

A Mean-Field-Game Approach to Overfishing

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- 1 Brief review of historical and modern overfishing
- 2 The mathematical analysis
- 3 Ongoing and future works

1 Brief review of historical and modern overfishing

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First documented case of overfishing

Early 1800s: Overfishing was first documented when people discovered that whale fat could be used to create lamp oil.



"Grand Ball Given by the Whales." Drawing. April 20, 1861. Vanity Fair.

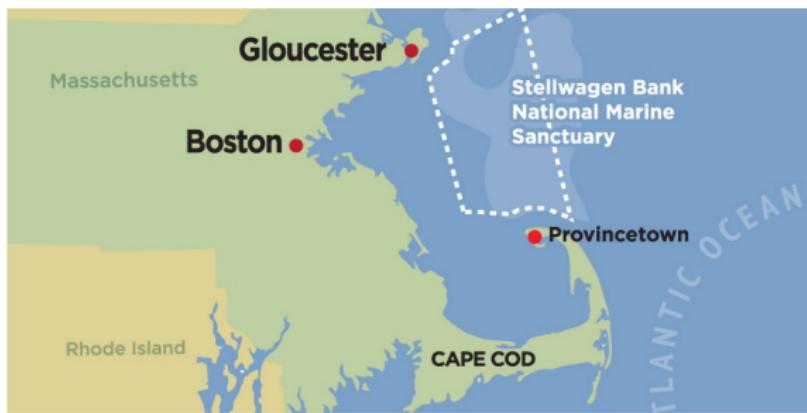
Thomas Henry Huxley:

*I believe then that the cod fishery, the herring fishery, the pilchard fishery, the mackerel fishery, and probably all the great sea **fisheries are inexhaustible**: that is to say that **nothing we do seriously affects the number of fish**. And any attempt to regulate these fisheries seems consequently from the nature of the case to be useless*

T.H.Huxley (1884) Inaugural Address, Fisheries Exhibition Lit., 4,1-22, London

Huxley did not predict the raise in fishing capacity and technology (e.g., motorised vessels, SONAR)

The Cape Cod study case [Du et al. 2021]

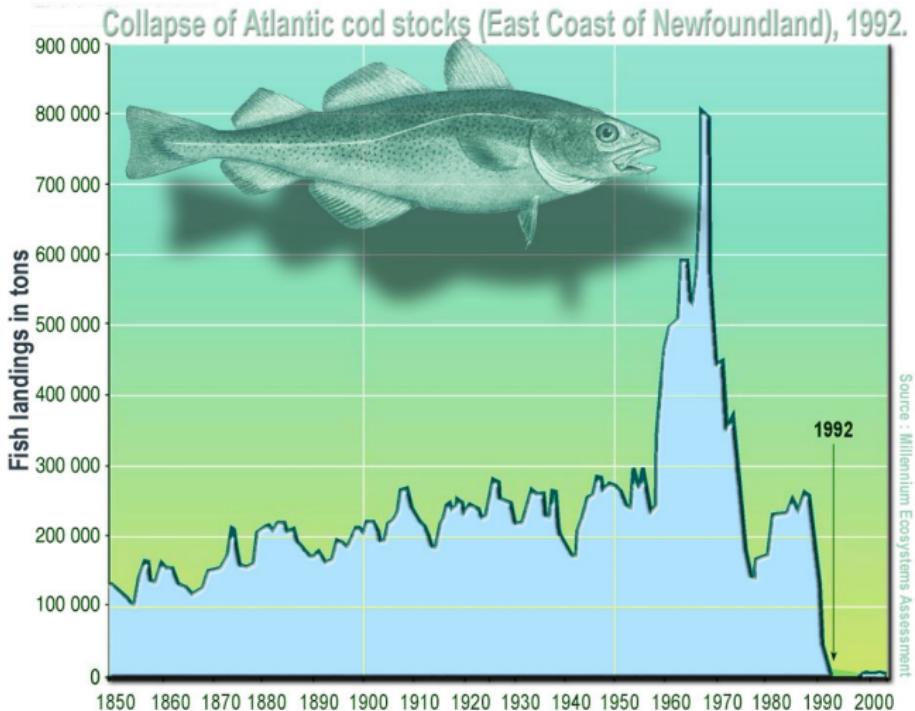


Predators have been overfished in the 20th century

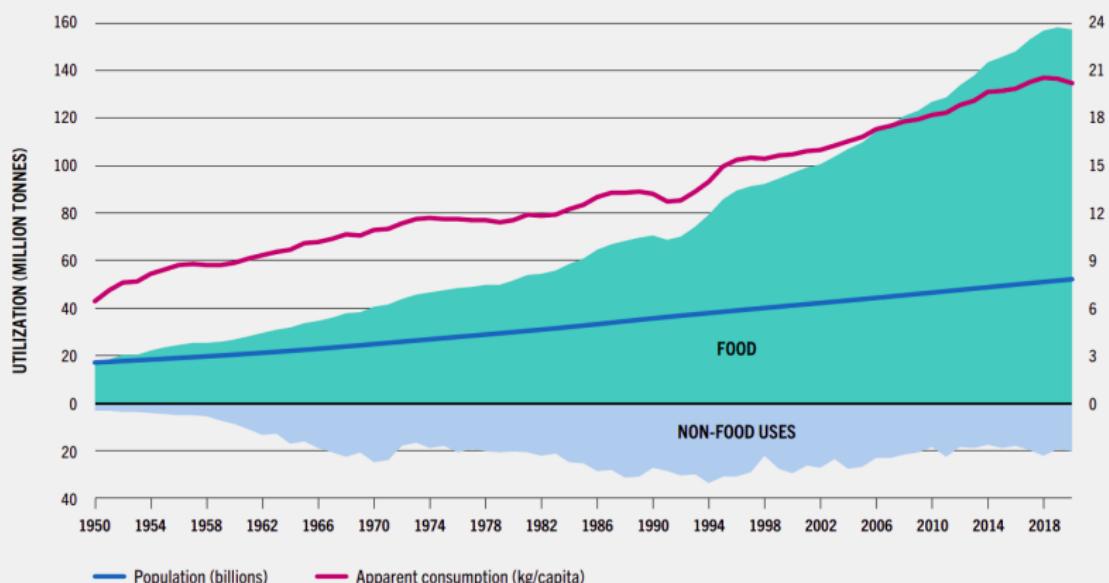
- ↳ Sesarma crabs prospered and eat salt marsh grasses
- ↳ Marshes were decimated, while they :
 - filtered pollutants,
 - protected coastline from erosion,
 - act as nurseries for the young of many species.
- ↳ the biodiversity and local economy declined

