Carrier Allocation Strategy for MC-CDMA uplink system

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***Abstract*— The abstract goes here.**

I. Introduction

The thermal noise, received from a radio channel, has the unique property to be perfectly adapted to the transmission environment. In a traditional communication system, the information is carried by an artificial signal that is usually designed to fully exploit the available radio channel. The technique described in this paper employs a scaled and delayed version of the received noise to carry mutual information between two terminals.

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II. The Noise-Loop Transmission Chain

In the analized system there are two terminals exchanging information: terminal no. 1 and terminal no. 2. Two different channels are considered: one for the link from terminal 1 to terminal 2 and one for the reverse link. The two channels are considered on different frequency bands and the thermal noise on one link is considered uncorrelated with the other. Each link is modeled as a conventional AWGN channel.

A. Symbols in the paper

The following symbols have been adopted in the paper:

- b_i — binary antipodal information signal originating form terminal i,
- $n_i(t)$ — Gaussian, white, continuos time random processes modelling the received noise at the receiver on terminal i, characterized by zero mean and variance σ_n^2,
- α_i — global link gain for the signal generated from terminal i. It includes transmission gain, path loss and channel response. It is also supposed to be known by the receivers,
- τ_p — propagation delay for the channel. It is assumed without loss of generality that both forward (1 to 2) and reverse (2 to 1) links have the same delay,
- $y_i(t)$ — baseband received signal available at the terminal i

B. Transmission Chain

In this simple transmission system the terminal operations are described in the fig. **??**. The signal from the inbound channel is modulated by the information and re-transmitted on the outbound channel. The reception is obtained by extracting the sign of the $2\tau_p$-delayed autocorrelation term of the incoming signal, multiplied by the own informative bit. The process is explained in details in the following sections.

C. Stationary signal analysys in unlimited bandwidth

In this section a stationary condition on the system inputs is analysed[1]. Due to the additive nature of the model, the received signals $y_1(t)$ available at terminal 1 is defined by the following series:

$$y_1(t) = \sum_{j=0}^{\infty}(b_1 b_2 \alpha_1 \alpha_2)^j n_1(t - 2j\tau_p) + \sum_{j=0}^{\infty}(b_1 b_2 \alpha_1 \alpha_2)^j b_2 \alpha_2 n_2(t - (2j+1)\tau_p) \quad (1)$$

An analogue expression can be obtained for $y_2(t)$ simply exchanging the subscript 1 and 2 in (**??**).

The first term of (**??**) represents the recursive contribution of the received noise $n_1(t)$ through the transmission loop. The second series on the other hand, is due to the injection of the noise process $n_2(t)$ by the other terminal. We also note that each term appears with a different delay on the received signal.

If the noise processes $n_i(t)$ are white on a unlimited bandwidth, then:

$$\mathrm{E}[n_i(t) n_j(t-\tau)] = \begin{cases} \delta(\tau)\sigma_n^2 & i = j \\ 0 & i \neq j \end{cases} \quad (2)$$

The structure of the signal in (**??**) draw our attention in the shifted correlation term

$$y_1(t - 2\tau_p) y_1(t) \quad (3)$$

[1]By stationary we intend a constant behavior over time of the information bits b_i, of the link gains and of the statistic parameters of the random processes involved

By resorting the terms obtained by the expansion of the above expression we obtain:

$$\begin{aligned}
&\mathrm{E}[y_1(t-2\tau_p)y_1(t)] = \\
&\qquad b_1 b_2 \sigma_n^2 (1+\alpha_2^2) \sum_{j=0}^{\infty} (\alpha_1\alpha_2)^{2j+1} \\
&\qquad + \mathrm{E}[\text{residual cross correlation terms}] \qquad (4)
\end{aligned}$$

The last term in (**??**) is null for an ideal AWGN channel, so the described autocorrelation term is dominated by the information bearing $b_1 b_2$ term, weighted by a term which is constant in the stationary case. The term contains the information of both terminals. Since we are interested in the information from terminal no. 2 only, a post-multiplication by b_1 is performed. The receiver at terminal 1 is depicted in figure **??**.

III. Performance Analysis on ideal AWGN Channel

The term in **??**, represents the instantaneous decision metric for the mutual information term to be estimated: $\hat{b_1}\hat{b_2}$. The performance of the receiver in terms of bit error probability is related to the first and second order statistics of **??**. The distribution of the unpredictable noise process, however, is no longer Gaussian.

A. Auto-Correlation Noise

This specific case fall into the more general class of the resulting noise process derived by a product operation on two Gaussian processes. The application of the statistics theory bring to the integral formulation of the pdf resulting from the product of two Gaussian pdf. Let x and y be two Gaussian zero-mean processes with unitary variance, whose pdf's are:

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \qquad p_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \qquad (5)$$

The pdf of the stochastic process $z = xy$ is a zero-order modified Bessel function:

$$p_Z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-\frac{z^2}{2y^2} - \frac{y^2}{2}}}{|y|} \mathrm{d}y = \frac{1}{\pi} J_0(|z|) \qquad (6)$$

where $J_0(\cdot)$ is the modified Bessel function of order zero. The distribution in (**??**) is shown in the figure **??**.

