The green line is an upper bound for the number of available

positions for big balls; the orange line is the number of big balls we're required to place to form a solution.

For each n > 2, we form a triangle made up of N = n(n+1)/2 balls. The goal is to arrange the balls, numbered 1 through N, in the inverted triangle such that the ball directly underneath two other balls is the absolute difference of the two above balls. Number the rows 1 through n, where Row n has n balls. The top row is Row n and the bottom row, consisting of a single ball, is Row 1.

The ball directly below two other balls is called the child ball of the two upper balls, which are known as the parent balls. It's easily seen that no child ball can be larger than either of its parents. A child can be smaller than both parents however.

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The highest numbered ball, Ball N, sometimes called the high ball. Since the N Ball can have no parent, it must reside on the top row of any solution.

Big balls and small balls. The top n+1 balls are called *big balls*. In the n=5 case, the big balls are the ⑮, ⑭, ⑬, ⑫, ⑪, and ⑩.

Small balls are the lowest n balls, ball 1 through ball n. In the case of n=5, the small balls are 1,2,3,4,5.

Notice there are 6 big balls, one more than there are small balls. This is true in general: there are n+1 big balls and n small balls.

The child of two big balls is always (ironically) a small ball. Not all smalls balls have two big parents though. The only way to produce a big child is by one big-ball parent and one small-ball parent. Remember, balls on the top row do not have parents.

Let M_k be the maximum ball of Row k.
We have:

$$M_1 < M_2 < \cdots < M_n = N$$

child = larger parent - smaller parent larger parent = child + smaller parent We have M_1 + sp2 + sp3 + ... spn = N. i.e. n numbers chosen from 1..n that sum to N. This can only happen if those n numbers are {1,...n}. Thus we have exactly one small ball per row.
In particular, the bottom ball is small.

Upper bound for balls on Row k. Must be less than or equal to N - 1 - 2 - ... - (n-k)

Not much room for big balls. A row with 2 or more big balls must be below a parent row with 2 or more big balls that are not adjacent. Row n-1 can have two big balls but they cannot be adjacent. Thus Rows n-2 and below can have at most one big ball. If Row n-1 has only one big ball, then... If Row n-1 has exactly two big balls, then... The very bottom rows can't have any big balls. The top row can have at most 3 + (n-4)/3 balls