In 1992 Stellwagen Bank National Marine Sanctuary was created.

The Atlantic cod stocks



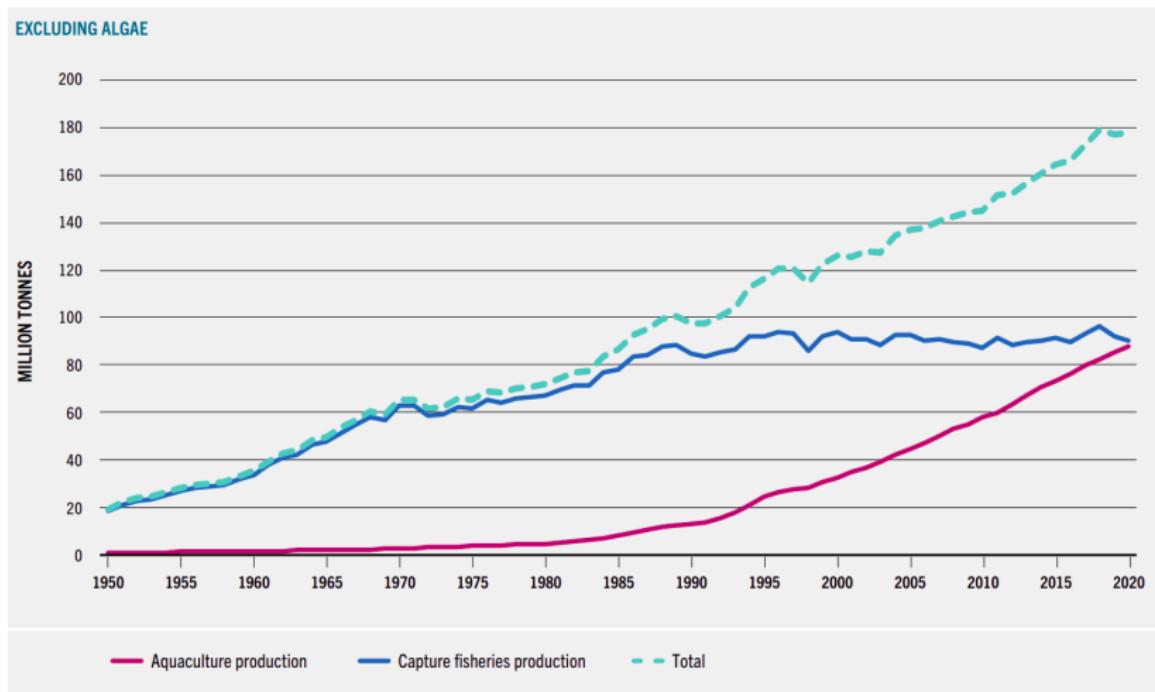
Raise in demands and offers (source: FAO)



NOTES: Excluding aquatic mammals, crocodiles, alligators, caimans and algae. Data expressed in live weight equivalent. For algae and apparent consumption, see Glossary, including Context of SOFIA 2022. Source of population figures: United Nations. 2019. 2019 Revision of World Population Prospects. In: UN. New York. Cited 22 April 2022. <https://population.un.org/wpp>

SOURCE: FAO.

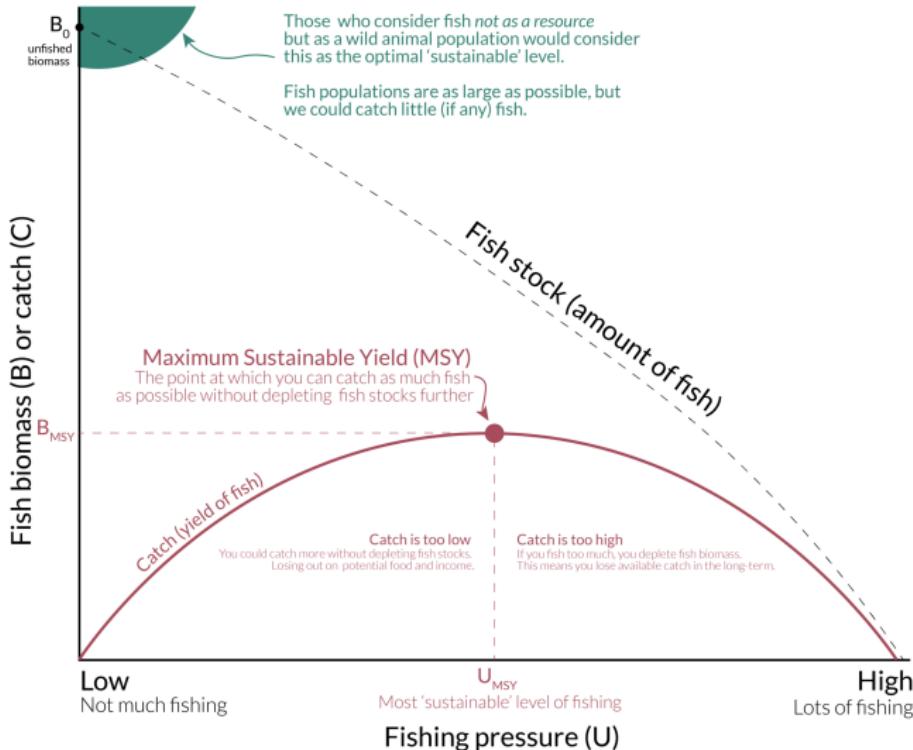
Capture vs aquaculture (source: FAO)



