

ICMT Division B Sample Problems

1. Suppose that two fair six-sided dice are rolled and the sum of the faces is 8. Compute the probability that at least one of the dice rolled a 4.
2. In rhombus $ABCD$, a circle is inscribed such that it is tangent to \overline{BC} at E , \overline{CD} at F , \overline{DA} at G , and \overline{AB} at H . If the area of $ABCD$ is 144 and the area of $\triangle DFG$ is 32, what is the area of $\triangle AGH$?
3. Find the product of all real numbers t such that there exists a polynomial, $f(x)$, with real coefficients such that

$$f(f(x)) + f(x) = x^4 + tx^2 + 1$$

4. A 3×3 grid of squares is colored with 3 colors (red, green or blue) so that no two squares that share an edge are the same color. In how many ways is this possible?
5. Let S be the portion of the surface defined by $z = x^2 + y^2$ that lies below the plane $z = 4$. Consider the function $f(x, y, z) = x^2yz$. Evaluate the maximum value of f on the surface S .
6. What is the expected number of positive divisors of an integer chosen uniformly at random from 1 to 1000000, inclusive? If A is your answer and C is the correct value, your score on this question will be $\left\lfloor 25 \cdot e^{-38|A-C|^2} \right\rfloor$.