The cumulative probability function in the positive semi-axis is also useful to the computation of the receiver performances. It has the same significance of the Q-function in the presence of Gaussian noise. It is defined for $x > 0$ by:

$$\text{(W-function)} \quad \mathrm{W}(x) = \frac{1}{\pi} \int_x^{\infty} J_0(z) \mathrm{d}z \quad (x > 0) \qquad (7)$$

B. Performance Evaluation

The decision term in (**??**) is characterized by a probability density function defined by (**??**) with a mean value of

$$\mathrm{E}[y_1(t-2\tau_p)y_1(t)]$$

and a second order moment of

$$var[y_1(t-2\tau_p)y_1(t)]$$

For binary antipodal signaling the bit error probability is related to the previously defined W-function by:

$$P_e = \mathrm{W}\left(\sqrt{\frac{\mathrm{E}[y_1(t-2\tau_p)y_1(t)]^2}{var[y_1(t-2\tau_p)y_1(t)]}}\right) = \mathrm{W}\left(\sqrt{\gamma_b}\right) \qquad (8)$$

where $\mathrm{W}()$ is the W-function in (**??**) and $var[]$ indicates the variance operator. The argument of **??**, γ_b, can be expanded into:

$$\gamma_b = \frac{\mathrm{E}[y_1(t-2\tau_p)y_1(t)]^2}{var[y_1(t-2\tau_p)y_1(t)]} = \frac{\sigma_n^4 (1+\alpha_2^2)^2 \left(\sum_i (\alpha_1\alpha_2)^{(2j+1)}\right)^2}{\sigma_n^4 \left[(1+\alpha_2^2)^2 \sum_i \sum_j (\alpha_1^2\alpha_2^2)^{j+l} + (1+\alpha_2^4) \sum_j (\alpha_1^2\alpha_2^2)^{2j+1}\right]} \qquad (9)$$

Where all the series, unless explicitely defined, are intended ranging from zero to infinity. The γ_b term does not depends on the noise variance. The observation can be explained with the consideration that the higher is the noise floor, the stronger will be the injection of modulated signal in the loop. The signal-to-noise ratio depends from the closed loop total gain which is lesser than unity for stability. In figure **??** is plotted the value of γ_b function of $\alpha = \alpha_1 = \alpha_2$.

The probability of error for the proposed trasmission system can be evaluated as a function of the signal-to-noise ratio (γ_b) numerically. The figure **??** shows the P_e values vs. SNR for the proposed thermal noise modulation and a traditional BPSK modulation.

1) *Subsubsection Heading Here:* Subsubsection text here.

IV. Conclusion

The conclusion goes here.

Appendix A

The partial expansion of the signal at terminal 1 is

$$\begin{aligned}
y_1(t) = {} & n_1(t) \\
& + b_2\alpha_2 n_2(t-\tau_p) \\
& + b_1 b_2 \alpha_1 \alpha_2 n_1(t-2\tau_p) \\
& + b_1 b_2^2 \alpha_1 \alpha_2^2 n_2(t-3\tau_p) \\
& + b_1^2 b_2^2 \alpha_1^2 \alpha_2^2 n_1(t-4\tau_p) \\
& + b_1^2 b_2^3 \alpha_1^2 \alpha_2^3 n_2(t-5\tau_p) \\
& + b_1^3 b_2^3 \alpha_1^3 \alpha_2^3 n_1(t-6\tau_p) \\
& + b_1^3 b_2^4 \alpha_1^3 \alpha_2^4 n_2(t-7\tau_p) + \ldots
\end{aligned}$$

and the 2τ-delayed version:

$$
\begin{aligned}
y_1(t-2\tau_p) = n_1(t-2\tau_p) \\
+ b_2\alpha_2 n_2(t-3\tau_p) \\
+ b_1 b_2 \alpha_1 \alpha_2 n_1(t-4\tau_p) \\
+ b_1 b_2^2 \alpha_1 \alpha_2^2 n_2(t-5\tau_p) \\
+ b_1^2 b_2^2 \alpha_1^2 \alpha_2^2 n_1(t-6\tau_p) \\
+ b_1^2 b_2^3 \alpha_1^2 \alpha_2^3 n_2(t-7\tau_p) \\
+ b_1^3 b_2^3 \alpha_1^3 \alpha_2^3 n_1(t-8\tau_p) \\
+ b_1^3 b_2^4 \alpha_1^3 \alpha_2^4 n_2(t-9\tau_p) + ...
\end{aligned}
$$

The instantaneous 2τ-delayed auto-correlation term is given by :

$$
\begin{aligned}
y_1(t)y_1(t-2\tau_p) = \\
+ \mathbf{b_1 b_2 \alpha_1 \alpha_2 n_1(t-2\tau_p) n_1(t-2\tau_p)} \\
+ \mathbf{b_1 b_2^3 \alpha_1 \alpha_2^3 n_1(t-3\tau_p) n_1(t-3\tau_p)} \\
+ \mathbf{b_1^3 b_2^3 \alpha_1^3 \alpha_2^3 n_1(t-4\tau_p) n_1(t-4\tau_p)} \\
+ ...
\end{aligned}
$$

V. Appendix B

Acknowledgment

The authors would like to thank...

References

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.