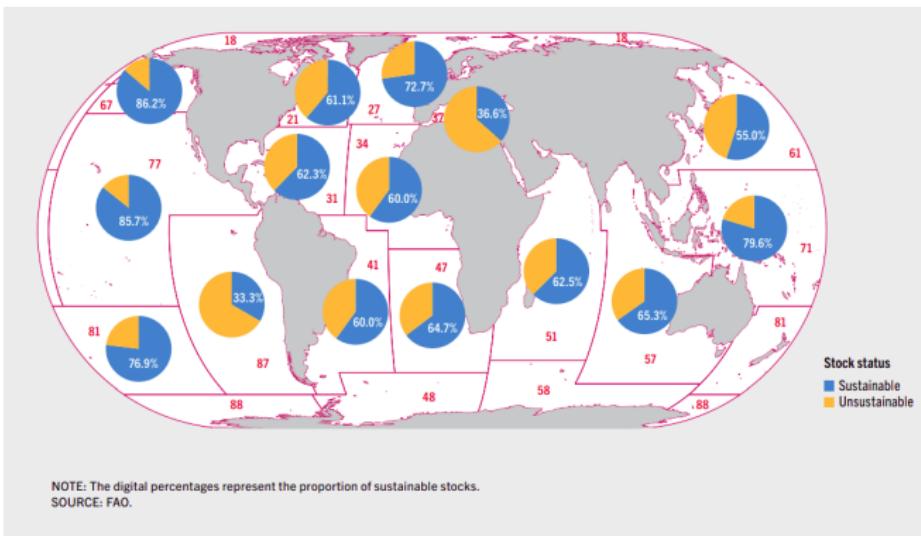
What does sustainable fishing mean?

Fishing pressure and catch are flow variables. Fish stock is a stock variable.

Our World
in Data



Part of overfished stocks



- FAO expects by 2030:
 - a 15% increase in aquatic-food needs,
 - a 10% decrease of fishing fleets.
- Restoring overfished stocks could increase fisheries production by 16.5 million tonnes (\approx 20% of captures).
- COP15 (12/22), by 2030:
 - making 30% of seas and oceans protected,
 - restoring 30% of seas and oceans.

Some benefits from better understanding overfishing

- reducing species collapses (biomass < 10% of unfished biomass),
- reducing risk of trophic cascades,
↳ indirect changes on environment and biodiversity,
- increasing global fishery production,
- securing sustainable production for future,
- reducing fishing costs and improving quality and margins,
- reducing the size of fishing fleet.

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⇒ Need for better understanding of

- responses of fishes' population dynamics facing large-scale fishing,
- fishers' behaviours, and how they adapt to different regulations.

An instance of the tragedy of the commons

"The selfish action of individuals that have access to a common resource can lead to the depletion of the resource due to the uncoordinated action of the individuals. This is in contrast with the common management of the resource or when regulations are applied to avoid this behaviour."

William Forster Lloyd.

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Impact of competition between fishers (that will be modelled as MFG):

selfishness \Rightarrow overfishing \Rightarrow extinction \Rightarrow low profits.

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In contrast, fishing strategy for the common good should be respective of the environments.

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What I will be discussing

[KMFRB23] Z. Kobeissi, I. Mazari-Fouquer and D. Ruiz-Balet.
The tragedy of the commons: a Mean-Field Game approach to
the reversal of travelling waves



They won the 2022 ESMTB (Euro. Soc. for Math-Bio) prize.

1st component of our models: the dynamics of the fishes through reaction-diffusion equation.

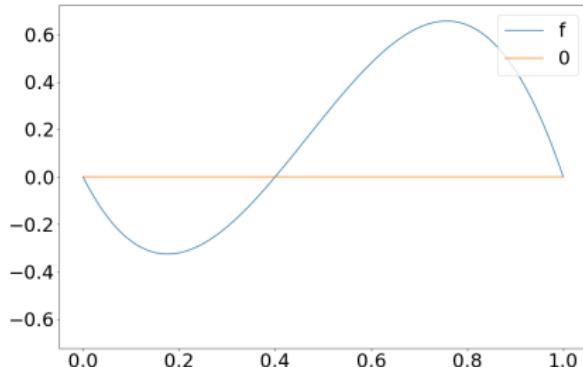
θ : fishes density, normalised so that $0 \leq \theta \leq 1$, for $(t, x) \in [0, \infty) \times \mathbb{R}$:

$$\partial_t \theta = \underset{\text{random diffusion}}{\partial_{xx}^2 \theta} + \underset{\text{natural birth and death}}{f(\theta)} - \underset{m: \text{ density of the fishers}}{m\theta}.$$

Boundary conditions: $\theta(0, -\infty) = 0$ and $\theta(0, +\infty) = 1$.

- RD equations are largely used in spatial ecology
see [Cantrell & Cosner 2004].
- Simple models with limited applicability
↳ allows mathematical analysis.
- Still complex enough to capture accurate qualitative behaviours.
- Extensive literature on the geometric properties of their solutions.

