has a single root 1/α outside the unit circle. The asymptotic stationary distribution can be shown to be:

$$x_\infty \sim N(\mu, \frac{\sigma^2}{1-\alpha^2}), \tag{6}$$

when σ>0. Considering the (state space like) simularities to the AR(1) process to the system of the system of (1) and (2); the question is what are the constraints (if any) that time varying process δ_t must meet to ensure stationary outcomes for the basefees.

5.2 Random Coefficient Autoregressive Models

Considering the problem statement in the previous section, we look to another relative to the AR(1) process, namely the Random Coefficient Autoregressive of order 1, or RCA(1) process. The RCA(1) is similar to the AR(1) process in the sense that the parameter α is allowed to vary with time. This is quite similar to the EIP 1559 system of (1) and (2). The RCA(1) model is given by:

$$x_t = \alpha + \beta_t x_{t-1} + \epsilon_t, \tag{7}$$

where ϵ_t is an independent sequence of random variables with 0 mean and variance $\sigma^2 > 0$; β_t is an independent sequence of random variables with mean μ_β and variance σ_β^2 of the random coefficient β_t, and variance σ^2 of the error ϵ_t. Given (3) and (4) it is obvious that when $\sigma_\beta^2 = 0$, the RCA(1) in (5) becomes a AR series, and becomes a random walk when $\mu_\beta = 1$ and $\sigma_\beta^2 = 0$, hence non-stationary.

EIP 1559 (1) fits into the framework of a Random Coefficient Autoregressive (RCA) model when ϵ is negligibly small. This is good news, as it allows us to apply the body of work on RCA processes to this problem. The next question is; under what conditions does an RCA(1) process exhibit stationary behaviour. If we know this, then we can apply this understanding to the EIP 1559 mechanism. To help with our understanding regarding this, we apply Wang's Theorem [7]; see Figure 4.

Theorem Consider RCA(1) model (5) with β_k, ϵ_k, which is identically normally distributed. Then the sufficient condition for the existence of a strictly stationary and ergodic solution is that:

$$\ln(\sigma_\beta^2) < \varsigma + \ln(2) - 2\int_0^1 \frac{1-\exp[-\lambda(1-w^2)]}{1-w^2}dw, \tag{8}$$

where $\varsigma \approx$ 0.57721, denotes Euler's constant and λ = μ_β^2/ $2\sigma_\beta^2$

Proof: See Theorem 2 in [7].

6 Conclusion

The purpose of this study was to understand under what conditions the EIP 1559 pricing mechanism would behave as a stationary process. We believe this as a critical first step to understanding the constraints to engineer stable system. Given the enormity of the Ethereum project, understanding these fundamentals are needed for setting up well-designed

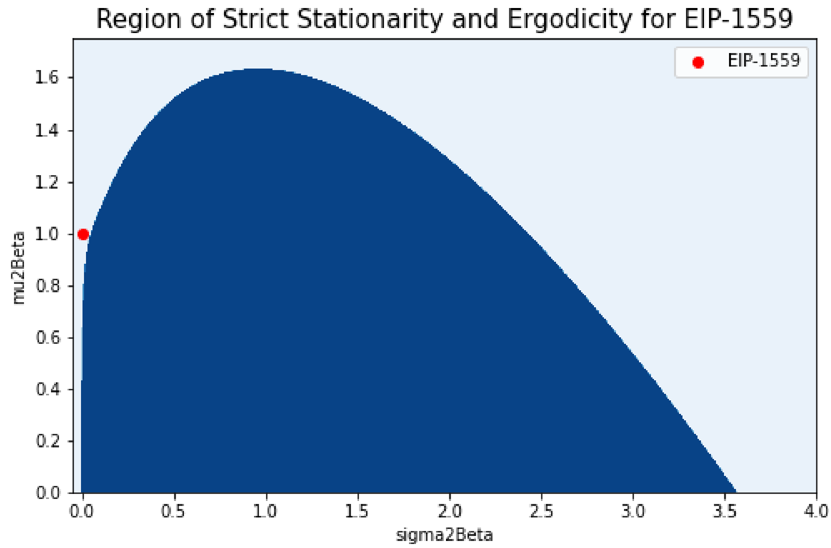

Figure 4: Region of strict stationarity and ergodicity for RCA(1) determined via (8). The annotated red dot represents where our setup (Table 1) is in relation to the region where an RCA(1) process behaves as a strictly stationary process. Based on this we can see that our setup was non-stationary.

simulation experiments to help capture all the edge cases that may arise so that the number of updates and patches are minimized prior to release.

It was determined that under the parameter setting used in this study that the outcomes behaved as a random walk, which is a non-stationary process. The RCA(1) was used as the mathematical framework to help understand this, as we can go to the literature to help guide our understanding of this kind of process. This conclusion was based on a series of simulations which we tested using the ADF, and Wang's Theorem indicating the region of stationarity for an RCA(1) process.

Ideally, we would like EIP-1559 to behave as a stationary process. Hence, more investigation is required to understand the setup that is necessary to achieve these conditions before proper simulation studies are implemented.

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