

ICMT Division B Sample Problems

July 10, 2025

1. Suppose that two fair six-sided dice are rolled and the sum of the faces is 8. Compute the probability that at least one of the dice rolled a 4.
2. In rhombus $ABCD$, a circle is inscribed such that it is tangent to \overline{BC} at E , \overline{CD} at F , \overline{DA} at G , and \overline{AB} at H . If the area of $ABCD$ is 144 and the area of $\triangle DFG$ is 32, what is the area of $\triangle AGH$?
3. A smooth real function $f(x)$ satisfies $f(x) + f'(x) + f''(x) = 2026$ for all x , and has a local minimum at $(26, 20)$. What is the value of $f^{(20)}(26)$?
4. Find the sum of all positive integers n less than 2030 such that $n^3 + 2n^2 + 2n + 1$ is divisible by 2029.
5. For a regular n -gon of side length 1 (where $n \geq 4$), define $f(n)$ as the sum of the distances from each vertex to the closest diagonal of the n -gon that does not contain that vertex. For example, $f(4) = 2\sqrt{2}$. Compute the limit $\lim_{n \rightarrow \infty} f(n)$.
6. Find the product of all real numbers t such that there exists a polynomial, $f(x)$, with real coefficients such that

$$f(f(x)) + f(x) = x^4 + tx^2 + 1.$$

7. A 3×3 grid of squares is colored with 3 colors (red, green or blue) so that no two squares that share an edge are the same color. In how many ways is this possible?
8. Let S be the portion of the surface defined by $z = x^2 + y^2$ that lies below the plane $z = 4$. Consider the function $f(x, y, z) = x^2yz$. Evaluate the maximum value of f on the surface S .
9. Compute the integral

$$\int_0^1 \int_0^1 \frac{1}{1-xy} dx dy.$$

10. What is the expected number of positive divisors of an integer chosen uniformly at random from 1 to 1000000, inclusive? If A is your answer and C is the correct value, your score on this question will be $\left\lfloor 25 \cdot e^{-38|A-C|^2} \right\rfloor$.