Main assumption : bistable non-linearity



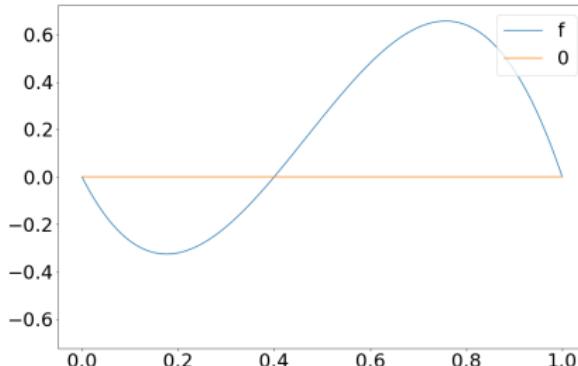
f is assumed to be bistable:

- $f(0) = f(1) = 0,$
- $f'(0), f'(1) < 0.$

$$\int_0^1 f > 0.$$

Allee effect: when the density is too low, the population decreases.

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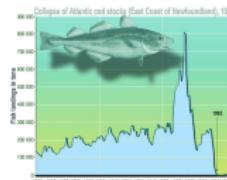
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Allee effect: when the density is too low, the population decreases.

Very helpful for modelling the tragedy of the commons

↳ We are going to use travelling waves

↳ Possible explanation of the brutal collapse of Atlantic cods.



Without fisher:

$$\partial_t \theta = \partial_{xx}^2 \theta + f(\theta), \quad \lim_{x \rightarrow -\infty} \theta(0, x) = 0, \quad \lim_{x \rightarrow +\infty} \theta(0, x) = 1,$$

Theorem (Fife 1979)

There exists a unique travelling wave solution: $\theta(t, x) = \Theta(x - ct)$ with

$$-\Theta'' - c\Theta' = f(\Theta), \quad \Theta(-\infty) = 0, \quad \Theta(+\infty) = 1.$$

*The fact that $\int_0^1 f > 0$ implies that $c < 0$: we say that 1 is invading.
Moreover, it is globally attractive.*

Dynamics of the fisheRs

2nd component of our models: the dynamics of the fisheRs using MFG

The position of one fisheR satisfies

$$\frac{dx_t}{dt} = \alpha_t,$$

and they maximise their profit

$$v(t_0, x_0) = \max_{\alpha} \left\{ \int_{t_0}^{\infty} \underbrace{e^{-\lambda(t-t_0)}}_{\text{discount factor}} \left(\underbrace{\theta(t, x_t)}_{\text{fishing}} - \underbrace{L(\alpha_t)}_{\text{moving cost (oil)}} \right) dt \right\}.$$

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v satisfies the HJB equation

$$\lambda v - \partial_t v - H(\partial_x v) = \theta,$$

with $H(p) = \sup_{\alpha} \{ p \alpha - L(\alpha) \}$ and

$$\alpha^*(t, x) = H'(\partial_x v(t, x)).$$

The MFG system

The whole system is

$$\begin{cases} \lambda \textcolor{red}{v} - \partial_t \textcolor{red}{v} - H(\partial_x \textcolor{red}{v}) = \theta, \\ \partial_t \textcolor{green}{m} + \partial_x (H'(\partial_x \textcolor{red}{v}) \textcolor{green}{m}) = 0, \\ \partial_t \theta - \partial_{xx}^2 \theta = f(\theta) - \textcolor{green}{m}\theta. \\ \theta(0, -\infty) = 0 \text{ and } \theta(0, +\infty) = 1 \end{cases}$$

The MFG system

The whole system is

$$\begin{cases} \lambda v - \partial_t v - H(\partial_x v) = \theta, \\ \partial_t m + \partial_x(H'(\partial_x v)m) = 0, \\ \partial_t \theta - \partial_{xx}^2 \theta = f(\theta) - m\theta. \\ \theta(0, -\infty) = 0 \text{ and } \theta(0, +\infty) = 1 \end{cases}$$

We look for a travelling wave solution, i.e., for $s = x - ct$,

$$v(t, x) = V(s), \quad m(t, x) = M(s), \quad \theta(t, x) = \Theta(s),$$

such that

$$\begin{cases} \lambda V + cV' - H(V') = \Theta, \\ H'(V') = c \text{ on } \text{supp}(M), \\ -c\Theta' - \Theta'' = f(\Theta) - M\Theta, \\ \Theta(-\infty) = 0 \text{ and } \Theta(+\infty) = 1 \end{cases}$$

Monotonous solutions

We look for solutions with a nondecreasing Θ and $M \not\equiv 0$.

↳ c should be positive \rightsquigarrow the fishes go extinct! (0 is invading).

Assumptions:

- f is C^1 ,
- f is bistable,
- $\int_0^1 f > 0$.

- L is C^2 ,
- $0 = L(0) = \min L(0)$,
- L is strictly convex,
- $\lim_{|\alpha| \rightarrow \infty} \frac{L(\alpha)}{1+|\alpha|} = \infty$.

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Theorem (KMFRB 2023)

There exists a unique TW solution at velocity c if

$$0 < c < c_0(\lambda) \text{ for some nonincreasing } c_0 : (0, \infty) \rightarrow (0, \infty),$$

and one of the three following condition holds:

- ① L is D -strongly convex and $\lambda \geq \sqrt{\frac{M_0}{D}}$ for some $M_0 > 0$.
- ② c is small.
- ③ λ is small \rightsquigarrow the extinction can be fast.

Three main ingredients:

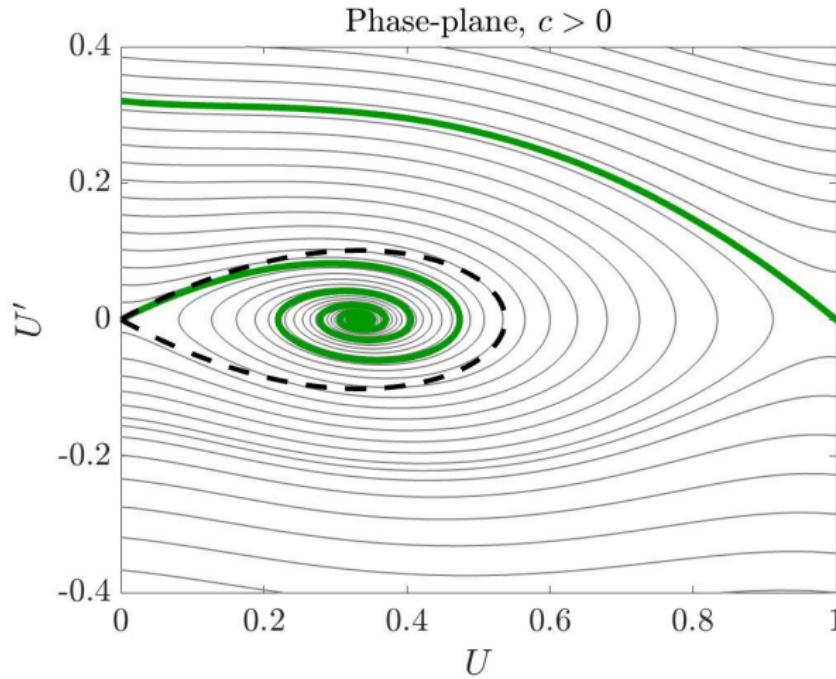
- ① for m fixed, a phase-portrait analysis on θ ,
- ② an explicit construction of reversed travelling wave profiles,
- ③ for θ fixed, optimality conditions investigated at an individual scale.

Phase plane without fisherR

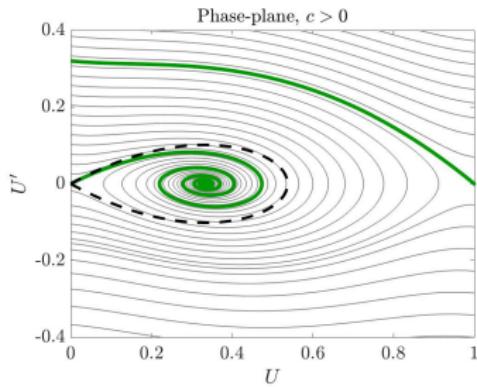
Without fisherRs, we get

$$-\Theta'' - c\Theta' = f(\Theta),$$

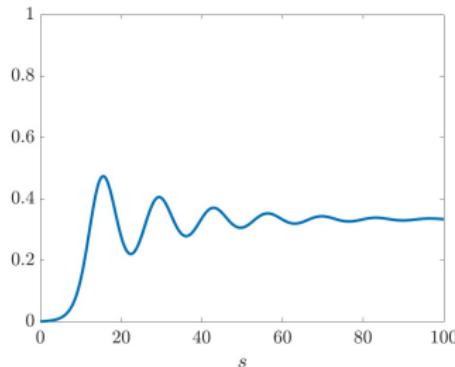
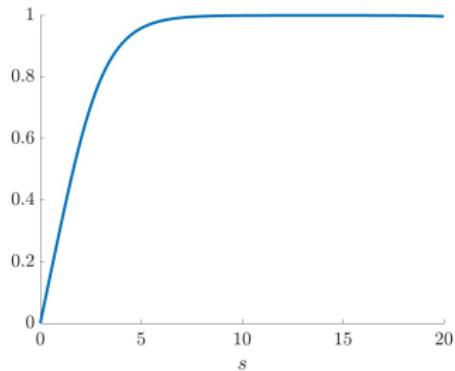
that might be analysed through the following phase portrait.



Phase plane

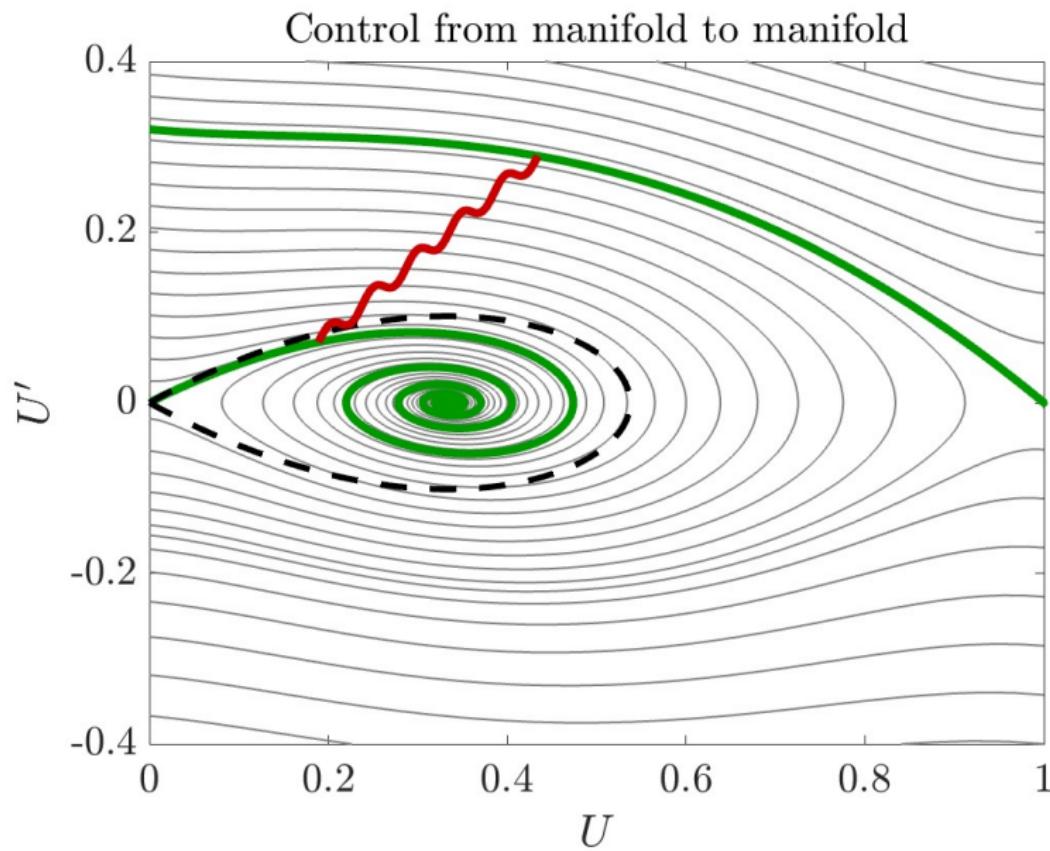


Instable solution from $(0, 1)$



Instable solution from $(0, 0)$

Adding fisheRs to the phase plane



Sketch of proof

On $\text{supp } m$, we have $\lambda V + cV' - H(V') = \Theta$ and $H'(V') = c$, implying

$$\lambda V(s) = L(c) - \Theta$$

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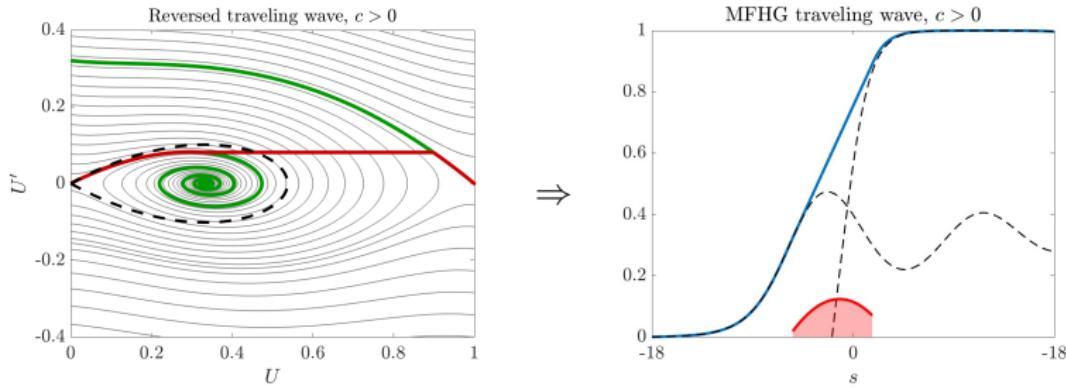
On $\text{supp } m$, we have $\lambda V + cV' - H(V') = \Theta$ and $H'(V') = c$, implying

$$\lambda V(s) = L(c) - \Theta \Rightarrow \lambda L'(c) = \lambda V' = -\Theta'$$

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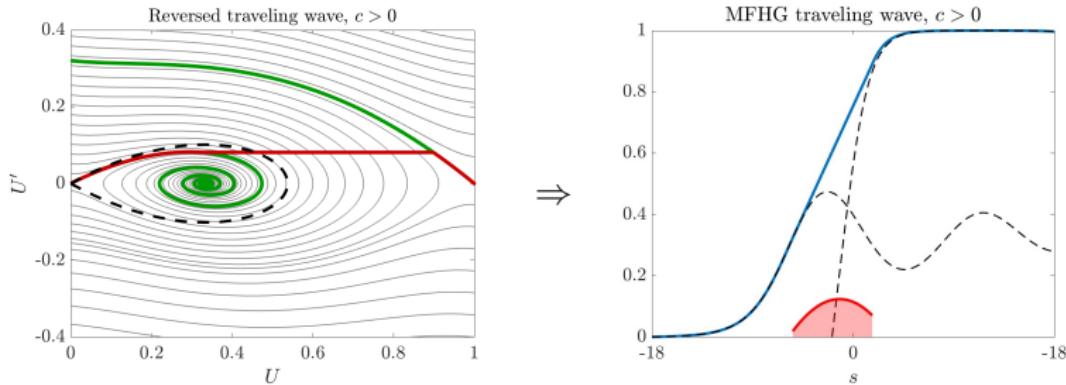
$$\lambda V(s) = L(c) - \Theta \Rightarrow \lambda L'(c) = \lambda V' = -\Theta' \Rightarrow \Theta'' = 0 \text{ & } \Theta' = \frac{m\Theta - f(\Theta)}{c}.$$



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It only remains the third part of the proof about individual optimality.

For $L = D \frac{|\alpha|^2}{2}$:

- ① we prove that $\alpha \equiv c$ is a critical point of the reward functional.
- ② for D large, the functional is strictly concave.

The general case (with L only strictly convex) is much more technical.

Adding coordination

Adding coordination may lead to:

- the survival of the species,
- better profits for all fisherRs.

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Theorem (KMFRB 2023)

For some (λ, L) , take (V, M, Θ) the TW solution with velocity $c = 1$.
There exists $\alpha : (0, \infty) \times \mathbb{R} \rightarrow \mathbb{R}$ such that, $\forall x \in \mathbb{R}$,

$$V(x_0) < v^\alpha(0, x_0) := \int_0^\infty e^{-\lambda t} (\theta^\alpha(t, x_t^\alpha) - L(\alpha_t)) dt,$$

where $\dot{x}_t^\alpha = \alpha(t, x_t^\alpha)$ and

$$\begin{cases} \partial_t \theta^\alpha = \partial_{xx} \theta^\alpha + f(\theta^\alpha) - \theta^\alpha m^\alpha, & \theta^\alpha(0, x) = \Theta(x), \\ \partial_t m^\alpha + \partial_x(\alpha m^\alpha) = 0, & m^\alpha(0, x) = M(x). \end{cases}$$

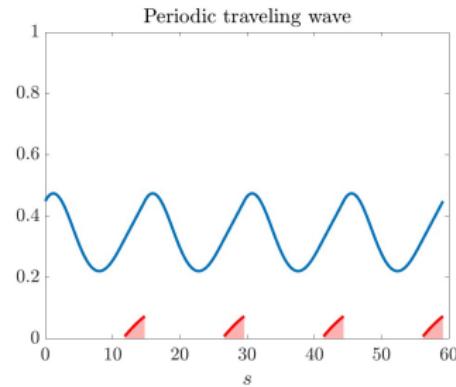
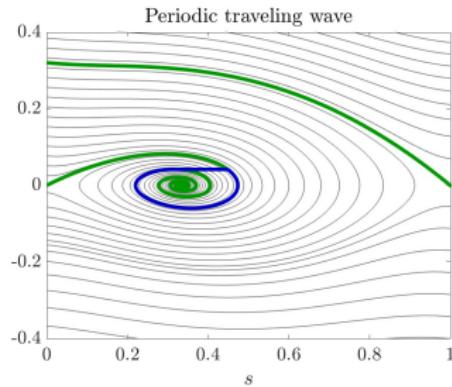
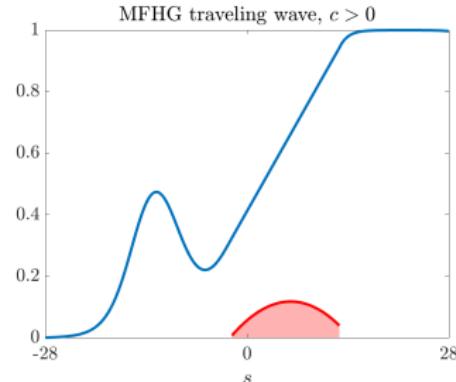
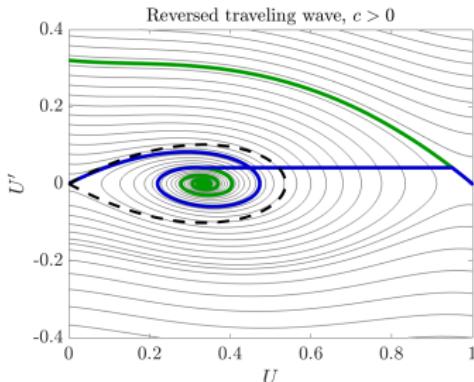
Moreover, we have

$$\theta^\alpha(t, x) \xrightarrow[t \rightarrow \infty]{} 1,$$

while, in the competitive setting: $\theta(t, x) = \Theta(x - ct) \rightarrow 0$.

(Typically λ is small and $L \approx |\alpha|^q$ with a large q).

More surprising solutions



Summary of the main contributions

- ➊ An original model of fishes/fisheRs dynamics
 - FisheRs are usually static, here **they can move**
 - MFG approach \rightsquigarrow **computations with a large number of fisheRs**.
 - propose a **possible explanation** to some experienced scenarios.
- ➋ A striking example of the tragedy of the commons
 - theoretical proof and numerical simulations,
 - competition may lead to **low profits** and **extinction of the fishes**,
 - coordination may **protect the fishes** while **increase the profits**.
- ➌ Only the second MFG model with travelling wave solutions.

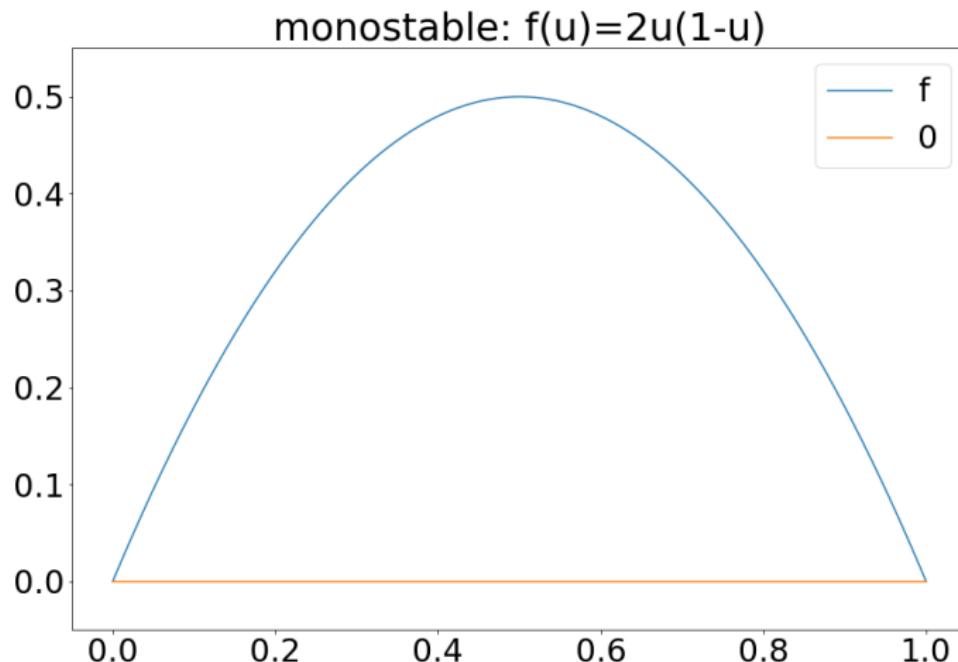
Important open question: are the monotonous solutions stable?

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Going back to the MF system

No more travelling waves from now on!

Replace $f(\theta) - m\theta$ by $\theta(K(x) - \theta) - \varepsilon\rho \star m\theta$, with $\varepsilon > 0$ and $\int_{\mathbb{T}^d} m = 1$.



The Mean-Field system

Recall the MF system, this time on $[0, T] \times \mathbb{T}^d$,

$$\begin{cases} -\partial_t V - \nu \Delta V - H(x, \nabla_x V) = \rho \star \theta \\ \partial_t m - \nu \Delta m + \operatorname{div}(H_p(x, \nabla_x V)m) = 0, \\ \partial_t \theta - \mu \Delta \theta - \theta(K - \varepsilon \rho \star m - \theta) = 0, \\ V(T) = 0, \quad m(0) = m_0, \quad \theta(0) = \theta_0, \end{cases} .$$

for $\nu \geq 0$, $\mu > 0$,

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Recall the MF system, this time on $[0, T] \times \mathbb{T}^d$,

$$\begin{cases} -\partial_t V - \nu \Delta V - H(x, \nabla_x V) = \rho \star (\theta(1 - \eta)) \\ \partial_t m - \nu \Delta m + \operatorname{div}(H_p(x, \nabla_x V)m) = 0, \\ \partial_t \theta - \mu \Delta \theta - \theta(K - \varepsilon \rho \star m - \theta) = 0, \\ -\partial_t \eta - \mu \Delta \eta - \eta(K - \varepsilon \rho \star m - 2\theta) = \gamma \varepsilon \rho \star m, \\ V(T) = 0, \quad m(0) = m_0, \quad \theta(0) = \theta_0, \quad \eta(T) = 0. \end{cases}$$

for $\nu \geq 0$, $\mu > 0$, and $\gamma \in \{0, 1\}$, with

- $\gamma = 0$: competitive case (MFG),
- $\gamma = 1$: cooperative case (MFC).

According to us, this is the first MFG system with four equations which are in duality pairwise.

The function $m \mapsto \theta$ is regularizing so the existence is easy.

Theorem

Under standard assumptions, there exists a solution for any $\gamma \in [0, 1]$.

The proof relies on classical results on parabolic second-order PDEs and a priori estimate from [Cirant & Goffi, 2021].

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Recall that uniqueness usually holds in two regimes:

- the Lasry-Lions monotonicity condition,
- an assumption on the smallness of some parameters, here it would be of the form $\varepsilon \leq e^{-CT}$ for some $C > 0$.

Existence and uniqueness results

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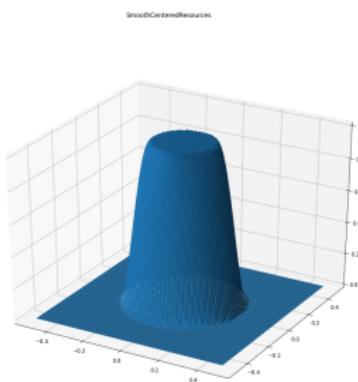
There exists ε_0 such that the uniqueness holds for all $T > 0$ and $\varepsilon < \varepsilon_0$. Moreover, we can prove convergence to the ergodic MFG when $T \rightarrow \infty$.

The proof does not rely on Banach fixed point theorem.

Numerical simulations (1/2): the parameters

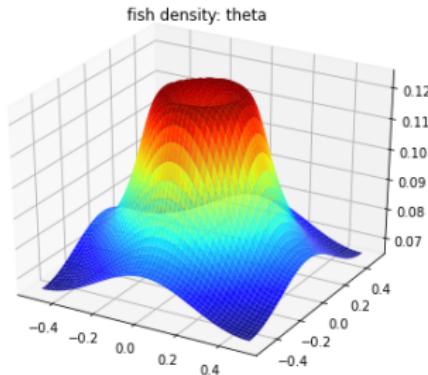
In \mathbb{T}^2 , we consider the discounted infinite-time MF systems, with:

- Discount factor: $\lambda = 0.01$
- Viscosities: $\nu = \mu = 0.01$
- Fishing capacity: $\varepsilon = 0.1$

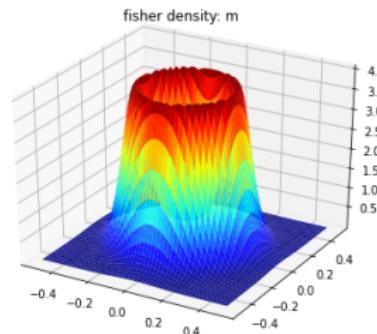
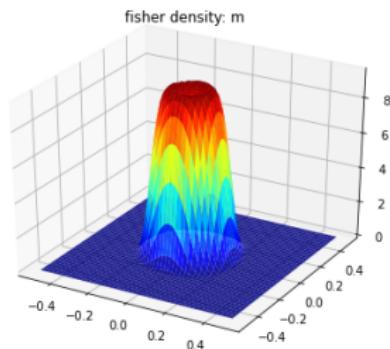
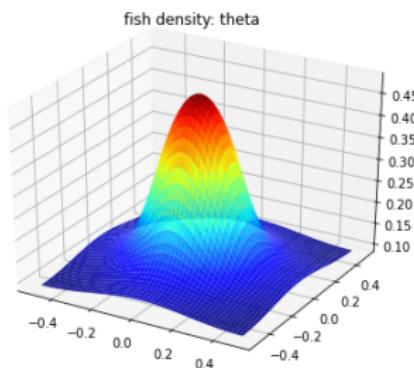


Numerical simulations (2/2): the solutions

Competition/MFG ($\gamma = 0$)



Cooperation/MFC ($\gamma = 1$)



- ➊ An original model of fishes/fisheRs dynamics.
- ➋ A new prove of uniqueness for some MFG systems.
- ➌ Interesting insights:
 - there exists a public policy forcing selfish fisheRs to be eco-friendly,
 - protected areas may be almost optimal for common good.
- ➍ Numerical solutions may help drawing protected areas.

[Mazari-Fouquet & Ruiz-Balet, 2022]: Game formulation of fishery management.

↳ static game with a finite number of fishing companies.

[Bressan et al. 2022]: Existence of reversed travelling waves.

↳ minimal cost to exterminate a population (mosquitoes).

[Porretta & Rossi, 2021] MFG travelling waves.

↳ game of knowledge diffusion (second order and nonlocal).

Thank you for your attention!